THE DAMPING OF DIFFERENTIAL ROTATION IN THE CORES OF NEUTRON STARS

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Differential rotation could significantly increase the upper mass limit of neutron stars. By adjusting the angular velocity at each point in the star to balance the local pressure gradient, it may be possible to stabilize massive stars against collapse. However, we shall show that differential rotation damps to a state of rigid body rotation in a time-scale of a few days and, consequently, cannot affect neutron star masses on longer time-scales.

We shall consider dissiptative mechanisms in two regimes: the formation of a hot differentially rotating neutron star—the usual supernovae hypothesis; and a less likely alternative, an initial state of a cold differentially rotating neutron star. In both cases, we take the rotational velocity to be of order c, the speed of light, since it is necessary to have large velocities to significantly increase the mass of a star near gravitational collapse.

HOT NEUTRON STARS

If neutron stars are born hot, either Ekman pumping or turbulent mixing will spin down a rapidly rotating core. A hot neutron star with KT $\geq$ 50–100 MeV cools to 1 MeV, the superconducting energy gap,[1] in 10^{-3} years.[2] For $\rho < 10^{15}$g/cm^{-3} neutron matter at these high temperatures behaves as a normal fluid, and if it can be shown that the time-scale for mixing is less than 10^{-3} years, only normal fluids need be considered.

The time-scale for viscous diffusion to damp out differential rotation in a hot neutron star is about 10 million years. For spin-down to occur on a shorter time-scale, it is necessary to show that a macroscopic flow in the interior is established. If the interior consists of a rapidly rotating core or if there are any discontinuities in the angular velocity, for example shell structure, an Ekman flow can develop which would mix the interior in $\sim 10^{-3}$ sec. Barring this, turbulent mixing will spin down the interior on about the same time-scale.

Using a simple, highly idealized model of a differentiatlly rotating neutron star, we can estimate the Ekman spin-down time of the core. Our model consists of a neutron star of radius R_s subdivided into N shells of equal radial increments $R = R_s/N$. The outer radius of the n^{th} shell, R_n, is given by $R_n = nR$. Each shell rotates as a rigid body at an angular velocity $\Omega_n = c/nR$, choesen so that the equatorial velocity of the outer edge of each shell is of order c but is not ultra-relativistic, so that we may neglect $\gamma^{-1} = \sqrt{1 - (v/c)^2}$. A more realistic model would have constant angular velocity on cylinders to satisfy the Proudman-Taylor requirement that the angular velocity be only a function of radius. However, the Ekman flow will not be significantly different in the two cases.

The classic example used to illustrate Ekman flow is a stirred cup of tea. The fluid flow is evident from the motion of the tea leaves. It is observed that fluid along the side walls of the cup moves down and then along the bottom, depositing tea leaves at the center of the cup. This may be understood by considering an initial state in which the fluid rotates as a rigid body and is in equilibrium. Fluid in the boundary layer at the wall experiences a drag and slows down, with a resulting decrease in the centrifugal acceleration. Because the pressure gradient is almost the same as before the differential rotation was established, fluid in the boundary layer now flows toward the axis of rotation. In our model of a differentially rotating star, a turbulent Ekman boundary layer is established. Elsewhere[3] we have estimated the spin-down time of the core by finding the time that it takes all the fluid in the core to flow through the turbulent boundary layer. This time has been found to be

$$T_{\text{turb}} \sim \frac{\Omega_1}{\zeta(\Omega_1 - \Omega_2)^2}.$$

The dimensionless parameter ζ is defined by the relation $\sigma = \zeta \rho V^2$, where σ is the surface stress exerted by the fluid on the turbulent boundary, ρ is the fluid density, and V is the relative fluid-boundary velocity difference. The turbulence parameter ζ has been found to be $\zeta \sim 0.1$ using similarity arguments[4] based on measurements of the discharge of a free turbulent jet into a fluid-filled space.

In terms of the total radius of the star and the number of shells that we arbitrarily chose to divide the star into,

$$T_{\text{turb}} \sim \frac{4R_s}{\zeta c N}.$$

By choosing $N = 2$ we can set up upper limit on the spin-down time. Taking $R_s = 10$ km, the spin-down time is less than a millisecond—or less then 10^{-7} of the cooling time. Thus, if there is an Ekman flow our estimate would have to be in error by a factor of 10^6 for this model to be inapplicable. While the Ekman flow is a solution to the given constraints, it is not clear that an Ekman circulation *must* develop. It is also possible that density stratification could stop the Ekman flow. However, because the interior is definitely turbulent—the Reynolds number $\sim 10^{17}$—if an Ekman flow does not develop, turbulent mixing will occur. Turbulence will mix the interior of the star in a time-scale $T_{\text{mix}} \sim r/c$ since r is the only length scale, and the velocity relative to that of rigid body rotation is of order c. Thus, $t_{\text{mix}} \sim R_s/c \sim 10^{-4}$ sec. If any magnetic fields were present, they would decrease the spin-down time. Magnetic field lines passing through a surface across which differential rotation takes place would further couple the two regions, tending to bring them to a state of rigid body rotation.

Differential Rotation in Cold Neutron Stars

At temperatures below 1 MeV and densities in the range $5 \times 10^{13}\text{g/cm}^{-3}$–$5 \times 10^{14}\text{g/cm}^{-3}$ neutron matter will be superfluid. Laboratory experiments using

He^4 superfluids show that only below the so-called "critical velocity" does a superfluid have zero viscosity. If superfluid helium is placed in a bucket initially at rest which subsequently starts to rotate with a wall velocity above the critical velocity, the superfluid quickly begins to rotate as a rigid body. The spin-up time is about what one would expect assuming a laminar Ekman boundary layer[5] based on an order of magnitude argument[6] for the superfluid eddy viscosity. The viscosity is due to tangled vortex lines and must be of the form $\nu \sim u\ell$, where ν is the kinematic viscosity, u is the turbulent velocity, and ℓ is the size of the dominant eddies. For a tangled array of vortices, ℓ cannot be less than the spacing between vortex lines, and u cannot be less than $n/\pi m \ell$ because of the quantization condition for vortices, $\oint \vec{v} \cdot d\vec{\ell} = n(h/m)$. Therefore, $\nu \sim h/\pi m \sim 10^{-3} cm^2 s^{-1}$ Macroscopically, the fluid behaves normally; it is only a superfluid microscopically—between vortex lines. The value, $\nu \sim 10^{-3} cm^2 s^{-1}$, coincidently is about the same as for hot neutron matter, and so a superfluid neutron star is also subject to Ekman pumping and turbulent mixing.

As the density increases above $5 \times 10^{14} g/cm^{-3}$, the fraction of matter that is superfluid is decreasing, and we shall consider matter to be degenerate. The viscosity of degenerate matter is quite large, principally because the degenerate particles can travel long distances between collisions. The kinematic viscosity of neutron matter at $\rho = 4 \times 10^{14} g/cm^{-3}$ for $T = 10^8$ deg K has been calculated[7] to be $\nu = 5 \times 10^4 cm^2 s^{-1}$. This viscosity is large enough so that the viscous diffusion time-scale $T_{diff} \sim R^2/\nu$, which is the longest spin-down time-scale, is quite short. For a 1-km core the viscous diffusion spin-down time is about 2 days.

We may conclude that an initial state of differential rotation is transformed into a state of rigid body rotation in a time-scale of less than a few days. Rigid body rotation cannot effectively increase the mass of extremely relativistic stars. This is the case because the pressure gradient increases so rapidly near the center of the star, due to the relativistic contribution of pressure to the gradient, that rigid body rotation spins off mass from the outer stellar surface before it can effectively balance the gradient near the center. For a massive neutron star the mass increase has been found to be less than 30%.[8,9]

These results have an important bearing on the neutron star upper mass limit.[10-13] We shall assume that general relativity is correct in the following discussion, though Ken Brecher earlier at this meeting has discussed some of the consequences of alternative relativity theories. The most massive model of a neutron star is a constant density model. We have argued elsewhere[11] that because neutron matter calculations become uncertain above densities of $2 \times 10^{14} g/cm^{-3}$, we must choose this to be the core density of our most massive star. Including a 30% increase above the non-rotating limit[11] due to rigid body rotation,[8,9] we find the neutron star upper mass limit to be 11 $M_\odot$. This conclusion has been discussed in more detail elsewhere.[3]

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