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Technical Notes

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Stacked Packaging Laminar-Convection-Cooled Printed Circuit Using the Entropy Generation Minimization Method

Takahiro Furukawa* and Wen-Jei Yang†
University of Michigan, Ann Arbor, Michigan 48109
and
Shuichi Torii‡
Kumamoto University, Kumamot 860-8555 Japan

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Nomenclature

A_b = heat-sink-base surface area
A_c = cross-sectional area of the fin
C_p = specific heat of the air
EI = Elenbaas number
g = gravity
H = fin height
h = heat transfer coefficient around fin
k_a = thermal conductivity of the air
k_h = thermal conductivity of the heat sink
L = fin length
N = total number of fins of the heat sink
P = plate-air parameter
P_e = perimeter of the fin
p_f = fin pitch
q_b = heat transfer rate to the heat sink base
q_f = heat transfer rate to the fin
R_{fin} = thermal resistance of the fin
R_{hs} = overall thermal resistance of the heat sink
$\dot{S}_{\mathrm{gen}}$ = entropy generation rate
$\dot{S}_{\mathrm{gen,hs}}$ = entropy generation rate from the heat sink
T_∞ = ambient temperature
t_b = base thickness of the heat sink
t_f = fin thickness of the heat sink
W = heat sink width
z = fin space
β = thermal expansion coefficient
η_f = fin efficiency
$\bar{\theta}$ = average temperature difference between the heat sink and ambient air
θ_b = temperature difference between the heat sink base and ambient air
μ = viscosity of the air
ρ = density of the air

I. Introduction

THE method of entropy generation minimization (EGM) introduced by Bejan [1,2] is used as an optimization tool of thermodynamic systems in the present study. The theory of EGM states that all real systems suffer their thermodynamic imperfection due to heat transfer, fluid flow, and mass transfer irreversibility. Consequently, the entropy generation can be used as a measure of the system's departure from reversibility. Based on this theory, the minimization of the entropy generation in the system leads to the optimization of the system. The EGM method has been applied to a broad range of engineering fields [3–5] such as heat exchangers and solar power and refrigeration plants.

Recently, Culham and Muzychka [6] applied the EGM method to optimization of plate-fin heat sinks in electronics applications through simultaneous optimization of heat sink design parameters. Dealing with various geometric parameters, heat transfer rates, and material properties of heat sinks, the application of multiparameter optimization is more productive than an analytical approach with empirical equations and powerful numerical simulation that cannot simultaneously optimize more than two parameters.

The theory of the EGM method has been extensively applied by previous researchers, but its accuracy remains uncertain. In this study, the fin pitch of a plate-fin heat sink in a free-convection environment is optimized by the EGM method. Results are compared with both analytical and numerical optimization results. Fin width, length, and height and heat dissipation are changed in the fin-pitch optimization process.

II. Design Parameter

Figure 1 shows the design parameters of a plate-fin heat sink. Fin pitch was optimized for three different parameter combinations, as listed in Table 1. The heat-sink-base thickness t_b and fin thickness t_f are 5 and 2 mm, respectively. The gravity direction is along the heat sink channel, as shown in Fig. 1. The ambient temperature around the heat sink and thermal conductivity of the heat sink material are set to be 25°C and 200 W/mK, respectively.

III. Model Development for the EGM Method

The entropy generation rate for an extended surface under free convection was derived by Bejan [2] as follows:

$$\dot{S}_{\mathrm{gen}} = \frac{q_b \theta_b}{T_\infty^2} \tag{1}$$

The temperature difference between the heat sink base and the ambient air, θ_b, may be expressed in terms of the overall heat sink resistance R_{hs} as

$$\theta_b = q_b R_{\mathrm{hs}} \tag{2}$$

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*Project Manager, Department of Mechanical Engineering; currently, Project Manager, Corning Company, Shizuoka, Japan.
†Professor, Department of Mechanical Engineering.
‡Professor, Department of Mechanical System Engineering, 2-39-1 Kurokami.

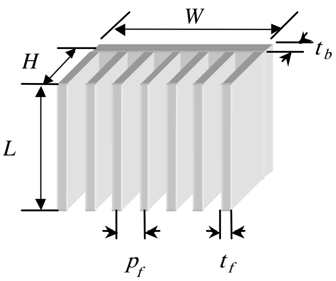

Fig. 1 Design parameters of a plate-fin heat sink.

Therefore, the entropy generation rate from a plate-fin heat sink under free convection is rewritten as

$$\dot{S}_{\text{gen,hs}} = \frac{q_b^2 R_{\text{hs}}}{T_\infty^2} \tag{3}$$

The overall heat sink resistance R_{hs} is defined as

$$R_{\text{hs}} = \frac{1}{(N/R_{\text{fin}}) + hL(W - t_f N)} + \frac{t_b}{kLW} \tag{4}$$

$$N = \frac{W - t_f}{p_f} + 1 \tag{5}$$

where N is the total number of fins, and R_{fin} is the fin resistance and is defined as [7]

$$R_{\text{fin}} = \frac{1}{\sqrt{hP_e k_h A_c}\tanh(mH)} \tag{6}$$

where

$$m = \sqrt{\frac{hP_e}{k_h A_c}} \tag{7}$$

$$P_e = 2(L + t_f) \tag{8}$$

$$A_c = Lt_f \tag{9}$$

Elenbaas [8] gave the heat transfer coefficient around the heat sink h as

$$h = \frac{k_a}{z}\left[\frac{576}{(EI)^2} + \frac{2.873}{(EI)^{1/2}}\right]^{-1/2} \tag{10}$$

where EI is the Elenbaas number, defined as

$$EI = \frac{\rho^2 \beta g C_p z^4 \bar{\theta}}{\mu k_a L} \tag{11}$$

in which

$$z = p_f - t_f \tag{12}$$

and $\bar{\theta}$ is the average temperature difference between the heat sink and the ambient air, defined as

$$\bar{\theta} = \overline{T_{\text{hs}} - T_\infty} \tag{13}$$

Substituting Eqs. (4–13) into Eq. (3), one gets the entropy generation rate in terms of geometric parameters, heat generation, and base temperature as

$$\dot{S}_{\text{gen,hs}} = f(L, W, H, t_f, t_b, p_f, q_b, T_\infty, \bar{\theta}) \tag{14}$$

The entropy generation rate equation (14) can be used to optimize any parameter in a plate-fin heat sink:

$$\dot{S}_{\text{gen,hs}} = f(L, W, H, t_f, t_b, p_f, q_b, T_\infty, \bar{\theta}) = f(x_i) \tag{15}$$

where $i = 1, 2, \ldots, n$, and n is the number of parameters.

Using the Newton–Raphson method [9], the desired parameter is optimized by minimizing the entropy generation rate from the heat sink as

$$\frac{\partial \dot{S}_{\text{gen,hs}}}{\partial x_i} = 0 \tag{16}$$

IV. Fin-Pitch Optimization of the Plate-Fin Heat Sink Using the EGM Method

As previously mentioned, the entropy generation rate of the plate-fin heat sink can be written as the function of fin length, fin height, base width, fin thickness, base thickness, fin pitch, heat transfer rate to the heat sink, and ambient temperature. In electronics cooling applications, an overall heat sink volume size is determined based on the size of the chip to which the heat sink is attached and the physically allowable space limit of the electronic device. The fin length and height and base width are specified in the early design stage. Fin thickness and base thickness might be fixed in terms of heat spreading resistance of the heat sink material. The fin base should have enough thickness so that heat can spread evenly across the heat sink base. The fin should be thick enough so that heat spreads evenly from the base to the fin tip. If the amount of heat-generation rate applied to the heat sink base is given, the most important design parameter of the plate-fin heat sink is its fin pitch. Therefore, all parameters in the heat sink design except the fin pitch are fixed, and the entropy generation minimization method is applied to the fin-pitch optimization of the plate-fin heat sink.

The temperature difference between the heat sink base and the ambient air [θ_b, defined in Eq. (2)] and the fin efficiency η_f have to be assumed before iteration starts. With the use of a high-thermal-conductivity material for the heat sink, it is assumed that fin efficiency is 0.99. The two assumed values must be verified after the iteration is converged:

$$\bar{\theta} = \eta_f \theta_b \tag{17}$$

where

$$\eta_f = \frac{\tanh(mH)}{mH} \tag{18}$$

A code for optimization using the EGM method was written by Wolfram [10].

For parameter set 1 in Table 1, the fin pitch and heat-sink-base temperature are initially assumed to be 4 mm and 77.2°C, respectively. The ambient temperature around the heat sink is set to be 25°C. The entropy generation rate from a heat sink decreases and converges, and Fig. 2 shows the entropy generation rate from a heat sink with respect to its fin pitch for parameter set 1. As can be seen from the figure, the entropy generation rate is minimized at the optimized fin pitch of 8.20 mm.

Table 1 Heat sink design parameter combinations

Parameter set	1	2	3
Heat dissipation, W	5	10	20
Heat sink width, cm	5	10	10
Fin length, cm	5	10	10
Fin height, cm	1.5	3	5

V. Optimization Using the Analytical Method

The analytical method used for fin-pitch optimization was introduced by Bar-Cohen and Kraus [11], based on the earlier work

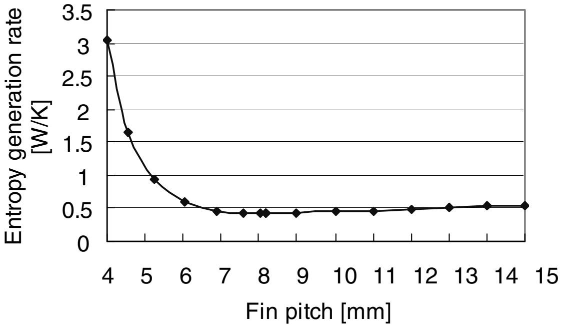

Fig. 2 Entropy generation rate from the heat sink with respect to its fin pitch for parameter set 1.

of Elenbaas [8]. The procedure is demonstrated later for parameter set 1.

First, the average fin temperature is assumed to be 80°C, with the average air temperature around the fin at 52.5°C. The temperature difference between the fin base and ambient air, θ_b, is 55°C. The thermal properties of the air at 52.5°C are used in the calculations. Similarly, with the use of a high-thermal-conductivity material for the heat sink, the assumed fin efficiency of $\eta_f = 0.99$ yields the average temperature difference between the fin surface and ambient air of

$$\bar{\theta} = \eta_f \theta_b = 0.99 \times 55 = 54.45°\mathrm{C} \tag{19}$$

The fin efficiency will be later verified by the result of the optimization process.

Bar-Cohen and Rohsenow [12] gave for the optimum fin spacing z_{opt} as

$$z_{\mathrm{opt}} = 2.714P \tag{20}$$

where P is called the plate-air parameter, with the thermal properties of the air evaluated at the average temperature between the plate and the ambient air, defined as

$$P = \left(\frac{\mu k_a L}{C_p \rho^2 g \beta \bar{\theta}}\right)^{1/4} \mathrm{m}$$

Computation yields P and z_{opt} for this plate fin as

$$P = 1.948 \text{ mm}, \qquad z_{\mathrm{opt}} = 5.287 \text{ mm} \tag{22}$$

Therefore, the optimum fin pitch $p_{f,\mathrm{opt}}$ is

$$p_{f,\mathrm{opt}} = 7.287 \text{ mm} \tag{23}$$

This optimized fin pitch gives the total number of fins:

$$N = \frac{W}{\mathrm{p}_{f,\mathrm{opt}}} + 1 = 7.862 \approx 8 \tag{24}$$

With the use of eight fins, the designed fin space z and fin pitch p_f are obtained:

$$z = 4.857 \text{ mm}, \qquad p_f = 0.006857 \text{ mm} \tag{25}$$

Equation (10) yields the heat transfer coefficient for natural convection between vertical isothermal plates as

$$h = 6.296 \text{ W/m}^2 \cdot \mathrm{K} \tag{26}$$

For a fin with negligible heat transfer from the tip (in other words, for an adiabatic fin tip), the fin heat transfer rate q_f can be expressed as [4]

$$q_f = M \tanh(mH) \tag{27}$$

where

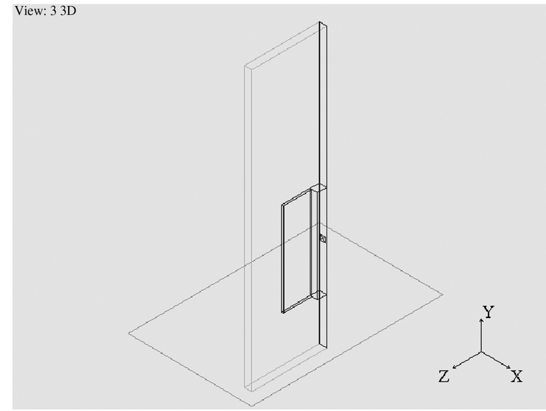

Fig. 3 Flotherm simulation model of a portion of the heat sink.

$$M = \sqrt{hP_e k_h A_c}\theta_b, \qquad m = \sqrt{hP_e/k_h A_c} \tag{28}$$

P_e is the fin perimeter, and A_c is the fin cross-sectional area. Thus, one finds

$$q_f = 0.539 \text{ W} \tag{29}$$

With the heat transfer coefficient known around the fins, the total heat dissipation from the plate fin with the base surface area A_b is found to be [13]

$$Q = Nq_f + hA_b\theta_b = 4.899 \text{ W} \tag{30}$$

The fin efficiency is already defined by Eq. (18). It gives the fin efficiency of

$$\eta_f = 0.998 \cong 0.99 \tag{31}$$

which agrees with the assumed value. Similarly, the same optimization is conducted for parameter sets 2 and 3.

VI. Optimization Using CFD Software

The fin pitch of the plate heat sink can be optimized using the computational fluid dynamics software Flotherm, for which the numerical scheme is based on the finite volume method. This software has been used in the electronics cooling design. The discretization method employed here uses a finite difference formulation. The grid system is changed from $100 \times 200 \times 50$ to $200 \times 400 \times 100$ to obtain a grid-independent solution: hence, a grid system of $200 \times 100 \times 50$ ($X \times Y \times Z$) nodal points with uniformly distributed nodal points. The optimization procedure is demonstrated later for parameter set 1. Figure 3 shows the simulation model for this plate heat sink. Because the arrays of straight plate fins are symmetric, it is sufficient to perform the computational simulation for only one portion of the arrays of fins. Let the number of fins of this model be seven and the fin pitch p_f be 8 mm. For the total heat transfer rate to the heat sink base (q_b) of 5 W, the fraction of heat input q_p attached to the fin base in this simulation is

$$q_p = q_b \frac{p_f/2}{W} = 0.4 \text{ W} \tag{32}$$

Table 2 Simulation result of plate-fin heat sink

Number of fins	Fin pitch, mm	Base temperature, °C
11	4.80	93.87
10	5.33	87.68
9	6.00	83.00
8	6.86	80.66
7	8.00	80.38
6	9.60	83.54
5	12.00	88.85

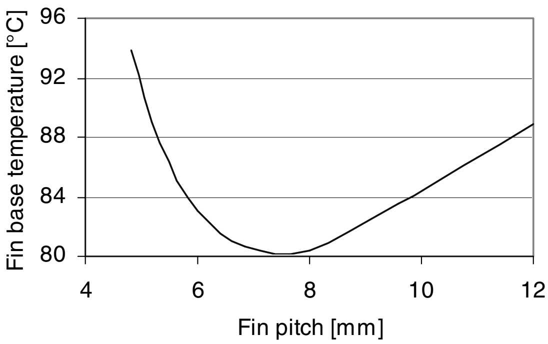

Fig. 4 CFD optimization: fin base temperature vs fin pitch for parameter set 1.

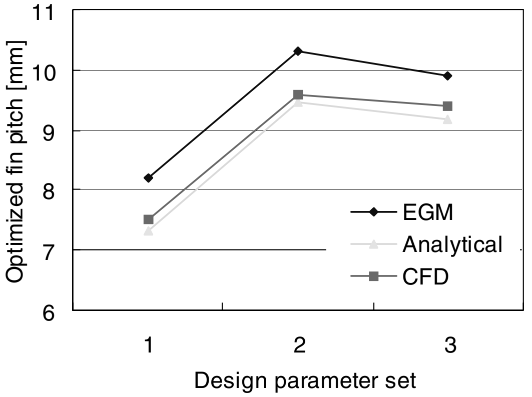

Fig. 5 Optimized fin-pitch results for the analytical, CFD, and EGM methods.

Let the direction of gravity be in the negative y direction. Simulations were repeated for the number of fins $N = 5$ to 11. Results are listed in Table 2, and Fig. 4 shows the fin base temperature vs fin pitch. An inspection of Table 2 and Fig. 4 reveals the optimized fin pitch to be 7–8 mm and the optimized number of fins to be 7 or 8.

VII. Comparison of EGM Optimization Results with Other Conventional Methods

Figure 5 compares the results of fin pitch optimized using analytical, CFD, and EGM methods. The optimized fin pitch of the EGM method is slightly higher than those of the analytical method and CFD, but it shows the same trend as with the other optimized fin pitches. The discrepancy between the analytical and EGM methods is within 7–12%, whereas that between the CFD and EGM methods is 5–9%. EGM and theoretical methods use the existing results to determine the heat transfer coefficient around the heat sink, whereas the corresponding value for CFD is obtained based on the predicted values of the velocity and temperature distributions. Thus, the discrepancy between EGM and the other two results is ascribed to the heat transfer coefficient around the heat sink.

VIII. Conclusions

The EGM method predicted slightly higher fin pitch than the analytical and CFD methods, but with the same trend. Even though the three methods are based on completely different theory, the maximum discrepancy in the 12% range is small and acceptable. This is because three different methods show the same trend with the other optimized fin pitches. Therefore, the EGM method is very reliable to use in the fin-pitch optimization of the plate-fin heat sink subject to free convection.

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