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FUEL CONFINEMENT AND STABILITY IN THE GAS CORE NUCLEAR PROPULSION CONCEPT

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Abstract

One of the most promising approaches to advanced propulsion that could meet the objectives of the Space Exploration Initiative (SEI) is the open cycle gas core nuclear rocket (GCR). The energy in this device is generated by a fissioning uranium plasma which heats, through radiation, a propellant that flows around the core and exits through a nozzle, thereby converting thermal energy into thrust. Although such a scheme can produce very attractive propulsion parameters in the form of high specific impulse and high thrust, it does suffer from serious physics and engineering problems that must be addressed if it is to become a viable propulsion system. Among the major problems that must be solved are the confinement of the uranium plasma, potential instabilities and control problems associated with the dynamics of the uranium core, and the question of startup and fueling of such a reactor.

In this paper, we focus our attention on the problems of equilibria and stability of the uranium core, and examine the potential use of an externally applied magnetic field for these purposes. We find that steady state operation of the reactor is possible only for certain core profiles that may not be compatible with the radiative aspect of the system. We also find that the system is susceptible to hydrodynamic and acoustic instabilities that could deplete the uranium fuel in a short time if not properly suppressed.

Introduction

A propulsion scheme that was first introduced in the sixties, and recently revived with the expectation that it might meet the needs of the Space Exploration Initiative (SEI) of the next century is the open cycle gas core nuclear rocket⁽¹⁾ (GCR), shown in Fig. (1). If successfully developed, it has the potential of meeting the objectives of SEI of returning to the moon, and on to Mars with manned missions in the first half of the twenty-first century. Since space travel is hazardous, and man is unable to endure long journeys without experiencing physical and mental degradation, it is imperative that such missions be completed in the shortest possible time. GCR has the potential of meeting these requirements. The principle of operation in this system involves a critical uranium core in the form of a gaseous plasma that heats, through radiation, a hydrogen propellant which exits through a nozzle, thereby converting thermal energy into thrust as illustrated in Fig. (1).

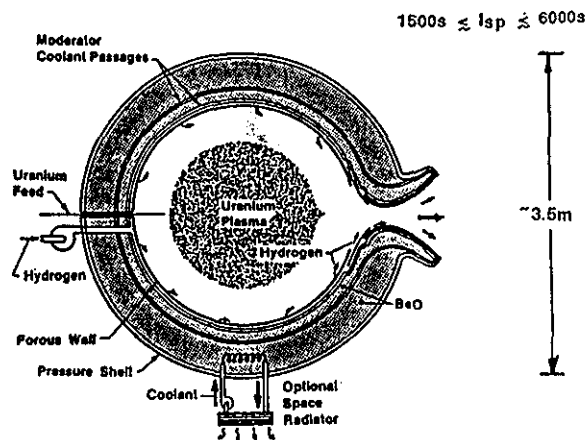


Fig. 1.
High Specific Impulse, Porous Wall Gas Core Engine.
(Courtesy of NASA, Lewis Research Center)

The temperature limitations imposed by material melting encountered in solid core thermal reactors is avoided in GCR since the nuclear fuel is allowed to exist in a high temperature ($10^4 - 10^5$ K) partially ionized state. In this so-called gaseous or "plasma core" concept, the sphere of fissioning uranium plasma functions as the fuel element of the reactor. Nuclear heat released within the plasma and dissipated as thermal radiation from the surface is absorbed by a surrounding envelope of seeded hydrogen propellant, which is then expanded through a nozzle to generate thrust. With the gas core rocket concept, specific impulse values ranging from 1500 to 7000 seconds appear to be feasible⁽¹⁾. This reactor concept requires a relatively high-pressure plasma (500 - 1000 atm) to achieve critical mass. At these pressures, the gaseous fuel is sufficiently dense for the fission fragment stopping distance to be comparable to or smaller than the dimensions of the fuel volume contained within the reactor cavity. The hydrogen propellant is injected through the porous wall with a flow distribution that creates a relatively stagnant non-recirculating central fuel region in the cavity. The question immediately arises as to whether this hydrodynamic containment is compatible with the performance requirements placed on GCR as a propulsion device. This paper is aimed at addressing some of these questions, and identifying perhaps the major obstacles that could seriously detract from its propulsion capabilities.

Basic Equations and Analysis

Assuming a singly ionized uranium plasma that remains so at all times, the appropriate conservation equations for the system may be expressed by:

$$\frac{\partial N}{\partial t} + \vec{\nabla} \cdot (N\vec{V}) = 0 \quad (1)$$

$$\frac{\partial \vec{V}}{\partial t} + \vec{\nabla} \cdot \frac{\partial \vec{V}}{\partial x} + \frac{1}{MN} \frac{\partial}{\partial x} [2NKT] = 0 \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \vec{\nabla} \cdot \frac{\partial}{\partial x} \right) [3NKT] + [5NKT] \frac{\partial}{\partial x} \cdot \vec{V} = P_f + \frac{\partial}{\partial x} \cdot \left(K_R \frac{\partial T}{\partial x} \right) \quad (3)$$

where N is the number density of uranium ions, \vec{V} the fluid velocity, T the temperature, and M the mass of the uranium ion. The above equations become a closed system when we specify the fission power density, namely

$$P_f = NnV\sigma Q = \alpha\phi\rho = q \quad (4)$$

and the radiation diffusion coefficient

$$K_R = \frac{16\sigma_B T^3}{3k_R} \quad (5)$$

In the above equations, n is the number density of neutrons, V their average velocity, ϕ their flux, σ the cross section for neutron-induced fission, and Q the energy released as fragment kinetic energy per fission. In the second form of P_f , the quantity α is in effect a constant, while $\rho = MN$ is the mass density of the uranium. Eq.(5) gives an effective diffusion coefficient for the radiative energy term shown at the end of Eq.(3). The quantity σ_B is the black body constant and k_R the mean Rosseland absorption coefficient. If the expansion term (second term on the left hand side) in the energy equation (3) is ignored, the radiative term is replaced by a simple heat conduction term with a constant coefficient $k = k_R$, and a constant C_p (specific heat at constant pressure) term is introduced, then Eqs.(1-3) can be replaced by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho\vec{V}) = 0 \quad (6)$$

$$\rho \left\{ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla}) \vec{V} \right\} + \vec{\nabla} P = 0 \quad (7)$$

$$\rho C_p \left\{ \frac{\partial T}{\partial t} + (\vec{V} \cdot \vec{\nabla}) T \right\} = q + k \nabla^2 T \quad (8)$$

which must be supplemented by the equation of state, i.e.

$$P = \rho RT \quad (9)$$

where R is the familiar gas constant. A quick glance at Eqs.(1-5) shows that due to the non-linearity of some of the terms, it is difficult to analytically obtain the dynamic equilibrium of the system as represented by the spatial profiles of the density, temperature, and velocity of the uranium core. In order to address some of the questions raised earlier, it is convenient to deal with the approximate but simpler set of conservation equations represented by Eqs.(6-9).

If we limit our analysis to a one-dimensional system (e.g. slab), consider steady state so that $\partial/\partial t = 0$, and further assume a constant heat generation $q = q_0 = \text{constant}$, then for a system with $\vec{V} \cdot \nabla = 0$, Eq.(6) yields

$$\rho V = \rho(0)V(0) = a = \text{Constant} \quad (10)$$

where zero subscript denotes the value at the origin. The energy equation (8), under these conditions, assumes the form

$$\frac{\partial^2 T}{\partial z^2} - \frac{C_p a}{k} \frac{\partial T}{\partial z} = -\frac{q_0}{k} \quad (11)$$

which, when solved subject to the conditions $T(0) = T_0$ and $T(z=L) = T_L$, yields the temperature profile:

$$T(z) = T_0 + \frac{q_0 z}{a C_p} + \frac{[T_L - T_0 - q_0 L / (a C_p)] \left\{ \exp\left(\frac{a C_p z}{k}\right) - 1 \right\}}{\left[\exp(a C_p L / k) - 1 \right]} \quad (12)$$

The momentum Eq.(7) along with Eqs.(9) and (10) can be put in the form

$$a \frac{\partial V}{\partial z} + \frac{\partial}{\partial z} (\rho RT) = 0 \quad (13)$$

which can be readily integrated to yield

$$V(z) = \frac{1}{2} \left(V_0 + \frac{RT_0}{V_0} \right) + \left\{ \frac{1}{4} \left(V_0 + \frac{RT_0}{V_0} \right)^2 - RT(z) \right\}^{1/2} \quad (14)$$

with $T(z)$ given by Eq.(12). The remaining equilibrium profiles are given by

$$\rho(z) = \frac{a}{V(z)} = \frac{\rho_0 V_0}{V(z)} \quad (15)$$

$$P(z) = R \rho(z) T(z) \quad (16)$$

The above results were obtained on the premise that the uranium core undergoes motion ($V \neq 0$) in equilibrium, which is represented by the velocity profile given by Eq.(14). If the equilibrium is static (i.e. $V = 0$), and the heat generation is also uniform (i.e. $q = q_0$), then the corresponding profiles in this case are obtained from Eqs.(6-9) to be

$$P = P_0$$

$$\rho_0 = \frac{P_0}{RT(z)} \quad (17)$$

$$T(z) = T_0 - \frac{q_0 z^2}{2k} + \left[\frac{T_L - T_0}{L} + \frac{q_0 L}{2k} \right] z$$

The above equilibria, approximate as they are, will be used to examine the hydrodynamic stability of the system.

It is a known fact that when a fluid of density ρ_2 moves with velocity V_2 past another fluid of density ρ_1 , which is stationary, in the presence of a gravitational force, the (sharp) boundary between them will, upon perturbation, undergo oscillations which under certain conditions can become unstable. This instability, known as the Kelvin-Helmholtz instability⁽²⁾, can lead to turbulent diffusion of material from one region into the other, and in the case of GCR, this could mean substantial flow of uranium from the core into the hydrogen and out through the nozzle. Not only will the loss of uranium affect the criticality of the system if not appropriately replaced, but also the flow of the hydrogen into the core will affect its composition and ultimately its criticality.

To assess the importance of this phenomenon, we apply it to a GCR design⁽³⁾ in which the radius r of the uranium core is 1 meter, the pressure is 1000 atm, and the hydrogen temperature is about 17,500°K, which suggests that the fuel temperature is about 35,000°K⁽⁴⁾. Our preliminary analysis of this 7500 MW system shows that the mean velocity of the hydrogen, which is commensurate with a cited mass flow rate of 4.5 kg/sec, is approximately 5 m/sec. If we first consider the static equilibrium case represented by Eq.(17), then the system under consideration may be viewed as consisting of a fluid (H) of density ρ_2 and velocity V_2 moving past a stationary fluid (U) of average density ρ_1 (obtained by averaging the expression in (17)) under the influence of a gravitational acceleration g . The instability condition can be written as⁽³⁾

$$V_2^2 > \frac{g(\rho_1 - \rho_2)}{k_V \rho_1 \rho_2} \approx \frac{g \rho_1}{k_V \rho_2} \quad (18)$$

where we have taken advantage of the fact that, for the temperature and pressure under consideration, the uranium density is much larger than that of the hydrogen. The above equation reveals that the minimum wave number, k_V , of the oscillation has the value

$$k_V = \frac{g \rho_1}{V_2^2 \rho_2} \quad (19)$$

and the corresponding growth rate γ of the instability is of the form

$$\gamma = V_2 k_V \sqrt{\frac{\rho_2}{\rho_1}} = \frac{g}{V_2} \sqrt{\frac{\rho_1}{\rho_2}} \quad (20)$$

The diffusion coefficient D for the uranium flow into the hydrogen can be approximated by

$$D = \frac{V}{k_V^2} \quad (21)$$

from which we can write the particle flux as

$$F = \frac{D \rho_1}{r} \quad (22)$$

where r is the radius of the spherical uranium core introduced earlier. The amount of uranium escaping per second by this diffusion process, U_L , can finally be written as

$$U_L = 4\pi r^2 F = 4\pi r D \rho_1 \quad (23)$$

or as a fraction of the total uranium, U_s , present in the sphere,

$$\frac{U_L}{U_s} = \frac{4\pi r D \rho_1}{4\pi r^3 \rho_1 / 3} = \frac{3D}{r^2} = \frac{3\gamma}{r^2 k_V^2} \quad (24)$$

At a pressure of 1000 atm, a hydrogen temperature of 17,500°K, and a uranium temperature of 35,000°K, the densities of hydrogen and uranium are, respectively, $4.6 \times 10^{-4} \text{ g/cm}^3$ and $5.53 \times 10^{-2} \text{ g/cm}^3$. With these values, and $V_2 = 5 \text{ m/sec}$, Eq.(23) yields about 7 kg/sec uranium loss, while Eq.(24) shows that approximately 3% of the fuel escapes per second. Clearly, these quantities are unacceptably large, and well over the 1% of the hydrogen mass flow rate (i.e. 45 g/sec) often cited as the loss due to turbulent mixing. In addition, this loss is far greater than the uranium burnup rate of 0.1 g/sec of U235 in a 7500 MW reactor. As can be seen from Eq.(20), the growth rate for a fixed wave number (i.e. a fixed wavelength) is smaller for smaller hydrogen flow velocity. But decreasing this velocity beyond a certain value may not be compatible with the mass flow rate dictated by heat transfer needs.

It may be argued that the above description of a hydrogen propellant flowing past a stationary uranium core is not an adequate description, since there might exist a thin boundary layer of uranium which is moving with the same velocity as the hydrogen, and thus no relative motion and correspondingly no instability. This, however, is not true since the same instability can arise in a fluid in which both the density and the velocity are continuously variable. In this case, an important parameter, J , known as the Richardson number, defined by

$$J = \frac{g}{\rho} \frac{dp/dz}{(dV/dz)^2} \quad (25)$$

emerges as the critical parameter for the Kelvin-Helmholtz instability. It represents the ratio of the buoyancy force to the inertial force, and must have a value of $J > 1/4$ for stability. When applied to the equilibrium represented by Eqs.(12-16), we see that $J < 0$ and hence the system remains unstable. Unless an

equilibrium with the appropriate density profile is found, GCR will continue to suffer from this hydrodynamic instability.

If profiling effects cannot be achieved or sustained, then perhaps the use of magnetic fields to suppress this instability may not be totally avoidable. It can be shown that if a magnetic field B is introduced in the direction of propellant flow, then it can act as a "surface tension" type of force that provides stability if the following condition is satisfied:

$$\frac{\rho_1 \rho_2}{(\rho_1 + \rho_2)} V^2 \leq \frac{B^2}{8\pi} \quad (26)$$

We see for the example addressed earlier that a minimum magnetic field strength of 54 Gauss is required. The shape of such a field is likely to be "Mirror-like" in order to accommodate the flow around the spherical uranium core. Although such a field can bring about stabilization, it is much too small to confine a plasma at 1000 atm. pressure, but might be adequate to respond to pressure fluctuations that may also occur in the system.

Another problem of major concern in GCR has to do with acoustic instabilities that might arise as a result of fluctuations in the density and temperature of the fissioning plasma. The mechanism for the generation of such oscillations can be described as follows(5): we imagine a standing sound wave to exist in a bounded region of the fissioning plasma that includes a constant background density of thermal neutrons; in the wave compressions, the fission power density increases due to the increased uranium density, while in the rarefactions the power decreases. This results in an increased pressure gradient associated with the wave, which in turn leads to a transfer of fission power to the wave. But competing with this process is the fact that radiation also tends to transport the extra thermal energy out of the wave compressions. Moreover, radiation diffusion tends to smooth out the temperature fluctuations of waves more rapidly as their wavelengths become shorter. This results in a critical wavelength below which waves are stable, and above which they are unstable. If the characteristic dimension of the system, such as the core radius, is larger than the critical wavelength, then the system will be unstable to these modes, and that could precipitate significant pressure fluctuations which could present serious control problems for GCR. Moreover, such unstable waves could also give rise to a significant uranium loss from the core which, eventually, will find its way out through the nozzle.

We assess the impact of the acoustic instability by returning to the basic equations (1-5), and carrying out a perturbation analysis. We take the equilibrium state to be that with $\vec{V} = 0$ and assume that all perturbed quantities are of the plane wave form, i.e.

$$N_1, T_1, V_1 \sim \exp i(\vec{k}_w \cdot \vec{x} - \omega t) \quad (27)$$

where k_w and ω are the wave number and frequency of the oscillation, respectively. A dispersion equation relating k_w to ω is then obtained which yields(5), upon solution, the linear growth rate γ , i.e.

$$\gamma = \frac{(2P_f/M - k_w^2 K_s [V_s^2 - 2KT_0/M]/K)}{6N_0 V_s^2} \quad (28)$$

where in this case, N_0 and T_0 represent the equilibrium density and temperature of the uranium core. The sound speed in the plasma, V_s , is given by

$$V_s = \left(\frac{10KT_0}{3M} \right)^{1/2} \quad (29)$$

with K denoting the Boltzmann constant and M the mass of the uranium atom. We note from Eq.(28) that a positive numerator gives rise to an instability (i.e. a wave with growing amplitude) while a negative value denotes a damped (stable) wave. The transition from one to the other is characterized by a critical wave number k_c given by

$$k_c = \left[\frac{5KP_f}{MV_s^2 K_s} \right]^{1/2} = \left[\frac{3P_f}{2T_0 K_s} \right] \quad (30)$$

which, upon substitution in Eq.(28), yields

$$\gamma = \frac{K_s}{15KN_0} \left\{ k_c^2 - k_w^2 \right\} \quad (31)$$

For the reactor example presented earlier, the above equation reduces to

$$\gamma = 3.7 \times 10^2 \left\{ k_c^2 - k_w^2 \right\} \quad (32)$$

and upon inserting the appropriate parameters, we find that the critical wave number is $k_c = 0.084$, and the critical wavelength is $\lambda_c = 75$ cm. Since the radius of the core is 1 m, it is clear that such a system will support acoustic instabilities, and for wave numbers corresponding to this dimension, Eq.(32) shows that the e-folding time is 0.9 seconds. Although detailed non-linear analysis is required to assess the impact of these instabilities, one can estimate the loss of fuel from the core due to these oscillations by using Equations (23), (24), and (32). One finds for the case at hand that about 9% of the uranium plasma per second will be transported out of the core, and that corresponds to a fuel mass flow rate of about 20 kg/sec. Needless to say, losses of this magnitude make the problem of refueling GCR a formidable one indeed.

Conclusion

The preliminary analysis presented in this paper shows in a rather dramatic fashion that hydrodynamic confinement of the fissioning fuel in the gas core nuclear rocket is quite difficult to achieve. It is shown to be subject to the Kelvin-Helmholtz instability and the acoustic instability; both of which could lead to turbulent mixing and rapid loss of fuel. Although profiling effects can in principle alleviate the Kelvin-Helmholtz problem, they are very difficult to implement in practice, and quite often run contrary to properties placed on the system by desirable performance characteristics. The acoustic instability can also be addressed by appropriate geometric scaling, but that also may run contrary to desired performance objectives. It appears in this connection that the use of externally applied magnetic fields may be feasible so long as they are aimed at stabilizing these modes instead of providing total confinement of the plasma core itself.

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