Broadside Radar Echoes from Ionized Trails

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An approximate method to compute rough estimates of the broadside radar return from axially symmetric ionized gas columns is presented and applied to an illustrative example The method uses a continuum approach and takes into account the electron-neutral collision frequency and both axial and radial variation of electron density The local reflectivity is determined as a function of axial position along the column, and these values of reflectivity are used, in turn, to obtain radar cross section estimates

I Introduction

COMPUTATION of the magnitude of the reradiation of an incident electromagnetic wave from a column of ionized gas, such as a wake, depends on a knowledge of the number density of electrons N_e and of v, their collision frequency for momentum transfer to neutrals, as functions of position in the column With the present state of knowledge, these are obtainable theoretically for wakes only from very idealized models or inferred from indirect experimental measurements (for example, such as were carried out during re-entry of Glenn's satellite capsule[2]) combined with hypothesized wake structure and accompanying theory In either case, one has only estimates of the correct values However, in computation of the radar reflection, one is at present no better off than in theoretical wake parameter computations, and one can obtain only rough estimates of the return that a given wake model would yield, particularly if it is desired to take into account the axial variation of N_e, and if collision effects are not negligible It is the purpose of this paper to give an approximate approach that can be applied for such an estimate in the limited situation of broadside incidence of an electromagnetic wave whose wavelength is long compared to turbulent irregularities in the plasma column Approximate reflectivities for the wake as functions of downstream distance are obtained, and these are shown to limit the effective length of the column as a reflector The reflectivity computation is then combined with the diffraction theory for thick cylinders of finite length to estimate radar returns The methods used are quite approximate, but the numerical work can be done reasonably on a desk calculator They should, perhaps, be contrasted to the extensive digital machine computations carried out by de Ridder and Edelberg[3] using relatively exact procedures but corresponding to the less realistic problem consisting of broadside incidence on a symmetrical infinite cylindrical wake with no axial variation in its properties Such results are appropriate only to infinite-line antennas parallel to the cylinder axis, unless they too are used as inputs to an approximate theory

The procedure is illustrated by application to a model plasma column As a convenient model, N_e values are used (which are close to those given by Feldman[4]) which would prevail for the electron distribution in a laminar wake behind a spherical body at 100,000 ft alt moving with a velocity of 20,000 fps on the assumption of thermodynamic equilibrium and neglecting boundary-layer effects Although such wake models have been studied extensively as witnessed by Ref 4 and the work of Lykoudis,[5] they should not be taken as realistic models for actual wakes under these flight conditions; such wakes, even behind simple smooth objects, are not expected to be either laminar or in thermodynamic equilibrium (as was, in fact, pointed out in Ref 4) They also clearly do not represent wakes such as that described in Ref 2 occurring behind a large body with various protuberances, firing retrorockets, etc In any case, a collision-frequency distribution, self-consistent with this assumed model, is also needed for the electromagnetic computations To obtain this, note that the pressure in such a hypothetical trail drops as it expands downstream until it reaches the pressure of the unperturbed atmosphere, and beyond this axial point it remains constant This region would constitute almost all of the trail, and only the N values appropriate to this region are used Then reference to extensive experimental results correlated by Shkarovsky et al[6] in the temperature range appropriate to the wake model being used here, $1800°K \leq T \leq 3400°K$, shows that v will be approximately $7 \times 10^{10} \times$ (pressure in atmospheres), leading to a constant collision frequency consistent with this wake model of $v = 7\ 9 \times 10^8$ cps We thus have a self-consistent, though admittedly unrealistic, wake model to use as a plasma column model on which to illustrate the electromagnetic computations

Position x measured along the trail from the point where constant pressure commences is, according to the analysis of such a trail, most naturally expressed in terms of a normalized parameter, which for the altitude and speed considered is

$$\bar{x} \sim (7\ 5 \times 10^{-2})(x/r_0^2)$$

with x and r_0, the sphere radius, to be expressed in centimeters Using $\bar{x}$ as a parameter, Fig 1 presents a family of curves of N vs r/r_0 From these data, a set of curves of $N/N_{m\ x}$ vs r/r_0 was computed These curves are shown in Fig 2 On each curve is superposed one or two curves of $S_1 = \operatorname{sech}^2[(1/2\sigma)/(r/r_0)]$ for values of σ as indicated on the curves These will be used to aid in estimating radar return

II Reflectivity and Radar Return

The fields of electric intensity $\bar{E}$ and magnetic induction $\bar{B}$, which compose the radar beam and its echo, are related by Maxwell's equations:

$$\operatorname{curl}\bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \operatorname{curl}\bar{B} = \frac{n^2}{c^2}\frac{\partial \bar{E}}{\partial t} \tag{1}$$

Here c is the velocity of light in free space and n is the index

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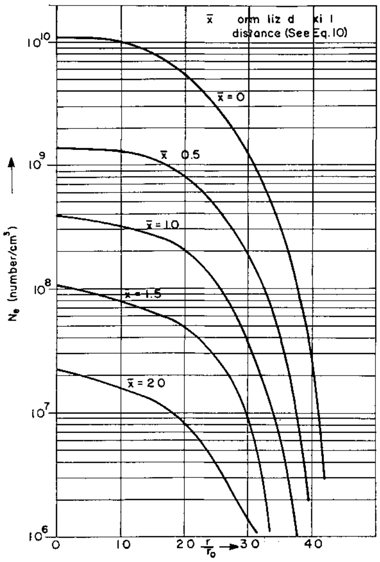

Fig 1 Electron number density as a function of radial distance from axis

of refraction which depends on N and v Time periodic fields of radian frequency ω will be assumed, so that time variation can be separated out in a factor $e^{-i\omega t}$ By not having n^2 operated on by $\partial/\partial t$, it is tacitly assumed that time variations of n^2 are negligible over a period of the order of time a radar pulse spends in the wake In addition, Eqs (1) assume that the medium is isotropic and that the permeability is constant everywhere

For a given radian frequency ω, the index of refraction is given by

$$n^2 = 1 - [X/(1 + iZ)] \tag{2}$$

where $Z = v/\omega$, $X = \omega_P^2/\omega^2$, and $\omega_P = [N\ e^2/(m\epsilon_0)]^{1/2}$ is the plasma frequency Here e is electron charge, m electron mass, and ϵ_0 the permittivity of free space

The commonly applied idea of perfect reflectivity of incident radiation at a constant N surface corresponding to $\omega = \omega_P$ is of no validity when $0\,01 \lesssim |Z| \lesssim 100$ To illustrate, a "natural" extension might be to consider surfaces of constant $Re\ n^2 = 0$ Thus, for example, reflectivity at a plane vacuum-plasma interface, where, in the plasma, $Re\ n^2 = 0$ leads to the values shown in Table 1

The procedures below will be applied in examples for $Z \simeq 8$ and $Z \simeq 40$ For a constant Z, the actual reflectivity is a function of the gradient of N in the direction of propagation Even in the case of one-dimensional variation with distance z of N or of X and a constant Z, it is a considerable mathematical and/or numerical problem to determine the reflectivity and the related transmissivity[7] In general, asymptotic

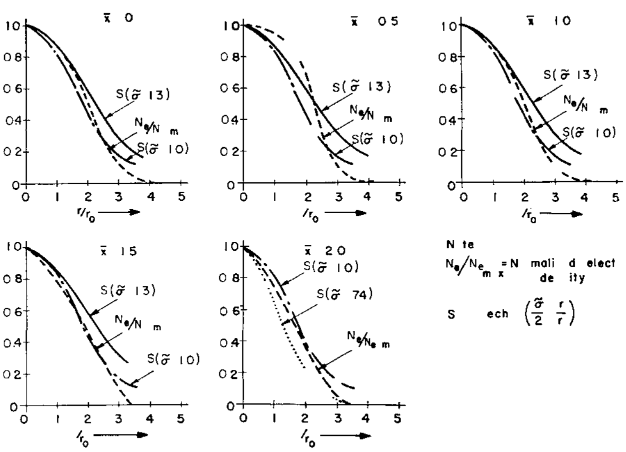

Fig 2 Normalized electron number density and sech² curves

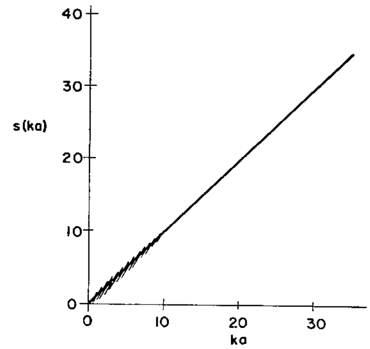

Fig 3 Cross-sectional factor, $s(ka)$

procedures must be used to pick out and relate appropriate solutions of the electromagnetic field equations In particular, solutions have been obtained for a wide class of X vs z curves known as Epstein profiles, plus generalizations of these by Rawer[8] Included in this class of profiles are the so-called "sech²" profiles:

$$n^2 = 1 - \frac{X_m}{1 - iZ}\operatorname{sech}^2\left(\frac{z}{2\sigma}\right)\left\{\frac{1}{2\sigma}(z - z_m)\right\} \tag{3}$$

so that

$$X = X_m \operatorname{sech}^2\left\{\frac{1}{2\sigma}(z - z_m)\right\} \tag{4}$$

This is a profile, symmetric about $z = z_m$, representing a plasma layer The reflection factor for $\bar{E}$ parallel to the N = const surfaces is given by

$$P = -\frac{\Gamma(-2ik\sigma C + 1)\Gamma(2ik\sigma C - \gamma - \frac{1}{2})\Gamma(2ik\sigma C + \gamma + \frac{1}{2})}{\Gamma(2ik\mu C + 1)\Gamma(-\gamma + \frac{1}{2})\Gamma(\gamma + \frac{1}{2})} \tag{5}$$

where

$$-\frac{16k^2\sigma^2 X_m}{1 - iZ} + \frac{1}{4} = \gamma^2$$

Here $\Gamma(x)$ is the gamma function, and C is the cosine of the angle between the incident propagation direction at $z = -\infty$ and the z direction This formula has been investigated extensively numerically by Rawer, who gave curves of $|P|$ vs ω/ω_1 for various values of v/ω_1 and S_1, where $\omega_1 = \omega_P\ (z = z_m)$ and $S_1 = 2\omega_1\ \sigma/c$ His results clearly illustrative the sensitivity of P to v and to the N gradients In the present context, besides illustrating the sensitivity of reflectivity to the details of the plasma layer, the Rawer results can be used to give local reflectivity as a function of axial distance along the trail The computations of the previous section yielded approximate X vs r distributions, where r is radial distance from the trail axis One can equate r with $z - z_m$ of Eq (3) and use the parameter σ to fit the distribution (4) for X to that of the computed X distribution This was done, although the P's were computed directly using tables of the complex gamma function[9] The results are described in Sec III

These reflectivities are suggestive of the ability of the various parts of the trail to reflect incident radiation They really, however, represent reflectivity of infinite plane waves from infinite surfaces To obtain a radar cross section for the wake, one must still take into account the variation in range and angle from the actual antenna to points on the trail and the effect of the wake's roughly cylindrical shape

What is done in the following is to realize that the wake is of a fairly restricted effective length when viewed near

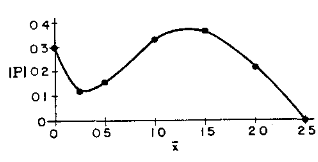

Fig 4 Magnitude of reflection factor, $|P|$ ($f = 2 \times 10^7$)

Table 1 Power reflectivities for normal incidence

Z	0 002	0 005	0 02	0 08	1 0	2 0	8	200	450
$\|P\|^2$	0.96	0.82	0.67	0.465	0.17	0.20	0.40	0.82	0.89

broadside and hence to avoid the difficulties in interpreting infinite cylinder results The numerical results in Sec III for reflectivity $|P|$ show that, for $\bar{x}$ up to some fairly well-defined value $\bar{x}_m$, the reflectivity does not vary radically, and then for $\bar{x} \gtrsim \bar{x}_m$ it dives to negligible values The trail for $x > \bar{x}_m$ will be neglected, whereas up to $\bar{x} = \bar{x}_m$ it will be considered to be cut into elementary segments $d\bar{x}$ These segments have no well-defined ends of discontinuous electromagnetic properties which would be dominant scatterers, combining to give pronounced resonant effects depending on the length Hence, such cylinders, if perfectly conducting, could be treated as so-called thick cylinders, as done in detail by Mentzer [10] Their radar cross section is roughly

$$\sigma = (4/\pi)(dx)^2 s(ka) \tag{6}$$

for broadside incidence, where $s(ka)$ oscillates in the shaded region of Fig 3 about the line of 45° slope, converging asymptotically to this line The upper bound is reached for electric field, the lower for magnetic field parallel to the symmetry axis Here k is 2π/free-space wavelength, and a is cylinder radius

Recalling the basic definition for radar cross section σ,

$$\sigma = 4\pi \lim_{\rho \to \infty} \rho^2 |E/E^m|^2 \tag{7}$$

where ρ is distance from the scatterer and E and E^m are the scattered and incident electric field strengths, one sees that the magnitude of the scattered field is then proportional to $\sigma^{1/2}$ To take into account the fact that the wake is a nonperfect conductor so that Eq (6) cannot be used unmodified, two approximative steps will be made, with only tenuous justification, to get an effective scattered field

The first problem concerns what to use for radius a In those cases where the surface $r = r$ corresponding to $X = 1$ is a very highly reflecting surface, one could use $a = r$ To do this requires that operating frequencies corresponding to a considerable range around $Z = 1\ 0$ be excluded, since, within this frequency range, there is no reason to consider a critical density contour to behave in any way like a perfect conductor

When frequency is not such that $a = r$ has much physical significance, what is probably the next best choice is based on noting that the N vs r curve must have an inflection point at some value $r = r_i$ The major part of the reflectivity must come from the region $r \leq r_i$, since it is in this region that there exist the steepest gradients of N This suggests using $a = r_i$ and is what we do in Sec III The second modification is to multiply $\sigma\ (kr_i)$ by P to take into account reflectivity Justification for simply multiplying σ by P is suggested, for example, by theoretical arguments by Franz and Beckmann[11] for large ka cylinders (of a uniform lossy material) and by the experimental results of Severin and Beckmann[12] for ka somewhat greater than unity These papers show that a good approximation to the backscattering radar cross section is obtained by multiplying the radar cross section for a perfectly conducting object of the same shape and size by the power reflection factor for a half-space of the actual material In Ref 12, for example, for dielectric disks the agreement between values measured and values computed by the foregoing means for both amplitude and phase of the backscattered field was extremely close for the smallest value tested of k times disk radius, namely, 2π

It is worth pointing out that, for order of magnitude results, which is, after all, all one might safely expect from the procedures being suggested here, one should be able to push this procedure down to ka slightly below unity for a low-reflectivity plasma column, that is, rule out only the Rayleigh region The reason for this is because the difference from the peak resonant return at $ka = 1$ to the geometric optics ($ka \gg 1$) results should not exceed the factor of about 4, which is found for a perfectly conducting object This is because one must expect much smaller "creeping wave" contributions than for a perfect conductor,[11 13] and it is the interference of these waves and the "geometrically reflected waves" which leads to the deviations from the geometrical optics results that occur in the resonance region (ka near unity) For thick, perfectly conducting cylinders, the range of these deviations is shown by the cross-hatching in Fig 3 Likewise, one would not expect for plasma columns any enhancement from internal focussing, as might occur for well-defined dielectric columns of index of refraction exceeding unity, since in the present case the incoming waves are refracted to deviate away from the line of sight

The result of all this is to approximate the radar cross section of the column from $\bar{x} = 0$ to $\bar{x} = \bar{x}_m$ by

$$\sigma = \frac{4}{\pi} \left| \int_0^{x_m} [s(kr_i)]^{1/2} |P| \exp[\arg P + k\Delta] dx \right|^2 \tag{8}$$

In addition to a phase of P, the phase $k\Delta$ due to variation in path length from the source to the integration points $\bar{x} > 0$ is included in this integral, whereas the effect of antenna gain pattern can be included with $|P|$

If the effective trail is long enough, the σ for off-broadside incidence should be used This is available in Ref 10

The contribution to returned power from regions in the wake where $P \sim 0$ will not be zero, although such echoes will be relatively small per unit length of trail compared to regions where Eq (8) is usable However, under some cases, such regions may dominate the net return, for example, because the denser wake regions are not being irradiated These regions may be handled by consideration of the individual scatterers, as is done, for example, in meteor trail work [14]

III Reflectivity and Radar Cross Section Numerical Results

For the assumed model plasma column, the reflectivity factor P given by Eq (8) was computed as a function of $\bar{x}$ for two frequencies, $f = 10^8$ and 2×10^7 cps Figures 4–6 give $|P|$, $\ln|P|$, and $\arg P$ for $f = 2 \times 10^7$ cps, whereas Figs 7–9 do the same for $f = 10^8$ cps In each case there is

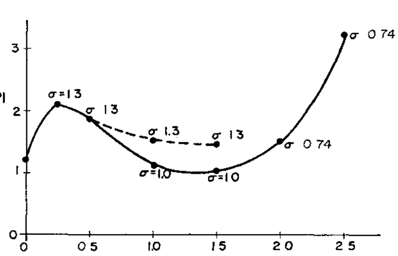

Fig 5 Magnitude of reflection factor illustrating sensitivity to $\sigma(f = 2 \times 10^7)$

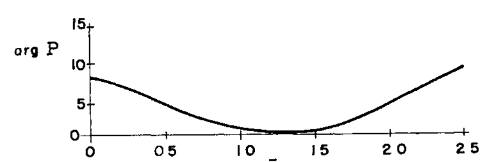

Fig 6 Phase of reflection factor, arg$P$ ($f = 2 \times 10^7$)

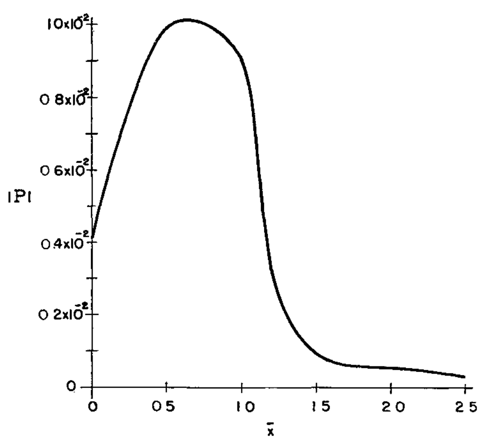

Fig 7 **Magnitude of reflection factor, $|P|$ (f = 10^8)**

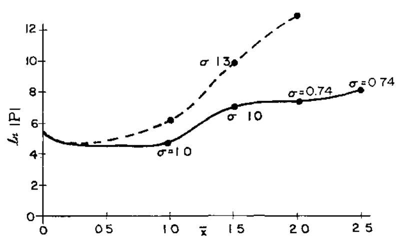

Fig 8 Magnitude of reflection factor illustrating sensitivity to σ ($f = 10^8$)

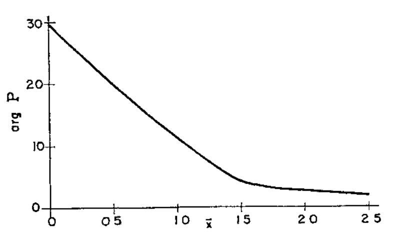

Fig 9 Phase of reflection factor, arg$P$ ($f = 10^8$)

a substantial region $\bar{x} < \bar{x}_m$, wherein $|P|$ does not vary by more than an order of magnitude Then for larger $\bar{x}$ values, as stated in the text, it very rapidly decreases to insignificance, which clearly limits the effective length of the column as a reflector However, even over the region where $|P|$ is not insignificant, its phase arg$P$ varies widely so that there is a considerable tendency for the echoes from successive sections of the trail to be out of phase with each other and partially to cancel their respective contributions to the net return

To obtain the corresponding radar cross section results, we have used Eq (8), assuming that $k\Delta$ variation can be neglected by virtue of a large range From the inflection point of the curves in Figs 2, a was taken to vary from 2 25 r_0 to 2 8 r_0 as $\bar{x}$ increased from $\bar{x} = 0$ to $\bar{x} = \bar{x}_m$ For this rough work, one could simply take an average $a \sim 2$ 6 r_0 and hence factor $s(ka)$ out of the integral The integration was carried out in terms of the dimensionless variable $\bar{x}$ defined in Sec I Thus the actual formula evaluated for σ was

$$\sigma = 220\, r_0^4\, s(ka)\, |I|^2$$

where

$$|I|^2 = \left| \int_0^{x_m} |P| \exp(\arg P) d\bar{x} \right|^2$$

and lengths are in centimeters

The results of the integrations and corresponding cross sections are given in Table 2 for r_0 = 30 cm and therefore for a = 78 cm This is about the smallest value of a for which use of the theory could be pushed so as to obtain order-of-magnitude estimates as discussed in Sec II For any larger r_0, the cross sections increase as r_0^5, while at the same time the expected accuracy of the results improves

Table 2

f, cps	$\|I\|^2$	σ, m^2
2×10^7	0 09	600
10^8	0.01	320

If the plasma column model were truly taken to represent the wake, then to these σ values should be added the return from the trail region between the body and the beginning of the expansion region and the return from the body and surrounding plasma itself The return from the trail for $x < 0$ could, if desired, be computed in the same manner, using the N_e values numerically obtained by Feldman in Ref 4 However, both of these contributions will be negligible compared to the σ values given in Table 2 Scattering from the 30-cm sphere itself will yield $\sigma < 0$ 5 m^2, whereas the trail region for $x < 0$ is only about 10 r_0 in length, according to Ref 4 Thus it, too, can have only a negligible effect on σ

The trail length for the r_0 = 30 cm case, as measured to the position where N_e falls to $N \sim 10^8/\text{cm}^3$ ($X \sim 1$), is $\sim$ 3000 m, whereas the radius within which $N > 10^8/\text{cm}^3$ exceeds $2r_0$ Thus, as one would expect considering the low reflectivity, the corresponding geometric cross section of the trail (with this arbitrary but often used criterion for trail length) viewed broadside exceeds its radar cross section Note that the effective trail length by the reflectivity criterion that we have used is much shorter

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