A NEW FLEXIBLE BODY DYNAMIC FORMULATION FOR BEAM STRUCTURES UNDERGOING LARGE OVERALL MOTION THE THREE-DIMENSIONAL CASE

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Abstract

A previously developed flexible body dynamic formulation for simulating beam structures undergoing large overall motion, restricted to two-dimensional prescribed motion, is extended to the study of both three-dimensional prescribed motion and non-prescribed motion. The formulation, called the Augmented Imbedded Geometric Constraint (AIGC) approach, is restricted to small elastic deformations of the beam structure. The overall motion is characterized by six degrees of freedom, while the elastic deformation is characterized by the superposition of a number of assumed global shape functions, developed using substructuring techniques. This extension allows the approach to be applied to the study of a broader spectrum of problems involving systems with applied forces/ torques where the overall motion is not known. Thus, the formulation can be used to study the two-way interaction between overall motion and local deformation which is fundamental to a general purpose flexible body dynamic formulation. The AIGC approach describes the elastodynamic behavior with a set of differential equations linearized in terms of the deformation degrees of freedom (but not in the degrees of freedom describing the overall motion), and a number of, possibly nonlinear, constraint equations describing the physical attachments of the ends of the beam. The beam characterization is extended to include the effects of torsion, rotatory inertia, and an offset between the centroid and shear center of the beam (eccentricity). Transverse shear deformation is neglected as it is generally not of importance in flexible beam dynamics. The validity of the extended formulation is demonstrated by comparison of solutions for two validation problems to independently obtained solutions.

Introduction

Interest in flexible body dynamics, the coupling between large overall motion and local deformation in structural dynamics, has increased significantly since the early 1960's. Initially motivated by problems seen in the aerospace industry, the interest has spread to other industries, including robotics and ground transportation. Efforts to develop general purpose analysis tools to address flexible body dynamics can be broken into two general classifications, nonlinear finite element approaches, and rigid body dynamic approaches modified to allow local flexibility. The former strategy is used by Simo and Vu Quoc[1] and Christensen and Lee[2], while the later is the approach adopted by Kane, Ryan and Banerjee[3]. In the work

in reference [3], the authors discussed the need to develop element-specific approaches in order to include interrelationships between the components of local deformation. They developed a beam specific formulation, often referred to as the Imbedded Geometric Constraint (IGC) approach. A similar formulation was developed by Yoo[4], referred to as the Nonlinear Strain Displacement (NSD) approach, to overcome the inability of the IGC approach to accurately solve problems where the lateral deformation of the beam structure is dominated by membrane stiffness. However, the NSD approach does not reliably solve problems where lateral deformation of the beam structure is dominated by bending stiffness. The inability of these two approaches to accurately solve both classes of problems prompted the authors[5] to develop another formulation, which is capable of accurately solving both classes of problems. This new formulation is referred to as the Augmented Imbedded Geometric Constraint (AIGC) Approach.

In this paper, the AIGC approach is extended to the solution of general beam dynamics problems, with the assumption of small elastic local deformation. This is accomplished by allowing for three dimensional motion and deformation, and also allowing the overall motion to be unknown. The removal of the latter restriction allows the AIGC approach to study the two-way interaction between overall motion and local deformation, which is a fundamental issue in flexible body dynamics. As in the two-dimensional prescribed motion work in reference [5], the development presented in this paper involves the dynamics of a single beam. The extension of the AIGC approach presented in this paper closely follows the original IGC development in reference [3], and a subsequent extension[6] of that work to non-prescribed motion. As a result, this paper will concentrate on the major differences between the two approaches, and will leave out many of the details covered in references [3] and [6]. Interested readers can also obtain a detailed presentation in the Ph.D. Dissertation of Haering[7], which covers the work presented in this paper and in reference [5].

As discussed in reference [5], the primary difference between the AIGC and IGC approach is that the differential equations of motion from the IGC approach, referred to as the system differential equations in the AIGC approach, have constraint equations added to enforce the physical boundary conditions for the beam structure. This set of differential equations with algebraic constraints forms the equations of motion for the AIGC approach. In addition, a set of general modal functions is employed in the AIGC approach to ensure the ability to

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satisfy any boundary conditions and prevent inadvertently imposing boundary conditions which are not correct for any given problem.

The plan of the paper is as follows. First, the physical system to be studied is introduced, the basic mechanics of the problem are introduced, and the system differential **equations** are given. Second, a discussion of the constraint equations and choice of modal functions is presented. Third, the equations of motion are presented and the technique for their solution is discussed. Finally, solutions for two verification problems, obtained with the AIGC approach, are presented and compared with independently obtained solutions.

Physical Model and System Differential Equations

The three-dimensional beam model is shown in Figure 1. A flexible beam B is attached to a rigid base A at point **O**. The mass of body A is given by m_A, and the center of mass is located at a point, A^* (not shown in Figure 1). A dextral set of unit vectors, $\vec{a}_1, \vec{a}_2, \vec{a}_3$, is fixed in A, with the $\vec{a}_1$ direction aligned with the undeformed elastic axis of the beam. Following Kane *et al* [3], $\vec{a}_2$ and $\vec{a}_3$ are aligned with the principal area moment of inertia axes of the beam. Prior to deformation, a point on the elastic axis contained within a generic cross section, dB, is located at point C_0, at a distance x measured along the undeformed elastic axis. After deformation, that point on the elastic axis within cross section dB, is located at point C, at a distance of x+s measured along the deformed elastic axis. An additional set of dextral unit vectors, $\vec{b}_1, \vec{b}_2, \vec{b}_3$, are fixed in cross section dB and are aligned with, $\vec{a}_1, \vec{a}_2, \vec{a}_3$, respectively, when the beam is undeformed. The centroid of section dB is defined by point P which is offset from the point C on the elastic axis by the eccentricity vectors given by

$$\vec{e} = e2 \, \vec{b}_2 + e3 \, \vec{b}_3 \tag{1}$$

Allowing general three-dimensional deformation, the position vector from the attachment point, **O**, to the centroid P of the generic section dB is given by

$$\vec{p}^{\,\overrightarrow{OP}} = (x + u_1) \, \vec{a}_1 + u_2 \, \vec{a}_2 + u_3 \, \vec{a}_3 + e_2 \, \vec{b}_2 + e_3 \, \vec{b}_3 \tag{2}$$

where $u_1 \, \vec{a}_1$, $u_2 \, \vec{a}_2$, and $u_3 \, \vec{a}_3$ represent orthogonal components of the beam deformation.

The orientation of section dB relative to body A can be described by three successive rotations of amounts θ_1, θ_2, and 63 about lines parallel to unit vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$, respectively. It is one of **24** possible descriptions, and is chosen to facilitate the introduction of beam torsion, and rotatory inertia. The relative orientation of dB and **A** is described by the following direction cosine matrix

$$^A C^{dB} = \begin{bmatrix} c_2 \, c_3 & s_1 \, s_2 \, c_3 - s3 \, c_1 & c_1 \, s_2 \, c_3 + s3 \, s_1 \\ c_2 \, s_3 & s_1 \, s_2 \, s_3 + c_3 \, c_1 & c_1 \, s_2 \, s_3 - c_3 \, s_1 \\ -s_2 & s_1 \, c_2 & c_1 \, c_2 \end{bmatrix} \tag{3}$$

where

$$^A C_{ij}^{dB} \equiv \vec{a}_i \cdot \vec{b}_j \, , \, (i,j=1,2,3) \tag{4}$$

and

$$s_i \equiv \sin \theta_i \qquad , \qquad c_i \equiv \cos \theta_i \quad , \quad i=1,2,3 \tag{5,6}$$
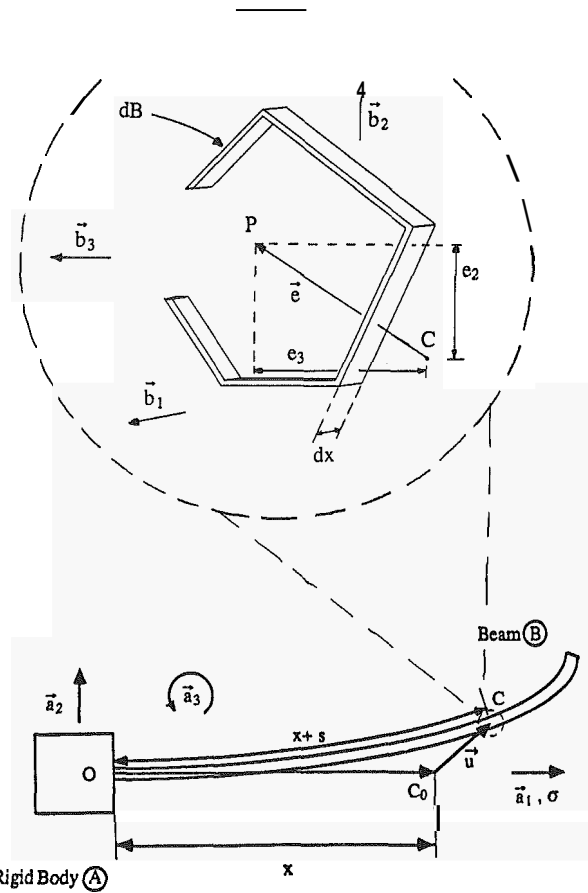

Figure 1 - Three-Dimensional Beam Model

The angular measure θ_1 is introduced to account for twist, and θ_2 and 63 complete the description of the arbitrary orientation of dB with respect to body A. The latter two angular measures allow shear deformation to be introduced (see Kane, *et al* [3] or Haering [7]).

Shear deformation is generally negligible in long beams and it will not be included in this development. With shear deflection neglected, the angles θ_2 and 63 can be related to the spatial derivatives of the deformations u_2 and u_3 as

$$\frac{\partial u_3}{\partial x} = -\theta_2 \qquad \qquad \frac{\partial u_2}{\partial x} = \theta_3 \tag{7,8}$$

Thus the symbols s_i and c_i (i=2,3) are now given by

$$s_2 = -\sin u_3' \qquad\qquad s_3 = \sin u_2' \tag{9,10}$$

$$c_2 = \cos u_3 \qquad\qquad c_3 = \cos u_2 \tag{11,12}$$

where a prime denotes a derivative with respect to x.

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The deformation measures s, u_2, u_3, and θ_1 are represented as follows:

$$s = \sum_{j=1}^{\nu} \phi_{1j}(x)\, q_j(t) \qquad u_2 = \sum_{j=1}^{\nu} \phi_{2j}(x)\, q_j(t) \quad (13,14)$$

$$u_3 = \sum_{j=1}^{\nu} \phi_{3j}(x)\, q_j(t) \qquad \theta_1 = \sum_{j=1}^{\nu} \phi_{4j}(x)\, q_j(t) \quad (15,16)$$

where $\phi_{ij}(x)$ $(i=1,...,4$, $j=1,...,\nu)$ are modal functions, the selection of which will be discussed later. The q_j's, $(q_j$, $j=1,2,...,\nu$) are the first ν generalized coordinates.

The system differential equations are developed using Kane's[8] approach. The contribution to the generalized active force from internal forces acting in the beam, can be obtained from the following strain energy function, V, which takes into account axial stretching, transverse bending, and torsion:

$$V = \frac{1}{2}\left\{ \int_0^L E\,A_0\left[\frac{\partial s}{\partial x}\right]^2 dx + \int_0^L G\,\kappa'\left[\frac{\partial\theta_1}{\partial x}\right]^2 dx \right.$$

$$\left. + \int_0^L E\,I_2\left[\frac{\partial^2 u_3}{\partial x^2}\right]^2 dx + \int_0^L E\,I_3\left[\frac{\partial^2 u_2}{\partial x^2}\right]^2 dx \right\} \quad (17)$$

Where E, A_0, G, κ' ,I_2, and I_3 are the modulus of elasticity, cross sectional area, shear modulus, effective torsional constant, and the area moments of inertia about the $\vec{b}_2$ and $\vec{b}_3$ axes, respectively.

Four measures of the beam deformation have been introduced, s, u_1, u_2, and u_3 ,but due to the nature of beam deformation, only three are independent. The following relationship exists between the four deformation measures (see references [4 or 7] for details):

$$x + s = \int_{J_0}^x \left[\left(1+\frac{\partial u_1}{\partial\sigma}\right)^2 + \left(\frac{\partial u_2}{\partial\sigma}\right)^2 + \left(\frac{\partial u_3}{\partial\sigma}\right)^2\right]^{\frac{1}{2}} d\sigma \quad (18)$$

where σ is a dummy variable of integration. This relationship can be reduced to the following more useful form [see reference [7]).

$$u_1 = \sum_{j=1}^{\nu} \phi_{1j}\,\dot{q}_j - \frac{1}{2}\sum_{j=1}^{\nu}\sum_{k=1}^{\nu}\left(\beta_{22}^x\right)_{jk} q_j q_k$$

$$- \frac{1}{2}\sum_{j=1}^{\nu}\sum_{k=1}^{\nu}\left(\beta_{33}^x\right)_{jk} q_j q_k \quad (19)$$

where $\left(\beta_{km}^x\right)_{ij}$ is defined as

$$\left(\beta_{km}^x\right)_{ij} = \int_0^x \phi'_{ki}\,\phi'_{mj}\,d\sigma \quad , \quad i,j=1,2,...,\nu \ , \ k,m=2,3 \quad (20)$$

Note that $\left(\beta_{km}^x\right)_{ij}$ $(i,j=1,2,...,\nu$, $k,m=2,3)$ are indefinite integrals. For the shape functions that will be introduced latter, they can be determined explicitly.

In this paper, only external forces applied to body A will be considered. It should be noted that reference [6] contains a development including external forces applied to the beam structure itself and gravitational forces, and that development is directly applicable to the AIGC approach.

Forces and torques applied to body **A** can be replaced by an equivalent set, including a force acting through the mass center, $\left(\vec{R}\right)^{A^*}$, and a torque, $\left(\vec{T}\right)^A$ as follows

$$\left(\vec{R}\right)^{A^*} = R_{1A}\,\vec{a}_1 + R_{2A}\,\vec{a}_2 + R_{3A}\,\vec{a}_3 \quad (21)$$

$$\left(\vec{T}\right)^A = T_{1A}\,\vec{a}_1 + T_{2A}\,\vec{a}_2 + T_{3A}\,\vec{a}_3 \quad (22)$$

The discussion above highlights the essential mechanical ingredients of the theory. The derivation of the system differential equations is quite lengthy and details will not be given here. As discussed earlier, the details can be found in references [3] and [6] (transverse shear deformation included), and also in reference [7] (both with and without transverse shear deformation included).

The system differential equations for non-prescribed motion are given by the form

$$\sum_{j=1}^{\nu+6} \widehat{M}_{ij}\,\dot{u}_j^* + \sum_{j=1}^{\nu+6} \widehat{G}_{ij}\,u_j^* + \sum_{j=1}^{\nu+6} \widehat{K}_{ij}\,q_j$$

$$= \widehat{F}_i \quad , \quad i=1,2,...,\nu+6 \quad (23)$$

The fiist ν q_j's, $(q_j$, $j=1,2,...,\nu$)were discussed above. The last six q_j's, $(q_j$, $j=\nu+1,...,\nu+6$)are angular and translational measures used to describe the orientation and location of the rigid base, body A, in the Newtonian reference frame. The first ν generalized speeds $\left(u_j^*$, $j=1,2,...,\nu\right)$ are the time derivatives of the corresponding generalized coordinates. The remaining six generalized speeds $\left(u_j^*$, $j=\nu+1,...,\nu+6\right)$ describe the overall motion of the beam. The $\nu+6$ unknowns $\dot{u}_j^*$, $\left(\dot{u}_j^*$, $j=1,2,...,\nu+6\right)$are the time derivatives of the generalized speeds. The matrices $\widehat{M}, \widehat{G}$, and $\widehat{K}$ contain, in addition to constants (resulting from integrals over the domain of the beam), terms involving the generalized coordinates and/or generalized speeds. The column matrix $\widehat{F}$ contains additional nonlinear terms in the generalized coordinates and generalized speeds. It should be noted that because of the nature of the matrices $\widehat{M}, \widehat{G}, \widehat{K}$, and $\widehat{F}$ the set of differential equations in equation 23 are nonlinear.

Constraint Equations and Modal Function Selection

The use of constraint equations to enforce the beam boundary conditions, and proper selection of the corresponding global shape (modal) functions, are the primary differences between the AIGC approach and the IGC and NSD approaches (references [3,4,and 6]. As demonstrated in reference [5], the use of constraints and proper modal function selection, allows the AIGC approach to accurately solve beam dynamics problems where the lateral deformations of the beam are dominated by either bending or membrane stiffness. This

stems from the ability of the AIGC approach to enforce boundary conditions which cannot be explicitly defined in the deformation measures chosen.

In particular, the IGC approach uses the s, u_2, and u_3 deformation measures and can only ensure satisfaction of boundary conditions explicitly defined in those deformation measures. As a result, the IGC approach fails to accurately solve beam problems where the lateral deformations are dominated by membrane stiffness, which includes a boundary condition explicitly described in terms of the u_1 deformation measure. Conversely, the NSD approach uses the u_1, u_2, and u_3 deformation measures and therefore can only ensure satisfaction of boundary conditions explicitly defined in those deformation measures. Thus, the NSD approach fails to accurately solve some beam problems where the lateral deformations are dominated by bending stiffness, which includes boundary conditions explicitly described in terms of the s deformation measure. The AIGC approach overcomes these limitations by enforcing boundary conditions which are not explicitly defined in terms of the chosen deformation measures through the use of constraint equations. Furthermore, the general nature of the AIGC approach allows the solution of problems where the dominant elastic effects are not known before hand.

The physical boundary conditions for the beam can be related to the deformations at the beam ends; this is accomplished directly with the relationships in equations 13-16, or 19 for deformation conditions, or with spatial derivatives of those relationships for force or moment conditions.

Specifically, consider a beam cantilevered to a rigid mass, as the one shown in Figure 1. The boundary conditions for that beam are those for a cantilever beam with the built-in end located at $x=0$, and the free end at $x=L$, and are given by the following equations.

$$s\big|_{x=0} = 0 \qquad \text{no axial deformation at the built-in end} \qquad (24)$$

$$u_2\big|_{x=0} = 0 \qquad \text{no lateral deformation in the } \vec{a}_2 \text{ direction at the built-in end} \qquad (25)$$

$$u_3\big|_{x=0} = 0 \qquad \text{no lateral deformation in the } \vec{a}_3 \text{ direction at the built-in end} \qquad (26)$$

$$\theta_1\big|_{x=0} = 0 \qquad \text{no twist at the built-in end} \qquad (27)$$

$$\frac{\partial u_2}{\partial x}\bigg|_{x=0} = 0 \qquad \text{no bending slope in the } \vec{a}_3 \text{ direction at the built-in end} \qquad (28)$$

$$\frac{\partial u_3}{\partial x}\bigg|_{x=0} = 0 \qquad \text{no bending slope in the } \vec{a}_2 \text{ direction at the built-in end} \qquad (29)$$

$$\frac{\partial s}{\partial x}\bigg|_{x=L} = 0 \qquad \text{no axial load at the free end} \qquad (30)$$

$$\frac{\partial^3 u_2}{\partial x^3}\bigg|_{x=L} = 0 \qquad \text{no shear force in the } \vec{a}_2 \text{ direction at the free end} \qquad (31)$$

$$\frac{\partial^3 u_3}{\partial x^3}\bigg|_{x=L} = 0 \qquad \text{no shear force in the } \vec{a}_3 \text{ direction at the free end} \qquad (32)$$

$$\frac{\partial \theta_1}{\partial x}\bigg|_{x=L} = 0 \qquad \text{no axial torque at the free end} \qquad (33)$$

$$\frac{\partial^2 u_2}{\partial x^2}\bigg|_{x=L} = 0 \qquad \text{no bending moment in the } \vec{a}_3 \text{ direction at the free end} \qquad (34)$$

$$\frac{\partial^2 u_3}{\partial x^2}\bigg|_{x=L} = 0 \qquad \text{no bending moment in the } \vec{a}_2 \text{ direction at the free end} \qquad (35)$$

Corresponding to these 12 boundary conditions are 12 constraint equations relating the deformation generalized coordinates, $q_j(t)$ ($j=1,...,\nu$)

$$\sum_{j=1}^{\nu} \phi_{kj}(0)\, q_j = 0 \quad , \quad k=1,2,3,4 \qquad (36)$$

$$\sum_{j=1}^{\nu} \phi'_{kj}(0)\, q_j = 0 \quad , \quad k=2,3 \qquad (37)$$

$$\sum_{j=1}^{\nu} \phi'_{kj}(0)\, q_j = 0 \quad , \quad k=1,4 \qquad (38)$$

$$\sum_{j=1}^{\nu} \phi''_{2j}(L)\, q_j = 0 \quad , \quad k=2,3 \qquad (39)$$

$$\sum_{j=1}^{\nu} \phi'''_{kj}(L)\, q_j = 0 \quad , \quad k=2,3 \qquad (40)$$

For simplicity, the 12 constraints expressed in equations 36-40 are represented as

$$\Theta_{1k} = 0 \quad , \quad k=1,2,...,12 \qquad (41)$$

The modal functions, $\phi_{ij}(x)$ ($i=1,...,4$, $j=1,...,\nu$), are developed using the substructuring techniques developed by Craig and Bampton[9] and are discussed in depth in references [4 and 7]. This technique subdivides the modal functions into two subsets called dynamic and static modal functions. For the sake of completeness, that procedure will be briefly discussed here.

The same modal functions are used to describe the lateral deformation measures, u_2, and u_3. On the other hand, the axial stretch, s, and the twisting deformation, θ_1, are described by a different single set of modal functions The same modal functions are used for the axial stretch and twist, because for a simple rod axial and twisting behavior is governed by wave equations of the same basic form.

The lateral dynamic modal functions, the dynamic functions are developed from an eigenanalysis for lateral vibration of a non-rotating beam with boundary conditions of zero displacement and zero slope at both ends. The lateral static modes are obtained by applying unit displacements (or rotations) in the directions held fixed in developing the dynamic modes. While enforcing each unit displacement or rotation, the other displacements or rotations are held fixed. For example, one such shape function satisfies the conditions $\phi(0) = 1$, and $\phi'(0) = \phi(L) = \phi'(L) = 0$.

For the stretch/twist modal functions, the dynamic functions are developed from an eigenanalysis for axial vibration of a non-rotating rod with boundary conditions of zero displacement at both ends. The static modes are obtained by applying unit displacements at each end while holding the other end fixed.

This method of selecting the modal functions serves two purposes. First, the set of modal functions are general enough to satisfy any boundary condition. Second, they do not satisfy any particular end condition, thus they prevent satisfying any boundary condition which is incorrect for the problem at hand.

Equations of Motion and Solution Technique

The final equations of motion are obtained by combining the system differential equations (equation 23) with the constraint equations describing the appropriate boundary conditions (equation 41 for the bending stiffness dominated problem being considered) to form a set of DAEs. Thus, for the case being considered (non-prescribed motion and bending dominated lateral deformations), the equations of motion are

$$\sum_{j=1}^{v+6} \widehat{M}_{ij} \dot{u}_j^* + \sum_{j=1}^{v+6} \widehat{G}_{ij} u_j^* + \sum_{j=1}^{v+6} \widehat{K}_{ij} q_j = \widehat{F}_i \quad , \quad i=1,2,...,v+6$$

$$\Theta_{1k} = 0 \quad , \quad k=1,2,...,12 \tag{42}$$

Reference [7] describes the development of equations of motion for the case of prescribed motion, which for the same boundary conditions yield the following equations of motion:

$$\sum_{j=1}^{v} M_{ij} \ddot{q}_j + \sum_{j=1}^{v} G_{ij} \dot{q}_j + \sum_{j=1}^{v} \left(K_{ij} + K_{ij}^* \right) q_j$$

$$= F_i + F_i^* \quad , \quad i=1,2,...,v$$

$$\Theta_{1k} = 0 \quad , \quad k=1,2,...,12 \tag{43}$$

Note that there are six less system differential equations for the prescribed motion case because the overall motion is assumed to be known. It is also worth noting that the system differential equations are linear (but with time varying coefficients) for the case of prescribed motion.

For the results which are about to be discussed, the equations of motion were solved using Baumgarte's[10] approach. The values of the constraint stabilization parameters a, and β were chosen such that no significant constraint drift was noticed over the range of simulation.

Verification

The validity of the equations of motion just presented will be demonstrated by comparing simulations for two distinct problems. First, a three-dimensional prescribed motion problem for a beam with an offset of the shear center and centroid, commonly referred to as non-compact, will be investigated. This will demonstrate the ability to capture the multi-axial coupling not present in the two-dimensional work in reference[5], and describe the coupling introduced by the offset of the shear center and centroid. Second, a non-prescribed motion problem will be investigated, thus demonstrating the ability of the AIGC approach to capture the two-way coupling between overall motion and local deformation associated with this type of problem.

The three-dimensional prescribed motion problem is illustrated in Figure 2, and the constants characterizing the beam are given in Table 1. Links L_1 and L_2 are assumed rigid (length of 8 m), link L_3 (undeformed length 8 m), possessing a channel section, is characterized as a flexible beam with the AIGC approach. The motion of the joints is characterized by the following descriptions of their respective angles of rotation.

$$\psi_1(t) = \begin{cases} \pi - \dfrac{\pi}{2T}\left[t - \left(\dfrac{T}{2\pi}\right)\sin\left(\dfrac{2\pi t}{T}\right)\right] \text{ (rad), if } 0 \le t \le T \\ \dfrac{\pi}{2} \text{ (rad)} \qquad\qquad\qquad\quad , \text{ if } t > T \end{cases} \tag{44}$$

$$\psi_2(t) = \begin{cases} \pi - \dfrac{3\pi}{4T}\left[t - \left(\dfrac{T}{2\pi}\right)\sin\left(\dfrac{2\pi t}{T}\right)\right] \text{ (rad), if } 0 \le t \le T \\ \dfrac{\pi}{4} \text{ (rad)} \qquad\qquad\qquad\quad , \text{ if } t > T \end{cases} \tag{45}$$

$$\psi_3(t) = \begin{cases} \pi - \dfrac{\pi}{T}\left[t - \left(\dfrac{T}{2\pi}\right)\sin\left(\dfrac{2\pi t}{T}\right)\right] \text{ (rad) , if } 0 \le t \le T \\ 0 \text{ (rad)} \qquad\qquad\qquad\quad , \text{ if } t > T \end{cases} \tag{46}$$
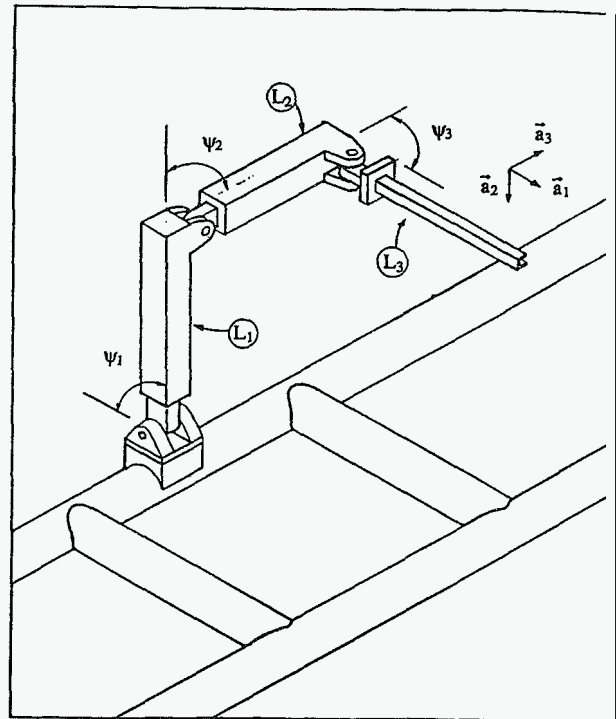
where T = 15 seconds.

Figure 2 - Physical System for the Three-Dimensional Prescribed Motion Problem

This problem, similar to one studied by Kane, *et al* [3], was simulated using the AIGC approach (Equation 43) and a discrete ADAMS[11] model. The results for the axial, twisting, and lateral deformations of the beam tip are given in Figures 3, 4, 5, and 6. The solutions are similar, although some differences (primarily peak magnitudes) exist. **As** no additional independent solutions are available, the differences are

accepted. Thus, this simulation demonstrates the ability of the AIGC approach to accurately solve a three-dimensional problem involving torsion and eccentricity.

Mass per Unit Length	$\rho = 2.02$ kg/m
Area Moments of Inertia	$\begin{cases} I_2 = 4.8746 \times 10^{-9} \text{ m}^4 \\ I_3 = 8.2181 \times 10^{-9} \text{ m}^4 \end{cases}$
Length	$L_3 = 8$ m
Cross Sectional Area	$A_0 = 7.3 \times 10^{-5}$ m^2
Elastic Modulus	$E = 1.0 \times 10^{10}$ N/m^2
Shear Modulus	$G = 5 \times 10^9$ N/m^2
Eccentricity Measures	$\begin{cases} e_2 = 0 \\ e_3 = 0.0185 \text{m} \end{cases}$
Effective Torsional Constant	$\kappa' = 2.446 \times 10^{-11}$ m^4

Table 1 - Flexible Beam Characterization for the Three-Dimensional Prescribed Motion Problem
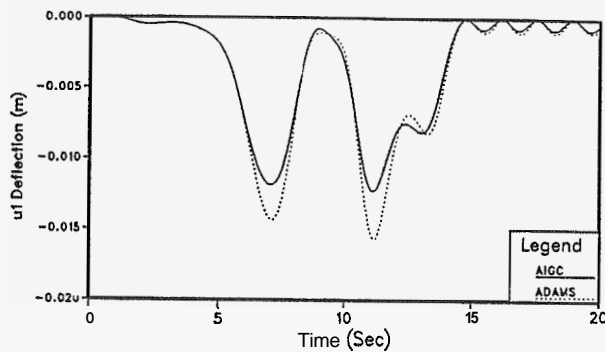
Figure 3 - AIGC and ADAMS Axial Deflection Solutions of the Beam Tip for the Three-Dimensional Prescribed Motion Problem
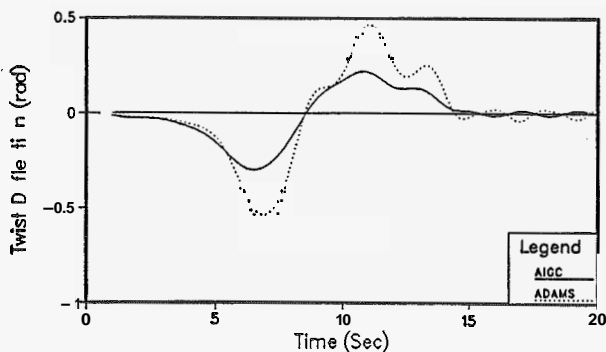
Figure 4 - AIGC and ADAMS Axial Twist Deflection Solutions of the Beam Tip for the Three-Dimensional Prescribed Motion Problem
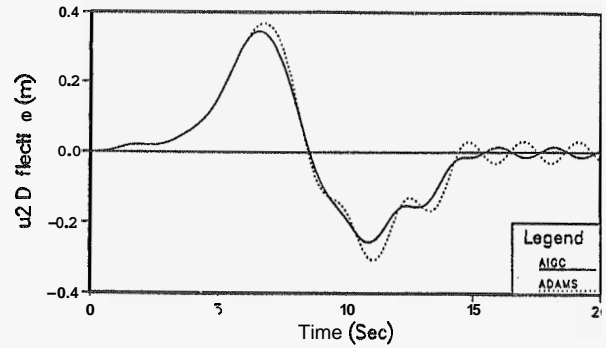

Figure 5 - AIGC and ADAMS Lateral (u_2) Deflection Solutions of the Beam Tip for the Three-Dimensional Prescribed Motion Problem
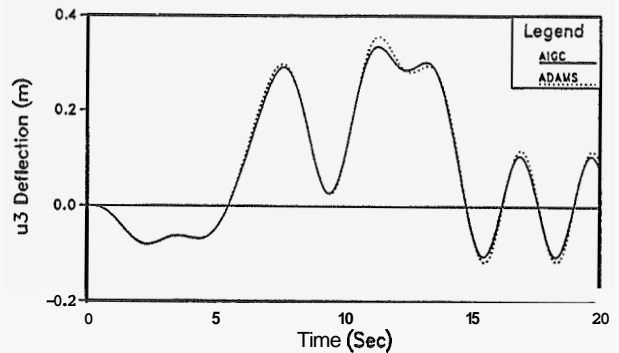

Figure 6 - AIGC and ADAMS Lateral (u_3) Deflection Solutions of the Beam Tip for the Three-Dimensional Prescribed Motion Problem

The ability to accurately describe non-prescribed motion problems is demonstrated by simulating the system shown in Figure 7. This problem was studied by Ryan [6], and is analogous to ones arising for some satellites. The mass and inertia properties of the rigid base, and the characteristics of the beam are given in Table 2. The applied torque (defined in equation 22) is given by the following relationship:

$$T_{3A} = \begin{cases} 0.1 \text{ N m} & , 0 < t \le 5 \text{ (sec)} \\ 0 \text{ N m} & , \text{if } t > 5 \text{ (sec)} \end{cases} \tag{47}$$
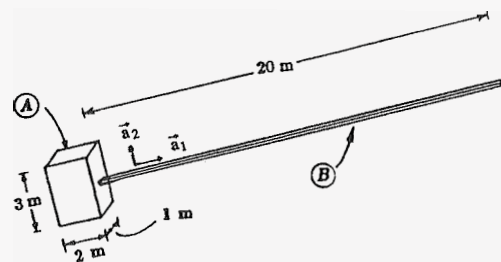

Figure 7 - Physical System for the Non-Prescribed Motion Problem

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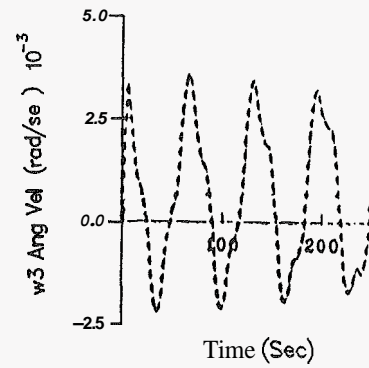|	
Mass of **Rigid** Base	$m_A = 120$ kg
Mass Moments of Inertia of **Base**	$\begin{cases} J_{11} = 100 \text{ kg m}^2 \\ J_{22} = 50 \text{ kg m}^2 \\ J_{33} = 130 \text{ kg m}^2 \\ J_{12} = J_{13} = J_{23} = 0 \end{cases}$
Flexible **Beam Length**	$L = 20$ m
Mass per Unit **Length**	$\rho = 0.2$ kg/m
Elastic **Modulus**	$E = 1.0 \times 10^{10}$ N/m^2
Cross Sectional **Area**	$A_0 = 9.30 \times 10^{-2}$ m^2
Area Moments of Inertia	$I_2 = I_3 = 5 \times 10^{-10}$ m^4
Effective Torsional Constant	$\kappa' = 1.2 \times 10^{-9}$ m^4
Shear Modulus	$G = 5 \times 10^9$ N/m^2
Eccentricity Measures	$e_2 = e_3 = 0$

Table 2 - Rigid Base and Beam Characterization for the Non-Prescribed Motion Problem

The results of Ryan's work (reference[6]) are compared to simulation results from the AIGC approach , generated using equation 42. The $\vec{a}_3$ measure of the angular velocity, is shown in Figures 8 (AIGC) and 9 (Ryan's IGC solution). The local deformation, described by the lateral $(\vec{a}_2)$ measure of the tip deflection, is given in Figures 10 (AIGC) and 11 (Ryan). Excellent agreement is seen between the AIGC and IGC (Ryan's) approaches. Also the effect of the local deformation on the overall motion is clearly seen by comparison of the rigid and flexible beam results for the angular velocity. This second simulation demonstrates the ability of the AIGC approach to accurately solve simultaneously for overall motion and local deformation when forces/torques are applied.
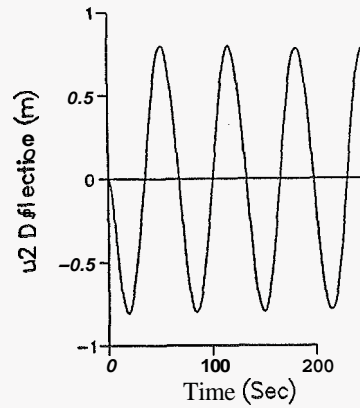

Figure 9 - Ryan's IGC Solution of the Overall Beam Angular Velocity for the Non-Prescribed Motion Problem

Figure 10 - AIGC Solution of the Lateral Beam Tip Deflection for the Non-Prescribed Motion Problem
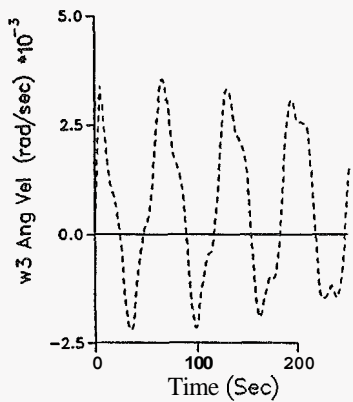

Figure 8 - AIGC Solution of the Overall Beam Angular Velocity for the Non-Prescribed Motion Problem
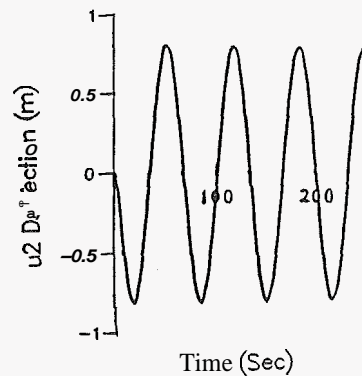

Figure 11 - Ryan's IGC Solution of the Lateral Beam Tip Deflection for the Non-Prescribed Motion Problem

Additional Classes of Problems

All problems that have been addressed in this paper and in reference[5] have beam **boundary** conditions which are time invariant, and are explicitly zero. The AIGC approach could be easily adapted to problems where the above restriction does not apply. Consider the system shown in Figure 12; in this case, neither the bending moment, transverse shear load, nor the axial strain are zero at the right hand end, but are related to the acceleration and angular acceleration (time varying) of the attached mass. The mass at the right hand end has to be treated differently from the one at the left (rigid base), because of the coordinate system employed. The AIGC approach could be extended to such problems by rewriting the constraints used to enforce the boundary conditions.
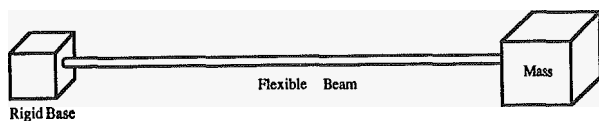

Figure **12** - Non-Constant, Non-Zero Boundary Condition: Beam With Attached Mass

Summary

In this paper, the AIGC approach has been extended to three-dimensional motion and deformation, and the overall motion is no longer restricted to a known function of time. This extended capability has been demonstrated by investigating the following two problems: 1) a three-dimensional prescribed motion problem with torsion and eccentricity, 2) a problem with an applied torque (non-prescribed motion), exhibiting two-way coupling between local deformation and overall motion.

This paper, in combination with reference[5], has shown that the AIGC approach can be applied to general flexible body dynamic problems involving beam structures (restricted to small elastic local deformations). Reference [5] demonstrated the ability of the AIGC approach to solve problems where the lateral deformation of the beam is dominated by either bending or membrane deformation, which no other known approach, using assumed global shape functions, is capable of. The development in reference [5] was limited to two-dimensional problems with known overall motion as a function of time. That restriction has been removed through the development in this paper.

References

1 Simo, J. C., Vu Quoc, L., "A Finite Strain Beam Formulation. The Three-Dimensional Dynamics Problem. Part I,," *Computer Methods in Applied Mechanics and Engineering,* Vol. 49, 1985, pp. 55-70.

2 Christensen, E. R., and Lee, S. W., "Nonlinear Finite Element Modeling of the Dynamics of Unrestrained Flexible Structures," *Computers and Structures,* Vol. 23, 1986, pp. 819-829.

3 Kane, T. R., Ryan, R. R., and Banerjee A. K., "Dynamics of a Cantilever Beam Attached to a Moving Base," *Journal of Guidance, Control, and Dynamics,* Vol. 10, March-April, 1987, pp. 139-151.

4 Yoo, H. H., "Dynamic Modeling of Flexible Bodies in Multibody Systems," Ph.D. Thesis, The University of Michigan, 1989, University Microfilms Inc.

5 Haering, W. J., Ryan, R. R., and Scott, R. A., "A New Flexible Body Dynamic Formulation for Beam Structures Undergoing Large Overall Motion," AIAA 92-1162, 33rd Structures, Structural Dynamics, and Materials Conference, Dallas, TX, April 1992. (submitted for publication in Journal of Guidance, Control, and Dynamics)

6 Ryan, R. R., "Simulation of Actively-Controled Flexible Spacecraft," *Journal of Guidance, Control, and Dynamics,* Vol. 13, July-August, 1990, pp. 691-702.

7 Haering, W. J., "A New Flexible Body Dynamic Formulation for Beam Structures Undergoing Large Overall Motion," Ph.D. Thesis, The University **of** Michigan, May 1992, University Microfilms Inc.

8 Kane, T. R., and Levinson, D. A., *Dynamics, Theory and Applications,* McGraw-Hill Book Co., New York, N.Y., 1983.

9 Craig, R. R., Jr., and Bampton, M. C. C., "Coupling of Substructures for Dynamic Analysis," **AIAA** *Journal,* Vol 3, 1968, pp. 1313-1319.

10 Baumgarte, J. W., "Stabilization of Constraints and Integrals of Motion in Dynamical Systems," *Computational Methods in Applied Mechanics and Engineering,* Vol. 1, 1972, pp. 1-16.

11 ADAMS 6.0 User's Manual, Mechanical Dynamics Inc., Ann Arbor, MI, April 1991.