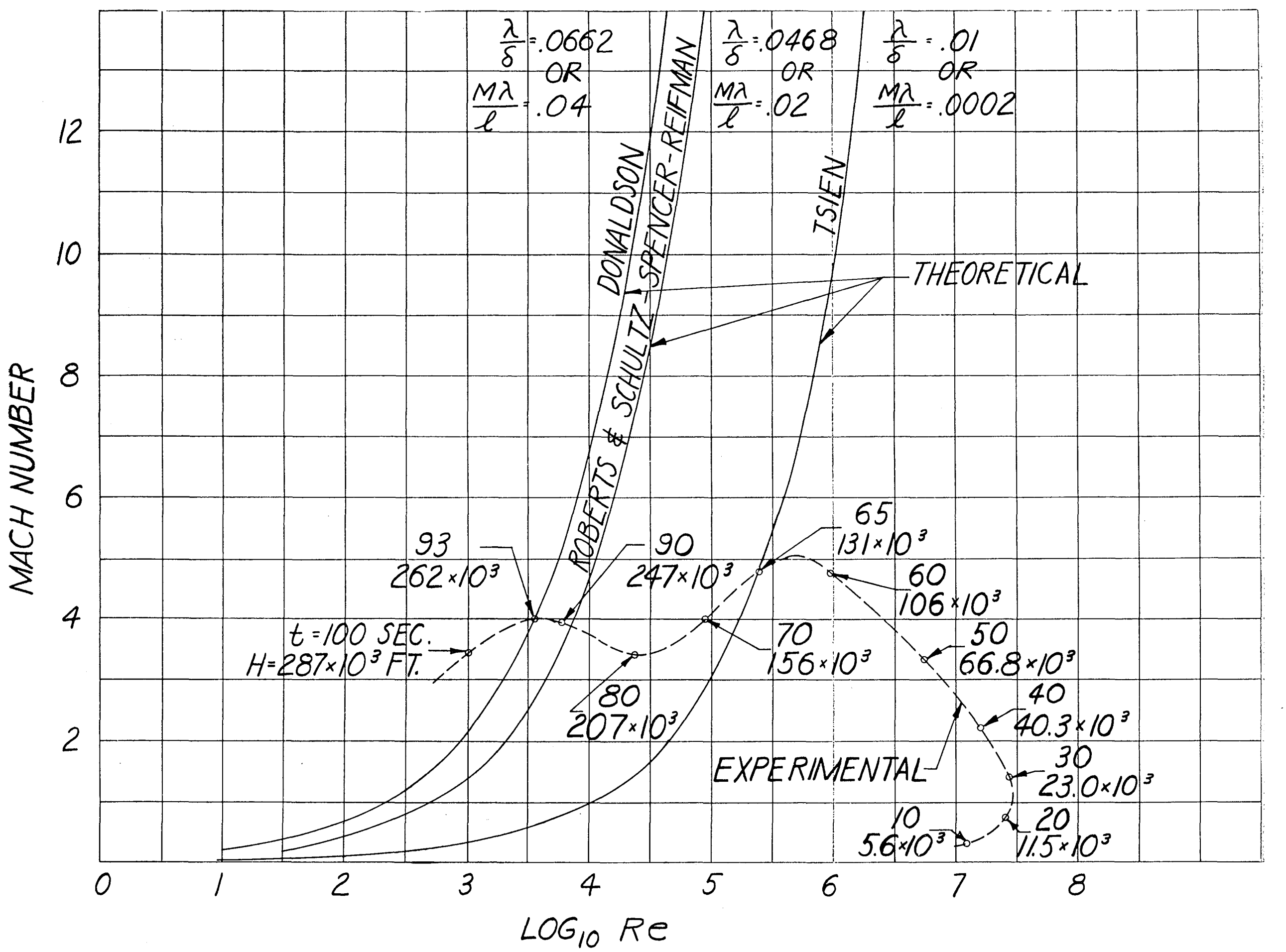

FIG. 1. Comparison of slip flow criteria.

Boundaries of Fluid Mechanics

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In October's Aeronautical Engineering Review, Mr. Roberts points out a new criterion[1] suggested by H. W. Liepmann to distinguish the realms of fluid mechanics. In the literature there already are criteria that attempt to define the regions of fluid mechanics. This note compares these criteria.

Boundary Between Continuum Analysis and Slip Flow

M = Mach Number
δ = boundary layer thickness
l = effective length
R = Reynolds Number
λ = mean free path
v = velocity
ρ = density
μ = viscosity coefficient
c = mean molecular velocity for air
a = velocity of sound

Subscript 0 signifies sea-level conditions.

(1) Donaldson:[2]

$$M\lambda/l = 0.04$$

(2) Tsien:[3]

$$\lambda/\delta = M/\sqrt{R}, \quad R \gg 1$$
$$\lambda/\delta = M/R, \quad R \ll 1$$
$$\lambda/\delta = 0.01$$

(3) Schultz, Spencer, and Reifman:[4]

$$\lambda/\delta = 0.438(M/\sqrt{R}) \cong M/2\sqrt{R}$$
$$\lambda/\delta = 0.05$$

(4) Roberts:[1]

$$R/M^2 = 100$$

We can compare these criteria in two ways: first,

$$\frac{M}{R} = \frac{M(v/a)}{\rho l v/\mu} = \frac{M}{l}\frac{\mu}{\rho a} = \frac{M\lambda}{l}\frac{\mu_0(T/T_0)^{0.75-0.5}}{\lambda_0 a_0}$$

In the region of the atmosphere under consideration T/T_0 can be considered unity in an order of magnitude analysis.

$$\frac{M^2}{R} = \frac{M\lambda}{l}(0.438)\left(\frac{T}{T_0}\right)^{0.25} \cong \frac{M\lambda}{l}\frac{1}{2}$$

Second, an extension of Donaldson's Appendix A,[2]

$$\frac{M^2}{R} = \frac{M^2}{\rho v l/\mu} = \frac{M^2}{(vl/\lambda)(\rho\lambda/\mu)}$$

but $\mu = 0.499\rho\, c\, \lambda$ and $c = 1.462a$; hence, $\mu = 0.73\rho a\lambda$ and

$$\frac{M^2}{R} = \frac{M^2}{vl/0.73\lambda a} = 0.73\frac{M\lambda}{l} \cong \frac{3}{4}\frac{M\lambda}{l}$$

Let us use $M^2/R = (M\lambda/l)(1/2)$ to compare the criteria.

(1) Donaldson:

$$M\lambda/l = 0.04$$

(2) Tsien:

$$M\lambda/l = 0.0002$$

(3) Schultz, Spencer, and Reifman:

$$M\lambda/l = 0.02$$

(4) Roberts:

$$M\lambda/l = 0.02$$

It thus appears that in any order of magnitude discussion that criteria (1), (3), and (4) are equivalent.

Lo[5] has superimposed Tsien's criteria on a graph of M versus $\log R$ for the flight of V-2 missile No. 21, fired on March 7, 1947. We have added the other criteria to this curve. We note in Fig. 1 that there is a spread of 131,000 ft. between application of these criteria.

DIFFERENT CRITERIA FOR THE LOWER BOUND OF FREE MOLECULE FLOW

(A) Roberts:

$$R/M^2 = 0.01$$
$$M/\sqrt{R} = 10$$

(B) Schultz, Spencer, and Reifman:

$$\lambda/\delta = 10$$
$$\lambda/\delta = 0.438(M/\sqrt{R})$$

(C) Tsien:

$$\lambda/\delta = \lambda/l = M/R = 10$$

One notes that, in the intended region of application of criterion (B), it was thought that $R \gg 1$, while criterion (C) implied that $R \ll 1$. Thus we can make no real comparison between these criteria.

Of course, we do not know which criteria are the best, since this answer must be determined by experiment.

REFERENCES

[1] Roberts, Howard E., *The Earth's Atmosphere*, Aeronautical Engineering Review, Vol. 8, No. 10, p. 19, October, 1949.

[2] Donaldson, Coleman duP., *An Approximate Method for Estimating the Incompressible Laminar Boundary-layer Characteristics on a Flat Plate in Slipping Flow*, N.A.C.A. R.M. No. L9C02, May 2, 1949.

[3] Tsien, Hsue-Shen, *Superaerodynamics*, Journal of the Aeronautical Sciences, Vol. 13, No. 12, pp. 653–664, December, 1946.

[4] Schultz, F. V., Spencer, N. W., and Reifman, A., *Atmospheric Pressure and Temperature Measurements between the Altitudes of 40 and 110 Kilometers*, Upper Air Research Program Report No. 2, Engineering Research Institute, University of Michigan, July 1, 1948.

[5] Lo, Hsu, *Determination of Transient Skin Temperature of Conical Bodies during Short-time, High-speed Flight*, N.A.C.A. T.N. No. 1725, p. 33, October, 1948.

The Pressure on a Slender Body of Nonuniform Cross-Sectional Shape in Axial Supersonic Flow

(Continued from page 175)

Fig. 3 shows a sketch of the distorted element, and Fig. 4 gives the ordinates and pressure coefficients for lines on the surface defined by various planes passing through the flow axis. The importance of the longitudinal shape of the element is indicated by the pressure reversals that occur for $\theta = 0°$ and $\theta = 90°$. The pressure reversals also indicate qualitatively that a drag force acts on the forward half of the element and that a negative drag force acts on the rear half. This tendency to eliminate the resultant drag is consistent with the conclusion previously stated—that no resultant drag could be predicted, within the limits of accuracy of the theory, for a body ending in a cylinder.

REFERENCES

[1] Graham, E. W., *Pressure and Drag on Smooth Slender Bodies in Linearized Flow*, Douglas Aircraft Company Report No. SM-13417, April, 1949.

[2] Ward, G. N., *Supersonic Flow Past Slender Pointed Bodies*, The Quarterly Journal of Mechanics and Applied Mathematics, Vol. II, Part I, p. 75, March, 1949.

[3] Graham, E. W., *The Pressure on a Slender Body of Non-Uniform Cross-Sectional Shape in Axial Supersonic Flow*, Douglas Aircraft Company Report No. SM-13346, August, 1948.

[4] Graham, E. W., *The Pressure on Slender Bodies of Uniform Cross-Sectional Shape in Axial Supersonic Flow*, Douglas Aircraft Company Report No. SM-13377, August, 1948.