INDEPENDENT DISCOVERIES IN GRAPH THEORY*

Frank Harary

Department of Mathematics
University of Michigan
Ann Arbor, Michigan 48109

I am very honored to have been designated the "Scientist-in-Residence" of the New York Academy of Sciences for the current week. This is an honor comparable to any of those which I have already enjoyed.

By definition, an *independent discovery in graph theory* occurs when two people or groups of people working independently discover essentially the same result. Usually these discoveries occur at about the same time, but they can be separated by periods of many years, as you will see from some of our examples. I should make it clear at the outset that I really mean independent discoveries *in* graph theory, not independent discoveries *of* graph theory. The latter, of course, refers to discoveries of the entire field, which I shall mention briefly before moving on.

The first recorded discovery of graph theory was made by the great Swiss mathematician Leonhard Euler [7] when he developed a theorem on graphs to solve the famous problem of the Seven Bridges of Königsberg in 1736. This theorem is now known as the characterization of eulerian graphs. The discovery of graph theory which has had the greatest impact on modern life was by Georg Kirchhoff in 1847 in his study [16] of electrical networks. Cayley, in 1857, while considering the problem of changes of variables in differential calculus, discovered trees [5], and some seventeen years later found in them the ideal tool for counting chemical isomers [6]. The great Irish mathematician and physicist, Sir William Hamilton, was led into graph theory as a result of a game he invented in 1859. The game, which essentially required the finding of a spanning cycle in the skeleton of a dodecahedron, was never a commercial success (except to Hamilton, who sold the idea for £25). However, it resulted in Hamilton's name being attached to one of the more recalcitrant problems in graph theory, that of characterizing hamiltonian graphs, which have a cycle containing all the points. Another famous problem which has led many people into graph theory is the Four Color Conjecture, only recently proved by Haken *et al.* [10] (with a computer). This by no means exhausts the list of discoverers of graph theory. In very many cases and in disciplines in the physical sciences, the social sciences, computer science, and the humanities, graphs frequently occur as a natural, useful, and intuitive mathematical model. The consequence is that those investigators who were not aware of the existence of graph theory as a study in its own right were led to rediscover it in order to apply it.

One of the most fundamental theorems is that of Kasimierz Kuratowski characterizing planar graphs [18]. When it appeared in 1930, two American mathematicians, Orrin Frink and Paul A. Smith, had already submitted (independently, and independently of each other) papers containing precisely the same theorem, which they promptly withdrew.

Now I would like to share with you a number of cases of independent discoveries *in* graph theory. These are classified as win, lose, or draw according to

* Adapted by J. A. Kabell from the recording of the address delivered at the New York Academy of Sciences on 5 May 1977.

0077-8923/79/0328-0001$1.75/1 © 1979, NYAS

chronological priority. A former student of mine, Lowell Beineke, published [1] a technique for partioning complete graphs into line-disjoint paths. It was only later that we learned that the great French number theorist and collector of puzzles, E. Lucas, had published exactly the same construction in 1887 [19]. Another loser was the characterization of planar graphs in terms of contractions (dualizing Kuratowski's theorem) which Bill Tutte and I published in 1965 [15], only to find later that Klaus Wagner had obtained the same result in 1937 [22].

Beineke and I, in 1965, calculated and published [2] the genus of the n-cube. Unfortunately for us, Gerhard Ringel [20] had published the same result ten years earlier. Another example of a loser is a manuscript I wrote with my first doctoral student, Bob Norman. In this note we showed that the sum of the point independence and covering numbers, and likewise the sum of the line independence and covering numbers, is always just the number of points in the graph ($\alpha_0 + \beta_0 = \alpha_1 + \beta_1 = p$). The only problem was that Tibor Gallai [9] had done precisely the same work a few years earlier. Fortunately, the referee to whom our paper was sent by Paul Halmos was familiar with Gallai's result, so Bob and I were saved some embarrassment.

Now that we have seen some losers, let me tell you about one *bona fide* draw. Pavol Hell and I, at an AMS meeting in Las Vegas, happened to hit on the idea of the ramsey number of a directed graph. Eventually, we wrote up and published [14] our results. The very same month in which our paper appeared, another journal had a paper by Jean-Claude Bermond [4], defining exactly the same concept and proving many of the same results! That can truly be called a draw.

Here are two more draw stories both involving the Japanese graph theorist, Jin Akiyama. By way of background, I met in person both Dragos Cvetković and his doctoral student S. Simic on arrival at the Belgrade airport in 1974. At about the same time, Akiyama and his student K. Kaneko were also deriving graph equations for line graphs and nth power graphs. Friendly correspondence between Tokyo and Belgrade led to a triply joint paper which is to appear in 1980.

Akiyama was also doing the research in another Ph.D. thesis elsewhere in the world when he derived a forbidden subgraph characterization of iterated line graphs. The referee recommended that Jin write to D. G. Akka of Shahabad, India, who had also obtained the same result, with the suggestion that they get together and write one joint article, and Jin has agreed to do so.

Now let us look at a few winners. In 1958, Claude Berge published the second book ever written [3] on graph theory (the first was, of course, König [17], in 1936), and *Mathematical Reviews* (MR) sent me a copy to review. It concluded with a section of fifteen unsolved problems. The last four were disguised versions of Four Color Conjecture which happily has since been proved, as mentioned above. Problem 11, however, was simple enough for me to solve at once. It asked for the maximum connectivity of a graph with a given number of points and lines, and I included the result in my review in MR. Shortly thereafter, I received several papers to referee in which it was solved. Therefore I decided to publish this straightforward result [13].

Another winner I would like to present tonight actually concerns a real world application of graph theory rather than a new theorem. The idea of signed graphs and balance, which I introduced in 1953 [11], grew naturally out of attempts to describe both positive and negative psychological and social interactions in graphical terms. As a war involves essentially the same types of interactions, but between countries rather than individuals, I analyzed the Mideast conflict of 1956 using signed graphs and the tendency toward balance. The resulting paper [12] was published in the *Journal of Conflict Resolution* in 1962. In 1969, I received a frantic long-distance telephone call from Greece, from a man named George. First he spoke to me in Greek and I spoke to him in English, with neither of us understanding a word of the other language. Then he tried Turkish and I tried Spanish with the same result, and we finally settled on very fractured French. He was calling to tell me that he had just found out that his Ph.D. thesis in political science was essentially subsumed by this article of mine and needed to have my written permission to receive his doctorate from the University of Athens. Naturally I was so delighted to learn that my approach was worthy of a Ph.D. in political science that I immediately gave my blessing.

Here is an example of multiple independent discovery. It is a well-established result that any planar graph can be drawn in the plane in such a way that all its edges are straight line segments. This was published by Fáry [8] in 1948 and is often called Fáry's Theorem. It was rediscovered by Sherman Stein [21] in 1951 and by several others since then. Once again, however, Klaus Wagner [23] anticipated everyone by publishing the same theorem in 1936.

An amusing incident concerns two papers received not long ago by the Managing Editor of the *Journal of Graph Theory*, my good friend and colleague Gary Chartrand. The result, which was submitted nearly simultaneously as a joint note by two mathematicians in one university and also by one chap in another university, was a novel observation which stated that if a graph satisfied three very simple sounding conditions, it didn't exist. Of course, Gary wrote to these three gentlemen and suggested that they submit one joint paper which we would be glad to publish. The sole author agreed, but the two coauthors did not, so unfortunately we had to decline both notes.

These anecdotes do not by any means exhaust the examples of independent discovery in graph theory. In fact, independent discovery is probably more the rule than the exception, particularly in a field which is growing as rapidly as graph theory. One of the reasons is that we now have results on problems of interest circulated widely among people of varying backgrounds. It is not at all unreasonable that more than one person should have a similar insight at about the same time.

References

1. Beineke, L. W. 1964. Decompositions of complete graphs into forests. Magyar Tud. Akad. Mat. Kutato Int. Kozl. **9:** 589–594.
2. Beineke, L. W. & F. Harary. 1965. The genus of the *n*-cube. Canad. J. Math. **17:** 494–496.
3. Berge, C. 1958. Theorie des Graphes et ses Applications. Dunod, Paris.
4. Bermond, J. C. 1974. Some ramsey numbers for directed graphs. Discrete Math. **9:** 313–321.
5. Cayley, A. 1857. On the theory of the analytical forms called trees. Philos. Mag. **13:** 19–30.
6. Cayley, A. 1874. On the mathematical theory of isomers. Philos. Mag. **67:** 444–446.
7. Euler, L. 1736. Solutio problematis ad geometriam situs pertinentis. Comment. Acad. Sci. I. Petropolitanae **8:** 128–140.
8. Fáry, I. 1948. On straight line representation of planar graphs. Acta Sci. Math. Szeged. **11:** 229–233.
9. Gallai, T. 1959. Über extreme Punkt- und Kantenmengen. Ann. Univ. Sci. Budapest, Eötvös Sect. Math. **2:** 133–138.
10. Haken, W., K. Appel & J. Koch. 1977. Every planar map is four-colorable. Illinois J. Math. **21:** 429–567.
11. Harary, F. 1953. On the notion of balance of a signed graph. Michigan Math. J. **2:** 143–146.

12. HARARY, F. 1961. A structural analysis of the situation in the Middle East in 1956. J. Conflict Resolution. **5:** 167–178.
13. HARARY, F. 1962. The maximum connectivity of a graph. Proc. Nat. Acad. Sci. U.S.A. **48:** 1142–1146.
14. HARARY, F. & P. HELL. 1975. Generalized ramsey theory for graphs V: The ramsey number of a digraph. Bull. London Math. Soc. **6:** 175–182; **7:** 87–88.
15. HARARY, F. & W. T. TUTTE. 1965. A dual form of Kuratowski's theorem. Canad. Math. Bull. **8:** 17–20, 373.
16. KIRCHHOFF, G. 1847. Über die Auflosung der Gleichungen, auf welche der linearen Verteilung galvanischer Strömme geführt wird. Ann. Phys. Chem. **72:** 497–508.
17. KÖNIG, D. 1936. Theorie der endlichen und unendlichen Graphen. Akademische Verlagsgesellschaft M.B.H. Leipzig. Reprinted Chelsea, New York (1950).
18. KURATOWSKI, K. 1930. Sur le problème des courbes gauches en topologie. Fund. Math. **15:** 271–283.
19. LUCAS, E. Récréations Mathématiques, I-IV, Paris, 1882–1894.
20. RINGEL, G. 1955. Über drei kombinatorische Probleme am *n*-dimensionalen Würfel und Würfelgitter. Abh. Math. Sem. Univ. Hamburg. **20:** 10–19.
21. STEIN, S. K. 1951. Convex maps. Proc. Amer. Math. Soc. **2:** 464–466.
22. WAGNER, K., 1936. Bemerkungen zum Vierfarbenproblem. Jber. Deutsch. Math.-Verein. **46:** 26–32.
23. WAGNER, K. 1937. Über eine Eigenschaft der ebenen Komplexe. Math. Ann. **114:** 570–590.