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Higher derivative effects on η/s at finite chemical potential

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We review the recent developments on the study of KSS bound violation in the presence of higher derivative corrections. Special emphasis is given to the cases with finite R-charge chemical potential.

1 Introduction

AdS/CFT correspondence is one of the most successful examples of strong-weak type duality in particle physics. Given the validity of the correspondence, it provides a powerful tool to study strongly coupled gauge theory in the perspective of weakly coupled gravitational theory. Kovtun, Son and Starinets (KSS), using a weakly coupled gravity dual description, showed that the shear viscosity to entropy ratio for strongly coupled quark-gluon plasma (sQGP) takes a universal value

$$\frac{\eta}{s} = \frac{1}{4\pi} , \tag{1}$$

which is pretty close to the experimental data. Also, based on the universality of the result and the viscosity to entropy ratio for known materials in nature, KSS proposed a lower bound of the ratio:

$$\eta/s \geq \frac{1}{4\pi} . \tag{2}$$

In this contribution, we review the recent progress on the study of the violation of KSS bound due to (the supersymmetrization of) curvature squared corrections in the gravity side, which can be understood as $1/N$ corrections in the gauge theory side.

2 KSS bound and its violation

In the rest of this contribution, we consider $\mathcal{N} = 1$ superconformal field theory in the gauge theory side for concreteness. The gravity dual description is given by type IIB string theory on $\mathrm{AdS}_5 \times X_5$ background, where X_5 is a five-dimensional Sasaki-Einstein manifold. By the dimensional reduction on X_5 and taking the low-energy limit, we obtain a five-dimensional gauged supergravity with eight supercharges.

Physical quantities associated with sQGP at temperature T can be extracted from weakly coupled gravity calculations on AdS_5-Schwarzschild black hole background at Hawking temperature T. The shear viscosity η can be derived using Kubo formula:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega i} \left[G^A_{xy,xy}(\omega, 0) - G^R_{xy,xy}(\omega, 0) \right] , \tag{3}$$

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where $G^A_{xy,xy}(\omega,\mathbf{k})$ and $G^R_{xy,xy}(\omega,\mathbf{k})$ are the advanced and retarded two-point correlation functions for stress tensor T_{xy} in momentum space, respectively. Note that these two-point functions are Lorentzian quantity, so Euclidean AdS/CFT cannot be used in the calculations. There has been no known rigorous formulation of AdS/CFT correspondence in Lorentzian signature, but Son and Starinets proposed in [2] that the retarded two-point function for an operator $\mathcal{O}(k)$, which is dual to a boundary field $\phi(k,z)$, be obtained by

$$G^R(\omega,\mathbf{k}) = -2\mathcal{F}(k)|_{\partial\mathrm{AdS}_5}, \quad I[\phi_0] = \int \frac{d^4k}{(2\pi)^4}\phi_0(-k)\mathcal{F}(k)\phi_0(k)|_{\partial\mathrm{AdS}_5}, \tag{4}$$

where $\phi_0(k) = \phi(k,z)|_{\partial\mathrm{AdS}_5}$, $I[\phi_0]$ is the action evaluated on-shell and $\phi(k,z)$ satisfies incoming wave boundary condition on the horizon.

The entropy density s can be calculated by using the Bekenstein-Hawking formula:

$$s = \frac{a_5}{4G_5}, \tag{5}$$

where a_5 is the area of the black hole horizon. By taking the ratio between these two quantities, KSS obtained the result (1) at two-derivative order approximation in supergravity. Now, we are interested in leading order higher derivative corrections to this result.

Since the configuration we are interested in is purely gravitational in five-dimensional perspective, the leading order corrections to the gravity side consist of the curvature squared terms. The general form of those terms is given by

$$\mathcal{L}^2_R = \alpha_1 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^2 \tag{6}$$

The second and third terms can be eliminated by redefinition of the metric, so we expect that the physical quantities such as η/s depends only on the value of α_1. This coefficient α_1 can be related to the central charges in CFT via trace anomaly [3–5] as

$$\alpha_1 = \frac{1}{16}\frac{c-a}{a} \tag{7}$$

a and c coincide at leading order for large N with $a = c = \mathcal{O}(N^2)$, but differ for finite N. So, this type of correction can be regarded as $1/N$ corrections.

The authors of [6, 7] cosidered the corrections to η/s from these curvature squared terms and obtained

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{c-a}{a}\right], \tag{8}$$

Whether the KSS bound is violated or not depends on the sign of $c-a$. However, for any known fully-interacting superconformal field theory, $c-a$ is positive at leading order in $1/N$ expansion [8]. So, we conclude that the violation of KSS bound is a general phenomenon in the presence of curvature squared corrections.

3 Chemical potential for the R-charge

The introduction of chemical potential in the presence of the $1/N$ corrections is an interesting subject to consider in both phenomenological and theoretical perspectives. In real experiments of sQGP, the chemical potential for baryon number is not zero but finite. Therefore, the corrections from finite chemical potential make the calculation of η/s more accurate. Also, at leading order in large N, the chemical potential for R-charge is known not to change the η/s [9–11] and it would be interesting to discuss if that conclusion is altered in the presence of higher derivative corrections.

We need two more ingredients to compute the shear viscosity to entropy ratio with finite chemical potential and higher derivatives:

1. Gravity dual description for the R-charge chemical potential. That is identified as the difference of zeroth component of graviphoton between the horizon and the boundary [12, 13]. Since the graviphoton should be sourced by a black hole in the bulk, we consider an AdS R-charged black hole as a gravity background.
2. Four derivative terms for R-charge gauge field. They can be obtained by supersymmetrizing an R^2-term. The explicit form of the terms is obtained in [14, 15].

Hence, we know all the ingredients for calculating η/s. After some lengthy calculations, we obtain a remarkably simple result [16, 17]:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{c-a}{a}(1+q)\right], \tag{9}$$

where q is the R-charge of the black hole and defined to be positive. This result implies that the introduction of R-charge chemical potential leads to an even larger violation of the KSS bound.

4 Conclusion

We reviewed the recent developments on the study of KSS bound in the presence of higher derivative terms. The curvature squared corrections generally violate the KSS bound and the violation is enhanced by finite chemical potential for the R-charge.

One interesting future direction would be to consider the chemical potential for baryon number. Also, it would be interesting to clarify whether or not $c - a > 0$ always holds for any $\mathcal{N} = 1$ SCFT and, if it doesn't hold, when $c - a$ can be zero or negative.

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References

[1] P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005) [arXiv:hep-th/0405231].
[2] D. T. Son and A. O. Starinets, J. High Energy Phys. **0209**, 042 (2002) [arXiv:hep-th/0205051].
[3] S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. A **15**, 413 (2000) [arXiv:hep-th/9903033].
[4] M. Blau, K. S. Narain, and E. Gava, J. High Energy Phys. **9909**, 018 (1999) [arXiv:hep-th/9904179].
[5] M. Fukuma, S. Matsuura, and T. Sakai, Prog. Theor. Phys. **105**, 1017 (2001) [arXiv:hep-th/0103187].
[6] Y. Kats and P. Petrov, J. High Energy Phys. **0901**, 044 (2009) [arXiv:0712.0743 [hep-th]].
[7] M. Brigante, H. Liu, R. C. Myers, S. Shenker, and S. Yaida, Phys. Rev. D **77**, 126006 (2008) [arXiv:0712.0805 [hep-th]].
[8] A. Buchel, R. C. Myers, and A. Sinha, J. High Energy Phys. **0903**, 084 (2009) [arXiv:0812.2521 [hep-th]].
[9] D. T. Son and A. O. Starinets, J. High Energy Phys. **0603**, 052 (2006) [arXiv:hep-th/0601157].
[10] J. Mas, J. High Energy Phys. **0603**, 016 (2006) [arXiv:hep-th/0601144].
[11] K. Maeda, M. Natsuume, and T. Okamura, Phys. Rev. D **73**, 066013 (2006) [arXiv:hep-th/0602010].
[12] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, Phys. Rev. D **60**, 064018 (1999) [arXiv:hep-th/9902170].
[13] M. Cvetic and S. S. Gubser, J. High Energy Phys. **9904**, 024 (1999) [arXiv:hep-th/9902195].
[14] K. Hanaki, K. Ohashi, and Y. Tachikawa, Prog. Theor. Phys. **117**, 533 (2007) [arXiv:hep-th/0611329].
[15] S. Cremonini, K. Hanaki, J. T. Liu, and P. Szepietowski, J. High Energy Phys. **0912**, 045 (2009) [arXiv:0812.3572 [hep-th]].
[16] S. Cremonini, K. Hanaki, J. T. Liu, and P. Szepietowski, Phys. Rev. D **80**, 025002 (2009) [arXiv:0903.3244 [hep-th]].
[17] R. C. Myers, M. F. Paulos, and A. Sinha, J. High Energy Phys. **0906**, 006 (2009) [arXiv:0903.2834 [hep-th]].