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The Role of Slotting Fees in the Coordination of Assortment Decisions

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Large numbers of new products introduced annually by manufacturers may strain the relationship between retailers and manufacturers regarding assortments carried by retailers. For example, many retailers in the grocery industry will agree to broaden their assortments only if the manufacturer agrees to pay slotting fees for the new products. We investigate the role played by slotting fees in coordinating the assortment decisions in a supply chain. To do so, we study a single-retailer, single-manufacturer supply chain, where the retailer decides what assortment to offer to end customers. Double marginalization results in a discrepancy between the retailer's optimal assortment and the assortment that maximizes total supply chain profits. We consider a payment scheme that is analogous to slotting fees used in the grocery industry: the manufacturer pays the retailer a per-product fee for every product offered by the retailer in excess of a certain target level. We show that, if the wholesale price is below some threshold level, this payment scheme induces the retailer to offer the supply-chain-optimal assortment and makes both parties better off.

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1. Introduction

Consumer goods manufacturers introduced more than 31,000 items in 2000, and the typical grocer stocked about 40,000 items in 2001, double the number of a few years earlier (Nelson 2001). A byproduct of this growth in product variety is the increased number of choices available to consumers. For example, in 2001, a consumer could choose from 16 different flavors of Kellogg's Eggo waffles and from nine different kinds of Kleenex tissue (Nelson 2001). Such broad assortments put a strain on the relationship between retailers and manufacturers when it comes to the level of variety offered by retailers. For example, many retailers in the grocery industry have been asking manufacturers to pay slotting fees in order to carry new products. According to a Federal Trade Commission (FTC) report, retailers report receiving slotting fees on 60% of new items, whereas manufacturers report paying slotting fees on 80–90% of new items. The same FTC report says that the average slotting fee can range from \$65 per store for bread to \$92 per store for hot dogs (Klie 2004). The topic of slotting fees is a rather controversial one among the players in the grocery industry, spurring multiple FTC reports and Senate committee hearings. In fact, the topic creates such tension in the retailer–manufacturer relationship that when the Senate Small Business Committee held hearings on the matter in 1999, out of 200 manufacturers interviewed, only three testified, and two did so hidden behind a screen so as to remain anonymous (Schoenberger 2000).

In this paper, we investigate the role played by slotting fees in coordinating the assortment decisions in a supply chain. We consider a single-retailer, single-manufacturer supply chain, where the retailer is in charge of deciding what assortment to offer to end customers. We explicitly model consumer choice and the effect of product variety on the inventory costs borne by the retailer. In a decentralized supply chain where the retailer and the manufacturer are independent, double marginalization results in a discrepancy between the assortment of products the retailer chooses to offer (hereafter, the retailer–optimal assortment) and the assortment that maximizes total supply chain profits (hereafter, the supply-chain-optimal assortment). In this setting, we address two main questions: Can slotting fees induce the retailer to offer the supply-chain-optimal assortment? Is it ever in the best interest of the manufacturer to pay slotting fees?

In our model, customers choose from the set of products offered by the retailer. In modeling the demand and consumer choice, we use the multinomial logit (MNL) model as used by van Ryzin and Mahajan (1999), who examine the inventory management and assortment planning problem faced by a retailer. On the inventory control side, in the decentralized supply chain of our model, the retailer faces an infinite-horizon, periodic-review inventory control problem for each of the products in the assortment. The retailer places an order with the manufacturer for each product at the beginning of each period. The manufacturer builds to order. In order to maintain tractability, we assume that the cost and price parameters are the same for all products. This assumption holds in cases where the items differ from each other only in attributes such as color, scent, or flavor.

We consider a contract under which the manufacturer pays a fee to the retailer for every product the retailer carries in its assortment in excess of a certain target level. The target level is set to be the number of products in the assortment the retailer is offering in the status quo. This contract is of particular interest, since it strongly resembles the slotting fees used in the grocery industry: the manufacturer pays the retailer a fee for every additional product the retailer adds to its assortment, as required by slotting fees. We show that if the wholesale price in the supply chain is below a threshold level, this contract will induce the retailer to offer the supply-chain-optimal assortment. In addition, this contract guarantees that both the retailer and the manufacturer will be better off, provided that the wholesale price is below the threshold level. Interestingly, once the wholesale price exceeds the threshold level, it will not be possible to find a fee that will induce the retailer to choose the supply-chain-optimal assortment.

Our results have some implications regarding the usage of slotting fees in the grocery industry. According to Bloom et al. (2000), there are two distinct opinions on slotting fees: proponents argue that slotting fees are tools that enhance channel efficiency, while opponents maintain that slotting fees are anti-competitive and merely a source of extra revenue for retailers. One of the arguments in favor of slotting fees cited by Bloom and colleagues is that these fees induce retailers to offer products that would otherwise not make it to the market because of an over saturation of the market with product proliferation. This argument is further supported by a recent empirical study: Israilevich (2004) finds that retailers carry some products that would be unprofitable for them had they not received slotting fees to stock these products. Our results give qualified support to this claim. We find that there exist fees that will induce the retailer to carry otherwise unprofitable products, thereby improving the manufacturer's profit as well. Furthermore, we find that the supply-chain-optimal level of variety is higher than which that the retailer would be willing to offer, but a contract that resembles slotting fees may induce the retailer to offer the supply-chain-optimal assortment. These observations suggest that slotting fees may indeed improve supply chain efficiency.

Here, we model the supply chain as a bilateral monopoly (i.e., consisting of a single retailer and a single manufacturer), ignoring the competition at both tiers of the supply chain. The assumption of bilateral monopoly is mainly for analytical tractability, but it also approximates the not-so-rare scenario where a manufacturer holds such a large market share in a product category that it comes close to being a monopoly (e.g., Ehrbar 2005, reports that Eggo has 67% market share in the frozen waffle category and Gerber owns 80% of the baby food market). In addition, our model captures supply relationships in which a product line is sold exclusively at a retailer (see Pereira 2001, for examples from the toy industry). Of course, a bilateral monopoly is the exception rather than the rule. This simplifying assumption allows us to highlight the role slotting fees can play in coordinating assortment decisions. One would need to model manufacturer competition to investigate the claim that slotting fees are anti-competitive.

In the next section, we discuss the related literature and position our work with respect to existing research on product line selection, supply chain coordination, and slotting fees. This is followed by a discussion of the centralized supply chain in Section 3. Section 4 describes how the decentralized supply chain differs from the centralized one. In Section 5, we examine a contract analogous to slotting fees. We analyze the effect of this contract on the manufacturer's profits, and we find the conditions under which this contract coordinates the decentralized supply chain so as to match the centralized supply chain's performance. In addition, we examine how the coordinating ability of the contract depends on problem parameters such as inventory costs and demand variability. A number of changes to the proposed contract and their effects are discussed in Section 6. Finally, we conclude with a discussion of future research directions in Section 7. All proofs are provided in the Appendix.

2. Related Literature

Kök et al. (2008) provide an excellent review of both the industry practices in assortment planning and the academic research on the topic. As they point out, there is little work that incorporates supply chain considerations into assortment planning. We provide a contribution to the analysis of assortment planning problems in supply chains, by highlighting the effect of slotting fees on the assortment decisions in a two-tier supply chain. As such, there are three streams of research related to the problem we study: (i) product line selection and pricing, (ii) supply chain coordination, and (iii) slotting fees. We discuss the relationship of our work to each of these research streams.

2.1. Product Line Selection and Pricing

The majority of the work in product line selection and pricing is in the marketing literature. In early treatments of the problem, Mussa and Rosen (1978) and Moorthy (1984) focus mainly on how cannibalization within a firm's product line affects the product offerings and the prices charged. Dobson and Kalish (1988) formulate the product line selection and pricing problem as a mixed integer linear program, and devise heuristics for solving this problem. Chen and Hausman (2000) show desirable mathematical properties of a product line selection and pricing problem where products are modeled as a collection of attributes. There is also some research on product line selection and pricing in the existence of design and manufacturing costs. Most of the work in this group follows the approach taken in Dobson and Kalish (1988), i.e., modeling the product line selection and pricing problem as a mixed integer linear program, and enriching this model in order to account for manufacturing complexity associated with product variety. We refer the reader to Yano and Dobson (1998) for a detailed review of this literature.

Another line of work in operations management investigates assortment decisions in the presence of demand uncertainty and associated inventory considerations. Van Ryzin and Mahajan (1999) consider the assortment planning and inventory management problems of a retailer using the MNL choice model. Rao et al. (2004) analyze a firm's joint inventory and assortment decisions in a setting with downward product substitution, where more costly products can be used to meet customer demand for less costly products. It is interesting to note that Rao et al. (2004) incorporate fixed costs for the inclusion of a product, which would admit an interpretation as a slotting fee. Gaur and Honhon (2006) address the joint assortment and inventory decisions of a retailer using a locational choice model, which leads to results that are qualitatively different from those obtained under the MNL choice model. Zhaolin (2007) extends van Ryzin and Mahajan to allow for unequal unit prices and costs across variants.

Our demand model based on the MNL choice model follows that of van Ryzin and Mahajan (1999). Many recent papers also use consumer choice and demand models that are based on the MNL model to address assortment-related problems in operations management (see, e.g., Cachon et al. 2005, Hopp and Xu 2005, Cachon and Kök 2007, and Maddah and Bish 2007).

2.2. Supply Chain Coordination

The premise of supply chain coordination is that decentralized decision-making in a supply chain results in inferior performance of the chain compared to one where decision-making is centralized. For a recent review of the literature on supply chain coordination, see Cachon (2003). In related work, Kurtuluş and Toktay (2005) investigate if and when a retailer should delegate category management decisions (i.e., assortment selection and pricing) to a manufacturer. Akçay and Tan (2008) identify the conditions that favor independent producers coming together to combine their assortments into one. In this paper, we focus on the effect of decentralization on the assortment of products offered to the customers. To the best of our knowledge, the only work that has considered the inefficiency introduced to the product line selection activity as a result of decentralized decision-making is that of Villas-Boas (1998). He considers a decentralized supply chain where the manufacturer first decides what assortment to offer to the retailer, and the retailer then decides which of these products to carry. The manufacturer charges the retailer a transfer price and the retailer is free to set the retail price. Villas-Boas (1998) determines the equilibrium assortment, transfer prices, and retail prices. He focuses on how the assortments offered by the centralized and decentralized supply chains are different, and does not address in detail how a decentralized supply chain can be coordinated. We explicitly model inventory costs associated with demand uncertainty, while Villas-Boas (1998) analyzes a setting where all demand is met, eliminating the need for inventory decisions. However, unlike Villas-Boas (1998), we take the prices and the manufacturer's assortment to be exogenously given.

2.3. Slotting Fees

In addition to the empirical work on slotting fees (e.g., Bloom et al. 2000, Israilevich 2004), there have been some model-based explanations in the literature for the existence of slotting fees. Most of these explanations come from marketing and economics literature. Chu (1992) considers a single-manufacturer, single-retailer channel with information asymmetry regarding the demand for a prospective product: the manufacturer knows whether the new product will have high or low demand, but the retailer does not. Chu shows that the manufacturer of a high-demand product may agree to pay a slotting fee upon a retailer's request. Lariviere and Padmanabhan (1997) show that if there is information asymmetry about the demand for the prospective product and the retailer incurs a fixed cost for carrying the product, then the manufacturer may willingly offer the retailer a slotting fee for the new product. Desai (2000) shows that slotting fees may arise even in the absence of information asymmetry when there is intense competition at the retail level. Unlike all of this earlier work, we model a situation where variants in an assortment cannibalize each other's demand (as opposed to considering the introduction of a single product in isolation). Furthermore, in our model, the manufacturer and the retailer share the same level of uncertainty about the demand for the products, and there are inventory-related costs associated with this demand uncertainty. Our result suggests that even when there are no issues regarding demand signaling and no competition at either echelon of the supply chain, slotting fees may still arise, and they may be in the interest of both parties in the supply chain.

3. The Centralized Supply Chain

In this section, we describe the model and formulate the assortment planning and inventory management problem of a centralized (e.g., vertically integrated) supply chain. In the centralized supply chain, a single decision-maker, referred to as "the firm," both manufactures the products and sells the assortment to the customers.

3.1. Consumer Choice Model

The firm has n potential products, each of which can be included in the assortment. We assume that the price and cost parameters are the same across all potential products. Let p denote the unit retail price and c the unit procurement cost. All n products serve the same general purpose for the customer, so each arriving customer observes the assortment offered by the firm, $S \subseteq \{1,\ldots,n\}$, and decides which product to purchase, if any. We follow van Ryzin and Mahajan (1999), who use the MNL model of consumer choice. The MNL model is a utility-based consumer choice model in which the stochastic utility of a consumer for product i, denoted by U_i, is given by $U_i = \alpha_i + \varepsilon_i$, where α_i is a fixed term and ε_i is a random error term that has a Gumbel distribution with mean zero and shape parameter μ. Here, α_i can be interpreted as the average utility of a customer for product i, assumed to be the same for all customers. In addition, the customer has the option of not purchasing from the assortment. A customer's utility for the no-purchase option is denoted by U_0 and given by $U_0 = \alpha_0 + \varepsilon_0$, where α_0 is a fixed term and ε_0 is again a random error term with a Gumbel distribution of mean zero and shape parameter μ. We assume that, for any given customer, ε_i's are independent across products. Under the MNL model, a customer chooses the product that maximizes her utility, and the probability that the customer will choose product i from assortment S, $q_i(S)$, is given by (see, for example, Guadagni and Little 1983)

$$q_i(S) = \frac{\exp\left(\frac{\alpha_i}{\mu}\right)}{\sum_{j \in S \cup \{0\}} \exp\left(\frac{\alpha_j}{\mu}\right)},\ i \in S \cup \{0\}.$$

Let $v_j = \exp\left(\frac{\alpha_j}{\mu}\right)$ for $j = 0,\ldots,n$. Following the terminology in van Ryzin and Mahajan (1999), we refer to v_j as the preference of the customer population for product j. Now, we can write

$$q_i(S) = \frac{v_i}{\sum_{j \in S \cup \{0\}} v_j},\ i \in S \cup \{0\}. \tag{1}$$

A key property of this choice model is that the more products there are in assortment S, the smaller the probability that an arriving customer will not purchase any of the products, i.e., as S gets larger, $q_0(S)$ decreases.

3.2. Inventory Model

We formulate the firm's inventory control problem as an infinite-horizon problem with periodic review, where future cash flows are discounted at a rate of γ per period. This model is similar to the "replenishable merchandise" model of Mahajan and van Ryzin (1998). We assume that there is a leadtime of L periods for the arrival of an order. Following Mahajan and van Ryzin (1998), we assume that the per-period demand for product i is normally distributed with a mean of $\lambda q_i(S)$ and variance of $(\lambda q_i(S))^{2\beta}$ for $0 < \beta < 1$, where $q_i(S)$ is given by (1). In addition, given the assortment, we assume that the demands are independent across products and periods. If $\beta = 0.5$, this demand model reduces to a normal approximation of Poisson arrivals of customers with a rate of λ per period, where each customer chooses product i with probability $q_i(S)$. The sequence of the events is as follows:

(1) At the beginning of a period, the firm observes the inventory level and the outstanding orders for a product. The order that was placed L periods ago arrives.
(2) The firm places an order for each product.
(3) The demand for each product is realized. If there is leftover inventory for a product, then a holding cost is incurred. If there is unmet demand for a product, then the unmet demand is backordered, and a penalty cost is incurred.

We assume that the holding and backorder costs are linear. Let h denote the unit holding cost per period and b the penalty cost per unit of backordered demand. In a grocery store setting, some customers, upon finding that their favorite product is out of stock, may wait until their next shopping trip to buy the item, which supports the assumption of backordering. Nonetheless, there will be other customers who would rather purchase whatever variant is in stock. Therefore, a more realistic model would be one where part of the excess demand for one product spills over to another. Unfortunately, even the basic task of determining the optimal stock levels is a challenge when there is such substitution upon stock-outs. Therefore, in the interest of analytical tractability, we assume full backordering of unmet demand. For a detailed discussion of alternative assumptions on the effect of stock-outs on consumer choice, see Mahajan and van Ryzin (1998). In addition, we assume that the shelf space allocated to the product category is large enough to allow the retailer to stock optimal quantities.

3.3. The Optimal Assortment

The problem described above is a standard inventory control problem, and it is well known that the firm's expected discounted holding and backorder costs can be minimized using a stationary order-up-to policy for each product. Given assortment S, under the optimal inventory control policy, the per-period optimal expected inventory holding and backordering cost for product i is given by

$$(b+h)\phi\left(\Phi^{-1}\left(\frac{b}{b+h}\right)\right)(\lambda(L+1)q_i(S))^{\beta},$$

where ϕ and Φ are, respectively, the pdf and cdf for the standard normal distribution. Let $G(S)$ be the optimal expected discounted holding and backordering cost over the infinite horizon, summed across all products in assortment $S \subseteq \{1,\ldots,n\}$. We can now write $G(S)$ as:

$$G(S) = \sum_{i\in S}\sum_{t=1}^{\infty}\gamma^{t-1}(b+h)\,\phi\left(\Phi^{-1}\left(\frac{b}{b+h}\right)\right)(\lambda(L+1)q_i(S))^{\beta}.$$

Throughout the remainder of the paper, we let

$$\phi^* := \phi\left(\Phi^{-1}\left(\frac{b}{b+h}\right)\right), \sigma(S) := \sum_{i\in S}(\lambda(L+1)q_i(S))^{\beta},$$

and $\Gamma = \dfrac{1}{1-\gamma}$.

Using this notation, we can now write

$$G(S) = \Gamma(b+h)\phi^*\sigma(S).$$

The expected number of units sold in a period (summed over all products) is $\lambda\sum_{i\in S} q_i(S)$ (recall that we assume unmet demand is backordered). Since the contribution per item sold is $p-c$, the expected gross profit per period is $\lambda(p-c)\sum_{i\in S} q_i(S)$. Subtracting from this the inventory cost $G(S)$, the net profit of the firm over the infinite horizon for assortment $S \subseteq \{1,\ldots,n\}$ is given by

$$\Pi^c(S) = \Gamma\left(\lambda(p-c)\sum_{i\in S} q_i(S) - (b+h)\phi^*\sigma(S)\right). \quad (2)$$

The firm's problem is to choose the assortment that maximizes profits:

$$\max_{S\subseteq\{1,\ldots,n\}} \Pi^c(S).$$

The result below, regarding the form of the optimal assortment, follows from Mahajan and van Ryzin (1998).

PROPOSITION 1. *(Mahajan and van Ryzin 1998) Assume the firm has n products to choose from in order to compose its assortment, and the products are indexed so that* $v_1 \geq v_2 \geq \cdots \geq v_n$. *The optimal assortment consists of the first k products for some* $k \in \{1,\ldots,n\}$.

In plain language, Proposition 1 says that the optimal assortment consists of a number of products with the highest customer appeal.

4. The Decentralized Supply Chain

Consider now the decentralized supply chain where the manufacturer and the retailer are separate entities. In the decentralized supply chain, the retailer holds inventories of the products to meet end customer demand, whereas the manufacturer builds to order. As in the centralized supply chain, the retailer incurs a holding cost of h per unit per period and a penalty cost of b per unit of backordered demand. Let w denote the wholesale price per unit of product, charged by the manufacturer to the retailer, and assume that $p \geq w \geq c$. We assume that all information is common to both parties in the supply chain.

In our analysis, we assume that the retailer selects her assortment from the same set of n products available to the centralized supply chain. This assumption is not necessarily innocuous. The manufacturer of the decentralized supply chain might prefer to offer only a subset of the potential products available in the centralized supply chain. In Section 6, we provide a condition under which the manufacturer will find it optimal to offer the same set of potential products as the centralized supply chain. When this condition is not satisfied, the manufacturer may indeed choose to offer a smaller set of products to the retailer, and we provide such an example in Section 6.

Let $\Pi^d_r(S)$, $\Pi^d_m(S)$, and $\Pi^d_{sc}(S)$ denote the expected profit, respectively, for the retailer, the manufacturer, and the supply chain when the retailer offers assortment $S \subseteq \{1,\ldots,n\}$. These profit functions are given by

$$\Pi_r^d(S) = \Gamma\left(\lambda(p-w)\sum_{i\in S} q_i(S) - (b+h)\phi^*\sigma(S)\right), \quad (3)$$

$$\Pi_m^d(S) = \Gamma\lambda(w-c)\sum_{i\in S} q_i(S), \quad (4)$$

$$\Pi_{sc}^d(S) = \Gamma\left(\lambda(p-c)\sum_{i\in S} q_i(S) - (b+h)\phi^*\sigma(S)\right). \quad (5)$$

The profit maximization problem of the retailer is

$$\max_{S\subseteq\{1,\ldots,n\}} \Pi_r^d(S).$$

The retailer's optimal assortment will follow the same form as that of the centralized supply chain, described in Proposition 1. Throughout the remainder of the paper, we assume that the products are indexed so that $v_1 \geq v_2 \geq \cdots \geq v_n$. Let A_k denote the assortment consisting of products 1 through k. Given Proposition 1, the centralized supply chain's optimal assortment will be A_{k_c} for some $k_c \in \{1,\ldots,n\}$ and the retailer's optimal assortment will be A_{k_d} for some $k_d \in \{1,\ldots,n\}$. The following result compares the retailer-optimal and supply-chain-optimal assortments.

PROPOSITION 2. *The centralized supply chain's optimal assortment is at least as large as the assortment chosen by a retailer in the decentralized supply chain, i.e.,* $k_c \geq k_d$.

This result is intuitive, and it is a manifestation of the double marginalization in the decentralized supply chain. One implication of Proposition 2 is that customers have more variety to choose from in the centralized supply chain.

Note that, in our model, the only type of misalignment between the centralized and decentralized supply chains is the assortment offered to the customers. In particular, if the retailer in the decentralized supply chain can be induced to offer the supply-chain-optimal assortment, then the retailer will find it optimal to use the same order-up-to levels as in the centralized supply chain (i.e., the products' inventory levels will be the same in both supply chains). This is because the unit *underage* and *overage* costs that drive the inventory levels are the same in both supply chains, due to our assumption of full backordering; both the centralized and decentralized supply chains have the same cost of missing one unit of demand (the unit backorder cost, b) and the same cost of having an extra unit of inventory at the end of a period (the unit holding cost per period, h). Alternatively, one could assume that all unmet demands become lost sales, in which case the cost of missing one unit of demand would be $p-c$ for the centralized supply chain and $p-w$ for the retailer in the decentralized supply chain. In such a case, the two supply chains would differ not only in the assortment decisions but also in inventory levels. Such a supply chain could be coordinated by a revenue sharing contract that sets $w=(1-\alpha)c$ for some $0<\alpha<1$ and requires the retailer to give the manufacturer a fraction α of the revenue from each unit sold (see Cachon and Lariviere 2005 for a detailed discussion of revenue sharing contracts). In our model, we focus on the full backordering case to limit our attention to the misalignment in assortment decisions. Since we wish to emphasize the role of slotting fees, we focus on a certain type of contract, one that requires the manufacturer to pay a fee for each additional product carried by the retailer. In the next section, we check if the manufacturer is willing to pay such fees and we examine how the assortment decisions can be coordinated through such a contract.

5. The Decentralized Supply Chain with Slotting Fees

In the status quo in the decentralized supply chain, the retailer is offering assortment A_{k_d}, the assortment consisting of products 1 through k_d. We now consider a contract where the manufacturer pays the retailer a fee K for each product offered by the retailer in excess of k_d. Note that this contract strongly resembles slotting fees that retailers require from manufacturers in order to carry new products offered by manufacturers: in the status quo, the retailer is offering k_d products, and the contract rewards the retailer with a fee of K per every additional product the retailer agrees to carry, as slotting fees would do.

Given a fee K, the retailer decides what assortment it wishes to offer. We impose an additional restriction on the contract: the retailer's choice is limited to assortments $A_j, j\in\{1,\ldots,n\}$. In other words, the retailer can offer any assortment as long as it consists of the most popular j products for some $j\in\{1,\ldots,n\}$. In spirit, this restriction is similar to what retailers already do when choosing assortments. For example, Nelson (2001) reports that Wal-Mart (who does not charge slotting fees) "pressures its store managers to drop products that don't sell well and pushes manufacturers to supply the most popular varieties." In Section 6, we discuss why this restriction is necessary, and we provide conditions under which this restriction could be removed. In addition, in Section 6, we show that some of our results carry over when the fee is product-specific instead of being the same for all products.

With the contract in place, the profit of a retailer that offers products 1 through j is given by

$$\begin{cases} \Pi_r^d(A_j) + K(j-k_d), & \text{if } j \geq k_d, \\ \Pi_r^d(A_j), & \text{if } j < k_d. \end{cases}$$

Under such a contract, the retailer will never offer an assortment with less than k_d products (since assortment A_{k_d} is optimal for the retailer in the absence of fees). Of course, any fee $K>0$ is always agreeable to the retailer. We next analyze if and when a given fee $K>0$ is agreeable to the manufacturer.

5.1. The Effect of the Contract on the Manufacturer

Consider assortment A_k, $k>k_d$, and suppose that there exists a fee K that induces the retailer to offer assortment A_k. In this section, we analyze if and when the manufacturer is willing to pay such a fee K.

To begin with, we characterize the conditions that a given fee K must satisfy in order to induce the retailer to offer assortment A_k. The fee K will achieve that result if the following set of inequalities hold:

$$\Pi_r^d(A_k) + K(k-k_d) \geq \Pi_r^d(A_j) + K(j-k_d),\ j \in k_d, \ldots, n.$$

Alternatively, one can write the above set of inequalities as

$$K \leq \overline{K}(w,k) := \min_{j:k<j\leq n} \left\{ \frac{\Pi_r^d(A_k) - \Pi_r^d(A_j)}{j-k} \right\}, \tag{6}$$

$$K \geq \underline{K}(w,k) := \max_{j:k_d\leq j<k} \left\{ \frac{\Pi_r^d(A_j) - \Pi_r^d(A_k)}{k-j} \right\}. \tag{7}$$

Given the wholesale price, w, the first inequality requires that K is not so large that the retailer will offer more than k products. Likewise, the second inequality requires that K is not so small that the retailer will offer less than k products. Note that $\overline{K}(w,k)$ need not be larger than $\underline{K}(w,k)$ in general. If $\overline{K}(w,k)<\underline{K}(w,k)$, then there exists no fee $K>0$ that will induce the retailer to offer assortment A_k. On the other hand, at a given wholesale price w, if $\overline{K}(w,k) \geq \underline{K}(w,k)$, then any fee $K \in [\underline{K}(w,k), \overline{K}(w,k)]$ will induce the retailer to offer assortment A_k. The manufacturer will agree to pay a fee in this range only if the benefit accrued to the manufacturer from the larger assortment A_k outweighs the cost of fees to be paid by the manufacturer. That is, the manufacturer will agree to pay a fee $K \in [\underline{K}(w,k), \overline{K}(w,k)]$ only if the following inequality is satisfied:

$$\Pi_m^d(A_k) - K(k-k_d) \geq \Pi_m^d(A_{k_d}). \tag{8}$$

Equivalently, the fee $K \in [\underline{K}(w,k), \overline{K}(w,k)]$ must satisfy

$$K \leq K_m(w,k) := \frac{\Pi_m^d(A_k) - \Pi_m^d(A_{k_d})}{k-k_d}. \tag{9}$$

In summary, as long as $\overline{K}(w,k) \geq \underline{K}(w,k)$, the retailer can be induced to offer assortment A_k and, if $K_m(w,k)>\underline{K}(w,k)$, there exist a range of fees, specifically $K \in [\underline{K}(w,k), K_m(w,k)]$, that the manufacturer is willing to pay to induce the retailer to offer assortment A_k.

The following proposition states that, as long as there exists a range of fees that induce the retailer to offer assortment A_k instead of A_{k_d} and the total supply chain profit is better under assortment A_k than under any smaller assortment, then a subset of such fees are agreeable to the manufacturer as well.

Proposition 3. *Consider assortment A_k where $k>k_d$. Suppose that the total supply chain profit is strictly higher under assortment A_k than under any smaller assortment, that is, $\Pi_{sc}^d(A_k)>\Pi_{sc}^d(A_j)$ for $j<k$. Furthermore, suppose that there exists a range of fees that induce the retailer to offer assortment A_k, that is, $\overline{K}(w,k) \geq \underline{K}(w,k)$. Then, $K_m(w,k)>\underline{K}(w,k)$. Consequently, the manufacturer is willing to pay any fee K between $\underline{K}(w,k)$ and $\min(K_m(w,k), \overline{K}(w,k))$.*

Proposition 3 suggests that slotting fees may arise in the interest of both parties in the supply chain. In particular, our model suggests that slotting fees may lighten the burden of additional inventory cost borne by the retailer due to higher variety, thereby making it profitable for the retailer to carry a larger assortment and benefiting the manufacturer at the same time. For this to happen, however, Proposition 3 poses an important condition: It is not enough that assortment A_k improves the total supply chain profit compared to assortment A_{k_d}; the larger assortment A_k must be better for the supply chain than any smaller assortment $A_j, j<k$. Indeed, when this condition does not hold, it is easy to find examples where the retailer can be induced to offer assortment A_k, but the manufacturer is not willing to pay any of the fees that achieve such inducement.

There exists at least one assortment that is larger than A_{k_d} and better for the supply chain than any smaller assortment: the supply-chain-optimal assortment, A_{k_c}. One consequence of Proposition 3 is that if there exists a range of fees that will induce the retailer to offer the supply-chain-optimal assortment, then some of those fees are agreeable to the manufacturer. We next analyze if and when such *coordinating fees* exist.

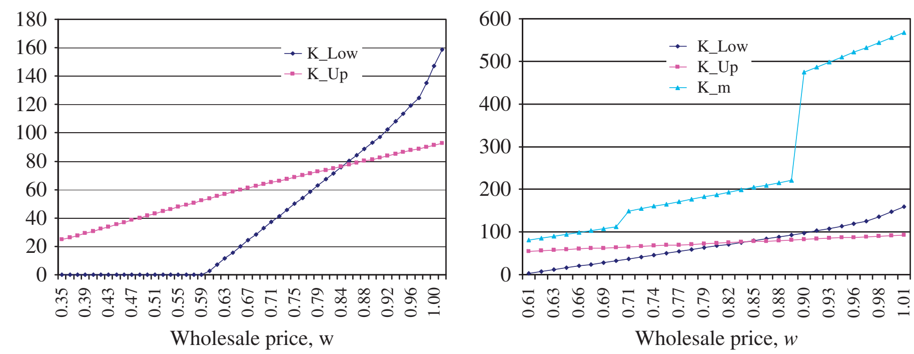
5.2. Coordination through Slotting Fees

At a given wholesale price w, in order for a fee K to induce the retailer to offer assortment A_{k_c}, we must have $\overline{K}(w,k_c) \geq \underline{K}(w,k_c)$ and $K \in [\underline{K}(w,k_c), \overline{K}(w,k_c)]$. In preparation for the main result of this section, the following lemma characterizes the behavior of $\overline{K}(w,k_c)$ and $\underline{K}(w,k_c)$ with respect to w.

Lemma 1. *There exists a $\tilde{w} \in [c,p)$ such that*

(a) *The supply chain is trivially coordinated, i.e., $k_d = k_c$, for $w \in [c, \tilde{w}]$.*

Figure 1 The Left Panel Depicts $\overline{K}(w, k_c)$ and $\underline{K}(w, k_c)$ as a Function of the Wholesale Price. The Right Panel Focuses on the Region of Wholesale Prices Where the Supply Chain is not Trivially Coordinated and Shows $K_m(w, k_c)$ in Addition to $\overline{K}(w, k_c)$ and $\underline{K}(w, k_c)$. The Parameter Values are: $\lambda = 1430$, $\Gamma = 500$, $p = 1.024$, $c = 0.349$, $n = 7$, $b = 0.3$, $h = 0.015$, $L = 4$, $\beta = 0.5$, $v_0 = 0.5$, $v_1 = 32.937$, $v_2 = 23.287$, $v_3 = 14.056$, $v_4 = 13.217$, $v_5 = 6.014$, $v_6 = 5.664$, $v_7 = 4.825$



(b) $\overline{K}(w, k_c) > 0$ *and* $\underline{K}(w, k_c) \geq 0$ *for* $w \in (\tilde{w}, p]$.
(c) $\lim_{w \to \tilde{w}^+} (\overline{K}(w, k_c) - \underline{K}(w, k_c)) > 0$.
(d) $\overline{K}(w, k_c) - \underline{K}(w, k_c)$ *is decreasing in* w *for* $w \in (\tilde{w}, p]$.

When the wholesale price w is very close to the production cost c, double marginalization is not strong enough to cause a difference between the retailer-optimal and supply-chain-optimal assortments. That is why the supply chain is trivially coordinated when the wholesale price is less than or equal to some threshold $\tilde{w}$. Now, once the wholesale price exceeds $\tilde{w}$, a strictly positive fee will be needed to induce the retailer to choose the supply-chain-optimal assortment. Lemma 1(b) and (c) together indicate that $\underline{K}(w, k_c)$ starts below $\overline{K}(w, k_c)$, but catches up as w increases beyond $\tilde{w}$. This observation implies that when the wholesale price is not too high compared to $\tilde{w}$, there exist coordinating fees (i.e., the fees $K \in [\underline{K}(w, k_c), \overline{K}(w, k_c)]$), but we may not be able to find such a coordinating fee once the wholesale price in the supply chain becomes sufficiently high. The following proposition formalizes this result and more.

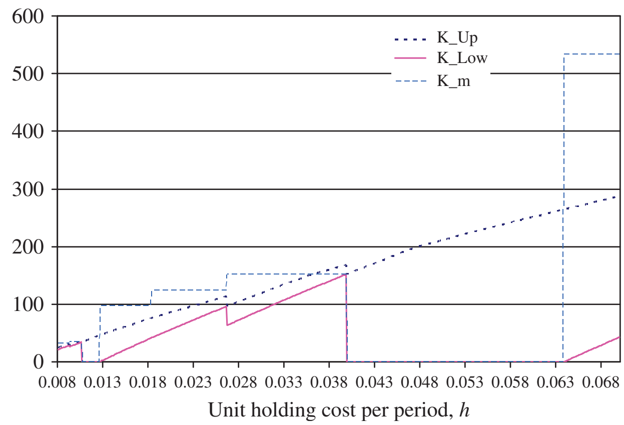
Proposition 4. *There exists a threshold wholesale price* $w^* \in (\tilde{w}, p]$ *such that if* $w \in (\tilde{w}, w^*]$, *then there exists a fee* $K > 0$ *that the manufacturer is willing to pay and that induces the retailer to offer the supply-chain-optimal assortment,* A_{k_c}.

Proposition 4 implies that the lower the wholesale price, the more likely we are to encounter coordinating fees that also make the manufacturer better off. Hence, when the wholesale price is not too high, slotting fees may enhance supply chain efficiency by inducing the retailer to choose the supply-chain-optimal assortment, while making the manufacturer better off as well.

An example is depicted in Figure 1. The problem parameters for this figure are based loosely on the data that Israilevich (2004) provides for the average weekly sales and prices (in 1989 dollars) of several kinds of bath tissue at a Chicago supermarket chain. We took similarly priced, four-roll packages as our potential set of products. This set consists of seven distinct products.[1]

In this example, $k_c = 4$. The left-panel plots $\overline{K}(w, k_c)$ and $\underline{K}(w, k_c)$. When the wholesale price is less than roughly 0.6 (the production cost and retail price are 0.34 and 1.02, respectively), we have $k_c = k_d$, i.e., the supply chain is trivially coordinated (in this region, we plot $\underline{K}(w, k_c) = 0$; note that $\underline{K}(w, k_c)$ is in fact undefined in this region). For any wholesale price larger than 0.6, a positive fee will be needed to induce the retailer to offer the supply-chain-optimal assortment. We observe from Figure 1 that $\overline{K}(w, k_c) \geq \underline{K}(w, k_c)$ for wholesale prices less than roughly 0.84. Consequently, if the wholesale price is between 0.6 and 0.84, this supply chain can be coordinated by the use of a per-product fee. On the other hand, if the wholesale price is above 0.84, then there exists no fee that will achieve coordination. In this example with $p = 1.02$ and $c = 0.34$, a wholesale price of 0.84 implies a profit margin division of 50:18 for the manufacturer/retailer, or about 3/4 of the entire profit margin going to the manufacturer. As long as the manufacturer's portion of the profit margin is smaller than about 3/4, it will be possible to find a fee that will induce the retailer to choose the supply-chain-optimal assortment. It is interesting to note that, according to the data we are adapting from Israilevich (2004), the average wholesale price of the seven items in our set of potential products is 0.81, which is just below the threshold wholesale price. The right-panel of the figure focuses on the region of wholesale prices where the supply chain is not trivially coordinated and plots the largest fee the manufacturer is willing to pay, K_m. As already established in Proposition 4, at any wholesale price where the retailer can be induced to offer the supply-chain-optimal assortment, there exists a range of fees

Figure 2 $\overline{K}(w, k_c)$, $\underline{K}(w, k_c)$ and $K_m(w, k_c)$ as Functions of h. All Parameter Values are the Same as in Figure 1, Except that h is Allowed to Vary and w is Fixed at 0.6865



that the manufacturer is willing to pay so that the retailer will offer the supply-chain-optimal assortment. In fact, in this example, it so happens that the fees that induce the retailer to offer the supply-chain-optimal assortment, $K \in [\underline{K}(w, k_c), \overline{K}(w, k_c)]$, are well below the maximum fee that the manufacturer would be willing to pay to achieve that result, $K_m(w, k_c)$.

5.3. Sensitivity Analysis

We first discuss the effect of inventory-related parameters on the range of coordinating fees. We then discuss the effect of such parameters on the threshold wholesale price.

5.3.1. Coordinating Fees at a Given Wholesale Price. Figure 2 shows how $\overline{K}(w, k_c)$, $\underline{K}(w, k_c)$ and $K_m(w, k_c)$ depend on the unit holding cost per period, h, at a given wholesale price, w. The behaviors with respect to the unit backorder cost, b, and the leadtime, L, are similar.

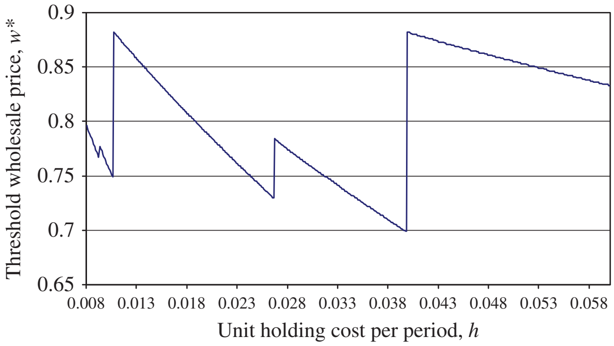
First, leaving aside the discontinuities for now, we observe that a marginal increase in h results in an increase in both $\overline{K}(w, k_c)$ and $\underline{K}(w, k_c)$. To understand this behavior, note that when h increases, carrying a product becomes more costly and, therefore, the minimum fee that will induce the retailer to offer the supply-chain-optimal assortment, $\underline{K}(w, k_c)$, increases. Likewise, since carrying a product becomes more costly, the manufacturer can give the retailer larger fees without the retailer choosing to offer an assortment larger than the supply-chain-optimal assortment. Hence, a marginal increase in h causes an increase in $\overline{K}(w, k_c)$. The discontinuities in $\underline{K}(w, k_c)$ correspond to changes in the supply-chain-optimal assortment. As h increases and the supply-chain-optimal assortment shrinks, the disparity between the retailer-optimal and supply-chain-optimal assortments is reduced. This makes it easier to induce the retailer to offer the supply-chain-optimal assortment, which explains the downward jumps in $\underline{K}(w, k_c)$. The same discontinuities occur in $\overline{K}(w, k_c)$ as well, even though they are harder to detect from the figure.

Notice from the figure that a marginal increase in h does not change the largest fee that the manufacturer is willing to pay, $K_m(w, k_c)$. This fee is a measure of the improvement in the manufacturer's profit when the retailer switches from the retailer-optimal assortment to the supply-chain-optimal assortment. This improvement in the manufacturer's profit does not depend on the holding cost h unless the retailer-optimal assortment or the supply-chain-optimal assortment changes in reaction to a change in the holding cost. Indeed, the upward jumps in $K_m(w, k_c)$ occur because the supply-chain-optimal assortment shrinks at those points, reducing the gap between retailer-optimal and supply-chain-optimal assortments. Hence, the number of additional products that the manufacturer is pushing onto the retailer decreases, which causes an increase in the per-product fee the manufacturer is willing to pay.

Given that a marginal increase in h pushes up the range of coordinating fees while leaving unchanged the largest fee that the manufacturer is willing to pay, one would expect that a marginal increase in h may narrow down the range of fees that coordinate while leaving the manufacturer better off. An example occurs in this figure when h is in the neighborhood of 0.038. In that region, $\underline{K}(w, k_c)$ gets progressively closer to $K_m(w, k_c)$. In fact, when h is slightly above 0.038, the interval $[\underline{K}(w, k_c), K_m(w, k_c)]$ almost vanishes, and only a tiny range of fees will coordinate the supply chain while making the manufacturer better off.

Another interesting observation from Figure 2 is the behavior when h is roughly in between 0.04 and 0.06. In that region, both $\underline{K}(w, k_c)$ and $K_m(w, k_c)$ are zero, indicating that the supply chain is trivially coordinated, that is, the retailer-optimal and supply-chain-optimal assortments are the same. This region of h, however, is preceded by a region where it would take a positive fee to coordinate the supply chain. Thus, a higher holding cost does not always translate to a requirement of higher coordinating fees. This observation sheds further light on the use of slotting fees in the grocery industry. An FTC report finds that slotting fees are the highest for frozen and refrigerated items. Retailers explain these high slotting fees by noting that (i) the cost of operating the shelf space and the cost of storage are higher for such items, and (ii) shelf space is less readily available for such items (Klie, 2004). In other words, retailers claim that they should charge high slotting fees for refrigerated items, because these items incur

Figure 3 **The Threshold Wholesale Price w^* as a Function of h. All Parameter Values are the Same as in Figure 1, Except that h is Allowed to Vary**



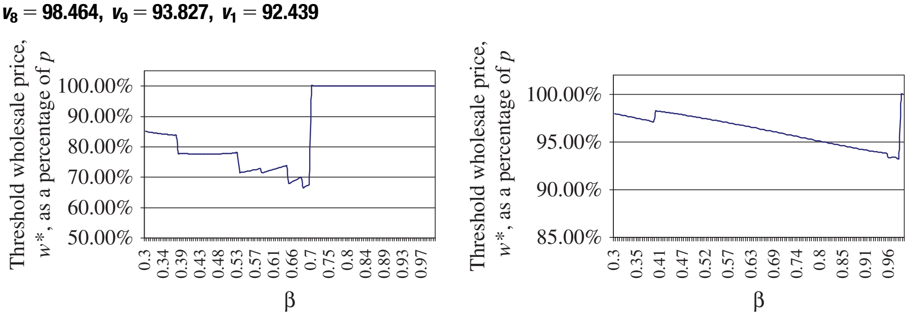
high inventory-related costs. As shown in Figure 2, a small increase in the unit holding cost, h, does push up the range of coordinating fees, seemingly supporting the retailers' claim. Nonetheless, the lesson from Figure 2 is that, when it comes to the size of coordinating fees, the level of disparity between the retailer-optimal and supply-chain-optimal assortments matters just as much, and this disparity is not necessarily getting worse as the inventory holding costs increase. Therefore, if the slotting fees required by retailers are meant to enhance channel efficiency (as opposed to merely improving the retailers' revenues), then it is not enough for the retailers to claim that the slotting fees for refrigerated items are high because of the high inventory-related costs. They also need to demonstrate that the disparity between the retailer- and supply-chain-optimal assortments is large enough to justify the large slotting fees.

5.3.2. The Threshold Wholesale Price. According to our model, the effect of per-product fees on a supply chain is critically dependent on the wholesale price in the supply chain. If the wholesale price is above a threshold level, then such fees may fail to induce the retailer to choose the supply-chain-optimal assortment. In addition, at any wholesale price below the threshold wholesale price, a subset of the fees that achieve coordination also make the manufacturer better off. Next, we discuss how the threshold wholesale price depends on inventory-related parameters.

Figure 3 shows how the threshold wholesale price, w^*, depends on the unit holding cost per period, h. The behaviors with respect to the unit backorder cost, b, and the leadtime, L, are similar. Observe from the figure that a marginal increase in h results in a decrease in the threshold wholesale price, w^*. Therefore, after a marginal increase in h, coordination is possible for a narrower range of wholesale prices. Nonetheless, as h continues to grow, the supply-chain-optimal assortment shrinks, rendering the retailer- and supply-chain-optimal assortments closer to each other. This makes it easier to find fees that coordinate the assortment decisions in the supply chain. Hence, every time the supply-chain-optimal assortment shrinks, the threshold wholesale price, w^*, jumps upward, indicating a growth in the range of wholesale prices under which coordination is possible.

We next discuss the behavior of the threshold wholesale price, w^*, with respect to β, which determines the coefficient of variation of demand. Note that in a supply chain that offers assortment S, the coefficient of variation for the demand of product $i \in S$ is given by $(\lambda q_i(S))^{\beta-1}$. Therefore, as β increases, the coefficient of variation for product i's demand may increase or decrease, depending on whether $\lambda q_i(S) > 1$ or not. Because an increase in β can move the coefficient of variation in either direction, one can observe a large variety of behavior regarding the effect of β on w^*. Figure 4 shows two different examples. In the example shown in the left-hand panel, as β increases, the supply-chain-optimal assortment gets larger. Every time the supply-chain-optimal assortment gets larger, the disparity between the retailer- and supply-chain-optimal assortments grows,

Figure 4 **The Threshold Wholesale Price w^* (Expressed as a Percentage of the Retail Price, p) as a Function of β. In the Left-Hand Panel, $\lambda = \Gamma = 1$, $p = 70$, $c = 20$, $n = 10$, $b = 5$, $h = 1$, $L = 2$. In the Right-Hand Panel, $\lambda = 10$, $\Gamma = 1$, $p = 70$, $c = 65$, $n = 10$, $b = 5$, $h = 1$, $L = 2$. Both Panels Use $v_0 = 80$ with the Following Randomly Generated v_i Values for $i = 1, \ldots, 10$: $v_1 = 178.997$, $v_2 = 177.385$, $v_3 = 162.962$, $v_4 = 150.005$, $v_5 = 132.970$, $v_6 = 113.711$, $v_7 = 122.136$, $v_8 = 98.464$, $v_9 = 93.827$, $v_1 = 92.439$**



which makes it more difficult to induce the retailer to offer the supply-chain-optimal assortment. This growing disparity results in a smaller w^*, indicating that coordination can be achieved for a narrower range of wholesale prices. This behavior changes only when β becomes as large as 0.7. Once β becomes that large, the optimal action for the supply chain is to offer all products, and there is no longer an upper bound on the coordinating fee (since there is no longer a concern that the retailer will offer more than the supply-chain-optimal assortment), and one can always find a fee large enough to induce the retailer to offer all the products. On the other hand, in the example shown in the right-hand panel, w^* makes an upward jump early on. This upward jump occurs at a point where the supply-chain-optimal assortment shrinks, rendering the retailer- and supply-chain-optimal assortments closer to each other.

6. Alternative Contracts

In this section, we discuss how the results would change under certain changes to the contract proposed in Section 5.

6.1. Restriction to Offer an Assortment A_j

The contract proposed in Section 5 restricts the retailer to offer an assortment A_j for some $j \in \{1, \ldots, n\}$ (recall that products are numbered in descending order of preference values, and A_j denotes the assortment containing products 1 through j). The form of the optimal policy described in Proposition 1 does not imply that the best way to choose j products is to select the j products with the highest preference values; i.e., although the optimal solution will be assortment A_j for some $j \in \{1, \ldots, n\}$, the optimal assortment of j products for some $j \in \{1, \ldots, n\}$ is not necessarily A_j. If the contract does not require that the retailer choose assortment A_j for some $j \in \{1, \ldots, n\}$, then we might run into situations where the contract induces the retailer to offer k_c products, but the retailer chooses an assortment of k_c products other than A_{k_c}. Consider the example shown in Table 1. In this example, $k_d = 1$ and $k_c = 2$. By requiring the retailer to offer assortment A_j for some $j \in \{1, \ldots, n\}$ and using a fee $K = 20$, the contract induces the retailer to offer assortment A_2, which is the supply-chain-optimal assortment. If the retailer were not limited to assortments A_j, then the retailer could choose to carry products 1 and 5, which is an improvement over assortment A_2 for the retailer. Such an action will also cause the manufacturer's profit to worsen, since the additional revenue the manufacturer makes from product 5 is outweighed by the fee the manufacturer needs to pay to the retailer. The contract discussed in the previous section avoids such a possibility by limiting the retailer to offering A_j forsome $j \in \{1, \ldots, n\}$. Note that, even with such a limitation, the retailer is better off under the contract. Furthermore, the following proposition states sufficient conditions on problem parameters under which the retailer will choose an assortment of the form A_j for some $j \in \{1, \ldots, n\}$, even when it is not explicitly required to do so.

Table 1 **If the Retailer is not Restricted to Choose an Assortment of the Form $\{1, \ldots, j\}$ for Some j, the Contract May Result in the Retailer Choosing an Assortment of k_c Products Other than $\{1, \ldots, k_c\}$**

Assortment	Retailer profit	Manufacturer profit	Supply chain profit
Profits without any fees			
{1}	1001.52	909.09	1910.61
{1,2}	983.99	941.18	1925.17
{1,2,3}	955.57	952.38	1907.95
{1, . . ., 4}	929.49	956.52	1886.01
{1, . . ., 5}	917.98	956.90	1874.87
Profits with fee $K = 20$			
{1}	1001.52	909.09	1910.61
{1,2}	1003.99	921.18	1925.17
{1,2,3}	995.57	912.38	1907.95
{1, . . ., 4}	989.49	896.52	1886.01
{1, . . ., 5}	997.98	876.90	1874.87
{1,5}	1005.90	890.70	1896.60

In this example, $\Gamma = 100$, $\lambda = (b+h)\phi^* = L = 1, p - w = 12.5, w - c = 10, v_0 = 5, v_1 = 50, v_2 = 30, v_3 = 20, v_4 = 10, v_5 = 1$.

Proposition 5. *Let* $\Delta = \sum_{i=1}^{n-1}(v_i - v_n), M = \lambda(p - w)$, *and* $Z = (b+h)\phi^*(\lambda(L+1))^\beta$. If $\frac{M}{\beta Z} > \left(1 + \frac{\Delta}{v_0}\right)\left(n + 1 + \frac{\Delta + v_0}{v_n}\right)^{1-\beta}$, *then the optimal assortment of j products is* A_j.

The condition of Proposition 5 guarantees that, for any $j \in \{1, \ldots, n\}$, the optimal assortment with j products is A_j. Therefore, if the condition holds, then the contract does not need to limit the retailer to choose an assortment of the form A_j, as it is already in the retailer's best interest to do so. The larger $M = \lambda(p - w)$ is with respect to $Z = (b+h)\phi^*(\lambda(L+1))^\beta$, the more likely the condition of Proposition 5 is to be satisfied, i.e., the larger the profit margin compared to the inventory-associated cost, the more likely it is that the condition will be satisfied.

Another way to remove this restriction would be to impose an alternative restriction on the manufacturer: If the manufacturer of the decentralized supply chain lets the retailer choose from only products 1 though k_c, then the retailer will offer assortment A_{k_c} as long as the fee K exceeds a certain limit.

6.2. Product-Specific Fees

In our earlier analysis, we assume that the retailer is paid a fee K for each product offered in excess of assortment A_{k_d}. Suppose now that the manufacturer announces a fee K_i for product $i \notin A_{k_d}$ and promises to pay the retailer K_i if the retailer adds product i to its assortment. In particular, let $K_i = K + \delta_i$ for $i \notin A_{k_d}$, where $K \geq 0$ can be interpreted as a base fee and $\delta_i \geq 0$ can be interpreted as an additional fee, specific to product i. Can one find K and δ_i for $i \notin A_{k_d}$ so that the retailer will offer the supply-chain-optimal assortment? The following proposition answers this question in the affirmative.

PROPOSITION 6. *Suppose the products are indexed in descending order of v_i's, i.e., $v_1 \geq v_2 \geq \cdots \geq v_n$ and the retailer is required to offer an assortment of the form $A_j = \{1, \ldots, j\}$. Suppose the manufacturer pays the retailer a fee $K + \delta_i$ for offering product $i \notin A_{k_d}$. There exists a $w^* \in [\tilde{w}, p]$ such that if $w \leq w^*$ then there exist $K \geq 0$ and $\delta_i \geq 0$ that will induce the retailer to offer the supply-chain-optimal assortment A_{k_c}.*

Note that one way to pick δ_i's is to choose $\delta_{k_d+1} \geq \delta_{k_d+2} \geq \cdots \geq \delta_n$, in which case the manufacturer offers larger fees for products with larger preference values. In some cases, such a flexibility may actually help remove from the contract the restriction to offer an assortment of the form A_j. For instance, in the example shown in Table 1, if the manufacturer picks δ_2 to be sufficiently larger than δ_5, then the retailer will have no incentive to offer assortment $\{1,5\}$ any longer and will offer assortment $\{1,2\}$ instead. Such an outcome could be achieved, for example, by setting $K = 0$, $\delta_1 = \delta_3 = \delta_4 = \delta_5 = 0$, and picking any $\delta_2 > 17.53$.

6.3. Paying a Fee for Each Product Offered

In the contract of the previous section, the retailer receives a per-product fee only for products offered in excess of assortment A_{k_d}. One could change the contract so that the manufacturer pays the retailer a per-product fee for *all* products offered by the retailer. In such a case, the result summarized in Proposition 4 continues to hold, i.e., there still exists a per-product fee that induces the retailer to offer k_c products, provided that the wholesale price is below a threshold. With such a payment scheme, the manufacturer will now agree to pay a coordinating fee $K \in [\underline{K}(w, k_c), \overline{K}(w, k_c)]$ only if the following inequality is satisfied:

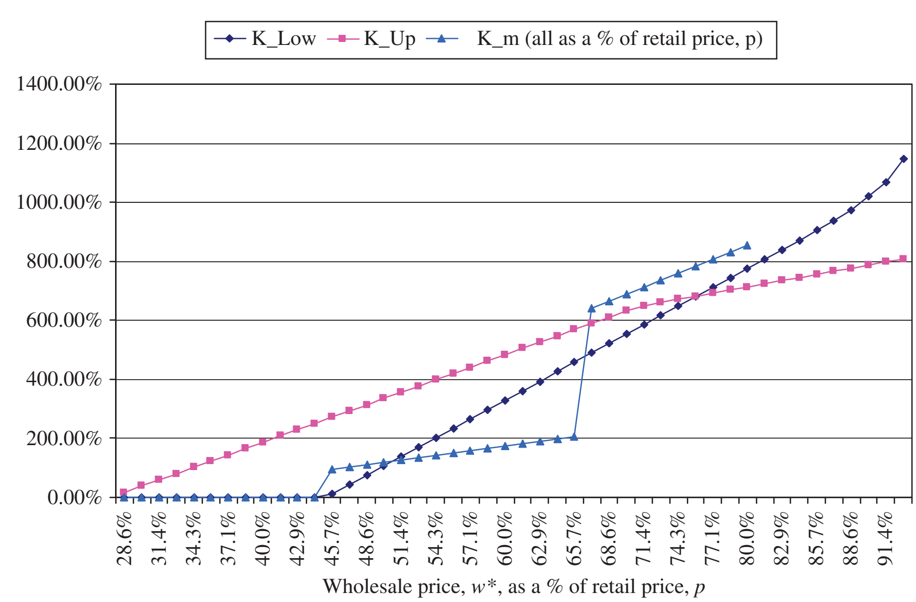
$$\Pi_m^d(A_{k_c}) - K \cdot k_c \geq \Pi_m^d(A_{k_d}).$$

Equivalently, the fee $K \in [\underline{K}(w, k_c), \overline{K}(w, k_c)]$ must satisfy

$$K \leq K_m(w) := \frac{\Pi_m^d(A_{k_c}) - \Pi_m^d(A_{k_d})}{k_c - k_d}. \qquad (10)$$

Figure 5 depicts a numerical example where $\overline{K}(w, k_c)$ and $\underline{K}(w, k_c)$ and the updated $k_m(w)$ are plotted as functions of the wholesale price (expressed as the percentage of the retail price). Notice from the figure that the existence of coordinating fees no longer guarantees that some of those fees will make the manufacturer better off. In this example there exist two ranges of wholesale prices (from 45% to 51% and from 66% to 77%) where there is a fee K that induces the retailer to offer assortment A_{k_c} while making the manufacturer better off, i.e., a fee K that satisfies both

Figure 5 **When the Wholesale Price is Between Roughly 51% and 66% of the Retail Price, There Exist Fees that Induce the Retailer to Offer Assortment A_{k_c}, but None of Those Fees Make the Manufacturer Better Off. In This Example, $\lambda = 1$, $\Gamma = 1000$, $p = 70$, $c = 20$, $n = 10$, $b = 10$, $h = 1$, $L = 2$, $\beta = 0.5$, $v_0 = 80$, $v_1 = 226.362$, $v_2 = 202.766$, $v_3 = 197.944$, $v_4 = 173.863$, $v_5 = 171.919$, $v_6 = 168.678$, $v_7 = 165.169$, $v_8 = 127.962$, $v_7 = 117.164$, $v_{10} = 94.885$**



$\overline{K}(w, k_c) \geq K \geq \underline{K}(w, k_c)$ and $K \leq K_m(w)$. However, these two ranges are separated by a range of wholesale prices (from 51% to 66%) where the retailer can be induced to offer assortment A_{k_c}, but none of the fees that achieve this result make the manufacturer better off.

6.4. Strategic Manufacturer

One of our assumptions in Section 5 is that there exists a given set of n products from which the retailer makes her assortment selection. Another approach would be to assume that the manufacturer has a set of n potential products, and the manufacturer decides which of these n products to let the retailer choose from, i.e., the manufacturer acts as the Stackelberg leader in deciding what products to offer to the retailer. In such a case, the manufacturer would not necessarily make all n products available to the retailer. Consider the example depicted in Table 2.

In this example, the manufacturer decides which of the eight potential products to make available to the retailer. If the manufacturer lets the retailer choose from all eight products, the retailer will offer products 1 through 4 in its assortment. However, if the manufacturer decides not to make product 2 available, then the retailer will offer products 1, 3, 4, 5, and 6, which provides the manufacturer with a profit greater than the assortment of products 1 through 4. Therefore, in the example shown in Table 2, the manufacturer will never make all products available to the retailer. Under certain conditions, however, it will always be optimal for the manufacturer to make all potential products available. The following proposition states such a sufficient condition.

PROPOSITION 7. *Suppose the products are indexed in descending order of v_i's, i.e., $v_1 \geq v_2 \geq \cdots \geq v_n$ and the retailer is required to offer an assortment of the form $A_j = \{1, \ldots, j\}$. If $v_i > \sum_{j=i+1}^{n} v_j$ for $i = 1, \ldots, n-1$, then the manufacturer will offer the retailer all the potential products, i.e., the manufacturer will let the retailer choose from $\{1, \ldots, n\}$.*

Given the set of potential products $\{1, \ldots, n\}$, Proposition 7 can be interpreted as saying that if there is a large gap between the preference values of the products, then the manufacturer is best off by letting the retailer choose from all potential products. If this condition were to hold, product i in assortment S would have a larger purchase probability than all the less popular products in assortment S combined. Hence, this condition may be too stringent in many practical cases. Nonetheless, given this condition, it is not too surprising that the example depicted in Table 2 leads to a situation where the manufacturer would like to eliminate some of the products: in that example, the v_i values are very close for many products.

Table 2 **An Example Where the Manufacturer Would Not Make All Potential Products Available**

Assortment	Retailer Profit	Manufacturer Profit
Scenario 1: Manufacturer offers all eight products		
{1}	1595.2	829.8
{1,2}	1716.6	905.9
{1,2,3}	1750.5	934.4
{1, . . ., 4}	1754.1	945.6
{1, . . ., 5}	1752.9	953.2
{1, . . ., 6}	1749.0	958.8
{1, . . ., 7}	1743.2	962.6
{1, . . ., 8}	1735.7	964.4
Scenario 2: Manufacturer removes Product #2 from the offering		
{1}	1595.2	829.8
{1,3}	1714.4	904.8
{1,3,4}	1735.9	926.6
{1,3,4,5}	1743.4	939.8
{1,3,4,5,6}	1744.5	948.7
{1,3,4, . . ., 7}	1741.2	954.5
{1,3,4, . . ., 8}	1734.4	957.2

In this example, $\Gamma = 100$, $\lambda = \mathrm{L} = 1$, $(\mathrm{b}+\mathrm{h})\phi^* = 0.5$, $\mathrm{p} - \mathrm{w} = 20$, $w - c = 10$, $\beta = 0.5$, $v_0 = 8$, $v_1 = 39$, $v_2 = 38$, $v_3 = 37$, $v_4 = 25$, $v_5 = 24$, $v_6 = 23$, $v_7 = 20$, $v_8 = 11$.

7. Conclusion

In this paper we considered the coordination of assortment planning in a supply chain. We showed that, as one would expect, the centralized chain offers at least as many products as the decentralized one, since the centralized supply chain does not suffer from double marginalization. We then considered the effects of a contract that requires the manufacturer to pay a fee to the retailer per product the retailer carries in excess of a certain target level. This payment scheme is significant, since it strongly resembles slotting fees used in the grocery industry. We showed that if the wholesale price in the decentralized supply chain is below some threshold level, then such fees will induce the retailer to offer supply-chain-optimal assortment. Furthermore, the contract is guaranteed to make both parties better off. Interestingly, once the wholesale price exceeds the threshold level, there exists no fee that will induce the retailer to offer the supply-chain-optimal assortment.

The model of this paper is particularly applicable in the case of retailers whose profit margins are not too high compared to their inventory-related costs, e.g., grocery stores. The low profit margins of grocery stores make it less attractive to carry large assortments unless the manufacturer provides an extra incentive.

Therefore, it is not too surprising that slotting fees appear to be more widely used by grocery stores than other types of retailers. Interestingly enough, some powerful retailers avoid slotting fees despite their bargaining power, e.g., Wal-Mart. One possible explanation is that such retailers are able to obtain low wholesale prices, which then makes it attractive for them to carry a large assortment without any need for slotting fees. In fact, Jacoby (2004) comments that Wal-Mart tells its suppliers that ''they earn shelf space with rock-bottom wholesale prices,'' as opposed to requiring slotting fees but accepting higher wholesale prices. In general, slotting fees appear to be less common in retail settings where retailer's margins are significantly high, e.g., fashion apparel. This is probably because the high profit margins in such settings provide enough incentive for the retailer to offer large assortments in the first place.

While in our model the wholesale price is exogenously fixed, it may be possible in practice for the manufacturer to set the wholesale price. When choosing the wholesale price, the trade-off for the manufacturer is that a larger wholesale price increases its unit profit margin, but decreases the size of the assortment chosen by the retailer. Hence, even if the wholesale price were optimally chosen, the manufacturer would still leave some profit margin to the retailer and double marginalization would continue to exist. Thus, the discrepancy between the retailer-optimal and supply-chain-optimal assortments would still be around and the manufacturer may still stand to gain from slotting fees. Nonetheless, it is possible that the fees that are agreeable to the manufacturer would be smaller when the manufacturer is able to choose the wholesale price.

We assumed that the retailer bears all the inventory holding and shortage costs. One could change the contract so that the supply chain uses consignment, i.e., the manufacturer incurs the inventory holding and shortage cost instead of the retailer. In such a case, even in the absence of any fees, the retailer would choose to offer all available products. This follows from the fact that sales volume increases with variety, and the only cost in our model that is keeping the retailer from offering all available variety is the inventory cost. However, such an outcome is not necessarily desirable, unless the supply-chain-optimal solution happens to be to offer all products.

How the manufacturer should design its set of potential products is an interesting question. In order to address this problem, future research may consider the use of models where the assortment is a continuum of products as opposed to our model where products are discrete; such a model of product differentiation is likely to render the problem more tractable.

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Appendix

We first provide the proofs of the propositions. This is followed by the proof of Lemma 1, which is stated in the body of the paper. Lemma 2 is stated and proved here in the Appendix.

PROOF OF PROPOSITION 2: Throughout the proof, recall that the products are indexed so that $v_1 \geq v_2 \geq \cdots \geq v_n \geq 0$. Substituting for $\Pi^c(A_k)$ and $\Pi_r^d(A_k)$ from Equations (2) and (3), we can write the difference of the two profit functions as

$$\Pi^c(A_k) - \Pi_r^d(A_k) = \Gamma\lambda(w - c)\sum_{i=1}^{k} q_i(A_k).$$

Note that $\sum_{i=1}^{k} q_i(A_k) = \frac{\sum_{i=1}^{k} v_i}{\sum_{i=0}^{k} v_i} = 1 - \frac{v_0}{\sum_{i=0}^{k} v_i}$ is increasing in k. Hence, $\Pi^c(A_k) - \Pi_r^d(A_k)$ is increasing in k. Now, the proof is by contradiction. Suppose $k_d > k_c$. Then, since $\Pi^c(A_k) - \Pi_r^d(A_k)$ is increasing in k, it follows that $\left(\Pi^c(A_{k_d}) - \Pi_r^d(A_{k_d})\right) - \left(\Pi^c(A_{k_c}) - \Pi_r^d(A_{k_c})\right) > 0$. Regrouping the terms, we can write

$$\left(\Pi^c(A_{k_d}) - \Pi^c(A_{k_c})\right) - \left(\Pi_r^d(A_{k_d}) - \Pi_r^d(A_{k_c})\right) > 0.$$

However, this leads to a contradiction, since A_{k_d} maximizes Π_r^d and A_{k_c} maximizes Π_c. Therefore, we cannot have $k_d > k_c$, which concludes the proof. ■

PROOF OF PROPOSITION 3: In order to prove the proposition, it suffices to prove that $K_m(w,k) \geq \underline{K}(w,k)$. From the definitions of $K_m(w,k)$ and $\underline{K}(w,k)$, given by (9) and (7), respectively:

$$K_m(w,k) - \underline{K}(w,k) = \frac{\Pi_m^d(A_k) - \Pi_m^d(A_{k_d})}{k - k_d} - \max_{j:k_d \leq j < k}\left\{\frac{\Pi_r^d(A_j) - \Pi_r^d(A_k)}{k - j}\right\}.$$

Notice that, for any assortment S, $\Pi_r^d(S) = \Pi_{sc}^d(S) - \Pi_m^d(S)$. Therefore, we can write:

$$K_m(w,k) - \underline{K}(w,k) = \frac{\Pi_m^d(A_k) - \Pi_m^d(A_{k_d})}{k - k_d} - \max_{j:k_d \leq j < k}\left\{\frac{\Pi_{sc}^d(A_j) - \Pi_m^d(A_j) - \Pi_{sc}^d(A_k) + \Pi_m^d(A_k)}{k - j}\right\}.$$

It follows from the condition that $\Pi_{sc}^d(A_k) \geq \Pi_{sc}^d(A_j)$ for all $k>j$:

$$K_m(w,k) - \underline{K}(w,k) \geq \frac{\Pi_m^d(A_k) - \Pi_m^d(A_{k_d})}{k-k_d} - \max_{j:k_d \leq j<k}\left\{\frac{\Pi_m^d(A_k) - \Pi_m^d(A_j)}{k-j}\right\}. \tag{A1}$$

Now, from the definition of $\Pi_m^d(A_k)$ given by (4), one can check that, for $j<k$,

$$\frac{\Pi_m^d(A_k) - \Pi_m^d(A_j)}{k-j} = (w-c)\Gamma\lambda v_0 \frac{\sum_{i=j+1}^{k} v_i}{(k-j)\sum_{i=0}^{j} v_i \sum_{i=0}^{k} v_i}.$$

Clearly, $\sum_{i=0}^{j} v_i$ is increasing in j. Also, $\frac{\sum_{i=j+1}^{k} v_i}{k-j}$ is decreasing in j (to see why, note that $\frac{\sum_{i=j+1}^{k} v_i}{k-j}$ is the average value of v_{j+1} through v_k, and v_i's are in descending order, i.e., $v_1 \geq v_2 \geq \cdots \geq v_n$). Therefore, $\frac{\Pi_m^d(A_k)-\Pi_m^d(A_j)}{k-j}$ is decreasing in j, and:

$$\max_{j:k_d \leq j<k}\left\{\frac{\Pi_m^d(A_k) - \Pi_m^d(A_j)}{k-j}\right\} \leq \frac{\Pi_m^d(A_k) - \Pi_m^d(A_{k_d})}{k-k_d}. \tag{A2}$$

Now, combining (A1) and (A2), we can write:

$$K_m(w,k) - \underline{K}(w,k) \geq 0,$$

which concludes the proof. ■

Proof of proposition 4: First, note from Lemma 1(b) that $\overline{K}(w,k_c)>0$ and $\underline{K}(w,k_c) \geq 0$ for $w>\tilde{w}$. In addition, Lemma 1(c) shows that $\lim_{w\to\tilde{w}^+}\overline{K}(w,k_c) > \lim_{w\to\tilde{w}^+}\underline{K}(w,k_c)$. Now, since $\overline{K}(w,k_c) - \underline{K}(w,k_c)$ is decreasing in w (by Lemma 1(c)), there exists $w^* \in (\tilde{w},p]$ such that $\overline{K}(w,k_c)>\underline{K}(w,k_c) \geq 0$ for $w<w^*$. Therefore, at any wholesale price w such that $w<w^*$, the retailer can be induced to offer the supply-chain-optimal assortment A_{k_c} by picking a fee $K \in [\underline{K}(w,k_c), \overline{K}(w,k_c)]$. By definition, the supply-chain-optimal assortment A_{k_c} is better for the supply chain than any smaller assortment $A_j, j<k_c$. Hence, for $w<w^* \in (\tilde{w},p)$, it follows from Proposition 3 that the manufacturer is willing to pay any fee $K \in \left[\underline{K}(w,k_c), \min\left(\overline{K}(w,k_c), K_m(w,k_c)\right)\right]$. ■

Proof of proposition 5: The proposition holds trivially when $j=n$. Suppose S is the best assortment with $j<n$ products, and assume for a contradiction that $S \neq \{1,\ldots,j\}$. Then there must exist a pair of products i and k such that $i \notin S$, $k \in S$ and $v_n \leq v_k < v_i$. We know from Lemma 2 that if we replace k with i, then $\Pi_r^d(S)$ will improve. Therefore, S cannot be the optimal assortment with j products, yielding a contradiction, which concludes the proof. ■

Proof of proposition 6: In order for the retailer to offer the supply-chain-optimal assortment A_{k_c}, K and δ_i for $i \notin A_{k_d}$ must satisfy the following:

$$\Pi_r^d(A_{k_c}) + K(k_c-k_d) + \sum_{i=k_d+1}^{k_c}\delta_i \geq \Pi_r^d(A_k) + K(k-k_d) + \sum_{i=k_d+1}^{k}\delta_i, k \in \{k_d,\ldots,n\}.$$

Alternatively, we can write the above set of inequalities as

$$K \leq \overline{K}'(w,k_c): = \min_{k:k_c<k\leq n}\left\{\frac{\Pi_r^d(A_{k_c}) - \Pi_r^d(A_k) - \delta_{k_c+1} - \ldots - \delta_k}{k-k_c}\right\}, \tag{A3}$$

$$K \geq \underline{K}'(w,k_c): = \max_{k:k_d\leq k<k_c}\left\{\frac{\Pi_r^d(A_k) - \Pi_r^d(A_{k_c}) - \delta_{k+1} - \ldots - \delta_{k_c}}{k_c-k}\right\}. \tag{A4}$$

Now, an analog of Lemma 1 can be proven for $\overline{K}'(w,k_c)$and $\underline{K}'(w,k_c)$. Here, we provide only a sketch of the proof. To see why $\overline{K}'(w,k_c) \geq 0$ and $\underline{K}'(w,k_c) \geq 0$ for $w \in (\tilde{w},p]$ (analogous to Lemma 1(b)), note that we can always pick δ_i values small enough to satisfy this condition. Now, given any set of fixed δ_i values, the result that $\overline{K}'(w,k_c) - \underline{K}'(w,k_c)$ is decreasing in w (analogous to Lemma 1(d)) follows from the same line of reasoning as in Lemma 1(d). Given an analog of Lemma 1 holds for $\overline{K}'(w,k_c)$ and $\underline{K}'(w,k_c)$, the existence of w^* below which the retailer can be induced to offer the supply-chain-optimal assortment follows. ■

Proof of proposition 7: Let A denote the entire set of products, $\{1,\ldots,n\}$, and let $S^* = \{1,\ldots,k\}$ be the optimal assortment chosen by the retailer when the retailer is offered A by the manufacturer. Suppose the manufacturer removes sets B and C from the set of products offered to the retailer (i.e., the manufacturer lets the retailer choose from $A/(B \cup C)$) where $B \subseteq S^*$ and $C \subseteq A/S^*$. The retailer will now choose $(S^*/B) \cup D$ for some $D \subseteq A/(S^* \cup C)$. Therefore, the manufacturer will prefer to offer the retailer $A/(B \cup C)$ instead of A only if $\Pi_m^d((S^*/B) \cup D) > \Pi_m^d(S^*)$, which will be the case if $\sum_{j\in D} v_j > \sum_{j\in B} v_j$ (this can be verified using the definition of $\Pi_m^d(\cdot)$ in (4) and noting that $\sum_{i\in S} q_i(S) = 1 - \frac{v_0}{v_0+\sum_{i\in S} v_i}$). However, if $v_i > \sum_{j=i+1}^{n} v_j$ for $i = 1, \ldots, n-1$, then there does not exist $B \subseteq S^*$, $C \subseteq A/S^*$ and $D \subseteq A/(S^* \cup C)$ such that $\sum_{j\in D} v_j > \sum_{j\in B} v_j$ (to see why this is true, recall that products are indexed so that $v_1 \geq v_2 \geq \ldots \geq v_n$, and note that the products in set B have lower indices than those in D). Hence, if $v_i > \sum_{j=i+1}^{n} v_j$ for $i = 1, \ldots, n-1$, then there does not exist $B \subseteq S^*$ and $C \subseteq A/S^*$ such that the manufacturer will prefer to offer the retailer $A/(B \cup C)$ instead of A. ■

PROOF OF LEMMA 1: First, note that the existence of $\tilde{w}$ follows from the fact that $k_d = k_c$ for $w = c$. Next, we prove parts (b) and (c) of the lemma.

PROOF OF (b): To see why $\overline{K}(w, k_c) > 0$ for $w \in (\tilde{w}, p]$, first note that, using Equations (3), (4) and (5), we can write

$$\Pi_r^d(A_{k_c}) - \Pi_r^d(A_k) = \Pi_{sc}^d(A_{k_c}) - \Pi_{sc}^d(A_k) + \Pi_m^d(A_k) - \Pi_m^d(A_{k_c}).$$

Now, $\Pi_{sc}^d(A_{k_c}) - \Pi_{sc}^d(A_k) \geq 0$, since assortment A_{k_c} maximizes Π_{sc}^d. Also, for $k > k_c$, $\Pi_m^d(A_k) - \Pi_m^d(A_{k_c}) > 0$ follows from the fact that $\sum_{i=1}^{k} q_i(A_k)$ is strictly increasing in k. Therefore, $\Pi_r^d(A_{k_c}) - \Pi_r^d(A_k) > 0$ for $k > k_c$. Hence, it follows from the definition of $\overline{K}(w, k)$, given by (6), that $\overline{K}(w, k_c) > 0$.

It remains to show $\underline{K}(w, k_c) \geq 0$. Note that by definition of $\tilde{w}$, $k_c \neq k_d$ for $w \in (\tilde{w}, p]$. Therefore, $\Pi_r^d(A_{k_d}) - \Pi_r^d(A_{k_c}) \geq 0$. Now, it follows from the definition of $\underline{K}(w, k)$, given by (7), that $\underline{K}(w, k_c) \geq 0$ for $w \in (\tilde{w}, p]$.

PROOF OF (c): First, we show that $\lim_{w\to\tilde{w}^+} \overline{K}(w, k_c) > 0$. From the definition of $\overline{K}(w, k)$, given by (6), we have:

$$\lim_{w\to\tilde{w}^+} \overline{K}(w, k_c) = \lim_{w\to\tilde{w}^+} \min_{j:k_c<j\leq n} \left\{ \frac{\Pi_r^d(A_{k_c}) - \Pi_r^d(A_j)}{j - k_c} \right\}$$
$$= \lim_{w\to\tilde{w}^+} \min_{j:k_c<j\leq n} \left\{ \frac{\Pi_{sc}^d(A_{k_c}) - \Pi_m^d(A_{k_c}) - \Pi_{sc}^d(A_j) + \Pi_m^d(A_j)}{j - k_c} \right\}.$$

Now, noting that $\Pi_{sc}^d(A_{k_c}) - \Pi_{sc}^d(A_j) \geq 0$ for $j \neq k_c$ and $\Pi_m^d(A_j) - \Pi_m^d(A_{k_c}) > 0$ for $j > k_c$, it follows from the above equation that $\lim_{w\to\tilde{w}^+} \overline{K}(w, k_c) > 0$.

Next, we show that $\lim_{w\to\tilde{w}^+} \underline{K}(w, k_c) = 0$. From the definition of $\underline{K}(w, k)$, given by (7), we have

$$\lim_{w\to\tilde{w}^+} \underline{K}(w, k_c) = \lim_{w\to\tilde{w}^+} \max_{j:k_d\leq j<k_c} \left\{ \frac{\Pi_r^d(A_j) - \Pi_r^d(A_{k_c})}{k_c - j} \right\}. \tag{A5}$$

Observe that, once the wholesale price w is $\tilde{w} + \varepsilon$ for an arbitrarily small $\varepsilon > 0$, the retailer-optimal assortment in the decentralized supply chain, A_{k_d}, differs from the supply-chain-optimal assortment by only one product, that is, $A_{k_d} = A_{k_c-1}$ as $w \to \tilde{w}^+$. Applying this observation to (A4) yields:

$$\lim_{w\to\tilde{w}^+} \underline{K}(w, k_c) = \lim_{w\to\tilde{w}^+} \Pi_r^d(A_{k_c-1}) - \Pi_r^d(A_{k_c}).$$

Now, observe that the retailer is indifferent between the supply-chain optimal assortment A_{k_c} and the smaller assortment A_{k_c-1} at $w = \tilde{w}$ (here is a sketch of the proof of this claim: Let $A^*(w)$ denote the retailer's optimal assortment at a given w. Because (i) $\Pi_r^d(A_j)$ is linear decreasing in w, and (ii) the retailer's profit from the optimal assortment, $\Pi_r^d(A^*(w))$, is the upper envelope of the functions $\Pi_r^d(A_j)$, $j = 1, \ldots, n$, it follows that (iii) $\Pi_r^d(A^*(w))$ is continuous in w. Therefore, at $w = \tilde{w}$, where the retailer's optimal assortment changes from A_{k_c} to A_{k_c-1}, the retailer's profits from the two assortments are equal). Hence, as $w \to \tilde{w}^+$, we have $\Pi_r^d(A_{k_c-1}) - \Pi_r^d(A_{k_c}) \to 0$.

So far we have proved that $\lim_{w\to\tilde{w}^+} \overline{K}(w, k_c) > 0$ and $\lim_{w\to\tilde{w}^+} \underline{K}(w, k_c) = 0$. The result now follows.

PROOF OF (d): Let us first define some notation that will be helpful in the proof. Let

$$\overline{K}_j(w, k_c) = \frac{\Pi_r^d(A_{k_c}) - \Pi_r^d(A_j)}{j - k_c}$$

so that $\overline{K}(w, k_c) = \min_{j:k_c<j\leq n} \{\overline{K}_j(w, k_c)\}$. Similarly, define

$$\underline{K}_j(w, k_c) = \frac{\Pi_r^d(A_j) - \Pi_r^d(A_{k_c})}{k_c - j}$$

so that $\underline{K}(w, k_c) = \max_{j:k_d\leq j<k_c} \{\underline{K}_j(w, k_c)\}$. Define $\Psi(j)$ for $j \neq k_c$ such that

$$(w - c)\Psi(j) = \frac{\Pi_m^d(A_j) - \Pi_m^d(A_{k_c})}{j - k_c}. \tag{A6}$$

Finally, define $\Delta(j)$ for $1 \leq j \leq n$ as

$$\Delta(j) = \Pi_{sc}^d(A_{k_c}) - \Pi_{sc}^d(A_j). \tag{A7}$$

It follows from the definitions above and Equations (3), (4) and (5), that

$$\overline{K}_j(w, k_c) = \frac{\Delta(j)}{j - k_c} + (w - c)\Psi(j) \text{ for } k_c < j \leq n$$

and

$$\underline{K}_j(w,k_c) = -\frac{\Delta(j)}{k_c - j} + (w-c)\Psi(j) \text{ for } k_d \le j < k_c.$$

We now establish some properties of $\Psi(j)$ that will be useful in the proof. From the definition of $\Pi_m^d(A_j)$ in (4), one can check that

$$\Psi(j) = \Gamma\lambda v_0 \frac{\sum_{i=\min\{k_c,j\}+1}^{\max\{k_c,j\}} v_i}{|j-k_c| \sum_{i=0}^{j} v_i \sum_{i=0}^{k_c} v_i}, \text{ for } k_c < j \le n.$$

Clearly, $\sum_{i=0}^{j} v_i$is strictly increasing in j. Also, $\frac{\sum_{i=\min\{k_c,j\}+1}^{\max\{k_c,j\}} v_i}{|j-k_c|}$ is decreasing in j (to see why, note that $\frac{\sum_{i=\min\{k_c,j\}+1}^{\max\{k_c,j\}} v_i}{|j-k_c|}$ is the average value of v_{k_c+1} through v_j or the average value of v_{j+1} through v_{k_c}, and v_i's are in descending order, i.e., $v_1 \ge v_2 \ge \cdots \ge v_n$). Therefore, $\Psi(j)$ is strictly decreasing in j.

Now we are ready to show that $\overline{K}(w,k_c) - \underline{K}(w,k_c)$ is decreasing in w for $w \in (\tilde{w}, p]$. Note that $\overline{K}(w,k_c)$ and $\underline{K}(w,k_c)$ are not necessarily continuous and differentiable in $w \in (\tilde{w}, p]$. However, at $w \in (\tilde{w}, p]$ where $\overline{K}(w,k_c)$ is continuous and differentiable,

$$\frac{\partial \overline{K}(w,k_c)}{\partial w} \le \max_{k_c < j \le n} \{\Psi(j)\}, \quad \text{(A8)}$$

since $\overline{K}(w,k_c) = \min_{k_c<j\le n}\{\overline{K}_j(w,k_c)\}$ and $\frac{\partial \overline{K}_j(w,k_c)}{\partial w} = \Psi(j)$ (to see why $\frac{\partial \overline{K}_j(w,k_c)}{\partial w} = \Psi(j)$, note that $\overline{K}_j(w,k_c) = \frac{\Delta(j)}{j-k_c} + (w-c)\Psi(j)$ for $k_c < j \le n$, and $\Delta(j)$ and k_c do not depend on w). Similarly, at $w \in (\tilde{w}, p]$ where $\underline{K}(w,k_c)$ is continuous and differentiable,

$$\frac{\partial \underline{K}(w,k_c)}{\partial w} \ge \min_{k_d \le j < k_c} \{\Psi(j)\}, \quad \text{(A9)}$$

since $\underline{K}(w,k_c) = \max_{0<j<k_c}\{\underline{K}_j(w,k_c)\}$ and $\frac{\partial \underline{K}_j(w,k_c)}{\partial w} = \Psi(j)$. Therefore, at $w \in (\tilde{w}, p]$, if both $\overline{K}(w,k_c)$ and $\underline{K}(w,k_c)$ are continuous and differentiable, then

$$\frac{\partial \underline{K}(w,k_c)}{\partial w} \ge \min_{k_d \le j < k_c}\{\Psi(j)\} \ge \max_{k_c<j\le n}\{\Psi(j)\} \ge \frac{\partial \overline{K}(w,k_c)}{\partial w}, \quad \text{(A10)}$$

where the first inequality follows from (A9), the last inequality follows from (A8), and the second inequality follows from our earlier observation that $\Psi(j)$ is decreasing in j.

Now, take any $(a,b] \subseteq (\tilde{w}, p]$. If both $\overline{K}(w,k_c)$and $\underline{K}(w,k_c)$ are continuous and differentiable in $(a,b]$, then $\overline{K}(w,k_c) - \underline{K}(w,k_c)$ is decreasing over $(a,b]$, since $\frac{\partial \underline{K}(w,k_c)}{\partial w} \ge \frac{\partial \overline{K}(w,k_c)}{\partial w}$ by (A10). If $\underline{K}(w,k_c)$ is discontinuous at b, then the jump in $\underline{K}(w,k_c)$ is upwards, causing $\overline{K}(w,k_c) - \underline{K}(w,k_c)$ to decrease (to see why this is true, note that a discontinuity in $\underline{K}(w,k_c)$ will arise at $w=b$ only if $\arg\max_{k_d\le j<k_c}\{\underline{K}_j(w,k_c)\}$ changes at b. Moreover, k_d is non-increasing in w, which implies that the domain $k_d \le j < k_c$ over which $\underline{K}_j(w,k_c)$ is maximized is not going to get smaller as w increases). Similarly, if $\overline{K}(w,k_c)$ is discontinuous at b, then the jump in $\overline{K}(w,k_c)$ is downwards, causing $\overline{K}(w,k_c) - \underline{K}(w,k_c)$ to decrease. The entire interval $(\tilde{w}, p]$ can be divided into a number of such (a,b) intervals. Thus, $\overline{K}(w,k_c) - \underline{K}(w,k_c)$ is decreasing in w for $w \in (\tilde{w}, p]$. ■

Lemma 2. *Suppose the products are indexed so that $v_1 \ge v_2 \ge \cdots \ge v_n$. For an assortment $S \subset \{1,\ldots,n\}$, define $S'(\delta)$ as the assortment S augmented with a product whose preference is δ. Let $\Delta = \sum_{i=1}^{n-1}(v_i - v_n)$, $M = \lambda(p-w)$ and $Z = (b+h)\phi^*(\lambda(L+1))^\beta$. If $\frac{M}{\beta Z} > \left(1+\frac{\Delta}{v_0}\right)(n+1+\frac{\Delta+v_0}{v_n})^{1-\beta}$, then $\frac{d\Pi_r^d(S'(\delta))}{d\delta} \ge 0$ for any $S \subset \{1,\ldots,n\}$ and δ such that $v_n \le \delta$.*

Proof of lemma 2: Using (1) and (3), we can write $\Pi_r^d(S'(\delta))$ as

$$\Pi_r^d(S'(\delta)) = \Gamma M \frac{\sum_{j\in S} v_j + \delta}{v_0 + \sum_{j\in S} v_j + \delta} - \Gamma Z \frac{\sum_{j\in S} v_j^\beta + \delta^\beta}{(v_0 + \sum_{j\in S} v_j + \delta)^\beta}.$$

For ease of notation, let $V = \sum_{j\in S} v_j$ and $X = \sum_{j\in S} v_j^\beta$. Then, $\Pi_r^d(S'(\delta))$ can be written as

$$\Pi_r^d(S'(\delta)) = \Gamma M\left(1 - \frac{v_0}{v_0+V+\delta}\right) - \Gamma Z \frac{X+\delta^\beta}{(v_0+V+\delta)^\beta}.$$

After some algebra, one can show that

$$\frac{d\Pi_r^d(S'(\delta))}{d\delta} = \Gamma \frac{v_0}{(v_0+V+\delta)^2} \times \beta Z\left(\frac{M}{\beta Z} - \frac{1}{v_0}\left(\frac{v_0+V+\delta}{\delta}\right)^{1-\beta}\left(v_0+V-X\delta^{1-\beta}\right)\right). \quad \text{(A11)}$$

Since $\delta \ge v_n$ and $v_1 \ge v_2 \ge \cdots \ge v_n$, we can make the following two observations: (i) $\frac{v_0+V+\delta}{\delta} \le 1 + \frac{v_0+V}{v_n} = n+1+\frac{v_0+\Delta}{v_n}$, (ii) $v_0 + V - X\delta^{1-\beta} \le v_0 + V - Xv_n^{1-\beta} \le v_0 + \Delta$. Applying these two observations to (A11) yields the lemma. ■

Notes

[1]Not all of these seven products are produced by the same manufacturer; we assume that to be the case for our illus-

trative purposes. The preference values are based on the market shares as represented by the weekly sales quantities. The price of the product is estimated as the average of the seven prices, which are similar to begin with. The production cost estimate is based on the assumption that it costs a manufacturer \$1080 to produce one metric ton of a paper product (http://www.risiinfo.com/blogs/Blog-Tissue-production-whos-the-best.html, last accessed on May 26, 2008). We assume each period is a week long and the lead-time is 4 weeks. The weekly holding cost is assumed to be about 4% of the unit production cost and the backorder cost is chosen so that the optimal stockout probability in a week is slightly $<5\%$. The weekly rate for discounting cash flows is assumed to be 0.005.

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