Pitot Pressure in Hypersonic Flow with Condensation

D. D. McBride* and P. M. Sherman†
The University of Michigan, Ann Arbor, Mich.

The way in which the diameter of a Pitot tube affects the measurement of Pitot pressure is shown by a series of measurements as well as by calculations for the flow of superheated zinc vapor in a helium carrier gas. The mixture is expanded in a supersonic nozzle and thereby supercooled and accelerated to a point well beyond where saturation and subsequent condensation occur. Calculations show that Pitot pressure may be either larger or smaller than the pressure would be without condensation, and they show useful limits to consider in making measurements. The analysis was kept general so that it could provide solutions for both a condensation and evaporation discontinuity in a two-component stream. The measurements were made near the exit plane of a nominal Mach 25 (helium)nozzle. They show no change in Pitot pressure for tubes greater than 0.125 in. in diameter but a steady decrease in pressure for tubes down to 0.008 in. in diameter. A method for decreasing the lag time in some transducers is also indicated.

Nomenclature

B = vapor saturation curve constant
C_p = specific heat at constant pressure
f = mass fraction of inert carrier gas
L = latent heat (of vaporization or sublimation depending upon T)
p = pressure
q = mass fraction of uncondensed vapor
R = universal gas constant
T = temperature
u = velocity
α = ratio of molecular weights (μ_v/μ_a)
γ = specific heat ratio
μ = molecular weight
ρ = density
Φ = parameter defined by Eq. (6)

Subscripts

a = property of inert carrier gas
c = property of mixture defined at chamber conditions
v = property of condensable vapor
1 = property defined just upstream of discontinuity
2 = property defined just downstream of discontinuity
∞ = indicates vapor saturation pressure

Introduction

When flow expands through a supersonic nozzle, the temperature and pressure drop, and when they are low enough condensation may occur (since the saturation curve is steeper than an isentrope). In supersonic flow without condensation, the Pitot pressure is simply the stagnation pressure downstream of a normal shock, the shock in front of the Pitot tube. In most cases, the size of the Pitot tube is not pertinent. However, in a flow containing condensed particles, size becomes important because it affects the degree of revaporization of particles between the bow shock in front of the Pitot tube and the pitot tube. (The particles will always tend to revaporize if they condense in the nozzle, that is, if the vapor was superheated upstream.) The temperature will increase downstream of the shock, finally reaching the original stagnation chamber temperature. Since there is a stagnation pressure loss across the shock, the vapor reaches a high-temperature, low-pressure state or a higher degree of superheat than in the arc chamber.

For some very "small" tube, the bow shock is so close to the tube that there is no time for any revaporization or appreciable momentum transfer between particles and gas, and the pressure measured would be the stagnation pressure downstream of a normal shock without further phase change. For all smaller-diameter tubes, the Pitot measurement would remain essentially the same. As the diameter of the tube increases, the shock standoff distance increases so that condensed particles may have time to evaporate. As the time for revaporization increases with greater shock standoff distance, the pressure measured will increase because of greater evaporation. When a "large" tube diameter is reached such that all of the particles will evaporate between the shock and the tube, then further increase in the tube diameter will not change the pressure recorded. The pressure will then be the pitot pressure downstream of a normal shock with total revaporization.

It had been pointed out early[1] that condensation should affect Pitot pressure. However, it was later found that, at moderate conditions, Pitot pressure was unaffected by condensation.[2-4] As a result, there has been a tendency to assume, without careful consideration of conditions, that, in general, Pitot pressure is insensitive to condensation. More recently measurements were made[5] which are sometimes referred to as "Pitot-tube measurements" with the incorrect inference that Pitot pressure was measured. Actually, the effect of tube size was employed to infer the size of ice crystals, but Pitot pressure measurements were not made. Instead, Pitot-tube-like probes were used with continuous flow through them for sampling the flow to measure the humidity in the sample.

The purpose of this paper is to present a solution for flow of a two-component mixture with either a condensation or evaporation discontinuity that can be employed to determine useful pressure limits for flow through a nozzle, and to show measurements that demonstrate Pitot pressure limits.

Analysis

Pitot pressures for the limiting cases of "small" and "large" tubes were computed. A solution similar to the earlier "condensation shock" solution[2-4,6] was employed. The solution given here, although similar in principle, is necessarily different in that it is for a mixture of two gases, one

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* Technical Staff Member, Aerothermodynamics Projects Department, Sandia Laboratories, Albuquerque, N. Mex.

† Associate Professor, Department of Aerospace Engineering. Member AIAA.

condensable and one inert. Also, the solution has been kept perfectly general to allow its use for either a "condensation jump" (a discontinuous heat addition through condensation) or a "vaporization shock" (a discontinuous heat subtraction in conjunction with a normal shock).

The solution assumes the following:

1) The flow is steady and one-dimensional.

2) The gaseous components are thermally and calorically perfect.

3) In the two-phase mixture, the particles and gaseous components have equal temperatures and equal velocities.

4) The density of the condensed phase is much greater than the density of the gaseous phase, allowing neglect of its volume.

5) The latent heat is constant.

The governing equations are simply the difference equations across a discontinuity at constant area:

Conservation of Mass

$$\rho_1 u_1 = \rho_2 u_2 \tag{1}$$

Conservation of Momentum

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{2}$$

Conservation of Energy

$$C_{p_c} T_1 + (u_1^2/2) + q_1 L = C_{p_c} T_2 + (u_2^2/2) + q_2 L \tag{3}$$

Equation of State

$$p_2 = (f + q_2)\rho_2 (R/\mu_2) T_2 \tag{4}$$

A simultaneous solution of these equations yields

$$p_2 = [-\Phi_2 \pm (\Phi_2^2 - 4\Phi_1\Phi_3)^{1/2}]/2\Phi_1 \tag{5}$$

where

$$\Phi_1 = \frac{1}{2\rho_1^2 u_1^2}\left[1 - \frac{2\gamma_c}{\gamma_c - 1}\left(\frac{1 - f + f\alpha}{q_2 + f\alpha}\right)\right] \tag{6a}$$

$$\Phi_2 = \frac{p_1 + \rho_1 u_1^2}{\rho_1^2 u_1^2}\left[\frac{\gamma_c}{\gamma_c - 1}\left(\frac{1 - f + f\alpha}{q_2 + f\alpha}\right)\right] - 1 \tag{6b}$$

$$\Phi_3 = \frac{(p_1 + \rho_1 u_1^2)^2}{2\rho_1^2 u_1^2} - C_{p_c} T_1 - \frac{u_1^2}{2} - (q_1 - q_2)L \tag{6c}$$

In Eq. (5), the sign of the radical will be (+) if the discontinuity contains only condensation or evaporation and (−) if a normal shock is also present. The stagnation chamber value of the specific heat ratio γ_c can be evaluated from the convenient relationship given by Goldin[7]:

$$\gamma_c = \gamma_v \left[\frac{(\gamma_a/\gamma_v)[(\gamma_v - 1)/(\gamma_a - 1)]f\alpha + 1 - f}{[(\gamma_v - 1)/(\gamma_a - 1)]f\alpha + 1 - f}\right] \tag{7}$$

A simultaneous solution of Eqs. (1, 2, and 4) yields T_2:

$$T_2 = \mu_v p_2 (p_1 - p_2 + \rho_1 u_1^2)/R\rho_1^2 u_1^2 (q_2 + f\alpha) \tag{8}$$

The downstream mass fraction of zinc vapor q_2 can be supplied by requiring that the flow return to saturated equilibrium downstream of the discontinuity. The integrated Clausius-Clapyron relationship between the pressure and temperature of a saturated vapor, used in conjunction with Dalton's law of partial pressures, yields the equation for satisfying this condition:

Clausius-Clapyron Equation

$$p_{2_\infty} = \exp(B - L\mu_v/RT_2) \tag{9}$$

Dalton's Law

$$p_{2_v} = q_2 p_2/(q_2 + f\alpha) \tag{10}$$

A solution may now be found by solving Eqs. (5 and 8–10) for incremental jumps of q_2 until $p_{2_v} = p_{2_\infty}$, a simple task using any digital computer. u_2 and ρ_2 may then be found from Eqs. (3) and (2).

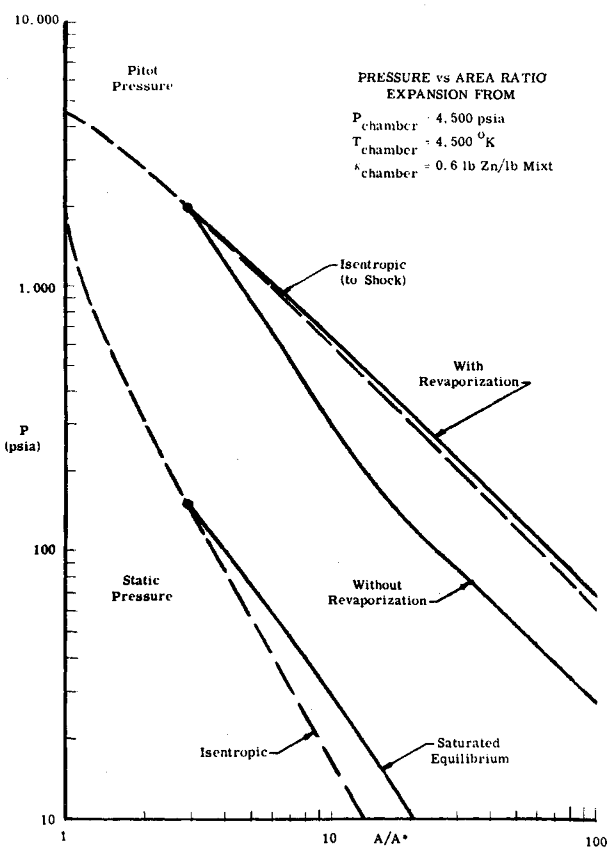

Fig. 1 Computed pressures as a function of area ratio for flow of zinc vapor in helium through a supersonic nozzle.

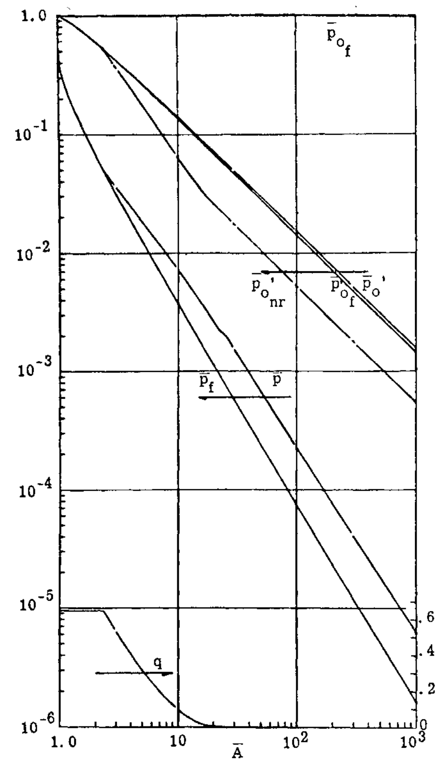

Fig. 2 Computed pressure ratios as a function of area ratio for flow of zinc vapor in helium. P_c = 4000 psia, T_c = 4000°K, f = 0.35 ($\bar{p}_f \equiv p/p_c$ for no condensation, $\bar{p} \equiv p/p_c$ for equilibrium condensation, $\bar{p}_0'{}_{nr} \equiv p_0'/p_c$ for no revaporization $\bar{p}_{0f}' \equiv p_0'/p_c$ for no condensation, $\bar{p}_0' \equiv p_0'/p_c$ for complete revaporization).

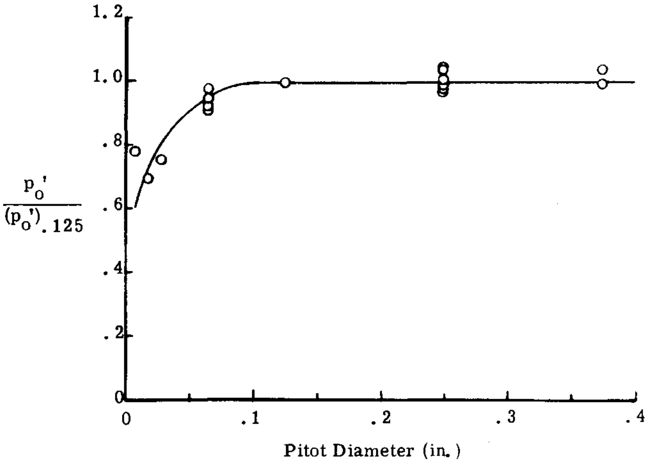

Fig. 3 Ratio of Pitot tube pressure measured to that measured with a tube diameter of 0.125 in. as a function of Pitot tube diameter.

When the discontinuity is a normal shock in a condensed flow whose stagnation conditions are superheated, total revaporization would occur for equilibrium between gas and vapor. In this case, the solution is closed form, since $q_2 = 1 - f$, and p_2 is thus specified completely in terms of the upstream properties. Pitot pressure is calculated from the definition of stagnation pressure as a function of velocity, temperature, and pressure.

It should be noted, that, if it is desired to release the assumption of constant latent heat, Eqs. (3, 6, and 9) must be changed in the preceding discussion:

$$C_{pc}T_1 + u_1^2/2 - (1 - f - q_1)L_1 = C_{pc}T_2 + u_2^2/2 - (1 - f - q_2)L_2 \quad (3')$$

$$\Phi_3 = (p_1 + \rho_1 u_1^2)^2/2\rho_1^2 u_1^2 - C_{pc}T_1 - u_1^2/2 - q_1L_1 + q_2L_2 + (1 - f)(L_1 - L_2) \quad (6')$$

$$p_{2_\infty} = \exp(B - L_2\mu_v/RT_2) \quad (9')$$

In addition to incrementing q_2 until $p_{2_v} = p_{2_\infty}$, the solution now requires that at each increment of q_2 some type of iteration be performed on L_2, since $L_2 = L(T_2)$ and L_2 must be used in the calculation of T_2.

Figures 1 and 2 show examples of pressures calculated for conditions of interest in experiments.[8] In Fig. 1, the curve showing the lowest pressure at a given area ratio is the usual static pressure curve for isentropic flow with no condensation. Static pressure for flow with equilibrium condensation[9] is presented just above, showing an increase in static pressure of about 70% at an area ratio of 10. The three upper curves are Pitot pressures computed for the limiting conditions of a large Pitot tube (complete revaporization), small Pitot tube (no revaporization), and the usual "frozen" flow or Rankine-Hugoniot solution with no condensation. The highest Pitot pressure curve is the large-tube case. It is about 10% higher at the higher area ratios than the dashed line showing the Pitot pressure for flow (isentropic to shock wave) with no condensation. The lowest Pitot pressure curve, which is down by a factor of ~2.5 from the no-condensation case (at the higher area ratios), is the Rankine-Hugoniot solution for equilibrium condensation conditions upstream of the shock and no revaporization. For no revaporization, $q_1 = q_2$, so that the equations are in the same form as for $q = 0$ and therefore yield the Rankine-Hugoniot solution.

Figure 2 shows qualitatively the same pressure changes as Fig. 1 but extended to higher area ratios. It can be seen that all of the curves are close to straight lines between an area ratio of 100 and 1000.

Measurements

Figure 3 shows the results of measurements with tubes of different sizes. Pitot probes varying in diameter from 0.008 to 0.375 in. were used to determine the variation of Pitot pressure with Pitot tube diameter. A diameter of 0.125 in. appeared to be the minimum Pitot diameter consistent with the assumption of equilibrium behind the Pitot shock. It had been hoped that the lower limit to the curve could be found indicating the point at which no relaxation at all had occurred, but difficulties associated with the miniature sizes and accompanying lag times made it too difficult. To minimize lag time, the 0.008-in.-diam Pitot was made a minimal length and attached to the transducer by epoxy glue as shown in Fig. 4. The transducer, a Hidyne variable reluctance gage, was also degassed under vacuum and filled with a low-vapor-pressure liquid, *n*-dibutyl phthalate, to reduce the transducer internal volume substantially. The latter approach was effective as long as the liquid was not permitted to overflow the transducer cavity into the tube and was kept entirely inside the transducer.

For conditions employed (2000 psia $< p_c <$ 4500 psia, 2000°K $< T_c <$ 4500°K, $0.3 < f < 0.65$, and an effective area ratio of approximately 800), the condensed particles in the stream are roughly 100–200 Å in diameter,[8] and the shock standoff distance for a $\frac{1}{16}$-in.-diam tube could be very roughly estimated as about $\frac{1}{64}$ in. Although an analysis of the flowfield between the shock and the Pitot tube is beyond the scope of this paper, a very rough estimate[10] of the distance for evaporation of the zinc particles is the $\frac{1}{64}$ in. and is at least not inconsistent with Fig. 3. The scatter in Fig. 3 is at least in part due to the variations in the flow conditions. (The curve drawn through the points is merely an "eyeball" curve.)

It had been suggested that the decrease in pitot pressure might be a Reynolds number effect. Although it is true that Pitot pressure first decreases as Reynolds number based on pitot size decreases (before it increases for very low Reynolds numbers), the effect has been shown to be less than ~3%. It therefore would not in itself account for the present decrease in Pitot pressure.

The measurements were made at the exit of a nominal Mach 25 (helium) nozzle for flow of zinc vapor in helium as a carrier gas. The facility employed was The University of Michigan hypersonic tunnel. The facility is an inductance-type hot-shot tunnel that was modified to generate the superheated metal vapor desired.[8] The nozzle exhausted

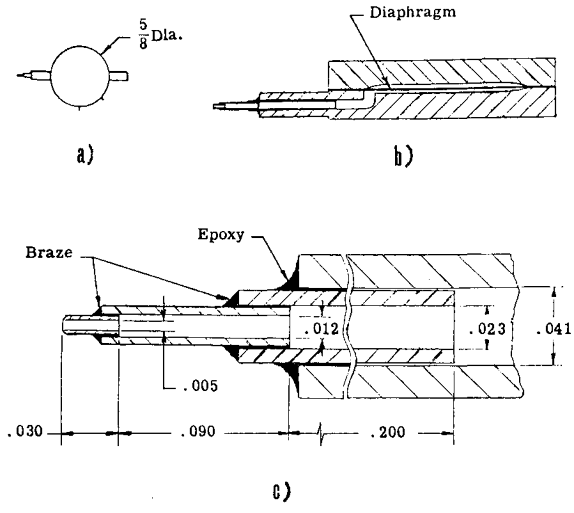

Fig. 4 Miniature Pitot tube arrangement. a) Pitot-tube-transducer assembly, b) section through transducer showing tube installed, c) section showing Pitot tube in transducer entrance (all dimensions in inches).

as a freejet into a cylindrical test section upstream of the large vacuum tank. The total included angle of the conical nozzle was 15°. The mass fraction of zinc could not be predetermined.

Conclusions

The discontinuity analysis for flow with condensation and either no revaporization or complete revaporization downstream of a shock is useful in determining some limits that might be expected in making Pitot tube measurements. In the low Mach number region, it appears that the pressure for the large-diameter tube is close to the pressure for the flow without condensation. It has been pointed out[2] that Pitot pressure at low Mach numbers is unaffected by condensation. For high Mach numbers, however, the difference between Pitot pressure with no condensation in the stream and some condensation in the stream could be significant.

The measurements demonstrate that the large-diameter pitot tube limit is reached with a rather small (0.125 in.) Pitot, and it is not clear whether the small-diameter limit could ever be reached. It appears that the tube could approach mean free path size before the shock standoff distance would be small enough to permit substantially no relaxation or revaporization. Although the limits of the Pitot tube diameter depend on such things as the degree of superheat and the size of particles condensed, the measurements do demonstrate that the Pitot pressure measured does decrease as the tube diameter decreases below the limiting size.

It should be noted that, for condensed particles that have grown "large," there may be separation between the fluid velocity field and that of the particles. This, combined with a large shock standoff distance or large diameter of Pitot tube, could yield higher Pitot pressures than that for small particles that follow the flow.

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