Engineering Notes

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Oscillation of High-Altitude Balloons

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Nomenclature

c_a = aerodynamic damping coefficient
f = frequency, Hz
f_{vb} = Väisälä-Brunt frequency, Hz
$h(t)$ = altitude of balloon
k_b = spring rate due to buoyancy
$L(t)$ = lift due to buoyancy
m_h = mass of helium contained in balloon
m_s = mass of structure of balloon, including payload
m_v = virtual mass of balloon and payload
p = pressure
R = gas constant
T = temperature
V = total volume of balloon gas
γ = ratio of specific heats
$\Delta(\ )$ = incremental operator
ρ = density

Subscripts
a = air
h = helium
0 = sea level
1 = reference altitude (before perturbation)
2 = perturbed altitude

Superscript
' = derivative

Introduction

THERE is a class of research balloons that operates at near-zero internal gauge pressure through the use of vents to the atmosphere at the bottom of the balloon. These balloons are partially filled with helium at launch and expand in volume with altitude gain. Many such balloons reach a "float" altitude of 37,000 m (120,000 ft), where helium volume is on the order of 600,000 m^3 (22,000,000 ft^3). The balloon envelope is polyethylene film of approximately 15 μ (0.0006 in.) thickness.

Helium-filled zero-pressure balloons have been observed to exhibit an oscillatory motion about the mean climb path (Fig. 1). The oscillation occurs over a broad range of altitude and indicates that the buoyancy of the balloon provides a "spring rate" that allows the balloon to act as an oscillator about the mean path. Aerodynamic damping is effective at suppressing motion at altitudes lower than 9100 m (30,000 ft) but has decreasing effect at higher altitudes.

The altitude given in Fig. 1 has been measured by pressure transducers in the balloon payload and referred to the standard atmosphere. Altitude perturbations as high as 300 m (1000 ft) are observed. The erratic climb of the balloon has been called "stair stepping." To date, such oscillations have been a minor curiosity and are not controlled. In the future, however, it may be desirable to apply flight control to provide a more regular climb behavior.

There are several types of disturbances present in the atmosphere that might promote oscillation of the balloon during climb. The most interesting are "atmospheric" waves[2,3] (also called "internal" waves) at the Väisälä-Brunt frequency. Indeed, in the tropopause, the Väisälä-Brunt frequency is close to a helium balloon's natural buoyant frequency. Because the balloon's amplitude of oscillation is far larger than the typical atmospheric wave amplitudes, it appears that resonance occurs, rather than the balloon merely riding the atmospheric wave.

Balloon buoyant oscillations and the atmospheric waves have been separately studied in earlier work.[1,3] The two problems will be reviewed in the next two sections.

As a related problem, balloons have also been observed to oscillate in altitude as the float altitude is first reached, due to an overshoot phenomena.[1] This is a dynamic transient; the phenomenon is well understood and is a different type of forced oscillation from that during steady climb or descent.

Balloon Buoyant Frequency

The balloon is modeled as a simple linear oscillator, with the balloon structure acting as a rigid body:

$$[m_s + m_h + m_v]\ddot{h}(t) + [c_a]\dot{h}(t) + [k_b]h(t) = L(t)$$

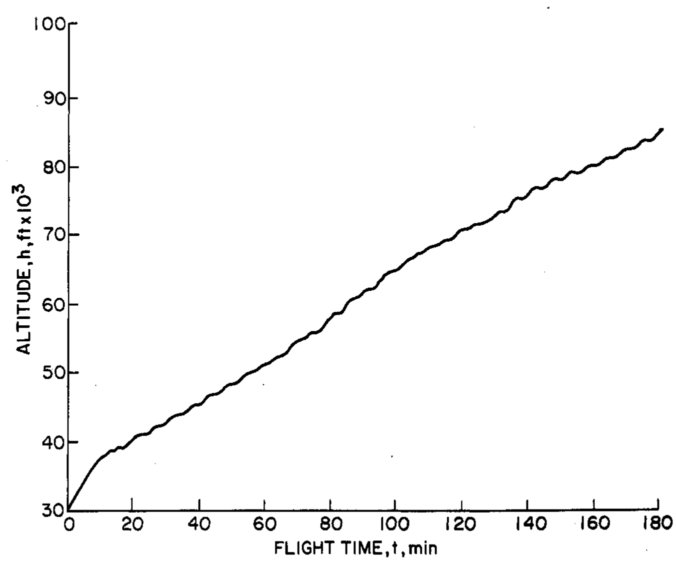

Fig. 1 Altitude profile for high-altitude helium balloon.

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The damping term is a viscous model of aerodynamic drag and will be neglected in our frequency discussions.

The float altitude is determined by the lift term $L(t)$. When the lifting gas can provide no excess lift, the float altitude has been obtained. For many flights, this altitude is where the balloon reaches its maximum capacity (becomes "overinflated" with gas) and starts to exhaust helium through the vents.

The oscillations of the balloon are governed by the buoyancy spring rate and the mass terms. The mass of the balloon is increased by approximately 50% due to the virtual mass of air surrounding the balloon, i.e., $m_v = 0.5(m_s + m_h)$. (Such a relation is exact for spheres and only approximate for similar shapes.) Experimental studies are underway to determine the virtual mass for small-scale balloons in free flight.

Observations show that during the perturbation from the mean position, the balloon may change altitude by a thousand feet and require several minutes for a period of oscillation. This period is long by structural standards but is small compared to the time required to reach thermal equilibrium in the helium. The helium will, therefore, undergo reversible, adiabatic pressure oscillations.

A specific balloon size will be considered. The frequency results, however, are valid for similar zero-pressure helium balloons of arbitrary size. The balloon structure, including payload, weighs 4536 kg (10,000 lb). It is initially filled with 3203 m^3 (113,100 ft^3) of helium gas at sea level conditions.

The effective spring rate is − (change in lift)/(change in altitude) due to gas buoyancy. If this spring force opposes the perturbed motion, the balloon can maintain a stable flight path. If the buoyancy forces tend to promote an altitude perturbation, the unpressurized helium balloon will not be stable but will monotonically diverge. There can be no oscillation in such a case. It will turn out that there is a stable buoyancy spring rate for helium balloons at all altitudes. This is due to helium's relatively large ratio of specific heats $\gamma_h = 1.66$ (On the other hand, hydrogen balloons can be unstable or stable, depending on atmospheric conditions.)

If the spring rate is calculated about the overinflated float position, it will be nonlinear, since the venting of gas on the upward half of the cycle will make the spring stiffer than on the lower half-cycle. This case is not considered here.

Consider the helium balloon at a reference position 1 (mean altitude) and a perturbed position 2. The standard air temperatures, pressures, and densities are known for the reference altitude T_{a_1}, p_{a_1}, ρ_{a1} and for the perturbed altitude T_{a_2}, p_{a_2}, ρ_{a_2}. The gas constant for helium R_h = 2,098 (J/kgK) [12,540 (ft · lb/slug°R)].

The helium at the reference altitude is at standard temperature and pressure: $T_{h_1} = T_{a_1}$ and $p_{h_1} = p_{a_1}$. The density and total volume of helium at state 1 are

$$\rho_{h_1} = \left(\frac{p_{h_1}}{R_h T_{h_1}}\right)$$

$$V_1 = V_0 \left(\frac{p_0}{p_1}\right)\left(\frac{T_1}{T_0}\right)$$

The density of the helium in the perturbed state is

$$\rho_{h_2} = \rho_{h_1} \left(\frac{p_{h_2}}{p_{h_1}}\right)^{(1/\gamma_h)}$$

The perturbed volume of helium is

$$V_2 = V_1 \left(\frac{\rho_{h_1}}{\rho_{h_2}}\right)$$

The total lift of the balloon at the reference altitude is

$$L = g(\rho_{a_1} - \rho_{h_1})V_1$$

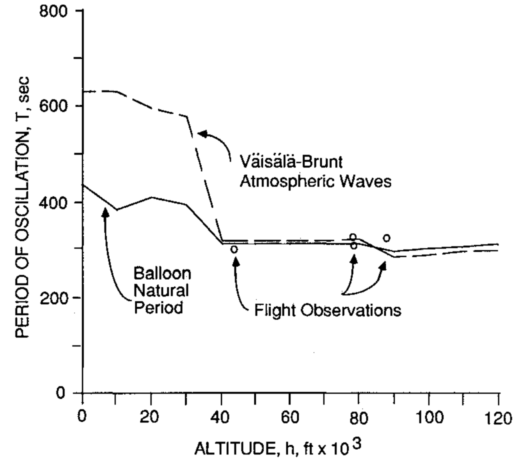

Fig. 2 Atmospheric, buoyancy, and observed periods.

Under a small perturbation of altitude, one finds an increment of lift (neglecting higher order terms):

$$\Delta L = g(\rho_{a_1} - \rho_{h_1})\Delta V + g(\Delta\rho_a - \Delta\rho_h)V_1$$

The increment is partly caused by the change in balloon volume and partly due to the change in densities of air and helium.

The spring rate of the buoyancy is given by the change in lift divided by the altitude perturbation. The negative sign is chosen so that a positive spring rate implies a stable restoring force:

$$k_b = -\Delta L/\Delta h$$

If the system is statically stable, the frequency of oscillation of the balloon is given by

$$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k_b}{m_s + m_h + m_v}}$$

The buoyant frequency does not depend on the size of the balloon because the spring rate is proportional to the displaced volume, which in turn is proportional to the weight (mass) of the balloon.

Atmospheric Waves

Any compressible fluid with a density gradient is a candidate for internal waves maintained by a balance between gravity and pressure effects (Lighthill, Ref. 2). If the waves are horizontally oriented, the frequency is given by the Väisälä-Brunt frequency:

$$f_{vb} = \frac{1}{2\pi}\sqrt{-\frac{g\rho_1'}{\rho_1} - \frac{g^2\rho_1}{\gamma_a p_{a_1}}}$$

where ρ_1' is the gradient of density with respect to altitude, evaluated at the reference position. This is actually a maximum frequency; waves with some vertical inclination would have a lower frequency. In figures given later, therefore, the period will be a lower limit for arbitrary inclinations of the wave.

The vertical amplitude of internal waves (up to 30 m) is an order of magnitude smaller than the amplitude of balloon oscillations that have been observed. The pressure perturbations due to the internal waves is a higher order effect on the pressure measurements used for altitude of the balloon.

Results

The analysis considered the buoyancy oscillation frequency of an unpressurized helium balloon in steady ascent or descent, or at float when no helium is venting. The spring rate of the buoyancy force for this size balloon varied from 1.6 N/m (0.11 lb/ft) to 3.5 N/m (0.24 lb/ft) for altitudes from sea level to 36,600 m (120,000 ft). At no altitude was the spring rate negative, which would have implied a divergent instability of the flight path. The magnitude of this restoring force was modest for a 4536-kg (10,000-lb) balloon and led to long periods, on the order of 5 min.

The buoyancy period of oscillation is shown as a function of altitude in Fig. 2. It is essentially constant in the tropopause. Experimental observations are also shown and are moderately close to the balloon's theoretical buoyancy periods.

When the Väisälä-Brunt period is added to the results in Fig. 2, one can see that such an atmospheric wave could easily excite the balloon at the Väisälä-Brunt frequency. The two periods (internal wave and balloon buoyancy) are so close that one must consider the possibility of a resonance phenomenon, particularly at altitudes greater than 11,600 m (38,000 ft).

Of course, there are other perturbations to the balloon flight, such as wind gusts or ballast drops, which could cause oscillations. The response to such perturbations should be in the form of a damped oscillation. A review of the rate of buildup or decay of oscillations in various flights could lead to a more rigorous evaluation as to which are resonant and which are the result of input pulses.

Conclusions

Helium balloon buoyancy oscillations can be excited by atmospheric waves at the Väisälä-Brunt frequency. These frequencies are close enough to the balloon's own natural frequency as to cause a near resonance condition, particularly from 11,600 to 24,400 m (38,000 to 80,000 ft). This near-resonance condition will hold for helium balloons of any size and payload, because the buoyant frequencies for the balloon are independent of size.

Designers of balloon flight controls should be aware of the buoyant frequencies involved, should anticipate vertical perturbations with period of approximately 5 min, and should not be drawn into an inappropriate means of maintaining steady rate-of-climb in fighting this oscillation. Dropping ballast or venting helium would be ineffective, unless properly phased in time.

Acknowledgments

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