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## MICROSCALES OF HYDROMAGNETIC CHANNEL FLOW

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Abstract

A two-length model based on the integral and Taylor scales is developed from the non-dimensionalization of the mean kinetic energy balance for homogeneous hydromagnetic turbulence. In the isotropic limit, the integral and Taylor scales approach the Kolmogorov microscale. The homogeneous mean kinetic energy balance reduces to a relation which defines an equivalent Kolmogorov scale for hydromagnetic channel flow. By combining this result with the definition of the turbulent friction coefficient, an implicit expression can be found in terms of integral scales. This expression is given by

$$f \approx \frac{1}{\text{Re}_\ell^{1/4}} \left( 1 + \frac{\text{Re}_\ell^{1/2}}{\text{Al}^2} \right)^{1/2}$$

where  $\text{Al} = \frac{(\rho\mu_0)^{1/2} U}{B}$  is known as the Alfvén number and  $\text{Re}_\ell$  is the Reynolds number based on an integral scale. A correlation for the friction factor is determined explicitly using turbulent experimental data for mercury channel flow from Murgatroyd (1953). The resulting correlation is

$$f = \frac{.290}{\text{Re}_\ell^{1/4}} \left[ 1 + 1.03 \left( \frac{\text{Re}_\ell^{1/2}}{\text{Al}^2} - 10^{-4} \right) \right]^{1/2},$$

for magnetic fields and Reynolds numbers in the range,

$$.4 < B < 1.5 \text{ Tesla}$$

and

$$3.0 \times 10^4 \leq \text{Re}_\ell \leq 1.3 \times 10^5.$$

When the magnetic field approaches zero, the friction coefficient reduces to

$$f = \frac{.290}{\text{Re}_\ell^{1/4}}, \quad B=0.$$

1. Introduction

In a series of articles, Arpaci (1990, 1987, 1986) and Arpaci and co-workers (1988, 1986, 1984) have extended classical works of Taylor (1935) and Kolmogorov (1941) on the microscales of turbulence to complex flows influenced by buoyancy, combustion and oscillation. The present study extends the foregoing microscales to hydromagnetic channel flow.

Following this introduction, Section 2 briefly develops an approximation for the mean magnetic energy of turbulent fluctuations. Section 3 introduces an equivalent Kolmogorov microscale for turbulent hydromagnetic flow developed from a two-length model. Section 4 incorporates this microscale into an implicit relation for the friction coefficient. Section 5 supports the approach used in finding this hydromagnetic Kolmogorov scale by correlating experimental data. Section 6 concludes the study.

2. Mean Magnetic Energy of Turbulent Fluctuations

For large Reynolds numbers, Taylor (1938), in an analogous equation for vorticity, showed that (see also Saffman 1963 for a recent study) the mean magnetic energy of turbulent fluctuations may be approximated by

$$\overline{h_i h_j s_{ij}} = \gamma_M \frac{\partial h_i}{\partial x_j} \frac{\partial h_j}{\partial x_i}. \quad (1)$$

On dimensional grounds, Eq. (1) yields

$$P_M \approx h^2 \frac{u}{\ell} \approx \gamma_M \frac{h^2}{\lambda^2} \approx \epsilon_M, \quad (2)$$

where  $h = \frac{b}{\rho\mu_0}$  represents the fluctuating part of the instantaneous magnetic density,  $\gamma_M = \nu\rho\mu_0$  is the magnetic diffusivity, and  $\approx$  is used to denote a scaling relation.

### 3. An Equivalent Kolmogorov Scale for Hydromagnetic Flow

Extending Taylor (1935), the balance of hydromagnetic energy for homogenous pure shear flow may be written as

$$P + P_M = \varepsilon, \quad (3)$$

where

$$P = -\overline{u_i u_j S_{ij}} \quad (4)$$

is the inertial production,

$$P_M = \frac{1}{\rho\mu_0} \overline{b_i b_j S_{ij}} \quad (5)$$

is the magnetic production, and

$$\varepsilon = 2\nu \overline{S_{ij} S_{ij}} \quad (6)$$

is the dissipation of turbulent kinetic energy. On dimensional grounds, Eq. (3) becomes

$$P + P_M \approx u^2 \frac{U}{\ell} + \frac{b^2}{\rho\mu_0} \frac{u}{\lambda} \approx \nu \frac{u^2}{\lambda^2} \approx \varepsilon. \quad (7)$$

A viscous estimate for the velocity in terms of the dissipation is given by

$$u \approx \lambda \left( \frac{\varepsilon}{\nu} \right)^{1/2}. \quad (8)$$

Then, Eq. (7) gives in terms of Eq. (8)

$$\frac{u^3}{\ell} \left( 1 + \frac{b^2 \ell}{\rho\mu_0 \lambda u^2} \right) \approx \nu \frac{u^2}{\lambda^2} \approx \varepsilon, \quad (9)$$

and

$$u \approx \frac{(\varepsilon \ell)^{1/3}}{\left( 1 + \frac{b^2 \ell}{\rho\mu_0 \lambda u^2} \right)^{1/3}}. \quad (10)$$

Elimination of the velocity between Eqs. (8) and (10) yields the dissipation scale

$$\lambda = \frac{\ell^{1/3} \left( \frac{\nu^3}{\varepsilon} \right)^{1/6}}{\left( 1 + \frac{B^2}{\rho\mu_0} \cdot \frac{\ell}{\lambda u^2} \right)}. \quad (11)$$

As the magnetic field approaches zero, Eq. (11) approaches the Taylor microscale,

$$\lambda = \ell^{1/3} \left( \frac{\nu^3}{\varepsilon} \right)^{1/6}. \quad (12)$$

In the isotropic limit as the integral and dissipative length scales approach the Kolmogorov length scale,  $\eta$ , and the velocity approaches the Kolmogorov velocity scale,  $v$ , Eq. (11) reduces to

$$\eta \approx \frac{\left( \nu^3 / \varepsilon \right)^{1/4}}{\left( 1 + \frac{B^2}{\rho\mu_0} \cdot \frac{1}{\nu^2} \right)^{1/2}}. \quad (13)$$

Substituting the Kolmogorov velocity scale,

$$\nu = (\nu \varepsilon)^{1/4}, \quad (14)$$

into Eq. (13) yields

$$\eta \approx \frac{\left( \nu^3 / \varepsilon \right)^{1/4}}{\left( 1 + \frac{B^2 / \rho\mu_0}{(\nu \varepsilon)^{1/2}} \right)^{1/2}}. \quad (15)$$

Incorporating the viscous dissipation into Eq. (15) and assuming  $u \approx U$ , yields,

$$\eta \approx \frac{\left( \nu^3 \ell^{1/4} / U^{3/4} \right)}{\left( 1 + \frac{B^2}{\rho\mu_0} \cdot \frac{\ell^{1/2}}{\nu^{1/2} U^{3/2}} \right)^{1/2}}. \quad (16)$$

For reasons that will become evident in section 4, it is convenient to write Eq. (16) in the form

$$\frac{\ell}{\eta} \approx \left( \frac{\ell^{3/4} U^{3/4}}{\nu^{3/4}} \right) \left( 1 + \frac{B^2 \ell^{1/2}}{\rho\mu_0 \nu^{1/2} U^{3/2}} \right)^{1/2}, \quad (17)$$

which can be written in terms of dimensionless parameters as

$$\frac{\ell}{\eta} \approx \text{Re}_\ell^{3/4} \left( 1 + \frac{\text{Re}_\ell^{1/2}}{\text{Al}^2} \right)^{1/2}. \quad (18)$$

#### 4. Turbulent Hydromagnetic Friction Factor

For turbulent flows the friction factor is defined as

$$f \approx \frac{\tau_w}{\frac{1}{2}\rho u^2} \approx \frac{\mu u/\eta}{\frac{1}{2}\rho u^2}, \quad (19)$$

which, in view of  $u \approx U$ , can be written as

$$f \approx \left(\frac{\nu}{U\ell}\right)\left(\frac{\ell}{\eta}\right). \quad (20)$$

Recalling the definition of the Reynold's number, the friction factor can be expressed as

$$f \approx \left(\frac{1}{\text{Re}_\ell}\right)\left(\frac{\ell}{\eta}\right). \quad (21)$$

Inserting Eq. (18) into the above relation yields an implicit expression for the friction factor,

$$f = \frac{1}{\text{Re}_\ell^{1/4}} \left[ 1 + \frac{\text{Re}_\ell^{1/2}}{\text{Al}^2} \right]^{1/2}. \quad (22)$$

Note that since  $\text{Al}^2 = \frac{\rho\mu_0 U^2}{B^2}$ , when the magnetic field vector is zero, Eq. (22) reduces to

$$f \approx \frac{1}{\text{Re}_\ell^{1/4}}, \quad B \rightarrow 0. \quad (23)$$

as expected.

#### 5. Turbulent Friction Correlation for Hydromagnetic Channel Flow

In 1953 Murgatroyd published experimental results of turbulent flow for mercury through a (.676 X 10.16) cm<sup>2</sup> channel. His data covered the range

$$3.0 \times 10^4 \leq \text{Re}_\ell \leq 1.3 \times 10^5,$$

where  $\text{Re}_\ell$  is the Reynold's number based on the channel hydraulic diameter.

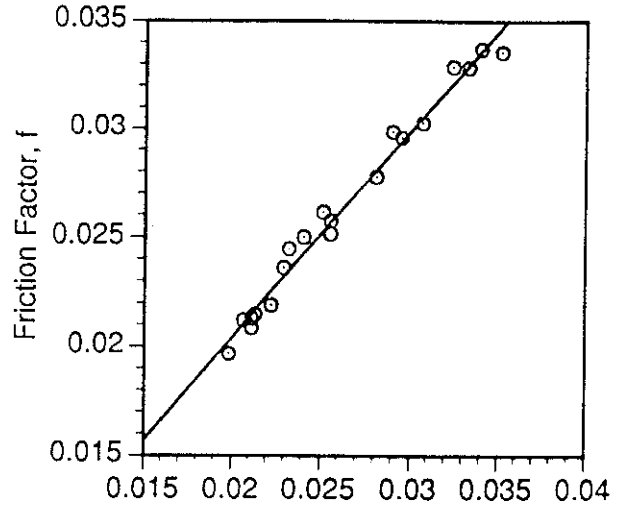
Coupling Murgatroyd's experimental data with the results of section 4, a turbulent friction correlation is obtained.

$$f = \frac{.290}{\text{Re}_\ell^{1/4}} \left[ 1 + 1.03 \left( \frac{\text{Re}_\ell^{1/2}}{\text{Al}^2} 10^{-4} \right) \right]^{1/2} \quad (24)$$

for the range (.4 < B < 1.5) Tesla, the product of the terms in parantheses being on the order of one. In the limiting case of zero magnetic field, the friction factor simplifies to

$$f = \frac{.290}{\text{Re}_\ell^{1/4}}, \quad B = 0 \text{ Tesla}. \quad (25)$$

Figure 1 below shows the foregoing correlation.



$$\frac{.290}{\text{Re}_\ell^{1/4}} \left[ 1 + 1.03 \left( \frac{\text{Re}_\ell^{1/2}}{\text{Al}^2} 10^{-4} \right) \right]^{1/2}$$

Figure 1: Turbulent Friction Correlation for Hydromagnetic Channel Flow

#### 6. Conclusion

This study extends the approach relating the microscales and the integral scale of turbulence to flow correlations. For hydromagnetic flow, the Kolmogorov scale is modified to account for the magnetic field. The resulting microscale is incorporated into the definition of the friction coefficient, yielding an implicit expression in terms of the Reynold's number based on an integral scale and the Alfvén number. The following turbulent friction correlations for hydromagnetic channel flow are found using experimental data from Murgatroyd (1953):

$$f = \frac{.290}{\text{Re}_\ell^{1/4}} \left[ 1 + 1.03 \left( \frac{\text{Re}_\ell^{1/2}}{\text{Al}^2} 10^{-4} \right) \right]^{1/2},$$

$$.4 < B < 1.5 \text{ Tesla}$$

and

$$f = \frac{.290}{\text{Re}_\ell^{1/4}}, \quad B = 0 \text{ Tesla.}$$

The foregoing correlations established for the range of Reynolds numbers,

$$3 \times 10^4 \leq \text{Re}_\ell \leq 1.22 \times 10^5,$$

support the microscale approach used in this work and several others by Arpaci (1990, 1987, 1986) and Arpaci and co-workers (1988, 1986, 1984).

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