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A METHOD FOR THE OPTIMAL DESIGN OF COMPOSITE CONTINUUM STRUCTURES*

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Introduction The model presented here comprises an application of the approach to formulation of optimal design problems where the modulus tensor of the elastic structural continuum material appears in the role of design variable [see e.g. Bendsøe et al (1994)]. A procedure used in conjunction with this model to generate results for optimal layout (topology), namely the *weighted unit relative cost method* described in Guedes & Taylor (1997), is exploited for present purposes to predict the configuration of reinforcing material in composite structures. The procedure leads from a design at the start having continuously varying material properties through a stepwise procedure to the result providing the layout in the composite of concentrated reinforcement. The stepwise procedure is described in the form of an algorithm. Experience with its implementation for computational treatment indicates that the method is dependable and robust. Results for application of the method to the design of a cylindrical structure with relatively low volume ratio (sparse), and to the prediction of an optimal composite beam are described.

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Analytical Model Among possible forms of variational problem statement for the combined analysis and design problem, the formulation is presented here as a maxmin problem. The inner part, which describes a min problem with quadratic objective and linear constraints, covers the mechanics of elastostatics. The design task is associated with a strictly linearly constrained max on the modulus tensor. The maxmin formulation may be interpreted conveniently to handle computations by iteration between the two parts, and accordingly one is free to make use of readily available means for discretization of the continuum structure.

With E^1_{ijkl} and E^2_{ijkl} symbolizing two constituent materials for the composite design, the system is modelled as though net material properties are given as they would be for a *simple mixture* of the two. Accordingly, the net properties E_{ijkl} are evaluated via:

$$E_{ijkl} = E^1_{ijkl} + E^2_{ijkl}$$

The analysis/design problem for this system is expressed as:

$$[\mathrm{D}] \qquad \max_{E^2_{ijkl}} \quad \min_{u_k(x)}{}' \left\{ \int_\Omega \left[\left(E^1_{ijkl} + E^2_{ijkl} \right) u_{i,j}\, u_{k,l} \right] dV \right\}$$

$$\begin{array}{ll} \textit{subject to} & \textit{subject to} \\ -E^2_{ijkl} \le 0 & \\ \underline{\Phi} \le \Phi \le \overline{\Phi} & \underline{W} - \left\{ \int_\Omega f_i\, u_i\, dV + \int_{\Gamma_t} t_i\, u_i\, dS \right\} \le 0 \\ \int_\Omega \omega\, \Phi\left(E^2_{ijkl}\right) dV - R \le 0 & \end{array}$$

The problem is stated in a form that is consistent with the intent to design a stiffer material E^2_{ijkl} in the setting where the less stiff material E^1_{ijkl} is specified and fixed. Data in problem [D] are the loads, the measure of material resource, the lower bound on compliance, the modulus tensor for the less stiff material, a specified weight on unit resource, and the domain of the structure, represented respectively by $f_i, t_i, R, \underline{W}, \omega, \& \ \Omega$. Argument $\Phi(E)$ of the isoperimetric (resource) constraint symbolizes a designated invariant measure of the modulus tensor (the trace, for example). A more detailed treatment of the maxmin form is given for a somewhat different problem, namely the design of optimal modification, in Taylor (1998).

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Computational Treatment With the weighting coefficient ω set identically to the value unity, the solution to problem [D] furnishes the optimal design for a structure with continuouslyly varying properties for 'material 2'. This result is used as the first step of the above mentioned algorithm, and as was indicated, the net properties are given pointwise simply by the sum of the properties of the two constituents. For subsequent steps, the weighting is systematically adjusted in a way to induce a reduction in the net material stiffness, represented by Φ, wherever it is relatively lower, and a consequent increase in this measure where it is higher. The stepwise procedure is continued until all the designable material is concentrated at either the upper or lower bounds on its invariant measure, and this result provides the sought for layout of optimally distributed, concentrated reinforcement. Note that at this stage the material properties everywhere outside of the part of the domain occupied by the reinforcing are given still by the sum of properties of the constituents. Thus when the lower bound on the stiffer material is taken to be relatively much smaller than the value for the specified less stiff material, the net properties in this part of the structure are essentially just those of the given, specified material. In this way the resulting design does in fact correspond to a two-material composite with the materials essentially fully separated.

Example Results The results shown in Figure 1 represent by the black part the optimal distribution of a reinforcing material in an end loaded, 3D cantilevered beam supported on the right. The balance of the original domain for the design, i.e. the lighter parts of the figure, are to be identified with the specified background material. The ratio of length to cross-sectional dimension is 2.5, the volume fraction of reinforcing material is .40 , and the ratio of effective stiffness of stiffer to stronger material is 3 . The thick walled cylinder of Figure 2 is loaded by uniform pressure on the outer surface. Direct application of the above-described computational procedure would lead to an axisymmetric design. The skeletal structure is obtained by inducing the larger void areas early in the procedure.

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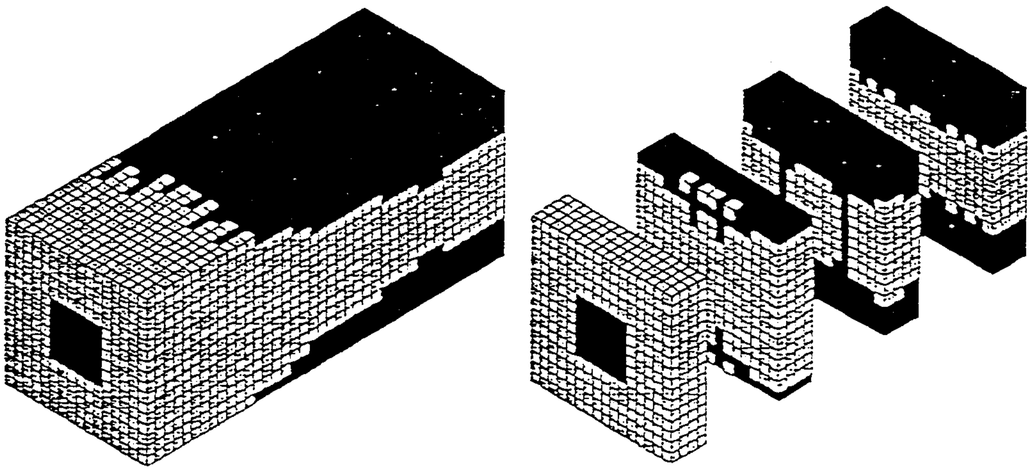

FIGURE 1 - Final design of a two-material composite cantilevered beam

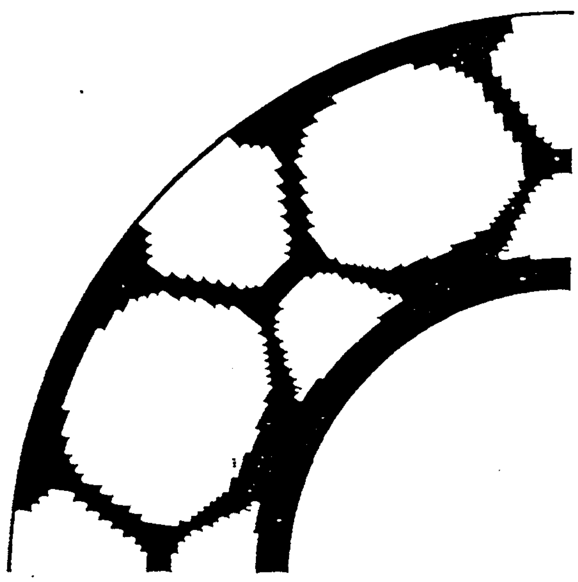

FIGURE 2 - Design of a cylindrical pressure vessel with induced light structure