$$u/2 + [1 + kc]^{3/2}/k = u/2 + (\omega/c)^3/k = \alpha \qquad (13)$$

$$-u/2 + [1 + kc]^{3/2}/k = -u/2 + (\omega/c)^3/k = \beta \qquad (14)$$

where (α, β) may be considered as generalizations of the usual Riemann invariants.

Consider a simple wave centered at the origin. On each characteristic $dx/dt = u + \omega$, (u, c, B) are constant, and these characteristics are straight lines in the (x, t) plane. The wave may be characterized by:

$$x = (u + \omega)t$$

$$\beta = \beta_0 = [1 + kc_0]^{3/2}/k$$

where c_0 is the stagnation sound speed.

From Eqs. (13) and (14):

$$u = \alpha - \beta_0 \qquad (15)$$

$$kc = [k(\alpha + \beta_0)/2]^{2/3} - 1 \qquad (16)$$

$$\omega = c[k(\alpha + \beta_0)/2]^{1/3} = (\alpha + \beta_0)/2 - [k(\alpha + \beta_0)/2]^{1/3} \qquad (17)$$

and

$$3\alpha/2 - \beta_0/2 - [k(\alpha + \beta_0)/2]^{1/3}/k = x/t \qquad (18)$$

Eqs. (15)–(18) give an implicit representation of the flow parameters in the simple wave as functions of (x, t).

An implicit representation could also be given for the non-centered simple wave.

REFERENCES

[1] Friedrichs, K. O., *Nonlinear Wave Motion in Magnetohydrodynamics*, Los Alamos Report No. 2105, 1957.

[2] Courant, R., and Friedrichs, K. O., *Supersonic Flow and Shock Waves*, Interscience Publishers, Inc., New York, 1948.

Formation of Rings in a Liquid Film Attached to the Inside of a Rotating Cylinder

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September 19, 1961

In an experimental investigation sponsored by the Technical Association of Pulp and Paper Industries we found that rings formed in a liquid film attached to the inside of a rotating cylinder, as the cylinder was suddenly stopped or slowed down. In the former case the rings (Fig. 1) lasted a few seconds before gravity pulled the liquid down to the lower part of the drum. (The axis of the drum was horizontal.) In the latter case the rings appeared for a shorter length of time but disappeared again, leaving a smooth film rotating at a smaller speed, so long as the final speed was great enough to withstand the pull of gravity.

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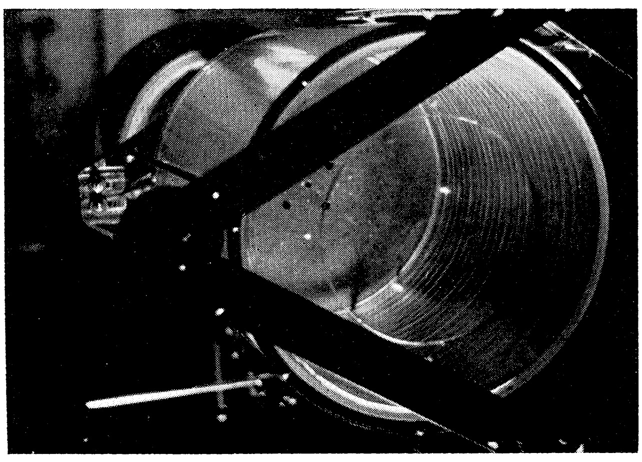

Fig. 1.

All this is really to be expected, since it is well-known that a revolving fluid is, generally speaking, unstable if the circulation decreases monotonically radially outward, as it indeed did in the present case. The investigation was carried out because the instability of the free surface of the film is very pertinent to the phenomenon of spouting of stock in the paper-making process. We wish to call attention to the ring formation here for a different reason. Professor S. F. Shen of Cornell University recently informed the second author that experiments by Donald Coles at the California Institute of Technology showed that Tollmien-Schlichting instability occurred in a fluid between rotating cylinders as the outer cylinder was stopped. We do not know the details of Coles' experiments, but understand that his inner cylinder was stationary, so that a large portion of the fluid was stable against Taylor instability (for revolving fluids). Only in a thin layer attached to the outer cylinder was Taylor instability possible, and there it was evidently dominated by Tollmien-Schlichting instability, so that only longitudinal ridges formed. In our case the free surface (or the inner boundary) rotated before the cylinder was stopped, and continued to do so at about the same speed shortly after it was stopped. Thus the bulk of the liquid was in our case unstable with regard to ring formation. The two cases are different, and apparently there must be intermediate cases in which Taylor instability and Tollmien-Schlichting instability are equally significant.

Inelastic Redundant Analysis and Test-Data Comparison for a Heated Ring Frame

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Adapting and modifying the procedures of Crichlow and Haggenmacher[1] to allow for temperature and inelastic effects, the virtual-work equations can be written in Peery's notation[2]

$$\left.\begin{aligned}
\delta_a &= \int \frac{M_o m_a}{EI}\,ds + \int \frac{S_o u_a}{AE}\,ds + \int \frac{K_T m_a}{c}\,ds + \\
&\quad \int e_T u_a ds + \int \frac{K_p m_a}{c}\,ds + \int e_p u_a ds + \\
&\quad X_a\delta_{aa} + X_b\delta_{ab} + \ldots + X_n\delta_{an} \\
\delta_b &= \int \frac{M_o m_b}{EI}\,ds + \int \frac{S_o u_b}{AE}\,ds + \int \frac{K_T m_b}{c}\,ds + \\
&\quad \int e_T u_b ds + \int \frac{K_p m_b}{c}\,ds + \int e_p u_b ds + \\
&\quad X_a\delta_{ba} + X_b\delta_{bb} + \ldots + X_n\delta_{bn} \\
\delta_n &= \int \frac{M_o m_n}{EI}\,ds + \int \frac{S_o u_n}{AE}\,ds + \int \frac{K_T m_n}{c}\,ds + \\
&\quad \int e_T u_n ds + \int \frac{K_p m_n}{c}\,ds + \int e_p u_n ds + \\
&\quad X_a\delta_{na} + X_b\delta_{nb} + \ldots + X_n\delta_{nn}
\end{aligned}\right\} \qquad (1)$$

where the temperature terms are[3]

$$K_T = \frac{M_T c}{EI} = \frac{c\int E\alpha T y dA}{\int E y^2 dA}, \quad e_T = \frac{S_T}{AE} = \frac{\int E\alpha T dA}{\int E dA}$$

The inelastic effects represented by K_p and e_p are calculated by the methods of Ref. 3 for the axial load and bending moment acting on any given cross section. For each cross section in