values of B. This value corresponds to natural convection from an isothermal flat plate embedded in a porous medium as obtained using the Oseen linearization.[5] A similarity (exact) solution to this problem was performed by Cheng and Mikowycz[9] yielding $Nu = 0.888\, Ra_{\kappa}^{1/2}$. In the other extreme, as A becomes very large, the overall heat transfer diminishes for all values of the wall conductivity and therefore the thermal communication between the pipe fluid and the porous material. In all cases, the counter-flow configuration yields a higher overall heat flux than the parallel-flow configuration. This effect, however, weakens as the parameter B decreases, such that for values of $B<0.1$ the overall heat transfer through the pipe is identical for both cases. Note that decreasing B while keeping A constant is equivalent to increasing the flow rate in the pipe.

Conclusions

In this technical Note, a simple yet reliable analysis was presented for the problem of counter-flow and parallel-flow convection in a vertical pipe surrounded by a porous material. Important results revealed interesting features of the temperature distribution of the pipe outer surface, of the mean fluid temperature in the pipe, and of the overall heat flux from the pipe to the surroundings. As the values of parameters A and B approach zero, the outer pipe surface approached an isothermal condition. A maximum was observed in the θ_0 distribution in the parallel-flow case. This maximum is more pronounced and occurs closer to the pipe inlet for larger values of B.

The overall heat flux through the pipe reaches a plateau as A decreases. This plateau corresponds to natural convection from an isothermal vertical wall embedded in a porous medium. The counter-flow configuration yields higher overall heat transfer than for the parallel-flow configuration. This feature diminishes as the pipe flow rate is increased (or the parameter B is decreased).

Acknowledgments

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Entropy Production in Boundary Layers

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Introduction

THE interpretation of the contemporary problems of thermomechanics in terms of entropy production is lately receiving increased attention. Because of its size, no attempt will be made here to survey the literature (see, for example, Bejan[1,2] for applications involving heat transfer and Arpaci[3,4] and Arpaci and Selamet[5,6] for applications involving radiation and flames). The following brief review on the local entropy production is for later convenience.

The development of the entropy production in moving media requires the consideration of the momentum, energy, and entropy balances. The fundamental difference,

$$\text{Total energy} - (\text{Momentum})\mathrm{v}_i - (\text{Entropy})T \tag{1}$$

may be rearranged to yield

$$\rho\left(\frac{Du}{Dt} - T\frac{Ds}{Dt} + p\frac{D\mathrm{v}}{Dt}\right) = -\left(\frac{q_i}{T}\right)\frac{\partial T}{\partial x_i} + \tau_{ij}s_{ij} + u''' - Ts''' \tag{2}$$

where s_{ij} is the rate of deformation. For a reversible process, all forms of dissipation vanish, and

$$\left(\frac{Du}{Dt} - T\frac{Ds}{Dt} + p\frac{D\mathrm{v}}{Dt}\right) = 0 \tag{3}$$

which is the Gibbs Thermodynamic relation. For an irreversible process, Eq. (3) continues to hold provided the process can be assumed in local equilibrium. Then, the local entropy production is found to be

$$s''' = \frac{1}{T}\left[-\left(\frac{q_i}{T}\right)\left(\frac{\partial T}{\partial x_i}\right) + \tau_{ij}s_{ij} + u'''\right] \tag{4}$$

where the first term in brackets denotes the dissipation of thermal energy into entropy (lost heat), the second term denotes the dissipation of mechanical energy into heat (lost work), and the third term denotes the dissipation of any (except thermomechanical) energy into heat. When radiation is appreciable, q_i denotes the total flux involving the sum of the conductive flux and the radiative flux

$$q_i = q_i^K + q_i^R \tag{5}$$

Neglecting contribution of viscous dissipation and assuming conductive and radiative heat fluxes to be in the transversal direction, Eq. (4) may be rearranged as

$$s''' = -\frac{1}{T^2}(q_y^K + q_y^R)\left(\frac{\partial T}{\partial y}\right) \tag{6}$$

Foregoing general considerations are applied below to a forced convection boundary layer.

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Radiation Affected Forced Convection

Consider the effect of radiation on the forced convection boundary layer over a horizontal flat plate. For heat transfer studies, rather than velocity profiles, a good approximation of these profiles near boundaries is convenient. This approach, in the absence of radiation, is well known and has been studied extensively (see Curle[7] for an early reference and Arpaci and Larsen[8] for a later reference). Also, the extension of the approach to the limiting cases of $Pr \ll 1$ and $Pr \gg 1$ are discussed in Arpaci and Larsen.[8] Since the case of $Pr \ll 1$ is for opaque fluids and has no application to radiation-affected problems and the case of $Pr \gg 1$ is known to approximate for all fluids with $Pr \gg 1$, here only the latter case is considered.

Replacing the longitudinal velocity by its tangent on the wall and using this velocity in the conservation of mass to determine the transversal velocity and including the radiation effect, the thermal energy balance gives

$$\rho\, c_p\left[y\left(\frac{\tau_w}{\mu}\right)\frac{\partial T}{\partial x}-\frac{1}{2}y^2\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\tau_w}{\mu}\right)\frac{\partial T}{\partial y}\right]=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_y^R}{\partial y} \tag{7}$$

subject to (Lord and Arpaci[9])

$$\frac{\partial q_y^R}{\partial y}=4\,\kappa_p\left[(E_b-E_{b\infty})-\frac{\epsilon_w}{2}(E_{bw}-E_{b\infty})E_2(\tau)\right] \tag{8}$$

where τ_w denotes the wall shear stress, κ_p the Planck mean absorption coefficient, E_b the emissive power, ϵ_w the wall emissivity, E_2 the second exponential integral, and τ the optical thickness. The boundary conditions to be satisfied are

$$T(0,y)=T_\infty,\qquad T(x,0)=T_w,\qquad T(x,\infty)=T_\infty \tag{9}$$

A similarity variable including both conduction and radiation is not feasible because of intrinsic lack of similarity between conduction and radiation. However, the effect of thin gas radiation on conduction is small. This fact suggests the use of the similarity variable for conduction by which the radiation effect can be treated locally similar.

Introducing $\eta = y/g(x)$ (see, for example, Arpaci and Larsen[8]), into Eq. (7) leads to the equation satisfied by $g(x)$,

$$\left(\frac{\tau_w}{\mu}\right)\frac{\mathrm{d}g^3}{\mathrm{d}x}+\frac{3}{2}g^3\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\tau_w}{\mu}\right)=\alpha$$

which readily gives

$$g(x)=\frac{\left[\alpha\int_0^x(\tau_w/\mu)^{1/2}\mathrm{d}x\right]^{1/3}}{(\tau_w/\mu)^{1/2}}$$

$$\text{and }\eta=\frac{(\tau_w/\mu)^{1/2}y}{\left[\alpha\int_0^x(\tau_w/\mu)^{1/2}\mathrm{d}x\right]^{1/3}} \tag{10}$$

In terms of Eqs. (10) and the approximation $E_2 \simeq \exp(-\sqrt{3}\tau)$, Eqs. (7) and (8) are combined to

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2}+\frac{1}{3}\eta^2\frac{\mathrm{d}\theta}{\mathrm{d}\eta}=\chi P\gamma^2x\left(\Theta^4-\frac{\epsilon_w}{2}e^{-\gamma x^{1/2}\eta}\right) \tag{11}$$

subject to $\theta(0)=1$ and $\theta(\infty)=0$. Here, $\chi=(\kappa_p/\kappa_R)^{1/2}$ is the weighted nongrayness, κ_R the Rosseland mean absorption coefficient and

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\qquad \Theta^4=\frac{T^4-T_\infty^4}{T_w^4-T_\infty^4},\qquad \gamma=\sqrt{3}\kappa_M G \tag{12a}$$

$$g=Gx^{1/2},\qquad G=\left[\frac{4\alpha/3}{0.332\,U_\infty(U_\infty/\nu)^{1/2}}\right]^{1/3} \tag{12b}$$

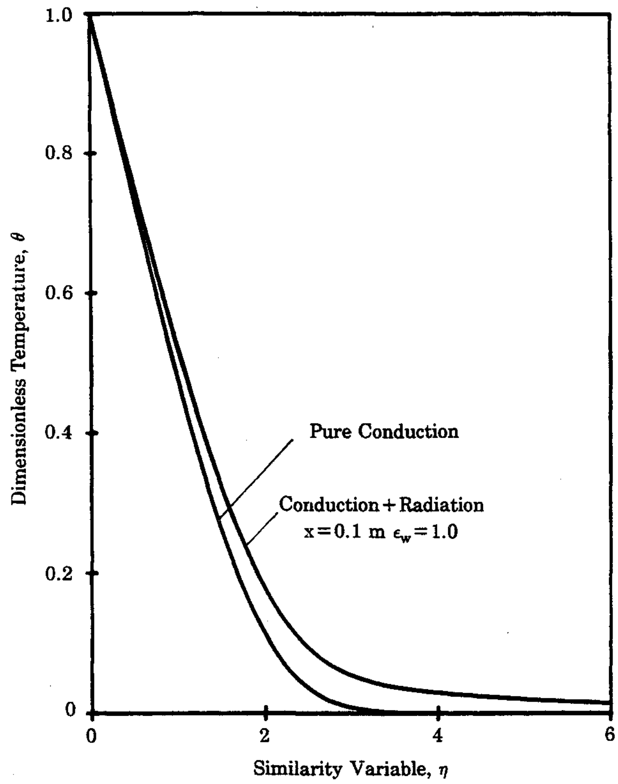

Fig. 1 Dimensionless temperature versus similarity variable.

$$P=\frac{4}{3}\frac{\sigma(T_w^4-T_\infty^4)}{k(T_w-T_\infty)\kappa_M}\simeq\frac{\text{Emission}}{\text{Conduction over }\kappa_M^{-1}} \tag{12c}$$

$\kappa_M=(\kappa_P\kappa_R)^{1/2}$ being the mean absorption coefficient. As $P\to 0$, the effect of radiation diminishes, and Eq. (11) reduces to the case of pure conduction, as expected.

Equation (11) was solved as a boundary-value problem by using the finite difference code PASVA3 developed by Lentini and Pereyra[10] and, employing the wall gradient of temperature obtained from PASVA3, as an initial-value problem by using the single step code DVERK based on a fifth- and sixth-order Runge Kutta-Verner approximation developed by Hull et al.[11] The results obtained separately from PASVA3 and DVERK are found to agree to five decimals. Figure 1 shows the variation of θ against η for pure conduction, which can be obtained by letting the right side of Eq. (11) equal to zero, and the combination of conduction and radiation as expressed by Eq. (11). The present study utilizes air properties at the film temperature and assumes $U_\infty = 2m/s$.

In terms of η and θ, the conductive constitution becomes

$$q_y^K=-\frac{k}{g}\frac{\mathrm{d}\theta}{\mathrm{d}\eta}(T_w-T_\infty), \tag{13}$$

where η and g are defined by Eqs. (10) and (12), respectively. Inserting T, the thin gas radiative heat flux, and the conductive heat flux expressed by Eq. (13) into Eq. (6), the volumetric local entropy production is found as

$$s_\eta'''=\frac{\left(-\dfrac{\mathrm{d}\theta}{\mathrm{d}\eta}\right)}{g\left(\theta+\dfrac{T_\infty}{T_w-T_\infty}\right)^2}\times\left[-\frac{k}{g}\frac{\mathrm{d}\theta}{\mathrm{d}\eta}+\epsilon_w\sigma(T_w+T_\infty)(T_w^2+T_\infty^2)\right] \tag{14}$$

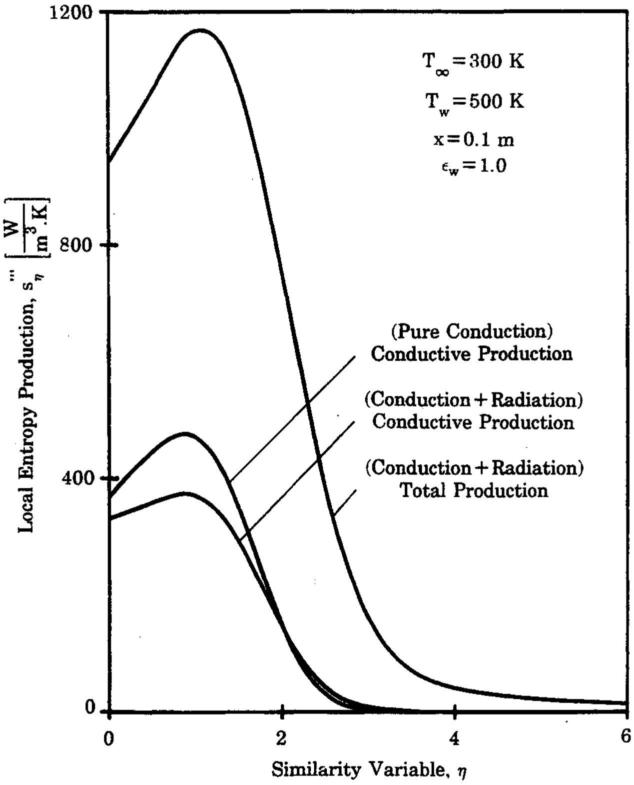

Fig. 2 Rate of local entropy production versus similarity variable.

For illustrative purposes, assuming a wall temperature of $T_w = 500$ K, Fig. 2 depicts the variation of s'''_η against η for pure conduction, conductive, and total (conductive + radiative) components in combined conduction and radiation problems.

Heat Transfer

The total heat flux on boundaries,

$$q_w = q_w^C + q_w^R \tag{15}$$

q_w^C being available from a usual boundary approach and q_w^R being the spectral average of the monochromatic wall heat flux to be evaluated next. From Özişik,[12] Siegel and Howell,[13] or Sparrow and Cess,[14]

$$q_w^R = \epsilon_w\left[E_{bw} - 2\int_0^\infty E_b\, E_2(\tau')\, d\tau'\right] \tag{16}$$

Split the interval into two domains: $[0, \tau_\Delta]$ and $[\tau_\Delta, \infty]$, τ_Δ denoting the thickness of the conduction boundary layer. Then the integration of Eq. (16) yields

$$q_w^R \simeq -2\epsilon_w \int_0^{\tau_\Delta} \frac{dE_b}{d\tau'} E_3(\tau')\, d\tau' \tag{17}$$

Assume a third-order polynomial in τ for E_b satisfying the apparent conditions,

$$E_b(0) = E_{bw}, \quad E_b(\tau_\Delta) \simeq E_{b\infty} \quad \text{and} \quad dE_b(\tau_\Delta)/d\tau \simeq 0 \tag{18}$$

and the limit of weak radiation,

$$d^2E_b(0)/d\tau^2 \to 0 \tag{19}$$

yields

$$\frac{E_b - E_{bw}}{E_{bw} - E_{b\infty}} = \frac{1}{2}\left(\frac{\tau}{\tau_\Delta}\right)^3 - \frac{3}{2}\left(\frac{\tau}{\tau_\Delta}\right) \tag{20}$$

In terms of Eq. (20), the wall heat flux from Eq. (17) yields

$$q_w^R = \epsilon_w(E_{bw} - E_{b\infty})\left(1 - \frac{3}{4}\tau_\Delta\right) \tag{21}$$

This relation apparently excludes the effect of conduction. To include this effect, reconsider the conditions given by Eq. (18), and, in place of Eq. (19), now utilize the wall balance of the thermal energy

$$k\left.\frac{d^2T}{dy^2}\right|_w = \left.\frac{dq_y^R}{dy}\right|_w \tag{22}$$

which in terms of Eq. (8) may be rearranged to give

$$k\left.\frac{d^2T}{dy^2}\right|_w = 4\kappa_P\left(1 - \frac{\epsilon_w}{2}\right)(E_{bw} - E_{b\infty}) \tag{23}$$

Also, from the (linearized) Stefan-Boltzmann law

$$\frac{d^2E_b}{dy^2} = 4\sigma T_M^3 \frac{d^2T}{dy^2} \tag{24}$$

where $T_M = [(\epsilon_w T_w^4 + T_\infty^4)/(\epsilon_w + 1)]^{1/4}$. The elimination of thermal curvature between Eqs. (23) and (24) gives

$$\left.\frac{d^2E_b}{d\tau^2}\right|_w = 12\chi\mathcal{P}\left(1 - \frac{\epsilon_w}{2}\right)(E_{bw} - E_{b\infty}) \tag{25}$$

where $\mathcal{P} = 4\sigma T_M^4/3kT_M\kappa_M$. Then, the polynomial approximation subject to Eqs. (18) and (25) yields

$$\frac{E_b - E_{bw}}{E_{bw} - E_{b\infty}} = \frac{1}{2}\left[-\left(3 + \frac{1}{2}\mathcal{P}_0\right)\frac{\tau}{\tau_\Delta} + \mathcal{P}_0\left(\frac{\tau}{\tau_\Delta}\right)^2 + \left(1 - \frac{1}{2}\mathcal{P}_0\right)\left(\frac{\tau}{\tau_\Delta}\right)^3\right] \tag{26}$$

where $\mathcal{P}_0 = 12\chi\mathcal{P}(1 - \epsilon_w/2)\,\tau_\Delta^2$. In terms of Eq. (26), Eq. (17) results in

$$q_w^R = \epsilon_w(E_{bw} - E_{b\infty})\left\{1 - \tau_\Delta\left[\frac{3}{4} - \left(1 - \frac{\epsilon_w}{2}\right)\tau_\Delta^2\chi\mathcal{P}\right]\right\} \tag{27}$$

which shows the explicit effect of conduction on the radiative heat flux. However, for the thin gas radiation, $\tau_\Delta\mathcal{P} \sim 1$, $\tau_\Delta \ll 1$, and, to first order, the explicit effect of conduction on the radiation flux is negligible, and Eq. (27) reduces to Eq. (21), which is the upper limit of the radiative flux obtained from strict radiative considerations. Now, in terms of this flux, the total heat transfer becomes

$$q_w = -k\left.\frac{\partial T}{\partial y}\right|_w + \epsilon_w(E_{bw} - E_{b\infty})\left(1 - \frac{3}{4}\tau_\Delta\right) \tag{28}$$

where, after neglecting the effect of thin gas radiation on the thermal boundary layer, $\tau_\Delta = \kappa_M\Delta = \kappa_M\delta/Pr^{1/3}$. From approximate studies on viscous boundary layers, $\delta \simeq 5.0x/Re_x^{1/2}$, and $\tau_\Delta = 5.0\tau_x/Re_x^{1/2}\,Pr^{1/3}$. Also, from thermal boundary-layer studies,

$$Nu_x = 0.629(-d\theta/d\eta|_w)Re_x^{1/2}Pr^{1/3} \tag{29}$$

which, for the pure conduction case

$$(-d\theta/d\eta|_w)^K = 0.538$$

gives

$$Nu_x^K = 0.339\, Re_x^{1/2}\, Pr^{1/3} \quad \text{and} \quad \tau_\Delta \simeq \frac{5}{3}\tau_x/Nu_x^K \tag{30}$$

Thus

$$\frac{Nu_x}{Nu_x^K} = \frac{(-\mathrm{d}\theta/\mathrm{d}y|_w)}{(-k\theta/\mathrm{d}y|_w)^K} + \frac{3}{4}\epsilon_w P\left(\frac{\tau_x}{Nu_x^K}\right)\left(1 - \frac{5}{4}\frac{\tau_x}{Nu_x^K}\right)$$

and the local thermal entropy production on the wall is

$$s_x''' = -\frac{1}{T_w^2}(q_w^K + q_w^R)\left(\frac{\partial T}{\partial y}\right)\bigg|_w \tag{31}$$

Introducing a wall local entropy production number, $\Pi_x = s_x''' x^2/k$, Eq. (31) may be arranged as

$$\Pi_x = \left(1 - \frac{T_\infty}{T_w}\right)^2\left(1 + \frac{q_w^R}{q_w^K}\right)\left[\frac{(\partial T/\partial y)|_w}{(T_w - T_\infty)/x}\right]^2 \tag{32}$$

With the definition of local Nusselt number

$$Nu_x = \frac{q_x^C}{q_w^K} = \frac{q_w^K}{q_x^K} = \frac{(\partial T/\partial y)|_w}{(T_w - T_\infty)/x} \tag{33}$$

Eq. (32) may finally be expressed as

$$\Pi_x = \left(1 - \frac{T_\infty}{T_w}\right)^2\left(1 + \frac{q_w^R}{q_w^K}\right)Nu_x^2 \tag{34}$$

Concluding Remarks

The radiation-affected forced convection over a flat plate is investigated in terms of thin gas. The distribution of entropy production within and outside the radiation-affected thermal boundary layer is evaluated. The retained nonlinearity of temperature in the entropy production leads to an extremum in this production within the boundary layer rather than on the boundary.

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Attenuating Thin Gas

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Introduction

A DERIVATION of the monochromatic intensity balance (transfer equation) under the influence of emission, absorption, and scattering is available in the literature.[1,2] The following brief review is for later convenience.

The monochromatic transfer equation integrated over the frequency domain gives

$$l_j\frac{\partial I}{\partial x_j} = \kappa I_o + \frac{\sigma_s}{4\pi}\int_{\Omega'} P(l_i', l_i) I(l_i')\,\mathrm{d}\Omega' - \beta I \tag{1}$$

where I is the intensity, I_o its equilibrium state, κ the absorption coefficient, σ_s the scattering coefficient, $\beta = \kappa + \sigma_s$ the extinction coefficient, Ω the solid angle, and $P(l_i', l_i)$ the phase function that satisfies

$$\frac{1}{4\pi}\int_{\Omega'} P(l_i', l_i)\,\mathrm{d}\Omega' = 1 \tag{2}$$

l_i being the direction of the optical energy balance and l_i' the direction of the scattering.

The first specular moment of Eq. (1) yields the radiative energy balance

$$\frac{\partial q_j^R}{\partial x_j} = 4\kappa E_b + \frac{\sigma_s}{4\pi}\int_\Omega\int_{\Omega'} P(l_i', l_i) I\,\mathrm{d}\Omega'\,\mathrm{d}\Omega - \beta J \tag{3}$$

where $q_i^R = \int_\Omega I l_i\,\mathrm{d}\Omega$ is the radiative heat flux in the x_i direction, $E_b = \pi I_o$ the equilibrium blackbody emissive power, and $J = \int_\Omega I\,\mathrm{d}\Omega$ the specular integrated intensity. In view of Eq. (2) and

$$\int_\Omega\int_{\Omega'} P(l_i', l_i) I\,\mathrm{d}\Omega'\mathrm{d}\Omega = \int_{\Omega'}\left[\int_\Omega P(l_i', l_i) I\,\mathrm{d}\Omega\right]\mathrm{d}\Omega' = 4\pi J \tag{4}$$

Eq. (3) may be rearranged as

$$\frac{\partial q_j^R}{\partial x_j} = \kappa(4E_b - J) \tag{5}$$

The second specular moment of the transfer equation leads to the radiative momentum balance

$$\frac{\partial \Pi_{ij}}{\partial x_j} = \frac{\sigma_s}{4\pi}\int_\Omega\int_{\Omega'} P(l_j', l_j) I l_i\,\mathrm{d}\Omega'\,\mathrm{d}\Omega - \beta q_i^R \tag{6}$$

where Π_{ij} is related to the radiative stress τ_{ij}^R by

$$\tau_{ij}^R = \frac{1}{c}\int_\Omega I l_i l_j\,\mathrm{d}\Omega = \frac{1}{c}\Pi_{ij} \tag{7}$$

c being the velocity of light.

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