where $k_1 = C(\beta_1 - 1)/\sigma\gamma(1 + \beta_1)A_0B_1^2$, $\zeta_1 = -(\beta_1 - 1)/\gamma$, and $X_1 = M^2 + \zeta_1$. This latter case represents constant-temperature magnetogasdynamic acceleration in a diverging nozzle.

From Eq. (20) it is seen that there is an upper limit to the Mach number which can be achieved in constant-area, constant-temperature flow. This occurs when the arguments of the logarithmic terms vanish and the channel length therefore approaches infinity. For the values used in the example, this Mach number is about 1.64. On the other hand, no such limit exists for the diverging-channel case, despite the logarithmic terms, as can be seen by examining Eq. (21).

Using the parameter values of the isothermal example in Ref. 2, we have calculated the significant quantities for the two exact solutions discussed above. These are plotted versus channel length in Figs. 1 and 2.

On the basis of the present analysis, it appears that in constant-temperature, constant-area MGD channel flow, the attainable Mach number will be limited to rather low values by the vanishing of the magnetic field. It appears, however, to be possible to overcome this difficulty by diverging the channel.

References

1 Resler, E. J., Jr., and Sears, W. R., *The Prospects for Magneto-Aerodynamics*, Journal of the Aeronautical Sciences, 25, No. 4, April 1958.

2 Berner, F., and Camac, M., *Air Scooping Vehicle*, AVCO-Everett Research Labs., Rsch. Rep., No. 76, Aug. 1959.

3 Kerrebrock, J. P., and Marble, F. E., *Constant-Temperature Magneto-Gasdynamic Channel Flow*, Journal of the Aero/Space Sciences, Vol. 27, No. 1, Jan. 1960.

4 Demetriades, S. T., *Magnetohydrodynamic Orbit Control for Satellites, AIEE*, Pacific General Meeting, San Diego, Calif., 9–12 Aug. 1960; Preprint No. CP 60-1087, Appendix A. Also Northrop Corp., Norair Div., Astro Sciences Rep., ASRL-TM-60-6.

5 Binder, R. C., *Advanced Fluid Dynamics and Fluid Machinery*, Prentice-Hall, Inc., New York, 1951.

6 Resler, E. L., Jr., and Sears, W. P., *The Prospects for Magneto-Aerodynamics—Correction and Addition*, Journal of the Aero/Space Sciences, May 1959.

7 Berlot, R. R., *On the Magnetic Field in MGD Channel Flow* (to be published).

Boundary-Layer Energy Equation

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February 9, 1962

THE BOUNDARY-LAYER ENERGY EQUATION for an incompressible fluid in general boundary-layer coordinates, such as those used by Michal[1] for the momentum equations, to the best of the author's knowledge, does not exist in the usual literature sources. It is the purpose of this note to derive and present such an energy equation.

The time-steady energy equation in general coordinates for an incompressible fluid with constant properties is

$$u^i \frac{\partial T}{\partial x^i} = \frac{k}{\rho c} g^{ij}\left[\frac{\partial T}{\partial x^i}\right]_{,j} + 2\frac{\nu}{c} g^{ik}g^{jl}\dot{e}_{ij}\dot{e}_{kl} \tag{1}$$

where

$$\dot{e}_{ij} \equiv (1/2)(u_{i,\,j} + u_{j,\,i}) \tag{2}$$

and $i = 0, 1, 2$. In Eq. (1) T is the fluid temperature at point x_i and u_i is the contravariant velocity component in the direction x_i. g^{ij} is the fundamental metric tensor of the space, a function of x_i. The comma denotes covariant differentiation with respect to the space coordinates x^i.

The coordinate system used by Michal has the coordinates x^1 and x^2 imbedded on the surface over which the flow takes place and the x^0 coordinate normal to it. Consequently

$$g^{00} = g_{00} = 1, \quad g^{0i} = g_{i0} = 0 \tag{3}$$

For boundary-layer application, Eq. (1) need only be valid in the same spatial region as the boundary-layer momentum equations because the thermal boundary-layer thickness is at most equal to the momentum boundary-layer thickness. This is known from the fact that the Prandtl number is a measure of the capacity of the fluid to diffuse momentum compared with its capacity to diffuse heat and that all gases and liquids, with the exception of liquid metals,* have Prandtl numbers which are approximately one or larger. In the boundary-layer momentum equations it has been assumed that in the region of validity the space metric components, $g_{ij}(i, j = 1, 2)$ can be replaced by the first fundamental metric of the surface $a_{\alpha\beta}$. That is,

$$a_{\alpha\beta}(x^1,x^2) = g_{ij}(0, x^1, x^2),\ i, j = \alpha, \beta = 1, 2 \tag{4}$$

In terms of this type of coordinate system Eq. (1) now becomes

$$u^\alpha \frac{\partial T}{\partial x^\alpha} + u^0 \frac{\partial T}{\partial x^0} = \frac{k}{\rho c}\left[a^{\alpha\beta}\left(\frac{\partial T}{\partial x^\alpha}\right)_{;\beta} + g^{00}\frac{\partial^2 T}{\partial (x^0)^2}\right] + 2(\nu/c)[\dot{e}_{\alpha\beta}\dot{e}_{\lambda\mu}a^{\alpha\lambda}a^{\beta\mu} + \dot{e}_{00}\dot{e}_{00}g^{00}g^{00} + 2\dot{e}_{0\alpha}\dot{e}_{0\beta}g^{00}a^{\alpha\beta}] \tag{5}$$

where the Greek indices denote the coordinates imbedded on the surface and range over 1, 2. The semicolon denotes covariant differentiation with respect to the surface coordinates.

The term $a^{\alpha\beta}(\partial T/\partial x^\alpha)_{;\beta}$ represents the thermal conduction parallel to the surface and will be neglected compared with the convection parallel to the surface. (This may also exclude the possibility of applying the equations to liquid metals.)

Because of the nature of the boundary layer, the principal contribution to the viscous dissipation term, the last term in Eq. (5), comes from the tangential shear in the plane parallel to the surface. Therefore the energy dissipation due to the normal stresses and the remaining shear stresses may be neglected. In the viscous dissipation term only the relation $2\dot{e}_{0\alpha}\dot{e}_{0\beta}g^{00}a^{\alpha\beta}$ contains the terms of the energy dissipation due to the tangential shear parallel to the surface. This relation also contains terms which are negligible and must be examined more closely. Using the definition of $\dot{e}_{\alpha\beta}$ the above relation becomes

$$(1/2)g^{00}a^{\alpha\beta}[u_{0;\,\alpha} + u_{\alpha,\,0}][u_{0;\,\beta} + u_{\beta,\,0}]$$

Only the second quantity in each set of brackets represents the dominant energy-dissipation terms. The first quantity in each set of brackets represents the tangential shears in planes normal to the surface and hence is neglected. The relation now becomes

$$(1/2)g^{00}a^{\alpha\beta}u_{\alpha,\,0}\,u_{\beta,\,0}$$

Because of the nature of the space metric defined in Eqs. (3) and (4), the covariant differentiation in the above relation reduces to the partial derivative of u_α with respect to x^0. The relationship is then

$$(1/2)g^{00}a^{\alpha\beta}(\partial u_\alpha/\partial x^0)(\partial u_\beta/\partial x^0)$$

or

$$(1/2)a_{\alpha\beta}(\partial u^\alpha/\partial x^0)(\partial u^\beta/\partial x^0)$$

With the above assumptions, the energy equation, Eq. (5), takes the following form for the incompressible boundary layer:

$$u^\alpha \frac{\partial T}{\partial x^\alpha} + u^0\frac{\partial T}{\partial x^0} = \frac{k}{\rho c}\left[\frac{\partial^2 T}{\partial (x^0)^2}\right] + \frac{\nu}{c} a_{\alpha\beta}\frac{\partial u^\alpha}{\partial \varphi}\frac{\partial u^\beta}{\partial \varphi} \tag{7}$$

With the following scale changes:

$$\varphi = x^0/\sqrt{\nu}, \qquad w = u^0/\sqrt{\nu}$$

Eq. (7) takes the form

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* The case of the fluid being a liquid metal is excluded throughout.

$$u^{\alpha}\frac{\partial T}{\partial x^{\alpha}} + w\frac{\partial T}{\partial \varphi} = \frac{k}{\mu c}\left[\frac{\partial^2 T}{\partial \varphi^2}\right] + \frac{1}{c}\,a_{\alpha\beta}\frac{\partial u^{\alpha}}{\partial x^{0}}\frac{\partial u^{\beta}}{\partial x^{0}} \qquad (8)$$

In Eq. (8) the velocity and coordinate variables correspond to those used by Michal in the momentum and continuity equations of the incompressible-boundary-layer equations.

The system of equations composed of the momentum and continuity equations from Michal and the above energy equation has been analysed recently by the author[2] for similarity solutions of the velocity and temperature profiles in the generalized coordinates.

References

[1] Michal, A. D., *Matrix and Tensor Calculus*, 1st Ed., pp. 103–110; John Wiley & Sons, Inc., New York, 1947.

[2] Winer, W. O., *An Analytical and Experimental Investigation in Similar Solutions of Three Dimensional Laminar Incompressible Boundary Layer*, Ph.D. Dissertation, University of Michigan; University Microfilm, Ann Arbor, Michigan, 1961.

Erratum—The Qualitative Dynamics of Atmospheric Entry and Re-Entry*

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December 4, 1961

For the second of the variables defined in the last paragraph read:

$$P' = V_{\theta}^{2}R^{m}$$

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Stagnation-Point Shock Detachment of Blunt Bodies in Supersonic Flow

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This note presents stagnation-point shock-detachment-distance values determined by the exact numerical method of Gravalos, Edelfelt, and Emmons.[1] The method represents a direct solution to the inviscid blunt-body problem in which the shock shape is calculated by a numerical iteration procedure for a given body shape. These results for spherical bodies[7] are compared with those from the methods of Van Dyke and Gordon,[2] Li and Geiger,[3] and Serbin,[4] and with experiment.[5, 6]

In Fig. 1, values of the ratio of the shock-detachment distance, Δ, to the nose radius, r_N, from the aforementioned theories are presented as a function of Mach number and are compared with experimental data. In this plot all of the results, except for a portion of those for the Gravalos method, were determined for an ideal gas with a ratio of specific heats, γ, equal to 1.4.

The agreement between the Van Dyke results and the test data (as previously indicated in Ref. 2) is very good. The results predicted by Serbin's method are slightly higher than the data presented, whereas the Li-Geiger results are appreciably low. The Gravalos ideal-gas ($\gamma = 1.4$) results are in excellent agreement with the Van Dyke results and with the experimental data in the Mach-number range where the comparisons can be made. As might be expected, the Gravalos real-gas-flow solutions are not in agreement with the ideal-gas solutions.

It is interesting, therefore, to observe this same comparison of shock-detachment distance (less the experimental data) when presented as a function of the ratio of free-stream density, ρ_∞, to density behind the bow shock, ρ_2, (Fig. 2) a procedure suggested by Hayes.[8] The real-gas values now correlate quite well within themselves; and furthermore, the Gravalos results form a smooth extension of the Van Dyke results to the small-density, high-Mach-number range. It is also observed that the method of Serbin (and also the Li-Geiger method, for a limited density range) appears to provide acceptable estimates of the shock-detachment distance at small values of density ratio.

The utility of this correlation is immediately obvious to those familiar with the Gravlos flow-field technique, since it provides a valuable aid in the estimation of the shock location. In addition, these calculated values of shock-detachment distance may be of use to the experimentalist in assessing the achievement of thermochemical equilibrium conditions for a body with a spherical nose in the hypersonic flow of dissociating air.

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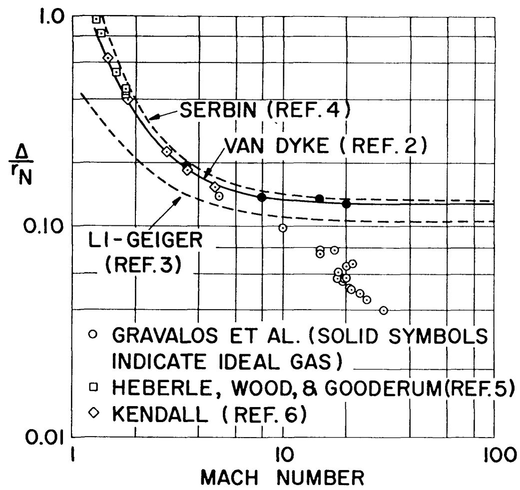

Fig. 1. Stagnation-point shock-detachment distance vs Mach number.

References

[1] Gravalos, F. G., Edelfelt, I. H., and Emmons, H., *The Supersonic Flow About a Blunt Body of Revolution for Gases at Chemical Equilibrium*, Proc. of the 9th Annual Congress, International Astronautical Federation, Aug., 25–29, 1958.

[2] Van Dyke, M. D., and Gordon, H. D., *Supersonic Flow Past a Family of Blunt Axisymmetric Bodies*, NASA TR R-1, 1959.

[3] Li, Ting-Yi, and Geiger, R. E., *Stagnation Point of a Blunt Body in Hypersonic Flow*, Journal of the Aeronautical Sciences, Vol. 24, No. 1, Jan. 1957.

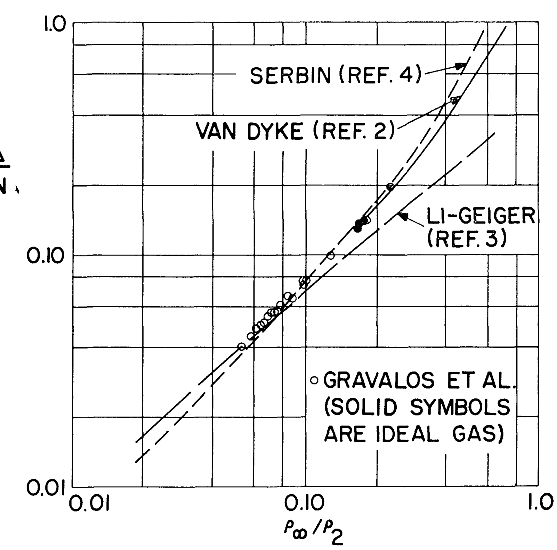

Fig. 2. Stagnation-point shock-detachment distance vs density ratio.