Latent variables in econometrics

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Unobservable variables in econometrics are represented in one of three ways: by variables contaminated by measurement errors, by proxy variables, or by various manifest indicators and/or causes. This paper contains a discussion of models involving each of these representations, and highlights certain interesting implications that have been insufficiently emphasized or completely unrecognized in the literature.

Key Words & Phrases: Unobservable variables, measurement errors, proxy variables.

1. Introduction

To the best of the author's knowledge the term "latent variables", as distinct from "observed variables", was first used in the econometric literature by KOOPMANS (1949) in reference to the stochastic disturbances in a standard simultaneous equation model of supply and demand.[1]
Although strictly speaking such disturbances are indeed always unobservable, they have not been labeled as "latent" in the subsequent econometric literature. Instead, the term "latent" has been reserved for unobservable variables *other* than stochastic disturbances. There are, in general, three main classes of such variables that may enter econometric models: (1) variables for which exact measurements are not available and which are represented by error-contaminated substitutes; (2) unobservable variables that can be represented only through closely related substitutes called "proxies"; and (3) variables that are *intrinsically* not measurable (and frequently not even properly defined) such as "permanent income" or "intelligence", but that are related to a number of measurable (manifest) variables such as age, educational attainment, etc. The term "latent" has been used by various authors to refer to *all* of the three above types of unobservable variables (see, e.g., AIGNER et al. 1984, or AIGNER and DEISTLER, 1989), while other authors have used this term only in reference to the intrinsically unmeasurable variables (see, e.g., GRILICHES, (1974), KMENTA, 1986, or GREENE, 1990).

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1. This reference was pointed out to the author by L.L. Wegge.

There are some important conceptual as well as practical differences between the three types of unobservable variables. The unobservable variables of the first type are typically well defined but imperfectly measured. The errors of measurement involved could, at least in principle, be reduced if not entirely eliminated if more resources were devoted to the task. For instance, it is generally recognized that the U.S. national income data are now considerably more precise than they were years ago. And while errors of measurement can afflict both the dependent (endogenous) variables as well as the explanatory (exogenous) variables, econometrically interesting cases are only those involving the explanatory variables. For the standard classical regression model, the well-known consequences of the presence of error-ridden explanatory variables are biasedness and inconsistency of the least-squares coefficient estimators.

The situation with the second type of unobservable variables represented by proxies is different from the error of measurement case in that there are no measurements at all available for the variables in question, but we have observations on closely related variables that can be used as surrogates. The most popular proxy variable in econometrics is a time trend, which is frequently taken to represent "technical progress" or other similar unobserved factors. Typically even if the unobservable variable is well defined, it could not be measured no matter how good the measurement instruments are. A good example of such a variable is capital stock as an input in a production function, which is typically represented by its monetary value. Proxy variables normally make their appearance as surrogates for explanatory variables, only rarely for dependent variables. The consequences of using proxy variables in place of their unobservable counterparts in a regression model are the same as in the case of measurement errors.

Finally, unobservable variables of the third type denoted as instrinsically latent represent concepts that are typically well understood but rarely rigorously defined such as "intelligence" or "ability". In econometrics they can appear as either dependent or explanatory variables. However, unlike in the case of the first two types, there are no single measurable counterparts for them. Their presence in econometric (and other) models can be handled by characterizing each latent variable by a number of observable (manifest) indicators, such as scores on intelligence tests, school grades, etc., or by a number of observable causes, such parents' IQ, schooling, etc. The best known latent variable in econometrics is probably "permanent income", which can never be exactly measured but which is determined by a number of measurable factors such as current income, age, etc. Unlike in the case of proxy variables, a latent variable is never represented by just one measurable factor.

Historically, the different types of unobservable variables appeared in econometrics in the same order in which they are listed above. In fact, models with variables tainted by measurement errors — the so-called "errors-in-variables" models — preceded models with correctly measured variables but confounded by the presence of a stochastic disturbance, the so-called "errors-in-equation" models. The development of the latter, begun by TINBERGEN

(1939) and the researchers affiliated with the Cowles Commission in the 1940's and early 1950's (see e.g., EPSTEIN, 1987), has pushed aside but never completely eliminated the concern of econometricians with errors of measurement. Models involving errors in variables were given a great boost in econometrics by the introduction of the "permanent income hypothesis" model of consumption by FRIEDMAN (1957), in which the difference between the current income and the "permanent" part of it (the so-called "transitory" income) can be formally treated as an error of measurement. Models involving proxy variables in econometrics have also been around for a long time, but the associated problem of biased estimation was largely ignored until the early 1970's when MCCALLUM (1972) and WICKENS (1972) raised the issue of the "proxy variable dilemma". The dilemma concerns a choice between including a proxy variable and committing an error of mismeasurement, or excluding it and committing an error of specification. Finally, models involving intrinsically latent variables have not been seriously considered in econometrics until the path-breaking papers of ZELLNER (1970) and GOLDBERGER (1972). In these papers the authors addressed the problem of representing permanent income in a considerably more fundamental way than by a simple analogy to the errors-in-variables models.

The purpose of this paper is to provide a broader perspective of models with unobservable variables from an econometric point of view, and to highlight certain implications that have been insufficiently emphasized or not at all discussed in the literature. There is no intention to provide a complete survey of the field, which has already been done by AIGNER et al. (1984), or to single out a few problems for thorough attention. The organization of the paper follows the classification of unobservable variables discussed above. Section 2 deals with models whose estimation involves variables measured with error, and contains some suggestions for a new approach to the problem of hypothesis testing. Models with proxy variables are discussed in Section 3, which also includes a new proposal for dealing with the "proxy variable dilemma". Section 4 contains some observations concerning models with intrinsically latent variables.

2. ERRORS OF MEASUREMENT

The standard representation of the measurement error problem in regression model with a single explanatory variable involves the relationship

$$Y_i = \alpha + \beta X_i^* + \epsilon_i \quad (i = 1, 2, \ldots, n) \tag{1}$$

where $\epsilon_i \sim N(0,\sigma^2)$, $E(\epsilon_i\epsilon_j)=0$ for all $i \neq j$, and X_i^* is an unobservable non-stochastic variable with a finite mean a finite variance for any sample size. However, instead of X_i^* we only observe X_i defined as

$$X_i = X_i^* + w_i \tag{2}$$

where w_i represents the error of measurement. It is assumed that $w_i \sim N(o,\sigma_w^2)$, $E(w_iw_j)=0$ for all $i \neq j$, and ϵ_i and w_j are independent for all

$i, j = 1,2,...,n$. Substituting (2) into (1), we obtain

$$Y_i = \alpha + \beta X_i + u_i \tag{3}$$

where $u_i = \epsilon_i - \beta w_i$.

Since X_i and u_i are correlated, the least squares estimators of α and β are biased and inconsistent. In particular, according to the well known textbook result concerning the least squares estimation of β (denoted by $\hat{\beta}$),

$$\begin{aligned} \text{plim}\ \hat{\beta} &= \beta - [\beta\sigma_w^2/(\sigma_*^2 + \sigma_w^2)] \\ &= \beta - \beta e \end{aligned} \tag{4}$$

where σ_*^2 is the limiting value (as $n \rightarrow \infty$) of the sample variance of X^* and $e = \sigma_w^2/(\sigma_*^2 + \sigma_w^2)$. A considerably less well known result concerns the asymptotic variance of $\hat{\beta}$ which, according to SCHNEEWEISS and MITTAG (1986), is given as follows:

$$\text{As. Var}[\sqrt{n}(\hat{\beta} - \text{plim}\ \hat{\beta})] = e_u - \beta^2 e^2 (3 - 4e + 2e^2) \tag{5}$$

where $e_u = (\sigma^2 + \beta\sigma_w^2)/(\sigma_*^2 + \sigma_w^2)$. The asymptotic distribution of $\hat{\beta}$ is normal. Thus $\hat{\beta}$ is asymptotically biased towards zero. As $\sigma_w^2 \rightarrow \infty$, both $\hat{\beta}$ and its asymptotic variance approach zero, so that the asymptotic distribution of $\hat{\beta}$ tends to collapse at zero. When $\sigma_w^2 = 0$, the asymptotic bias disappears and the asymptotic variance reduces to the standard expression σ^2/σ_*^2. The same result is obtained when $\beta = 0$, the only time when the presence of measurement errors does not have a detrimental effect on the properties of the least squares estimators. When $\sigma_w^2 < \sigma_*^2$, the asymptotic variance of $\hat{\beta}$ can be approximated as follows:

$$\text{As. Var}[\sqrt{n}(\hat{\beta} - \text{plim}\ \hat{\beta})] \approx e_u - 3\beta^2 e^2. \tag{6}$$

This approximation appears to be quite satisfactory for most practical purposes.

The small sample properties of the LSE of β have been derived by RICHARDSON and WU (1970). If the terms of order n^{-2} in the degree of smallness are dropped, the mean and the variance of $\hat{\beta}$ are

$$E(\hat{\beta}) = \beta - \beta e[1 - 2(e_*/n)] \tag{7}$$

and

$$\text{Var}(\hat{\beta}) = [e_\epsilon + \beta^2 \sigma_*^2 \sigma_w^2 (e_*^2 + e^2)]/(n-2) \tag{8}$$

where $e_\epsilon = \sigma^2/(\sigma_*^2 + \sigma_w^2)$ and $e_* = \sigma_*^2/(\sigma_*^2 + \sigma_w^2)$.

For sample sizes greater than 18 the approximation is accurate to within one-tenth of one percent. From (7) it is clear that the bias *increases* as sample size increases, and it is largest — at its asymptotic value given by (4) — when $n \rightarrow \infty$. On the other hand, the variance of $\hat{\beta}$ decreases with increases in sample size. It can also be shown that when $\sigma_w^2 < \sigma_*^2/3$, the mean square error of $\hat{\beta}$ decreases as n increases. When $\beta = 0$, the bias disappears and the variance reverts to its standard formula (except vor a small adjustment in sample size).

RICHARDSON and WU (1970) also show that $\hat{\beta}$ has a noncentral t-distribution.

The above findings suggest that the use of least squares estimators may be quite appropriate for the purpose of testing for the existence of a relationship between Y and X^* when the sample size is moderately large. This is so because under the null hypothesis that $\beta=0$, the LSE of β is unbiased and consistent, and its variance is given by the conventional formula. Further, when the size of the sample is moderately large, the distributions of the LSE of β should not be far from normal. In order to carry out the test, we need an estimate of $\mathrm{Var}(\hat{\beta})$. Let us consider

$$\text{Est.}\,\mathrm{Var}[\sqrt{n}(\hat{\beta}-\beta)] = (\Sigma\hat{u}_i^2/n)/(\Sigma x_i'^2/n)$$

where $\hat{u}_i=y_i'-\hat{\beta}x_i'$, and the primes indicate deviations from the respective sample means. Now

$$\begin{aligned}\text{plim}(\Sigma\hat{u}_i^2/n) &= \text{plim}\,\Sigma[-(\hat{\beta}-\beta)x_i'+u_i']^2\\ &= \text{plim}[(\Sigma u_i'^2/n)-(\hat{\beta}-\beta)^2(\Sigma x_i'^2/n)]\\ &= (\sigma^2+\beta^2\sigma_w^2)-\text{plim}\,\mathrm{Var}(\hat{\beta})(\sigma_*^2+\sigma_w^2)\\ &= \sigma^2\end{aligned}$$

when $\beta=0$. Further,

$$\text{plim}(\Sigma x_i'^2/n) = \sigma_*^2+\sigma_w^2.$$

Thus

$$\text{plim Est.}\,\mathrm{Var}[\sqrt{n}(\hat{\beta}-\beta)] = \sigma^2/(\sigma_*^2+\sigma_w^2)$$

so that the variance of $\hat{\beta}$, when $\beta=0$, can be consistently estimated by

$$\text{Est.}\,\mathrm{Var}(\hat{\beta}) = (\Sigma\hat{u}_i^2/n)/(\Sigma x_i'^2).$$

Therefore the presence of measurement errors poses no serious problems in testing the hypothesis that $\beta=0$ against $\beta\neq 0$ in the conventional way. It can be shown that this conclusion also holds when the regression model (3) is extended by including additional explanatory variables that are correctly measured.

When $\beta\neq 0$, consistent estimation becomes a problem. Its solution in the literature has typically been facilitated by the introduction of additional information, consisting of either specifying the value of σ_w^2 (or σ_w^2/σ_*^2) *a priori*, or by postulating a relationship between X and some specified instrumental variable Z. Without additional information, the problem of consistent estimation of β seems insolvable. The situation with respect to hypothesis testing looks somewhat brighter. Specifically, let us consider testing the hypothesis that two regression equations have the same slope, i.e., $H_0\colon\beta_1=\beta_2$, and let us replace the assumptions that X^* is nonstochastic by the assumption that X^* is normally distributed with variance σ_*^2. In this case our H_0 is equivalent to

$$H_0\colon \beta_1\sigma_*^2/(\sigma_*^2+\sigma_w^2) = \beta_2\sigma_*^2/(\sigma_*^2+\sigma_2^2)$$

or

$$H_0: \quad \beta_1^* = \beta_2^*.$$

The parameters β_1^* and β_2^* are consistently estimated by the LSE's of β_1 and β_2, respectively, and the only problem is to estimate their variances. The difficulty with the latter is that the formulas for the relevant asymptotic or finite sample variance given by (5), (6), or (8) involve β and not β^*. However, by reference to (6) we can see that when $\sigma_w^2 < \sigma_*^2$ — which is likely in most practical situations — the upper limit of $\mathrm{Var}[\sqrt{n}(\hat{\beta} - \mathrm{plim}\,\hat{\beta})]$ equals e_u defined as $(\sigma^2 + \beta\sigma_w^2)/(\sigma_*^2 + \sigma_w^2)$.
The latter can be consistently estimated by

$$e_u = [\Sigma \hat{u}_i^2/n]/[\Sigma(X_i - \overline{X})^2/n].$$

Also, as noted earlier, in large samples $\hat{\beta}_1$ and $\hat{\beta}_2$ are approximately normal. Using these results, we can carry out a conservative test (i.e., a test that will reject H_0 less often than the nominal size of the test indicates) of the hypothesis of equal slopes in two separate regression equations.

3. Proxy variables

Let us consider the following classical regression model:

$$Y = X_1\beta_1 + X_2\beta_2 + U \tag{9}$$

where

$Y \to (n \times 1), X_1 \to (n \times K_1), X_2 \to (n \times K_2), \beta_1 \to (K_1 \times 1), \beta_2 \to (K_2 \times 1),$

and

$$U \to (n \times 1).$$

We assume that $U \sim N(0, I_n)$, and that X_1 and X_2 are nonstochastic. (If X_1 and X_2 are stochastic, then our results are conditional on their sample values). Suppose that X_2 is not observable but can be replaced by a set of proxy variables Z that are related to X_2 and orthogonal to U. We then have a choice of using either a truncated model

$$Y = X_1\beta_1 + U_1 \tag{10}$$

or a proxied model

$$Y = X_1\beta_1 + Z\gamma + U_2 \tag{11}$$

where Z is the same dimension as X_2 and γ is a vector of constants. If we choose to estimate β_1 using (10), we are facing an error of specification, whereas if we use (11), we face the problem of mismeasuring X_2. The LSE of β_1 based on (10) will be biased unless X_1 and X_2 are orthogonal; and the LSE of β_1 based on (11) will be biased unless Z is a perfect proxy for X_2 (i.e., unless $Z = X_2A$ for some A). Both, orthogonality of X_1 and X_2 and perfect proxies, are rare in empirical econometric research. McCallum (1972) and

WICKENS (1972) have compared the asymptotic biases of the two least squares estimators of β_1 and concluded that using the proxied model is preferable to using the truncated model. However, using a mean square error criterion, AIGNER (1974), FROST (1979), KINAL and LAHIRI (1983), and TERÄSVIRTA (1987) found that under certain conditions it would be preferable to use the truncated model.

In this section we propose a criterion for deciding between the truncated and the proxied model that is different from that of asymptotic bias or mean square error[2]. It is well known that biased estimation distorts the test statistic involved in testing hypotheses about β_1 by turning the relevant central F distribution into a noncentral F distribution, characterized by two noncentrality parameters. It then seems reasonable to suppose that the larger the magnitude of the noncentrality parameters, the greater the difference between the model in question and the true model (9). Our suggestion is to estimate the two noncentrality parameters involved and to use these estimates in deciding between (10) and (11). Our point of departure is a test of the null hypothesis $H_0:\beta_1=\beta_1^*$ against $H_A:\beta_1 \neq \beta_1^*$, which we consider for each of the two competing models in turn.

In the case of the truncated model, the F statistic appropriate for the test is

$$F_1 = \frac{(\hat{\beta}_1-\beta_1^*)'(X_1'X_1)(\beta_1-\beta_1^*)/K_1}{\hat{U}_1'\hat{U}_1/(n-K_1)} \tag{12}$$

where

$\hat{U}_1 = Y-X_1\hat{\beta}_1 = M_1Y$, $\hat{\beta}_1 = (X_1'X_1)^{-1}X_1'Y$, and $M_1 = 1-X_1(X_1'X_1)-1X_1'$.

When H_0 is true, it is straightforward to show that F_1 has a doubly noncentral F distribution with noncentrality parameters $[\beta_2'X_2'(I-M_1)X_2\beta_2,\ \beta_2'X_2'M_1X_2\beta_2]$ and degrees of freedom $[K_1, n-K_1]$. Similarly, for the proxied model the appropriate test statistic is

$$F_2 = \frac{(\tilde{\beta}_1-\beta_1^*)'(X_1'M_ZX_1)(\tilde{\beta}_1-\beta_1^*)/K_1}{\tilde{U}_2'\tilde{U}_2/(n-K_1-K_3)} \tag{13}$$

where

$\tilde{U}_2 = Y-X_1\tilde{\beta}_1-Z\tilde{\gamma}$, $\tilde{\beta}_1=(X_1'M_ZX_1)^{-1}X_1'M_ZY$, $\tilde{\gamma}=(Z'M_1Z)^{-1}(Z'M_1Y)$, and $M_Z=I-Z(Z'Z)^{-1}Z'$.

Again, it can be shown that F_2 has a doubly noncentral F distribution with noncentrality parameters $[\beta_2'X_2'MX_2\beta_2,\ \beta_2'X_2'(M_Z-M)X_2\beta_2]$ and degrees of freedom $[K_1,\ n-K_1-K_2]$, where

$$M = M_ZX_1(X_1'M_ZX_1)^{-1}X_1'M_Z.$$

2 The following analysis relies heavily on Sections 2 and 3 of BHATTACHARYYA and KMENTA (1989).

The specification of the noncentrality parameters involves $X_2\beta_2$ which is unknown and has to be estimated.

The estimation of $X_2\beta_2$ can proceed as follows. First, we obtain the LSE's of β_1 and γ using the proxied model (11). Next, we form the following 'pseudo' residuals:

$$\hat{V} = Y - X_1\tilde{\beta}_1 \tag{14}$$

where

$$\tilde{\beta}_1 = (X_1'M_ZX_1)^{-1}X_1'M_ZY$$

as before.

Substituting for Y from (9) into (14) we find

$$\hat{V} = N(X_2\beta_2) + NU \tag{15}$$

where

$$N = I - X_1(X_1'M_ZX_1)^{-1}X_1'M_Z.$$

Thus we have

$$E(\hat{V}) = NX_2\beta_2.$$

Under the standard assumptions of the classical regression model, $N \to I$ as $n \to \infty$. The LSE of $(X_2\beta_2)$ can be obtained from (15) since N is observable. However, since N is singular, we have to minimize $\|V - N(X_2\beta_2)\|$ with respect to $(X_2\beta_2)$ by using a generalized inverse procedure. If the resulting estimate is denoted by Q, and if δ_{11} and δ_{12} are the noncentrality parameters of the respective numerator and denominator of F_1 in (12), and δ_{21} and δ_{22} are the noncentrality parameters of the respective numerator and denominator of F_2 in (13), then we have the following

$$\hat{\delta}_{11}: \quad Q'(I - M_1)Q$$

$$\hat{\delta}_{12}: \quad Q'M_1Q$$

$$\hat{\delta}_{21}: \quad Q'MQ$$

$$\hat{\delta}_{22}: \quad Q'(M_Z - M)Q$$

The above results can be used in choosing between the truncated and the proxied model. If we find that

$$\hat{\delta}_{11} < \hat{\delta}_{21} \text{ and } \hat{\delta}_{12} < \hat{\delta}_{22}$$

then, for the given set of data, the truncated model is closer to the true model than the proxied model. If the inequalities are reversed, the opposite conclusion holds. In other cases we can use the distance from the origin, i.e.,

$$\Delta_1 = \hat{\delta}_{11}^2 + \hat{\delta}_{21}^2 \text{ and } \Delta_2 = \hat{\delta}_{21}^2 + \hat{\delta}_{22}^2$$

and choose the model with a lower Δ value.

4. Intrinsically latent variables

Econometric models may contain variables which are intrinsically not measurable but which may be characterized by various manifest indicators and/or causes. Such models have not been given much attention until the appearance of the seminal papers by ZELLNER (1970) and GOLDBERGER (1972). Both papers deal with the same problem of estimating a model of consumption with "permanent income" as a latent explanatory variable. The model, formulated as a "multiple cause model," can be represented as follows:

$$y = x^* \beta + \epsilon, \tag{16a}$$

$$x^* = Z\pi, \tag{16b}$$

$$x = x^* + u, \tag{16c}$$

where y, x^*, x, ϵ, and u are $(n \times 1)$ vectors, β is a scalar, $Z \rightarrow (n \times K)$, and $\pi \rightarrow (K \times 1)$. The variable Y stands for consumption and x^* represents permanent income, which is unobservable but can be expressed as a linear combination of observable characteristics Z. The stochastic disturbance ϵ and the error term u satisfy all classical assumptions and are mutually independent. Their variance-covariance matrices are $\sigma^2 I$ and $\sigma_u^2 I$ respectively.

Both Zellner and Goldberger approaced the problem of estimating the unknown parameters of the model by forming the reduced form equations

$$y = Z\pi\beta + \epsilon,$$

$$x = Z\pi + u.$$

Zellner proposed a two-stage procedure, starting with the estimation of σ^2 and σ_u^2 by s^2 and s_u^2 which are defined as follows:

$$s^2 = (n-K)^{-1}(y - Z\hat{\gamma})'(y - Z\hat{\gamma}),$$

$$s_u^2 = (n-K)^{-1}(x - Z\hat{\pi})'(x - Z\hat{\pi}).$$

where $\hat{\gamma}$ and $\hat{\pi}$ are the least squares estimators of the respective parameters obtained by applying the LS method to

$$y = Z\gamma + \epsilon,$$

$$x = Z\pi + u.$$

The second stage involves minimizing

$$S = s^{-2}(y - Z\pi\beta)'(y - Z\pi\beta) + s_u^{-2}(x - Z\pi)'(z - Z\pi) \tag{17}$$

with respect to β and π. Goldberger, on the other hand, proposed estimating σ^2, σ_u^2. β, and π simultaneously by maximizing the following log-likelihood function:

$$L = \text{constant} - (n/2)\log\sigma^2 - (2\sigma^2)^{-1}(y - Z\pi\beta)'(y - Z\pi\beta)$$
$$- (n/2)\log\sigma_u^2 - (2\sigma_u^2)^{-1}(x - Z\pi)'(x - Z\pi). \tag{18}$$

AIGNER *at al.* (1984, p. 1361) dismiss Zellner's procedure because it is presumably based on 'limited information" whereas Goldberger's method is based on "full information." However, the fact of the matter is that both procedures lead to estimators with the same asymptotic properties, and Zellner's method is computationally easier. The asymptotic equality of the two estimators could be expected by analogy with the two-stage Aitken and maximum likelihood estimation of seemingly unrelated regressions, see LAHIRI (1973/1974). If one were to adopt the approach of Zellner but would iterate instead of stopping with the second stage, the procedure would convergence to the MLE's of Goldberger, see OBERHOFER and KMENTA (1974). If the values of σ^2 and σ_u^2 were known *a priori*, the two methods would yield identical estimates, as can easily be seen by comparing (17) and (18).

The maximum likelihood estimators proposed by Goldberger are computationally difficult. Their calculation could be considerably simplified by rewriting (16) as

$$y = x\beta + \nu, \tag{19a}$$

$$x = Z\pi + u, \tag{19b}$$

where $\nu = \epsilon - u\beta$, and recognizing that (19a) and (19b) form a triangular system. For such systems LAHIRI and SCHMIDT (1978) proved that when the variance-covariance matrix of the disturbances is known or is *efficiently* estimated, the standard generalized least squares estimator is identical to the full information maximum likelihood estimator. Their suggestion is to treat (19) as a system of seemingly unrelated regressions and to use the iterative procedure IZEF proposed by KMENTA and GILBERT (1968).

The introduction of the multiple cause model by Zellner and Goldberger was soon followed by more complicated models such as MIMIC and LISREL. These models are described in recent econometrics texts, and are thoroughly discussed in specialized volumes such as BOLLEN (1989). However, as a perusal of applied econometric literature clearly shows, models with latent variables have not gained a great degree of popularity among empirical researchers. The reasons for this are not clear, since concepts for which there are no directly observable variables are quite common in economics. The warning concerning the inaccuracy of economic measurements issued by MORGENSTERN (1950) in the early days of econometrics has apparently gone largely unheeded to this day.

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