MEDIEVAL DISTORTIONS: THE PROJECTIONS OF ANCIENT MAPS

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ABSTRACT. Estimates of the map projection employed for an ancient map is a prerequisite for a variety of other studies. The preliminary evaluation presented here has yielded empirical equations for the Hereford map and illustrated the agreement of a Portolan chart with an oblique Mercator projection.

THE study of ancient maps provides one of the fascinating aspects of historical geography. Such maps can be analyzed for many purposes and from several points of view. The following comments refer only to the estimation of the map projection implied by the ancient mappaemundi and portolan charts. Evaluation of the map projection of these old maps is of assistance in the determination of the accuracy of the maps, and may provide insight into their method of construction. Modern theories regarding the ancients' perception of the world also may require consideration of the map projection employed for maps.

The maps in the two classes under investigation do not contain any indication of the terrestrial graticule of latitude and longitude. This has led some students to conclude that the maps are not based on any map projection.[1] This point requires clarification. Certainly the lack of the graticule does not imply the absence of a projection. Even modern maps are occasionally published without this grid.[2] More telling is the high probability the sphericity of the earth was unknown to, or was not considered relevant by, the individuals who constructed the maps. If this is the case the maps would be constructed as though the earth were flat. Inconsistencies between the plotting and the observational information then might arise; these inconsistencies could be attributed to the (unavoidable) errors in one or the other, or both. For a small area the errors and inconsistencies might be quite small and could go unnoticed. Inconsistencies are not necessary or inevitable, however. No set of observational information specifying the location of any terrestrial position by not more than two independent measures will lead to inconsistencies when plotted. This is true whether the earth is considered round or flat. In either of the above events it is correct to say that the map is not based on a map projection *only* in the sense that the cartographer involved was not consciously employing a map projection.[3] But, as one learns from any elementary work on map making, every map requires a map projection. The ancient maps therefore are implicitly referred to some map projection.

The next difficulty, it seems, occurs if it is assumed that this implicit projection is one of the now-known projections. For example, the portolan charts have been compared with charts drawn on Mercator's projection and on the square projection.[4] Suppose that the match is sufficiently poor to conclude that the chart is not drawn on either of these two pro-

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[1] A. E. Nordenskiold refers to these maps as paratropical; *Periplus,* An Essay on the Early History of Charts and Sailing-Directions (Stockholm: Bather translation, 1897).

[2] R. E. Dahlberg, "Maps without Projections," *The Journal of Geography,* vol. 60 (1961), pp. 213–18.

[3] Similar comments apply to an engineering survey of a small area.

[4] H. Wagner, "Das Ratsel der Kompasskarten im Lichte der Gesamtentwickelung der Seekarten," *Verhandlungen,* XI Deutsches Geographentages, Bremen, 1895, pp. 65–87; E. Steger, "Untersuchung uber italienische Seekarten des Mittelalters auf Grund der kartometrischen Methode" Dissertation, Gottingen, 1896; M. Fiorini, *Le projezioni delle carte geografiche,* Bologna, 1881; A. Breusing, "Zur Geschichte der Kartographie," Zeitschrift fur Wissenschaftliche Geographie, II (1881), p. 168 ff; M. A. Clos-Arceduc, "L'Enigme des Portulans: Etude sur la Projection et le mode de construction des cartes a rumbs du XIV^{e} et du XV^{e} Siecle," *Bulletin,* Comite des Travaux Historiques et Scientifiques, Section de Geographie, LXIX (1956), pp. 215–31. The square projection is also known by the names plate carrée, simple cylindrical, and cylindrical equal-spaced projection.

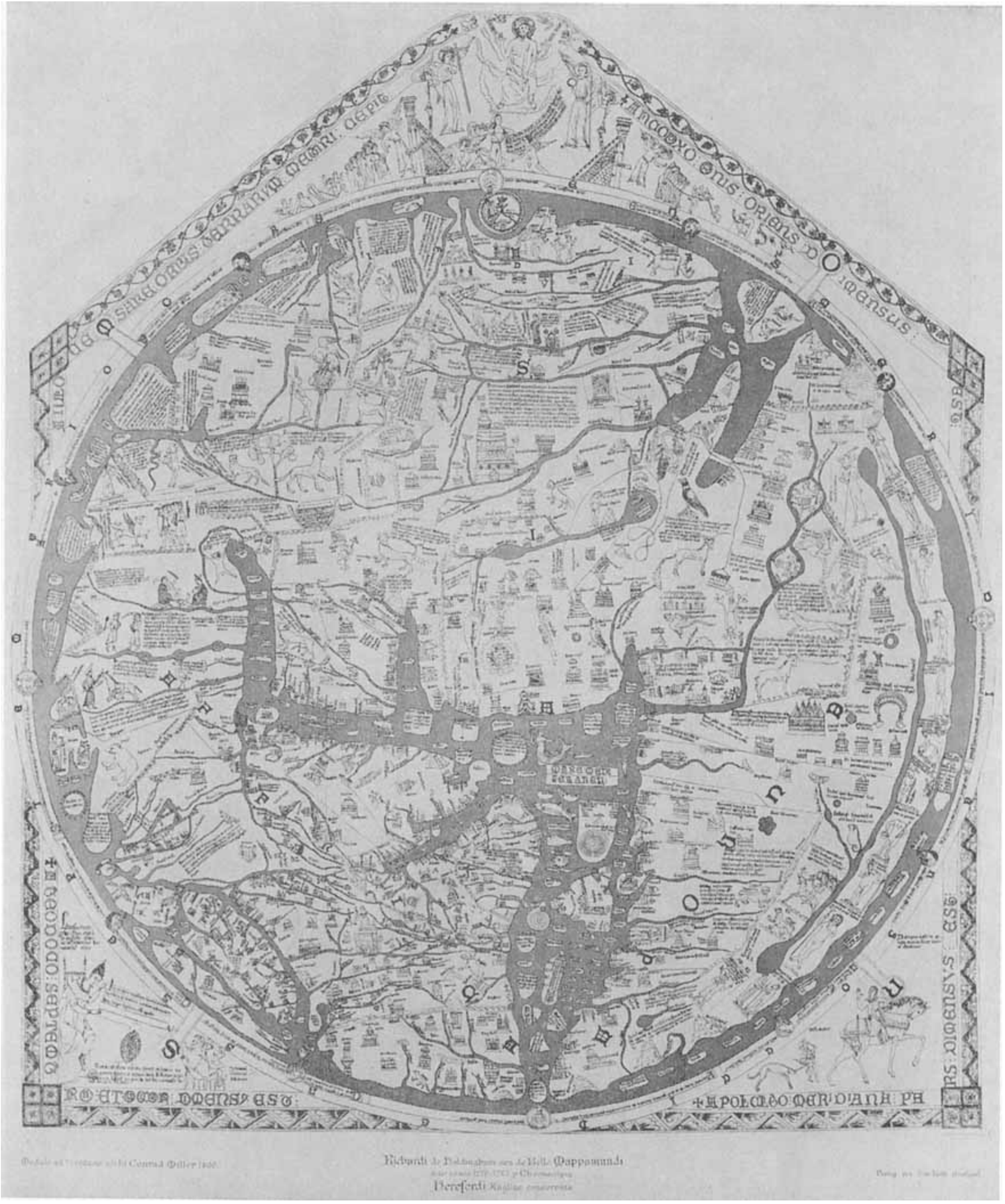

FIG. 1. Konrad Miller's edition of the Hereford Map.

jections. This does not prove that the chart is not based on a map projection; such a conclusion can in fact never be drawn if one accepts the notion of an implicit map projection. The search, then, must continue for a map projection, which may be any of the several hundred now known, or may be one which is completely unknown today. The problem can be attacked from several directions. Most helpful is an examination of the method of construction of the maps, if this is known. Typically this turns out to be inconclusive, and is in fact one of the questions which many have attempted to answer. But there are some hints which allow the range of possible projections to be narrowed down to a relatively limited few. A second obvious approach is to attempt to sketch the lines of latitude and longitude on the map, as estimated by identification of locations shown thereon. Examination of the graticule, its curvature and so on, should provide hints as to a reasonable family of projections.

A crucial point has now been reached. On the basis of some study it is postulated that "this" projection forms the basis for the map. How is this hypothesis to be tested? The test generally consists of superimposing a map drawn on the postulated projection over the original map, with a scale adjustment and shifting until the best average coincidence is obtained. Since one knows that the agreement of the two maps will not be perfect, the question is now one of deciding how much agreement is necessary before the hypothesis is to be accepted. The procedure outlined below does not answer this question, but it does allow one to say, with relative precision, how great the agreement is, and thus permits one to rank, from greatest to least agreement, all projections for which one cares to carry through the necessary operations. The method requires extensive observations and computations but is perfectly general and may be applied whenever it is desired to ascertain the agreement of a questionable map with a particular map projection. The necessary calculations may seem formidable but entail less than five minutes on a modern high-speed digital computer.

To begin it is necessary to identify a large number of points on the map. By identify is meant to record the modern latitudes and longitudes of these points. The next step is to record the map coordinates of these same points. Any system of coordinates will serve but rectangular (x, y) coordinates are the most convenient. It does not matter in the least what units are used for the coordinates, and it is not necessary to determine the scale of the chart or any distances thereon.[5] Nor does the orientation of the grid system matter. The postulated map projection will be defined by a mathematical relation of the form:

$$X = f(\phi, \lambda)$$
$$Y = g(\phi, \lambda),$$

where ϕ is the latitude and λ is the longitude, and X and Y are the corresponding map projection coordinates. Using this relation calculate the map projection coordinates of all of the points identified by latitude and longitude. This is where the points should be if the projection were to give a perfect fit. These calculated coordinates (X, Y) are now to be compared to the observed map coordinates (x, y). Since the coordinates employed for the recording of the observations were quite arbitrary, a different result would be obtained for each possible set of recording coordinates. It is therefore necessary to apply the mathematical equivalent of rotation and change of scale, as might be obtained by use of an optical reducing–enlarging instrument, to give the best possible overall average fit. This is given by a least squares Euclidean transformation, which brings the numbers given the recorded observations into the same units as the calculated map projection coordinates (see Appendix). The comparison of the map projection with the map is now made by calculating the difference between the observed and calculated locations for corresponding points. The correlation coefficient between the two sets of coordinates gives the amount of agreement, and areas of greatest disagreement may provide hints as to a more suitable map projection. With the same observational information the entire procedure can be repeated a second time for a new map projection, and so on.

As one already has recorded, for the foregoing operations, the latitudes and longitudes of a large number of places along with the map locations (in some arbitrary coordinate system) of these same identified points, one can continue by obtaining an empirical estimate of the equations defining the map projection. The problem can be phrased thus: find the equations which, when entered with any specific latitude and longitude value, result in the observed x and y coordinate values. The mathematical theory indicates that this can always be achieved with a sufficiently complicated equation. In practice it is

[5] Readers of the earlier treatises (especially Wagner, *op. cit.* footnote 4) will recognize this as a distinct advantage.

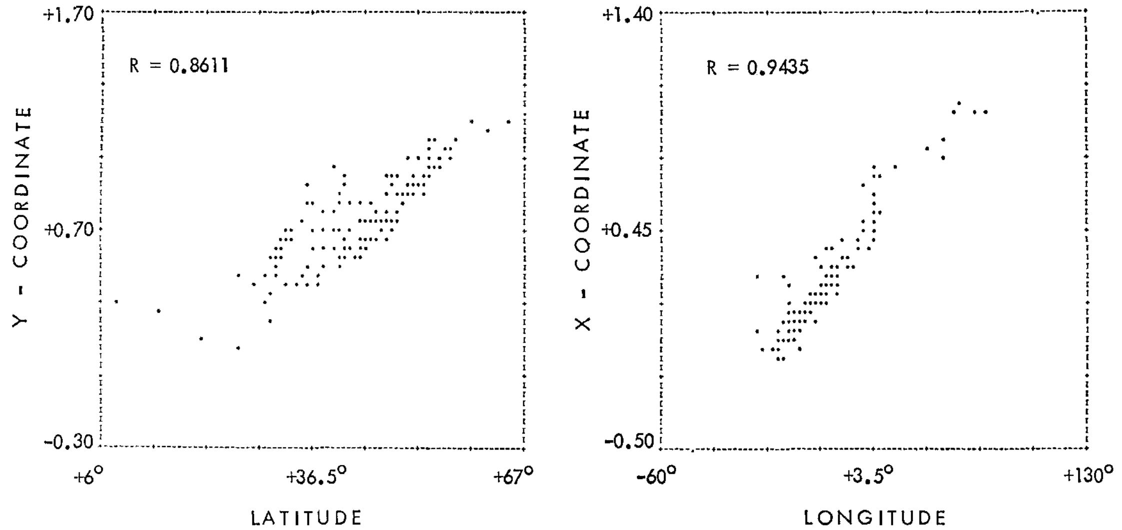

FIG. 2. Scatter diagrams illustrating the correlation between latitude and longitude and rectangular coordinates (after rotation and scaling) on the Hereford Map.

difficult to obtain a perfect fit and one is generally content with an equation which reproduces the observations with a high degree of reliability. This is reasonable since it is not desired to reproduce minor errors, such as might be caused by the shrinkage which has occurred on a 500-year-old map. The mathematical fitting procedure generally employed is the method of least squares polynomial or Fourier series curve fitting.[6] The equations obtained in this manner enable one to calculate, and draw, the latitude and longitude graticule at any desired interval. Equally importantly, they allow calculation of Tissot's measures of map projection distortion.[7] Formulation of the problem in terms of the distortion is independent of any hypothesis regarding the specific map projection, but can be employed to infer whether or not the projection has specific properties. This in turn may be of assistance in determining the projection. A word of caution is necessary here since the necessary differentiation of the empirical equations may be subject to large errors.[8] The same comment holds true if a graphic determination of Tissot's measures is applied to visually sketched lines of latitude and longitude.[9]

THE HEREFORD MAP

The Hereford map[10] is one of the most famous of the surviving medieval mappaemundi (Fig. 1). Certainly it is one of the largest (53 × 65 inches). It is a product of the later Middle Ages, *circa* 1283, and is still in the cathedral at Hereford, England. In many respects the map represents a culmination of 1,000 years of mapping efforts, having precedents from Roman times. The map has been the subject of at least one book, and several monographs and articles.[11] At least three large

[6] For a discussion of curve fitting see any elementary work on numerical analysis or an intermediate work on statistical methods. If there are some locations which can be identified with a greater degree of reliability than others, a weighting procedure may be employed.

[7] M. A. Tissot, *Memoire sur la Representation des Surfaces* (Paris: Gautier-Villars, 1881).

[8] Cf., B. Arden, *An Introduction to Digital Computing* (New York: Addison-Wesley, 1963).

[9] G. A. Ginzburg, "A Practical Method of Determining Distortion on Maps," *Geodezist* (Moscow), 10 (1935), pp. 49–57.

[10] The materials in this section are summarized from: S. Jones, "The Hereford Map Projection," M.A. thesis, Department of Geography, University of Michigan, Ann Arbor, 1964, 39 pp., and are used with the kind permission of Miss Jones.

[11] W. L. Bevan and H. W. Phillott, Medieval Geography; *An Essay in Illustration of the Hereford Mappa Mundi* (London: 1873); K. Miller, *Die Herefordkarte* (Stuttgart, 1903); G. R. Crone, *The World Map by Richard of Haldingham* (London: Royal Geographical Society, 1954) (with an extensive set of references).

reproductions of the map have been published. A detailed description of the map, therefore, is not necessary. The form of the map follows the typical circular T-in-O style, with Jerusalem in the center and Asia (paradise) at the top. In many respects the map is more a representation of religious reality than geographical reality. On the other hand, the map preserves most of the topological properties[12] of a map projection; if this were not the case the drawing would not be recognizable as a map.

The Hereford map is believed to have been prepared by copying some other ancient map, with supplementary information obtained from itineraries. This does not provide much assistance in an initial guess at a map projection. The many authors who have discussed the Hereford map have ignored, or rejected, the question of a map projection and no assistance can be gained here, though these authors are extremely valuable in providing identification of names. The form of the map, however, suggests an azimuthal projection, perhaps with a larger scale at the center of the map. The orthographic projection is one (of many) which has these properties. The empirical observations suggest that this is not a poor guess.

Konrad Miller's 3/7 size edition of the Hereford map was employed to identify 155 locations. This is a tedious operation and subject to error. Each identification consisted of map coordinates, and modern latitude and longitude.[13] The dispersion of the observations is rather uneven since it proved impossible to identify anything in some parts of the map, particularly the margins. Misidentifications may have occurred, in which event the consequent analysis will be slightly distorted.

Direct correlations between the latitude and the y coordinate, and between the longitude and the x coordinate are high (Fig. 2). This is to be expected since the Hereford map preserves most of the topological properties of any map projection. The results of further computations indicate that the square projection provides a seventy-three percent match to the Hereford map, and that an oblique orthographic projection centered at Jerusalem provides an eighty-four percent match. Using a polynomial approximation the locus of lines of latitude and longitude for the Hereford map can be reproduced with a fidelity of ninety-five percent. A computer print of these calculated lines in fact agrees quite well with a manually interpolated graticule. The details of these results are given in the Appendix.

Comparison of the Hereford map with only two known map projections cannot be said to provide an exhaustive study. Nor can it be said that the map is drawn on the orthographic projection, though this provides a better fit than does the square projection. Examination of the interpolated lines of latitude and longitude does, however, reinforce the hypothesis of European antecedents for the map, on the grounds that the larger scale will be in the vicinity of areas with which the cartographer is most familiar. The only apparent exception is the Jerusalem region.

PORTOLAN CHARTS

One of the most interesting classes of maps in the venerable history of cartography are the early sailing charts depicting the vicinity of the Mediterranean Sea. The oldest existent map is estimated to have been drawn in the latter portion of the thirteenth century. The fame of these representations rests in part on their accuracy relative to other European maps of the same period. When contrasted with the contemporaneous T-in-O maps, for example, the charts appear outstandingly more correct. The earliest of these sailing charts do not contain any indication of the terrestrial grid but carry an extensive set of symmetrical, criss-crossing lines. A voluminous though somewhat controversial, literature is available concerning the antecedents, method of compilation, construction, and employment of these charts.[14]

Several authors have come to the conclusion that the portolan charts are not based on a

[12] R. H. Bing, "Elementary Point Set Topology" *American Mathematical Monthly,* 67, 7, part II (Aug./Sept., 1960).

[13] See Table 2.

[14] K. Kretschmer, *Die italienischen Portolane des Mittelalters,* Heft 13 of the Veröffentlichungen des Institutes fur Meereskunde und des Geographischen Institutes an der Universitat Berlin, 1909, 687 pp; A. Cortesao, *Cartografia e Cartografos portugueses dos seculos XV e XVI* (Lisboa: 1935); Y. Kamal, *Hallucinations scientifiques* (Leiden: E. J. Brill, 1937), 95 pp; for additional references see: W. W. Ristow, and C. E. LeGear, *A Guide to Historical Cartography,* 2nd ed. (Washington: Library of Congress, 1960).

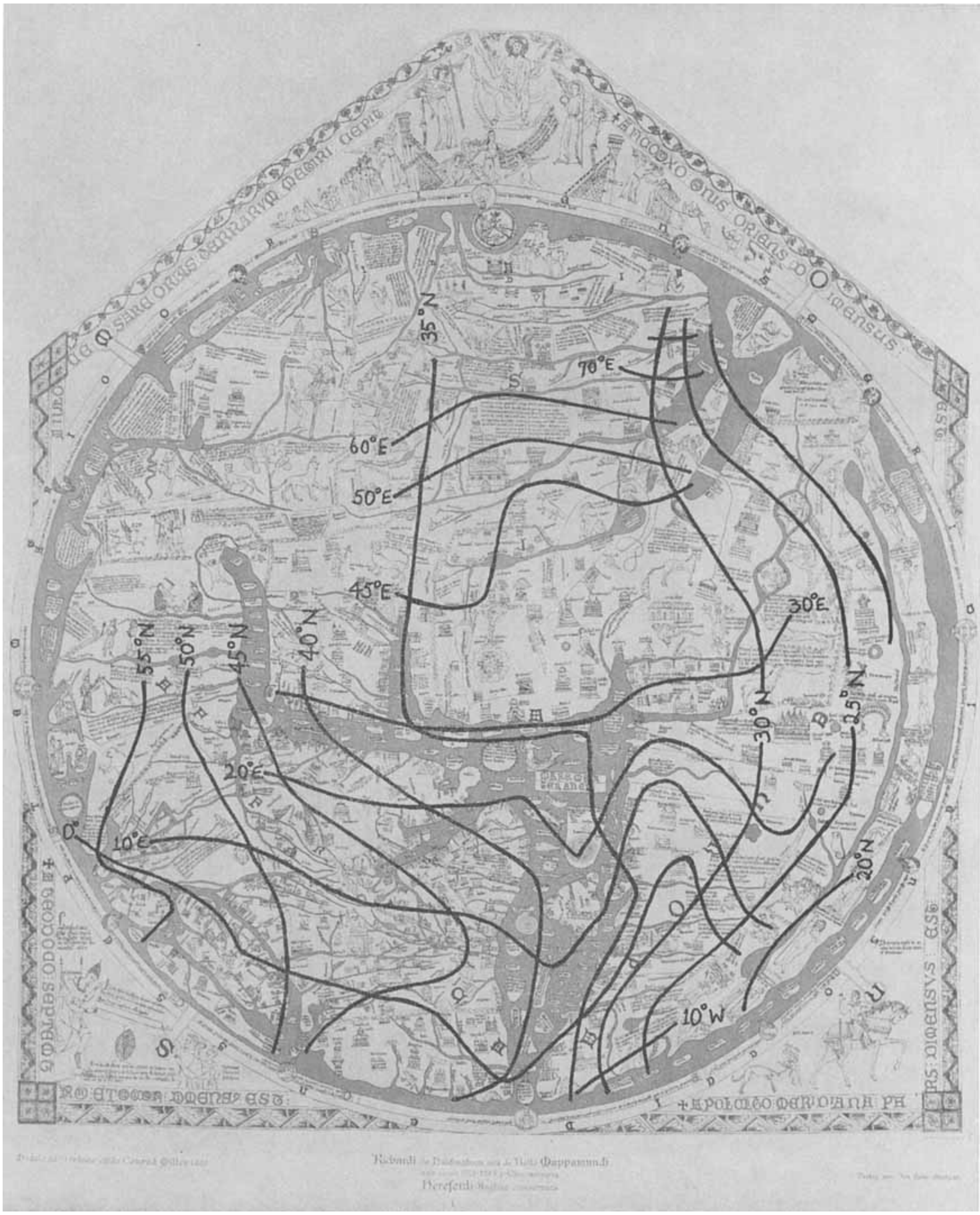

FIG. 3. Sketch of the latitude and longitude graticule as interpolated for the Hereford Map.

map projection.[15] This view has already been rejected here on *a priori* grounds, in accordance with the modern interpretation of map projections.[16] In contradistinction to the T-in-O maps, where the suggestion is usually rejected

[15] Among these are Nordenskiold, Wagner, Bagrow, and M. Eckert.

[16] Cf. Clos-Arceduc, *op. cit.*, footnote 4.

out of hand, the very great accuracy of the portolan charts has prompted questions concerning the map projection of these charts. The method of construction of the portolan charts, though not definitely known, does provide some hints. Perhaps the compass was involved and the observational information available for compilation consisted of loxodromic directions. This suggests Mercator's projection. The magnetic meridians[17] are not coincident with the geographical meridians, however, so that a magnetic error should exist on the maps. A difference between "rose north" and true north can in fact be observed on the charts. Breusing then reasoned that the compass, if it contributed to the development of these charts, would yield readings resulting in a projection with curved parallels. This he took to require a conic projection. Fiorini came to the conclusion that the projection should be an oblique azimuthal equidistant, since both directional and distance information seem to have been available, and since the central rose provides a convenient point of departure for the plotting of the map. Another projection which has been proposed is the square projection. This choice is apparently derived from the subsequent use of this projection for sailing charts.

It should be possible to perform the operations previously outlined and applied in the case of the Hereford map to all of the foregoing projections relative to a portolan chart. The necessary identification is difficult since the maps themselves are rare (even facsimilies are rare), much of the script is difficult to read and translate, and determination of the modern equivalent locations is often impossible, or at best tedious. Unfortunately some previous scholars published only their conclusions, not the detailed observations employed in the analysis. This is especially true of Wagner's otherwise excellent study.[18] Wagner sketched lines of latitude and longitude on detailed tracings of several portolan charts and demonstrated that the length of mile employed differs along the west coast of Europe from that employed in the interior of the Mediterranean Sea.[19] He also illustrated an abrupt jog in the path of the parallels through Greece on the map and attributed this to a method of construction in which the map is made by piecing together separate, local charts. He concluded that the portolan charts are not based on a map projection, thus rejecting the notion of an implicit map projection. Wagner presented a careful analysis of all of these points. Steger proceeded in a less detailed but similar manner and concluded, on the basis of map interpolations, that the meridians and parallels are straight lines when sketched in on the charts. He thus refuted Breusing's suggestion. Steger's demonstration, however, can be disputed on two grounds. His study did not include the Black Sea, which is where the curvature might be most noticeable, and his meridians and parallels are virtually all determined by only two points. A more recent study of the Portolan Charts by Clos-Arceduc compared them to a Mercator chart at approximately the same scale. His reasoning was that Mercator's projection provides a better fit than does the square projection. He further illustrated that the eastern portion of the charts is too far north, only, it must be noted in relation to Mercator's projection.

Computation by the method employed for the Hereford map would allow estimation of the degree of association between the charts and each of the projections. Calculation of Tissot's measures of distortion from an empirical equation should also be of assistance. Thus, if Fiorini's postulate of an oblique azimuthal equidistant projection is correct, one should find that the one value of the linear distortion in some direction is always unity at every point on the map. Similarly if the charts are loxodromic they should be conformal, or very nearly so. A result demonstrating very minor and randomly distributed angular error would provide additional evidence for the use of the compass in the construction of the charts. This procedure, via Tissot's theorem, has the distinct advantage that it is independent of any particular *a priori* hypothesis regarding the nature of the map projection.

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[17] Cf. A. N. Strahler, *The Earth Sciences* (New York: Harper & Row, 1963), Figure 9.15, p. 152.

[18] Wagner, *op. cit.*, footnote 4; and also see his "The Origin of the Mediaeval Italian Nautical Charts" (London: Report of the Sixth International Geographical Congress, 1895).

[19] For a plausible explanation of this phenomena which differs from that given by Wagner, see Clos-Arceduc, *op. cit.*, footnote 4, p. 226.

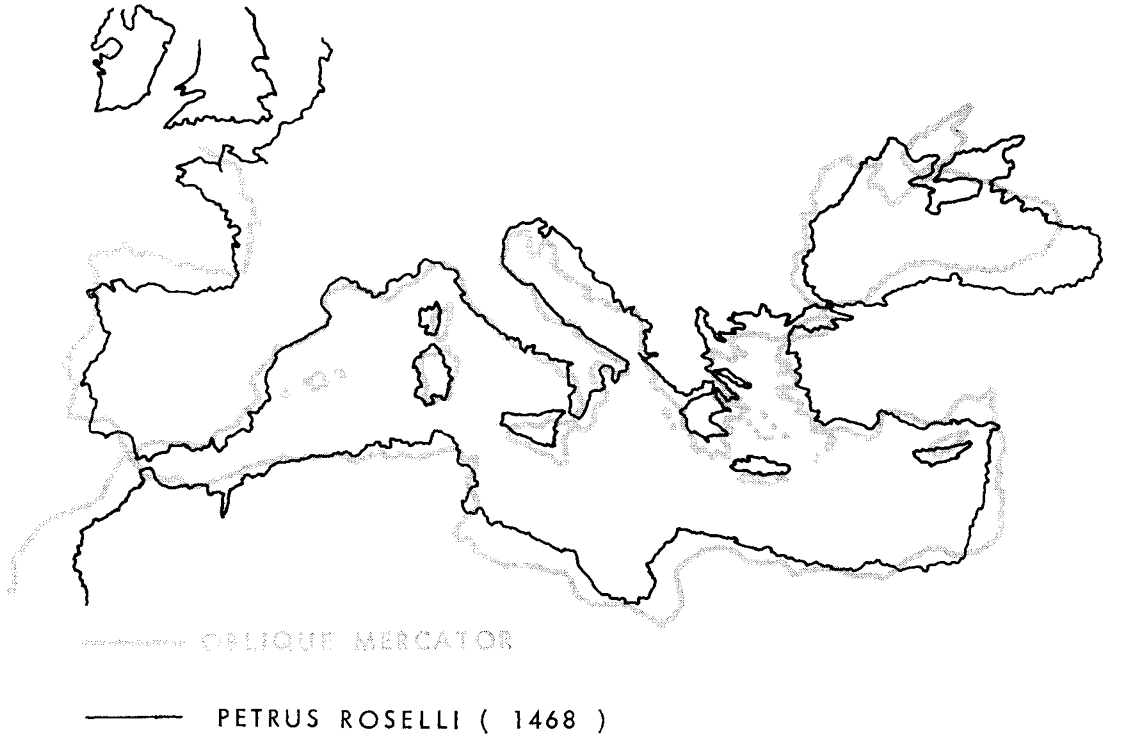

FIG. 4. Comparison of the outline from the Petrus Roselli portolan chart of 1468 with an outline on an oblique Mercator projection.

Two additional projections can be proposed for the portolan charts. The first of these is an oblique Mercator projection. The difference between the rose north and true north can be taken as a guide to the obliquity. One such projection[20] is illustrated and compared with an outline from an early portolan chart in the figure. The discrepancy at the eastern extremity of the map is less than for the previously proposed projections. The meridians converge slightly as required by Breusing. An oblique conical or square projection would yield similar results, however. Comparison of several small tracings of photographs of early portolan charts further indicates that there exists a considerable variation between individual portolan charts. Suppose one were to take all the known portolan charts which do not contain any indication of latitude and longitude and compare them with some standard chart, perhaps the *Carte Pisane.* In each case the correlation will be somewhat less than perfect. A postulated map projection should not be expected to perform better. Perhaps an appropriate strategy would be to assume that the map projection of the portolan charts is determined if the variance between the postulated map projection is less than or equal to the variance between individual portolan charts.

A final postulated projection might be referred to as an "oblique magnetic Mercator projection." This can be conceived of as a Mercator projection based on magnetic meridians and parallels.[21] On such a map the geographical grid would appear distorted in the vicinity of local magnetic anomalies. This is a very appealing hypothesis, but it is apparently not possible to determine the locus of

[20] The oblique Mercator projection has been chosen so that the great circle passing through 37.77° N, 1° W and 37° N, 30° E becomes the projection equator. The axis of the projection is the great circle arc connecting the oblique pole at 51.55° N, 168.36° W with the center of map at 38.41° N, 14.58° E. The angle between grid north and the geographical meridian is 1°50′ E at the center of the map. An earlier attempt with an 8.25° obliquity (as suggested by Clos-Arceduc) provided a poorer fit.

[21] This postulate can also be found in Clos-Arceduc, *op. cit.,* footnote 4, p. 222.

TABLE 1. – POLYNOMIAL APPROXIMATION TO THE HEREFORD MAP. BASED ON 155 OBSERVATIONS.

Standard errors of the estimate, in radians : $S_x = 0.05176$; $S_y = 0.05785$

$$X = 0.2174 + 0.4752\lambda - 0.3383\,\varnothing^5\lambda + 0.01529\,\lambda^6 + 0.4530\,\varnothing^6 - 0.5129\,\varnothing^2 - 0.5587\,\varnothing\lambda^5 - 1.258\,\varnothing^4\lambda^2 + 1.657\,\varnothing^2\lambda^4 + 0.6047\,\varnothing\lambda - 1.179\,\varnothing^3\lambda^3 + 0.09247\,\lambda^2 + 0.01191\,\varnothing$$

$$Y = 0.4399 + 2.956\,\varnothing^2 - 2.023\,\varnothing\lambda \text{ [illegible] } \varnothing^4\lambda^2 + 0.0\text{[illegible]}\,\varnothing^5\lambda - 1.712\,\varnothing + 2.\text{[illegible]}\,\varnothing^2\lambda^4 + 0.0591\,\varnothing\lambda^2 + 2.305\,\varnothing^2\lambda + 0.\text{[illegible]}\,\lambda^6 \text{ [illegible] } \varnothing\lambda^4 - 1.186\,\lambda^2 \text{ [illegible] } 0.5789\,\varnothing^4 + 0.4399\,\varnothing^3\lambda^3$$

TABLE 2

OBSERVED AND CALCULATED VALUES FOR THE HEREFORD MAP

NUMBER	MAP NAME	MODERN NAME	GEOGRAPHICAL COORDINATES		MAP COORDINATES[1]		SQUARE PROJECTION COORDINATES[2]		ORTHOGRAPHIC PROJECTION COORDINATES[3]		POLYNOMIAL COORDINATES[4]		VALUES FROM TISSOT'S INDICATRIX[5]			
			LAT.	LONG.	X	Y	X'	Y'	X''	Y''	X'''	Y'''	A	B	S	2W
1	ORCADES	ORKNEYS	59 00	-3 00	.21142	1.18544	-.05236	1.02974	.08039	1.02353	.18555	1.1451	3.15594	.56482	1.78255	88.2768
2	YSLAND	ICELAND	65 00	-18 00	.24464	1.18423	-.31416	1.13446	.03088	1.14561	.25106	1.1637	4.70039	1.23602	5.80979	71.4054
3	FAREIE I	FAROES	62 00	-6 30	.25216	1.16867	-.11345	1.08210	.07493	1.07755	.26736	1.2077	3.90939	.13095	.51195	138.5137
4	FL WISARA	WESER R.	53 30	8 25	.14869	1.11671	.14690	.93375	.15938	.91888	.17807	1.0364	2.64599	.36401	.96317	98.5998
5	FL ALBANA	ELBE R.	53 50	9 20	.16565	1.11339	.16290	.93957	.16966	.92428	.19032	1.0597	2.68643	.26637	.71559	110.0856
6	FL FISTULA	VISTULA R.	54 20	19 00	.18261	1.11008	.33161	.94830	.27108	.93534	.23625	1.2272	2.71272	.16376	.44423	124.7843
7	ABERTENE	ABERDEEN	57 09	-2 09	.11828	1.09624	-.03752	.99746	.07607	.99131	.13570	1.0919	2.80404	.78659	2.20563	68.3699
8	FL EMISA	EMS R.	53 20	7 15	.12581	1.08068	.12654	.93084	.14690	.91638	.16504	1.0177	2.60278	.46007	1.19747	88.7866
9	BREMA	BREMEN	53 05	8 49	.15832	1.08489	.15388	.92648	.16230	.91132	.17611	1.0253	2.63129	.38644	1.01684	96.1273
10	EDENBURGH	EDINBURGH	55 55	-3 10	.08316	1.06965	-.05527	.97593	.05985	.97195	.08664	1.0522	2.55505	1.03124	2.63487	50.2886
11	S. JOH	PERTH	56 25	-3 25	.10483	1.07246	-.05963	.98466	.06070	.98080	.09599	1.0672	2.63401	.98686	2.59940	54.1177
12	FARDIN	VERDEN	52 57	9 15	.16444	1.08018	.16144	.92415	.16637	.90884	.17823	1.0253	2.63553	.37568	.99010	97.2643
13	S. ANDR.	ST ANDREWS	56 20	-2 45	.11306	1.05148	-.04800	.98320	.06598	.97828	.10378	1.0654	2.64126	.94514	2.49637	56.4493
14	OLDELINBURGH	OLDENBURG	53 07	8 12	.17407	1.04836	.14312	.92706	.15596	.91219	.17103	1.0196	2.61600	.42107	1.10152	92.5580
15	BENCUR	BANGOR	53 12	-4 05	.00748	1.01577	-.07127	.92852	.03567	.92678	.01547	.9706	2.09751	1.38856	2.91252	23.4678
16	DIVELIN	DUBLIN	53 20	-6 15	.03528	1.01386	-.10908	.93084	.01662	.93298	-.01732	.9839	1.99932	1.54840	3.09575	14.6044
17	CARLUA	CARLISLE	54 55	-3 00	.08475	1.01476	-.05236	.95848	.05545	.95462	.06552	1.0215	2.39898	1.13412	2.72073	41.9550
18	ALBSTAD	HALBERSTADT	51 54	11 04	.19314	1.02879	.19315	.90583	.18315	.88996	.18269	1.0083	2.63678	.39710	1.04706	95.1607
19	CASTELLO NOVO	NEWCASTLE ON TYNE	54 55	-1 37	.10713	1.01215	-.02822	.95848	.06799	.95253	.08612	1.0223	2.45709	1.01807	2.50149	48.9237
20	BRAGA	PRAGUE	50 05	14 26	.25957	1.02638	.25191	.87412	.21753	.85774	.19871	.9876	2.68058	.46822	1.25510	89.2728
21	SNAWEDON	MT. SNOWDON	53 05	-3 55	.04351	.99289	-.06836	.92648	.03643	.92443	.01616	.9666	2.09120	1.38420	2.89463	23.4752
22	DUREM	DURHAM	54 45	-1 40	.07602	.99710	-.02909	.95557	.06655	.94968	.08181	1.0169	2.43089	1.03999	2.52810	47.2480
23	ARHMATA	ARMAGH	54 21	-6 42	-.00456	.98115	-.11694	.94859	.01915	.95132	-.00575	1.0146	2.14107	1.47444	3.15689	21.2500
24	CESTRIA	CHESTER	53 12	-2 55	.06187	.97874	-.05091	.92852	.04642	.92483	.03368	.9679	2.16016	1.29081	2.78835	29.1818
25	M. CLEOE	CLEE HILL	52 24	-2 38	.02674	.95215	-.04596	.91455	.04486	.91045	.02554	.9422	2.07979	1.32577	2.75731	25.5836
26	SNOTINGHAM	NOTTINGHAM	52 55	-1 10	.06468	.95706	-.02036	.92357	.06161	.91732	.05535	.9577	2.21711	1.16425	2.58128	36.2838
27	CELDARA	KILDARE	53 05	-6 55	-.04300	.93761	-.12072	.92648	.00906	.93006	-.03259	.9810	1.92704	1.61972	3.12127	9.9416
28	SCOBESBIRI	SHREWSBURY	52 42	-2 50	.01661	.94533	-.04945	.91979	.04459	.91598	.02695	.9521	2.10323	1.32199	2.78045	26.3686
29	BONONIA	BOULOGNE	50 45	1 40	.10401	.95113	.02909	.88575	.07989	.87542	.06781	.8886	2.15638	1.07569	2.31959	39.0680
30	WIRCESTR.	WORCESTER	52 10	-2 10	.00717	.93308	-.03782	.91048	.04813	.90571	.02934	.9339	2.08031	1.30143	2.70738	26.6319
31	LINCOLN	LINCOLN	53 15	-0 33	.07762	.94221	.00960	.92939	.07979	.92078	.08506	.9714	2.33874	.99137	2.31856	47.7324
32	FATNA	VIENNA	48 14	16 20	.26729	.96677	.28507	.84183	.23817	.82526	.21181	.9471	2.69619	.59818	1.61281	79.1141
33	PAZAEI	BUDAPESTH	47 30	19 05	.27813	.96817	.33307	.82903	.27102	.81400	.23468	.9677	2.77009	.63837	1.76835	77.4257
34	NORHANTON	NORTHAMPTON	52 15	-0 55	.04581	.93258	.01600	.91193	.07883	.90280	.07513	.9382	2.25149	1.03950	2.34043	43.2183
35	OXEFORD	OXFORD	51 45	-1 15	.03637	.92034	-.02182	.90321	.05494	.89695	.03754	.9195	2.08858	1.24994	2.61060	29.0971
36	ELY	ELY	52 25	0 20	.06347	.92384	.00582	.91484	.07370	.90657	.06935	.9426	2.23900	1.07540	2.40784	41.1059
37	CIMERACUM	CAMBRAI	50 10	3 11	.09056	.92735	.05556	.87557	.09345	.86347	.08281	.8733	2.19077	.99948	2.18963	43.8528
38	AGRIPPINA COLONIA	COLOGNE	50 57	6 57	.15017	.93507	.12130	.88925	.13635	.87441	.13509	.9263	2.42937	.70389	1.71000	66.8301
39	BATHE	BATH	51 25	-2 25	-.00016	.90459	-.04218	.89739	.04170	.89296	.01584	.9105	1.99227	1.36249	2.71445	21.6402
40	COLCESTRIA	COLCHESTER	51 52	0 52	.05403	.91160	.01513	.90524	.07661	.89616	.06940	.9250	2.21205	1.07015	2.36723	40.7187
41	REMESBURGH	REGENSBURG	49 02	12 04	.21660	.93265	.21060	.85579	.18900	.83855	.17579	.9121	2.55381	.62494	1.59597	74.7172
42	LONDONIA	LONDON	51 30	-0 07	.03306	.90338	.00204	.89884	.06733	.89066	.05434	.9114	2.13877	1.15328	2.46661	34.8377
43	LEODIUM	LIEGE	50 39	5 35	.11976	.91460	.09745	.88401	.12069	.86998	.11691	.9042	2.34590	.81070	1.90183	58.2011
44	BOSFORUS TRACIUS	ST. OF CONSTANTINOPLE	41 10	29 00	.38058	.94287	.50615	.71849	.41029	.71454	.37541	.9068	2.89968	1.01965	2.95665	57.3292
45	FLUENTIA	COBLENTZ	50 22	7 35	.14284	.90657	.13235	.87906	.14151	.86374	.13734	.9104	2.42121	.71350	1.72754	66.0183
46	FL SCHENE	SHANNON R.	52 30	-9 45	-.07794	.86696	-.17017	.91630	-.01989	.92624	-.08941	.9902	1.89541	1.65346	3.13399	7.8184
47	EXCESTRIA	EXETER	50 45	-3 30	-.02916	.87328	-.06109	.88575	.02763	.88297	-.00846	.8931	1.88103	1.46655	2.75861	14.2247
48	CANTURIA	CANTERBURY	51 15	1 00	.01419	.87889	.01745	.89448	.07517	.88509	.06389	.9043*	2.16316	1.09817	2.37552	38.1194
49	SALZEBURGH	SALZBURG	47 48	13 01	.23637	.90766	.22718	.83427	.19842	.81635	.18274	.8782	2.54457	.68956	1.75463	69.9998
50	CONSTANTINOPOLIS	CONSTANTINOPLE	41 02	29 00	.35559	.92310	.50615	.71617	.41065	.71199	.37703	.9011	2.89674	1.02196	2.96035	57.1645
51	WINTONA	WINCHESTER	51 05	-1 20	.01017	.86735	-.02327	.89157	.05090	.88529	.02836	.8978	2.02310	1.29039	2.61058	25.5508
52	MAGONTIA	MAINTZ	50 00	8 13	.14023	.88419	.14341	.87266	.14748	.85676	.14169	.9029	2.42870	.70794	1.71938	66.5419
53	METIS	METZ	49 09	6 10	.15649	.88630	.10763	.85783	.12251	.84266	.11323	.8539	2.27632	.87307	1.98738	52.9187
54	AUGUSTA	AUGSBURGH	48 23	10 53	.18428	.88439	.18995	.84445	.17411	.82684	.16251	.8720	2.47017	.71143	1.75734	67.1166
55	SAMARCAN	SAMARKAND	39 40	67 00	.86167	.97210	1.16937	.69231	.91028	.86358	.79679	1.0335	3.53398	.02754	.09732	159.8208
56	NAMETIS	NANTES	47 13	-1 33	.01158	.85652	-.02705	.82409	.03077	.81692	-.00237	.7738	1.84504	1.29581	2.39081	20.1412
57	WORMACIA	WORMS	49 37	8 21	.13692	.86724	.14573	.86597	.14806	.84988	.14084	.8900	2.41299	.72954	1.76038	64.7825
58	ROTOMAGUM	ROUEN	49 25	1 05	.05633	.85129	.01891	.86248	.06870	.85249	.04823	.8421	2.02798	1.17489	2.38266	30.8946
59	NOVITUM	NOYON	49 37	3 00	.08885	.85550	.05236	.86597	.08950	.85381	.07607	.8533	2.14200	1.03974	2.22712	40.5388
60	MONS OLIMPUS	MT OLYMPUS	40 03	22 20	.28393	.88076	.38979	.69900	.32001	.68146	.31581	.7329	2.63113	.97209	2.55770	54.8301
61	BASELLA	BASEL	47 33	7 35	.15598	.84766	.13235	.82990	.13407	.81294	.12371	.8069	2.25650	.88911	2.00628	51.5321
62	BRAGARIA	BRAGA	41 32	-8 26	-.05938	.80876	-.14719	.72489	-.07017	.72934	-.10585	.6792	2.19442	.67826	1.48839	63.7121
63	BAYONE	BAYONNE	43 30	-1 28	-.02686	.81297	-.02560	.75922	.01667	.74958	-.00433	.6583	1.84644	1.09348	2.01905	29.6795
64	M LAUDUN	LAON	49 35	3 35	.09777	.82911	.06254	.86539	.09549	.85253	.08353	.8544	2.17033	1.00353	2.17800	43.1387
65	BURDEGALA	BORDEAUX	44 40	-0 30	.00093	.81106	.00873	.77758	.04360	.76764	.02409	.6865	1.83122	1.17283	2.14771	25.3201
66	THESSALONICA	SALONIKA	40 35	22 55	.30580	.83951	.39997	.70831	.32735	.69211	.31766	.7647	2.68251	.96597	2.59123	56.1307
67	REMIS	RHEIMS	49 17	4 00	.07890	.80462	.06981	.86016	.09899	.84675	.08692	.8457	2.17198	.99214	2.15491	43.7864
68	ANDEGAVIS	ANGERS	47 30	-0 30	.04167	.79429	.00873	.82903	.05424	.81884	.02921	.7781	1.90123	1.23504	2.34810	24.5278
69	CABILLA	CHALON SUR SAONE	46 52	4 55	.12506	.78856	.08581	.81798	.10158	.80250	.08849	.7651	2.07525	1.03582	2.14958	39.0366
70	PARISIUS	PARIS	48 52	2 20	.06616	.77542	.04072	.85288	.07944	.84111	.06202	.8253	2.05744	1.11263	2.28917	34.6801
71	ANTIOCHIA	ANTIOCH	36 05	36 15	.53762	.83647	.63268	.62977	.53177	.64299	.52407	.8030	2.97358	1.09024	3.24192	55.2188
72	M CAUCASUS	HINDU KUSH	35 45	70 30	.95488	.89050	1.23046	.62396	.98828	.82612	.92848	.7865	3.63056	.21265	.77276	125.5556
73	LAYBOURNA	LIBOURNE	44 55	-0 15	-.00289	.75546	.00436	.78394	.04160	.77256	.02042	.6950	1.83339	1.18319	2.16924	24.8947
74	TOLOSA	TOULOUSE	43 35	1 28	.01879	.75827	.02560	.76067	.05127	.74636	.03900	.6504	1.81874	1.13344	2.06143	26.8453
75	VERONA	VERONA	45 27	11 00	.15968	.77651	.19199	.79325	.17084	.77359	.16214	.7639	2.32956	.87012	2.02699	54.2741
76	PICTAVIS	POITIERS	46 35	0 20	.03103	.74884	.00582	.81303	.04872	.80257	.02385	.7483	1.86668	1.23024	2.29647	23.7181
77	CARNOTUM	CHARTRES	48 25	1 30	.05271	.75164	.02618	.84503	.06866	.83406	.04782	.8087	1.98857	1.17503	2.33664	29.8026
78	LUGDUNUM	LYON	45 45	4 50	.10148	.75796	.08436	.79849	.09740	.78226	.08562	.7261	2.01208	1.06555	2.14398	35.8232
79	LARIS	LARISA	39 36	22 25	.34534	.78953	.39124	.69115	.32185	.67332	.32059	.7166	2.61128	.98002	2.55911	54.0303
80	LAC BENACUS	L GARDA	45 45	10 45	.14553	.75815	.18762	.79849	.16816	.77912	.15900	.7721	2.33146	.86347	2.01313	54.7066
81	BACTRUM	BALKH	36 44	66 58	.78498	.84095	1.16879	.64112	.93887	.81678	.88060	.8769	3.70442	.27322	1.01211	119.2250
82	CORINTHUS	CORINTH	37 55	22 55	.28783	.76556	.39997	.66177	.33169	.64300	.34091	.6594	2.53848	1.00210	2.54382	51.4348
83	FL MIMEUS	R. MINHO	41 52	-8 50	-.10164	.70962	-.15417	.73071	-.07290	.73651	-.11337	.6946	2.21927	.67994	1.50897	64.1393
84	TURONIS	TOURS	47 20	0 40	.05009	.72926	.01164	.82612	.05541	.81560	.03101	.7724	1.90106	1.22704	2.33268	24.8867
85	AUTISIODORUM	AUXERRE	47 49	3 35	.07719	.73277	.06254	.83456	.08940	.82090	.07376	.7927	2.05929	1.07388	2.21144	36.6623
86	FL PADUA	R. PO	45 00	12 30	.18015	.74611	.21817	.78540	.18874	.76526	.17977	.7654	2.38835	.85611	2.04469	56.3627
87	AURELIANUM	ORLEANS	47 55	1 57	.05692	.71913	.03403	.83630	.07169	.82455	.05171	.7923	1.98319	1.15953	2.29956	30.3879
88	MEDIOLANUM	MILAN	45 27	9 10	.13890	.72423	.15999	.79325	.14849	.77400	.14064	.7448	2.22810	.92110	2.05231	49.0418
89	RAVENNA	RAVENNA	44 25	12 13	.19309	.73125	.21322	.77522	.18467	.75469	.17795	.7403	2.34275	.88528	2.07399	53.6806
90	CIPRUS I	CYPRUS	35 00	33 30	.37734	.75511	.58469	.61087	.49542	.61424	.50147	.7117	2.83070	1.07049	3.03025	53.6409
91	LOSANNA	LAUSANNE	46 32	6 40	.10167	.71390	.11636	.81216	.12105	.79497	.11039	.7635	2.15005	.97057	2.08678	44.4152
92	VENICIA	VENICE	45 26	12 20	.21005	.72794	.21526	.79296	.18717	.77313	.17710	.7794	2.40113	.84012	2.01723	57.5811
93	FL DORIUS	R. DOURO	41 05	-8 40	-.10757	.67028	-.15126	.71704	-.07483	.72182	-.10742	.6712	2.20581	.64100	1.41392	66.6893
94	TOLETUM	TOLEDO	39 53	-4 02	-.02628	.68081	-.07039	.69610	-.02669	.68844	-.03129	.5762	1.93622	.78290	1.51586	50.1940
95	GARGANUS	TESTA DEL GARGANO	41 48	16 13	.21828	.70696	.28303	.72955	.23514	.70760	.23444	.6983	2.42218	.92473	2.23986	53.1558
96	ATHENE	ATHENS	37 54	23 52	.33890	.71156	.41655	.66148	.34533	.64423	.35245	.6749	2.58209	1.00833	2.60359	51.9938
97	AUGUSTUDUNUM	AUTUN	46 59	4 19	.07949	.67246	.07534	.82001	.09509	.80498	.08083	.7666	2.05055	1.06045	2.17451	37.1152
98	TIRUS	TYRE	33 17	35 11	.40463	.71456	.61406	.58090	.52759	.58843	.54182	.6554	2.82059	1.05383	2.97243	54.2594
99	VIENA	BIENNE	45 31	4 52	.09103	.66845	.08494	.79442	.09717	.77793	.08591	.7183	2.00197	1.06832	2.13874	35.4071
100	PLACENTIA	PIACENZA	45 02	9 41	.14522	.67546	.16901	.78598	.15390	.76613	.14719	.7348	2.23441	.92132	2.05861	49.1767
101	BONONIA	BOLOGNA	44 30	11 21	.16689	.67827	.19809	.77667	.17381	.75617	.16775	.7331	2.29893	.89975	2.06848	51.8791
102	TARENTUM	TARANTO	40 28	17 12	.23263	.68127	.30020	.70628	.24868	.68370	.25391	.6636	2.39945	.95219	2.28473	51.1646
103	AUGUSTA	AOSTA	45 44	7 19	.10869	.65971	.12770	.79820	.12674	.78003	.11786	.7397	2.14153	.97260	2.08285	44.0936
104	FLORENTIA	FLORENCE	43 47	11 15	.17372	.66813	.19635	.76416	.17165	.74297	.16834	.7059	2.25469	.92619	2.08827	49.3728
105	RODOS I	RHODES	36 30	28 00	.37492	.68868	.48869	.63705	.40883	.62673	.41599	.6891	2.68299	1.04375	2.80037	52.1899
106	PISA	PISA	43 43	10 24	.16639	.63963	.18151	.76300	.16091	.74196	.15828	.6947	2.20303	.94594	2.08394	47.0569
107	FL BOETIS	GUADALQUIVER R.	36 48	-6 23	-.10386	.59913	-.11141	.64228	-.06573	.63771	-.04775	.5265	1.99723	.52779	1.05411	71.1759
108	NARBONA	NARBONNE	43 11	3 00	.03703	.61737	.05236	.75369	.06839	.73709	.06162	.6369	1.83309	1.11384	2.04176	28.2537
109	TURRIS BABEL	BABYLON	32 45	44 30	.70970	.69897	.77667	.57160	.67042	.61699	.67578	.7159	3.23913	1.00313	3.24925	63.6166
110	NOLA	NOLA	40 55	14 33	.22339	.62498	.25395	.71413	.21270	.69056	.21979	.6428	2.27887	.95592	2.17842	48.2811
111	JERUSALEM	JERUSALEM	31 46	35 14	.47808	.65795	.61494	.55443	.53441	.56126	.55679	.5859	2.73917	1.03019	2.82186	53.9225
112	ARELAS	ARLES	43 39	4 41	.07847	.59519	.08174	.76184	.08997	.74396	.08435	.6549	1.90604	1.08876	2.07523	31.6736
113	MASSILIA	MARSEILLE	43 20	5 20	.10015	.59300	.09308	.75631	.09714	.73747	.09376	.6466	1.91883	1.07445	2.06169	32.7702
114	GENUA	GENOA	44 24	8 55	.12183	.60080	.15563	.77493	.14351	.75489	.13868	.7051	2.15731	.96263	2.07669	45.0293
115	MAR GALILEE	SEA OF GALILEE	32 50	35 25	.61496	.66466	.62105	.57305	.53558	.58169	.55172	.6393	2.81622	1.04757	2.95017	54.4842
116	ROMA	ROME	41 45	12 15	.19298	.60451	.21380	.72867	.18257	.70543	.18777	.6446	2.19692	.96254	2.11461	45.9955
117	SONFENTA	COSENZA	39 15	16 16	.29192	.60630	.28391	.68504	.23629	.66050	.25057	.6062	2.27295	.97462	2.21527	47.1295
118	CORDUBA	CORDOBA	37 54	-4 46	-.08671	.55176	-.08319	.66148	-.04242	.65385	-.03119	.5325	1.93895	.64788	1.25622	59.8798
119	FL HIBERUS	EBRO R.	40 44	0 50	-.05962	.55527	.01454	.71093	.03484	.69505	.03739	.5654	1.76956	1.01282	1.79225	31.5637
120	CORSICA I	CORSICA	42 30	9 00	.04877	.56930	.15708	.74176	.14137	.71987	.14424	.6397	2.05748	1.00265	2.06293	40.3273
121	DAMASCUS	DAMASCUS	33 35	36 28	.67196	.65000	.63646	.58614	.54593	.59840	.55610	.6855	2.89303	1.05943	3.06495	55.2801
122	ZAGINTHUS	ZANTE I.	37 45	20 40	.33126	.60037	.36070	.65886	.29957	.63670	.31500	.6165	2.41529	.99265	2.39755	49.3470
123	BETHLEEM	BETHLEHEM	31 41	35 11	.44576	.60969	.61406	.55298	.53418	.55934	.55682	.5815	2.73158	1.02877	2.81018	53.8512
124	NEAPOLIS	NAPLES	40 51	14 26	.23582	.57149	.25191	.71297	.21103	.68919	.21874	.6389	2.26826	.95812	2.17326	47.9168
125	SALERNA	SALERNO	40 41	14 43	.24666	.57289	.25685	.71006	.21499	.68625	.22308	.6366	2.27430	.95924	2.18161	47.9944
126	EBRON	HEBRON	31 32	35 05	.55013	.61218	.61232	.55036	.53323	.55628	.55673	.5736	2.71727	1.02619	2.78843	53.7109
127	SARDINIA	SARDINIA	40 00	9 00	.06452	.53277	.15708	.69813	.13794	.67360	.15418	.5565	1.91575	1.02557	1.96473	35.2331
128	JERICO	JERICHO	31 51	35 29	.58003	.59402	.61930	.55589	.53817	.56353	.55950	.5925	2.75687	1.03173	2.84435	54.1748
129	PALERNA	PALERMO	38 07	13 23	.14389	.51550	.23358	.66526	.19617	.63824	.22142	.5327	2.03367	.99813	2.02987	39.9436
130	CUZA	GAZA	31 29	34 27	.45820	.55620	.60127	.54949	.52365	.55311	.54838	.5644	2.68120	1.02472	2.74747	53.1005
131	BERSABEE	BEERSHEBA	31 15	34 45	.54560	.56201	.60650	.54542	.52928	.55001	.55450	.5572	2.68197	1.02115	2.73870	53.2940
132	PETRA	PETRA	30 20	35 30	.72985	.58587	.61959	.52942	.54477	.53587	.57231	.5235	2.66235	1.00548	2.67695	53.7093
133	UNNAMED	SICILY	37 30	14 10	.17309	.50275	.24725	.65450	.20718	.62692	.23531	.5207	2.03598	.99729	2.03046	40.0506
134	OPPIDUM	CEUTA	35 58	-5 23	-.03755	.46997	-.09396	.62774	-.05621	.61992	-.02730	.4926	1.91876	.53401	1.02464	68.7446
135	FL MALUA	MULUWI R.	35 20	2 30	.03290	.47909	.04363	.61668	.04391	.59274	.09015	.4123	1.58550	.84163	1.33440	35.6947
136	CARPATHAS I	SCARPANTO	35 40	27 10	.40618	.52751	.47415	.62250	.39870	.60952	.41359	.6409	2.59739	1.03570	2.69013	50.9162
137	CRETA I	CRETE	35 10	25 00	.27204	.50455	.43633	.61377	.36761	.59578	.39048	.5868	2.46100	1.02005	2.51034	48.9055
138	CARTAGO MAGNA	LATER CARTHAGE	37 00	10 20	.19095	.44997	.18035	.64577	.15320	.61742	.18807	.4715	1.79636	1.01299	1.81970	32.3824
139	FORTUNATE IS	CANARY AND MADEIRA IS	30 45	-15 45	-.05763	.41227	-.27489	.53669	-.19697	.55949	-.12268	.5697	2.23093	.07801	.17403	137.6333
140	MONS MERCURII	CAPE BON	37 05	10 50	.27835	.45577	.18908	.64723	.16023	.61880	.19424	.4775	1.86642	1.01183	1.84802	33.3574
141	CATAPAS	GABES	33 58	10 03	.31699	.45527	.17541	.59283	.14781	.56095	.20304	.3834	1.57989	.98103	1.54991	27.0472
142	ALEXANDRIA	ALEXANDRIA	31 12	29 52	.45247	.47281	.52127	.54454	.45312	.53394	.48756	.4992	2.41854	1.00814	2.43822	48.6102
143	MARE MORTUUM	DEAD SEA	31 30	35 30	.70174	.50509	.61959	.54978	.53995	.55714	.56280	.5765	2.73688	1.02587	2.80768	54.0944
144	OCEA	TRIPOLI	32 57	13 12	.38814	.45897	.23038	.57509	.19392	.54181	.25194	.3745	1.64465	.98084	1.61313	29.2902
145	CIRENE	CYREME	32 55	21 50	.41052	.45636	.38106	.57450	.32415	.54865	.36685	.4607	2.12036	.99350	2.10658	42.4320
146	UNNAMED	END OF PERSIAN GULF	30 00	58 00	.81293	.49745	1.01229	.52360	.88295	.64524	.88550	.5279	3.80183	.73159	2.78136	85.2579
147	ACESINES-HYDASPES	CHENAB-JHELUM R.	31 15	72 00	.99788	.51588	1.25664	.54542	1.04915	.76604	1.00000	.4992	3.68400	.24740	.91144	121.8866
148	POBBROTA	PATNA	25 35	85 15	.94580	.49261	1.48789	.44651	1.24058	.78918	.93873	.5119	4.62224	.63043	2.91400	98.9211
149	UNNAMED	END OF RED SEA	28 00	34 00	.73866	.43273	.59341	.48869	.53037	.48820	.56691	.4105	2.39910	.96245	2.30901	50.6027
150	M HESPERUS	CAPE VERDE	14 30	-17 30	.01461	.32245	-.30543	.25307	-.24605	.27899	.00596	.3293	1.01130	.09324	.09429	112.4383
151	MEMPHIS	MEMPHIS	29 50	31 15	.50182	.34696	.54542	.52069	.47952	.51274	.51660	.4602	2.38692	.99119	2.36590	48.8081
152	TAPHANA I	CEYLON	8 00	81 00	.98291	.37620	1.41372	.13963	1.33246	.47302	.98567	.3684	7.58899	.62368	4.73309	116.0152
153	BABYLONIA	CAIRO	30 02	31 21	.64712	.28864	.54716	.52418	.48049	.51669	.51659	.4692	2.40854	.99473	2.39585	49.0926
154	TROCODITE	TIBESTI RANGE	20 10	17 30	.22112	.21695	.30543	.35197	.27854	.30841	.37518	.1380	.82977	.59167	.49095	19.2851
155	GAMERA	OASES OF FEZZAN	25 30	15 00	.36502	.16946	.26180	.44506	.22802	.40406	.32032	.2124	1.08473	.86057	.93349	13.2338

1) AFTER ROTATION AND SCALING TO GIVE BEST FIT TO THE SQUARE PROJECTION
2) IN RADIAN UNITS
3) OBLIQUE ORTHOGRAPHIC PROJECTION CENTERED AT 31°46' N, 35°14' E. COORDINATES ROTATED AND SCALED TO GIVE BEST FIT TO THE MAP COORDINATES.
4) OBTAINED FROM EXPANSION OF THE UNWEIGHTED REGRESSION POLYNOMIALS.
5) COMPUTED FROM THE UNWEIGHTED POLYNOMIALS. VALUES OF K, H, AND Ø ALSO COMPUTED BUT NOT LISTED HERE.

the magnetic meridians in the years 1200 to 1300 A.D. so that the empirical curve fitting procedure appears appropriate. Such a magnetic Mercator should be approximately conformal and investigation of the angular distortion is suggested.

Without performing the actual analysis, it is not possible to draw further conclusions concerning the portolan charts. Any such analysis would need to be quite carefully done, since the charts will probably fit several map projections very closely.

CONCLUSIONS

It has been possible to answer only a few of the substantive questions concerning the possible methods of preparation of medieval maps. The strategy developed in this preliminary analysis, however, seems to offer some improvement over procedures employed in previous studies. For the student of the history of geography the techniques also allow estimates of the rate of cartographic progress. Imhof, for example, has recently published small illustrations of two old maps of the Canton of Zurich.[22] On the basis of these rather small illustrations, the simple calculations imply an improvement of twenty-eight percent in the mapping of Zurich between 1566 and 1667, and an improvement of only two percent between 1667 and 1965. This result in turn suggests the hypothesis of an S-shaped growth curve for the history of positional accuracy on maps. The same approach as outlined here can be applied, with obvious modifications, to maps which do contain the latitude and longitude graticule, either to determine the perhaps unknown map projection, or to make estimates of the accuracy of the maps if the projection is known.

Appendix

The procedure employed in estimating the amount of correlation between the observed map locations and a specific map projection is given here. Let the complex number

$$W_j = x_j + iy_j, \qquad (i^2 = -1)$$

be the map location of the jth point of identified latitude and longitude. Let

$$Z_j = X_j + iY_j$$

be the location of this same point on some map projection. Transform the arbitrary map recording coordinates to a new system,

$$\hat{W}_j = \hat{x}_j + i\hat{y}_j,$$

by applying a translation, rotation, and change of scale. This is no more than an assignment of new numbers to the recording coordinates and in no way changes the interrelations of the points. The transformation is of the form

$$\hat{W} = A + BW,$$

where A and B are complex numbers determined by application of the least squares criterion in a manner such that the residual

$$\sum_{J=1}^{N} |\hat{W}_j - Z_j|^2$$

is a minimum.

The amount of correlation between the transformed observational locations and the locations on the postulated map projection (calculated from the estimated latitudes and longitudes) is then given as the ratio of the regression variance to the total variance, as in ordinary correlation methods. The square of this complex correlation times 100 provides a measure of the percentage agreement between the two maps. This entire procedure is repeated for each postulated map projection. In the case of the Hereford map there were three comparisons; the square projection, the oblique orthographic projection, and the polynomial approximation. The map projection computations were all performed for a sphere of unit radius.

The coefficients of the unweighted polynomial approximation ($R^2 = 0.9450$) to the Hereford map are given in Tables 1 and 2. It is cautioned that terms cannot be dropped from this equation without changing the coefficients. The latitude (ϕ) and longitude (λ) values are to be entered in radian units. The computations were performed with the assistance of the University of Michigan computing center.

[22] E. Imhof, "Beitrage zur Geschichte der topographischen Kartographie" *International Yearbook of Cartography*, 4 (1964), pp. 129–53.