SUBOPTIMAL GUIDANCE WITH LINE-OF-SIGHT RATE ONLY MEASUREMENTS

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Abstract

A new suboptimal guidance law is presented. It uses proportional navigation with phase lead compensation. The guidance gain, the amount of phase lead and the frequency at which the maximum phase lead occurs are decided from the dynamic lags of missile and target. The implementation of the suboptimal guidance law(SOG) does not require any information other than line-of-sight rate. The performance of SOG is compared with that of the classical proportional navigation(PN) using simulation studies of a linear homing system and a realistic nonlinear homing system. The following results are obtained: 1) For a launch error or a constant target acceleration in the linear homing system, the minimum time-to-go for which the miss is negligible is about half as large when SOG is used rather than PN. 2) The magnitude of the acceleration produced with SOG is less than or approximately the same as that with the PN. 3) Since SOG is more sensitive to measurement noise than PN because of the phase lead compensation, the noise filter must be carefully designed in accordance with the measurement noise level. 4) The results of the simulation of the realistic nonlinear homing system show the same features as those of the linear homing system. From all these results, the SOG is shown to be a very effective guidance law for a high maneuverability target.

Introduction

It is well known that classic proportional navigation(PN) is an adequate missile guidance law for a low maneuverability target.[1] The higher maneuverability of modern fighters, however, makes PN unsatisfactory in terminal air-to-air missile engagements.[2] In order to yield better performance than PN, a number of optimal guidance systems have been investigated.[3,4,5] Though these optimal guidance laws are attractive from the mathematical viewpoint, there are many difficulties in ease of implementation, robustness and cost effectiveness.[6] In general, the mechanization of the optimal guidance law requires information on time-to-go, range, range rate and missile acceleration. These can be estimated using a Kalmam filter in the case of a radar homing seeker. However, because range and range rate are not accurately known in a system using an infrared seeker, it is difficult to estimate accurately the variables needed for the implementation of the optimal guidance law for this type of missile. Unfortunately, many short-range air-to-air missiles are of the infrared homing type.

In this paper the authors derive a suboptimal guidance law, starting from the optimal guidance formulation. The implementation of this suboptimal law requires only line-of-sight rate (LOS rate). The performance of the suboptimal guidance law is compared with that of PN using simulation studies of both linear and realistic nonlinear homing systems.

Derivation of the Suboptimal Guidance Law

We will consider the kinematics of the missile and target trajectories in two dimensions. Fig. 1 shows the geometry of the intercept. M_0 and T_0 indicate the positions that a missile and a target would have had respectively at time t in the reference trajectory case. M and T display the actual positions of a missile and a target at time t, respectively. It is assumed that both the target and missile maneuver with accelerations normal to their respective velocity vectors, and their dynamics are modeled with first-order transfer functions. When Θ_0 and Ψ_0 are 0 and π radians, respectively, (this corresponds to a head-on attack), we obtain the following state space equations of motion:[4]

$$\frac{d}{dt}\begin{bmatrix} y_d \\ \dot{y}_d \\ a_t \\ a_l \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -\lambda_t & 0 \\ 0 & 0 & 0 & -\lambda_m \end{bmatrix}\begin{bmatrix} y_d \\ \dot{y}_d \\ a_t \\ a_l \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda_m \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} w \quad (1)$$

Here y_d is the miss distance given by $y_t - y_m$, y_t and y_m are the perpendicular displacements, respectively, of target and missile from the reference LOS, a_t is the target acceleration, a_l is the missile acceleration, λ_t is the target maneuver bandwidth, w is white noise with a power spectral density of $2\lambda_t\beta^2$, β is the rms acceleration level of the target, λ_m is the missile maneuver bandwidth and u is the commanded missile acceleration.

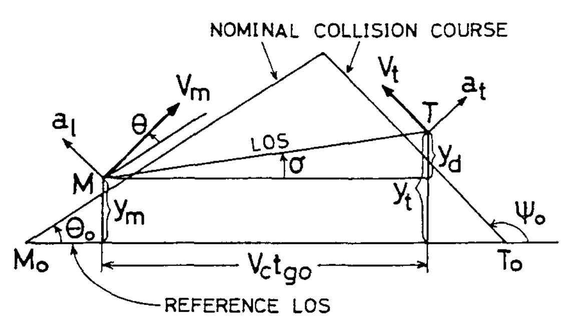

Fig. 1 Intercept geometry

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The performance index to be minimized is given by

$$J = E\left\{ y_d^2(t_f) + \rho\int_0^{t_f} u^2(t)\,dt \right\} \quad (2)$$

where t_f is the intercept time and ρ is the weighting on control effort. The solution to this problem is called an optimal guidance law and is given by[4]

$$u = c_1 y_d + c_2 \dot{y}_d + c_3 a_t + c_4 a_l \quad (3)$$

where $t_{go} = t_f - t$ and is called the time-to-go,

$$c_1 = \frac{n}{t_{go}^2} \quad (4)$$

$$c_2 = \frac{n}{t_{go}} \quad (5)$$

$$c_3 = n\frac{e^{-\lambda_t t_{go}} + \lambda_t t_{go} - 1}{\lambda_t^2 t_{go}^2} \quad (6)$$

$$c_4 = -n\frac{e^{-\lambda_m t_{go}} + \lambda_m t_{go} - 1}{\lambda_m^2 t_{go}^2} \quad (7)$$

$$n = \frac{3t_{go}^2\left(t_{go} - \frac{1-e^{-\lambda_m t_{go}}}{\lambda_m}\right)}{3\rho + \frac{3}{2\lambda_m^2}(1-e^{-2\lambda_m t_{go}}) + \frac{3t_{go}}{\lambda_m^2}(1-2e^{-\lambda_m t_{go}}) + t_{go}^2\left(t_{go} - \frac{3}{\lambda_m}\right)} \quad (8)$$

If the time lag of the missile dynamics is neglected, that is, $\lambda_m = \infty$, and ρ approaches zero, the resulting navigation ratio n becomes constant with a value of 3. Thus let

$$\bar{n} = 3 \quad (9)$$

When Eqs. (4) and (5) with $n = \bar{n}$ are substituted into Eq. (3), u becomes

$$u = \bar{n}\frac{y_d + \dot{y}_d t_{go}}{t_{go}^2} + c_3 a_t + c_4 a_l \quad (10)$$

From Fig. 1 the LOS angle σ is given by

$$\sigma = \frac{y_d}{V_c t_{go}} \quad (11)$$

where $V_c = -d(V_c\, t_{go})/dt$ = closing velocity. Differentiation of Eq. (11) leads to the following formula for the LOS rate $\dot{\sigma}$:

$$\dot{\sigma} = \frac{y_d + \dot{y}_d t_{go}}{V_c t_{go}^2} \quad (12)$$

From Eqs. (10) and (12), u can be written as

$$u = \bar{n} V_c \dot{\sigma} + c_3 a_t + c_4 a_l \quad (13)$$

From Eq. (1), we have

$$\ddot{y}_d = a_t - a_l \quad (14)$$

From Eq. (11), y_d is given by

$$y_d = V_c t_{go} \sigma \quad (15)$$

Differentiating Eq. (15) twice, we obtain

$$\ddot{y}_d = V_c t_{go} \ddot{\sigma} - 2V_c \dot{\sigma} \quad (16)$$

From Eqs. (14) and (16), a_t is given by

$$a_t = V_c(t_{go}s - 2)\dot{\sigma} + a_l \quad (17)$$

Substituting Eq. (17) into Eq. (13), we obtain

$$u = V_c(\bar{n} - 2c_3 + c_3 t_{go}s)\dot{\sigma} + (c_3 + c_4)a_l \quad (18)$$

From Eq. (1), a_l can be written by

$$a_l = \frac{\lambda_m}{s + \lambda_m}u \quad (19)$$

Substituting Eq. (19) into Eq. (18) and solving for u, we obtain

$$u = \frac{(\lambda_m + s)(\bar{n} - 2c_3 + c_3 t_{go}s)}{(1 - c_3 - c_4)\lambda_m + s}\dot{\sigma} \quad (20)$$

With the assumption that

$$s^3\sigma = 0 \quad (21)$$

Eq. (20) leads to

$$u = V_c\frac{\{-2\dot{c}_3 + \lambda_m(\bar{n} - 2c_3)\} + (\bar{n} - 3c_3 + \dot{c}_3 t_{go} + \lambda_m c_3 t_{go})s}{(1 - c_3 - c_4)\lambda_m + s}\dot{\sigma} \quad (22)$$

where

$$\dot{c}_3 = \frac{\bar{n}}{\lambda_t^2 t_{go}^3}\{(\lambda_t t_{go} + 2)e^{-\lambda_t t_{go}} + \lambda_t t_{go} - 2\} \quad (23)$$

Eq. (22) can be rewritten as

$$u = \Lambda V_c \frac{1 + Ts}{1 + \alpha Ts}\dot{\sigma} \quad (24)$$

where

$$\Lambda = \frac{-2\dot{c}_3 + \lambda_m(\bar{n} - 2c_3)}{(1 - c_3 - c_4)\lambda_m} \quad (25)$$

$$T = \frac{\bar{n} - 3c_3 + \dot{c}_3 t_{go} + \lambda_m c_3 t_{go}}{-2\dot{c}_3 + \lambda_m(\bar{n} - 2c_3)} \quad (26)$$

$$\alpha = \frac{1}{T(1 - c_3 - c_4)\lambda_m} \quad (27)$$

Since Λ, T and α are functions of t_{go}, we must estimate t_{go} in order to implement the guidance law given by Eq. (24). If Λ, T and α can be considered as constant parameters, however, t_{go} need not be known for implementation of the

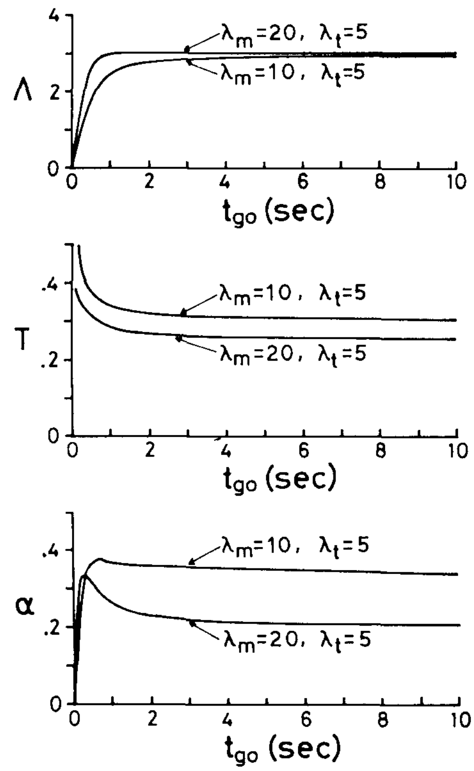

Fig. 2 Optimal guidance gain Λ and parameters T, α as a function of time-to-go(n=3)

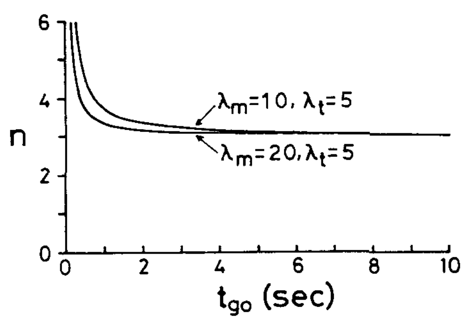

Fig. 3 Optimal navigation gain as a function of time-to-go

guidance law. The values of Λ, T and α are illustrated in Fig. 2 as functions of t_{go} for representative λ_m and λ_t. The figure shows that the values of Λ, T and α remain almost constant in each case for t_{go} greater than about two seconds time-to-go but change rapidly for t_{go} less than one second in order to make the final miss distance approach zero. Because most anti-aircraft missiles use proximity fuses, they do not necessarily require a zero miss. In this case, we can approximate Λ, T and α as constant parameters. This then avoids the difficult requirement of measuring or estimating t_{go}, the time-to-go, and therefore helps greatly the practical implementation. We call the guidance law given by Eq. (24), with constant values of Λ, T and α the suboptimal guidance law(SOG). Since PN is expressed by

$$u = N V_c \dot{\sigma} \tag{28}$$

where N is the effective navigation ratio, SOG given by Eq. (24) can be viewed as proportional navigations with phase lead compensation. The implementation of SOG requires only the LOS rate $\dot{\sigma}$. Fig. 3 displays the true value of n as a function of time-to-go for the same λ_m and λ_t considered in Fig. 2. The figure shows that n remains almost 3 until the time-to-go is less than one second. The reason for this rapid change in n for small t_{go} is similar to that for the rapid change in Λ, T and α.

Performance Comparison in Linear Homing Systems

SOG is compared with PN in terms of performance measured by the miss distances and the required missile lateral accelerations. The comparison is made using the linearized homing system shown in Fig. 4. For the case where $\lambda_m = 20$ and $\lambda_t = 5$ in Fig. 2, the SOG parameters are chosen to have the following values:

$$\Lambda = 3, \quad T = 0.26, \quad \alpha = 0.215 \tag{29}$$

The switch S is positioned according to the type of

Table 1 System parameters

$a_0 = 747336.7$	$a_1 = 62507.3$	$a_2 = 1443.72$
$a_3 = 58.9$	$b_0 = 746214.8$	$b_1 = 143.75$
$b_2 = 65.2$	$T_H = 0.1$	$V_c = 896$ (m/s)

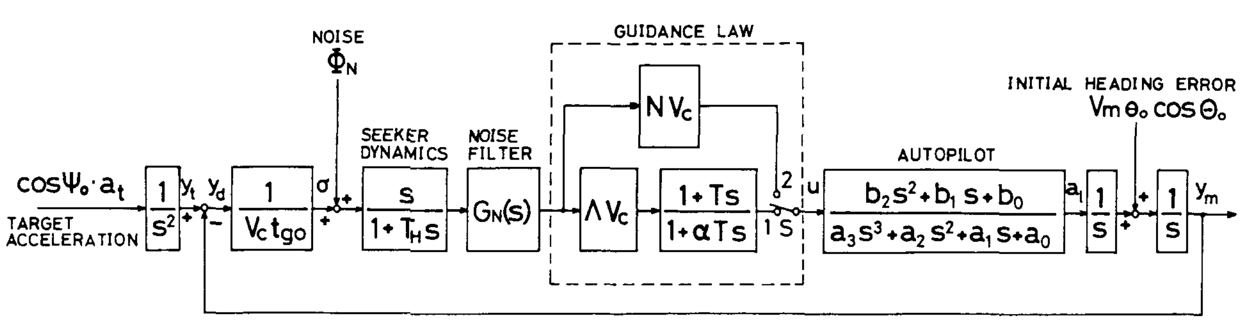

Fig. 4 Block diagram for the linear homing system

guidance law being evaluated; position 1 is for SOG and position 2 is for PN. The autopilot parameters, the time constant of the seeker dynamics and the closing velocity are listed in Table 1. The LOS angle measurement noise is caused by many different error sources. Some of these are

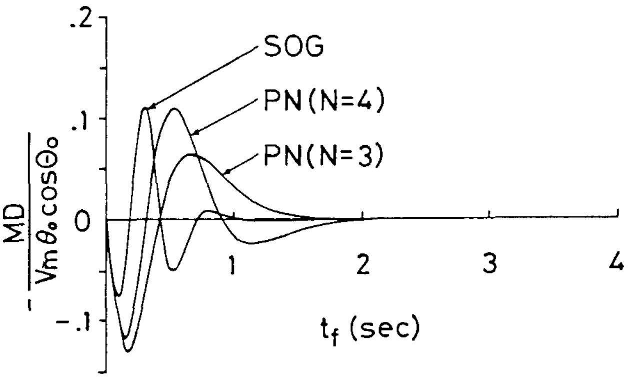

Fig. 5 Miss due to a launch error

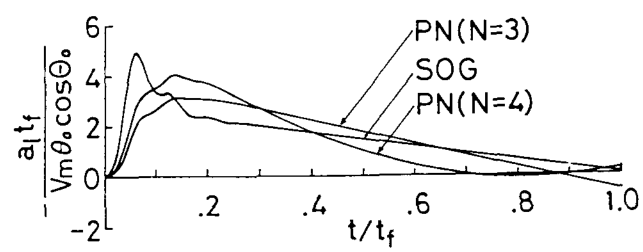

Fig. 6 Missile acceleration due to a launch error

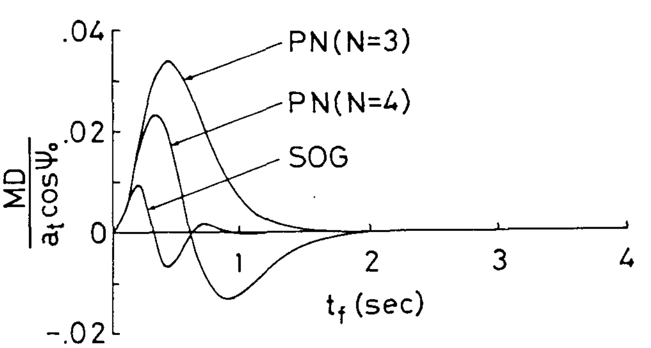

Fig. 7 Miss due to a target acceleration

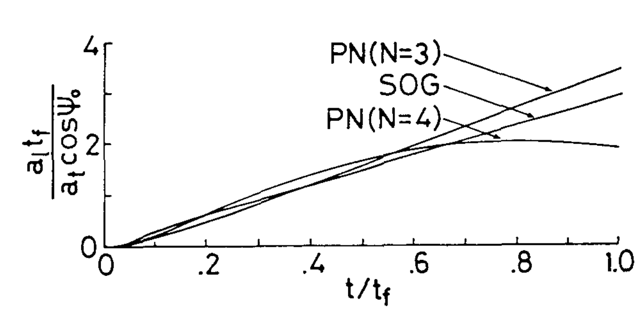

Fig. 8 Missile acceleration due to a target acceleration

range dependent and non-white. However, only angular noise which is independent of range and white with spectral density Φ_N is considered here.

Miss and Acceleration Due to a Launch Error

The miss distance MD and the missile lateral acceleration due to a launch error are computed, assuming no target acceleration, no noise and $G_N(s)=1$. For comparison, two navigation constants of PN, N=3 and 4, are chosen. The results are depicted in Figs. 5 and 6 in normalized form. Fig. 5 shows that SOG requires at least 1.2 sec intercept time to make the miss distance negligible, whereas PN requires about 1.8 sec for $N=3$ and about 2 sec for $N=4$. On the other hand, Fig. 6 shows that the maximum acceleration of SOG is larger than that of PN, while the response of SOG is faster than that of PN. From these results it can be concluded that SOG makes a closer attack possible because of its smaller intercept time and faster response compared with PN.

Miss and Acceleration Due to a Step Target Acceleration

The miss distance and the missile lateral acceleration due to a step target acceleration are computed, assuming no launch error, no noise and $G_N(s)=1$. The results are depicted in Figs. 7 and 8. Fig. 7 shows that SOG produces negligible miss for intercept times down to approximately half those of PN. The maximum miss produced with SOG is about two fifth of that with PN of $N=4$. On the other hand, Fig. 8 shows that there are no large differences between the accelerations produced by SOG and PN. These figures show that SOG is a very effective guidance law for a target with high transverse acceleration.

Miss Due to Noise

The miss distance due to white noise is computed using the monte calro simulation technique with 100 runs. It is assumed that the launch error and target lateral acceleration are both zero, and a noise filter is not considered. The effective navigation constant of PN was set to 4 rather than 3 in order to compensate for dynamic lags of both a seeker head and an autopilot. The results are

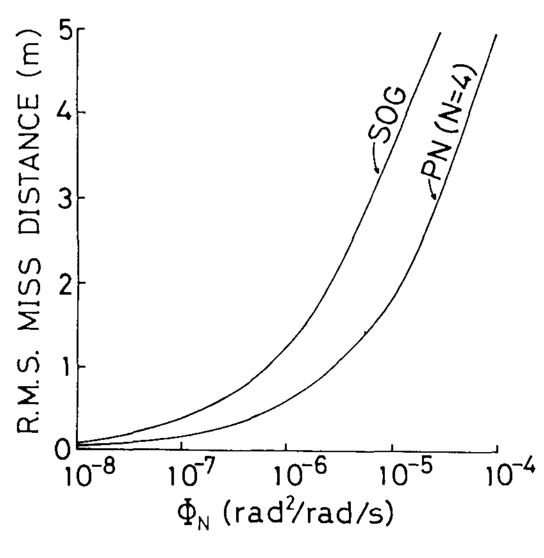

Fig. 9 Miss due to noise(No noise filter)

plotted in Fig. 9 as a function of spectral density, Φ_N. Since linear equations are used, the rms miss is proportional to noise amplitude and hence to the square root of Φ_N. The figure shows that the rms miss produced with SOG is approximately double that with PN. In order to suppress measurement noise, a low-pass noise filter must be considered. In general a first-order low-pass filter is used in proportional navigation guidance systems.[6] This filter has the transfer function

$$G_N(s) = \frac{\Omega_c}{s+\Omega_c} \tag{30}$$

where Ω_c is the cut-off frequency. The high frequency gain of the SOG system becomes larger than that of the PN system because of the phase lead compensation represented in Eq. (24). For this reason the SOG system uses a second-order Butterworth filter with transfer function given by

$$G_N(s) = \frac{\Omega_c^2}{s^2+1.4\Omega_c s+\Omega_c^2} \tag{31}$$

To investigate the effect of the noise filter, the rms miss due to noise with $\Phi_N = 10^{-7}$ rad^2/rad/s, a value often used in design studies of guidance systems,[7] is computed. The results are plotted in Fig. 10 as a function of cut-off frequency Ω_c.

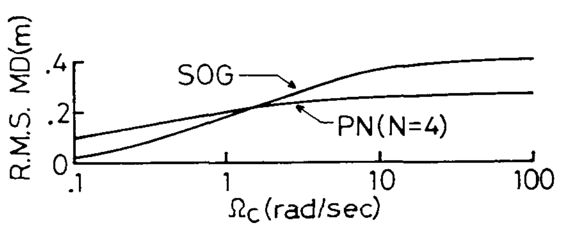

Fig. 10 Miss due to noise as a function of the cut-off frequency of the noise filter

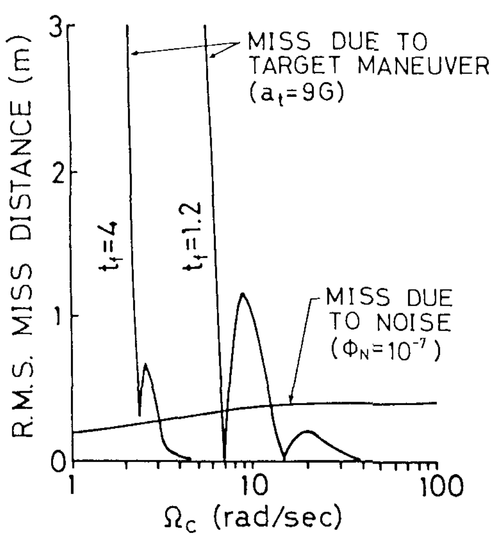

Fig. 11 Miss as a function of cut-off frequency of the noise filter (Suboptimal guidance law)

To keep the rms miss less than 0.1 meter, the figure shows that the cut-off frequency must be less than 0.2 rad/s for the SOG system and less than 0.1 rad/s for the PN system. However, these values of cut-off frequency are too low for a missile to track accurately a maneuvering target. Figs. 11 and 12 display the rms miss due to either a target 9G acceleration or a measurement noise as a function of the cut-off frequency of the noise filter. The optimum cut-off frequency is selected to minimize the total miss distance, that is, the miss due to noise plus the miss due to the target maneuver. From Fig. 11 the optimum cut-off frequency of SOG is about 3 rad/s for $t_f = 4$ sec and 35 rad/s for $t_f = 1.2$ sec. On the other hand, Fig. 12 shows that the optimum cut-off frequency of PN is about 6 rad/s for $t_f = 4$ sec and 40 rad/s for $t_f = 2$ sec. From these results, we choose 35 rad/s for the cut-off frequency. Though this value for Ω_c seems too high for adequate noise

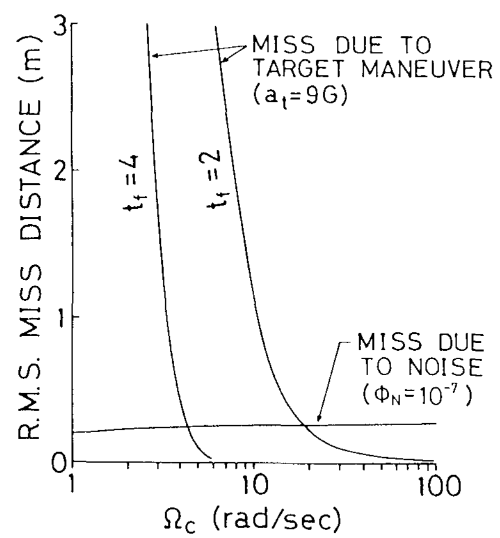

Fig. 12 Miss as a function of cut-off frequency of the noise filter (Proportional navigation)

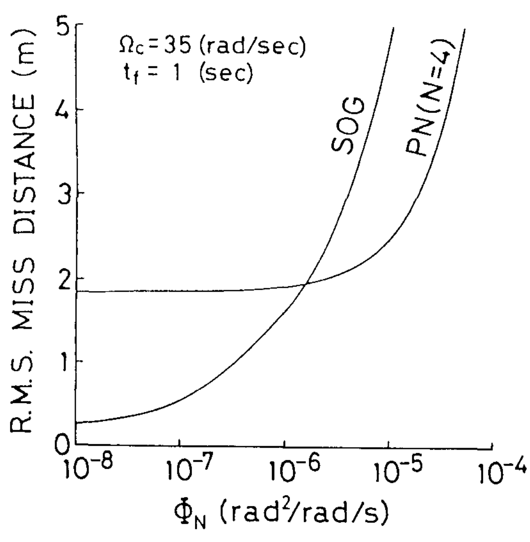

Fig. 13 Miss due to target's 9G maneuver and noise as a function of spectral density of noise

Table 2 Miss and acceleration of the missile intercepting a 7G constant turn target

Initial Aspect Angle (deg)	Initial Distance (m)	Suboptimal Guidance			Proportional Navigation(N=4)		
		Miss(m)		Maximum Load Factor (G)	Miss(m)		Maximum Load Factor (G)
		Noise Free	Noise(rad^2/Hz) $\Phi_N = 10^{-7}$		Noise Free	Noise(rad^2/Hz) $\Phi_N = 10^{-7}$	
0 (Tail Chase)	1500	0. 02	0. 90	6. 43	0. 05	0. 50	5. 77
90 (Broadside)	1500	0. 06	0. 96	33. 18	0. 20	0. 60	36. 73
180 (Head-on)	1000	0. 13	0. 88	15. 16	3. 10	3. 27	18. 19

suppression, a lower cut-off frequency reduces the effectiveness of the guidance law against a high maneuverability target. Fig. 13 illustrates the miss due to 9G target lateral acceleration and angular noise of various spectral densities. If $\Phi_N < 10^{-6}$ rad^2 /rad/s, the miss achieved with SOG is smaller than that with PN. If $\Phi_N > 2\times 10^{-6}$ rad^2 /rad/s, the reverse is true. This is because the cut-off frequency was chosen for angular noise of 10^{-7} rad^2 /rad/s. It is clear that Ω_C should be selected carefully based on the measurement noise levels.

Application to the Infrared Homing Short-Range AAM

SOG and PN are applied to the model of a fictitious infrared homing short-range air-to-air missile(AAM), for which aerodynamic coefficients and geometrical and inertial data are estimated from present missiles such as the AIM-9. The aerodynamic coefficients are functions of the Mach number. The position of the center of gravity and the inertial moments are changed in accordance with the consumption of fuel. The simulation contains the mathematical model that describes the equations of motion of a missile, the detailed aerodynamic coefficients, and the detailed nonlinear mathematical models of major missile subsystems. These subsystems include the seeker, noise filter, autopilot, disable time of the control surfaces, rolleron and propulsion systems. For the example simulated before, the altitude is 5000 meters, the velocity is Mach 2, the time elapsed after launch is 2. 8 sec and the angular velocity of the rotor of the rolleron is 2400 rad/s. The overall system is the one shown in Fig. 4 with the autopilot parameters listed in Table 1. The cut-off frequency is set to 35 rad/s. The target model is a point mass which has three-degrees-of-freedom. The initial launch conditions are selected as follows: 1) missile and target are at the same speed(0. 9Mach); 2) missile and target are at the same altitude at launch(5000m); 3) the initial off-boresight angle is 0 degrees.

Intercepting a Target Performing a Constant-G Horizontal Turn

Table 2 displays the results of simulations for the AAM intercepting a target which is turning at 7G in the horizontal plane. The simulations were performed using a monte calro method with 100 runs. For the case of no noise, all misses achieved with SOG are smaller than those with PN. But when noise is added, for the broadside attack and tail chase cases, SOG produces larger misses than PN, with noise the main contributor to the miss. On the other hand, for the head-on attack PN produces a much larger miss than SOG. As mentioned before, these results show that SOG is more sensitive to noise than PN but SOG is much more effective for a high maneuverability target.

Intercepting a Target Performing a High-G Barrel Roll

As the high-G barrel roll(HGB) is considered one of the most effective evasive maneuvers of a fighter against a missile,[8] some simulations for the AAM against a target performing a 7G barrel roll

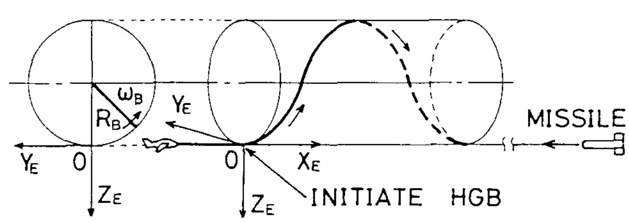

Fig. 14 HGB flight pattern and nomenclature for coordinate axes

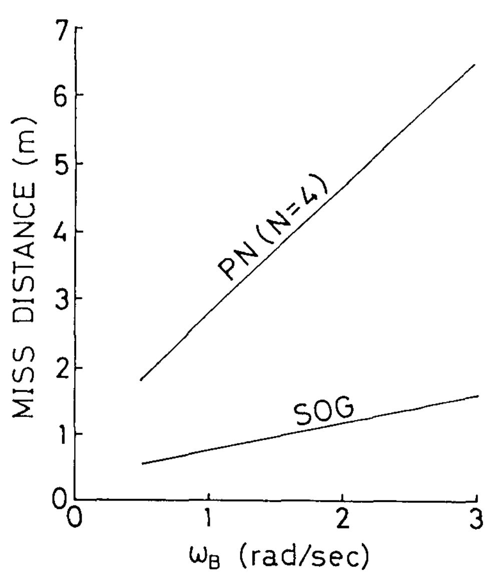

Fig. 15 Miss vs barrel roll rate

were run. Fig. 14 shows the flight pattern of a target when performing the HGB maneuver, as well as the initial position of the missile. Assuming the target is a point mass, the flight path for the HGB is given by

$$X_E = a_B R_B \omega_B t$$
$$Y_E = -R_B \sin(\omega_B t)$$
$$Z_E = -R_B + R_B \cos(\omega_B t)$$

where R_B is the radius of the barrel roll, ω_B is the barrel-roll rate and a_B is the pitch of the spiral. The target load factor is

$$n_B = \frac{R_B \omega_B^2}{g}$$

It is assumed that the engagement geometry is head-on, as displayed in Fig. 14. The initial relative range is 5000 meters and the target flies straight until initiating the HGB. Fig. 15 shows the miss vs barrel-roll rate for a target which starts the barrel roll at a point 3000 meters from

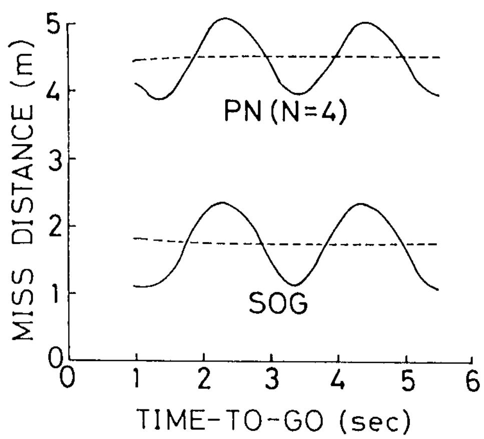

Fig. 16 Miss vs time-to-go(ω_B= 3 rad/sec)

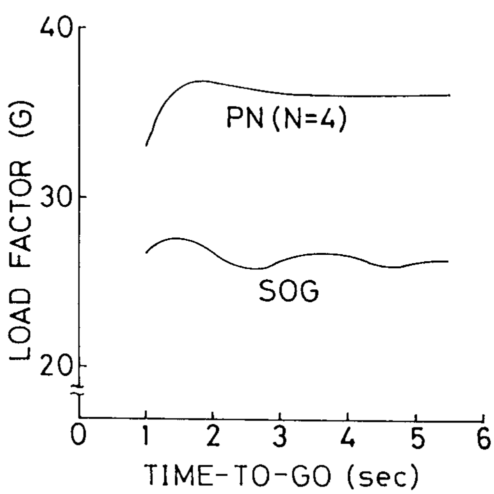

Fig. 17 Load factor vs time-to-go(ω_B= 3 rad/sec)

the AAM. From this figure, if the effective miss for target kill is 3 meters, the target performing the HGB with more than 1.2 rad/s barrel-roll rate can evade the missile guided by PN. However, for all barrel-role rates shown in the figure, the target cannot evade the missile guided by SOG. The solid lines in Fig. 16 illustrate miss vs time-to-go when the target initiates a barrel roll with a rate of 3 rad/s. The miss changes oscillatorily because of the effect of gravity. If the autopilot is modified to compensate for gravity, the oscillation of miss disappears, as shown by the broken lines in Fig. 16. In this case the miss does not depend on the maneuver initiation time. The maximum load factor in each case is depicted in Fig. 17, which shows that the load factor of the missile with SOG is smaller than that with PN.

Conclusion

A new suboptimal guidance law which uses only LOS rate for implementation has been presented. It uses proportional navigation with additional phase lead compensation. The guidance gain, the amount of phase lead and the frequency at which the maximum phase lead occurs are decided from the dynamic lags of both missile and target. The simulation study for the linear homing system shows the following results: 1) For either a launch error or a constant target acceleration, SOG produces negligible miss for a minimum time-to-go which is about half the minimum time-to-go for PN. 2) The magnitude of the missile acceleration produced with SOG is approximately the same as that with PN. 3) Though SOG is more sensitive to noise than PN because of the phase lead compensation, the miss due to noise is not a serious problem if an adequate noise filter is designed in accordance with the noise level.

SOG was then applied to a realistic short-range air-to-air missile and simulations were run for interceptions of a target performing either a constant-G horizontal turn or a high-G barrel roll. These show that the missile with SOG can shoot down the target from all directions, even when the target performs the high-G turn or the HGB. When the missile uses PN, the simulations show that the target performing a high-G turn cannot be shot down in a head-on attack, nor can it be shot down when performing a HGB with roll rate more than about 1.2 rad/s.

These results show that SOG is a very effective guidance law for a high maneuverability target.

Some of the simulations for this research were performed on the AD-100 computer of Applied Dynamic International in Ann Arbor, Michigan.

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