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A Wave-Model-Based Refinement Criterion for Adaptive-Grid Computation of Compressible Flows

H. Paillére, K. G. Powell and D. De Zeeuw
Department of Aerospace Engineering
The University of Michigan
Ann Arbor, MI USA

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Henri Paillère *
Kenneth G. Powell †
Darren De Zeeuw ‡

The University of Michigan
Department of Aerospace Engineering
Ann Arbor, MI 48109-2140

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Abstract

A criterion for adaptive mesh refinement for compressible-flow calculations is presented. The criterion is based on a two-dimensional wave-model for the Euler equations, and can separately detect regions in which acoustic waves are important, and regions in which shear waves are important. This ability to detect regions of compressibility and rotationality separately gives better resolution of features of disparate strength. Results for representative test cases are presented, and compared to results from a grid-aligned wave-model-based criterion, and more classical criteria, such as undivided differences of pressure or density.

Introduction

Computation of the inviscid compressible flow around a body requires discretization of the computational domain into a mesh of points. Whether structured or unstructured, body-fitted or Cartesian, the mesh must satisfy two conditions:

1. it must be appropriately refined around the body;

2. it must be appropriately refined in regions in which the solution is changing rapidly in space.

A straightforward, but computationally expensive, way to achieve this resolution is to use global mesh refinement, i.e. to reduce the mesh-spacing uniformly over the domain until the resolution near the body and in high-gradient flow regions is satisfactory. A more sophisticated and efficient way is to use an adaptive-refinement method, in which the mesh is refined only in regions where added resolution is deemed necessary. The primary added difficulty in adopting an adaptive-refinement approach is the necessity of a more sophisticated data structure: connectivity information must be stored, and updated as the mesh is adapted.

Appropriate resolution of the body may be assured by clustering boundary points in high-curvature regions of the body. This may be done from the start, by discretizing the body in a satisfactory manner and introducing those points into the initial mesh. Alternatively, it may be done in a separate step, by beginning with a very coarse mesh of the flow domain, and refining cells in the vicinity of high-curvature regions of the body until a satisfactory mesh is obtained. A quadtree-based generation of a body-cut Cartesian mesh, for example, is carried out in three steps. First, a quadrilateral that encloses the entire flow domain is generated, and enough levels of children cells are spawned that a user-specified cell size is reached throughout the flow domain. The intersections of this coarse mesh with the body are computed. Next, all of the cells which are cut by the body spawn children cells, until a user-specified maximum

*Graduate Student, Research Assistant
†Assistant Professor, Senior Member AIAA
‡Doctoral Candidate, Member AIAA

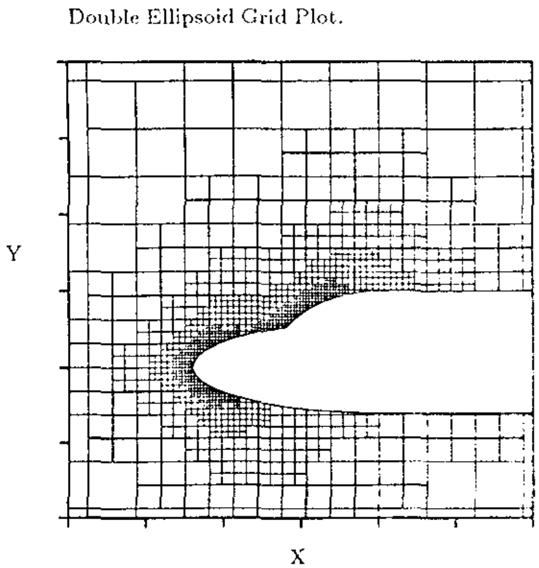

Figure 1: Example Initial Mesh

cell size on the body is reached. Finally, slopes of neighboring faces on the body are compared, with children cells spawned until the differences in slopes between each pair of neighboring faces on the body fall below a user-specified value. The resulting mesh, generated with only three simple criteria set by the user, is shown in Figure 1.

Appropriate resolution of the flow-field requires an adaptation criterion, i.e. some method of detecting regions in which the flow is under- (or over-) resolved. Ideally, this adaptation criterion should have the following properties:

1. it should be inexpensive to compute;
2. it should be able to detect a variety of flow features (e.g. shocks, wakes, rapid expansions);
3. it should be sensitive enough to detect weak features, and should *not* exclusively refine discontinuous regions of the solution at the expense of smooth high-gradient regions.

The first requirement, that the adaptation criterion be inexpensive to compute, is particularly vital in unsteady-flow calculations, in which the "refinement front" must follow (or, actually, precede) the propagation and evolution of flow features at each iteration of the solver. In steady-flow calculations, refinement is typically carried out infrequently, and the cost of the calculation of the adaptation criterion, relative to the cost of one iteration of the solver, becomes less vital. In this work, a very inexpensive adaptation criterion, and a fairly inexpensive thresholding technique, are described.

The second requirement, that the adaptation criterion be able to detect a variety of flow features, requires an understanding of what flow features can arise in the solution of a given set of governing equations, and of the way in which the various flow features manifest themselves in the solution. In inviscid compressible flows, there are two types of discontinuities:

- shocks, across which

$$\begin{aligned}[\rho] &\neq 0\\ [u_n] &\neq 0\\ [p] &\neq 0\\ [u_t] &= 0\,,\end{aligned}$$

- and shear layers/contact contact discontinuities across which

$$\begin{aligned}[\rho] &\stackrel{?}{=} 0\\ [u_n] &= 0\\ [u_t] &\stackrel{?}{=} 0\\ [p] &= 0\,,\end{aligned}$$

  where the symbol $\stackrel{?}{=}$ denotes that a quantity might or might not change across the discontinuity.

In addition, there are "smooth" flow features, such as expansion fans and vortices, which should be captured. There are two approaches that can be taken in order to detect such a variety of smooth and non-smooth features; either a single "catch-all" criterion can be designed, or a combination of simpler criteria, each of which is designed for detecting a certain feature, can be used in conjunction with each other. In this work, the second approach is taken, and some advantages of this approach are described.

The third requirement, that the criterion should detect weak features as well as strong ones, and smooth features as well as non-smooth ones, was pointed out in a sobering fashion by Warren et al [WATK91]. They showed an example in which adaptation criteria geared towards detecting non-smooth flow features could lead to a converged solution with an incorrect shock location. This was due to the criterion flagging the (incorrectly located) shock for refinement, at the expense of the resolution of the trailing-edge region. They noted that adaptation criteria based on derivatives of flow quantities become unbounded at a discontinuity as resolution increases, and adaptation criteria based on undivided differences remain bounded but finite at a discontinuity as resolution increases. To avoid the problem of discontinuities acting as "refinement magnets" at the expense of other important regions of the flow, the adaptation criterion must be one that tends to vanish at a discontinuity as it becomes resolved. Two methods of ensuring that smooth regions are not ignored are used in this work, and described in this paper.

Many modern algorithms for compressible flow make use of an approximate solution to the Riemann problem [Roe81] to define interface fluxes for the solution update. This paper outlines an adaptation criterion based on the the wave strengths from Roe's approximate Riemann solver. The resulting criterion is inexpensive to compute, and does a relatively good job of resolving various flow features. A better criterion can be designed, however, by accounting for waves moving in arbitrary directions, through use of a wave model for the Euler equations [Roe86b]. The resulting criterion is simple, inexpensive, and very effective in resolving a wide variety of flow features.

In the following sections, the framework for wave models to the Euler equations is described, and two adaptation criteria resulting from this framework are put forward. The first is based on a grid-aligned wave model; the second on a more general wave model. A technique for determining refinement and coarsening criteria is described, and results for both criteria are presented.

Traveling Wave Solutions to the Euler Equations

The Euler equations in two dimensions may be written in primitive variables as

$$\mathbf{W}_t + \mathbf{A}\mathbf{W}_x + \mathbf{B}\mathbf{W}_y = 0 \qquad (1)$$

where

$$\mathbf{W} = \begin{pmatrix} \rho \\ u \\ v \\ p \end{pmatrix},$$

and

$$\mathbf{A} = \begin{bmatrix} u & \rho & 0 & 0 \\ 0 & u & 0 & 1/\rho \\ 0 & 0 & u & 0 \\ 0 & \gamma p & 0 & u \end{bmatrix}$$
$$\mathbf{B} = \begin{bmatrix} v & 0 & \rho & 0 \\ 0 & v & 0 & 0 \\ 0 & 0 & v & 1/\rho \\ 0 & 0 & \gamma p & v \end{bmatrix}.$$

Wave-like solutions, of the form

$$\begin{aligned} \mathbf{W} &= \mathbf{W}(\mathbf{k}\cdot\mathbf{x} - \lambda t) \\ &= \mathbf{W}(\xi) \end{aligned}$$

may be sought, with $\mathbf{k}$ and $\mathbf{x}$ defined as

$$\mathbf{k} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$
$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

This leads to the eigenvalue problem

$$[\mathbf{A}\cos\theta + \mathbf{B}\sin\theta - \lambda\mathbf{I}]\,\mathbf{W}'(\xi) = 0 \qquad (2)$$

which admits non-trivial solutions only if $\mathbf{W}'(\xi)$ is a right eigenvector of $(\mathbf{A}\cos\theta + \mathbf{B}\sin\theta)$, with λ its corresponding eigenvalue. These eigenvalues are, for a given θ:

$$\begin{aligned} \lambda_1 &= u\cos\theta + v\sin\theta\,; \\ \lambda_2 &= u\cos\theta + v\sin\theta\,; \\ \lambda_3 &= u\cos\theta + v\sin\theta + a\,; \\ \lambda_4 &= u\cos\theta + v\sin\theta - a\,; \end{aligned} \qquad (3)$$

where a is the acoustic speed $a = \sqrt{\gamma p/\rho}$. The corresponding right-eigenvectors are:

$$\begin{aligned} \mathbf{r}_1 &= [0, -a\sin\theta, a\cos\theta, 0]^T\,; \\ \mathbf{r}_2 &= [\rho, 0, 0, 0]^T\,; \\ \mathbf{r}_3 &= \left[\rho, a\cos\theta, a\sin\theta, \rho a^2\right]^T\,; \\ \mathbf{r}_4 &= \left[\rho, -a\cos\theta, -a\sin\theta, \rho a^2\right]^T\,; \end{aligned} \qquad (4)$$

which represent respectively a shear wave, an entropy wave, an acoustic wave propagating in direction θ and an acoustic wave propagating in direction $\theta + \pi$. The eigenvalues represent the speed of propagation of the waves.

It is the determination of the wave angles θ that separate a grid-aligned approach from a multi-dimensional approach. In a grid-aligned approach, the waves are assumed to propagate normal to the grid faces; in a multi-dimensional approach, the values of θ are determined from local flow data. Adaptation criteria based on these two approaches are described in the following two sections.

A Refinement Criterion Based on a Grid-Aligned Wave-Model

Roe's approximate Riemann solver [Roe81, Roe86a] expresses the flux difference across a face in the mesh as the effect of four waves, all moving normal to the cell face. That is, the flux normal to each interface is given by

$$\boldsymbol{\Phi}_n(\mathbf{U}_L, \mathbf{U}_R) = \frac{1}{2}(\boldsymbol{\Phi}_{nL} + \boldsymbol{\Phi}_{nR}) - \frac{1}{2}\sum_{k=1}^{4}\left|\hat{\lambda}_k\right|\Omega_k\hat{\mathbf{R}}_k ,$$

where $()_R$ and $()_L$ denote the states right and left of the interface. The waves are taken to be moving normal to the cell face, in a direction θ_g, with the flux vector defined as

$$\boldsymbol{\Phi}_n = \begin{pmatrix} \rho u_n \\ \rho u_n u + p\cos\theta_g \\ \rho u_n v + p\sin\theta_g \\ \rho u_n H \end{pmatrix} ,$$

where

$$\begin{aligned} u_n &= u\cos\theta_g + v\sin\theta_g \\ u_t &= -u\sin\theta_g + v\cos\theta_g . \end{aligned}$$

The wave speeds $\hat{\lambda}_k$ are those of Equation 3, with θ taken to be the grid angle θ_g. The eigenvectors $\hat{\mathbf{R}}_k$ are the conserved-variable form of the eigenvectors given in Equation 4. They are evaluated at an average state which satisfies certain flux properties; the Roe-average state, defined by

$$\begin{aligned} \omega &= \sqrt{\rho_L}/(\sqrt{\rho_L} + \sqrt{\rho_R}) \\ \hat{\rho} &= \sqrt{\rho_L \rho_R} \\ \hat{u} &= u_L\omega + u_R(1-\omega) \\ \hat{v} &= v_L\omega + v_R(1-\omega) \\ \hat{H} &= H_L\omega + H_R(1-\omega) \\ \hat{a} &= \sqrt{(\gamma-1)\left[\hat{H} - \frac{1}{2}(\hat{u}^2 + \hat{v}^2)\right]} \end{aligned}$$

The wave strengths Ω_k are given by:

$$\begin{aligned} \Omega_1 &= \frac{\Delta u_t}{\hat{a}} \\ \Omega_2 &= \frac{1}{\hat{\rho}\hat{a}^2}(\hat{a}^2\Delta\rho - \Delta p) \\ \Omega_3 &= \frac{1}{2\hat{\rho}\hat{a}^2}(\Delta p + \hat{\rho}\hat{a}\Delta u_n) \\ \Omega_4 &= \frac{1}{2\hat{\rho}\hat{a}^2}(\Delta p - \hat{\rho}\hat{a}\Delta u_n) . \end{aligned}$$

These wave strengths seem to form an attractive criterion for solution-adaptive refinement:

- they are necessary for the flux calculation, and are therefore pre-computed;
- since each wave strength is associated with a certain type of wave (shear, entropy, "fast" acoustic, "slow" acoustic), they should make it possible to detect regions of compressibility and rotationality separately, using Ω_1 to detect regions of rotationality, and, for instance, $|\Omega_3| + |\Omega_4|$ to detect regions of compressibility.

The only apparent weakness of using these wave strengths to form an adaptation criterion is that the wave model is inherently one-dimensional. A pure shear, lying oblique to the grid, will not be recognized as such, but instead be interpreted as the sum of a shear and two acoustics [RvLR91b, RvLR91a]. A more general wave model, which is independent of the grid direction θ_g, might be desirable.

A Refinement Criterion Based on a More General Wave-Model

A wave-model that is less dependent on the grid geometry may be defined by *solving* for wave propagation directions, rather than assuming them to be normal to cell-interfaces of the grid. Roe has developed several models which combine acoustic, entropy and shear waves to represent the solution [Roe86b]. The most promising appears, at this point, to be made up of:

- four acoustic waves, of strengths $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, propagating in orthogonal directions $\theta, \theta + \pi/2, \theta + \pi, \theta + 3\pi/2$;
- an entropy wave of strength β propagating in a direction ϕ;
- a shear wave of strength σ propagating in a direction μ;

and solving, in each cell, for the wave strengths and directions, in terms of local values of the solution gradient. The equations for the derivatives of the primitive variables may be written, using Equation 1 and the model defined above, as

$$
\begin{aligned}
\rho_x &= \sum_{i=1}^{4} \rho\alpha_i \cos\theta_i + \rho\beta\cos\phi \\
\rho_y &= \sum_{i=1}^{4} \rho\alpha_i \sin\theta_i + \rho\beta\sin\phi \\
u_x &= \sum_{i=1}^{4} \alpha_i a \cos^2\theta_i - \sigma a \sin\mu\cos\mu \\
u_y &= \sum_{i=1}^{4} \alpha_i a \cos\theta_i \sin\theta_i - \sigma a \sin^2\mu \\
v_x &= \sum_{i=1}^{4} \alpha_i a \sin\theta_i \cos\theta_i + \sigma a \cos^2\mu \\
v_y &= \sum_{i=1}^{4} \alpha_i a \sin^2\theta_i + \sigma a \sin\mu\cos\mu \\
p_x &= \sum_{i=1}^{4} \alpha_i \rho a^2 \cos\theta_i \\
p_y &= \sum_{i=1}^{4} \alpha_i \rho a^2 \sin\theta_i \; .
\end{aligned}
$$

This set of eight equations in nine unknowns defines a one-parameter family of wave models. The most physically justifiable model of the family is the one for which

$$
\begin{aligned}
\tan 2\theta &= \frac{v_x + u_y}{u_x - v_y} \\
\mu &= \theta \pm \frac{\pi}{4} \; .
\end{aligned}
$$

For this model, the acoustic waves propagate along the principal axes of the strain-rate tensor, and the shear wave propagates at an angle of 45° to them [Roe91].

Defining

$$R = \sqrt{(v_x + u_y)^2 + (u_x - v_y)^2}\;,$$

the wave angles and strengths are given by

$$
\begin{aligned}
\sigma &= \frac{v_x - u_y}{a} \\
\phi &= \arctan \frac{a^2\rho_y - p_y}{a^2\rho_x - p_x} \\
\beta &= \frac{1}{\rho a^2}\sqrt{(a^2\rho_x - p_x)^2 + (a^2\rho_y - p_y)^2} \\
\theta &= \frac{1}{2}\arctan\frac{v_x + u_y}{u_x - v_y} \\
\alpha_1 &= \frac{1}{2}\left[\frac{u_x + v_y + R - a\sigma}{2a} + \frac{p_x\cos\theta + p_y\sin\theta}{\rho a^2}\right] \\
\alpha_2 &= \frac{1}{2}\left[\frac{u_x + v_y - R - a\sigma}{2a} - \frac{p_x\sin\theta - p_y\cos\theta}{\rho a^2}\right] \\
\alpha_3 &= \frac{1}{2}\left[\frac{u_x + v_y + R + a\sigma}{2a} - \frac{p_x\cos\theta + p_y\sin\theta}{\rho a^2}\right] \\
\alpha_4 &= \frac{1}{2}\left[\frac{u_x + v_y - R + a\sigma}{2a} + \frac{p_x\sin\theta - p_y\cos\theta}{\rho a^2}\right]
\end{aligned}
$$

From the above, it can be seen that the sum of the α_i's is directly proportional to the divergence of the velocity:

$$\nabla \cdot \mathbf{u} = a(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)\,. \qquad (6)$$

This, and the definition of σ, define a cell-centered refinement criterion: regions of compressibility may be detected by examining $u_x + v_y$; regions of rotationality may be detected by examining $u_y - v_x$. This simple criterion can be justified even without the wave model described above; the divergence of the velocity is a local measure of compressibility, and the curl of the velocity is a local measure of the rotationality. This criterion again has the advantage of being easily computed; velocity gradients are necessary for any high-order scheme.

Proper Scaling of Adaptation Criteria

Gradient-based criteria will tend to *increase* at a discontinuity as it becomes more resolved; criteria based on undivided differences will remain bounded but finite at a discontinuity as it becomes more resolved. Either of these situations will cause discontinuities to act as "refinement magnets," with the discontinuity being resolved at the expense of other regions of the flow. As pointed out by Warren et al [WATK91], this situation can lead to erroneous results.

To prevent this from happening, two steps are taken. First, criteria based on undivided differences (as in the grid-aligned wave model) are multiplied by $\sqrt{L}$, where L is a length scale for a cell (e.g. $\sqrt{A_{cell}}$); criteria based on gradients (as in the general wave model) are multiplied $L^{3/2}$. This acts to de-emphasize the importance of the shocks and shears as they become more and more resolved. Second, a *minimum length scale for a cell is set; cells of scale* $L < L_{min}$ are refined no further. The minimum length scale L_{min} is based on a global length scale for the problem.

Setting the Threshold

For each computational cell, the rotationality criterion, τ_r, and the compressibility criterion, τ_c, are calculated, using either the grid-aligned wave model or the general wave model. Then the standard deviation about zero for both criteria, σ_r and σ_c, are calculated using only those cells which have a length scale $L > L_{min}$. Thus, if a cell is already at its smallest allowable size, it will not be used in determining the standard deviations of the adaptation criteria. The effect of this is that, once a flow feature has been resolved, the refinement will look for other, weaker features to resolve. The cell may, however, be flagged for coarsening, since the flow feature that was once there may have moved or disappeared. Cells that have a length scale $L > L_{min}$ are refined if either the rotationality or compressibility criterion lie more than one standard deviation away from zero, the value for uniform flow. Cells are flagged for coarsening if both the compressibility and rotationality criteria lie within one-tenth of a standard deviation from zero.

For the general wave model, for example, σ_r and σ_c are determined by

$$\tau_r = |\nabla \times \mathbf{U}|\, L^{\frac{3}{2}} \quad , \quad \sigma_r = \sqrt{\frac{\sum_{i=1}^{n} \tau_{s_i}^2}{n}}$$

$$\tau_c = |\nabla \cdot \mathbf{U}|\, L^{\frac{3}{2}} \quad , \quad \sigma_c = \sqrt{\frac{\sum_{i=1}^{n} \tau_{c_i}^2}{n}} \qquad (7)$$

Cells will be flagged for refinement if

$$L > L_{min} \quad \text{and} \quad (|\tau_c| > \sigma_c \quad \text{or} \quad |\tau_r| > \sigma_r) \ . \qquad (8)$$

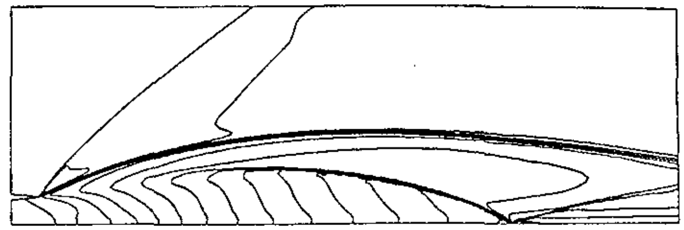

Figure 2: Jet Flow Mach Contours

Cells will be flagged for coarsening if

$$|\tau_c| \leq \tfrac{\sigma_c}{10} \quad \text{and} \quad |\tau_r| \leq \tfrac{\sigma_r}{10} \ . \qquad (9)$$

Results of Using the Criteria

Results of using the grid-aligned criterion and the general wave-model criterion are depicted here for three test cases. The computations were carried out using an adaptively-refined Cartesian mesh code that uses linear reconstruction to obtain high-resolution solutions [DP91].

The flow in the first case is the result of the interaction of an axisymmetric jet $M_j = 2.0$ with a supersonic stream of Mach number $M_s = 2.0$. The total pressure ratio between the two streams is $p_{0_j}/p_{0_s} = 20$; the total temperature ratio is unity. This flow has the following features:

- a strong expansion out of the nozzle of the jet;
- a curved shock emanating from the lip of the nozzle, which reflects at the axis of symmetry;
- an oblique shock through which the outer stream passes;
- a strong shear separating the jet and the outer stream.

Figure 2 shows the Mach number contours for this flow. The solution used to produce these contours was calculated on a moderately resolved grid, refined twice using the divergence/curl criterion.

Figures 3–5 show grids resulting from refinement based on

1. difference of density across a face ($\Delta\rho$);

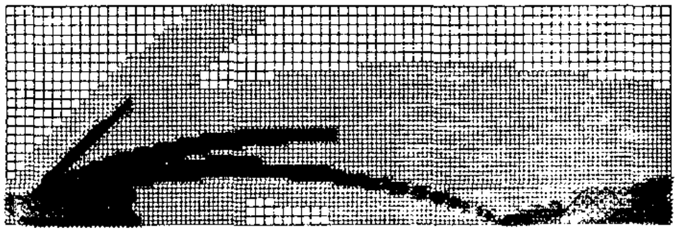

Figure 3: Grid Resulting from Refinement Based on Density Difference (15% of cells flagged for refinement)

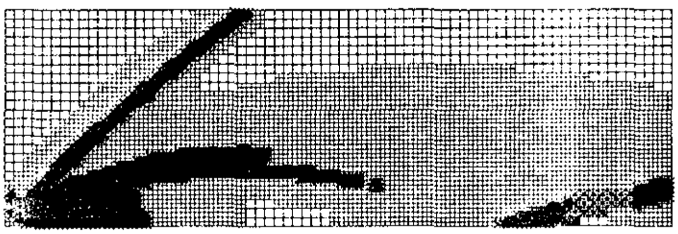

Figure 4: Grid Resulting from Refinement Based on Pressure Difference (15% of cells flagged for refinement)

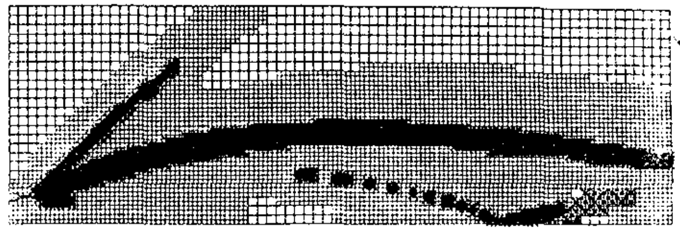

Figure 5: Grid Resulting from Refinement Based on Flow-Speed Difference (15% of cells flagged for refinement)

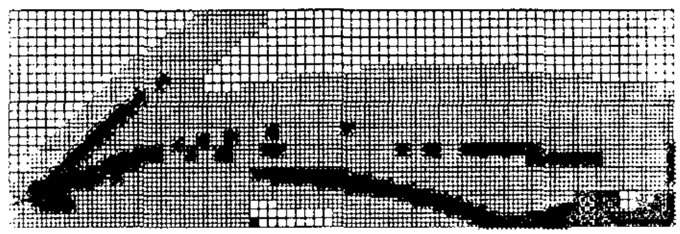

Figure 6: Grid Resulting from Refinement Based on Grid-Aligned Shear Waves (15% of cells flagged for refinement)

2. difference of pressure across a face (Δp);

3. difference of flow speed across a face ($\Delta\sqrt{u^2 + v^2}$);

Thresholds were set so that the same number of cells were refined with each criterion. As can be seen, the density difference (Figure 3) detects the shocks and the expansion region, but misses most of the jet boundary. The pressure difference (Figure 4) fares even worse, detecting the external shock and the expansion region, but missing some of the shock emanating from the lip of the nozzle, and *all* of the jet boundary. Only the flow-speed difference (Figure 5) detects the jet boundary sufficiently well, but it misses much of the curved shock.

The grid-aligned wave model does no better. Refinement based on the shear wave strength is shown in Figure 6; refinement based on the acoustic wave strength is shown in Figure 7. The shocks are captured by the compressibility criterion, but the shear is missed by the rotationality criterion.

The general wave model behaves the best for this case, as shown in Figure 8, which shows the grid resulting from refinement based on divergence and curl. Again, the threshold is set so that the same number of cells are flagged for refinement as when using the other criteria. All of the features are resolved, with the curl flagging only the jet boundary, and the divergence flagging the shocks and the expansion.

The way that the criterion based on the general wave model acts to separate the effects of compressibility and rotationality may be seen in the computation of the steady flow over a double-wedge, with relative turning angles of 10° and 14°, and incoming Mach number $M_\infty = 3.0$. The Mach contours are shown in Figure 9. There is a shock from each of th

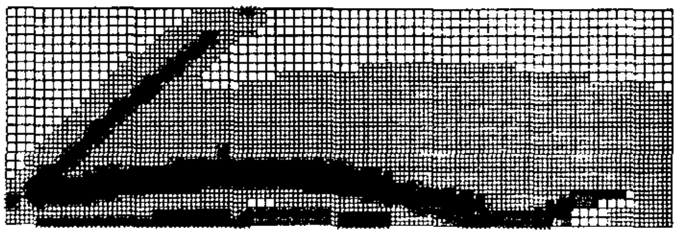

Figure 7: Grid Resulting from Refinement Based on Grid-Aligned Acoustic Waves (15% of cells flagged for refinement)

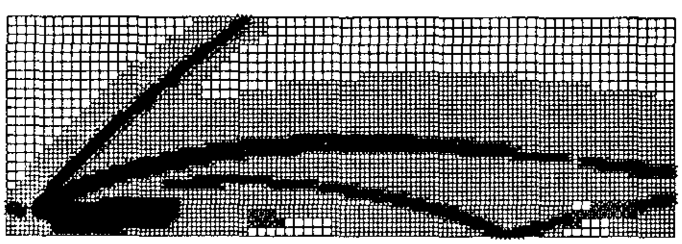

Figure 8: Grid Resulting from Refinement Based on Wave-Model-Based Acoustic and Shear Waves (15% of cells flagged for refinement)

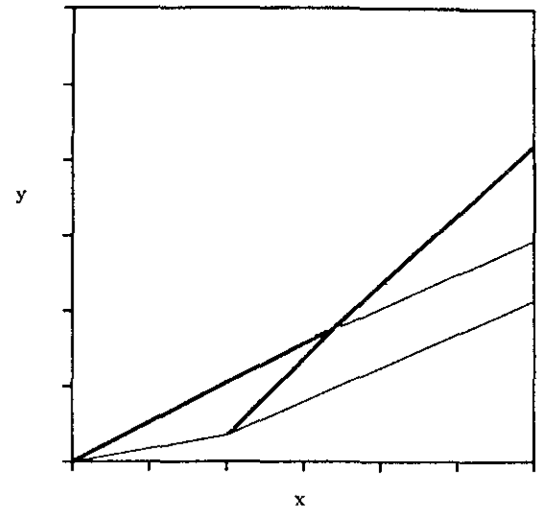

Figure 9: Mach Contours for Double Wedge Case

compression corners, and the two shocks interact, resulting in a shock, a contact and an expansion. The expansion can not be seen from the Mach contours. The expansion reflects from the wall.

Figure 10 shows how the divergence criterion, with a threshold set so that 20% of the cells are flagged, detects the features associated with compressibility, i.e. the shocks and expansion. The effects of both divergence and curl, with the threshold set so that 15% of the cells are flagged, are shown in Figure 11. Adding the curl to the refinement criterion causes the contact to be captured, as well as the shocks.

The divergence/curl criterion also behaves well for smooth flows; the grid resulting from calculation of a purely subsonic flow is shown in Figure 12. The leading-edge stagnation point and expansion are detected, as is the trailing-edge stagnation point.

Concluding Remarks

Two criteria for adaptive refinement based on wave models for the Euler equations have been proposed and studied. In the first, the wave strengths from Roe's approximate Riemann solver, which constitute a grid-aligned wave model, are used to detect regions of compressibility and rotationality separately. In the second, the wave strengths from a more general wave model are used, resulting in a simple velocity-

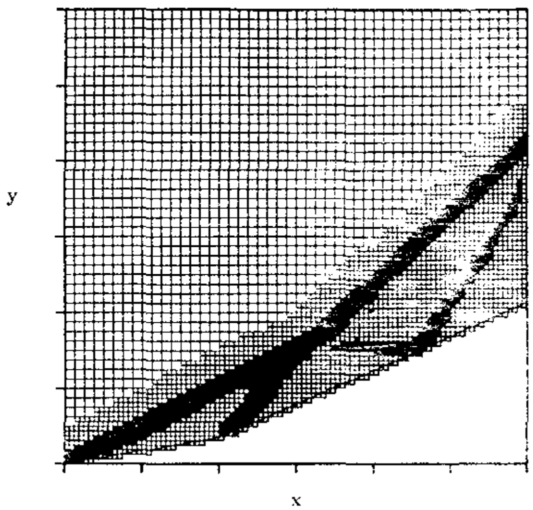

Figure 10: Grid Resulting from Refinement Based on Wave-Model-Based Acoustic Waves (20% of cells flagged for refinement)

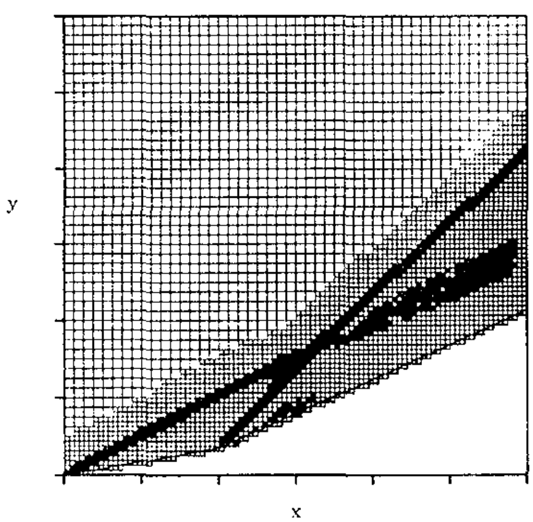

Figure 11: Grid Resulting from Refinement Based on Wave-Model-Based Acoustic and Shear Waves (15% of cells flagged for refinement)

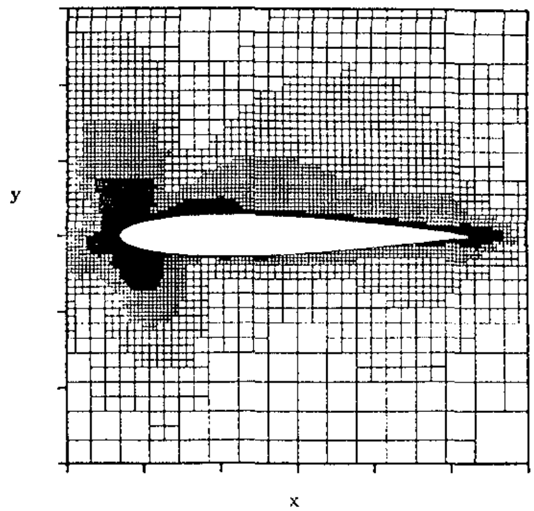

Figure 12: Grid Resulting from Refinement Based on Wave-Model-Based Acoustic and Shear Waves

divergence criterion for detection of regions in which compressibility is important, and a velocity-curl criterion for detection of regions in which rotationality is important. The more general wave model has been shown to do a better job of separating the effects of compressibility and rotationality. A method for flagging cells for refinement and coarsening, based on the standard deviation of the divergence and curl of the velocity, has been described, and shown to perform very well.

It should be noted that the divergence/curl criterion proposed could be deduced without the use of a wave model. The wave model serves, however, to give physical justification for the choice of refinement criterion, and to suggest more sophisticated criteria in which not only the wave strengths, but the wave directions and speeds, are used, in order to determine the propagation of flow features in unsteady-flow computations. An example of the use of wave strengths and speeds in refining for unsteady flow is given by Chiang et al [CvLP92], for a grid-aligned wave model.

Finally, it should be noted that the refinement criterion presented in this paper is not proposed as the be-all and end-all of refinement criteria. It would not, for example, work well in incompressible potential flow. The only legitimate test for a refinement criterion is whether or not it works for the class of problems on which it is being used; there is no one "right" criterion for a given class of problems. The divergence/curl criterion presented here is specifically geared towards computation of steady, compressible flow.

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