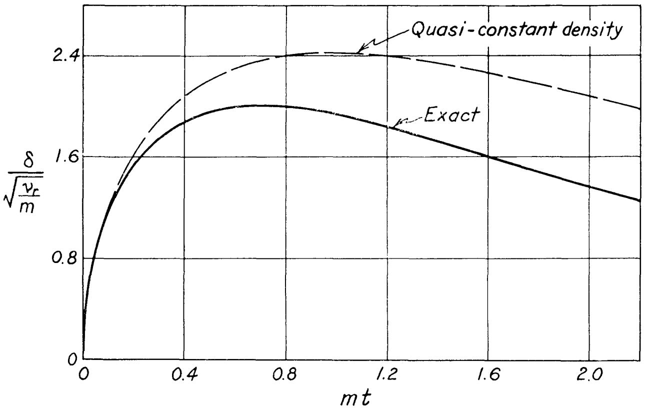

FIG. 1. Variation of boundary-layer thickness with time on a suddenly moved flat plate with exponential density variation.

$$\eta = (\rho/\rho_r)y \tag{9}$$

$$\tau = \int(\rho/\rho_r)dt \tag{10}$$

where ρ_r is a reference density. Eq. (7) becomes

$$\partial u/\partial\tau = \nu_r(\partial^2 u/\partial\eta^2) \tag{11}$$

where $\nu_r = \mu/\rho_r$. This, of course, is the well known one-dimensional heat equation, numerous solutions of which are recorded in the literature. Any of these can be used for arbitrary variations of density with time, provided only that these variations make sense physically. The mathematical requirements on the function $\rho(t)$ are simply that it must be integrable and that it must remain positive (or negative) so that the transformation given by Eq. (10) exists and is one to one. Since in all cases η is proportional to y at any time t, the instantaneous velocity profile—that is, the variation of u with y—always has the characteristics of the profile in the constant density case, where η can be identified with y.

For example, a solution given in reference 1 is written in our notation

$$u = u_0 \operatorname{erfc} \zeta = u_0(1 - \operatorname{erf} \zeta) \tag{12}$$

where

$$\zeta = \eta/2\sqrt{\nu_r\tau} \tag{13}$$

which gives a boundary-layer like flow for small τ. Eq. (13) may represent the solution for a suddenly moved infinite flat plate in a region where the density may be supposed to be varied in an arbitrary manner by the vertical motion of a plane at a distance from the suddenly moved plate which is large compared to the thickness of the boundary layer.

As an interesting case of this solution, assume that the density varies in an exponential manner

$$\rho = \rho_r e^{mt} \tag{14}$$

where m is a constant. The boundary-layer thickness, δ, can be taken as the value of y for which $\zeta = 2$. But, from Eqs. (9), (10), (13), and (14), we find

$$\zeta = e^{mt}y/2\sqrt{(\nu_r/m)\ (e^{mt} - 1)}$$

and, therefore,

$$\delta = 4\sqrt{\nu_r/m}\ e^{-mt}\ \sqrt{e^{mt} - 1}$$

This relation is plotted in Fig. 1. It is seen that the thickness actually decreases at large values of t. A comparison can be made with what might be called the solution for quasi-constant density—that is, when ζ is given by

$$\zeta = y/2\sqrt{\nu t}$$

with

$$\nu = \nu_r(\rho_r/\rho) = \nu_r e^{-mt}$$

This gives

$$\delta = 4\sqrt{\nu_r/m}\ e^{-mt/2}\ \sqrt{mt}$$

which is also plotted in Fig. 1. This too shows a decrease for large t, but the effect is considerably delayed and weaker than in the exact solution.

These results may be interpreted as follows. Initially the boundary layer on the suddenly moved plate grows rapidly due to the high shear and low kinematic viscosity. After a time, however, the increasing kinematic viscosity causes a reduction in the boundary-layer thickness. This thinning is larger in the exact case as a result of the convection of the outer layers of the boundary layer toward the plate, as required by continuity.

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On the Character of the Instability of the Laminar Boundary Layer Near the Nose of a Blunt Body*

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RECORDS WERE OBTAINED of the hot-wire response to velocity of fluctuations in the boundary layer of a body of revolution

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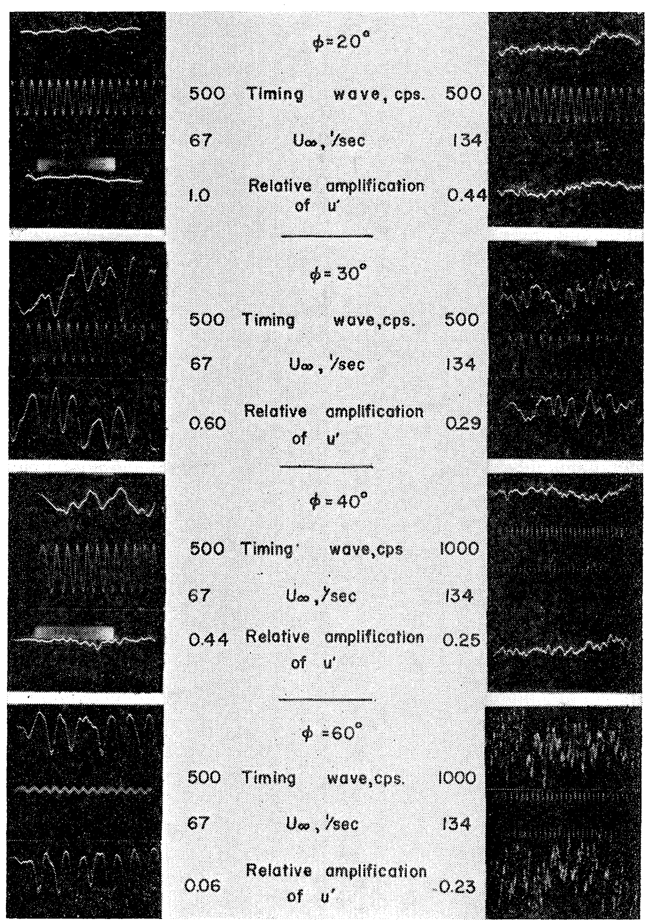

FIG. 1.

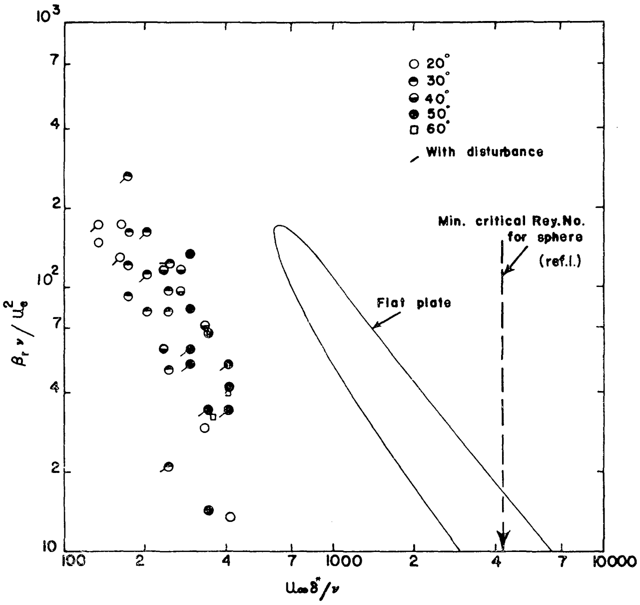

FIG. 2.

with a hemispherical nose 11.5 in. in diameter, at wind speeds up to 134 ft./sec. in the University of Michigan 5 × 7 ft. low-turbulence tunnel. Sample records at various angles from the nose along the same meridian plane are shown in Fig. 1. Two sweeps are shown for each condition. Approximate relative amplification factors for u' are indicated. The disturbances shown were probably produced by surface roughness since they did not appear at all meridian planes. The bursts of approximately sinusoidal fluctuations indicate that the instability is probably of the Tollmien-Schlichting type.

Comparison of the reduced frequency $\beta_r \nu / u_e^2$ where $\beta_r = 2\pi$ (frequency of the disturbances) and u_e is the velocity at the edge of the boundary layer versus $u_e \delta / \nu$ with the flat-plate (Schlichting) and with sphere stability theory[1] is shown in Fig. 2. Some of the points refer to records made with a rough "button" at the nose.

The fact that the disturbances appear and grow at Reynolds Numbers far below the critical value given by theory indicates the existence of a source of instability that is neglected in the theory. I suggest that this hitherto unrecognized source is the stretching of the vortex filaments in the region of diverging flow near the stagnation point.

Approximate analysis[2] of the destabilizing influences of stretching and surface cooling compared with the stabilizing influence of curvature indicates that near the nose the net effect is destabilizing. The analysis uses the Taylor-Görtler vortices (which stretch in the favorable pressure gradient) as a model, but it is pointed out[2] that analogous effects exist for those of the Tollmien-Schlichting type. Qualitative agreement between theory and the present experiment is obtained for the condition of no heat transfer to the body.

It is pointed out further[2] that, while the effect of cooling a flat plate is known to be stabilizing to Tollmien-Schlichting disturbances, when convex curvature is introduced the centrifugal force acting on a displaced particle causes a destabilizing influence as well. It seems reasonable, therefore, that if the curvature is large enough the effect of cooling a surface may be destabilizing to Tollmien-Schlichting disturbances.

A theoretical and experimental program is under way.

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Relative Importance of Free-Stream Vorticity and Self-Induced Pressure Gradient on a Flat-Plate Boundary Layer

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THE PROBLEM of the incompressible laminar boundary layer on a flat plate in a free stream containing a relatively small amount of vorticity, ω_0, has recently been discussed by Glauert[1] and Li.[2] In reference 1, Glauert refutes an earlier claim by Li[3] that the free-stream vorticity introduces a pressure gradient which should be considered in the solution for the effect of free-stream vorticity on the boundary-layer development. However, Glauert did not make an order-of-magnitude estimate of the self-induced pressure gradient and, consequently, Li remained unconvinced.[2] The purpose of this note, in support of Glauert, is to point out that if a self-induced pressure gradient must be considered, it is the pressure gradient associated with the displacement thickness of the zero-order ($\omega_0 = 0$) solution. (The effect of the free-stream vorticity on the self-induced pressure gradient is of higher order and is negligible.) Criteria for evaluating the relative importance of free-stream vorticity as compared with the self-induced pressure-gradient effect are also developed herein for both incompressible and supersonic flows. Unless otherwise specified, the notation of references 1 and 2 is followed.

Consider first the laminar boundary layer on a flat plate in an incompressible uniform stream. Various attempts have been made to improve the zero-order (Blasius) solution by expanding in inverse powers of the local Reynolds Numbers. Kuo[4] has shown that a first-order correction may be obtained by including, in the usual boundary-layer equation, the pressure gradient induced by the displacement thickness of the zero-order solution. The form of the stream function is then

$$\psi = \sqrt{u_0 \nu x}\,[f_0(\eta) + (1/\sqrt{Re_L})\sqrt{L/x}\,F_1(x, \eta) + \ldots] \qquad (1)$$

where $Re_L = u_0 L/\nu$, L = plate length, and $F_1(x, \eta)$ is of order 1. The form of $F_1(x, \eta)$ is obtained by computing the pressure distribution due to the zero-order displacement thickness at the plate and in the wake. If a small amount of vorticity is in the free stream it will have a higher-order effect on the self-induced pressure distribution and therefore does not influence $F_1(x, \eta)$.

Now, consider the incompressible free stream to have a small amount of vorticity. The stream function can be expressed as[1]

$$\psi = \sqrt{u_0 \nu x}\,[f_0(\eta) + (\omega_0 L/u_0\sqrt{Re_L})\sqrt{x/L}\,f_1(\eta) + \ldots] \qquad (2)$$

In order for this perturbation scheme to be valid it is necessary that

$$\omega_0 L/u_0\sqrt{Re_L} \approx \text{vorticity in free stream/vorticity in boundary layer} \ll 1 \qquad (3)$$

The relative importance of the self-induced pressure gradient and small free-stream vorticity can now be evaluated. Considering x/L, $f_1(\eta)$, and $F_1(x, \eta)$ to be of order 1, the ratio of the free-stream vorticity effect to the self-induced pressure effect is

$$R \equiv \frac{\omega_0 L/u_0\sqrt{Re_L}}{1/\sqrt{Re_L}} = \frac{\omega_0 L}{u_0} \qquad (4)$$

If R is large, only the free-stream vorticity effect need be considered. If R is small, only the self-induced pressure gradient effect need be considered. If R is of the order 1, both effects are equally important. However, in this case the resulting solution is simply the linear superposition of the pressure-gradient effect (as computed by Kuo) and the free-stream vorticity effect (as computed by Glauert or Li for the case of zero induced pressure).

A similar discussion can be made for the case of supersonic flow. Assuming a linear viscosity-temperature law, $\mu/\mu_0 = C(T/T_0)$, the zero-order displacement thickness can be expressed as