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Surface pressure distributions are derived when gas is injected through a strip at the surface of a thin wedge in uniform flow at high Mach number. The blowing velocities are such that the flow separates ahead of the blowing region, but the layer of blown gas remains thin. Asymptotic descriptions of the separation region and the blowing region are reviewed and extended, for weak laminar viscous interaction and a cooled surface.

Introduction

Surface pressure distributions on a body in high-speed flow can be drastically altered if gas is injected at the body surface, through the distortion of the flow field as well as through the momentum flux at the surface. One case of blowing from a strip on a flat plate was studied by Smith & Stewartson.[1] In this case the supersonic laminar boundary layer separates somewhat ahead of the blowing region and moves away from the surface as a free shear layer at nearly constant pressure. Between the separation region and the location where blowing begins, a small part of the injected gas moves upstream at a low velocity to supply the mass needed for entrainment in the lower part of the shear layer (Stewartson[2]). In the neighborhood of blowing a favorable pressure gradient turns the blown gas toward the downstream direction. The layer of blown gas is still thin but gradients are small enough that viscous stresses are small, and the flow here is described by "inviscid boundary-layer equations" (Cole & Aroesty[3]). Downstream of the blowing region the pressure is assumed constant and equal to its undisturbed value, with no reversed flow near the wall.[2]

The flow properties in the laminar free interaction at separation are nearly independent of downstream conditions, and are described asymptotically at large Reynolds numbers by a variation of the "triple-deck" theory. The formulation relevant to "self-induced separation" at supersonic speeds was given by Stewartson & Williams[4] and by Neiland[5]. In effect the length scales are reduced near separation and locally the flow is approximately a rotational inviscid flow with a new thinner boundary layer close to the wall. The asymptotic form of the solution somewhat downstream of separation, as the appropriate scaled variable becomes large, was described by Stewartson and Williams.[6]

As the Mach number increases, the length of the local-interaction region grows, as does the thickness of the viscous sublayer, until the interaction is no longer local, when the hypersonic viscous interaction parameter is no longer small. In different terms, if the Mach number is large, the boundary layer has only a small effect on the external flow at points sufficiently far downstream: the interaction is "weak." But closer to the leading edge the streamline slopes are no longer small in comparison with the slopes of characteristics in the external flow, and the boundary-layer thickness can not be neglected in a first approximation: the interaction is "strong."

If separation occurs in the weak-interaction region, the asymptotic description is still local, as explained by Brown, Stewartson, & Williams[7]. Moreover, the scale of the interaction shrinks as the wall temperature decreases. This was noted first by Neiland[8] and has been discussed further by Brown, Cheng, & Lee[9]. At high Mach numbers, if the wall is cooled, the first approximation to the boundary-layer solution satisfies a condition of zero wall temperature. This of course must be corrected at small distances from the wall, as explained by Neiland[8] and later by Seddougui, Bowles, & Smith,[10] in a discussion of the effect of wall cooling on stability. In a particular limit it is found that the triple-deck solution must be augmented, since the displacement effect of the changes in the main boundary layer is no longer of higher order than that of the sublayer.[7,8,9] For still smaller wall temperatures, the scales continue to decrease; in this case a suggestion for the asymptotic form of the pressure somewhat downstream of separation was given by Gajjar & Smith.[11] When the streamwise length

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scale is no longer large in comparison with the boundary-layer thickness, the transverse pressure gradient must be restored in the description of the main boundary layer and analytical solutions are no longer possible.

In the blowing region, the pressure changes must be compatible in the subsonic blown gas and the supersonic outer flow. If the transverse pressure gradient remains small, the problem can be reduced to solution of an integral equation for the pressure. Results for the pressure distribution and for the location of separation were given in Ref. 1 in the case of uniform blowing.

In the present work the analysis of Smith & Stewartson[1] is extended to flow past a wedge at high Mach number. A description of separation from a cooled wall similar to that of Brown, Cheng, and Lee[9] is given in terms of the physical coordinate rather than the Dorodnitsyn-Howarth variable. A complete asymptotic flow description leads to a simpler expression for a constant in the pressure-displacement relation. The formulation for the blowing region is the same as in Ref. 1, and is applied to some specific examples, including a case with blowing through two strips. Some representative pressure distributions are shown.

Formulation

A thin wedge of length L and small half-angle $\alpha<<1$ is placed at zero incidence in a uniform hypersonic flow at a Mach number $M_\infty>>1$. A perfect gas is assumed, with constant specific heats. Coordinates x^* and y^* are measured along and normal to the upper wedge surface, respectively. Gas is injected in a direction normal to the surface from a slot occupying the region $x_0^*<x^*<x_1^*$, where $x_0^*>0$ and $x_1^*<L$ (Fig. 1). The blowing velocity is such that the flow separates from the surface at a location $x^*=x_s^*$ upstream of the slot. It is assumed that the added mass is not so large that the separation point has moved to the wedge vertex; i. e., $0<x_s^*<x_0^*$. Separation of the laminar boundary layer occurs through a hypersonic free interaction in a small neighborhood of $x^*=x_s^*$, where the pressure rises to a constant plateau value. The pressure begins to drop at $x^*=x_0^*$, and for $x_0^*<x^*<x_1^*$ the blown gas is turned downstream by the favorable pressure gradient.

The velocity, pressure, density, temperature, enthalpy, and viscosity coefficient are u, p, ρ, T, h, and μ, respectively, with corresponding undisturbed values u_∞, p_∞, ρ_∞, T_∞, h_∞, and μ_∞. The Reynolds number based on x_s^* and free-stream quantities is $Re=\rho_\infty u_\infty x_s^*/\mu_\infty$. The viscosity is assumed to vary as a power of the temperature $\mu/\mu_\infty=(T/T_\infty)^\omega$; at high temperatures the Sutherland law is recovered for $\omega=1/2$, whereas comparisons with certain existing results can be made if $\omega=1$.

The problem formulation is given in terms of four small dimensionless parameters: the wedge half-angle α, the reciprocal $1/M_\infty$ of the free-stream Mach number, the reciprocal Re^{-1} of the Reynolds number, and a nondimensional surface temperature $\theta=T_w/(M_\infty^2T_\infty)$. A limiting case is considered where $M_\infty\to\infty$, $\alpha\to0$, $Re\to\infty$, and $\theta\to0$, such that $M_\infty\alpha$ is fixed (hypersonic small-disturbance theory) and $M_\infty^{2+\omega}Re^{-1/2}\to0$ (weak viscous interaction); the order of magnitude of θ is chosen later.

The undisturbed wedge flow has shock-wave angle $\beta\sim\alpha\beta_0$, nondimensional pressure $p/p_\infty\sim M_\infty^2\alpha^2p_0$, and Mach number $M\sim\alpha^{-1}M_0$ found in the hypersonic small-disturbance limit from

$$\beta_0 = \frac{\gamma+1}{4} + \left\{\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{M_\infty^2\alpha^2}\right\}^{1/2} \tag{1}$$

$$p_0 = \gamma\beta_0 + \frac{1}{M_\infty^2\alpha^2} \tag{2}$$

$$\frac{1}{M_0^2} = \frac{\beta_0 - 1}{\beta_0} p_0, \tag{3}$$

where γ is the ratio of specific heats and $\beta_0/(\beta_0-1)$ is the density ratio across the shock wave. Near separation these quantities characterize the uniform undisturbed wedge flow external to the boundary layer; similarly, in the blowing region the undisturbed flow above the separation streamline is the uniform wedge flow. While outgoing waves will be reflected at the shock wave, the strength of the reflected waves is numerically small in comparison with that of the incident waves; although these reflections are not weak in an asymptotic sense for the limiting case considered here, they will nevertheless be neglected.

The separation region

The boundary-layer equations, plus the equation of state and the viscosity law, can be written as

$$\frac{\partial(\rho u)}{\partial x^*} + \frac{\partial(\rho v)}{\partial y^*} = 0 \tag{4}$$

$$\rho\left(u\frac{\partial u}{\partial x^*} + v\frac{\partial u}{\partial y^*}\right) + \frac{dp}{dx^*} = \frac{\partial}{\partial y^*}\left(\mu\frac{\partial u}{\partial y^*}\right) + \ldots \tag{5}$$

$$\rho\left(u\frac{\partial h}{\partial x^*} + v\frac{\partial h}{\partial y^*}\right) - u\frac{dp}{dx^*} = \frac{\partial}{\partial y^*}\left(\frac{\mu}{Pr}\frac{\partial h}{\partial y^*}\right) + \mu\left(\frac{\partial u}{\partial y^*}\right)^2 + \ldots \tag{6}$$

$$p = \rho RT, \qquad \mu/\mu_\infty = (T/T_\infty)^\omega \tag{7}$$

where Pr is the Prandtl number and R is the gas constant. At the wedge surface $y^*=0$ it is required that

$$u=v=0, \quad T=T_w=\text{const.} \tag{8}$$

For large M_∞, the boundary layer has thickness equal to the displacement thickness δ^*, since the mass flow in the high-temperature boundary layer is small[12]. The interaction parameter χ measures the ratio of a typical streamline slope in the boundary layer to the slope of a characteristic in the inviscid wedge flow. Here χ is defined for a length x_s^* by

$$\chi = \frac{M_0}{\alpha}\delta, \qquad \delta = \frac{M_\infty^{\ \omega}}{\alpha p_0^{1/2} Re^{1/2}} \tag{9}$$

The displacement thickness is $\delta^*=(\text{const.})x_s^*\delta$; if $\omega=1$, the constant factor is $0.332(\gamma-1)$. The product $M_\infty^2\alpha^2$ is considered fixed in the limit, but it is possible to recover the results for a flat plate when $M_\infty\alpha\to 0$, since in that case $M_\infty^2\alpha^2 p_0 \to 1$ and $M_0/\alpha \to M_\infty$. Thus for a flat plate χ as defined here reduces to the usual expression $M_\infty^{2+\omega}/Re^{1/2}$. If χ is small, the boundary-layer thickness is small in comparison with the wedge thickness except at points very close to the leading edge, and the interaction between the boundary layer and the external inviscid flow is weak, except near the vertex. In the following, it will be assumed that the interaction is weak; that is, $\chi\to 0$. The size of θ in terms of χ will be chosen later.

The undisturbed boundary-layer profiles are given by

$$u/u_\infty = U_0(Y) + \ldots, \qquad T/T_\infty = M_\infty^2 T_0(Y) + \ldots \tag{10}$$

where $Y=y^*/(x_s^*\delta)$ is the boundary-layer coordinate. Since the boundary-layer thickness may be set equal to the displacement thickness, the edge of the boundary layer is located at $Y=\delta^*/(x_s^*\delta)$. In the inviscid flow outside the boundary layer, $u/u_\infty \sim 1$ and $T/T_\infty = O(1)$. Thus $U_0\to 1$ and $T_0\to 0$ as $Y\to\delta^*/(x_s^*\delta)$, while, from Eq. (8), $U_0\to 0$ and $T_0\to 0$ as $Y\to 0$; if Pr=1, for example, then $T_0=(\gamma-1)U_0(1-U_0)/2$. As $Y\to 0$, it follows from Eqs. (5) and (6) that $T_0^{\ \omega}U_0' \sim \lambda$=constant and $T_0^{\ \omega}T_0' \sim \lambda_T$=constant, so that $T_0 \sim [(\omega+1)\lambda_T Y]^{1/(\omega+1)}$ and $U_0 \sim (\lambda/\lambda_T)T_0$ as

$Y \to 0$. On the other hand, at the wall $T=T_w << M_\infty^2 T_\infty$, and the profiles must be modified when $Y=O(\theta^{\omega+1})$ to have the form

$$u/u_\infty = \theta \bar{U}_0(\bar{Y}) + \ldots, \qquad T/T_\infty = \theta M_\infty^2 \bar{T}_0(\bar{Y}) + \ldots, \tag{11}$$

where $\bar{Y}=Y/\theta^{\omega+1}$ and

$$\bar{T}_0^{\,\omega} \bar{U}_0' = \lambda, \qquad \bar{T}_0^{\,\omega} \bar{T}_0' = \lambda_T, \qquad \bar{U}_0(0) = \bar{T}_0(0) - 1 = 0 \tag{12}$$

so that

$$\bar{T}_0 = [1+(\omega+1)\lambda_T \bar{Y}]^{1/(\omega+1)}, \qquad \bar{U}_0 = (\lambda/\lambda_T)(\bar{T}_0 - 1) \tag{13}$$

This formulation appears to have been given first by Neiland[8]; these modifications near the surface have also been noted by Seddougui, Bowles, & Smith[10] and by Brown, Cheng, & Lee[9].

In the neighborhood of separation the reference length in the flow direction is small, and the proper streamwise coordinate is

$$x = \frac{1}{\Delta}\left(\frac{x^*}{x_s^*} - 1\right), \tag{14}$$

where $\Delta=\Delta(\chi,\theta)<<1$ and is to be determined. In most of the boundary layer, for the case to be considered here, the perturbations in u and p are chosen to be of the same order of magnitude, say $O(\varepsilon)$, where $\varepsilon=\varepsilon(\chi,\theta)$. A consequence of this choice will be a relation between the orders of magnitude of θ and χ. The flow variables are then expanded for $x=O(1)$ and $Y=O(1)$ in the form

$$\frac{u}{u_\infty} = U_0(Y) + \varepsilon U_1(x,Y) + \ldots, \qquad \frac{v}{u_\infty} = \frac{\delta}{\Delta} \varepsilon V_1(x,Y) + \ldots, \tag{15}$$

$$\frac{p}{M_\infty^2 \alpha^2 p_0 p_\infty} = 1 + \varepsilon \gamma P_1(x,Y) + \ldots \tag{16}$$

$$\frac{T}{M_\infty^2 T_\infty} = T_0(Y) + \varepsilon T_1(x,Y) + \ldots, \qquad \frac{\rho}{\alpha^2 p_0 \rho_\infty} = \rho_0(Y) + \varepsilon \rho_1(x,Y) + \ldots \tag{17}$$

The perturbation quantities satisfy linear differential equations

$$\rho_0(U_{1x} + V_{1Y}) + U_0 \rho_{1x} + \rho_0' V_1 = 0 \tag{18}$$

$$U_0 U_{1x} + U_0' V_1 + P_{1x}/\rho_0 = 0, \qquad P_{1Y} = 0 \tag{19}$$

$$U_0 T_{1x} + T_0' V_1 - (\gamma-1) U_0 P_{1x}/\rho_0 = 0 \tag{20}$$

and the equation of state gives

$$\rho_0 T_0 = 1, \qquad \gamma P_1 = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0} \tag{21}$$

Thus the viscous forces are of higher order than the convection and pressure terms, and the equations for the first approximation describe small disturbances to an inviscid rotational flow.

The solutions to Eqs. (18) - (21) are

$$\frac{V_1}{U_0} = P_1' \int_0^Y \left(\frac{T_0}{U_0^2} - 1\right) dY - A_1' \tag{22}$$

$$U_1 = -P_1 U_0' \int_0^Y \left(\frac{T_0}{U_0^2} - 1\right) dY - P_1 \frac{T_0}{U_0} + U_0' A_1 \tag{23}$$

$$T_1 = -P_1 T_0' \int_0^Y \left(\frac{T_0}{U_0^2} - 1\right) dY + (\gamma-1) T_0 P_1 + T_0' A_1 \tag{24}$$

where $P_1=P_1(x)$ and $A_1=A_1(x)$; $U_0/T_0^{1/2}$ is equal to the local Mach number. The integral exists, since U_0 and T_0 are $O(Y^{1/(\omega+1)})$ as $Y \to 0$; thus as $Y \to 0$,

$$V_1 \sim -U_0(Y)A_1'(x), \qquad U_1 \sim U_0'(Y)A_1(x), \qquad T_1 \sim T_0'(Y)A_1(x) \tag{25}$$

When $Y=O(\theta^{\omega+1})$, the proper coordinate is $\bar{Y}$ rather than Y, and the profiles $U_0(Y)$ and $T_0(Y)$ must be replaced by $\theta\bar{U}_0(\bar{Y})$ and $\theta\bar{T}_0(\bar{Y})$. The convection terms are now the largest terms in Eqs. (19) and (20), so that the solutions for $\bar{Y}=O(1)$ have the same form as in Eq. (25) but in terms of the barred profiles. Then as $\bar{Y} \to 0$

$$\frac{v}{u_\infty} \sim -\frac{\delta}{\Delta}\varepsilon\theta^{-\omega}\lambda Y A_1', \qquad \frac{u}{u_\infty} \sim \theta^{-\omega}\lambda(Y+\varepsilon A_1), \qquad \frac{T-T_w}{M_\infty^2 T_\infty} \sim \theta^{-\omega}\lambda_T(Y+\varepsilon A_1) \tag{26}$$

As in the conventional triple-deck theory, viscous forces can no longer be neglected near the surface, and different asymptotic representations are required in a thin sublayer where Y and $\bar{Y}$ are small. An inner variable y is defined by

$$y = \frac{Y}{\zeta} = \frac{1}{\zeta}\frac{y^*}{x_s^*\delta} \tag{27}$$

where $\zeta=\zeta(\chi,\theta)<<1$, and it will be found that ζ also satisfies the stronger condition $\zeta<<\theta^{\omega+1}$; thus the sublayer is thin enough that the solutions there should match with the solutions given by Eq. (26). The limit process for the sublayer is chosen such that all terms in the boundary-layer momentum equation are of the same order, the largest terms in the solutions for u are matched, and the solutions for p are matched. Since the pressure perturbation is $O(\varepsilon)$ in the sublayer as well as in the main boundary layer, these conditions lead to expansions in the form

$$\frac{u}{u_\infty} = \varepsilon^{1/2}\theta^{1/2}u_1(x,y) + \ldots \tag{28}$$

$$\frac{v}{u_\infty} = \lambda\delta\varepsilon^{-1/2}\theta^{1/2}v_1(x,y) + \ldots \tag{29}$$

$$\frac{p}{M_\infty^2\alpha^2 p_0 p_\infty} = 1 + \varepsilon\gamma p_1(x,y) + \ldots \tag{30}$$

$$\frac{T}{M_\infty^2 T_\infty} = \theta + \ldots \tag{31}$$

where the scales Δ and ζ are found in terms of ε as

$$\Delta=\lambda^{-2}\varepsilon^{3/2}\theta^{\omega+1/2}, \qquad \zeta=\lambda^{-1}\varepsilon^{1/2}\theta^{\omega+1/2} \tag{32}$$

The perturbation quantities satisfy the incompressible boundary-layer equations

$$u_{1x} + v_{1y} = 0 \tag{33}$$

$$u_1u_{1x} + v_1u_{1y} + p_{1x} = u_{1yy}, \qquad p_{1y} = 0 \tag{34}$$

with boundary conditions $u_1=v_1=0$ at $y=0$ and initial condition $u_1 \sim y$ as $x \to -\infty$. As $y \to \infty$,

$$u_1 \sim y+A, \quad v_1 \sim -A'y \tag{35}$$

where the function $-A(x)$ implies an effective shift of the origin for y, and so represents a scaled change in displacement thickness of the sublayer. The form of the expansions defined by Eqs. (28) through (32) is independent of the relative sizes of the small parameters χ and θ.

The interaction of the boundary layer with the external flow determines a relation between p_1 and A_1, as well as a definition for ε and therefore expressions for Δ and ζ. In the flow outside the boundary layer, the pressure and the velocity components satisfy a linear wave equation, for $x^*-x_s^*=O(x_s^*\Delta)$ and

$y^*=O(x_s^*\alpha\Delta/M_0)$. If only outgoing waves are present locally, the result at the edge of the boundary layer is the usual linear-theory relation between the pressure perturbation and the streamline slope. Since these quantities are continuous at the edge of the boundary layer, the solutions in the main boundary layer evaluated at $Y=\delta^*/(x_s^*\delta)$ must satisfy this condition. It follows that

$$\varepsilon P_1 + ... = \chi \frac{\varepsilon}{\Delta} V_1 + ... \tag{36}$$

at $Y=\delta^*/(x_s^*\delta)$, and so $\Delta=O(\chi)$. Matching the second terms in u from Eqs. (26) and (35) gives $\varepsilon=O(\theta^{2\omega+1})$ and A_1=(const.)A. Combining with Eq. (32) then shows that $\theta=O(\chi^{1/(4\omega+2)})$ for the case considered. It is convenient to introduce constant factors in such a way that the pressure-displacement relation found from Eqs. (36) and (22) contains a single parameter Q. Since also $p_1 = P_1$, the results are

$$p_1 = -A' - \frac{1}{Q} p_1' \tag{37}$$

$$\varepsilon = \lambda^{1/2}\chi^{1/2}, \qquad A_1(x) = \theta^{\omega+1/2}\lambda^{-5/4}\chi^{-1/4}A(x) \tag{38}$$

where the parameter Q is defined by

$$Q\int_0^\infty T_0\left(1 - \frac{T_0}{U_0^2}\right)d\eta = \theta^{\omega+1/2}\lambda^{-5/4}\chi^{-1/4} \tag{39}$$

The integral in Eq. (39) has been rewritten in terms of a Dorodnitsyn-Howarth variable η defined by $dY=T_0 d\eta$; as $\eta\to 0$, both T_0 and U_0 are $O(\eta^{-1/\omega})$. The streamwise length scale $\Delta=O(\chi)$ can be made specific by the choice $\Delta=Q\chi$, for convenience in recovering the case of large Q, as noted below; it follows also that $\zeta=Q\varepsilon$. Eq. (37) serves as an additional boundary condition for Eqs. (33) and (34) as $y\to\infty$.

Thus the special case in which the displacement effects of the viscous sublayer and the main part of the boundary layer are of the same order of magnitude corresponds to a limit such that $\chi\to 0$ and $\theta\to 0$ with $\theta^{\omega+1/2}/\chi^{1/4}$ held fixed. In this limit, the scalings in Eqs. (14), (27), and (32) are

$$\Delta=Q\chi, \qquad \zeta=Q\varepsilon \tag{40}$$

and so the coordinates become

$$(x^*-x_s^*)/x_s^* = Q\chi x, \qquad Y = Q\varepsilon y \tag{41}$$

The flow regions are sketched in Fig. 2. The length scales for the local external flow are the same as for the boundary layer: for $x^*-x_s^*=O(x_s^*\Delta)$, disturbances in the outer flow extend only to a distance $y^*=O(\delta^*)$, and not to a still larger distance as in the usual triple-deck theory. Since $\zeta=O(\chi^{1/2})$ and $\chi=O(\theta^{4\omega+2})$, it is seen that $\zeta<<\theta^{\omega+1}$, as anticipated following Eq. (27). Thus the interaction occurs in a streamwise length $O(\chi x_s^*)$, and the sublayer thickness is $O(\chi^{1/2}\delta^*)$; if $\omega=1$, the nondimensional wall temperature is $\theta=O(\chi^{1/6})$, and the region where corrections to U_0 and T_0 are required has thickness $O(\chi^{1/3}\delta^*)$.

Parameters equivalent to Q have also been given in Refs. 7 through 10; in Refs. 7 and 10 the exponent ω was taken equal to one. The result (37) has been given by Neiland,[8] by Brown, Stewartson, & Williams,[7] and by Brown, Cheng and Lee.[9] In the second of these, the coefficient of p_1' has the same dependence on χ, θ and on the profile shapes, and is of order one because γ-1<<1 rather than θ<<1. Brown, Cheng, & Lee[9] used a Dorodnitsyn-Howarth variable, obtained a divergent integral, and have given a more complicated right-hand side.

For a constant-pressure boundary layer with Pr=1, the integral in Eq. (39) can be evaluated by setting $T_0=(\gamma-1)U_0(1-U_0)/2$ and taking numerical values from the Blasius solution:

$$\int_0^\infty T_0\left(1-\frac{T_0}{U_0^2}\right)d\eta=\frac{1}{4}(\gamma^2-1)\cdot 0.664-\left(\frac{\gamma-1}{2}\right)^2\cdot 1.721 \tag{42}$$

If γ=1.4, the value is 0.0905. Thus for this profile the main boundary layer behaves as a supersonic flow, since from Eq. (22), with interaction omitted, the displacement thickness decreases as the pressure increases. On the other hand, since -A is a sublayer displacement thickness, the first term -A' is positive in Eq. (37). Values of plateau pressure p_{1f} given in Ref. 9 are $p_{1f}=p_1(\infty)$=1.809, 1.681, 1.564 when the value of Q^{-1} in Eq. (37) is ∞, 1, 0.5 respectively.

For large Q, the forms of the sublayer expansions in Eqs. (28) through (31) are unchanged. In the main boundary layer, the pressure perturbation remains $O(\varepsilon)$, but the velocity, temperature, and density perturbations in Eqs. (15) and (17) are $O(Q\varepsilon)$, because it follows from Eqs. (38) and (39) that $A_1=O(Q)$, and the choice $\Delta=Q\chi$ gives $V_1(x,\delta^{-1}\delta^*/x_s^*)=QP_1(x)$. As a consequence, the pressure-displacement relation in Eq. (37) becomes $p_1=-A'$. If ω=1, the formulation for the first approximation is then identical to that of conventional triple-deck theory in the limit as $M_\infty\rightarrow\infty$.

On the other hand, Eq. (37) requires modification if $Q\rightarrow 0$. In this case the interaction has to be studied on two different length scales. The separation region can be expected to become smaller as Q decreases, since the temperature near the wall decreases. But a length of order $x_s^*\chi$ in the x* direction should also remain important, since it characterizes a special limit such that the length scale in the y* direction, namely $y^*=O(\delta^*)$, is the same for the outer inviscid flow as for the boundary layer.

Earlier studies[7,8] suggest a balance between the two terms on the right-hand side of the pressure-displacement relation given by Eq. (37) in the region close to separation when $Q\rightarrow 0$. Near separation the sublayer equation should continue to express a balance among inertia, pressure and viscous forces. In the main boundary layer the approximate differential equations should again contain all the terms shown in Eqs. (18) through (21), and the solutions should again have the form of Eq. (26) as $\bar{Y}\rightarrow 0$. If the solutions for v and the second terms in the solutions for u are matched, but the condition given by Eq. (36) is omitted, it is found that $\varepsilon=O(\theta^{2\omega+1})$. If Q is held fixed in the limit, Eq. (37) is recovered. But for $Q\rightarrow 0$ the streamtube area changes are not only of the same order in the sublayer and in the main boundary layer, but also are equal and opposite in their integrated effect, so that the streamline inclination at the edge of the boundary layer is of higher order than at points closer to the surface. That is, the increased displacement thickness of the viscous sublayer is offset by a decreased streamtube area in the inner part of the main boundary layer. In this case $\Delta=O(\theta^{4\omega+2})$, $\zeta=O(\theta^{2\omega+1})$, and $p/p_\infty=O(\theta^{2\omega+1})$; in the sublayer $u/u_\infty=O(\theta^{\omega+1})$ and $v/u=O(\delta/\theta^{2\omega+1})$. The streamline slope at the edge of the boundary layer is proportional to the product of the pressure perturbation with δ/χ, and so is small in comparison with v/u in the sublayer. The orders of magnitude imply a rescaling such that x is replaced by Q^3x, A by QA, and p_1 by Q^2p_1; the left-hand side of Eq. (37) is then smaller than the right-hand side by a factor $O(Q^4)$.

As x increases downstream of the separation point, the boundary layer begins to move away from the surface and reversed flow is present near the wall. If it is assumed when Q is small that the pressure grows such that $p/p_\infty=O(x^n)$ as $x\rightarrow\infty$, for some n>0, then the values of v/u at the separation streamline and at the edge of the boundary layer will be of the same order when $x=O(\chi/\theta^{4\omega+2})$, i. e., when $(x^*-x_s^*)/x_s^*=O(\chi)$. Then an equation of the same form as Eq. (37) relates the pressure perturbation to a thickness of the flow region below the separated shear layer. Gajjar & Smith[11] have proposed a value for the exponent n, by equating the mass in the backflow to the mass entrained in the shear layer. Thus to reach a constant plateau pressure it is not sufficient that $x=(x^*-x_s^*)/(x_s^*\Delta)\rightarrow\infty$, but instead the stronger condition $\theta^{4\omega+2}x/\chi\rightarrow\infty$ is required. This feature may be related to difficulties encountered[7,9] in obtaining a plateau pressure from numerical calculations for small Q.

The blowing region

Gas is injected from the surface in the range $x_0^*<x^*<x_1^*$, with separation occurring further upstream, at $x^*=x_s^*$, where $0<x_s^*<x_0^*$. Downstream of separation the separated boundary layer moves away from the surface as a thin free shear layer at nearly constant pressure. A low-speed backflow,[2] accelerated by a higher-order pressure gradient, provides the small amount of mass required for entrainment in the thin shear layer for $x_s^*<x^*<x_0^*$. Most of the added mass, however, is turned downstream by the favorable pressure gradient for $x_0^*<x^*<x_1^*$, as indicated in Fig. 3. Since the mass entrained is small in comparison with the total mass added at the wedge surface, the streamlines entering the shear layer originate at points very close to the beginning of blowing.

The slope of the separated shear layer is $dy^*/dx^*=\lambda^{1/2}\chi^{1/2}p_{1f}\alpha/M_0$. At $x^*=x_0^*$, the beginning of blowing, the distance from the surface to the shear layer is $y^*=\tilde{\delta}_0$, where

$$\tilde{\delta}_0=\frac{\alpha}{M_0}\lambda^{1/2}\chi^{1/2}p_{1f}(x_0^*-x_s^*) \tag{43}$$

Suitable coordinates for the blowing region are therefore

$$\tilde{x}=\frac{x^*-x_0^*}{x_0^*-x_s^*}\;, \qquad \tilde{y}=\frac{y^*}{\tilde{\delta}_0} \tag{44}$$

where $0<\tilde{x}<\tilde{x}_1$ and $\tilde{x}_1$ is the value of $\tilde{x}$ at $x^*=x_1^*$. The shear layer has thickness small in comparison with $\tilde{\delta}_0$ and is defined by $\tilde{y}=\tilde{\Delta}(\tilde{x})$, with $\tilde{\Delta}(0)=1$. The solutions are expanded in the form

$$\frac{u}{u_\infty}=\frac{\tilde{T}_w^{\,1/2}}{M_\infty}\lambda^{1/4}\chi^{1/4}\,p_{1f}^{\,1/2}\,\tilde{u}_1(\tilde{x},\tilde{y})+\ldots \tag{45}$$

$$\frac{v}{u_\infty}=\frac{\tilde{T}_w^{\,1/2}}{M_\infty}\frac{\tilde{\delta}_0}{x_0^*-x_s^*}\lambda^{1/4}\chi^{1/4}\,p_{1f}^{\,1/2}\,\tilde{v}_1(\tilde{x},\tilde{y})+\ldots \tag{46}$$

$$\frac{p}{M_\infty^2\alpha^2p_0p_\infty}=1+\lambda^{1/2}\chi^{1/2}\gamma p_{1f}\tilde{p}_1(\tilde{x},\tilde{y})+\ldots \tag{47}$$

$$\frac{T}{T_\infty}=\tilde{T}_w+\ldots \tag{48}$$

where the temperature is nearly constant because the local Mach number is small. If the temperature of the injected gas were the same as that of the surface upstream of the slot, then $\tilde{T}_w$ would be equal to $M_\infty^2\theta$. The differential equations for the flow perturbations are inviscid boundary-layer equations:

$$\frac{\partial\tilde{u}_1}{\partial\tilde{x}}+\frac{\partial\tilde{v}_1}{\partial\tilde{y}}=0 \tag{49}$$

$$\tilde{u}_1\frac{\partial\tilde{u}_1}{\partial\tilde{x}}+\tilde{v}_1\frac{\partial\tilde{u}_1}{\partial\tilde{y}}=-\frac{\partial\tilde{p}_1}{\partial\tilde{x}}\;, \qquad \frac{\partial\tilde{p}_1}{\partial\tilde{y}}=0 \tag{50}$$

The surface boundary condition in the blowing region is

$$\tilde{v}_1=\tilde{v}_{1w}(\tilde{x}) \qquad \text{at } \tilde{y}=0, \quad 0<\tilde{x}<\tilde{x}_1 \tag{51}$$

A stream function is defined by

$$\frac{\partial\psi}{\partial\tilde{y}}=\tilde{u}_1 \qquad \frac{\partial\psi}{\partial\tilde{x}}=-\tilde{v}_1 \tag{52}$$

Integration of the first of Eqs. (52) gives

$$\tilde{y} = \int_{\psi_w}^{\psi} \frac{d\psi}{\tilde{u}_1} = -\int_{\tilde{x}}^{\tilde{x}_w} \frac{\tilde{v}_{1w}}{\tilde{u}_1}\, d\tilde{x}_w \tag{53}$$

where a streamline is denoted by the location $\tilde{x}_w(\psi)$ at which it leaves the wall, and so $d\psi/d\tilde{x}_w=-\tilde{v}_{1w}$, with $\tilde{x}_w(0)=0$. Eq. (50) then implies the Bernoulli equation

$$\tilde{p}_1(\tilde{x}) + \tfrac{1}{2}\tilde{u}_1^{\,2}(\tilde{x},\tilde{x}_w) = \tilde{p}_1(\tilde{x}_w) \tag{54}$$

Outgoing waves that originate in the free-interaction region will be reflected from the shock wave, but the reflections will be numerically quite weak. For simplicity these reflected waves will be ignored; their effects could be added later if it were desired. (Similarly, the reflection of outgoing compression waves from the separation region is neglected.) Since the pressure above the shear layer is then linear in the slope,

$$\tilde{\Delta}(\tilde{x}) = 1 + \int_0^{\tilde{x}} \tilde{p}'_1 d\tilde{x} \tag{55}$$

and $\tilde{p}_1(0)=1$, since the pressure at the beginning of blowing is equal to the plateau pressure achieved downstream of separation. Combining Eqs. (53) and (54), also

$$\tilde{\Delta}(\tilde{x}) = \frac{1}{2^{1/2}} \int_0^{\tilde{x}} \frac{\tilde{v}_{1w}(\tilde{x}_w)\, d\tilde{x}_w}{\{\tilde{p}_1(\tilde{x}_w) - \tilde{p}_1(\tilde{x})\}^{1/2}} \tag{56}$$

For the special case $\tilde{v}_{1w}$=constant, the solution[1] can be found as a series in the inverse form

$$\frac{d\tilde{x}}{d\tilde{p}_1} = -\sum_{n=0}^{\infty} A_n (1-\tilde{p}_1)^{(n-1)/2} \tag{57}$$

where $A_0=2^{1/2}/(\pi\tilde{v}_{1w})$ and A_n satisfies a recursion relation for n>0, namely

$$A_n = \frac{(2/\pi)^{1/2}}{\tilde{v}_{1w}} \frac{\Gamma(n/2)}{\Gamma((n+1)/2)} (A_{n-1}-A_{n-3}) \tag{58}$$

with A_k taken to be zero for k<0. Evaluation of Eq. (55) gives

$$\tilde{\Delta}(\tilde{x}) = 1 + \sum_{n=0}^{\infty} \frac{4}{(n+1)(n+3)} A_n \left(1+\frac{n+1}{2}\tilde{p}_1\right)(1-\tilde{p}_1)^{(n+1)/2} \tag{59}$$

Following Smith & Stewartson,[1] it is assumed that there is no separation at the end of blowing, where $\tilde{x}=\tilde{x}_1$. Then further downstream, for $\tilde{x}>\tilde{x}_1$, the flow must be parallel to the surface, for there would otherwise appear to be a contradiction: either the low-speed flow adjacent to the surface decelerates and the pressure begins to rise again, or this flow accelerates and the separation streamline from $x^*=x_s^*$ has negative slope. It is therefore assumed that $\tilde{p}_1=0$ at $\tilde{x}=\tilde{x}_1$. Integration of Eq. (57) then gives

$$\tilde{x}_1 = \frac{x_1^*-x_0^*}{x_0^*-x_s^*} = 2\sum_{n=0}^{\infty} \frac{1}{n+1} A_n \tag{60}$$

Since the A_n depend on $\tilde{v}_{1w}$, Eq. (60) specifies the location of separation $x^*=x_s^*$ in terms of the scaled blowing velocity $\tilde{v}_{1w}$. If $\tilde{v}_{1w}$ is small, separation occurs at a point very close to the beginning of blowing:[1]

$$\frac{x_0^*-x_s^*}{x_1^*-x_0^*} = \frac{3\tilde{v}_{1w}}{4\sqrt{2}} \exp\left\{\frac{-2}{3\pi\tilde{v}_{1w}^{\,2}}\right\} + \ldots \tag{61}$$

If instead $\tilde{v}_{1w}$ is large, a first approximation for the location of separation is

$$\frac{x_s^*}{x_0^*} = 1 - \frac{x_1^*-x_0^*}{x_0^*}\frac{\pi\tilde{v}_{1w}}{2^{3/2}}\left(1+O(\tilde{v}_{1w}^{-1})\right) \tag{62}$$

where it is understood that the length $x_1^*-x_0^*$ of the blowing region is $O(\tilde{v}_{1w}^{-1}x_0^*)$ and is small enough that $x_s^*>0$. Thus for large $\tilde{v}_{1w}$ the distance from separation to the slot is proportional to the product of blowing velocity and slot width; i. e., $x_0^*-x_s^*$ is proportional to the rate at which mass is added. The corresponding pressure distribution in the blowing region $x_0^*<x^*<x_1^*$ is found from

$$\tilde{p}_1 = 1 - \left(\frac{\tilde{x}}{\tilde{x}_1}\right)^2\left\{1 - \frac{1}{2^{3/2}\tilde{v}_{1w}}\left[4\frac{\tilde{x}}{\tilde{x}_1} - \left(\frac{\tilde{x}}{\tilde{x}_1}\right)^3 - 3\right]\right\} + O(\tilde{v}_{1w}^{-2}) \tag{63}$$

Thus the pressure is constant from separation until the beginning of blowing and to a first approximation for large $\tilde{v}_{1w}$ is quadratic in $x^*-x_0^*$ in the blowing region; the length $O(x_s^*\chi)$ of the separation region is small in comparison with the other lengths.

The interaction parameter χ has been defined by Eq. (9) in terms of the distance x_s^* from the wedge vertex to the separation point, where x_s^* depends on the location and strength of the blowing and is obtained as part of the solution. Thus the definitions of the quantities $\tilde{\delta}_0$, $\tilde{u}_1$, $\tilde{v}_1$, and $\tilde{p}_1$ in Eqs. (43), (45), (46), and (47) also depend on the value of x_s^*, which is unknown in advance. For example, the initial value of $\tilde{p}_1$ is $\tilde{p}_1(0)=1$, whereas the actual pressure perturbation at the beginning of the strip is proportional to $\chi^{1/2}\tilde{p}_1(0)$, which increases as x_s^* decreases. For some purposes it is preferable instead to think in terms of quantities that depend on a specified length, say the distance x_1^* from the wedge vertex to the end of the strip within which mass is added. This can be accomplished by replacing χ with an interaction parameter χ_1 based on x_1^* instead of x_s^*. Then each of the quantities $\tilde{\delta}_0$, $\tilde{u}_1$, $\tilde{v}_1$, and $\tilde{p}_1$ is multiplied by a power of $\chi/\chi_1=(x_1^*/x_s^*)^{1/2}$, as indicated in some of the figures described below.

Pressure distributions found from Eq. (57) for various values of the scaled blowing velocity $\tilde{v}_{1w}$ are shown in Fig. 4, for $x_0^*/x_1^*=0.5$, by plots of $(\chi/\chi_1)^{1/2}\tilde{p}_1$ vs. x^*/x_1^*; these curves are equivalent to those given in Ref. 1. On the scale of the wedge length the pressure increase near separation appears as a jump, and the location of separation is seen to move upstream as $\tilde{v}_{1w}$ increases. The magnitude of the pressure jump increases as separation moves forward, since the jump is proportional to $\chi^{1/2}$ and χ is proportional to $(x_s^*)^{-1/2}$. When $\tilde{v}_{1w}=0.2$, separation occurs very close to the beginning of blowing, as predicted by Eq. (61). For $\tilde{v}_{1w}=1.0$, the pressure is close to the form for large $\tilde{v}_{1w}$ given by Eq. (63). The scaled location of the separation streamline is plotted against x^*/x_1^* in Fig. 5, again for several values of $\tilde{v}_{1w}$. The slope is constant between separation and the beginning of blowing, and decreases to zero at the end of the blowing region. The value of the initial slope is proportional to the pressure increase at separation, and therefore increases as $\tilde{v}_{1w}$ increases.

The mass flow rate $\dot{m}$ per unit span is

$$\dot{m} = \int_{x_0^*}^{x_1^*}[\rho v]_{\tilde{y}=0}dx^* = (M_\infty^2\alpha^2 p_0\rho_\infty)\frac{\alpha}{M_0}\frac{u_\infty}{M_\infty\tilde{T}_w^{1/2}}\lambda^{3/4}\chi_1^{3/4}\gamma p_{1f}^{1/2}x_1^*\dot{m}_1 \tag{64}$$

where the nondimensional mass flow $\dot{m}_1$ is defined by

$$\dot{m}_1 = \frac{\chi^{3/4}}{\chi_1^{3/4}} \frac{x_1^*-x_0^*}{x_1^*} \int_0^1 \tilde{v}_{1w} d(\tilde{x}/\tilde{x}_1) \tag{65}$$

The integrated force change and the location of separation are shown in Fig. 6 in terms of $\dot{m}_1$. The distance between the separation point and the beginning of blowing is very small when $\dot{m}_1$=0.2, as also is evident in Figs. 4 and 5. This distance increases as $\dot{m}_1$ increases, and becomes nearly linear in $\dot{m}_1$ when $\dot{m}_1$ is greater than about 0.6, in agreement with Eq. (62). The added pressure force acting on the wedge surface has the form

$$\Delta F = \int_{x_0^*}^{x_1^*} \Delta p \, dx^* = M_\infty^2 \alpha^2 p_0 p_\infty \lambda^{1/2} \chi_1^{1/2} \gamma p_{1f} x_1^* F_1 \tag{66}$$

in terms of the interaction parameter χ_1 based on x_1^*; here Δp is the difference between the surface pressure and the undisturbed wedge pressure. The scaled force change F_1 is

$$F_1 = \left(\frac{1}{\tilde{x}_1} + \int_0^1 \tilde{p}_1 d(\tilde{x}/\tilde{x}_1) \right) \frac{\chi^{1/2}}{\chi_1^{1/2}} \frac{x_1^*-x_0^*}{x_1^*} \tag{67}$$

where the first term arises from the constant pressure in $x_s^*<x^*<x_0^*$. From Eqs. (60) and (61), it is seen that F_1 is zero when $\tilde{v}_{1w}=0$; F_1 then increases as $\tilde{v}_{1w}$ increases, as in Fig. 6. At the larger values of $\tilde{v}_{1w}$ shown in the figure, the force change is approximately equal to the pressure increase in the plateau region multipled by the length of the separation region.

Fig. 7 shows the required mass flow vs. slot width for several values of integrated force. In the figure, the leading edge $x^*=x_0^*$ of the slot moves rearward from the wedge vertex as the coordinate x_0^*/x_1^* increases from zero, and the slot width approaches zero as x_0^*/x_1^* approaches one. The force change F_1 is seen to depend primarily on the mass flow $\dot{m}_1$, since the lines of constant force are nearly horizontal: if the slot width changes, but the mass flow is held constant, the variation in F_1 is very small. Lines of constant scaled blowing velocity $(\chi/\chi_1)^{3/4}\tilde{v}_{1w}$ are also shown; these are straight lines, since the mass flow is proportional to the slot width. The boundary at the left of the figure is a curve for which the blowing velocity or slot width is large enough that the separation point $x^*=x_s^*$ is very close to the wedge vertex, at $x_s^*=0.01x_1^*$.

Next it is supposed that gas is injected from two strips, defined by $x_0^*<x^*<x_1^*$ and $x_2^*<x^*<x_3^*$ (Fig. 8). Suitable coordinates for $x_0^*<x^*<x_1^*$ are $\tilde{x}^{(1)}$ and $\tilde{y}^{(1)}$, defined in the same way as $\tilde{x}$ and $\tilde{y}$ in Eq. (44), but now with a superscript added to emphasize use for the first strip. Similarly, the scaled pressure perturbation $\tilde{p}$ in Eq. (47) is now called $\tilde{p}^{(1)}$. The implicit solution for $\tilde{p}^{(1)}$ is found by integration of Eq. (57):

$$\tilde{x}^{(1)} = \sum_{n=0}^{\infty} \frac{2}{(n+1)} A_n (1 - \tilde{p}_1^{(1)})^{(n+1)/2} \tag{68}$$

where $A_0=2^{1/2}/(\pi\tilde{v}_{1w}^{(1)})$, with $\tilde{v}_{1w}^{(1)}$ identical to $\tilde{v}_{1w}$ in Eq. (46), and where A_n again satisfies the recursion relation given by Eq. (58) for n>0. Now, however, the pressure at the end of the strip remains higher than the external-flow value. The range for Eq. (68) is $0<\tilde{x}^{(1)}<\tilde{x}_f^{(1)}$ and $1>\tilde{p}^{(1)}>\tilde{p}_f^{(1)}$, where $\tilde{x}_f^{(1)}=(x_1^*-x_0^*)/(x_0^*-x_s^*)$ and $0<\tilde{p}_f^{(1)}<1$. A relation between $\tilde{x}_f^{(1)}$ and $\tilde{p}_f^{(1)}$ is found by evaluating Eq. (67) at the downstream end of the strip. If $\tilde{v}_{1w}^{(1)}$ is specified, this relation gives the location x_s^* of separation in terms of the still unknown $\tilde{p}_f^{(1)}$.

For the second strip, suitable coordinates are

$$\tilde{x}^{(2)} = \frac{x^*-x_2^*}{x_2^*-x_1^*} \quad , \qquad \tilde{y}^{(2)} = \frac{y^*}{\tilde{p}_f^{(1)}\tilde{\delta}_0} \tag{69}$$

The reference length for $\tilde{x}$ is the distance between the trailing edge of the first strip and the leading edge of the second strip; the reference length for $\tilde{y}$ is the distance from the wedge surface to the separation streamline at the beginning of the second strip. In Eqs. (46) and (47), $\tilde{v}_1$ and $\tilde{p}_1$ are replaced by $(\tilde{p}_{1f}^{(1)})^{3/2}\tilde{v}_1^{(2)}$ and $\tilde{p}_{1f}\tilde{p}_1^{(2)}$ respectively, where the superscript refers to the second strip. After the same change of subscripts is made, Eq. (68) becomes an implicit relation for $\tilde{p}_1^{(2)}$ in terms of $\tilde{x}_1^{(2)}$, when $\tilde{v}_{1w}^{(2)}$ is known. If no separation occurs, then as for the single-strip solution it is assumed that the pressure has returned to its external-flow value when the end of the second strip has been reached, and so $\tilde{p}_1^{(2)}=0$ when $\tilde{x}_1^{(2)}=\tilde{x}_{1f}^{(2)}$, where the known nondimensional length of the second strip is $\tilde{x}_{1f}^{(2)}=(x_3^*-x_2^*)/(x_2^*-x_1^*)$. This provides an implicit solution for $\tilde{p}_{1f}^{(1)}$ in terms of the geometry and the blowing velocities. The location of separation in then found from Eq. (68) as noted.

Some pressure distributions for two strips are shown in Fig. 9. The lengths of the strips are taken to be equal, with the distance between the strips equal to twice the width of one strip. In terms of the distance x_3^* from the wedge vertex to the end of the second strip, the first strip lies between $x^*/x_3^*=0.5$ and $x^*/x_3^*=0.625$, while the second strip extends from $x^*/x_3^*=0.875$ to $x^*/x_3^*=1.0$. The scaled blowing velocities $\tilde{v}_{1w}^{(1)}$ and $(\tilde{p}_{1f}^{(1)})^{3/2}\tilde{v}_{1w}^{(2)}$ are taken to be the same, so that the mass flow is the same for the two strips. As the mass flow is increased, the separation point is seen to move upstream, as expected, and the pressure rise at separation increases, since the Reynolds number Re decreases and the interaction parameter χ increases. Thus the solution is implicit in the sense that the scaled blowing velocity or mass flow is specified, and the corresponding dimensional values calculated after the location of separation has been determined.

Concluding Remarks

The solutions of Ref. 1 for strip blowing from a flat plate at supersonic speed have been extended relatively easily to wedge flow at high Mach number, if the hypersonic viscous interaction parameter is small. The flow near separation is described as a hypersonic free interaction for a cooled surface by extension and modification of results from Refs. 7, 8, and 9; this gives the value for the pressure at the beginning of blowing. The dependence of the integrated force on the parameters is then determined for a particular parameter range.

If the surface were not cooled, the free-interaction solution for supersonic flow would suffice, provided that the viscous interaction for the undisturbed wedge flow is weak. For the cooled wall considered here, it is assumed that the wall temperature is small in comparison with the maximum temperature in the boundary layer. From Eq. (39) it then follows that the parameter $Q\chi^{1/4}$ must be sufficiently small. The solution near separation, however, is not yet complete for small values of Q. It remains to be shown that the solution of Ref. 2 for the backflow ahead of the blowing region can be consistent with the solution of Ref. 11 for the backflow just downstream of separation from a highly cooled surface.

It has also been assumed that the flow does not separate at the end of the blowing region (or behind the second strip in the case of two strips). A proposal for describing the flow details near the end of blowing was given in Ref. 2, but the derivation seems based on an implicit assumption that separation does not occur. The present authors believe that a correct representation may include a shallow separation bubble, but thus far have not been able to complete a self-consistent flow description. Numerical solutions might be desirable for providing a guide to the proper analytical description; the role of downstream boundary conditions would have to be examined.

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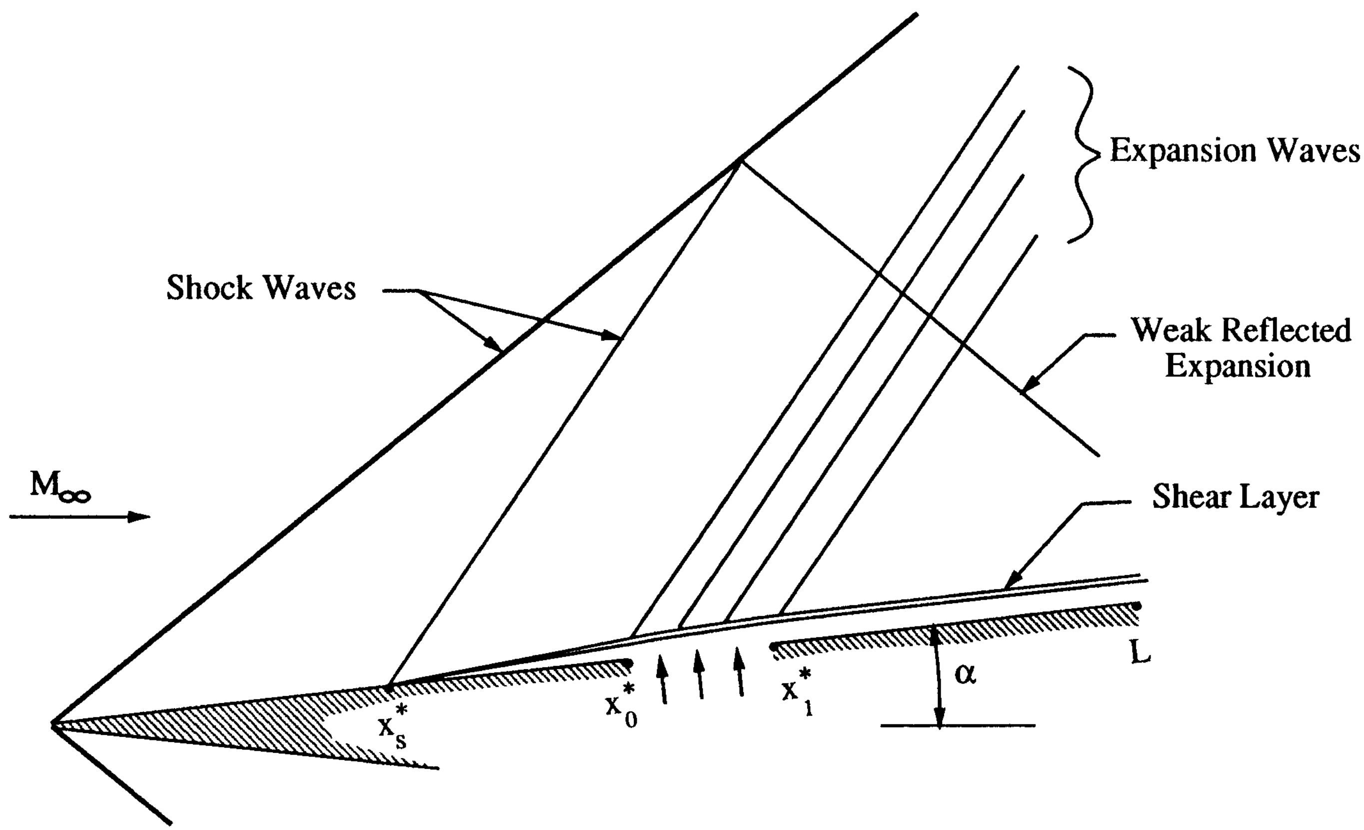

Fig. 1. Wedge flow with strip blowing

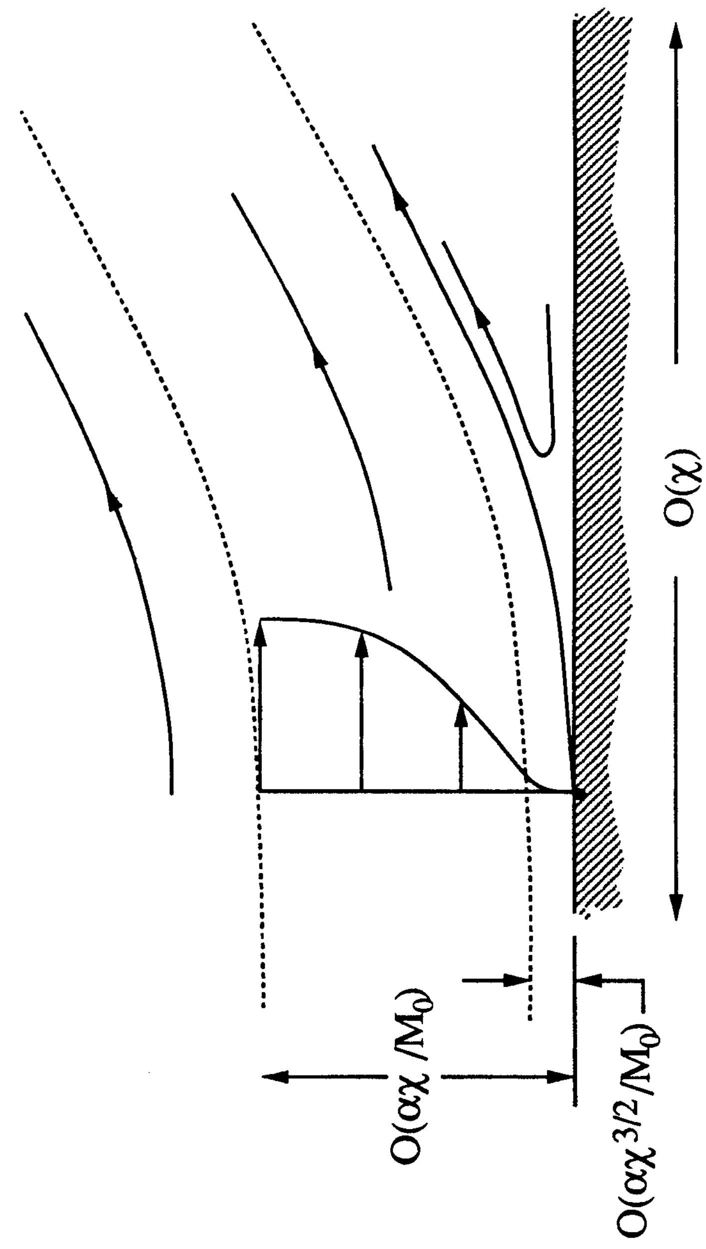

Fig. 2. Flow structure near separation

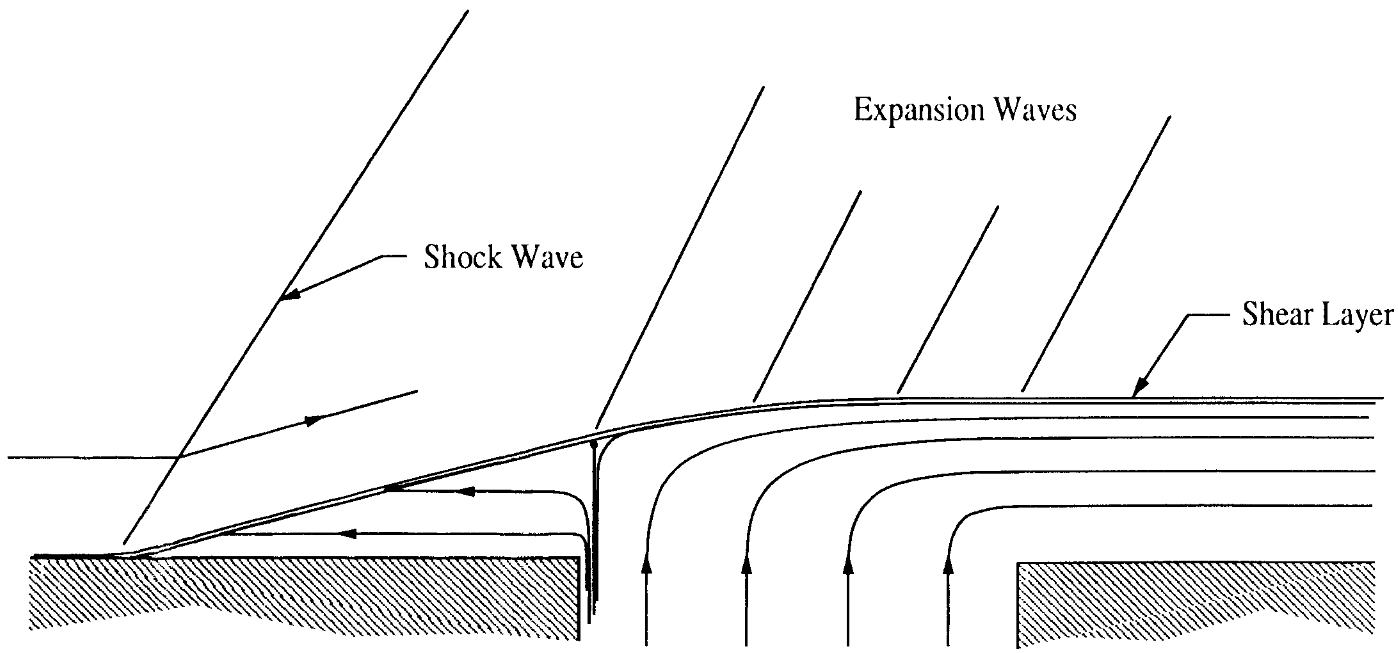

Fig. 3. Some flow details for strip blowing

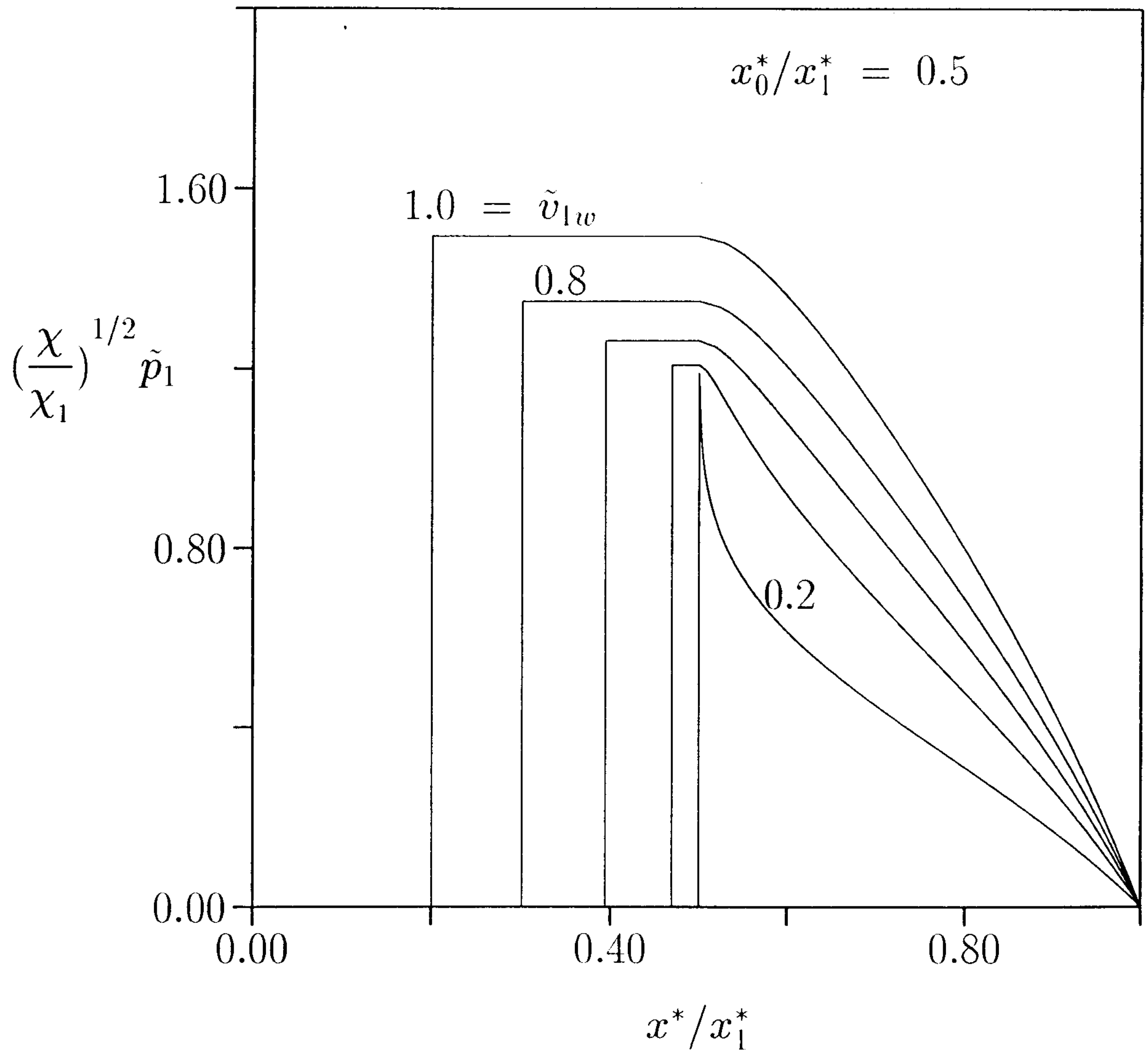

Fig. 4. Scaled pressure distribution for various scaled blowing velocities

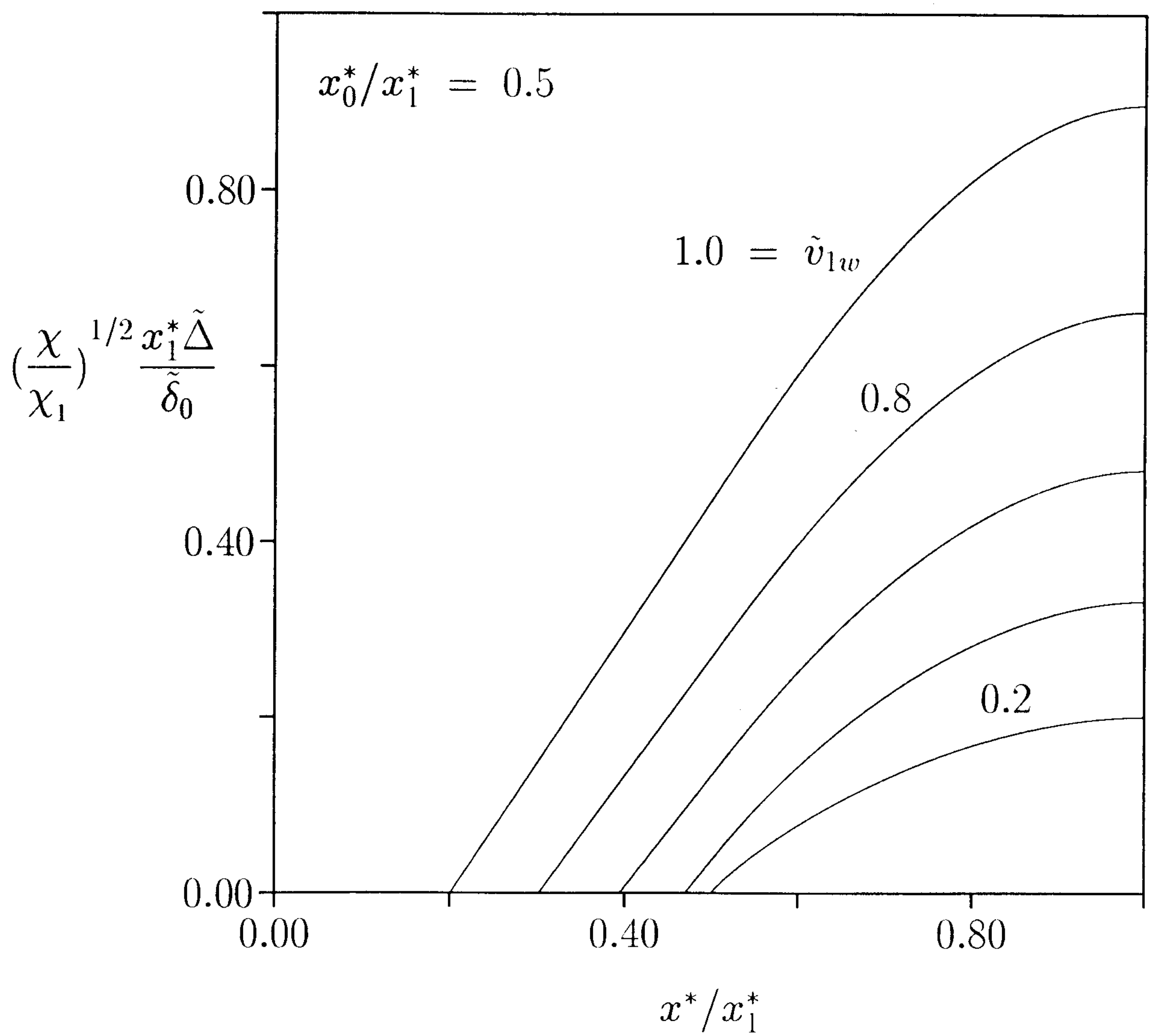

Fig. 5. Scaled location of separation streamline for various scaled blowing velocities

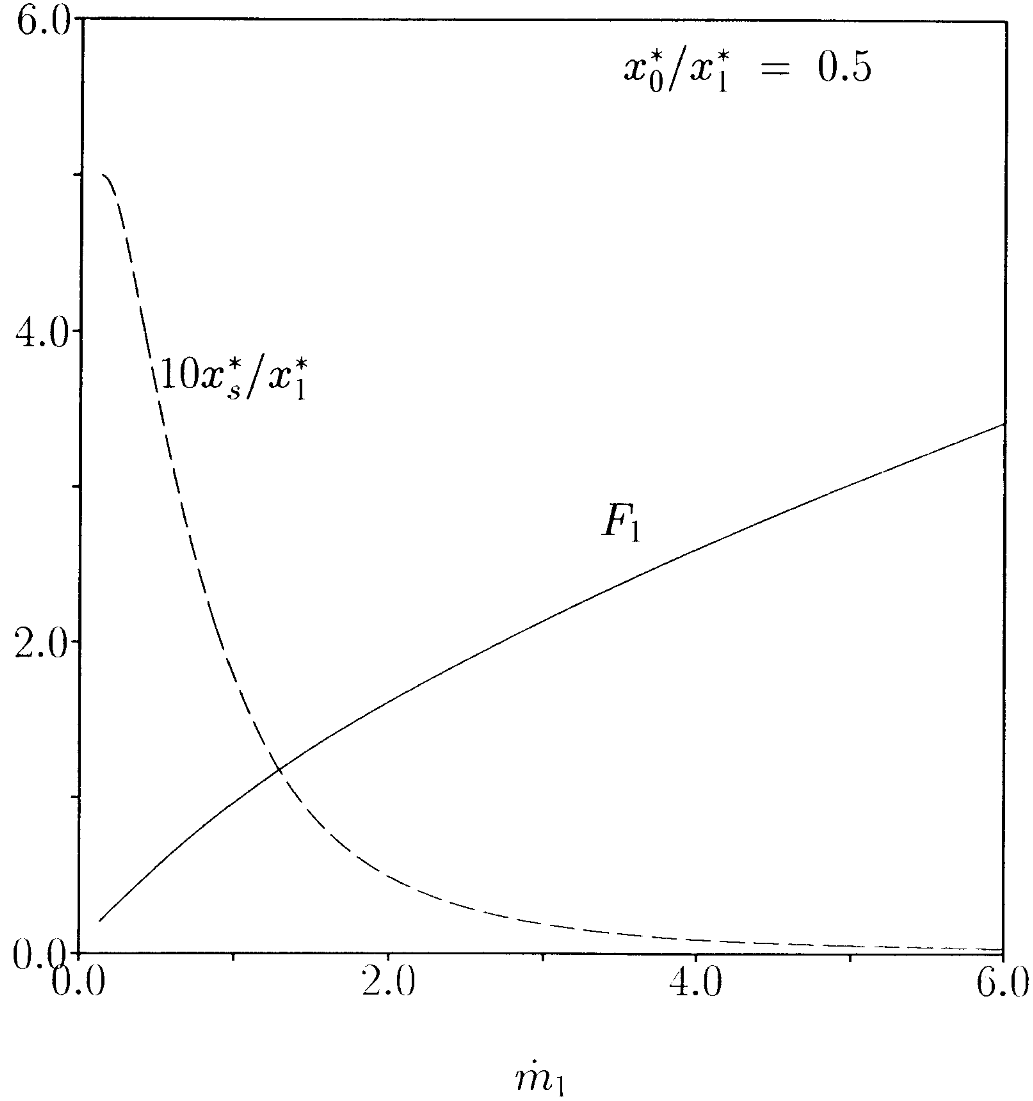

Fig. 6. Integrated force change and nondimensional location of separation, as functions of scaled mass flow rate

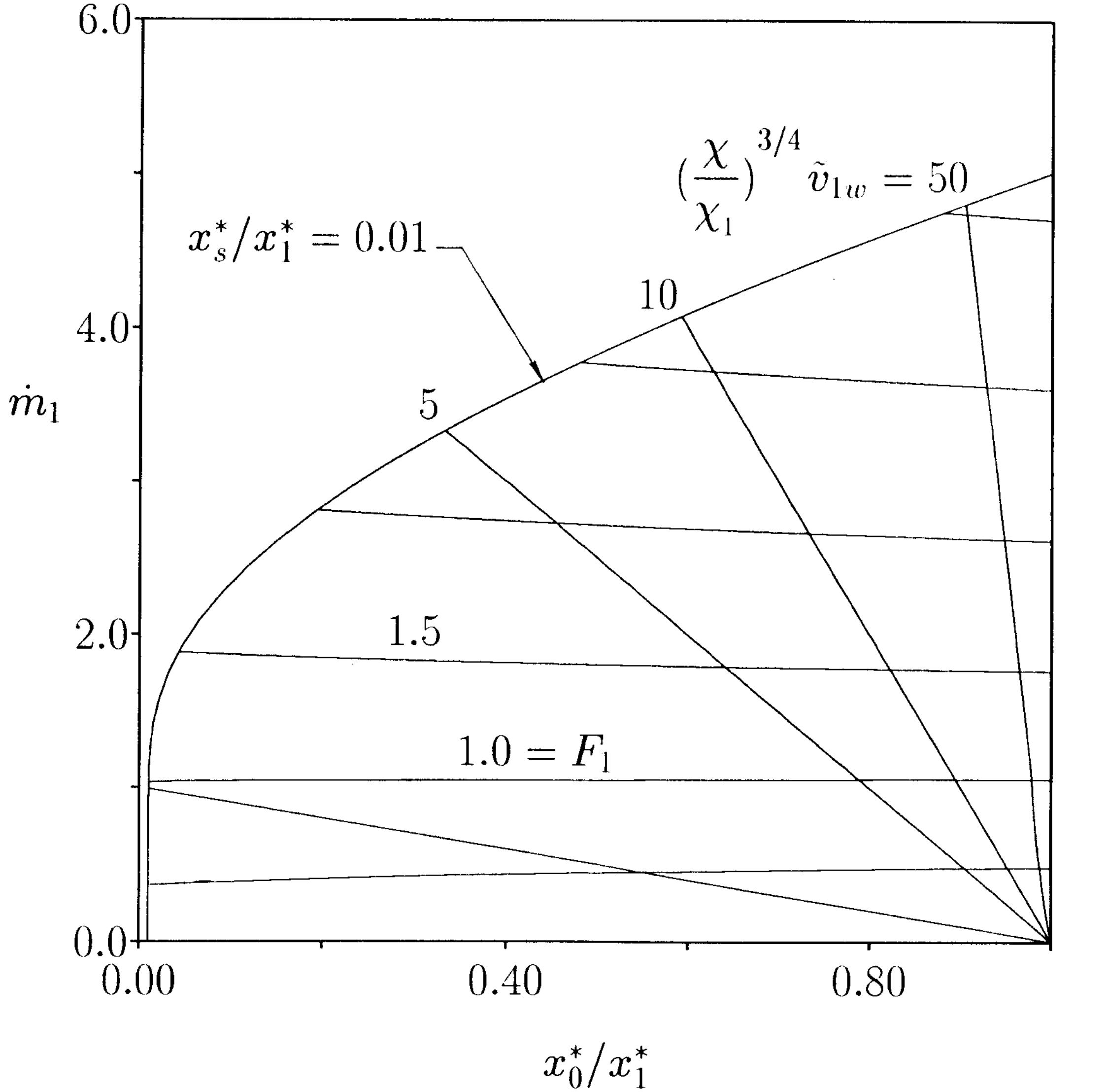

Fig. 7. Scaled rate of mass addition vs. nondimensional location of slot leading edge, for constant values of scaled force change or scaled blowing velocity

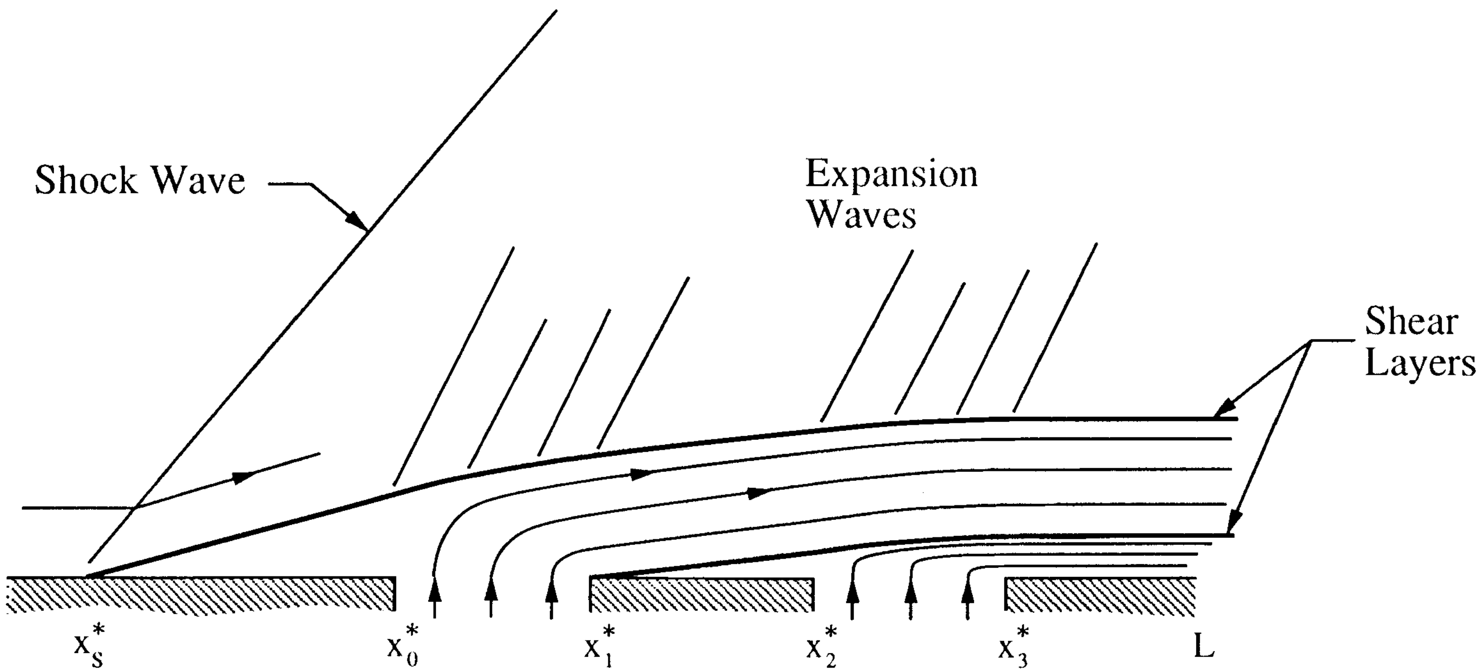

Fig. 8. Some flow details for blowing from two strips

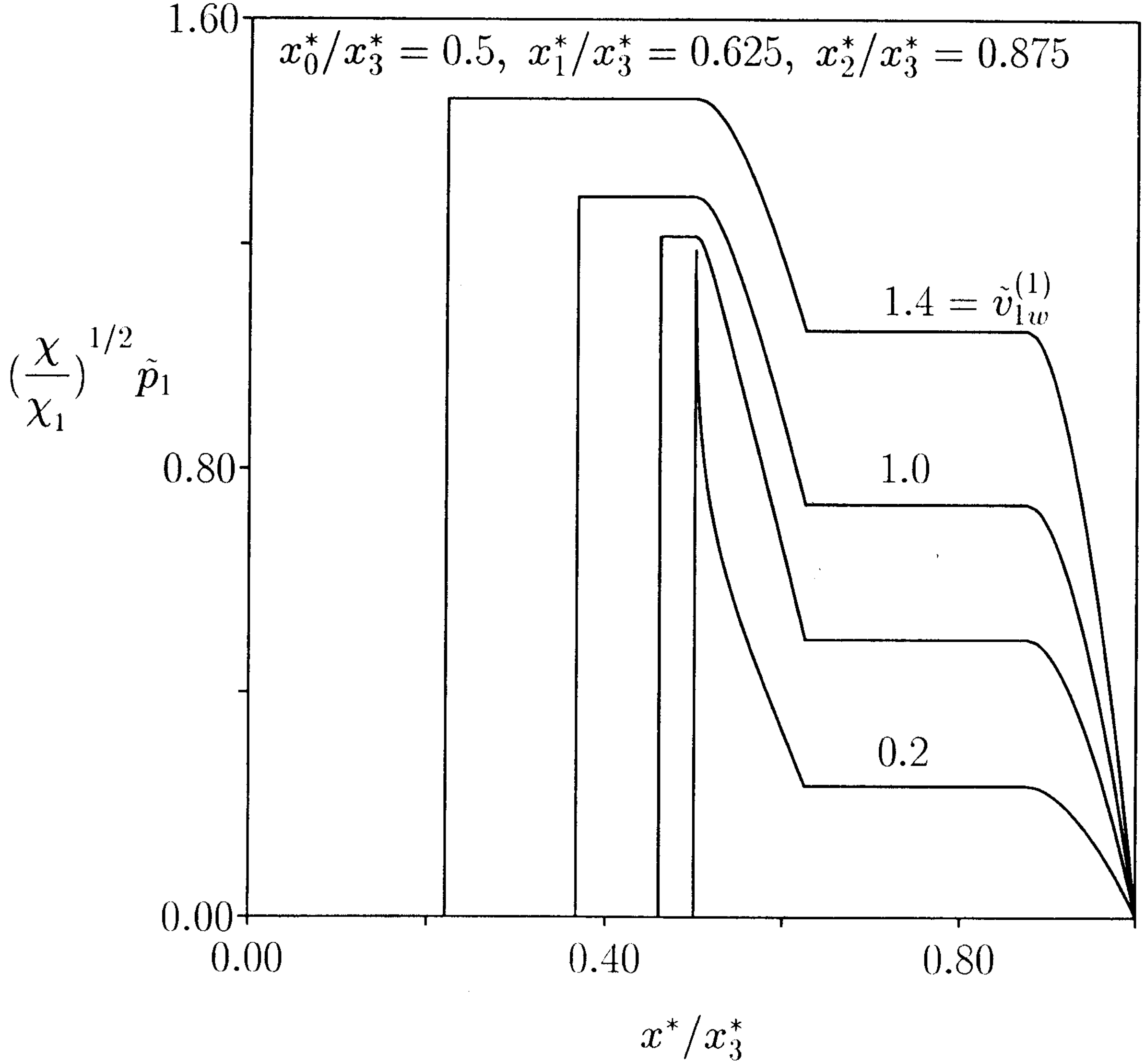

Fig. 9. Pressure distributions for blowing from two strips, for various rates of mass addition