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Natural Convection
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RADIATION AFFECTED
TURBULENT NATURAL CONVECTION

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ABSTRACT

The small-scale structure of turbulent natural convection based on the concepts introduced by Kolmogorov and Taylor and extended by Arpaci is further extended to incorporate the effects of radiation. On dimensional grounds, a radiation-affected thermal microscale for fluids with $Pr \geq 1$,

$$\eta_0 \sim (va^2/P_B)^{1/4} Pr^{-1/4}[1+\tau[1-\varepsilon_w/(2\tau)]\tau P]^{1/2}$$
$$[1+Pr+\tau[1-\varepsilon_w/(2\tau)]\tau P]^{1/4}$$

is proposed. Two limits of this scale without radiation effects for $Pr \sim 1$ and $Pr \rightarrow \infty$ are the Kolmogorov and Batchelor scales, respectively. An alternative form, expressed in terms of the buoyancy force rather than of the buoyant production of energy, is shown to be

$$\eta_0 \sim (va/g\beta\Delta T)^{1/3}Pr^{-1/3}[1+\tau[1-\varepsilon_w/(2\tau)]\tau P]^{1/3}$$
$$[1+Pr+\tau[1-\varepsilon_w/(2\tau)]\tau P]^{1/3}$$

A heat transfer model based on this scale for radiation affected turbulent natural convection, with a characteristic length l of the geometry, is shown to be

$$Nu_l \sim Ra_l^{1/3}Pr^{1/3}[1+\tau[1-\varepsilon_w/(2\tau)]\tau P]^{-1/3}$$
$$[1+Pr+\tau[1-\varepsilon_w/(2\tau)]\tau P]^{-1/3} + 4(\tau-\varepsilon_w/2)\tau P_l$$

NOMENCLATURE

a	thermal diffusivity, $k/(\rho c)$;
c	specific heat at constant pressure;
c_1, c_2, c_3	constants in the correlation of heat transfer for laminar flow;
E_b	black body emission;
g	gravitational acceleration;
G_x	local Grashof number, $g\beta\Delta Tx^3/v^2$;
k	thermal conductivity;
k_p	*Planck mean of absorption coefficients;*

k_R	Rosseland mean of absorption coefficients;
k_M	mean absorption coefficient, $(k_p/k_R)^{1/2}$;
Nu	local Nusselt number;
P	Planck number, $(E_{bw}-E_{b\infty})/(kk_p\Delta T)$;
P	inertial production;
P_B	buoyant production;
P_0	thermal production;
Pr	Prandtl number, v/a;
P_∞	ambient Planck number, $4\sigma T_\infty^4/(kk_pT_\infty)$;
q	heat flux;
S_{ij}, s_{ij}	mean and fluctuating strain rates;
T	temperature;
Ra_x	local Rayleigh number, $g\beta\Delta Tx^3/(va)$;
R_{Tc}	combined temperature difference and wall diffusive emissivity effects;
u, U	fluctuating and mean velocities in the x-direction;
v	Kolmogorov velocity scale;
x	variable along plate wall;
y	variable normal to plate wall.

Greek symbols

β	coefficient of thermal expansion;
Δ	difference of the corresponding value;
Γ_x	local dimensionless number, $\tau_x/(G_x/4)^{1/4}$;
δ	thickness of the momentum boundary layer;
δ_0	thickness of the thermal boundary layer;
ε	dissipation;
ε_w	diffusive emissivity of plate wall;
η	weighted nongrayness, $(k_p/k_R)^{1/2}$, or Kolmogorov length scale for turbulent flows;
η_0	thermal length scale for turbulent flows;
λ	temperature ratio, $(T_w-T_\infty)/T_w$;
v	kinematic viscosity;
ρ	density of the fluid;
σ	Stefan-Boltzman constant;

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τ   optical thickness, $k_p\eta_0$, for turbulent flows;

τ_x local Bouguer number, $k_p x$, for laminar flows;

θ,Θ temperature scale and mean temperature scale;

Superscripts

c   convection;
k   conduction;
R   radiation;
~   instantaneous value;
−   mean value.

Subscripts

l   quantity based on the length scale of geometry;
i,j   direction index;
o   reference value;
x   local value;
w   wall value;
∝   ambient value.

1. INTRODUCTION

The small-scale structure of turbulence has received intensive attention after Taylor[1] and Kolmogorov[2], who respectively proposed an inviscid estimate for dissipation in homogeneous turbulent flows and an isotropic estimate for kinetic scales for this dissipation. This fundamental idea was extended to the small scales for dynamically passive scalar contaminants in a turbulent flow by Oboukhov[3], Corrsin[4] and Batchelor[5]. Also, Priestley[6] modelled the turbulence in the lower atmosphere and Townsend[7] measured the turbulence in terms of these scales.

Although the small scales have been extensively used in the development of some energy and entropy spectra, their relevance to the foundations of heat and mass transfer correlations have been overlooked except for the recent studies by Arpaci et al.[8-12] who all showed that the microscales have significance in heat and mass transfer correlations. Yet the radiation affected microscales, which are very important to the modelling of some combustion phenomena such as large-scale fires, industrial furnaces and combustors[13,14], so far remained undisclosed.

The objective of this study is to demonstrate the effect of thermal radiation on the microscales of turbulent natural convection and also the relation between the small scale and the integral scale of turbulence. The paper begins with the problem of laminar natural convection which is used to illustrate the use of dimensional arguments. This is followed by the development of the relevant thermal scale and the heat transfer correlation based on this scale, and finally, by the conclusions of the study.

2. RADIATION AFFECTED LAMINAR NATURAL CONVECTION

Problem description

Consider a heated, semi-infinite, vertical plate in an infinite expanse of stagnant and radiating gas. To simplify the problem and to compare the results of the present study with that of Arpaci[15], the following assumptions are made: the gas is perfect, thin and gray. Radiation pressure, scattering, and the contribution of radiation to internal energy are negligible. Non-equilibrium effects other than diffusion and radiation are also neglected.

Radiation

The relevant radiative heat flux for one-dimensional thin gas is[16]

$$dq^R/dy = 4k_M\eta\left[(E_b-E_\infty)-\varepsilon_w(E_{bw}-E_{b\infty})\exp(-\sqrt{3}k_M y)/2\right] \qquad (1)$$

Here $k_M=(k_p k_R)^{1/2}$, $\eta=(k_p/k_R)^{1/2}$, where k_p and k_R denote the Planck mean and Rosseland mean of absorption coefficients, respectively. For the gray gas $(\eta=1)$, equation (1) is reduced to

$$dq^R/dy = 4k_p\left[(E_b-E_\infty)-\varepsilon_w(E_{bw}-E_{b\infty})\exp(-\sqrt{3}k_p y)/2\right]. \qquad (2)$$

Dividing this expression into two terms and considering the value close to wall, as follows:

$$dq^R/dy)_w = dq^R/dy)_1 + dq^R/dy)_2, \qquad (3)$$

where $dq^R/dy)_1=4k_p(E_b-E_{b\infty})$ denotes the radiative energy difference between the local element and ambient with a characteristic length δ which is relevant to the gradient of the radiative heat flux; and $dq^R/dy)_2=4k_p\left[-\varepsilon_w(E_{bw}-E_{b\infty})\exp(-\sqrt{3}k_p y)/2\right]$ the radiative energy difference between wall and ambient due to the attenuating effects noticed by Arpaci et al.[17] with a characteristic length $1/k_p$.

On dimensional grounds, linearizing $E_{bw}-E_{b\infty}$, assuming $\theta\sim\Delta T$, the radiative heat flux can be approximated as

$$q^R_w \sim 4k_p\sigma T_\infty^3\theta[\delta-2\varepsilon_w/k_p] \qquad (4)$$

and

$$dq^R/dy)_w \sim 4\sigma T_\infty^3\theta R_{T_b}, \qquad (5)$$

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where $R_{T\varepsilon}=k_p f(\lambda)(1-\varepsilon_w/2)$, with $f(\lambda)=(\lambda+2)(\lambda^2+2)$ and $\lambda=(T_w-T_\infty)/T_\infty$, describes the combined effects of the temperature difference and the wall emissivity.

Dimensional Analysis of governing equations

Based on the foregoing assumptions the momentum balance becomes, on dimensional grounds,

$$u\, u/x + v\, u/\delta^2 \sim g\, \Delta\rho/\rho \sim g\,\beta\,\Delta T, \qquad (6)$$

where δ is the thickness of the momentum buoyancy layer and β is the coefficient of thermal expansion of the fluid. Equation (6) states that the driving buoyancy force, is balanced by the inertial force and the viscous force (dissipation). Also the thermal balance is

$$u\, 0/x \sim 1/\rho c(k\, 0/\delta_0^2 + dq^R/dy) \qquad (7)$$

δ_0 being the thickness of the thermal boundary layer. Equation (7) states that the heat carried by the fluid comes from conduction and radiation. Following the assumption made by Squire[16] as far as the heat transfer is concerned, that is

$$\delta \sim \delta_0, \qquad (8)$$

and inserting equation (5) into equation (7) results in

$$u \sim ax/\delta_0^2 + a\tau_x P_\infty R_{T\varepsilon}, \qquad (9)$$

where $a=k/\rho c$ is the thermal diffusivity of the fluid, $\tau_x=k_p x$ is the local Bouger number and $p_\infty=4\sigma T_\infty^4/(kk_p T_\infty)$ is the ambient Planck number. Thus, from equations (6), (8) and (9), and with the assumption $\tau_x^4 P_\infty^2/G_x \ll 1$, one obtains

$$(x/\delta)^4(1+1/Pr)$$
$$+ (x/\delta)^2\tau_x^2 p_\infty R_{T\varepsilon}(1+2/Pr) \sim Ra_x, \qquad (10)$$

where $Pr=v/a$, $G_x=g\beta\Delta Tx^3/v^2$ and $Ra_x=g\beta\Delta Tx^3/va$, are the Prandtl number, the local Grashof number and the local Rayleigh number, respectively. To simplify the analysis, equation (10) is rearranged with the assumptions of $Pr\sim1$ and $\tau_x^4 p_\infty^2 \ll G_x$ to yield:

$$x/\delta_0 \sim G_x^{1/4}(1+\tau_x^2 p_\infty R_{T\varepsilon}/G_x^{1/2})^{1/2} \qquad (11)$$

With a truncated binomial expression, equation (11) gives

$$x/\delta_0 \sim G_x^{1/4}(1+\tau_x^2 p_\infty R_{T\varepsilon}/G_x^{1/2}/2). \qquad (12)$$

Heat Transfer

The energy balance at the wall gives

$$q^c_w = q^k_w + q^R_w \qquad (13)$$

The local Nusselt number is then as

$$Nu_x \equiv q^c_w/q^k_x = q^k_w/q^k_x + q^R_w/q^k_x \qquad (14)$$

Let the conduction near the wall be characterized by the thickness of the sublayer, δ_0. The Nusselt number thus becomes

$$Nu_x \sim (k\Delta T/\delta_0)/(k\Delta T/x) + q^R_w/(k\Delta T/x)$$

or,

$$Nu_x \sim x/\delta_0 + q^R_w x/(k0). \qquad (15)$$

Combining equations (4) and (12) with equation (15) and neglecting the higher order terms, equation (15) leads to

$$Nu_x/G_x^{1/4} \sim 1 + \varepsilon_w\Gamma_x p_\infty$$
$$-\Gamma_x^2 p_\infty(1+R_{T\varepsilon}/2), \qquad (16)$$

where $\Gamma_x=\tau_x/(G_x/4)^{1/4}$ is the dimensionless number describing the ratio of radiation to buoyancy. An equality corresponding to equation (16) is

$$Nu_x/(G_x/4)^{1/4} \approx c_1 + c_2\Gamma_x p_\infty$$
$$+ c_3\Gamma_x^2 p_\infty, \qquad (17)$$

where c_1, c_2 and c_3 are the numerical constants. The method of multiple regression was employed to obtain the best fit to Arpaci's results[15] for $\lambda=0.1$, $\varepsilon_w=1.0$ and $Pr=0.733$. The resulting numerical values for c_1, c_2 and c_3 are 0.53997 and 1.16017 and -1.251, respectively; and the standard errors for these coefficients are less than 1% and the correlation coefficient is 1.0. This excellent agreement between current study and Arpaci's[15] is shown in Fig. 1.

Without the radiation effects, equation (17) reduces to Squire's solution for laminar natural convection along a vertical plate. Although equation (17) was fitted without the effects of temperature difference, λ, it is still quite appropriate since the scaled Nusselt number, $Nu_x/(G_x/4)^{1/4}$, is a weak function of λ, embedded in $R_{T\varepsilon}$, which is also demonstrated in the Fig.2 of Arpaci's[15] with small Γ_x. Besides, the scaled radiative Nusselt number without the constant term which represents the contribution from the pure natural convection is proportional to the optical thickness, Γ_x, which is also observed by Arpaci[15] (if the second

order term is neglected). In the following section, this approximation will be used in the turbulent case to obtain the thermal scales and the heat transfer correlation.

3. RADIATION AFFECTED TURBULENT NATURAL CONVECTION

Radiation

Similar to the laminar case radiative heat flux in the turbulent flow can be written, on dimensional grounds, as:

$$q^R_w \sim 4\tau[1-\varepsilon_w/(2\tau)](E_{bw}-E_{b\infty}), \qquad (18)$$

where $\tau=k_p\eta_0$ is the optical thickness based on η_0 and $P=(E_{bw}-E_{b\infty})/(k_pk\Delta T)$ is the Planck number germane to the turbulent flow. Accordingly, the ratio of the radiative heat flux to the conductive heat flux is, with

$$q^k_w \sim k\Delta T/\eta_0,$$

$$q^R_w/q^k_w \sim \tau[1-\varepsilon_w/(2\tau)]\tau P. \qquad (19)$$

Again, equation (19) shows that this ratio is proportional to the optical thickness, τ, if τ is small.

Radiation Affected Turbulence

Following the usual practice, decompose the instantaneous velocity and temperature of a buoyancy-driven turbulent flow into a temporal mean and fluctuations

$$u_i = U_i + \tilde{u}_i \quad \text{and} \quad \theta_i = \Theta_i + \tilde{\theta}_i$$

where U_i and Θ_i are statistically steady.

For a homogeneous pure shear flow (in which all averaged quantities except U_i and Θ_i are independent of position and in which the field of strain rate is uniform), the balance of the mean kinetic energy of velocity fluctuations yields (see, for example, Tennekes and Lumley[18])

$$(-P_B) = P + (-\varepsilon) \qquad (20)$$

where $P_B=-g_ju_j\theta/\Theta_o$ is the buoyant production, $P=-u_iu_jS_{ij}$ is the inertial production and $\varepsilon=2vs_{ij}s_{ij}$ is the dissipation of turbulent energy, S_{ij} and s_{ij} being the mean and fluctuating strain rate, respectively. Also the balance of the root mean square of temperature fluctuations gives

$$P_\theta = \varepsilon_\theta^k + \varepsilon_\theta^R \qquad (21)$$

where $P_\theta=-u_j\theta\partial\Theta/\partial x$ is the thermal production, $\varepsilon_\theta^k=a(\partial\theta/\partial x_j)(\partial\theta/\partial x_j)$ is the conductive dissipation and $\varepsilon_\theta^R=(a/k)(\partial/\partial x_j)q_j^R$ is the radiative dissipation.

Equation (20) states that a part of buoyant production is converted into inertial production while the rest of it is dissipated, and in equation (21) the thermal production is dissipated through both the conductive and radiative dissipations.

Under isotropy, equations (20) and (21) respectively reduce, on dimensional grounds, to

$$P_B \sim v^3/\eta + vv^2/\eta^2 \qquad (22)$$

and

$$v_0\theta^2/\eta_0 \sim a\theta^2/\eta_0^2(1+q^R_w/q^k_w) \qquad (23)$$

where η, η_0, v and v_0 are the Kolmogorov scale, thermal microscale, Kolmogorov velocity scale and thermal velocity scale, respectively. Under the turbulent condition, the thickness of the thin, viscous layer close to the wall may be assumed to be η. Following the intuitive vortex model developed by Corrsin[19] and Tennekes[20], the dissipation may be estimated as vv^2/η^2. Similarly, the conductive dissipation is to be estimated as $a\theta^2/\eta_0^2$.

Again, following Squire's postulate for heat transfer, let

$$\eta \sim \eta_0 \quad \text{and} \quad v \sim v_0. \qquad (24)$$

That is, the appropriate length scale for equation (22) is η_0 when the equation is considered for heat transfer.

Thermal scale

Accordingly, letting $\eta \to \eta_0$ in equation (22) and substituting v obtained from equation (23) into this result, η_0 can be obtained for fluids with Pr≥1,

$$\eta_\theta \sim (va^2/P_B)^{1/4} Pr^{-1/4} (1+q^R_w/q^k_w)^{1/2}$$
$$(1+Pr+q^R_w/q^k_w)^{1/4} \qquad (25)$$

Substitution of equation (19) into equation (25) yields

$$\eta_0 \sim (va^2/P_B)^{1/4} Pr^{-1/4}\left[1+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{1/2}$$
$$\left[1+Pr+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{1/4} \qquad (26)$$

Without any radiation effects, equation (26) will reduce to the thermal scale proposed by Arpaci[21] who also demonstrated that this scale becomes the Kolmogorov scale with Pr~1 and the Batchelor scale with Pr>>1.

The dimensional approximation of buoyant production term is examined again to demonstrate the relation between this small scale and the integral scale used in the usual correlations of natural convection problems. With $\Theta_o^{-1} \approx \beta$, the production term becomes

$$P_B \sim gu\theta/\Theta_o \sim gu\beta\theta . \qquad (27)$$

Substitution of equation (27) into equation (26) and with u from equation (23), equation (26) becomes

$$\eta_\theta \sim (va/g\beta\Delta T)^{1/3}Pr^{-1/3}\left[1+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{1/3}$$
$$\left[1+Pr+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{1/3} \qquad (28)$$

or, in terms of a characteristic length l for geometry,

$$\eta_\theta/l \sim Ra_l^{-1/3}Pr^{-1/3}\left[1+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{1/3}$$
$$\left[1+Pr+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{1/3} \qquad (29)$$

where $Ra_l = g\beta\Delta T l^3/va$ is the Rayleigh number.

Heat Transfer

The energy balance at wall, based on l, is

$$Nu_l \equiv q^k_w/q^k_l + q^R_w/q^k_l \qquad (30)$$

Assuming that the heat transfer at wall is characterized by the thermal scale, η_θ, the average Nusselt number based on η_θ can be expressed as

$$Nu_l \sim l/\eta_\theta + q^R_w /q^k_l \qquad (31)$$

With equations (19) and (29), equation (31) reduces to

$$Nu_l \sim Ra_l^{1/3}Pr^{1/3}\left[1+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{-1/3}$$
$$\left[1+Pr+\tau[1-\varepsilon_w/(2\tau)]\tau P\right]^{-1/3} + (\tau-\varepsilon_w/2)\tau P_l \qquad (32)$$

where $P_l = (E_{bw}-E_{b\infty})l/(k\Delta T)$ is the Planck number based on the length of the geometry, l. When the radiation is neglected, the Nusselt number reduces to $Nu_l \sim Ra_l^{1/3}[Pr/(1+Pr)]^{1/3}$ which is also obtained by Arpaci[23]. However, the common experimental correlations are $Nu_l \sim Ra^n$ with n always less than 1/3. The reason for this disagreement is a result of the usually ignored Prandtl number effect.

4.CONCLUSION REMARKS

This study employs an intuitive approach to relate the microscales and the integral scale of turbulence to the heat transfer correlations. It constructs a heat transfer model without making any reference to the eddy viscosity which does not have any physical basis. The radiation effects on the turbulent natural convection problem were modeled following the success of the dimensional analysis on the laminar natural convection. A radiation-affected heat transfer model which is valid for Pr≥1 is proposed based on the radiation affected thermal microscale and the integral scale. In the absence of thermal radiation, the model reduces to the model proposed by Arpaci[21]. To date, there appears to be no experimental data in the literature on the radiation-affected turbulent natural convection.

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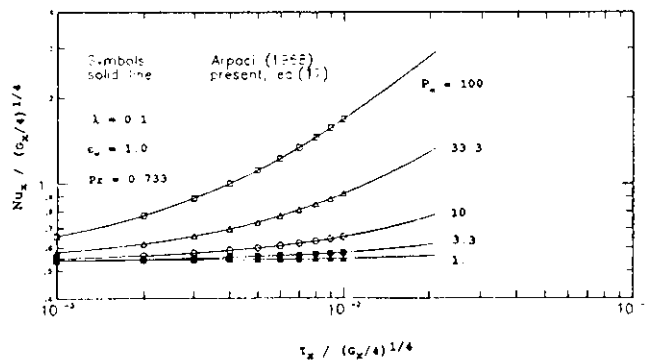
Fig. 1