

Finite element analysis of three- and four-unit bridges

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Summary

A two-dimensional finite element model of a mandibular quadrant was used to examine differences in magnitude of the principal stresses from the placement of three- and four-unit bridges. The area of interest spanned the first premolar to the second molar. Loading conditions were (i) vertical and distributed and (ii) 30° to the vertical and concentrated. The principal stresses were calculated and compared for: (i) the first molar removed with the remaining bone either cancellous or cancellous surrounded by a cortical shell; (ii) as in (i) but with the second premolar and first molar removed; (iii) a three-unit bridge spanning the second premolar to the second molar; and (iv) a four-unit bridge spanning the first premolar to the second molar. Each tooth was supported by periodontal ligaments, cortical and cancellous bone with each assigned the appropriate physical constants. Removal of the first molar resulted in considerable variation of the stresses especially when the cortical shell was replaced by cancellous bone. Because of the lower modulus of cancellous bone and its lower load-bearing capabilities the stresses were three to ten times lower and more uniform within the cancellous bone. Generally, the addition of a bridge resulted in lower and better distributed σ_{\min} stresses. The bridge also resulted in higher tensile stresses distal to the abutment teeth which theoretically could result in bone deposition. No significant differences in magnitude were observed between the three- and four-unit bridge. From a stress standpoint the bridges resulted in more uniform stress distribution around the abutments and an increase in the tensile stresses distal to the abutments. Such findings support the placement of a fixed bridge to maintain bone in an edentulous area.

Introduction

A two-dimensional finite element model of a mandibular quadrant was used to examine differences in magnitude of the principal stresses from the placement of three- and four-unit bridges. Farah, Craig & Meroueh (1988) used this model in an earlier study to determine the stresses and displacement throughout a similar but fully dentulous model.

The usefulness of the finite element method for studying design problems in dentistry has long been established by Hood, Farah & Craig (1975), Farah, Dennison & Powers (1977) and Peters (1981) among others for cases of single tooth cavity design, as well as multiple units of fixed and removable partial dentures.

Gupta, Knoell & Grenoble (1973) used a three-dimensional finite element model

of the mandible to evaluate stresses and displacements within the model, and compared these to stresses and displacements on actual mandibular specimens using holographic interferometry. Agreement was obtained within one order of magnitude in a comparison of the two techniques.

Materials and methods

The two-dimensional finite element model was described previously by Farah *et al.* (1988). In this portion of the study three states of edentulism were examined. In the first case, the first molar was removed, the model subjected to loading, and the stresses at the extraction site were evaluated as a function of cancellous bone or a combination of cancellous and cortical bone in that area. Secondly, a three-unit bridge was placed to span the second molar and the second premolar. Thirdly, the second premolar was removed as well as the first molar, and a four-unit bridge spanned the first premolar and the second molar.

In each case the loading was vertical and distributed or concentrated and at 30° to the distal of the vertical. The load was applied on the second molar and second premolar in the absence of a bridge or on the bridge as described for each respective case. The simulated bridge was made from type III gold with a Young's modulus of 0.83×10^7 N/cm² and Poisson's ratio of 0.33.

In all cases, a 100 N load was used, which was equivalent to the total chewing forces measured by Lundgren & Lourell (1986). Each tooth was supported by periodontal ligaments, cortical and cancellous bone, with each assigned the appropriate physical constants as shown in Table 1.

Table 1. Modulus (E) and Poisson's ratio (γ) of tooth and supporting structures

Structure	E (N/cm ²)	γ
Dentine	0.18×10^7	0.31
Enamel	0.84×10^7	0.33
Cementum	0.18×10^7	0.31
Cancellous bone	0.0025×10^7	0.30
Cortical bone	0.1×10^7	0.30
Periodontal ligament	0.69×10^3	0.45

For each of the above cases the maximum and minimum principal stresses were tabulated and compared. Special emphasis was placed on elements in the immediate vicinity of the teeth (level 1) mentioned above as shown in Fig. 1.

Results

The σ_{\min} and σ_{\max} stresses were plotted along the root surface of each tooth to facilitate the interpretation of the stress distribution in that area (Figs 2–6). The lowest and highest stresses at any point within a model are commonly denoted as σ_{\min} and σ_{\max} respectively. Often but not always σ_{\min} stresses are compressive while σ_{\max} stresses are tensile in nature. Upon removal of the first molar the stresses varied considerably, depending on whether the cortical bone and the periodontal ligament surrounding the first molar were removed and replaced by cancellous bone, as shown in Fig. 2. Figure 2 depicts two extreme clinical situations, one in which soon after

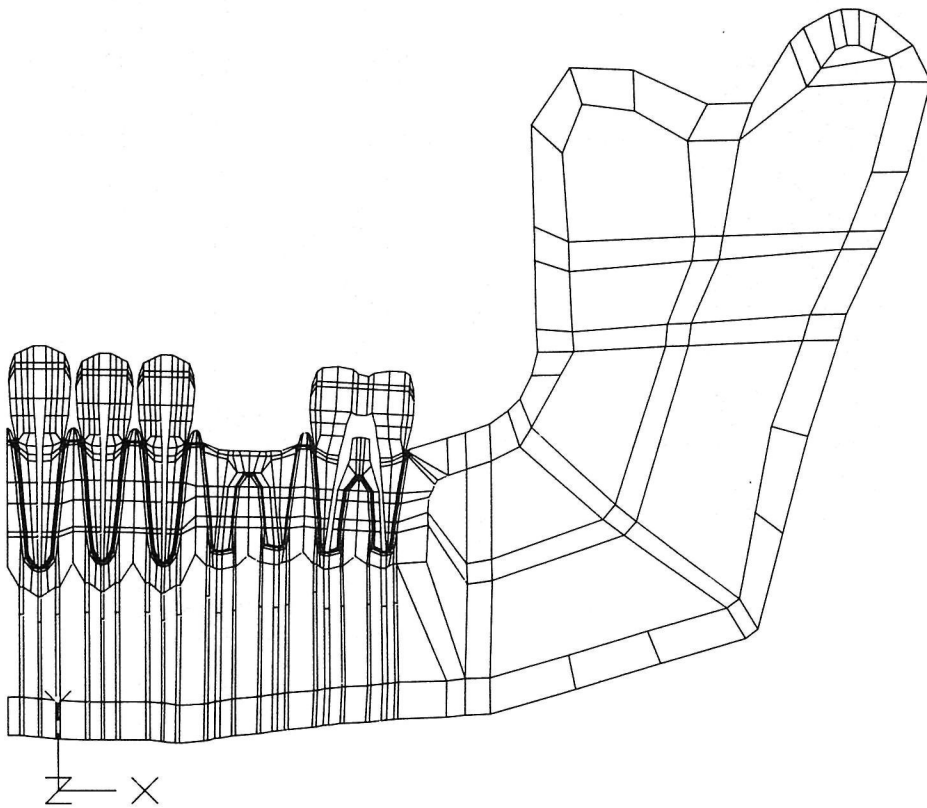


Fig. 1. Finite element model of a mandibular quadrant with the first molar removed.

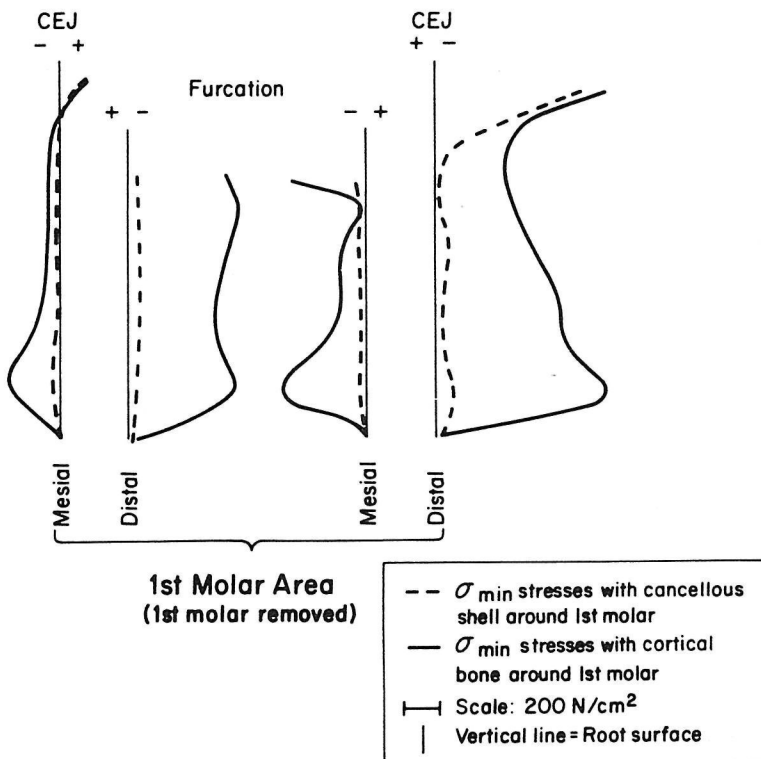


Fig. 2. Distribution of σ_{\min} stresses in an area of the first molar, with and without cortical shell, under a 100 N load distributed on the second molar.

extraction little or no healing has taken place and cancellous bone with a very low modulus remains at the extraction site, and the other where some healing and bone regeneration has occurred and a cortical shell has formed around the extraction site. Cancellous bone resulted in much lower and more uniform σ_{\min} and σ_{\max} stresses at

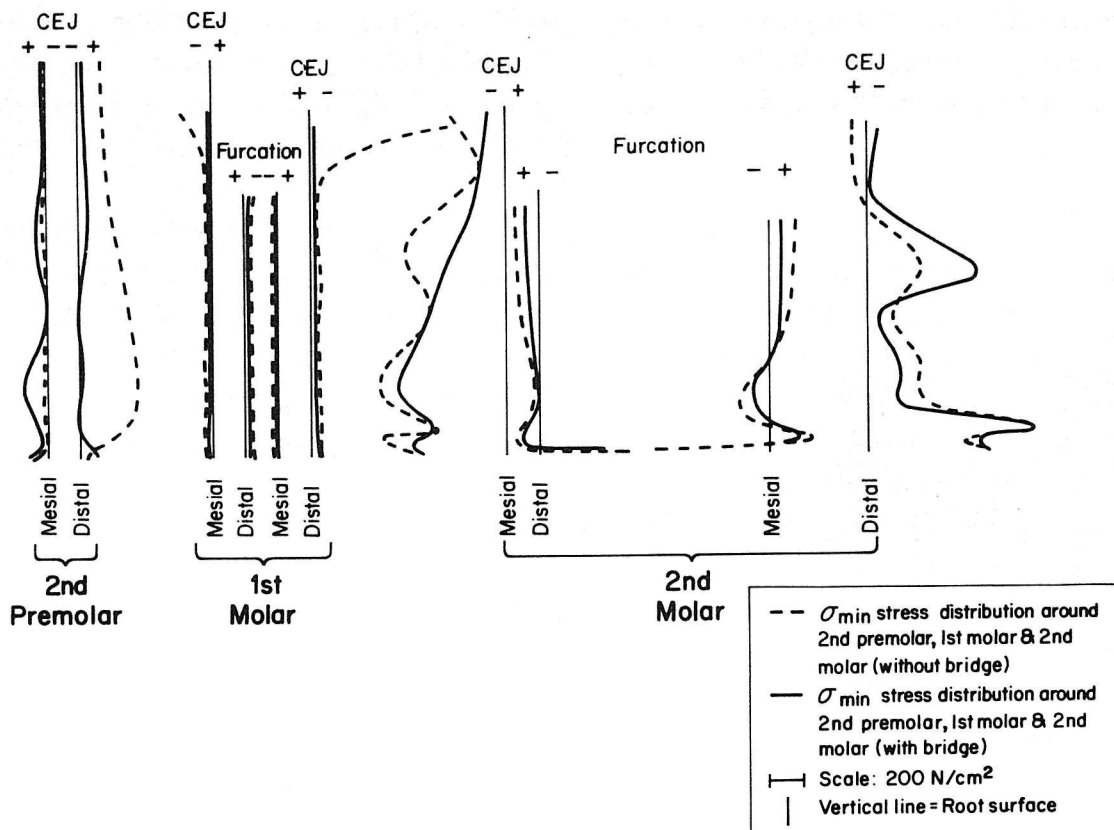


Fig. 3. Comparison of σ_{\min} stresses resulting from a case with a three-unit bridge and one without a three-unit bridge.

the extraction site. Generally, the σ_{\min} and σ_{\max} stresses were less than ± 100 N/cm², except at the alveolar crest area where a thin layer of cortical bone was maintained. In this instance σ_{\min} stresses reached a maximum of about 800 N/cm² at the distal area of the first molar. Higher σ_{\min} stresses were observed when the cortical shell was intact. The σ_{\min} stresses ranged from -50 to -600 N/cm² in the area distal to the mesial root to -50 to -1000 N/cm² in the area distal to the distal root.

Figure 3 compares the σ_{\min} stresses from the second premolar to the second molar with the first molar removed and a three-unit bridge spanning those teeth. The 100 N load was distributed on the second molar. The placement of the bridge resulted in higher compressive stresses on the mesial of the second premolar but lower compressive stresses on the distal of the same tooth. In the area of the first molar the σ_{\min} stresses were not significantly different with or without a bridge.

The placement of the bridge also resulted in lower overall σ_{\min} stresses around the second molar. Since the load was applied on the second molar, the σ_{\min} stresses were higher (from -100 to -1000 N/cm²) than the stresses around the anterior abutment (which range from -20 to $+350$ N/cm²).

Figure 4 compares the effect of the type of loading, concentrated or distributed, on a three-unit bridge. It should be noted that for both σ_{\min} and σ_{\max} the stresses were lower when the load was distributed as opposed to concentrated and at 30° to the vertical. The concentrated load resulted in 2- to 5-fold higher stresses than the distributed load.

For example, σ_{\min} on the mesial of the second premolar ranged from -10 to -180 N/cm² for the distributed load and from -20 to -640 N/cm² for the concentrated load. Similarly, σ_{\max} ranged from -10 to -400 N/cm² for the distributed load and -200 to -1440 N/cm² for the concentrated load.

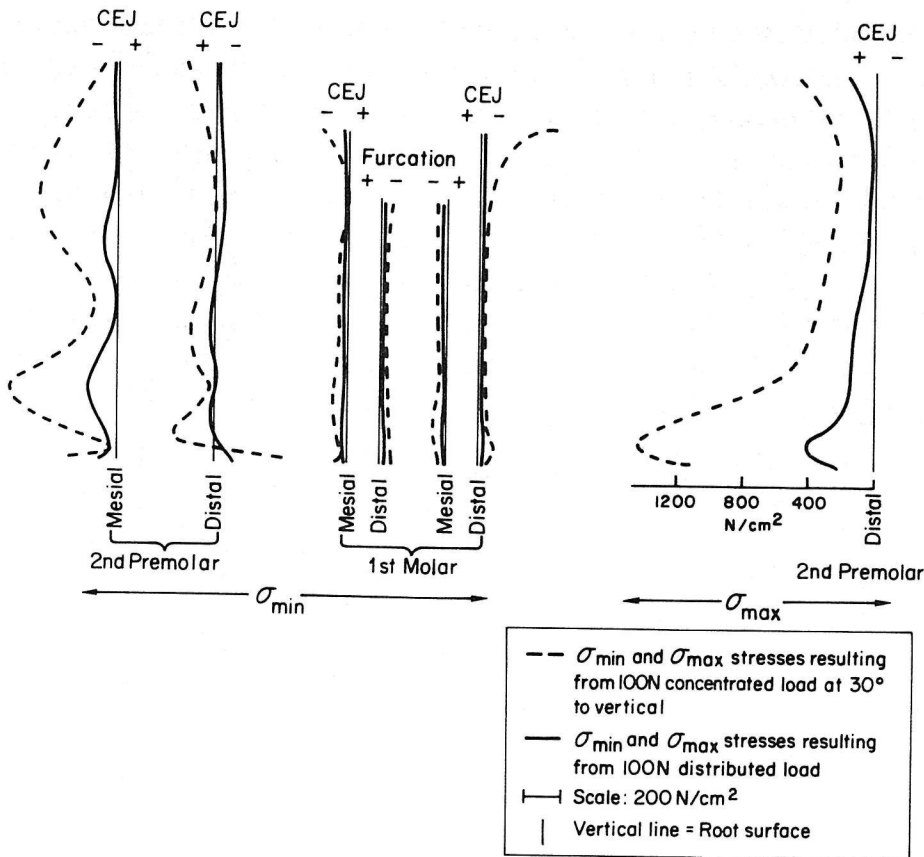


Fig. 4. Comparison of σ_{min} stresses resulting from a 100 N load concentrated on a three-unit bridge.

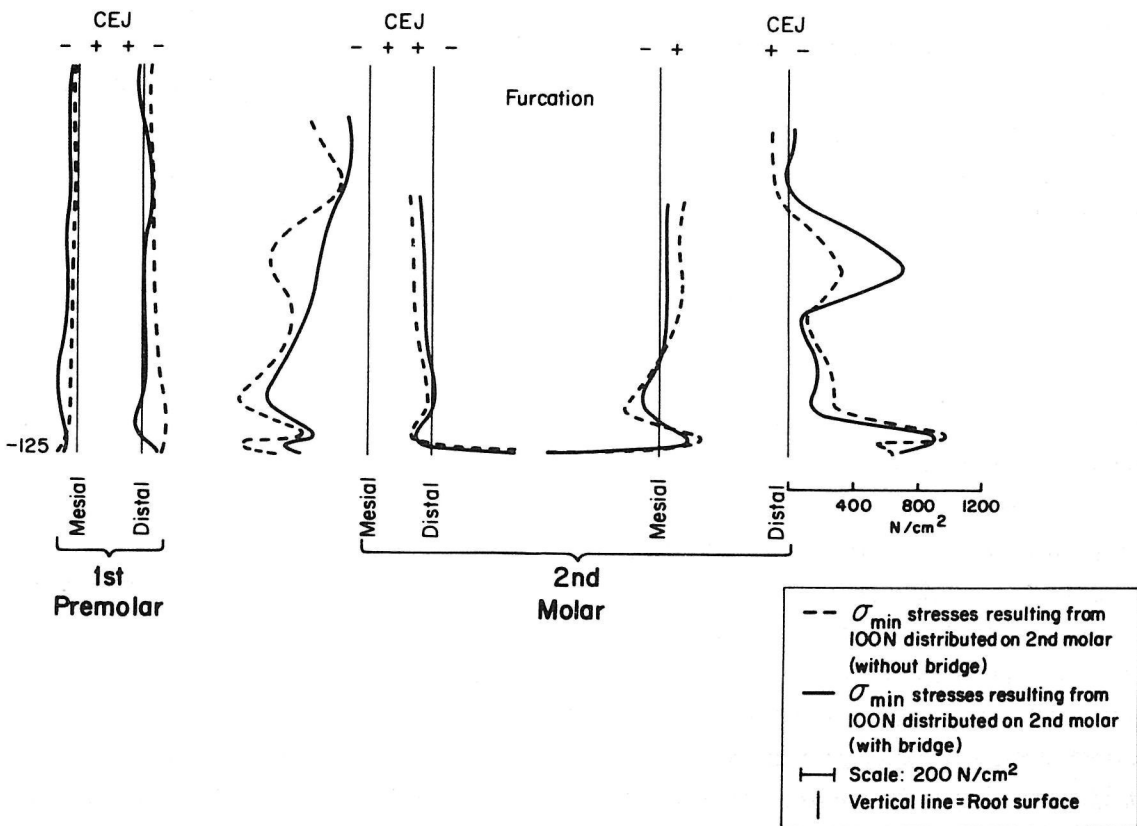


Fig. 5. Distribution of σ_{min} stresses around the abutments of a four-unit bridge.

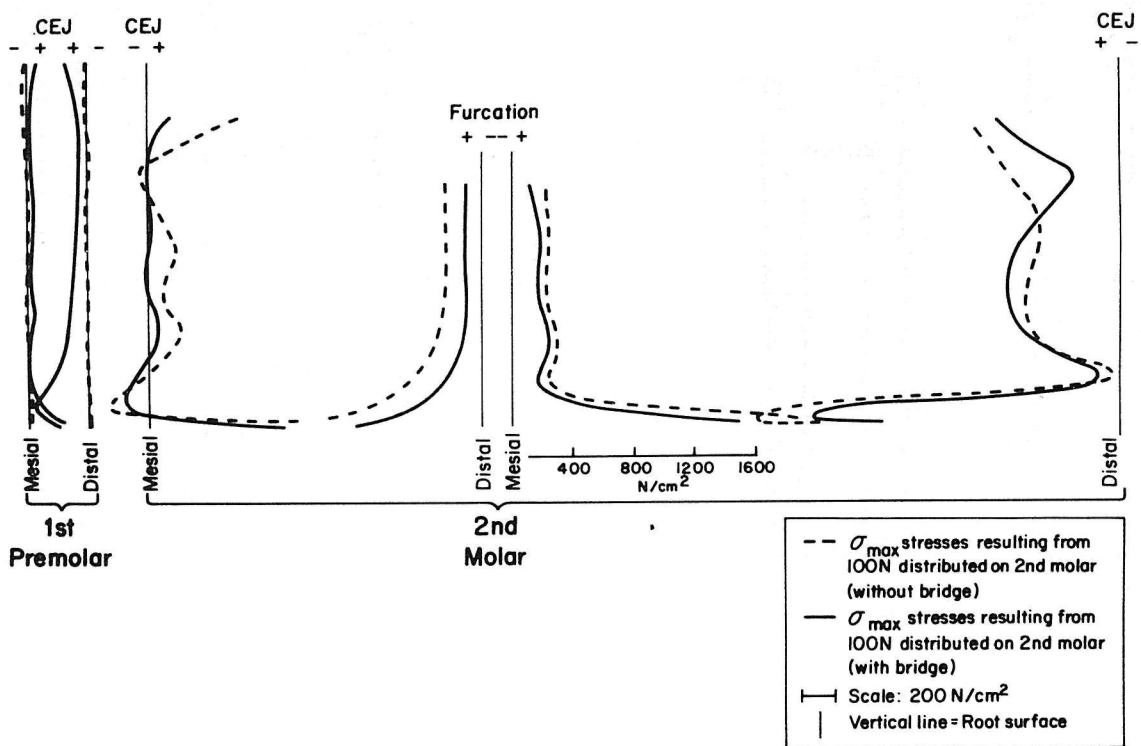


Fig. 6. Distribution of σ_{\max} stresses around the abutments of a four-unit bridge.

Figure 5 compares the σ_{\min} stresses around the abutment teeth of a four-unit bridge. The stresses at the mesial of the first premolar were higher when the bridge was placed, but they were lower on the distal. Around the second molar, placement of the bridge resulted in generally lower σ_{\min} stresses except at the distal cervical third of the second molar. The stresses around the extraction sites, although not shown in Fig. 5, were of much lower magnitude and were uniform.

Figure 6 compares the distribution of σ_{\max} stresses around the abutment teeth of the four-unit bridge. The addition of the bridge again resulted in lower stresses. The stresses were approximately 20% lower when the bridge was added. Interestingly, at the distal of the first premolar the addition of the bridge resulted in increased tensile stresses. In general the tensile stresses were approximately twice the magnitude of the compressive stresses around the abutment teeth.

Discussion

The results indicate that the stresses at the extraction sites were a function of the amount of bone removed at the time of extraction. For that reason, several cases were examined in which different amounts of bone, both cortical and cancellous, were maintained at the extraction site in order to learn about their effects on the surrounding stresses. It was found that the presence or absence of a cortical shell in that area had the most dramatic effect on the stresses (Fig. 2).

The cancellous bone having a relatively low modulus compared to the cortical bone resulted in a low, uniform stress distribution as shown in Fig. 2, except at the alveolar crest area where the bone was assumed to be cortical in nature. In general, the lower the modulus the lower the stress-bearing ability of that area. As more cortical shell formed around the extraction site, higher stresses were observed, because the higher modulus of the cortical bone lent more rigidity to the area. Under

similar loading conditions similar stresses will result, but depending on the modulus these stresses will be redistributed in different locations.

Figure 3 compares the σ_{\min} stresses resulting from the placement of a three-unit bridge to those stresses without a bridge. One can see that the addition of the bridge resulted in slightly higher σ_{\min} at the mesial root surface of the second premolar, and a 4- to 6-fold lower σ_{\min} at the distal of the second premolar. The bridge with a load caused the second premolar to be displaced mesially and thus resulted in higher σ_{\min} stresses at the mesial. σ_{\max} values, on the other hand, although not shown in Fig. 3, were lower at the mesial but higher at the distal of the second premolar. The fact that σ_{\max} was more positive at the distal of the second premolar could mean that more bone deposition will take place at the distal, which is a desirable feature in that area.

At the extraction site the difference between the stresses was minimal; overall, the σ_{\min} stresses at the extraction site were low in magnitude and uniform. This condition is particularly true because of the low modulus in that area. The case without a bridge did have high stresses at the alveolar crest, as shown in Fig. 3.

Around the second molar the addition of the bridge resulted in overall reduction of σ_{\min} as well as σ_{\max} , and in some cases the reduction approached 50% of the original stresses. At the distal of the second molar the stresses were less uniform. This condition is partially caused by the size of the elements within the mandible distal to the second molar, and the boundary effects in that area. Usually the larger the elements the more variation in the stresses.

Although σ_{\max} is not shown in Fig. 3, it was generally of the same magnitude as σ_{\min} , except at the distal of the second molar where σ_{\max} was about 30% higher than σ_{\min} . The nature of the loading in that area resulted in more tensile bending stresses.

Figure 4 compares the stresses resulting from a distributed 100 N load to those resulting from a 100 N concentrated load at 30° to the vertical. It can be seen that both the σ_{\min} and σ_{\max} stresses were substantially higher for the concentrated load applied at an angle. In this case, only stresses around the second premolar and first molar were plotted for comparative purposes. Stresses around the second molar were of greater magnitude and much less uniform.

On the mesial of the second premolar the σ_{\min} stresses range from zero to -160 N/cm^2 for the distributed load and from -40 to -640 N/cm^2 for the concentrated load. The σ_{\min} stresses from the concentrated load were also higher at the distal of the second premolar. The σ_{\max} stresses ranged from $+20$ to $+400 \text{ N/cm}^2$ for the distributed load and from 200 to 740 N/cm^2 for the concentrated load. It is obvious from the above that a distributed vertical load generally resulted in much lower σ_{\min} and σ_{\max} stresses.

The introduction of the bridge resulted in higher, more positive σ_{\max} values around the second premolar; in fact, σ_{\max} with the bridge was about twice the value of σ_{\max} without the bridge. Another interesting point is that σ_{\min} on the distal of the second premolar was positive ($+50$ to $+200 \text{ N/cm}^2$) with the bridge while without the bridge σ_{\min} was negative (-400 to -1700 N/cm^2). This situation again could have interesting implications in promoting bone growth around the second molar. In the area of the first molar the stresses were uniform and of low magnitude and the placement of the bridge resulted in a reduction of the stresses by one-half. Around the second molar the bridge resulted in even lower stresses. For example, σ_{\min} with the bridge ranged from -700 N/cm^2 at the apex to near zero at the alveolar crest, while σ_{\min} without the bridge ranged from -700 to -1700 N/cm^2 for the same area.

The addition of the bridge had a more profound effect on the stresses in the case of the concentrated load compared with the distributed load. Therefore, the placement of a bridge is especially important when the load is not distributed and is at a slight angle, a condition that often results clinically in the loss of a tooth. The tilt in the distal abutment will often result in a point loading and thus an increase in the magnitude of the stress.

Figures 5 and 6 represent the σ_{\min} and σ_{\max} stress distributions around the abutment teeth of a four-unit bridge. On comparing Figs 3 and 5, it becomes apparent that the σ_{\min} stress pattern around the abutment teeth for a three- and four-unit bridge is quite similar. The stresses around the second premolar, the three-unit bridge abutment, were slightly higher than those around the first premolar, which is the four-unit bridge abutment, and this is most probably a result of the fact that the second premolar was closer to the site of load application. The four-unit bridge, because of its longer span, would be expected to result in more bending and, therefore, more stresses around the abutment, but in this case the stresses were not higher, probably because the load was applied to the second molar abutment in both cases and thus the length of the span played a minor role. Had the load been applied to the pontics it is likely that more stress would have resulted around the first premolar.

As noted earlier in the case of a three-unit bridge, the addition of the bridge did result in some lowering of the stresses around the abutment teeth when compared to the no-bridge case. Again, as before, the bridge did result in more uniformity of the σ_{\min} stresses, except at the distal of the second molar where element size and edge effects seemed to distort the uniformity.

A similar decrease in the σ_{\max} stresses is shown in Fig. 6. Higher positive stresses resulted at the distal of the first premolar with the addition of the bridge and this, as pointed out earlier, can stimulate bone deposition. Around the second molar the addition of the bridge resulted in a general reduction of the stresses. The peaks of σ_{\max} stresses were lower with the bridge except at the distal cervical third of the second molar. It is apparent from this study that the addition of a three- or four-unit bridge does result in general in lowering of the σ_{\min} as well as the σ_{\max} stresses.

Conclusions

- (i) In an extraction area, the lower modulus results in lower stresses because of the lower stress-bearing ability of that area. However, the stresses are redistributed to the abutment areas.
- (ii) The addition of a bridge resulted in an overall better distribution of the stresses, i.e. they were more uniform and of lower magnitudes (in some cases up to 50% lower).
- (iii) The bridge resulted in higher σ_{\max} stresses especially at the distal of the second premolar and some at the mesial of the second molar. Higher tensile stresses could imply that more bone deposition would occur.
- (iv) Because of the element size and possibly edge effect, the stresses at the distal of the second molar were much less uniform and more erratic.
- (v) Concentrated loads at 30° resulted in much higher stresses than distal loads (in some cases up to ten times higher). Clinically, this can imply that it is better to place a bridge before the abutment teeth start tipping.

- (vi) No major difference in stresses was found between the three- and four-unit bridges, possibly because the load was on the second molar and bending of the bridge was not as pronounced.

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