



A99-16738

AIAA 99-0910

CFD Education: Past, Present, Future

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**37th AIAA Aerospace Sciences  
Meeting and Exhibit**  
January 11-14, 1999 / Reno, NV

# CFD Education: Past, Present, Future

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## 1 Introduction

Computational Fluid Dynamics has earned itself a worthy place beside the classical disciplines of theoretical and experimental fluid dynamics. The establishment of CFD as a major discipline has been marked by the appearance of CFD as a *teaching* subject at leading universities all over the world. This paper is dedicated to CFD education: its history, present state, and what challenges it faces in the future. This sounds more impressive than it really is. CFD education has pretty much stabilized, and does not appear to face more challenges than the rest of engineering education.

As CFD is a relatively young subject, the history of CFD education is still strongly linked to that of CFD itself. In turn, the evolution of CFD has in a significant way been steered by the evolution of the modern computer. The influence of hardware development makes itself felt in several ways:

1. by the actual benefits of faster computing and larger memory;
2. by architecture that favors certain numerical algorithms above other ones, as is the case with vector and parallel computing;
3. through architecture-based national funding policies, such as the current emphasis on software development for parallel machines.

In this rapidly changing computational scenery it is the task of the CFD education system to define the basis of CFD, to sift the trendy from the permanent, and yet to prepare its students for whatever the future has in store.

## 2 Milestones in the history of CFD

Contrary to popular belief among aerospace engineers, CFD was not born in the aerospace sciences. The first serious, but failed, attempt to simulate fluid motion with a discrete numerical model dates back to the beginning of this century, when the British meteorologist L. F. Richardson tried to realize his dream of numerical weather prediction. He actually ran into a problem as fundamental as maintaining numerical stability. His heritage includes the time-centered diffusion scheme, which is unconditionally unstable [1].

The problem of stability was addressed and understood by three applied mathematicians, R. Courant, K. O. Friedrichs and H. Lewy in their famous 1928 paper [2]. Their findings remained of little use, until the first programmable computer appeared on the scene. World War II was raging then, and the scientists at Los Alamos National Laboratory were not only developing the atomic bomb, but also the means to describe the violent flow created by such a device. John von Neumann gave us both the method of artificial viscosity, enabling the capturing of shocks whenever and wherever they appear, and a practical Fourier method of analyzing finite-difference schemes regarding their stability [3].

Initially, CFD development in the USA<sup>1</sup> remained largely in the hands of scientists at the national laboratories in Los Alamos and Livermore, location of the largest existing computers, and was therefore chiefly used for weapons research. (The only contributing university was New York University which boasted Courant and Friedrichs in its mathematics department and received major AEC funding.) During the 1950's large computer codes were created capable of dealing with any combination of deformable media; descriptions of the methodologies used can be found in the series of books "Methods in Computational Physics" edited by the Livermore scientists B. Alder, S. Fernbach and M. Rotenberg, and published by Academic Press. This series was succeeded in 1967 by the Journal of Computational Physics, under the same editors.

The core algorithm in these early codes is always modelled after the original Von Neumann-Richtmyer method [3] and is of first-order accuracy. An interesting detail is that in this period the astrophysical community benefited

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<sup>1</sup>For an account of parallel developments in the USSR, read S. K. Godunov's[4] recent review.

from the weapons codes by using them to simulate exploding and pulsating stars.

Second- and higher-order methods, such as the Lax-Wendroff scheme [5] were explored during the 1960's, with dubious success. The numerical oscillations created by such methods in the vicinity of a discontinuity can drive pressures, densities and concentrations negative, making them neither suited for computing flows with strong shocks nor for the mere advection of water vapor and trace elements in the atmosphere. In this decade the contribution to CFD by universities rose sharply as IBM and CDC computers became available to the wider academic community.

Non-oscillatory higher-order methods were introduced in the early 1970's by Boris [6] and Van Leer [7]. It is not accidental that both were trained in astrophysics, where the flows studied are more violent than in any other discipline. At the other end of the spectrum there was aeronautical science, where one tries to design streamlined bodies that cause minimal perturbations when immersed in a uniform flow. During the 1970's computational aerodynamics passed through a sequence of potential-flow models (small disturbance, transonic small-disturbance, full potential); around 1980 it was ready to take on the fully compressible Euler equations, which were the traditional basis for simulating high-energy flows.

During the 1980's, aerospace sciences truly took the lead in funding the development of powerful numerical techniques, applicable to flow problems in many disciplines. Much of this activity originated at ICASE, NASA Langley Research Center; for a historic and technical account of this development, and reprints of influential papers, the reader is referred to the recent anthology "Upwind and High-Resolution Schemes" [8].

From 1985 onward, numerical research in the aerospace sciences included the Navier-Stokes equations. By 1994, however, the aerospace interest in further developing CFD methods had largely dried up. CFD *methods* research today is erratically driven by HPCC funding programs, where the emphasis is on achieving sustained high FLOPS rates on large parallel machines. Research on improving the accuracy of methods, and on convergence acceleration, is on the back burner.

### 3 A short history of CFD teaching

CFD teaching in the USA originated in the mathematics department of New York University, although the subject was not recognized as such. It was embedded in the teaching of numerical methods for solving partial differential equations. The contacts of Courant, Friedrichs and the young Peter Lax with Los Alamos scientists such as R. D. Richtmyer guaranteed an influx of interesting lecturers. Richtmyer describes in the foreword to his book [3] on "Difference Methods for Initial-Value Problems" how one seminar series in 1953 became the basis for that book.

For a decade and a half NYU remained a leader in numerical education and research, but this role faded during the 1960's, when universities worldwide acquired their first mainframes and started to develop curricula in numerical analysis. The first courses did not go beyond computational linear algebra and the numerical integration of ODE's, but during the 1970's higher-level courses on solving PDE's followed.

In the late 1970's an initiative at NASA Langley Research Center for the first time made funding available for graduate student research in aeronautical CFD, at a small group of universities. The originators of this initiative were Vic Peterson, Al Gressow, Jerry South and others; the targeted aerospace engineering departments were those of Iowa State, Princeton (with a link to NYU that had no aero department), MIT, Stanford and the University of Cincinnati. Although the funding was modest, it was important in that CFD was recognized for the first time as an educational topic by itself.

The 1980's brought us the establishment of the first faculty positions designated to CFD (e. g. Earl Murman at MIT). Very soon all major universities hired new faculty that could teach CFD courses at the graduate and sometimes even the undergraduate level. These new hires initially were recruited from an older generation of workers in the field of CFD, but after a few years the first PhD's started to come off the assembly line and CFD teaching proliferated. Many course-packs were written during the 1980's, and textbooks began to appear, the earliest one being produced at Iowa State [1]. During this decade the new CFD teachers faced the task of selecting from an ocean of CFD methods and analysis techniques the ones that were fundamental enough to warrant inclusion in a course. It soon became clear that there was enough essential material for a basic two-term curriculum, and this has remained the consensus in the aerospace community.

## 4 The two-term curriculum

The first term of a two-term CFD curriculum deals with numerical-analysis techniques, with applications restricted to scalar model equations such as the linear advection-diffusion equation, Burgers' equation, and perhaps a small system such as the 1-D compressible Euler or 2-D incompressible Navier-Stokes equations. This course can be designed as an introduction to CFD for students of all engineering departments. At the University of Michigan, for example, this course is taught alternately by Aerospace faculty and Mechanical Engineering faculty, and further draws students from Civil & Environmental, and Nuclear Engineering, and from Atmospheric, Ocean & Space Sciences.

Not every aerospace department can afford the luxury of offering a second CFD term, at least not every year. There may simply not be enough students to justify the teaching; some degree of consolidation with other departments can help in this respect.

The second term is needed to cover higher-level numerical analysis and state-of-the-art CFD methods, and includes elaborate computer projects that mimic real-world problems, in so far as this can be done in one term. It is directed to research students who either wish to work in CFD method development (very few can be supported these days) or face a substantial, nontrivial flow-modeling task that stems from their own experimental work or that of others. At the University of Michigan this course is taught in the Aero department, but the computer projects may be taken from any discipline, or zoom in on an advanced numerical technique. Over the past 6 years the students have been asked to develop 2-D codes for:

- Steady flow over an airfoil, using an unstructured grid;
- Propagation of a shock wave in an L-shaped hydraulic transmission line (suggested by Ford Motor Company);
- Multifluid-shock interaction using the level-set method;
- Discharge of a basin through a breach in a dam (shallow-water description);
- Flow through a steam-release valve and duct in the Fermi-II nuclear power plant in Monroe, MI (adapted from a consulting task for Detroit

Edison);

- Experiments in convergence acceleration;

An outline of the second-term course is given in the Appendix. The course is backed up by an elaborate course pack plus the two-volume book by C. Hirsch [9]. Hirsch's opus is still the most authoritative source on CFD. Unfortunately it is more of a reference book than a textbook, although it does include problem sections, and therefore is hard to teach from. The CFD teaching community is still waiting for a true CFD textbook to come along, with the same scope and depth.

## 5 Recurrent questions

A often-heard recommendation, coming from industrial advisors, but also from faculty with strong ties to industry, is: to include in a CFD course some training of the students in the use of commercial flow codes, such as RAMPANT. CFD teachers appear to resist this mild pressure, and for good reasons. Speaking for myself, I find the educational gain of such instruction too small to warrant the sacrifice of several lecture hours. The goal of a university CFD course is to provide *insight* in CFD methods, which includes: enabling the students to analyze unexpected numerical trouble arising when working on a demanding application, and subsequently modify their code so as to remedy such trouble. Commercial codes are designed to *avoid* such a situation. Their source texts are already cluttered with special measures and fixes for special problematic cases, making them obscure and inaccessible - if the source is available at all.

Nevertheless, when used as a computational "laboratory" to enhance a fluids class, a commercial software package can be very useful. Here the emphasis is on understanding the physics, not on developing or validating a numerical method, so devoting some time to instructing the students in the use of the package makes sense.

On the other hand, a package of routines such as CLAWPACK, developed by Randy LeVeque at the University of Washington, is very well suited for use in a CFD class. This package consists of modules with clean, understandable, documented, basic CFD algorithms that can be used as building blocks for codes with a wide variety of applications. The teacher thus has the freedom to

indicate certain pre-programmed routines that may be copied by the student, while other parts of the code must be newly programmed. This lightens the programming and debugging task of the student and allows teacher and students to focus on selected, advanced numerical issues.

Another recurrent request is: to offer CFD in the undergraduate curriculum. This is problematic because even the first CFD course, interpreted as a course in the numerical solution of certain (nonlinear) PDE's, has a laundry list of mathematical subjects as prerequisite knowledge (besides fluid dynamics). Ideally it looks like this:

- Elliptic, parabolic, hyperbolic equations;
- Fourier analysis;
- Numerical interpolation, integration;
- Root finding;
- Eigenvalues, eigenvectors;
- Gaussian elimination
- Numerical integration of ODE's.

Some of these subjects may have been covered in a basic calculus series; the rest of them are found only in an advanced engineering mathematics course or a first numerical analysis course, both of which are often regarded themselves as junior- or senior-year electives. It is possible to treat some of these topics during the CFD course itself, but my experience is that including all of them makes the course too hard to digest for most students, besides taking time away from the teaching of CFD itself. It is therefore more practical to leave CFD in the graduate curriculum; senior students with a strong mathematical background and interest are welcome to take it as an elective.

Undergraduate CFD, as far as I can see, will remain limited to, e. g., the treatment of panel methods in a fluids class, with use of pre-coded or self-coded computer programs.



## 6 Who hires CFD graduates?

Graduates with CFD expertise are hired by industry, CFD consulting firms, government research labs and universities. Since undergraduate CFD teaching is limited and rather elementary, any employer seeking to hire CFD-knowledgeable persons must shop for graduates with at least a master's degree. Universities, of course, require a PhD for their faculty hires.

In contrast to the drop in the supply of CFD students around 1994, the job market for CFD has remained remarkably stable. Aerospace engineers with training or specialization in CFD, however, should not expect to be immediately employed in aerospace engineering. It therefore is crucial to make the CFD curriculum broad, that is, interdisciplinary, even when taught in an aerospace department. This makes the CFD graduate more marketable and potentially more successful in landing a satisfying job.

A number of universities offer an interdisciplinary doctoral program in Scientific Computing. The University of Michigan has one in place, run by the Laboratory for Scientific Computing (LaSC). This program broadens the course and exam requirements of the student's home department, adding requirements in the area of scientific computation in its greatest generality. The student can collect credits in scientific computation in any science or engineering department. This, again, makes the student more marketable.

In response to industrial demands there has been a gradual shift in engineering education, towards producing more masters degrees and reducing doctoral programs; this also affects CFD education. The first CFD course described above is well suited to be part of a masters program. The sequel course is less suited in general, but would fit into a special masters program in scientific computing; such programs I have not yet seen.

## 7 The future of CFD teaching

CFD education has quickly matured over the past two decades. There is little dispute about what should be considered classical material and what not, although the emphasis of each course depends on the favorite flow problems of the department in which it is taught. There is also consensus about what is basic and what advanced material. Thus, the Von Neumann stability analysis belongs in the introductory course, and TVD conditions are detailed in the

sequel course.

As CFD advances, the cutting-edge techniques of today are destined to become routine tomorrow, and they will find their way into the CFD curriculum. This is normal for any discipline. As workstations become more powerful, computer projects can be solved with greater resolution. Pre-programmed subroutines can be the stepping stones toward more efficient, more specialized and more advanced code development. within the limited time-frame of one academic term. Changes in hardware that require special ways of programming and favor special classes of algorithms, will have their influence on the material taught. None of this is surprising or requires uncommon adaptability of the curriculum.

Universities that wish to engage in such activity may include CFD as a subject for distance learning. CFD actually is very well suited for such a learning mode, since it operates in virtual reality anyway. While there may be financial incentives and rewards for those involved in the development of off-site courses, there are also obstacles. The bottom line is that university faculty have little time for such endeavors. The emphasis in the modern research university is on funded research and not on teaching, although administrators are loath to admit this publicly. Tenure, promotion and salary raises depend largely on research accomplishments, with exceptional teaching efforts at most regarded as mitigating circumstances. This policy discourages investment in advanced teaching concepts on the part of the faculty.

Besides for distance learning, there is also a market for short courses. For instance, the Von Kármán Institute for Fluid Dynamics in Rhode-St.Genèse, Belgium, offers one-week short courses throughout the year, including an introductory and an advanced CFD course. These include several speakers, each of whom typically gives three lectures. Such a task is within the reach of a university professor.

At this point it may be clear to the reader that, in my view, CFD as a teaching subject has become much like many other subjects such as thermodynamics or heat transfer, in the sense that it is a stable and indispensable part of the (aerospace) engineering curriculum. The teaching capacity for CFD, though, is not yet saturated: the numbers of students taking CFD courses will keep increasing until CFD learning within the engineering education has become as commonplace as that of fluid dynamics itself.

## Acknowledgement

Ken Powell is thanked for his careful reading of the manuscript, and constructive suggestions.

## References

- [1] J. C. Tannehill, D. A. Anderson, and R. H. Pletcher, *Computational Fluid Mechanics and Heat Transfer*. Taylor & Francis, 1997. second edition.
- [2] R. Courant, K. O. Friedrichs, and H. Lewy, "Ueber die partiellen Differenzgleichungen der mathematischen Physik," *Mathematische Annalen*, vol. 100, p. 32, 1928.
- [3] R. D. Richtmyer and K. W. Morton, *Difference Methods for Initial-Value Problems*. Interscience, 1967.
- [4] S. K. Godunov, "Reminiscences about difference schemes," *Journal of Computational Physics*, 1999. to appear.
- [5] P. D. Lax and B. Wendroff, "Systems of conservation laws," *Communications in Pure and Applied Mathematics*, vol. 13, pp. 217-237, 1960.
- [6] J. P. Boris, "A fluid transport algorithm that works," in *Computing as a Language of Physics*, pp. 171-189, International Atomic Energy Commission, 1971.
- [7] B. van Leer, "Towards the ultimate conservative difference scheme. I. The quest of monotonicity," *Lecture Notes in Physics*, vol. 18, pp. 163-168, 1973.
- [8] M. Y. Hussaini, B. van Leer, and J. H. van Rosendale, eds., *Upwind and High-Resolution Schemes*. Springer, 1997.
- [9] C. Hirsch, *Numerical Computation of Internal and External Flow*. Wiley, 1988-1990. two volumes.

## 8 Appendix

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AE 623 Winter 1998

### Computational Fluid Dynamics II

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**Outline** (Material to be selected from topics below)

#### I General considerations

##### Justification of CFD

- CFD vs. experimental and theoretical fluid dynamics
- Uses of CFD in science and engineering
- Code validation

##### Fluid models for CFD

- Reynolds-averaged Navier-Stokes equations (RANS)
- Sub-grid turbulence modeling
- Large-eddy simulation (LES)
- Computational effort for RANS and LES

##### A Best Buy: the Euler equations

- Differential form
- Characteristic form (1-D)
- Integral form
- Weak solutions
- The entropy condition
- Artificial dissipation vs. shock fitting
- Uses and limitations of the Euler equations

##### Related sets of equations

- The Lagrangean equations (1-D)
- The shallow-water equations
- The equations of magneto-hydrodynamics (MHD)
- The equations of elasticity

## II Numerical treatment of hyperbolic systems

### Basic schemes

- From  $q$ -schemes to  $Q$ -schemes
- Numerical flux functions
- Scalar function applied to a matrix
- Lax-Friedrichs, Courant-Isaacson-Rees, Lax-Wendroff schemes
- Stability
- Monotonicity
- Two-step forms of the Lax-Wendroff scheme (Richtmyer, MacCormack, Lerat)
- Roe's average of the flux Jacobian
- Roe's parameter vector

### Upwind differencing

- Two views of upwind differencing (CIR, Godunov)
- Godunov's method for hyperbolic systems
- Riemann's initial-value problem
- Exact Riemann solver
- Approximate Riemann solvers
- Approximate Riemann solver based on Roe's local linearization

### Fluctuation splitting, also called: flux-difference splitting

- Roe's linear decomposition
- A conservative upwind scheme that doesn't look it
- Representation of steady discontinuities
- Satisfying the entropy condition
- Osher's simple-wave decomposition
- O and P version of Osher's flux
- An odd bird: the random-choice method (Glimm-Chorin-Colella)

### Flux splitting, also called: flux-vector splitting

- The Boltzmann approach
- The Beam Scheme, or: Steger-Warming splitting
- Mathematical approach
- Van Leer's splitting
- Representation of steady discontinuities
- Numerical diffusion through flux splitting

- The rise and fall of flux splitting
- Advection Upstream Splitting Method (AUSM)

### III. One-dimensional discrete analysis

#### Non-oscillatory convection schemes

- Nonlinear schemes for the linear convection equation
- Monotonicity-preserving interpolation
- Non-conservative monotonicity-preserving Lax-Wendroff scheme
- Conservative non-oscillatory schemes: the MUSCL approach
- Projection – non-oscillatory reconstruction – evolution
- Limiters: limiting derivatives during reconstruction
- Van Leer and Sweby diagrams
- Artificial compression
- Piecewise-Parabolic Method (PPM)
- Flux-Corrected Transport (FCT)
- Danger in limiting during evolution (as in FCT)

#### Total-Variation-Diminishing (TVD) schemes

- Harten's sufficient condition for explicit schemes
- Harten's sufficient condition for implicit schemes
- Inadequacy of TVD requirement beyond one dimension
- Essentially Non-Oscillatory (ENO) schemes
- Numerical results from non-oscillatory schemes

### IV Numerical treatment of multi-dimensional flow

#### General considerations for Euler methods

- The Euler equations in curvilinear coordinates
- Conservative differential formulation
- Finite-volume formulation
- Simple multi-dimensional schemes
- Stability considerations
- Back to basics: two-dimensional advection
- Justification of operator splitting
- Second-order accuracy by operator splitting
- Strength and weakness of operator splitting
- Multi-dimensional flux functions
- Genuinely multi-dimensional schemes

**Time-accurate methods**

- The MUSCL approach
- A Best Buy: Hancock's predictor-corrector scheme
- Choice of state variables for reconstruction and predictor
- Time-accurate versus steady-state calculations
- Making Methods More Modular: the method of lines
- Time integration of ODEs
- Multi-step and multi-stage schemes
- TVD conditions on multi-stage schemes
- When to use an implicit method

**Marching to a steady state**

- General strategy
- Explicit versus implicit methods
- Multi-stage schemes
- Spatial discretizations for use with multi-stage marching
- $\kappa$ -family of reconstructions (Van Leer)
- k-exact reconstruction (Barth)
- Central differencing plus explicit stabilizing terms (Jameson-Schmidt-Turkel)
- MUSCL versus central differencing
- Matrix viscosity for use with central differencing
- A modest acceleration trick: local time-stepping
- Beyond local time-stepping:  $\Delta t$  becomes a matrix
- Implicit methods
- Classic relaxation methods
- Alternating-Direction-Implicit (ADI) and Approximate Factorization (AF) methods
- Computer architecture and the performance of relaxation methods

**Convergence acceleration**

- Basic principles and goals of multi-grid relaxation
- Multi-grid components: single-grid relaxation, restriction, prolongation
- Conjugate gradients, GMRES, Vector-Sequence Convergence Acceleration

**More about multi-grid**

- How to converge on all grids simultaneously: forcing terms
- Sawtooth, V, W and F cycle
- Correction Scheme (CS) and Full-Approximation Storage (FAS)

- Full Multi-Grid (FMG) method
- Design of high-frequency smoothers: optimizing multi-stage schemes
- The alignment problem
- Semi-coarsening and sparse-grid relaxation

#### Discrete solutions to the Navier-Stokes equations

- Conservative differential formulation
- Finite-volume form
- Spatial discretization
- Marching to a steady state
- Convergence problems of Navier-Stokes calculations

#### Numerical results for multi-dimensional inviscid and viscous flows

- Time-dependent flows
- Steady flows
- CFD folklore
- CFD bloopers

### V Encounters with boundaries

- Natural and artificial boundaries
- Boundary conditions and boundary procedures
- Boundary conditions for hyperbolic systems
- Radiation conditions
- Review of stability analysis
- Stability in the presence of boundaries
- Normal-mode analysis of Kreiss, Gustafsson and Sundström (KGS)
- Tadmor's theorem for dissipative schemes
- KGS analysis of various boundary procedures

#### Absorbing boundary conditions

- Engquist-Majda analysis
- Multi-dimensional KGS analysis

### VI Grid generation

#### Basics of grid generation

- Desirable features
- Structured-grid topologies: O, C and H grids



- Generation methods: conformal, algebraic, differential
- Generation by solving elliptic equations
- Generation by solving hyperbolic equations

#### Advanced topics in grid generation

- Unstructured grids
- Linked-list data structures
- Advancing-front techniques
- Methods based on Delaunay triangulation
- Solution-adaptive grids
- Tree-like data structures
- The cut-Cartesian adaptive approach

### VII Advanced topics in CFD

- Cell-vertex vs. cell-centered schemes
- Cell-vertex advection schemes
- Genuinely multi-dimensional schemes
- Local preconditioning of the Euler and NS equations
- Computing on vector and parallel processors
- Numerical flux formulas for real gases
- Numerical treatment of chemically reacting flows
- Treatment of stiff source terms
- Simulation of rarefied flow: extended hydrodynamics
- Lattice-gas dynamics
- Computational aero-acoustics
- Computational electro-magnetics (CEM)
- Computational MHD

## Computer Problem #1

*Building your own shock tube*

## Computer Problem #2

*The Fermi-II power plant problem:  
A 2D code for internal flow of steam*

## Problem Set #1

*Analysis and design of advection schemes*

## Problem Set #2

*Everything you always wanted to know about numerical flux functions  
but were afraid to ask*

## Problem Set #3

*Boundary conditions and procedures;  
multi-dimensional methods*

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