Identifying, Measuring, and Defining Equitable Mathematics Instruction

by

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DEDICATION

My princesses Bria and Naima

Remember that with faith, hard work, and God you can do anything!

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ABSTRACT

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Many scholars have studied the problem of persistent inequitable educational opportunities and outcomes in the U.S. They have presented analyses of the causes of these inequities and proposed solutions ranging from increasing school funding to studying participation structures in classrooms. This dissertation takes the perspective that inequities are produced inside of classrooms as well as through the complex interplay of social and economic factors and argues that instructional practice is an important site for study and intervention. Therefore, although there exist numerous definitions of and strategies for working toward equity for underrepresented minority students, serious attention to instruction is crucial. This is specifically accomplished by studying the mathematical knowledge and skills, along with cultural awareness and sensitivities that would produce equitable, high quality teaching. In this dissertation, equitable teaching is defined as focused on quality mathematics and distributed intentionally to ensure that all students learn.

This study probes the interplay in instruction of attention to equity and to the quality of the mathematical content, with a focus on what constitutes equitable mathematics instruction for students in elementary classrooms. Specific instructional

practices are evaluated to determine whether and how particular teaching practices provide leverage and create access to the mathematics content for different groups of learners. This study has two central features. The first details the construction of the set of Mathematical Quality and Equity codes, analytic video codes focused on issues of equity. The second section comprises analyses of three paradigmatic examples of instruction. One is of a teacher with high MKT (Karen); a second a teacher who has clear commitments to students and to equitable access (Rebecca), and a third a teacher who has both high levels of MKT and of commitment to students and to equity (Lauren). The analyses illustrate the central hypothesis in my dissertation, that teaching mathematics in equitable ways requires both attention to the quality of the mathematics combined with sensitivities to issues of equity and diversity for students.

This dissertation contributes to the empirical examination of instruction and its contributions to equity. The central contribution of this study is the characterization of instruction in which high quality mathematics is combined with specific attention to concerns for equity. Four features are proposed as central to this instruction. Both the features and the measures used to capture them will be useful to ongoing research on equitable instruction in three ways: (1) studies of effects of different forms of "equitable" instruction on student learning; (2) analyses of the resources (i.e., knowledge, skills, commitments) needed to produce equitable instruction; (3) effects of professional programs designed to help teachers develop the capacity for effective equitable instruction. Designing and enacting equitable instruction is paramount for improving academic achievement for all students, especially those most depending on school for learning, and is the most important challenge facing American public schools.

CHAPTER ONE:
THE RESEARCH PROBLEM

Introduction

The central argument for this dissertation is that, although there are many different definitions of equity and strategies for working toward equitable educational opportunity for underserved minority students in schools, attention to *instruction* is crucial (Cohen, Raudenbush, & Ball, 2002). Of interest is high quality teaching, defined as focused on quality mathematics and distributed intentionally to ensure that all students learn. Both the nature of this instruction and what it takes to produce it are the territory of this study.

Studying Equity and Mathematics Education

A broad view of studies of issues of equity and equality in American public schools. Results from National Assessment of Educational Progress (Silver & Kenny, 2000), TIMSS (Hiebert, et al., 2003), and other assessments show significant group-related inequities in mathematics achievement. Research over time indicates that certain factors such as school funding, per pupil expenditures, class size, teachers' expectations, and teacher knowledge, influence these differences in academic achievement between groups of students. In American public schools, educational achievement is linked to social groupings such as race, ethnicity, class and gender (Ball, et. al., 2003; Bowles &

Gintis, 1976). For example, studies reveal that inequitable instruction results in significant differences between the academic performance of students of color (particularly those attending urban schools and living in poverty), and their white, suburban, middle-class peers (RAND, 2003; Silver & Kenney, 2000; Tate, 1997). Research over the last 50 years has pointed to a variety of causes of inequities in achievement, including cultural, economic, and political, and for the failure of American public schools to educate poor and minority students (Bowles & Gintis, 1976; Coleman, 1966; Reyes & Stanic, 1988; Secada, 1989, 1992).

The evidence about inequitable school structures and low quality instruction is broad and persistent. Ethnic minority students (particularly African Americans) and students living in poverty are disproportionately placed in special education (Artiles, Trent, & Palmer, 2004; O'Connor & Fernandez, 2006). More specifically, as reported by Ladson-Billings (2006),

> In the 2005 National Assessment of Educational Progress results, the gap between Black and Latina/o fourth graders and their White counterparts in reading scaled scores was more than 26 points. In fourth-grade mathematics, the gap was more than 20 points (Education Commission of the States, 2005). In eighth-grade reading, the gap was more than 23 points and in eighth-grade mathematics the gap was more than 26 points. Even when we compare African Americans and Latina/os with incomes comparable to those of Whites, there is still an achievement gap as measured by standardized testing (National Center for Education Statistics, 2001)(p. 5).

These data support a belief held by many educators: that structure and bias in American public schools provides unequal access to learning opportunities and continues to perpetuate inequitable academic performance and achievement which contribute to gaps in achievement between minority students and their white counterparts. As Jenks (2004) describes, "conventional research on racial achievement gaps has focused on these issues of equality of educational opportunities and schooling conditions among different racial groups as key determinants of their achievement gaps" (p. 52). Similarly, a great deal of additional research has focused on equity issues by studying the "achievement gap" and looking at the distribution of educational resources across schools and examining what effect these disparities may have on the achievement of poor and minority students (Bowles & Gintis, 1976; Kozol, 2001, 2005; Lee, 2001, 2007; Lubienski, 2002). Findings from this line of research has sought to illustrate quantitatively differences in educational opportunities between groups of students as a way of better understanding current consequences of institutional racism, and as a means of arguing against claims of heritability of Intelligence Quotient scores (Jensen, 1969) and cultural deficit models (as cited in Lee, 2007). Alternatively, more recent work has argued against studying "*the* achievement gap" (Gutiérrez, 2008; Ladson-Billings, 2006). In an article based on her AERA Presidential address, entitled, "From the Achievement Gap to the Educational Debt: Understanding Achievement in U.S. Schools", Ladson-Billings argues that, "this all-out focus on the "Achievement Gap" moves us toward short-term solutions that are unlikely to address the long-term underlying problem" (p. 4). She instead argues for a new focus on how "the historical, economic, sociopolitical, and moral decisions and policies that characterize our society have created an education debt" (p. 5). Similarly, Gutiérrez argues against what she calls "gap gazing" claiming that, "deepening our knowledge in this arena is unlikely to advance the cause of marginalized students" (p. 357). In that article, she describes how these works often fail to focus on or capture students' gains in academic achievement and provide "little direction for eliminating the gap" (see also Lee, 2002; Tate, 1997). Similarly, other scholars have argued that an emphasis on "<u>the</u> 'achievement gap' might be better described as a deliberate effort to focus on the national ideology of Black intellectual inferiority" (Perry, Steele, Hilliard, 2003, p. 8). Collectively, these authors argue that sufficient statistical data has been produced that focuses on the deficits of these students and calls for a shift in focus from studying what these students lack to instead focusing research on promising strategies for remedying widespread inequities in education.

A number of scholars have also studied equity issues in the broader context of American public schools, by exploring the opportunities and experiences of "minority" students and examining the relationship between race, culture and learning (Lee & Wong, 2004; Rist, 1970; Tate, 1997). Some of these scholars studied social reproduction in schools and sought to better understand how power and social structures work to create segregated and socially stratified schooling experiences for students often leading to inequitable opportunities for learning (Delpit, 1988, 1995; Oakes, 1983). Still others sought to deliberately address dynamics of culture, race, class, and schooling that focus on students' educational experiences and opportunities (Boaler, 2002; Cobb & Hodge, 2002; Delpit, 1988; Heath, 1983; Ladson-Billings, 1994, 1995, 2006; Nasir, 2002). In a similar vein of research, these and other scholars also examined the interplay of students' outside of school lives with structures of schools (Boaler, 1993, 1998; Cohen & Lotan, 1997, 2002; Delpit, 1988; Khisty, 1995, 1997; Moschkovich, 1999; Nasir, 2002). Results from this work argues that schooling can perpetuate unequal outcomes, especially as evidenced by patterns of achievement that are predictable along the lines of race, social class, and ability. One example of this work is the practice of tracking in schools. As Delpit (1995) articulates, tracking practices promote inequities in schools by legitimizing low expectations and unequal outcomes of students, and also by supporting non-rigorous mathematical practices for some students. More specifically, because these students have been denied access to vital resources needed for learning, one solution for improving student achievement lies in re-allocating instructional resources by implementing culturally relevant pedagogy (Ladson-Billings, 1994, 1995). These and similar theories of culturally specific pedagogy (Gay, 2000; Nieto, 2004; Tate, 1995), offer descriptions about the ways in which cultural knowledge and perspectives can influence teachers to make teaching more culturally sensitive and relevant, thus increasing student participation and student achievement. These theories argue against studying issues of race, culture, and equity from a deficit approach and instead focus on better understanding the role cultural knowledge plays in designing and implementing culturally responsive teaching. Drawing from previous theories and practices of multicultural education, the theories of culturally relevant and responsive teaching, and teaching for social justice (i.e. Gutstein, 2003) provide suggestions to educators and researchers alike of promising strategies and practices of equitable instruction (i.e. Ladson-Billings, 1997).

What has been largely missing from research and scholarship focused on intervening on patterns of inequity in student achievement has been a purposeful, detailed analysis on the role content knowledge plays in designing and enacting equitable instruction. Although multicultural education and other theories and frameworks claim that subject matter knowledge matters for equitable teaching, there has been little progress in specifying the depth and nature of this work. Later in this chapter, and throughout my dissertation, I attempt to contribute to this line of research.

Why Study Teaching?

Educational researchers and policy makers agree that there are often-cited problems with inequities in mathematics learning in this country—from access, to instructional quality, to achievement. There are many reasons for these inequities; many are systemic and cultural and related to enormous discrepancies in resources and opportunity.

Although many factors beyond the classroom clearly impact students' opportunities to learn, empirical research has established that the quality of instruction is directly related to teachers' knowledge and skill and that it makes a difference for students' learning (Darling-Hammond, 1999; Ingersoll, 2002; Whitehurst, 2002). Results from these and other studies reveal that a ten percentage difference between top quartile teachers and bottom quartile teachers results in significant differences in how much students gain when they have a more skillful teacher (Hill, Rowan, & Ball, 2005). Therefore, analyzing students' opportunities for learning mathematics offers significant value in seeking to improve the quality of instruction, particularly for students most dependent on public school for access to higher education and social mobility.

Defining Equity and Equitable Instruction

The RAND Mathematics Study Panel's publication, *Mathematical Proficiency for All Students*, defined and described equity as working to:

> …improve proficiency in mathematics and eliminating the gaps/ differences in levels of mathematics proficiency among students in different social, cultural, and ethnic groups. Because students' opportunities to develop mathematical proficiency are shaped within classrooms through their interactions with teachers and with specific content and materials, the proposed program addresses issues directly related to teaching and learning (RAND, 2003).

Furthermore, Kilpatrick, Swafford, and Findell (2001) define "proficiency as having five components: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition" (p. 115-155). It is important to note that throughout this dissertation I am presuming that the goal of teaching is to help students develop mathematical proficiency. Building on this work, I define the expression "equitable mathematics instruction" as those teaching practices that create a fair distribution of opportunities to learn mathematics among students, with special emphasis on the learning of students who are members of ethnic and social groups currently "underperforming" in mathematics, and those students who depend on schools for their primary access to learning. "Distribution" refers to the practices or occasions in which the teacher takes deliberate steps to engage every student in the mathematical work. Scholars suggest that default patterns of instruction are drawn from unexamined assumptions about students, teaching, and learning that are based on teachers' own experiences and habits (Ball, et al., 2003). Furthermore, mathematics instruction that is not explicit, such as when the teacher "facilitates" classroom discussion may be more culturally congruent with some students' cultures and learning experiences than others, thus providing uneven access for participating in mathematical discourse (Ball, et al., 2005; Delpit, 1998; Heath, 1983; Lubeinski, 2000). Moreover, Ball and colleagues (Ball, Goffney, & Bass, 2005) argue that explicit guidance for learning complex skills or ideas is crucial if all students are to develop such capacities, and not leave the construction of these to chance or to cultural differences in discursive norms (p. 4). The degree to which teachers are explicit in their teaching can be a significant resource for students' access to complex mathematical concepts and ideas. As such, explicitness is a feature of equitable mathematics instruction.

Notions of equity versus equality. Recent legislature and policy documents, reveal efforts to both articulate and call for national attention to issues of equity in mathematics teaching and learning (National Council of Teachers of Mathematics, 2000; No Child Left Behind, 2001; RAND report, 2003). However, there are many different meanings for the concept of the term "equity." Gutiérrez (2002) indicates that "the ways in which we define equity directly relates to how we seek to both measure and achieve it in our schools" (p. 152). She further argues that a first step is to distinguish notions of equity from ideas about equality, where equality is measured in terms of equal treatment or outcomes, which are usually based on standardized test scores and are often based on comparisons between demographic groups. She explains the many varied interpretations for what "equity" means, and what it would look like to be achieved. She describes how the work done by researchers ranges from emphasizing concepts like educational access (e.g., equal resources, quality teachers, opportunity to learn, etc.) to focusing on students' mathematical literacy (e.g., the ability to apply knowledge in new domains, the ability to make sense of data, etc.)(p. 152).

Similarly, Secada (1989) describes “classical notions of equity” where equity is considered in terms of equality (or inequality) of educational opportunity. Secada argues that equality is considered a quantitative construct, centered on the inputs, processes, or outcomes of education (1989). He asserts that discussions of equality in education have often focused on equality of outcomes, including achievement as measured by scores on standardized tests, or equality of inputs, such as the money spent per pupil or the proportion of credentialed teachers in a student’s school or district. Therefore, distinguishing between notions of equity versus equality is a critical step toward defining equity in elementary mathematics instruction.

The terms equity and equality have often been used interchangeably implying that they have equivalent meanings. Many of the educational policy documents birthed from the Civil Rights efforts of the 1960’s focused on the notion of equality, namely in providing the same resources to both White and Black students. More recently, however, researchers have sought to explain how notions of equality are not sufficient for working on increasing access to high quality teaching, if the goal is to have equal participation in higher education and better opportunities for social mobility. Secada (1986) contextualizes the definition of equity as follows:

> The historical denial of educational opportunity is but one in a long list of injustices that have been visited on the original members of marginalized groups and their descendents. Depending on one’s stance to American history, equity demands some forms of specialized opportunity as a means of:
>
> a. Compensating for the larger social injustices visited on members of these groups (such as segregated housing, unacceptably low job opportunities, poor health care, discrimination in voting and other forms of social participation)
> b. Compensating for the more-narrowly defined denial of educational opportunity

c. Dismantling the social structures that continue to impact the lives of members of marginalized groups (p.19-20).

Similarly, Gutiérrez (2002) describes equity as:

> assum[ing] neither equal approaches (e.g., same treatment of students, same resources) nor equal outcomes (e.g., same achievement). Instead, both approaches and outcomes should be equitable, not equal. This argument rests on the assumption that there exists natural variance between people in terms of goals, strengths, and interests. Therefore operating under a just system we could expect to see students achieve in school and aspire to do a variety of things. There would be natural variation among any given group—girls, boys, those in poverty, those in middle-class. I emphasize the goal of being unable to predict student patterns (e.g., achievement, participation, the ability to critically analyze data or society) based solely on characteristics such as race, class, ethnicity, sex, beliefs and creeds, and proficiency in the dominant language. Being unable to predict mathematics patterns based solely on certain student characteristics addresses issues of power (p. 153).

Secada and Gutiérrez help describe and illustrate notions of equity in the context of the historical differentiated academic achievement of marginalized students. Secada's definition includes a portrait of how these inequities have had broad implications for the lived experiences of African Americans and other marginalized groups in terms of housing, employment opportunities and healthcare. His work helps to show how these social structures affect negatively the lived experiences of the children from these groups thus implying that schools can offer "specialized opportunities," and that this choice either perpetuates these inequities or works to mediate them. Gutiérrez argues against defining equity as equality, noting that although reasonable and natural variation will exist between students' performances, this variation should not be predicted by the social indicators Secada describes. Allexsaht and Hart (2001) state that their definition for equity:

> …begins with the premise that all students, regardless of their race, ethnicity, class, gender, or language proficiency will learn and use mathematics…. Equity in mathematics education requires equitable distribution of resources to schools, students, and teachers; equitable quality of instruction; and equitable outcomes for students. Equity is achieved when differences among subgroups of students in these three areas are decreasing or disappearing (p. 93).

Although these distinctions are useful for differentiating work that focuses on striving to achieve equality from those that focus on moving toward equity, these definitions lack specificity and therefore can't provide guidance for identifying characteristics of equitable teaching in elementary mathematics classrooms, nor do these definitions offer resources for teachers or teacher educators for avoiding or eliminating these patterns of inequities for their students. In this dissertation study, I focus my analyses on equity at the instructional level in mathematics classrooms.

Working definition of equitable instruction for this dissertation. Although equity can be described in many different contexts, understanding the relationships between teaching and learning at the classroom level is paramount for the initiatives towards increasing educational opportunities in public schools. For example, Cobb and Hodge (2002) describe equity in the context of "access to opportunities to develop forms of mathematical reasoning that have 'clout' (Bruner, 1987) and enable students to participate in out-of-school practices in relatively substantial ways" (p. 252). Their definition is "concerned with how continuities and discontinuities between out-of-school and classroom practices play out in terms of access" (p. 252). Similarly, Esmonde (2007) defines equity as "the fair distribution of opportunities to learn in a classroom or cooperative group" (p. 9). Building on the definitions described in this section, I also define equity in terms of access and opportunities for learning.

To define “opportunities to learn,” I draw on Esmonde’s description (2006) stating that each student should have access to the resources and forms of participation he or she needs to move forward in their thinking and promote their understanding of the mathematics being taught. Research suggests that default patterns of instruction are often not effective at distributing opportunities to learn math equitably, and this may lead to only a few students being successful in learning mathematics proficiently, thus creating gaps in mathematics achievement (Ball, et al., 2003; RAND, 2003). By way of counterexample of default patterns of instruction, some teachers seek to provide equitable instruction by designing multiple forms of classroom participation and encouraging a range of mathematically productive contributions from their students (Boaler 2002, 2005; Darling-Hammond, 1998). These kinds of instructional considerations can broaden what it means for a student to be successful in mathematics and can thus promote equity by providing and supporting multiple ways for students to be cognitively engaged and contribute to the mathematical work of the class. Teachers vary in their ability to listen and interpret student thinking. Cohen and Lotan (1997; 1999) have noted that assigning competence by making mathematical thinking and work public includes labeling students' work as important and valuable to the collective mathematical work of the class. Teachers who are skillful at attending to the mathematically significant elements of a student’s explanation, attending to the integrity of the discipline of mathematics, while also recognizing and respecting the differences students bring to learning mathematics, generate mathematics content that is accessible to more students, thus promoting equity. In this way, these practices are considered to be features of equitable instruction.

For this dissertation, equitable instruction is defined as the practices that provide equitable access to challenging and meaningful opportunities for learning mathematics through instruction that increases participation and academic success, especially for marginalized students. I draw from two theoretical perspectives and interconnected frameworks for examining the relationship between mathematics, teaching, and equity. One focuses on the content knowledge needed for equitable instruction (Mathematical Knowledge for Teaching) and the other focuses on the cultural knowledge and sensitivities needed for equitable instruction (Cultural Modeling).

Theoretical Framework: Cultural Considerations of Mathematical Knowledge for Teaching

Many scholars in the last twenty years have studied classrooms as a context in which to intervene on issues of equity, especially related to uneven schooling experiences and educational outcomes based on race, culture, and social class. Early theories responded to the academic achievement differences between African American students and their White counterparts. Bank's work on multicultural education has expanded to also consider theories of culturally relevant and culturally responsive teaching (Banks, 2004; Gay, 2000; Irvine, 2003; Ladson-Billings, 1994; Nieto, 2002). These scholars argue that cultural knowledge can play an important role in designing and enacting equitable teaching. For example, Irvine's (2003) work describes how the notion of caring and being able to build important relationships with parents and community members support teaching and learning that is more culturally congruent with students' outside of school experiences. However, centrally missing from this set of theories are details about the depth and nature of disciplinary knowledge needed for high quality and equitable teaching.

Teacher knowledge, particular content knowledge, plays a critical role in high quality instruction (Hill, Rowan, & Ball, 2005). It is argued throughout this dissertation that equitable teaching requires equitable access to mathematics content. I also argue that providing equitable access requires a cultural perspective about who the students are and recognizes the experiences, resources, and perspectives they bring to learning. More specifically, I argue that in order to design and enact high quality equitable instruction, teachers must build bridges between what the students know and what they need to learn which requires knowledge of students, culture, and content. Thus, to provide equitable instruction, teachers must rely both on a solid knowledge of the subject matter as well as knowledge of their students' cultural lived experiences, and bring sensibilities and awareness of issues related to equity. In this way, the theoretical frameworks of Cultural Modeling and Mathematical Knowledge for Teaching provide the foundation for this dissertation.

Cultural Modeling. Cultural Modeling offers a unique framework for considering the dimensions of equitable teaching and the resources it requires. This framework provides an examination of the role of culture in considering how to design and enact equitable instruction. The Cultural Modeling framework is also useful in this dissertation because of it relies on African American students as examples given that although, "historically they are the most negatively stereotyped groups in this country… they share many issues with Latinos/as, Native Americans, Asian Americans, Pacific Islanders, as well as European Americans who experience persistent intergenerational poverty" (Lee, 2003, p.7) In the Cultural Modeling framework, students' cultural perspectives, experiences, and practices (especially African American children and youth) are not seen as deficits as they generally are in most American classrooms, rather they are seen as strengths that can be leveraged as resources for learning subject matter knowledge. As Lee (2007) explains that Cultural Modeling is a framework for designing instruction that makes explicit connections between students' everyday knowledge and the demands of subject-matter learning (p.123). One central purpose of this framework is to organize participation structures in classrooms that allow students to "communicate in ways that are culturally familiar" while studying the knowledge and skills required to be academically successful in a particular discipline. For example, the Cultural Modeling framework scaffolds and supports students work to examine texts by using their everyday experiences as leverage for learning strategies that are needed to examine other "canonical" texts (e.g., Lee, 2001, 2007). The Cultural Modeling framework contributes to the analyses in this dissertation by articulating a dimension of cultural knowledge that (1) makes teaching relevant to students' life experiences and (2) builds connections with the required content to be learned. Moreover, this framework is based on the premise that students bring to the classroom a rich array of knowledge that is useful for learning. Different than the theories of multicultural education and other previous work on diversity and equity, Cultural Modeling deliberately analyzes the relationship between content knowledge, pedagogical content knowledge, and knowledge of culture. As Lee explains, "cultural modeling takes the position that one cannot imagine points of leverage between everyday experiences and subject matter learning without understanding the structure of the discipline in terms of both breadth and depth" (Lee, 2007, p. 111). Lee further explains that 'breadth' includes a "declarative knowledge of the range of topics, the range of strategies available for solving problems and the range of debates in the discipline"; and 'depth' "includes understanding what concepts are most generative, meaning that if you know these concepts well you can do a lot of work in the discipline" (p. 112). Although her work primarily focuses on English, she offers mathematical examples for illustrating what she means by depth:

> This includes understanding issues in a discipline (such as):
>
> - How are concepts related to one another (e.g. the inverse relationship between multiplication and division)?;
> - What are the procedures for determining and acting on those relationships (e.g. using repeated additions for multiplication and repeated subtraction for division)?;
> - Under what conditions are such procedures useful (e.g. distinguishing when the situation requiring divisions asks how many equal parts versus how many in each part)?;
> - What is a range of acceptable ways to operate on and with these concepts (e.g. solving a problem through estimation or through explicit computation; when is it sufficient to solve part of the problem to arrive at a reasonable response versus when the entire problem must be solved)?;
> - How does one extrapolate from these relationships to unknowns or less explored problems? (Lee, 2007, p.113)

Lee (2007) argues "from the perspective of Cultural Modeling, conceptualizing resources that students already bring with them from their experience outside of school is a fundamental element in the teachers' PCK (pedagogical content knowledge) toolkit" (p.120). She describes that

> …PCK involves understanding the following: developmental progressions, enduring misconceptions and naïve theories held by youth and novices generally, multiple routes to maximize opportunities to learn, how to assess what learners understand and don't understand, and the ability to design multiple routes to maximize opportunities to learn, and to design learning environments that allow learners to draw supports on multiple resources (Gutièrrez, et al., 1999; Lee, 2007, p. 120).

More specifically, when teachers "re-contextualize instruction in ways that draw on what students already know and value, it has leverage for all types of readers" (Lee, 2007, p. 123). Lee's work begins to illustrate the relationship between pedagogical content knowledge and attention to issues of equity. Now I turn to the second framework that helps to explore the practices of equitable mathematics instruction, Mathematical Knowledge for Teaching.

Mathematical Knowledge for Teaching. This dissertation is designed to explore what constitutes equitable mathematics instruction and, as such, must carefully consider the role of teacher knowledge for designing and carrying out mathematically skillful instruction that focuses on students' learning. Because all teaching brings together concern for the content with concerns for the students, teachers' knowledge plays a central role in what is taught, as well as, how students learn (Hill, Rowan, & Ball, 2005). Designing and implementing mathematics instruction for students who most depend on school for their access to learning the skills needed to develop mathematical proficiency requires special attention on the part of the teacher, thus the teacher needs specialized skills and resources to address their needs. Children who live in poverty, many of whom are children of color and are linguistically diverse, often face tough educational conditions. These students often receive education that is underfunded, lacks academic rigor, and occurs in schools with poor building structures, large classes, and poorly prepared teachers (Kozol, 2001; 2005; Lee, 2001). In addition, in the last twenty years there has been an increase in academic demands with more rigorous academic standards and higher expectations for student's academic performance. This shift requires students to move from simply meeting minimum competency standards to high proficiency standards for all students. In mathematics, this means ambitious teaching that requires all students to move toward mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). Teaching for deeper understanding of mathematical ideas places significant demands on what teachers need to know, about mathematics and about the students in their classrooms. Researchers over the last several years have argued that one of the most important resources for improving student learning is improving teachers' knowledge (Cohen, Raudenbush, & Ball, 2002; Hill, Rowan, & Ball, 2005; Ball, Hill, & Bass, 2005; Shulman, 1987) and as such, many efforts to improve students' achievement in mathematics have focused on improving teachers' content knowledge (Ball, Lubienksi, & Mewborn, 2001; Hill & Ball, 2004).

Building on Shulman's theory of pedagogical knowledge for teaching (1985), researchers at the University of Michigan began to refine and articulate a theory of teachers' mathematical knowledge rooted in analyses of the work of teaching, rather than in imperatives of what teachers "ought" to know. These scholars study the mathematical demands of teaching by studying instruction (Ball, 1990; Ball, Lubienski, & Mewborn, 2001; Ball & Bass, 2000, 2003, 2009; Ball, Hill & Bass, 2005; Ball, Sleep, Boerst, & Bass, 2009; Ball, Thames, & Phelps, 2008). Based on empirical analyses of the work of teaching, they developed taxonomy of tasks that teachers carry out routinely that seemed to be mathematical in nature (e.g., posing questions, analyzing students' errors, choosing representations) and then analyzed the kind of mathematical skill, knowledge and sensibilities required to carry out these tasks effectively. Mathematical knowledge for teaching (MKT) is defined as the mathematical knowledge demanded by the actual work of teaching (Ball & Bass, 2003; Ball, Thames & Phelps, 2008). Here, the word knowledge means something broader than just knowledge of facts and topics, it includes the skills, sensibilities, and habits of mind required for effective teaching. This includes not only the mathematical knowledge teachers directly teach students, but the subject matter knowledge that supports that teaching--for instance, why and how specific mathematical procedures work, or the ability to analyze unusual solution methods and student work. Work of teaching means more than just what the teacher does inside the classroom, it includes other tasks of teaching such as planning, grading homework, meeting with parents, or talking about student work which require mathematical knowledge (Ball, Thames, & Phelps, 2006; Hill, Rowan & Ball, 2005). Therefore, the theory of mathematical knowledge for teaching contributes to this study of equitable instruction in elementary classrooms by identifying and specifying the content-specific knowledge needed.

Summary. Cultural Modeling helps to explain how and why teachers need to know and use students' cultural perspectives, experiences and practices for teaching. Mathematical knowledge for teaching (MKT) helps to articulate the categories or domains of mathematical knowledge that are demanded by the actual work of teaching. These theoretical perspectives are grounded in the work that teachers have to do and they have illuminated how this work places demands on teachers' capacities. Therefore, the complementary frameworks of MKT and Cultural Modeling provide tools for studying the resources needed to produce equitable teaching in elementary mathematics. I hypothesize that equitable teaching requires specialized skills that are central to both the Cultural Modeling and MKT frameworks. These specialized skills include methods of teaching, ways of listening to students in ways that are not neutral or generic, but are culturally relevant and appropriate. Addressing issues of equity is clearly an instructional problem and instruction is about both subject matter and about learners. Therefore attention to instruction requires attending to the interplay between the content and the learners. This dissertation explores the relationship between the math and learners in trying to teach equitably. The studies in this dissertation explore a way to look at this relationship.

Dissertation Study

Many of the analyses in this dissertation focus on the development and use of a specific observational tool developed by the Learning Mathematics for Teaching (LMT) project co-directed by Deborah Ball, Hyman Bass, and Heather Hill and of which I was a member for several years. This tool, a detailed set of observational video codes, serves as a set of proposed indicators of equitable mathematics instruction. To construct the video codes, the first task involved reviewing the literature for specific features of instruction suggested as central for equitable mathematics instruction. Subsequently, these features were used to create an initial blueprint for this tool. Next, project researchers refined this blueprint by using it to guide observations of mathematics teaching with an eye to see if additional features of equitable instruction appeared to be missing from the blueprint, and if the features suggested by the literature seemed to be relevant and necessary for equitable mathematics teaching. The subsequent analyses in this dissertation use this observational tool to develop an argument for defining what constitutes equitable mathematics instruction.

Research Questions and Hypotheses

The central question for the study is: Can the mathematical quality of a teacher's instruction be measured together with attention to the equitableness of the instruction? Three sub-questions guide analyses in this dissertation:

1. What is known about equitable mathematics instruction in elementary schools? The purpose of this question is to guide the literature review about developing the equity video codes.

2. What is the relationship between a teacher's scores as measured by mathematical quality and equity (MQE) video codes and teacher's levels of mathematical knowledge for teaching (MKT)? The purpose of this question is to explore the relationship between practices of equitable mathematics instruction and teachers' mathematical knowledge.

3. What might constitute examples and counter examples of equitable high quality mathematics instruction? These examples and counter examples are described in greater detail in the case study analyses.

Data Sources

The data used in this dissertation were collected as a part of a larger project designed to develop and validate measures of teachers' mathematical knowledge (LMT, 2006). The data collection methods and sources are described briefly in this section and in greater detail in chapter 2.

Ten teachers, nine from elementary classrooms and one from a middle school classroom, were recruited to participate in the Learning Mathematics for Teaching video study project based on their commitment to participate in a specific mathematics professional development institute (Hill & Ball, 2004). A range of data sources were collected and used for analyses in this dissertation including: results from paper-and-pencil survey measures designed to measure mathematical knowledge for teaching, video records from nine mathematics lessons, data from the curriculum materials used in those lessons, post-observation interviews audio and video taped after each lesson, and one general interview on teaching topics such as working with diverse learners and experiences in professional development workshops.

Measures of Mathematical Knowledge for Teaching (MKT), Mathematical Quality of Instruction (MQI), and Mathematical Quality and Equity (MQE).

In this section I briefly describe the measures of Mathematical Knowledge for Teaching (MKT), Mathematical Quality of Instruction (MQI), Mathematical Quality and Equity (MQE), and then briefly describe the methods used for the analyses in this dissertation.

I used previously calculated teachers' scores for MKT and MQI (Hill, et al., 2008). The measures for MQE are a central feature of this dissertation and are described in greater detail in Chapters 2 and 3. It is important to note that the Learning Mathematics for Teaching (LMT) measures of Mathematical Knowledge for Teaching (MKT) have been linked to gains in student achievement in a study of school improvement[1] (Hill, Rowan, & Ball, 2005). As such, the analyses in this dissertation extend and leverage the work produced in the LMT project to help for this study of equitable mathematics instruction.

Mathematical Knowledge for Teaching (MKT) score. The measures of MKT used for this dissertation are the paper and pencil items developed by the Learning Mathematics for Teaching project (e.g., Ball & Hill, 2004; Hill, Ball, & Schilling, 2008;

[1] As described in Hill, Ball, Blunk, Goffney & Rowan (2007), " students of teacher who answered more items correctly gained more over the course of a year of instruction...indicating that the above-average teacher 'added' an effect equivalent of that of 2 to 3 extra weeks of instruction to her students' gain scores." (p.109) (Hill, Rowan, & Ball, 2005)

Hill, Rowan, & Ball, 2005). The form of items used in these analyses were developed in 2002 and contained items measuring both common content knowledge and specialized content knowledge focused on number and operations, geometry and algebra (Hill, et al., 2007). IRT scores of teacher's MKT produced for a related study were used for the work in this dissertation.

Measures of Mathematical Quality of Instruction (MQI). The measures of MQI are described in greater detail throughout Chapter 2. Briefly, each lesson was coded in five minute segments using the Mathematical Quality of Instruction observational instrument which has 33 codes designed to represent aspects of mathematical quality. Lesson scores were generated calculating these codes and teacher level scores were generated that represent the average of all nine lesson scores (Hill, et al., 2008).

Measures of Mathematical Quality and Equity (MQE). This measure is the primary focus of this dissertation and was computed using straightforward calculations comparing the presence of specific equity codes with the total number of opportunities available in a particular lesson. The codes in this frame of the instrument are designed to represent teacher's use of mathematics to promote equity. As such, this collection of codes represents core ideas about both mathematical knowledge for teaching and cultural modeling.

Two different analytic methods are used in this dissertation. The first was a validation study in which I calculated an 'equity score' for each of the lessons in the second wave of data collection.[2] The MQE score was calculated by using the coding results of thirty lessons from Wave 2 of the data collection, and comparing instances of "present" features to the total number of codes available for a lesson. These results revealed quantitative differences in classrooms in terms of how frequently a teacher's instruction illustrated features of equitable instruction, and thus validating the claim that teachers who earn high a high "equity score" promote equity through their mathematics instruction.

The second analytic method involved a set of comparative case studies. The multiple-case design allowed the testing of hypotheses regarding the mathematical demands on the teacher to enact equitable mathematics instruction. This method was selected because, as Yin (2003, 2009) suggests, case studies can illuminate possible links about the resources needed to enact equitable mathematics instruction (p. 46). As such, the study design produced contrasting results, revealing both teachers' instruction that is equitable (or promotes equity) and teacher's instruction that is inequitable (or does not promote equity). In this way, evidence for supporting my hypothesis generalizes to theory, but not to a population of teachers.

A brief explanation of these cases and the assertions are included here. The first case is Karen who represents a case of high mathematical knowledge, as evidenced by her high scores on the Learning Mathematics for Teaching paper-and-pencil measures, but poor skills for working on equity, as measured by her low scores on the MQI equity video codes. Analyses of Karen's teaching provide evidence regarding how mathematical knowledge is an important resource for equitable mathematics instruction, but is not the only resource needed. The second case is Rebecca, a teacher who describes herself as trying to deliberately address issues of equity through her mathematics instruction. However, her scores on the LMT paper and pencil items indicate that she has low mathematical knowledge. For several of the equity video codes (such as 'expressed expectation that everyone will be able to do the work'), Rebecca scores higher than most of the other teachers, and her interviews indicate very deliberate and careful thinking about the needs of her diverse students. However, Rebecca's case helps to illustrate how beliefs and commitments alone are not enough of a resource to produce equitable mathematics instruction. The third case is Lauren, who earned high scores on both the LMT pencil-and-paper items and on the MQI equity video codes. Analyses of Lauren's instruction show significant attention to equity issues that arise in her classroom and also illustrate ways in which Lauren is able to leverage her substantial mathematical knowledge and her concerns for equity as resources to design and enact equitable mathematics instruction.

Cross Case Analysis

The cross cases analysis in Chapter 4 focuses on specific examples from each of these three case study teachers' instruction that represent either equitable or inequitable mathematics instruction in ways that distinguish features of instruction that promote or impede equitable engagement in the mathematics. The analyses discussed in Chapter 4 are organized around three themes of equitable instruction: mathematically rich and rigorous instruction, teachers' responses to students' mathematical ideas, and equitable participation. This chapter offers commentary and analysis of examples and counter examples of these three themes.

Organization of the Dissertation

This dissertation is organized into five chapters: In Chapter 1, I introduced the problem, framed the studies and analyses conducted in this dissertation, and provided details about the central focus and purpose of each chapter. Chapter 2 details the construction of a set of analytic video codes focused on issues of equity, including analyses from two smaller validation studies that investigate the mathematical quality of the codes. Chapter 3 offers three examples of deliberate study of instruction in the form of case study investigations. Chapter 4 provides a cross case analysis of these three teachers, focusing on three themes: equitable participation, rich mathematics and responding to students. Chapter 5, the conclusions chapter, summarizes the insights gleaned from these studies and sketches the implications of this work, its next phases, as well as some of the limitations of these studies.

Conclusions and Contributions

The central contribution of this dissertation is a set of features of mathematics instruction that characterize instruction in which high quality mathematics is taught with specific attention to concerns for equity. Results from this dissertation offer significant resources for the field of teacher education including suggestions for revising mathematics methods courses. Additionally, the use of the equity video codes as an observational tool can be extended for use as a tool to help practicing teachers evaluate the quality of their own mathematics instruction with special attention to issues of equity and using mathematics with diverse students.

CHAPTER TWO:

CONSTRUCTING AND VALIDATING THE LEARNING MATHEMATICS FOR TEACHING PROJECT MATHEMATICAL QUALITY AND EQUITY (MQE) VIDEO CODES

Introduction

Results from national assessments show large and persistent disparities in mathematics achievement related to social class and race (Lee & Wong, 2004; Lubienski, 2002). These differences in achievement are not necessarily due to innate deficiencies in students, but to the quality of the opportunities and instruction provided to them. These disparities are most pronounced in the "highest-need" urban classrooms where students are more dependent on school for access to higher education and social mobility.[3] Additionally, these differences in mathematical achievement have far reaching implications because they become a barrier for access to higher education and social mobility for students in underserved schools and classrooms, particularly for students of color. In effect mathematics has become a "gate keeping" subject for access to higher education (Moses & Cobb, 2001).

Policy makers, researchers, and general members of society at large contend that mathematics can be a critical lever for educational progress, instead of seeing

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[3] In *Abbeville et al v. State of South Carolina* (1999), Dr. Gloria Ladson-Billings testified, without explicitly referencing racial identity, how significant school was in the lives of some students. In particular, she claimed that students that attend high-need urban schools depend on school for access to learning, opportunities and access to higher education, and social mobility.

mathematics as culturally neutral, politically irrelevant, and mainly a matter of innate ability (Ball, Goffney, & Bass, 2005). As Adler (1999) suggests,

> …the promotion of equity in school mathematics is a function of the extent to which more diverse groups of student are able to construct and enact strong mathematical identities, which in turn is a function of pedagogical practices in mathematical practices. Equity, in mathematics education, demands a focus on social and pedagogical practices and the production of agency, identity, and ways of knowing (p. 48).

Other scholars have focused on culture as a resource for learning mathematics claiming that students bring a range of cultural and linguistic resources and perspectives to learning (Adler, 2001; Ball, Hoover, Lewis, Bass, & Wall, 2003; Cobb & Hodge, 2002). Studying teaching practices illuminates the ways which teachers must account for these of culture, perspective, and equity during instruction and how these cultural and linguistic resources and perspectives may promote student academic engagement and achievement, thus promoting equitable participation.

Access to quality mathematics instruction is directly related to issues of equity and social justice. Overall, this dissertation contends that mathematics, and the ways in which teachers teach it, is a key resource for building a socially just and diverse democracy (Ball, Goffney, & Bass, 2005). This argument illuminates and exemplifies the significant role that teachers' knowledge and use of skills plays in helping students navigate the bridge that would enable students, especially those most dependent on school for social mobility and access to higher education, to be successful in the classroom.

The above paragraph articulates how mathematics is important and how high quality instruction plays a crucial role for improving students' academic achievement. It also describes how and why teachers' knowledge of students' culture and outside of school experiences matters, especially for improved student participation. I turn now to consider another crucial factor for improving student achievement, teacher knowledge. More specifically, research has shown that high quality teaching requires a specific type of mathematical knowledge (Ball, 1993; Ball & Bass, 2000; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Hill, Rowan, Ball, 2005). Mathematical knowledge for teaching (MKT) is described in greater detail later in this chapter. I argue that teaching mathematics in equitable ways requires substantial mathematical knowledge, in addition to a kind of cultural knowledge and sensitivity to their students, that enables teachers to broaden notions of what it means to be successful in mathematics by helping every student to participate in learning the mathematics content being studied.

This chapter has three main sections. First is a detailed description of the Learning Mathematics for Teaching (LMT) video study. This section will provide details about study design, teacher sample, data collection, and a brief description of the instruments.[4] The second main section provides a detailed analysis of how the mathematical quality and equity video codes were developed, revised, and refined. The third and final section offers quantitative evidence about the validity of the mathematical quality and equity video codes and an explanation of how using the mathematical quality and equity video codes helps to reveal important differences in mathematics instruction in elementary classrooms.

[4] The data described in this section, and used throughout this dissertation study, was primarily collected for the Learning Mathematics for Teaching video study project. Many of analyses conducted, in particular those focused on equity and frame five of the MQI instrument, are uniquely for the purposes of this dissertation, but draw from previous work conducted by project members.

Study Description

Learning Mathematics for Teaching Project: Paper and Pencil Measures

The data collected for this study were a part of a larger measures-development project with multiple goals. This project, *Learning Mathematics for Teaching*, investigates the mathematical knowledge needed for teaching, and how such knowledge develops as a result of experience and professional learning (LMT, 2006). By studying instruction, these researchers developed a classification of tasks of teaching that teachers routinely carry out that are considered to be mathematical, and then analyzed the kind of mathematical skill, knowledge, and sensibilities required to do these tasks effectively. Therefore, Mathematical Knowledge for Teaching is defined as the "mathematical knowledge needed to carry out the recurrent tasks of teaching mathematics to students" (Ball and Bass, 2003; Ball, Bass, & Hill, 2005; Ball, Thames & Phelps, 2008), where "teaching" is defined as everything teachers do to support the learning of their students, including the interactive work of teaching lessons and the tasks that arise in doing that work.

One of the primary instruments used in this project is a set of paper-and-pencil multiple-choice measures of teachers' mathematical knowledge for teaching (MKT[5]). These paper-and-pencil measures are designed to reflect real mathematics tasks that teachers face in classrooms.

> …These measures are different from conventional mathematics tests in that they not only assess whether teachers can solve the problems they directly teach to children, but also how they navigate through some of the mathematical tasks unique to teaching—for instance, assessing student work, representing numbers and operations, and explaining common mathematical rules or procedures (Ball & Bass, 2003; Ball, Hill, & Bass, 2005).

Learning Mathematics for Teaching Project: Mathematical Quality of Instruction Observational Instrument

The LMT project conducted several studies to validate these pencil-and-paper measures of teachers' mathematical knowledge for teaching (Hill, Dean, & Goffney, 2007; Hill, Schilling, & Ball, 2004;). This work initially was designed to validate the paper-and-pencil measures, and to demonstrate that the paper-and-pencil scales do differentiate between teachers whose classroom performance differs. The video codes were designed to validate whether a teacher's performance on multiple choice assessment of mathematical knowledge for teaching was related to the mathematical quality of instruction. Members of the LMT project knew from past experiences as classroom teachers and as researchers observing classroom teaching that high scoring teachers generally offer mathematically rich instruction and low-scoring teachers tend to make more mathematical errors. The project sought to answer whether higher scores predicted better quality mathematics instruction and how higher or lower scores measure performance and how this might have related to students opportunities for learning (LMT, 2006). The LMT Mathematical Quality of Instruction (MQI) measures are an observation-based instrument designed to quantify the quality of the mathematics in instruction. Analyses of this work show that lesson characteristics highlighted with this tool are a significant indicator of an individual's mathematical knowledge (Hill, et. al., 2008). As such, a teacher's ability to enact these features in appropriate ways depends largely on the level of their mathematical knowledge. A central purpose of these measures has been to explore the effects of mathematics knowledge on classroom practice.

The Mathematical Quality of Instruction (MQI) observational instrument, developed by members of the Learning Mathematics for Teaching Project (LMT), comprises five frames: 1) Instructional format and content, 2) Teacher's knowledge of mathematical terrain of the enacted lesson, 3) Teacher's use of mathematics with students, 4) Curriculum's mathematical guidance for teachers, 5) Teacher's use of mathematics to teach equitably (Hill, et al., 2008). Elsewhere, members of the LMT project have designed studies and written about the first four frames in the MQI instrument (i.e. Hill, et. al., 2008). This study focuses on the fifth frame, 'teachers' use of mathematics to teach equitably' which consists of codes that focus on features of instruction that attend to mathematical quality and equity. It is important to note that the codes and glossary definitions are underspecified for this frame as compared to the codes and glossary descriptions about the other four frames in the MQI instrument. By "underspecified" I mean that, whereas the other frames require two layer coding (first indicating if the code is present or not-present, and a second layer that specifies if the code was appropriate or inappropriate), the equity video codes only indicate if the code were present. Also, two of the original equity video codes were removed from the final frame for validity reasons because members of the project were unable to use the codes reliably.[6] In this chapter, I describe how this analytic tool, consisting of video codes

I hypothesize that these codes were not used reliably for two main reasons. The first is that the feature being described is too complex for simply coding 'present' or 'not present'. The second reason is because many members of the LMT project had few experiences working with underserved minority and linguistically diverse students or deliberately studying issues of equity and therefore lacked expertise in designed to measure teachers' use of mathematics in order to teach equitably, was developed.

Dissertation Study Design

Overall, the video study component of the *Learning Mathematics for Teaching Project* (LMT Project) involved following a group of teachers over the span of two academic years and through their summer experience in a mathematics professional development institute. The mathematics lessons for this group of ten teachers were videotaped for three consecutive days, on three separate occasions: once, in 2003, before they attended a week-long mathematics-intensive professional development institute[7], and then again on two different occasions after they have attended the professional development, once in mid-fall and the other in late spring of the following year. Study of video data collected from these classrooms served as the context for validating the LMT paper and pencil measures. This validation work[8] led to the development of the Mathematical Quality of Instruction (MQI) observational tool and is described in greater detail later in this chapter.

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these areas. The deleted codes were 'teacher elicits, uses, and values multiple forms of participation and kinds of mathematical contributions' and 'teacher makes social comparisons'.

[7] Teachers attended Mathematics Professional Development Institutes in California. For more details, see Hill and Ball (2004).

[8] The video codes were developed as a validation study to determine if teachers who scored higher on the LMT paper and pencil items produced higher quality instruction. See publications cited in this chapter for additional details.

Research Questions and Hypotheses

The central question for this dissertation is, "Can the mathematical quality of a teacher's instruction be measured with attention to issues of equitable instruction?" Three sub-questions guide this analysis. The first question, "What is known about equitable mathematics instruction in elementary schools?" guides the literature review about developing the mathematical quality and equity video codes. The second question, "What is the relationship between a teacher's scores on the mathematical quality and equity video codes and the quality of their mathematics instruction?" frames analyses of two possible correlations. One analysis examines how teachers fared on the equity scores against the other four frames of this observational tool that measured high quality mathematics instruction (as measured by the Mathematical Quality of Instruction tool); and the second analysis explores the relationship between equitable instruction and teachers' mathematical knowledge.

Teacher Sample

Through previous work, researchers on the LMT project identified and recruited ten[9] elementary teachers to participate in this study based on their commitment to attend particular professional development workshops that were designed to increase teachers' mathematical knowledge. As such, this is a convenience sample—but one that was expected to represent a wide range of mathematical knowledge for teaching. Based on early results of the LMT paper and pencil measures their initial levels of mathematics

[9] Through previous work, a number of sites of professional development were identified by the Learning Mathematics for Teaching Project, that produced evidence of substantial increases in mathematics understanding of those teachers attending their institutes. Ten teachers who planned to attend two of these sites for math professional development were randomly selected to follow before, during and after their summer professional development experiences.

knowledge for teaching differed. These levels reflected a large range (22nd to 99th percentile) in their mathematical knowledge for teaching, and therefore supported the claim that this group of teachers does in fact reflect a wide range of MKT. The teachers in this study taught in elementary classrooms ranging from second to sixth, with the exception of one, Rebecca, who moved to 8th grade in the second year of the project. Seven teachers taught in districts serving families from a wide range of social, economic, and cultural backgrounds, including English language learners. The other three teachers taught in the same school in a small, middle to upper-class, primarily Caucasian district. The teachers in this study were not selected based on the quality of their teaching, but instead were identified based on their commitment to attend select professional development workshops and to participate in our study.

Data Collection

Teachers were taped by professional videographers from Lesson Lab[10] three times during the spring of 2003 (Wave 1), all prior to a week-long mathematics-intensive professional development, then three times in the fall of 2003 (Wave 2) after summer professional development, and again three times in the spring of 2004 (Wave 3). Therefore, the data set used in this study is comprised of 9 mathematics lessons for each of the ten teachers. The lessons vary by length by teacher and lesson, however, generally lessons ranged from 20-80 minutes. The spacing of the observation allowed me to see a range of each teacher's instruction across different subject matter content as well as any learning or changes in practice that may have developed from both professional

[10] The quality of the videos from Lesson Lab was quite high. These videos were used to generate transcripts that covered almost 100% of the teacher utterances and over 80% of student dialogue (p.114, Hill, et. al., 2007). These transcripts were a central data point for the analyses in this dissertation.

development and from classroom practices. Observing consecutive lessons shows how teachers develop and connect related mathematical material over time for instructional purposes. For each of these taped lessons, teachers completed a post-observation interview about the planning process and mathematics of the lesson. Additionally, for Waves 2 and 3, curriculum materials (i.e. printed materials used by the teacher for the lesson, such as copies from district adopted materials, quizzes, homework, etc.) were collected from the teachers for each of the lessons. During Wave 2, each teacher participated in one, three hour-long structured interview, that probed their ideas and beliefs about mathematics teaching, experiences in professional development, and ideas about equitable mathematics teaching. Teachers also completed the Learning Mathematics for Teaching pencil-and-paper measures at the beginning of the study, after their participation in professional development, and at the end of taping for Wave 3 lessons.

Developing the Mathematical Quality and Equity (MQE) Video Codes

The next section in this chapter provides a detailed analysis of how the mathematical quality and equity video codes were developed. There are two major categories in this section. The part I describes how the video codes were designed, including the purpose for designing this tool and how project members worked together for the initial design and refining of the codes. The second part elaborates on the mathematical quality and equity video codes by describing how each code or cluster of codes was created. The structure there follows a similar pattern, offering the name of the code and definition, a description of the supporting literature references, and the purpose of the code. This section also describes different versions of the code, and then offers classroom illustrations of examples or counter examples of the codes. This section of the chapter concludes with a brief summary describing the issues that arose with the design or use of the code or set of codes.

Why Develop an Observational Instrument?

As mentioned previously, the broader work of the Learning Mathematics for Teaching project sought to identify what mathematical knowledge is useful to teachers and when and how teachers draw upon and use such knowledge during instruction (Hill, Ball, & Schilling, 2008; Hill, et. al., 2008). In order to examine how teachers use their knowledge of mathematics through instruction, this project drew data from two primary sources, primary data on teaching and extant literature. These efforts to develop video codes are described in two phases of work, first efforts that focused on mapping the terrain and the second effort that focused on using literature to support video codes that emerged in the initial design in an effort to refine, validate, and revise this tool.

Mapping the Terrain

The Learning Mathematics for Teaching project is comprised of research mathematicians, former elementary teachers, research scientists, and graduate students. This diverse set of experiences provided a substantial set of resources to guide the initial efforts to develop video codes. The first strategy for developing codes involved systematically listing elements of mathematical knowledge for teaching and designing a scheme for “grading” particular elements in each lesson (Research meetings July 25, 2003, and August 25, 2003). Watching short segments of videos from the classrooms of the teachers in the LMT data set suggested categories less often described in existing mathematics education research. Many lessons, for instance, had widespread problems with mathematical language, from misuse of technical terms to failing to clarify differences between mathematical and non-mathematical definitions for terms. In other lessons, teachers demonstrated considerable skill in their teaching, such as carefully choosing numbers and sequencing mathematical tasks for scaffolding student learning. Observations also suggested that teachers vary in how they attend to, interpret and handle their students' oral and written productions (e.g. students' questions in class, difficulties and misconceptions, etc.) There were also considerable differences between teachers' abilities to make connections between classroom work (following a procedure; using manipulatives) and the mathematical idea or procedure the work was meant to illustrate. The initial goals of developing codes were three-fold; 1) to track on mathematical knowledge that shows up in teaching, including agility or fluency in its use, 2) to watch for places where the teachers encounter mathematical difficulties, and 3) to develop knowledge of the mathematical issues and problems that arise in teaching. After the first few meetings and some initial watching of videotape by most of the members of the LMT project, four main categories of codes emerged corresponding to the following questions:

1. What is the teacher's command of the mathematical terrain of this lesson?
2. How does the teacher know and use mathematical knowledge in dealing with students?
3. How does the teacher know and use mathematical knowledge in using the curriculum?
4. How does the teacher know and use mathematical knowledge for teaching equitably?

In an effort to design a tool that would answer the fourth question above, eight sub-codes were developed that focused on mathematical knowledge for teaching and equity. They are: contexts sensitive to students' experiences, forms of participation that either privileged or are inclusive, demand and support high expectations, valuing of students and mathematics, explicitness of key features of instruction, tasks presented in ways that enact high expectations, organization of instruction, and pacing (allowing adequate time for students and tasks). Each code was initially rated using a five-point Likert scale ranging from "not applicable" to "all of the time." During subsequent meetings, these initial draft codes were revised and refined after project members used them to code segments of classroom teaching from the LMT video study data set. This process of watching teaching while using the codes revealed nuances and new categories. Revisions to existing categories were made and are described in greater detail below.

Using the Literature to Refine, Validate, and Revise the Mathematical Quality and Equity Video Codes

One of the initial purposes of developing the system of codes to measure classroom instruction was one of a four-tiered effort to measure teachers' mathematical knowledge for teaching, however, early feedback suggested that this tool might be of use to the broader field of mathematics teacher education. Although the teachers in this study were conveniently sampled, as described in the study description section above, it was expected that the video data used in this study would be representative of the range of instruction in typical American classrooms, and thus a goal was to design a tool that would be generalizable. Through a series of research meetings over the next twenty-one months, LMT project members studied selected readings with an eye towards developing new codes, and revising and refining existing codes. In addition to revising the codes, two other important changes are worth noting. The first revision was a shift from using a Likert scale to using a coding system based on whether the actions indicated by the code were “present” (P) or “not-present” (NP). This coding system was insufficient because it was necessary to evaluate, 1) whether elements that were present were also mathematically acceptable, or 2) whether they contained errors or mathematical flaws, and 3) whether elements that were absent should have occurred in order for “instruction to reasonably proceed” (LMT, 2006). Therefore, for two frames in the final version of the Mathematical Quality of Instruction observational tool, codes were evaluated by four criteria: present/ appropriate (PA), present-inappropriate (PI), not present appropriate (NPA), not-present inappropriate (NPI). However, due to difficulties with reliability among project members, it was decided that the equity video codes, should be written in the positive indicating that the element of instruction should occur, therefore this frame only has two indicators, “present” (P) and “not-present” (NP). Based on evidence from literature and on multiple reliability analyses conducted by LMT project members (Blunk, 2007)[11], the final codes in this frame are: real world contexts or examples, explicit student tasks and work, explicit talk about the meaning and use of mathematical language, explicit talk about ways of reasoning, explicit talk about mathematical practices, instructional time is spent on mathematics, teacher encourages diverse array of mathematical competence, teacher emphasizes student effort and conveys message that effort will eventually pay off, teacher encourages and gives opportunities for student to work autonomously, and expressed expectation that everyone will be able to do the work. The remainder of this section offers examples of how scholarship was used to develop, inform, and validate the mathematical quality and equity video codes.

This particular segment of the chapter is organized into four major sections each representing different domains in which the mathematical quality and equity video codes were designed. The first section, area 1, focuses on teacher expectations for students' mathematical effort and competencies and is designed from a select review of research on teacher beliefs and motivation, providing insights into how teachers' perceptions and beliefs enacted in practice affect not only how students learn mathematics, but also which students are able to learn challenging mathematics. The next major section, Area 2, reviews select mathematics education research on attention to context and the mathematical rigor of tasks. This section will discuss recent scholarship about the use of contextualizing mathematics problems and instruction as a means of making mathematics more "interesting" and accessible for culturally diverse students. Mathematics practices and language: Use and implications for equitable mathematics instruction comprises Area 3. The final topic examined in this section, Area 4, explores the significance of access, participation structures and group work and the role attending to these issues plays in teaching and learning mathematics.

Area 1: Teacher expectations for students' mathematical effort and competencies. Many areas of research have offered contributions toward a collective understanding of the dynamics that influence motivation and achievement in education. Research has linked teachers' attitudes and beliefs to students' opportunities for learning (Oakes, 1983; Rist, 1970). The relationship between teachers' attitudes and beliefs with students' opportunities for learning is more pronounced when considering how teacher expectations for students' mathematical effort and competencies play out during classroom instruction (Ladson-Billings, 1994, 1997; Malloy, 1997). Teaching in equitable ways involves challenging teachers' beliefs about learning, where being successful in school is not based on innate abilities, but instead is focused on student effort. Moreover, pursuing complex and challenging mathematics content with all students depends on a teacher's ability to scaffold and enact high expectations for students' participation in class mathematical work and discourse (Irvine, 2003).

Teachers reflect their expectations for students' engagement, participation, and achievement in critical ways in elementary classrooms. Producing equitable instruction requires better understanding of the specific moves, strategies, and ways of treating students that will increase equitable participation and achievement (Ball *et. al*., 2005). Researchers studied the ways in which particular teaching practices and mathematical dispositions appeared to encourage students to take themselves seriously as mathematics learners. Results of this work prompted the design of codes for considering what teachers might to manage differences among students in ways that lead to students developing skills for becoming mathematically proficient.

The literature on teachers' expectations led to the following four codes: (1) "expressed expectation that everyone will be able to do the work, " (2)'teacher emphasizes student effort and conveys message that effort will eventually pay off,' (3) 'instructional time spent on mathematics,' and (4) 'teacher encourages and gives opportunities to work autonomously'. The third code was designed based Ladson-Billings' advice regarding teachers' efficient use of instructional time, and is defined as when the teacher spends most (over three-fourths of the segment) 'instructional time is spent on mathematics' or setting up a mathematical task, rather than on administrative or organizational matters, i.e. passing out papers, or resolving student confusion over poorly presented tasks, i.e. answering the question, 'what do I color?' (Ladson-Billings, 1994).

The fourth code is designed to track on how teachers prepare and support students in working on mathematics tasks autonomously.

Area 2: Attention to the context and mathematical rigor of tasks. Interventions promoting equity and diversity often assume that it is valuable to make mathematics relevant for learners by connecting mathematics to their every day lives (Boaler, 1993; Cobb & Hodge, 2002; Ladson-Billings, 1994). "Context," for the purposes of this study, describes the "situation in which a problem is embedded" (Borasi, 1986). These contexts are usually supplied by the text of the problem presented in the curriculum materials and are usually related to "real-world" experiences in an effort to motivate and connect students' life experiences with mathematical concepts. Paying attention to context is often viewed as an essential component for attending to issues of equity in mathematics instruction. While it seems important to consider what kinds of learning and activities students experience outside of school, there are good reasons to pay careful attention to the ways in which context is treated in the literature, specifically attending to the ways in which these contexts imply translation into instructional practice. Another key aspect of equitable instruction lies in the ways in which teachers are able to establish a balance between respecting the mathematics being taught with students' experiences with learning (Ladson-Billings, 1997; Malloy, 1997). As reported by Boaler (2005), students' opportunities for learning are significantly shaped by the curriculum used in classrooms and by the decisions teachers make as they enact curriculum and organize other aspects of instruction (Boaler, 2002, 2005; Darling-Hammond, 1998). More specifically, teachers' careful and explicit attention to the ways in which students' need to engage in the mathematical tasks requires a cultural awareness and sensitivity (Gutiérrez, 2002).

The need for teachers to explicitly attend to students' understanding of the ways they need to work is especially important in mathematics.

Recent scholarship suggests that real-world contexts and examples may motivate or afford students leverage into mathematics (Nasir, 2002; Adler, 2001; Boaler, 1993), in particular when context for problems are drawn from students' homes, neighborhoods or schools. These and other studies suggested that we develop video codes designed to capture examples of when contexts or materials exclude students from mathematics by using situations with which some students are not familiar or for which they do not have access (Lubienski, 2000). Additionally, the use of context in mathematical work must also maintain the integrity of the mathematics. The teacher's work in attending to contexts sensitively, mediating issues of social capital and focus on teaching moves that make the context and the mathematics accessible for all students (Ball, *et al*., 2003; Moschkovich, 2002).

The literature on context to led to designing codes for "real world problems or examples." This code has three parts, the first layer simply indicates if real world contexts and examples are present or not present in the lesson. The next layer of coding indicates if the context present in the lesson is sensitive or insensitive to students' experiences. The last layer of this code indicates if the context is appropriate or inappropriate for the lesson's mathematical goals[12].

Area 3: Mathematics practices and language- Use and implications for equitable mathematics instruction. Language is an important element of discourse in

[12] The glossary with detailed directions on coding is found in Appendix A and the coding framework is found in Appendix B.

mathematics (Pimm, 1987). Teachers' use of language, in particular mathematical terminology and meaning, is an essential resource for their students learning of mathematics. We read literature that focused on how teachers use mathematical language to help students learn skills and practices needed to develop mathematical proficiency and to the ways in which teachers are able to provide explicit mathematical practices for students' learning. For example, how do teachers represent mathematics through explicit talk about the ways of reasoning, meaning and use of mathematical language? What mathematical practices are used by the teacher to ensure that all students have access to features of mathematical reasoning? Boaler (2000) argues that teachers need to be able to explicitly attend to their students' understanding of the ways they need to work, and secondly, that teachers must provide students with a clear sense of the characteristics of high quality (mathematics) work. Moreover, to promote equity, access to explicit and rigorous mathematics instruction becomes especially important for students who are navigating in and between these multiple languages (Boaler, 1997, 2000; Cohen & Lotan, 1997; 1999; Gorgorio & Planas, 2001). Thus, one feature of inequitable mathematics instruction is when teachers fail to attend explicitly to the language used, either to his or her own language or that of the students.

Mathematics instruction, especially in elementary classrooms[13], is language-dependent; its meanings are carried through language. In Adler's (1999) description of the challenges of teaching in linguistically diverse classrooms she discusses how the dilemmas of teaching can be utilized as "explanatory tools and analytic devices for

[13] I specify elementary classrooms here because students in elementary classrooms are still developing their primary language skills such as reading, writing, and speaking. For example, students in elementary classrooms are learning to read English words, in addition to learning the mathematical meaning for particular words.

teaching" (p. 47). Khisty's work (1995) helps to illustrate the importance for attending to issues related to mathematical language. In particular, based on her work with Hispanic students, she argues that students who lack the academic or mathematical language skills needed for expressing mathematical ideas are eliminated from being able to participate in the mathematical tasks and activities in the class (p. 283). These and other authors argue that "minority" language learners are disadvantaged by the ways in which teachers and schools view students' language. Equitable teaching depends on teacher's mathematical knowledge and deliberate attention to issues of language use between the students and the mathematical tasks at hand during instruction. Research suggests that default patterns of instruction relies on using unmodified mathematical tasks from grade level curriculum and that these typically do not rise to a level of "explicitness" demanded by the descriptions for these video codes (Ball, *et. al.*, 2003). A teacher's ability to provide explicit talk about the meaning and use of mathematical language and ways of reasoning and or about mathematical practices requires a particular use of mathematical knowledge. More specifically, mathematics instruction that is not explicit, such as when the teacher "facilitates" classroom discussion may be more culturally congruent with some students' cultures and learning experiences, thus providing uneven access for participating in mathematical discourse (Ball, et al., 2005; Delpit, 1998; Heath, 1983; Lubeinski, 2000). In addition, Ball and colleagues (Ball, Goffney, & Bass, 2005) have argued that explicit guidance for learning complex skills or ideas is crucial if all students are to develop such capacities, and that leaving the construction of these to chance or to cultural differences in discursive norms is inappropriate (p. 4). Degrees of explicitness can serve as a significant resource for students' access to complex mathematical concepts and ideas.

This literature led to designing four codes focused on explicitness. The first, "explicit student tasks and work" is designed to capture if students are clear on what they are supposed to do in a particular segment of the lesson. The second code, "explicit talk about the meaning and use of mathematical language" attends explicitly to the language used by the teacher and students, for example if a teacher helps students to define and use technical mathematical language. A third code, "explicit talk about ways of reasoning" differentiates teaching that helps students learn to offer explanation or provide a proof or justification for their solution. The fourth and final "explicitness" code is "explicit talk about mathematical practices." The ninth code overall captures examples of when a teacher explicitly and deliberately helps students to use or develop mathematical practices, such as teaching that helps students learn to use representations, pose or use a mathematical definition, or respond to another students' mathematical argument.

Area 4: Access, participation structures, and group work. Seminal research on culturally based theories of teaching argue that high quality, culturally responsive and relevant teaching incorporates and includes diverse ways for students to contribute and participate in mathematics classrooms, thus suggesting that codes be developed that capture instances where teachers are able to include all students in the classroom mathematical work of the classroom (Irvine, 2003; Ladson-Billings, 1994, Malloy, 1994).

Scholars such as Cohen and Lotan, (1997; Cohen *et al.*, 2002) and Boaler (1998) have studied classrooms in an effort to better understand how teachers increase access to the mathematics content for culturally and linguistically diverse students. More specifically, they consider the practices associated with broadly defining what it means to be successful in mathematics classrooms. Illustrations from this work describe how teachers create opportunities for students by designing multiple forms of classroom participation and valuing different kinds of contributions in their classrooms. This work suggests that teachers who accept and encourage multiple forms of participation create a classroom culture that creates broad access to opportunities for learning complex mathematics. This kind of equitable instruction provides students with mathematical resources, particularly skills and practices, needed for complex problem solving and reasoning.

In light of these and similar studies, the last two codes were proposed to capture when a "teacher encourages a diverse array of mathematical competence" and also when the "teacher elicits and values multiple forms of participation and kinds of mathematical contributions."

Validation Study

Methods: Calculating a Mathematical Quality and Equity (MQE) Video Code Score

The section offers both quantitative and qualitative evidence to validate the mathematical quality and equity video codes as a tool for measuring equitable mathematics instruction. Using the coding results of thirty videotaped lessons, I generated an overall equity score for each lesson. I calculated equity scores by comparing the instances of "present" codes with the total number of codes for each five-minute clip in a lesson. To verify the accuracy of the codes[14], I watched more than half of the thirty lessons in Wave 2 to randomly verify that the codes generated by researchers for each of the lessons in this wave were accurate (see also Blunk, 2007).

[14] For details on analyses strategic efforts to achieve intercoder reliability see Blunk, M. (2007). *The QMI: Results from validation and scale-building*. Paper presented at the Annual Meeting of the American Educational Research Association.

These thirty lessons are in Wave 2 of our data set, meaning that these lessons occurred in the middle of the academic year, and occurred after teachers attended at least 40 hours of professional development during the previous summer. In these calculations, I excluded two of the original codes. "Teacher makes social comparisons" was not written in the positive (i.e. social comparisons were deemed an inequitable practice) and therefore, codes of NP (not present) would have counted for promoting equitable instruction. In other reliability and validity analyses, "classroom culture encourages a diverse span of mathematical competence" was not coded reliably, and therefore not included in these calculations. The code "Real world contexts and examples" was only used in the calculation if present and was calculated either as equitable (if the context was used sensitively) or as inequitable (if context was used insensitively); if there was no context, this code was not used in calculating an equity score the lesson.

Coding was a rigorous process. As described in other publications (see Learning Mathematics for Teaching, 2006 for additional details) our group spent a great deal of time and effort focusing on issues of reliability and validity with the video codes and glossary. A coder would sit and watch the entire lesson first, sometimes taking notes about interesting features of the lesson or teaching, before attempting to use the codes. The next step would be watching the same lesson in five minute clips and deciding for each particular code, if the feature was present or absent and then, for some codes, deciding if it was mathematically appropriate or inappropriate. Using the glossary of definitions and coding document, (see Appendix A and B), one would take detailed notes, later used as evidence to support his or her decisions about selected codes.

From these calculations, averages were generated that represent quantitative overall equity scores for each lesson[15]. These scores ranged from .821 for the highest scoring lesson to .111 for the lowest scoring lesson, with an average of .341. The range represents the variation of scores among these thirty lessons. As described in the examples later in this chapter, the higher scoring lessons have more features of equitable mathematics instruction than lower scoring lessons. More specifically, higher scoring lessons focus on important mathematics content and consist of equitable features of instruction. These calculations represent one strategy for differentiating instruction that represents equity and high quality from lessons that are lower quality and inequitable.

More specifically, these analyses also attend to the correspondence between the quantitative and qualitative results, thus providing evidence that mathematical quality and equity video codes are a valid tool for measuring equitable mathematics instruction. These quantitative results are described above as calculating a mathematical quality and equity video code score for each lesson. There are two versions of the qualitative results. The first is described in greater detail below offering classroom examples to illustrate the differences between a higher scoring lesson and a lower scoring lesson, and providing a classroom illustration of specific video codes. The second version of these qualitative results is offered in the case study analyses in the next chapter. Comparisons across the Wave 2 lessons in the LMT data set revealed important differences in the characteristics and practices of equitable and inequitable mathematics instruction. While some of these differences were obvious, such as the difference between a teacher who encourages students to listen to each other and one who uses student responses for public ridicule,

[15] In this equity framework, the language and description of the codes are positive therefore "present" codes indicate that the teacher is providing equitable instruction in this segment of the lesson on that particular topic.

many of these differences in instruction were more subtle. Closely studying the instructional practices in lessons that earned higher equity scores versus those that earned lower scores has important implications for efforts aimed at improving equitable participation and achievement in mathematics classrooms. As Gutiérrez argues, “If we believe that unequal opportunities in math occur partly through instruction, we must also explore the classroom as a unit of analysis” (2002). The analyses in this section illustrate how results from the mathematical quality and equity video codes help discriminate among teachers’ skills orientation to working on issues of equity. In the sections below I provide classroom vignettes that serve help illustrate both the differences between a lesson that earned the highest and one that earned the lowest equity scores. These vignettes also provide examples of the codes themselves.

Illustrations of Mathematical Quality and Equity Video Codes

The classroom vignettes below offer evidence of the differences between a lesson that promotes equity and a lesson that impedes equitable participation. There are three parts of this section. First, I present illustrations of the video codes from the highest scoring lesson; then I present a, counter-example illustrations of the video codes from the lowest scoring lesson in wave 2; and third, I present other significant examples that serve as illustrations of both the mathematical quality and equity video codes.

Highest scoring lesson: Factors of 24. The following two vignettes are from a lesson taught to a fourth grade class (Wave 2, Leson 24). As measured by the Mathematical Quality and Equity (MQE) video codes, this lesson generated an overall score of .821 and consisted of 8 video clips (a 40 minute mathematics lesson). The teacher is preparing students for work on examining factors of 24. During a post observation interview, the teacher describes the purpose of this lesson as “reinforcing the idea that numbers can break apart and be put together in different ways”. The lesson consisted of three main tasks: first the students reviewed working on multiplication by discussing their homework. Next the students worked on an individual task of writing four multiplication facts for the product 24. Third, the students compute the number of squares in an array that has several colored triangles.

Vignette 1: Orienting students to the task. The excerpt below is from the beginning of the lesson, during the first five-minute clip in day three of taping. In her introduction the teacher thanks the students for their patience for having visitors (i.e. videographer and observers) in the classroom for the last few days (this lesson occurs in day three of taping for this wave). She also says that she knows this is Friday and that with many special events occurring, it can be difficult to focus, but that it is important to focus and pay attention.

Teacher: So let me give a couple of directions on that before you start. Remember if you’re sitting at the end, you can work in a three-some, but if you’re gonna work in a three-some I want you to move your chair so you’re sitting right next to them--so everyone's participating in the conversation. Robbie, did you get that? Okay. So, your gonna go over your homework with partners today. I want you to share your answers. Okay, I'm gonna walk over to the points board….I want you to share your answers. What do you do if your answers are different? If you have different answers? Whose gonna remind us what we do when that happens, Sheeba?

Sheeba: Ummm like, after were done correct-like you always ask did you have any problems and then if we raise our hand, you will come over and help us.

Teacher: Okay, very good. So, if you have a disagreement, you're first gonna try and figure it out, right? You're gonna try and figure out why your answers are different. And you're gonna try and explain to each other why your answers are different. If you can't settle your

> disagreement, then you can raise your hand and ask me, right? And I'll come over and try and settle your disagreement for you. I'm gonna give you just a couple of minutes to do this, this morning. As groups are finishing up, you can quietly work on your mountain map or wait silently as, as other partners are finishing up. Does anybody not know who they're supposed to work with? Okay, remember I want you to really try and help each other if your answers are different. Work together. *(teacher leaves small group to work together while she continues to walk around and observe and assist other students.)*

Three video code examples: "Explicit student tasks and work", "Teacher encourages diverse array of mathematical competence" and "Expressed expectation that everyone will be able to do the work." In her initial directions, the teacher is supporting a classroom culture that encourages a wide range of mathematical competence. She provides explicit directions for the assigned talk about how to listen carefully to the ideas and ways of reasoning from their partners with a focus on understanding both how they solved the problem as well as how their partners worked mathematically on solutions to their homework. This segment illustrates how what is included in the "mathematical work" of the class is defined broadly and supports diverse student interactions and productions that represent an array of mathematical performances. Students, following the model established by the teacher, take the time to listen and try to understand how their peers worked to solve the problems on their homework. The teacher emphasizes effort rather than innate mathematical ability.

Vignette 2: Hasan's homework dilemma. This segment is about 11 minutes into the lesson and students are discussing how they worked on their homework assignment with their partner. As the teacher is walking around the room listening to students share their solution strategies, two students request her assistance. Apparently one student clearly knew the mathematics, but did not follow the directions on the homework. This dilemma has led to some confusion as this student tried to convince his partner how he solved the problem and how his answers were correct.

The students are working call her over to assist them.

Teacher: Okay, go ahead Hasan show him what you did, okay? I just want him to understand. Show him with pan A.

Hasan: I did. I did one, two, three. One, two, three, four, five. Five here and three here.

Teacher: Yeah.

Hasan: Multiply it by three times five is fifteen. Then fifteen plus fifteen is thirty, so two times. (*Student is pointing to the homework and counting the number of circles on an array that is 5 rows of 3 circles and explaining that using this array labeled Pan A on this worksheet is equal to 15 circles.)*

Teacher: Okay. So, you used this array (Pan A), counted the rows, and figured using pan A one time would give you fifteen and then you use pan A another time then it gives you 30. So this is another way of putting that. *(Showing that 15 + 15 equals 30 and this is the same as 15 × 2.)* Together- three times two, so Pan A is used two times. Okay.

Hasan: This is how I …(inaudible- student is trying to show how he used this same method for solving all the problems on the page.)

Teacher: Oh, I understand what you did. I totally understand and it-and it-makes (inaudible) your answer makes sense. It makes sense because your combinations add up to thirty.

Hasan: Okay.

Teacher: So it was just you just didn't understand the directions. So can you refigure it now? Okay, that would be great. Osman, go ahead and help Hasan okay, if he needs it.

Hasan: Do you want me to erase it?

Teacher: Actually, no. Can you use a pen, because you're supposed to have a pen? Just write the correct answers, the other answers right next to it. Your answer isn't wrong in terms of how you figured the thirty, you did a great job, and it's just in following the directions on the paper. So just put your answer in pen right next to it and don't worry about it.

Two examples of mathematical quality and equity video codes: "Explicit talk about ways of reasoning" and "Teacher elicits, uses and values multiple forms of participation and kinds of mathematical contributions." A lever for attending to equity issues for mathematics teachers lies in broadening what it means to be successful in mathematics classrooms. In the preceding segment, the teacher has created a classroom culture that encourages students to try to understand how they solved the problem while also being able to understand how their classmates generated solutions. By eliciting, using and valuing multiple forms of participation and kinds of mathematical contributions, teachers are able to attend to issues of inequities her math class. This segment illuminates key features of equitable mathematics instruction. The teacher has taken steps to understand an ambiguous student production. She also explicitly invites and supports broad participation by encouraging a wider range of mathematically relevant forms of work and talk. The teacher recognizes that this student (and others) may not always express mathematical ideas with precision. Therefore efforts to both understand and help students become more explicit and accurate about their ideas, support students in becoming mathematically proficient.

Lowest mathematical quality and equity video code score: Benchmark numbers lesson. Inequitable instruction, according to the LMT mathematical quality and equity video codes, occurs when features of high quality, sensitive instruction are absent indicating that the teaching is generally both mathematically weak and insensitive to student differences. Lessons are characterized as inequitable if they receive low overall equity scores from the quantitative analyses. These lessons at times have included teachers' poor use of unmodified curriculum tasks or a teacher's comments that make social comparisons between students in ways that marginalize and demean students' mathematical ideas. The following example illustrates an inequitable representation of two of the mathematical quality and equity video codes: explicit student tasks and work; and explicit talk about the meaning and use of mathematical language.

Vignette 3: Introduction to "benchmark" numbers. The following segment is the beginning of a math lesson on estimation for a class of third graders (Wave 2, Lesson 10). As measured by the MQE video codes, this lesson generated an overall score of .111 and consisted of 9 clips (approximately a forty-five minute lesson). The lesson begins by the teacher asking the students for "friendly" numbers. The objective of the lesson, as stated in the curriculum, is to use benchmarks to understand the relative magnitude of numbers. "Benchmark numbers", a part of the mathematical topic of estimation, are a common topic in early elementary curriculum in which students use number sense to estimate a part or multiples of quantities. The curriculum used by this teacher provided the following definition: "benchmark numbers- numbers that help you estimate the number of objects without counting them."

Teacher: Now we're gonna start a new kind of concept for our math today, so I want you to think of a what we might call a friendly number. A number that you like using. Oops. Shh. Excuse me. You're supposed to be thinking. Think of a friendly number, a number that you like to use. Okay. So Jaime what would be a friendly number to you.

Jaime: Uh- twenty-eight?

Teacher:	Twenty-eight is a friendly number to you? How do you, how did you come up with twenty-eight.
Jaime:	Because it has…. Because it's my favorite number.
Teacher:	Okay, 'cause it's your favorite number. Are we talking about a favorite number though at this time or are we talking about a friendly number that we can use in math?
Jaime:	Friendly number.
Teacher:	That we can use in?
Jaime:	Math.
Teacher:	Okay, so do you wanna still keep twenty-eight as a friendly number for you? Okay.
Mark:	Two?
Teacher:	Two?
Mark:	Twenty-nine?
Teacher:	Twenty-nine, and why is twenty-nine, I know why two is 'cause it's...
Mark:	Two is a friendly number to me and so is nine?
Teacher:	Okay. So you're putting them together and they become friendly numbers. Nathan?
Nathan:	Eight.
Teacher:	Eight? And why is eight a friendly number to you.
Nathan:	Because I use it a lot.
Teacher:	You use it a lot. Tell me how you would use it.
Nathan:	In math.
Teacher:	Well, besides in math. You gotta to be more specific. How would you use the number eight in your math.?
Nathan:	I don't know.
Teacher:	You're not sure, okay.

In the above segment, the purpose of the launch of this lesson was to set the context for the lesson's work on estimation and using "benchmark" numbers. Here the goal was for students to begin to consider how estimation is a useful construct. For example, if students were trying to estimate the number of jelly beans in a jar and a measurement of twenty-five was marked on the jar, students could use this as a benchmark to estimate where one hundred might be. Instead, the teacher led the students in an un-productive conversation where students were both distracted from the goal and central task of the discussion and where the mathematics was distorted. In this case, I am using distorted to mean that the mathematical discussion led the students to be un-focused instead of framing them for work on estimation.

Two counter examples of mathematical quality and equity video codes: "Explicit student tasks and work", "Explicit talk about mathematical practices." These video codes were developed to discern between instruction that provides explicit directions and guidance about the mathematical work of the lesson, and when students simply follow implied directions. In particular, this code was designed to track ways in which the mathematics is either ambiguous, distorted due to lack of purpose or goal for the tasks, or only accessible for students who bring cognitive, social, and linguistic resources to school and use them to infer appropriate links and connections between the mathematical purpose or goals and the tasks of the lesson. This example earned a "not present" coding for explicit tasks and student work. By not offering explicit guidance about what a "benchmark number" is, or about how the use of "benchmark numbers" connect with other lessons and ideas the students may have about estimation, this lesson distorts the central mathematical features of understanding number sense and estimation that were the purpose of this lesson.

Additional counter example of equity video code: "Explicit talk about the meaning and use of mathematical language." There are many language issues that arise in mathematics class. As described by researchers, one dilemma lies in language differences between students' home environment and school (Adler, 2001; Moschkvich, 2002; Schleppegrell, 2002). More specifically, recent scholarship notes that students are often navigating multiple languages while in school. This research describes how students are negotiating meaning and use of home language (including other languages, such as Spanish, as well as other versions of English, such as African American Vernacular English), school language, while also beginning to learn about technical disciplinary language (Delpit, 1988, 1995).

This segment was coded as "not-present" for "explicit talk about the meaning and use of mathematical language". This code was designed to track on if the teacher attends explicitly to language (used by the teacher or the students), for example when the teacher defines terms or demonstrates how to use terms precisely or highlights specific labels or names. In the preceding segment, the teacher was not explicit about the meaning or use of mathematical language, in particular distinguishing between the features of "what my favorite number might be" and "a number that is useful in estimating quantities." In framing the lesson, the teacher failed to define what a 'friendly number' was or offer guidance on how students might be able to describe or identify one. Furthermore, this distortion of the mathematical concept of estimation by not identifying how to define or use a 'friendly' number or 'benchmark' number provided students no mathematical resources with which to engage mathematically to understand the concept of estimation. These examples are typical characterizations of this teacher's lessons. Her lessons, in general, are both mathematically weak and inattentive to the students' needs or differences in their classrooms. Although she earned a score of .111 on the specific lesson described above, this teacher's average equity score for all three of her lessons in Wave 2 of the data collection is .208, the lowest of the ten teachers in the sample.

Other significant examples. This section will illustrate two additional important equity codes not described in the examples above. These two mathematical quality and equity video codes are "Real world contexts or examples" and "Instructional time spent on mathematics." Below I review the purpose of these code and offer an example and counter example for each code.

"Real world contexts or examples." Ball *et. al.* (2003) argue that without significant intervention, schools often reproduce rather than reduce inequity. They argue that "curricula are biased, dominant groups privilege their own version of knowledge, out-of-school support for in-school success is uneven, and, intentionally or inadvertently, teachers and students tend to reenact patterns of inequity as they interact in the classroom" (p. 2). One such example is the use of real-world contexts or examples. Many scholars (Adler, 2001; Boaler, 1993) have studied how contextualizing mathematics problems can promote equity by motivating or affording students leverage into mathematics. These include problems that designed to be sensitive to students' cultural values, ethnic identities, related to their homes, communities, or out-side-of-school experiences (Irvine, 2003; Irvine & Aremento, 2001; Ladson-Billings, 1995; 19947. However, contexts and materials may also exclude students from the mathematics by using situations with which some student are not familiar or do not have access. We used these codes are that designed to capture teachers' abilities to recognize and make unfamiliar context interpretable and usable for all students. Additionally, our coding definition maintains that these contexts also must be appropriate for the lesson's mathematical goal and not serve to distort or complicate the mathematics of the lesson (Lubienski, 2000). The following two segments illustrate contrasting examples of equitable and inequitable use of context in mathematics instruction. It is important to note that equitable use of context includes use of given contextualized mathematical problems in written curriculum that are used unmodified, as well as teacher generated uses of context in their teaching.

Vignette 5: Inequitable "use of contexts" in mathematics instruction. This is an excerpt from the very beginning of a lesson about plotting points on a coordinate plane in a fourth grade class (Wave 1[16], Lesson5). In her post-observation interview, the teacher stated that her goal was to teach students how to plot on the coordinate plane and that the lesson came from her district curriculum. The vignette below is the very start of the lesson.

Teacher:	And how to identify location of the point on a coordinate plane. Okay. Raise your hand if you've ever played a game called Battleship?
Students:	Oh yeah.
Teacher:	I asked for hands, not for "Oh yeah." All right. How do you play Battleship, how can you tell or what do you have to say when you're trying to … you know, when you're trying to see somebody's battleship, what is it that you're saying? What are you saying, Nathan?
Nathan:	You say like D-one or…
Teacher:	D one or what?

[16] Although this lesson was not from the Wave 2 data collection, I have chosen to include it here because I believe it is a good example about use of context. Many teachers try to incorporate common children's games as a way of motivating interest in particular mathematics topics. Although I am not arguing against this strategy, I use this example to show the types of considerations necessary when making such a choice.

Nathan:	E one.
Teacher:	E one.
Nathan:	A five.
Teacher:	Okay. What do those mean?
Nathan:	That means … if the ships on top of that thing that they said. The white thing turns into a red. And that means that they hit all of them and they are all red, they blow up and they won.
Teacher:	Okay, sounds like you've definitely played Battleship before. But the D one and the E one and E five, what are those talking about? It's not just a thing, what is it? It's telling you what? What is it telling you Sanjay?
Sanjay:	Like where to click it.
Teacher:	It's telling you where something is. In Battleship, it's where you're trying to guess or to figure out where your opponent's ships are. So you're trying to figure out the location. Well we use those in Math also. Not to try to sink battleships but when we're trying to talk about where a particular point is. Okay. We're gonna make something … kind of close to what you're using Battleship. It's gonna look a little differently … called coordinate plane. Okay. You're gonna need your ruler … wait, wait, wait.

The teacher begins the lesson by referencing the board-game, "Battleship". While it is clear that the teacher was attempting to draw students' interest into the mathematical lesson for the day by referencing an (assumed) familiar board-game for children, there are no links in her work to make this a familiar experience for the students in her class. The teacher did not bring in a visual example of the game, nor did she provide students with an opportunity to play the game before or after this introduction. In this way, referencing "Battleship" in the framing of her lesson that day likely isolates students unfamiliar with "Battleship" rules of play, game structure and strategies. Therefore, adding context --without the proper support and scaffolding— created additional layers, rather than mediated, issues of equity for students in her class.

Vignette 6: Equitable "use of contexts" in mathematics instruction. The following segment is from a fourth grade lesson on using arrays to identify prime and composite numbers. These students have been learning about working with factors and learning the definitions of prime and composite numbers. This segment is from the last part of the lesson where the teacher is transitioning from the main work of the lesson to individualized practice on homework worksheets. The worksheet asks students to figure out how many times to use different sized muffin pans (representing arrays) to make 30 muffins.

Teacher:	Make eye contact with me. Give me those beautiful smiles. Remember we talked a lot in here about a safe environment and being- being okay to share and ask questions. (Pause) Okay? Whether we think it's a silly question or not we need to be able to ask it. Right? We talked about that.
	Does anybody not know what a muffin pan is? Okay. You don't know what a muffin pan is?
Teacher:	Give her a round of applause. Thank you, Kay and Didi. Okay. I brought one in. This is a muffin pan.
Student:	Oh.
Teacher:	Okay? What do we cook in here?
Class:	Muffins.
Teacher:	You ever seen cupcakes?
Students:	Yeah.
Teacher:	In the store? Cupcakes. People put little- little wrappers in here sometimes and then they pour cake mix to make muffins, which is like a bread. It's like a food or a bread. So it's used for cooking and cupcake is- kind of looks like a muffin. Chocolate muffins, banana muffins, but it's a kind of bread used for cooking. Okay? (Pause) So there you have it.

Contexts in this example are considered equitable because of the way in which the teacher mediated an unfamiliar context for some of the students in her class. In this example, the teacher has take deliberate steps to ensure that the context of the worksheet familiar to each of her students. The worksheet comes from the district adopted curriculum and is designed on the assumption that all students would know the purpose of a muffin pan for baking in quantities. In this segment, the teacher's mathematical knowledge provided resources for being able to respond to the mathematical difficulties that arose when teaching multiplication with arrays. Because the teacher was familiar with students' outside of school experiences, she was able to anticipate difficulties that might arise with contexts used in the curriculum. The teacher's work to mediate this context for her students counted earned a code of "present" and "sensitive-appropriate" use of contexts.

Conclusion

This dissertation investigates the characteristics and features of mathematics teaching that may promote equity. This chapter has explored the efforts for designing this observational tool, including details on the initial blueprint, different strategies for coding instruction, the literature that contributed to the codes, and the process of refining and using the codes to differentiate between equitable and inequitable teaching practices. The last section of this chapter describes how using these codes reveal the differences between characteristics of equitable teaching and inequitable teaching. The next chapter uses these codes to craft three paradigmatic cases of teaching.

CHAPTER 3:

CASES OF TEACHING MATHEMATICS

Introduction

This chapter presents three comparative cases of teaching. The focus of the cases developed in this chapter is not on these particular teachers, but the teaching practices they enact in their classrooms. More specifically, these cases focus on the different ways in which attention to mathematics and to issues of equity play out during instruction. Use of Yin's (2009) multiple-case design allowed the exploration of hypotheses regarding the mathematical and pedagogical demands on the teacher in enacting equitable mathematics instruction. This method was selected because as Yin (2009) suggests, case methods can provide insight into the resources needed to enact equitable mathematics instruction (p. 141).

Methods

Measures of Mathematical Knowledge for Teaching (MKT), Mathematical Quality of Instruction (MQI), and Mathematical Quality and Equity (MQE)[17]

In this section I will briefly describe the measures of Mathematical Knowledge for Teaching (MKT), Mathematical Quality of Instruction (MQI), Mathematical Quality and Equity (MQE), and then describe the methods used for the analyses in this dissertation. For the analyses in my dissertation, I used previously calculated teachers' scores for MKT and MQI (Hill, et al., 2008). The measures for MQE are a central feature of this dissertation. It is important to note that the Learning Mathematics for Teaching measures of Mathematical Knowledge for teaching have been linked to gains in student achievement in a study of school improvement[18] (Hill, Rowan, & Ball, 2005). As such, the analyses in this dissertation extend and leverage this work in this study of equitable mathematics instruction.

Mathematical Knowledge for Teaching score. The measures of MKT used for this dissertation are the paper and pencil items developed by the Learning Mathematics for Teaching project (Ball & Hill, 2004; Ball & Rowan, 2004; Hill, Rowan, & Ball, 2005) using the form of items developed in 2002. As stated on the project website, these

[17] It would have been impossible for me to have done all the parts of this work individually because of the standards for the validation analyses. For example, the way in which we calculated an overall lesson score required inter-rater reliability pairs. As a member of this project, I was involved in each aspect of this work, collecting data, {i.e. conducting interviews, observing professional development workshops, observing teaching during video tape collection, etc.} developing and refining codes, coding lessons, checking reliability, developing scales (meta-codes), etc. In this section, I have deliberately noted which of the analyses were collectively generated by project members and which analyses I individually generated.

[18] As described in Hill, Ball, Blunk, Goffney & Rowan (2007), " students of teacher who answered more items correctly gained more over the course of a year of instruction...indicating that the above-average teacher 'added' an effect equivalent of that of 2 to 3 extra weeks of instruction to her students' gain scores." (p.109) (Hill, Rowan, & Ball, 2005)

multiple choice measures of teachers' mathematical knowledge for teaching are comprised of items:

> Items in each category capture whether teachers can not only answer the mathematics problems they assign students, but also how teachers solve the special mathematical tasks that arise in teaching, including evaluating unusual solution methods, using mathematical definitions, representing mathematical content to students, and identifying adequate mathematical explanations. (Some items also measure) teachers' knowledge of students and content…they probe whether teachers recognize common student errors and solution strategies, and can anticipate whether material will be difficult or easy for students (http://sitemaker.umich.edu/lmt/content).

As described in other publications, "scoring teachers' performance on our paper and pencil measures involved straightforward calculations of scores per teacher per domain" (Hill, Ball, Blunk, Goffney, & Rowan, 2007, p.114). The survey used contained items representing two elements of MKT: teachers' common content knowledge and specialized content knowledge. Of the 34 items on the assessment, 12 problems were in number and operations, 14 in geometry, and 8 algebra problems. Furthermore, "Item Response Theory (IRT) was used to create teacher level scale scores (Hill, et al., 2007, p. 116). This represents the MKT score used for the analyses throughout this dissertation.

Measures of Mathematical Quality of Instruction (MQI). Each lesson was coded in five minute segments using the Mathematical Quality of Instruction observational instrument which has 33 codes designed to represent aspects of mathematical quality: (absence of) mathematics errors; (absence of) responding to students inappropriately; connecting classroom practice to mathematics; richness of the mathematics; responding to students appropriately; and mathematical language (Hill, et al., 2008). Of the 33 codes, 12 are codes for teachers' mathematical knowledge, 8 codes for teacher's use of mathematics with students, 10 are equity codes. There are three additional codes for evaluating the teacher's knowledge as low, medium, or high. In this way, "this lesson score represents the coders' overall evaluation of the teacher's mathematical knowledge as manifested in the particular lesson. The overall lesson score for each teacher represents the average of all nine lesson scores (Hill, et al., 2008, p. 441)[19].

Measures of Mathematical Quality and Equity (MQE). The codes in this frame of the instrument represent teacher's use of mathematics to promote equity. As such, these ten codes represents core ideas about mathematical knowledge for teaching and cultural modeling. For example, the code "Explicit talk about the meaning and use of mathematical language" draws from core ideas from mathematical knowledge for teaching because it captures how the work a teacher does in helping students learn to accurately use mathematical language such as defining terms or showing students how to use a particular word. This code also represents themes of cultural modeling because it captures a teacher's efforts to offer explicit instruction, or guided practice for learning mathematical language. Some codes, such as "explicit talk about mathematical practices" are more closely aligned with mathematical quality drawing more from theory of Mathematical Knowledge for Teaching, and some codes, such as "use of real world contexts" or "teacher encourages diverse array of mathematical competence" draw more from the Cultural Modeling framework (Ball, 2008; Lee, 2007). As such, the frame representing MQE codes feature both mathematical knowledge and attention to issues of culture and equity and are designed to be features of teaching that create access to complex and rigorous mathematics content.

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[19] Although I was a member of the LMT project and participated in developing, refining and using the codes to generate these analyses, these measures represent collective work.

MQE scores represent whether features are "present" in each five minute segment of a lesson. This number is compared to the total number of codes possible (Thus, a 45 minute lesson, has 9 segments. Because there are 10 codes there are 90 possible codes that can be assigned to this lesson; if a teacher earns 27 "present" codes, then the MQE score would be 27/90 = .30). As indicated on Appendix F and described in greater detail later in this chapter, I calculated MQE scores for each lesson in Wave 2 (30 lessons). In an effort to generate a measure that would more accurately represent how a teacher fares on the MQE measures, I also calculated an average score using the three lessons taped in Wave 2. In this way, a teacher's MQE score does not solely depend on one particular lesson, but instead is an average across three lessons that were taped toward middle of the academic year[20].

Case Selection

Yin's advice (2009) was followed about case selection: "Each case [was] carefully selected so that it either (a) predicts similar results (a literal replication) or (b) predicts contrasting results (a theoretical replication)" (p. 53). More specifically, the cases of Lauren and Karen were selected to predict similar results because they both share the characteristic of high levels of mathematical knowledge. I used a "two-tail design" for selecting the cases of Rebecca and Karen. As Yin (2009) suggests, I used a "two-tail design" because "the rationale was derived from the prior hypothesizing of different types of conditions and the desire to have sub-groups of cases covering each type if contrasted results were predicted explicitly at the onset of investigation" (p. 59-60). The cases of Rebecca and Karen were predicted to produce contrasting results

[20] Although most of the members of the LMT project participated in coding lessons for validity and inter-rater reliability purposes, the analyses represent my original work generated for the purposes of the studies in this dissertation.

because Rebecca has the second highest Mathematical Quality and Equity (MQE) score and Karen's MQE score is in the bottom half. In this way, the three cases developed in this chapter were selected purposefully to represent combinations predicted to likely produce equitable instruction. I selected the two teachers who earned the highest scores on the paper-and-pencil assessment of mathematical knowledge for teaching (MKT) and the two teachers who earned the highest mathematical quality and equity (MQE) scores. This selection method produced three teachers for detailed case study analysis. The two teachers who earned the highest MKT scores were Karen and Lauren. The two highest equity video code scores were Lauren and Rebecca. Therefore, the final selection of cases for this multiple-case design is Karen, Rebecca, and Lauren.

Karen: Case Study 1. Karen earned the highest score on the MKT measures, (1.50). Karen's overall MQI score was 2.56, while her MQE score was .304. I hypothesized that studying Karen's teaching would help to illustrate how mathematical knowledge is an important resource for enacting practices of equitable mathematics instruction.

Rebecca: Case Study 2. Rebecca earned a high equity video code (MQE) score, (.41). Her MKT score was -0.71, and her overall MQI score was 1.40. I hypothesized that studying Rebecca's teaching would illustrate features of equitable instruction detected by the equity video codes and reveal important nuances about the characteristics of sensitive and equitable mathematics instruction.

Lauren: Case Study 3. Lauren has a high score on all three (MKT, MQI, and MQE) measures. Lauren's MKT score was 1.30, her MQE score was .698 and her overall MQI score was 2.78. I hypothesized that studying Lauren's teaching would represent features of equitable mathematics instruction.

Paradigmatic cases of teaching. As described in Chapter 2, two paradigms form the bedrock for the analyses in this dissertation. For decades, the field of education has been interested in improving educational experiences and academic achievement of students of color, poor students, and other groups of students traditionally not served well by schools. For some, working on improving academic achievement and equity lies solely in increasing teacher's content knowledge. These scholars often claim that if teachers simply knew more mathematics, then patterns of student achievement would increase. Karen's case is a paradigmatic representation for those in the mathematics education community who believe that the sole focus of improving student's mathematics achievement lies in increasing teachers' mathematical knowledge.

Another perspective is drawn from researchers who claim that knowledge of students and culture is most important. These scholars argue that if more teachers knew, understood and were committed to understanding students' outside of school experiences, then student achievement would increase. Many of these believe that a teacher's commitment to particular students is the most important resource teachers to bring to working towards equity. Rebecca's teaching is a paradigmatic example of those who believe that the sole strategy for improving student achievement is in recruiting teachers who are committed to working with culturally and linguistically diverse students.

A third perspective balances these two themes: math and cultural knowledge and commitment. The teacher for this case is one who represents a dual emphasis of mathematical knowledge and commitment to students outside of school experiences, students' culture, etc. The scholars and work in this area are somewhat newer and strive to collect promising teaching practices and strategies with this dual emphasis. Lauren's teaching represents a paradigmatic example of this dual emphasis. As illustrated in the case study of her teaching later in this chapter, Lauren thinks carefully about how to represent both serious attention to the mathematics content, while also being responsive to the needs, perspectives, and experiences of her students and their outside of school experiences.

Data

The data used for creating these three cases involved collecting: video records of nine mathematics lessons, post-observation interviews for each lesson, data from a general interview in Wave 2 of the data collection, and some curriculum materials for the taped lessons.

General Analytic Strategy and Analyzing Qualitative Data

As described by Yin (2009), I used three general analytic strategies for developing my cases: relying on theoretical proposition, developing a case study description, and using quantitative and qualitative data (p. 130-132). More specifically, the analytic techniques used in each of these cases were focused on “pattern matching” (p. 136) and “explanation building” (p. 141). For example, for “pattern matching”, I noted where a teacher’s mathematical knowledge appeared to support my hypothesis that mathematical knowledge is required to produce equitable instruction. Similarly, I also noted examples of when a teacher’s attention to issues of equity with her students appeared to influence her mathematics teaching.

Each case was developed using grounded theory methods (Glaser & Strauss, 1967). As described in Strauss and Corbin (1998), I viewed each case holistically, trying to better understand the patterns about the ways in which teachers’ practice differs according to their concern for equity and their level of mathematical knowledge. More specifically, I micro-analyzed the data using “open coding” techniques (p. 119) and then later used “selective coding” (p.141-160). The results of these analyses are represented in the construction of the in-depth cases described in this chapter.

Focus on Teaching, Not Teachers

It is important to note that the illustrations and claims in these cases are not about the teachers themselves, as individuals, but instead represent sets of teaching. More specifically, the analyses presented in these cases are about how attention to the mathematics and to issues of equity come together to produce teaching that is differently equitable.

Rebecca: A Case of High Mathematical Quality and Equity Video Code Scores and Low Mathematical Knowledge Score

Each case is presented in three sections. The first section offers a general description of Rebecca’s context for teaching, including a description of her school setting, classroom, and her planning. The second section explores how teaching promotes or impedes equity and the third section analyzes the mathematical quality of instruction

Rebecca's Students and Classroom

Rebecca teaches in what is considered to be a difficult school where many of the students are culturally and linguistically diverse. More than 90% of the students in her class are labeled as English language learners. These students also struggle with a variety of social issues, such as poverty and violence. Many of Rebecca's students come to her class with a wide range of mathematical skills, cultural perspectives and experiences, and with a negative and deficit perception about their own mathematical abilities.

Rebecca's classroom is well organized. She frequently uses both the overhead projector and the large white board at the front of her classroom as tools for demonstrating the steps for solving computational problems. Although students' desks are arranged into tables, which allows for easy access for students to work in groups, every student can easily see the front of the room where Rebecca focuses most of the attention during instruction. Additionally, her room arrangement provides spacing between the tables of desks which allows Rebecca to both circulate while the students are working, as she frequently does, and to work individually with students who are having difficulties.

Planning

Although Rebecca doesn't indicate in her interviews how much time she spends planning lessons, she does offer some details about how she prepares lessons and her reasoning for the structure of her lessons and the tasks she assigns. As described in her interviews, three main considerations guide Rebecca's planning process. First, Rebecca believes it is her responsibility to focus on building her students' mathematics skills.

Secondly, Rebecca believes her teaching should provide adequate time for practice. A third consideration is that she wants to be familiar with the concepts and topics she is teaching.

Rebecca's plans and enacts procedural lessons may be a symptom of her shallow mathematical knowledge. Because, as Heaton (1992) describes, "moving away from the textbook and developing one's own plans requires a flexible understanding of the content to be learned as well as ideas about how to help students learn," it is not surprising that Rebecca's lessons focus solely on learning mathematical procedures (p. 156); she appears to lack the content knowledge needed to develop or modify lessons in ways that focus on developing conceptual understanding or adaptive reasoning. Similar to the teacher in Heaton's study, when making decisions about how and when to use the district adopted curriculum, Rebecca alters the mathematical purpose of the lesson to focus solely on developing procedural knowledge, which often misrepresents the mathematics content in important ways (p. 158).

When asked about how she plans a typical lesson, Rebecca responded, "I go through my bags of knowledge and stuff in here [in her mind] and think of activities that *I* did to learn it *myself* and how to teach it. So I think back to my professional development and say, 'Oh was there anything from this?' And then if I want them to do it on their own, I'll use the book for that…" Rebecca's district adopted a standards-based textbook, which Rebecca says "feels 'lecture style.'" She describes how she doesn't think she can teach "lecture style" and prefers to use the district adopted curriculum primarily for choosing example problems for student practice. She further offers that in her teaching, "I like to do activities that allow the kids to explore," especially “activities” that are on her students’ level that will allow them to “understand and feel successful.” After using “activities,”, Rebecca describes that she “turns back to the textbook and maybe picks out a couple of problems to do on the board for them and then I have them pick out their own problems and allow them to explore and what happens is they start to conceptualize ‘Oh, this is how I do it, now if I want to use the book’….(I know how to do it).” (Wave 2, general interview) She adds that in her lessons, she also allows time for students to “work collaboratively, and sometimes having them work independently, having them go back to drill and skill because that’s what they need.” A final consideration when planning focuses on her own understanding of the mathematics procedures being taught on a particular day. She explains, “It’s just sometimes you have to go back over things before you are going to teach it and feel really confident when you’re up there…I make sure I remember how to do all these concepts beforehand.” (Wave 3, general interview)

General Overview of Rebecca’s Teaching

Rebecca’s lessons follow a structure similar to that found in many U.S. elementary classes (Wood, 2001). In this pattern, “typical of traditional mathematics teaching, the teacher presents a problem, asks for the solution, and then asks a series of questions intended to ensure that students know a mathematical idea and the written symbols.” (p. 110). Similarly, a typical lesson in Rebecca’s classroom begins by stating the objective for the day’s work and modeling a basic procedure. Classroom discussions are largely teacher directed. Many of her lessons begin with a “story” or “poem” that serve as a metaphor for a mathematical procedure (e.g. positive and negative numbers “don’t like each other”). The vast majority of the whole group time is spent having the students recite the work of following a procedure or describing the steps for solving a computational problem (illustrations from her classrooms are included below to illustrate her teaching style). Moreover, this whole group time is not spent focusing on the underlying mathematical principles of the procedure or emphasizing meaning or purpose of this concept or making connections between a procedure she just taught and larger mathematical ideas. Rather, Rebecca spends approximately 75-80% of the instructional time engaged in teacher directed work where she is positioned at the front of the classroom. She often models procedures and offers several similar examples for her students to try. During this time, students are expected to take notes and practice the procedure. Rebecca also uses this whole group time to reinforce the steps of the procedure and clarify student misconceptions or student errors. Consider, for example, the following segment at the very beginning of the lesson. Rebecca has assigned students to take care of distractions ("Jessica, you're going to answer the phone for me. Edward, open the door when somebody knocks.") and is now ready to start the lesson. The mathematical content of the lesson is algebra and the students are learning to multiply binomials. Rebecca is teaching the class a "happy face" strategy for multiplying binomials, which she refers to as "F.O.I.L[21]." (Wave 3, Lesson 10).

Rebecca: Oh, okay. All righty. So, before we get started, please make sure you're paying attention because I don't want to drag this lesson out for an hour. 20 minutes tops. I'm falling asleep, okay? All righty. So today we're going to work on multiplying, and because I'm really tired, we're multiplying binomials, not polynomials. So sorry. So, remember, was it Monday, we did *x* plus 2 plus *x* plus 4. Something like that. And then we said *x* plus 2 plus *x* plus 4. What's *x* plus *x*? *x* plus *x* is what?

[21] FOIL represents a saying applying the distributive property for multiplying binomials and stands for (multiplying) the first two, the outside numbers, the inside numbers, and then the last two numbers together.

Students: 2.

Rebecca 2. And then 2 plus 4 is what?

Students: 6.

Rebecca: 6. So that would be our answer, 2*x* plus 6. Well, today we're going to make it a little more difficult. But we're going to multiply. This is the fun part. You guys should pick on this easier.

Rebecca: So everybody on your paper, *x* plus 4 times *x* minus 3. *x* plus 4 times *x* minus 3. Thanks book. Hopefully it's on the right page. Now, I'm going to make a little face. You guys ready for my face.

(She begins teaching them how to multiply binomials using the F.O.I.L. method.) *Whatever I do on my board, you do on your paper.* Everything. All righty. So *x* plus 4 time *x* minus 3. So here's the first part of my not so great face. Here's the eyebrow of one part of the face. That is one eyebrow. Got the one eyebrow? Okay, so what's *x* times *x*?

Student: 1*x* .

Rebecca: Not one *x*, *x* what?

Student 2: Squared? (Pause) *x* squared.

Rebecca: *x* squared. So *x* times *x* is going to give me *x* squared.

This type of exchange is typical in Rebecca's classroom. Rebecca's lessons focus on learning procedures and she poses questions to students that require simple, usually one-word, responses. While the students are still in whole group, she provides additional numbers and examples for students to practice the procedure being taught. The students are expected to follow along.

Next, the students often transition into working in small groups, partners, or individually, to practice the procedure she demonstrated. Rebecca uses this time to circulate among students to clarify the steps in the procedures and answer questions from. If students have questions, Rebecca generally repeats the steps of the procedure and walks the students through additional examples. The lesson generally concludes by

Rebecca providing leading questions, a summary statement, and assigning additional homework problems for practice. The following is the summary of the lesson described above:

Rebecca:	Okay, well, the bell is going to ring so you have two options. We're going to let Julio go up and decide if we have homework. Go ahead Julio. Get it wrong, get it wrong. Okay, encourage. But if people aren't paying attention then I'll add on homework, because that's not fair to the people that are up at the front. Are you done?
Julio:	Yeah.
Rebecca:	You're sure?
Julio:	Yeah.
Rebecca:	Are you correct or are you wrong?
Julio:	I'm wrong.
Rebecca:	How many think he's correct?
Students:	I'm lost.
Students:	I don't know.
Rebecca:	You don't know? Well, I have to tell you guys that they did have the hardest one for the day. And their answer is correct. Unless for some reason I can't add or whatever. But Julio gave it to me orally so that means he did it all right. So you guys don't have any homework at all today. So make sure your names are on your papers and pass them forward.

As is typical in Rebecca's lessons, there is no summary about the mathematics topics covered in that lesson.

Exploring Rebecca's Teaching: How does Rebecca Promote or Impede Equity in Her Classroom?

Rebecca considers herself to be a teacher who cares about equity and diversity. More than being concerned with issues of equity, she desires to intervene on these patterns of achievement for the students in her class. This section explores four characteristics of Rebecca's teaching that reflect her deep commitment to working with diverse learners: (1) deep commitment to working with diverse learners, (2) diverse learners need different strategies for learning mathematics, (3) diverse learners need mathematics instruction that helps them learn important mathematics, and (4) diverse learners need a "safe" environment for learning mathematics.

Deep Commitment to Working with Diverse Learners

Many of Rebecca's comments in her post-observation and clinical interviews reveal that she has a deep commitment to working with diverse learners. For example, in her Wave 3 general interview she is responding to a series of questions about equity and diversity, where she describes her commitment to teaching in her school setting: "…only one or two of [of the students in her class] was not [an] English language learner…But in all honesty, people say, 'I want to teach GATE (Gifted And Talented Education) students because they'll do their projects so much nicer' and I'm going 'No, I don't want to deal with them. I would rather work with students that I can present projects in different ways and let them go ahead at it how they want to and bring in their cultural aspects and make the whole classroom like a family. So yeah, it's fun, though." (Wave 3, general interview) This comment reflects Rebecca's commitment to working with diverse learners and her desire to teach in ways that value and include their cultural perspectives that make them feel respected and appreciated, "like a family."

An additional example of Rebecca's commitment to working with diverse learners is from her Wave 2 interview where she discusses what curriculum she uses in her lesson planning. She responds that because her district gives her complete freedom in using materials other than the district adopted textbook series she uses a variety of materials including "internet resources, workbooks, [materials from] other teachers. I take it all. Whatever is going to work for those students, that's what I use" (Wave 2, general interview). This comment further reflects Rebecca's commitment to serving the needs of the students in her class. She strives to design and enact mathematics teaching that will encourage, value, and respect who they are and the perspectives they bring to learning, while also providing them with important mathematics content.

Diverse Learners Need Different Strategies for Learning Mathematics

In an interview, Rebecca described how she thinks that her students, and students similar to hers, need additional strategies for engaging in the mathematics. In particular, she seems to argue that students who do not have fluency with basic skills should still be able to participate in the mathematics and that teachers just need to allow them to use additional strategies. For example, in her interview about her experiences in the professional development workshop, she describes not only feeling isolated, but frustrated because of the other teachers' attitudes towards teaching students like those in her class. She comments,

> I mean if they are English Language learners, then you accommodate for that, but you don't basically water down your lesson and teach multiplication and fractions all day long. And even for those kids that don't know multiplication, find another way to teach it, you know? If they don't have it by memory, it's OK to just allow them to use other methods if need be, don't get hung up for nine months complaining that your students don't know how to multiply or that your students are English language learners and they're not going to learn math (Wave 2, general interview).

Here Rebecca is commenting that students who are not fluent (in English) are able to engage in learning mathematical tasks and concepts if their teachers create broad access into the mathematics for their students. Additionally, she argues that this broadening work should not compromise the integrity of the mathematics. She implies that broadening the ways that students are able to engage in the assigned mathematical tasks also fosters a sense of high expectations. The last part of her comment about teachers believing that English language learners are not going to learn math, suggests that Rebecca believes that if teachers were to take deliberate steps to include all the students into the mathematics work, then each student would be able to do the work. In this way, teachers would be able to enact high expectations if they create multiple ways that their students can access the mathematics.

Evidence of Rebecca's efforts to do this work is drawn from a lesson on multiples. In this lesson, Wave 2, Lesson 28, she guides the students in solving the first four steps in working on the "locker problem." In the locker problem, there are 1000 lockers in a school and the first student has opened every locker. The second student closes the even numbered lockers and the third student changes the lockers (open the locker if it's closed and close the locker if it is open) in multiples of three, and then four. This brief exchange takes place as the students are starting to work on the number three. "So, you're gonna decide if he opens or closes it for all your multiples of three. If you don't remember your multiples of three take out your multiplication chart that you guys have. Or that some of you guys have." In this case, Rebecca anticipated that some students might struggle with this problem because of weak multiplication skills and provided an easy remedy for this issue. Instead of allowing a mathematical weakness eliminate or reduce these students' opportunities for participating in the mathematics, Rebecca has found a way to allow them engage in the mathematical task. Using this type of scaffolding creates access into the mathematics by removing barriers for students who may have weak mathematical skills. As such, this practice is an example of equitable teaching.

Diverse Learners Need Mathematics Instruction that Helps Them Learn Important Mathematics

Rebecca reflects her commitment to teaching diverse learners by providing mathematics instruction that she believes will help her students learn important mathematics. Rebecca's interview data reveals she perceives that her students are best served by having a solid, basic, procedural understanding of mathematics in order to be successful (Wave 2 & 3 interviews). Rebecca comments that she is committed to teaching these kinds of students with a purpose of ensuring that they gain a solid mathematical foundation in her class. Rebecca is often able to predict and identify her students' difficulties, demonstrating a level of mathematical knowledge . However, as described earlier in this case analysis, evidence of her weak content knowledge is found because she doesn't seem to know which details to emphasize (teaching a procedure instead of helping students understand why it works) or how deeply to explore a particular topic. Her responses to the students in her class seem to try to balance wanting to have a very positive relationship with students while also helping them focus on the mathematics. Similar to teachers in Gloria Ladson-Billings' study (1994), Rebecca has a strong and positive rapport with the students in her class, speaking and managing the class with a business-like manner. This manner allows her to reach a student population who has not been served well by schools. Many of the students in her class are well below grade level, and are more reluctant to engage in mathematics classroom.

Responding to these students in a respectful manner is paramount for their motivation and engagement in the mathematics.

Diverse Learners Need a "Safe" Environment for Learning Mathematics

Rebecca reflects a commitment to working with diverse learners by creating a "safe" learning environment. In her Wave 2 interview, she comments,

> (As a teacher of mathematics) she tries to encourage kids to: 'Just do it.' Like not even just try, do it. And you will be successful. Just the confidence level, just building them up, because of my students may feel they can't do it or when they get in here it's like 'Oh no' it's going to be hard in here. But they get in here and they're like. 'Oh, this is a relatively relaxed classroom.' So the environment is safe and able to say how they feel or if they don't understand, it's OK.

This illustration reflects Rebecca's commitment to working with students who have struggled in schools and describes her deliberate effort to create a learning environment where they will be safe and can learn complex mathematics. In another interview, she is asked if her students have a great deal of difficulty learning mathematics, and she responds that over half of them do. Instead of blaming the students or expressing frustration that these students to do come to her class with the prior knowledge needed for doing the mathematics she has to teach (as many other teachers commonly do), she instead recognizes the important role teachers play in this dilemma, commenting that she believes "these students" have been taught multiplication for seven years, so they don't come have knowledge of order of operations and they don't have knowledge of fractions, they don't have that much" (Wave 2, general interview). Rebecca recognizes this dilemma and tries to deliberately intervene on behalf of these students by having high expectations and creating as "safe" learning environment for them.

One example of Rebecca's teaching that reflects this "safe" learning environment occurs in a lesson on areas of circles, triangles and rectangles. She is teaching this lesson to an eighth grade class. Within the last ten minutes of class, Rebecca is circulating while the students are working, asking follow up questions such as, "Why? How do you do this? Do you remember what to do?" As she is walking around, she comes to one group of students asking them "You guys done over here? Come on Jorge! Mr. Jorge, let's go! You falling asleep?" Jorge responds, "Yeah" to which Rebecca asks "Why?" The students in his group respond, "He don't really understand some of it. So he helped us on part of it." Rebecca clarifies, "He helped on certain parts and on other parts he doesn't understand? So what don't you understand, Jorge?" Jorge responds, "Do you times this and then you use this and turn this and then after that, you do it like, let's see three times this and then this will be divided by two?" Rebecca confirms that he does understand the problem and is on the correct path for solving the problem. In many classrooms, students who are choosing not to engage and work on the mathematics are penalized, either through class or school discipline practices or by being isolated from participating with the class by the teacher. Rebecca appears to have viewed this student's behavior as a symptom of being discouraged and frustrated with the problem and took the time and effort to help this student re-engage with his group and the mathematical tasks. In this way, Rebecca's strategies for creating a safe environment involved generously interpreting student's behavior and taking deliberate steps to re-engage him in the class. These practices of generously interpreting a student's behavior are a practice associated with Rebecca's goal of creating a "safe" learning environment and are an example of equitable instruction.

Exploring Rebecca's Teaching: What is the Mathematical Quality of Her Instruction?

Overview of the Quality of Rebecca's Mathematics Teaching

Rebecca's levels of mathematical knowledge are represented in the results from the LMT paper-and-pencil measures and results from coding her classroom instruction. Rebecca earned the lowest scores of the ten teachers in the LMT video study project (see Hill, et al., 2008). Rebecca's lessons are quite similar and her teaching strategies have very little variation among topics or concepts. In addition, her practice is not characterized by identifiable segments of teaching, such as warm-up or launching a major task or summarizing the mathematics in Rebecca's lessons. The illustrations used in this case are not as detailed and complex as the classroom illustrations in the other cases because each of the lessons in the data look the same—she demonstrates how to do a procedure, the students follow along, and then she offers additional examples both in whole group and for the students to do on their own. I discuss three features that illustrate the mathematical quality of her instruction. First, Rebecca's teaching frequently contains mathematical errors. Some of these errors are subtle in nature, such as description of a mathematical procedure that lacks the degrees of explicitness needed for a mathematical explanation, whereas some are more blatant errors, such as referring to π "three point one fourteen." A second feature focuses on the lack of emphasis on conceptual understanding. The third and final feature is a tendency to use mathematical contexts ineffectively. Although the purpose of using mathematical contexts is to motivate and interest students in the mathematics content, poor use of context occurs when the context obscures or misrepresents the mathematics focus of the lesson, as is common in Rebecca's teaching.

Common Errors with the Mathematics Content

Many of Rebecca's lessons contain mistakes. These errors range from obvious mathematical errors, such as "π" is "three point one fourteen" (Wave 2, Lesson 26) to her use of imprecise mathematical language, for example "canceling" and using "disappearing" when teaching about adding and subtracting positive and negative numbers (Wave 1, Lesson 2). She also makes mathematical errors that cause students confusion, such as when she makes up numbers to have students practice multiplying positive and negative numbers. Because she has given little thought to the development or purpose of her numerical examples, she even confuses herself when multiplying "negative six times negative six times negative six times negative six negative six times negative six times negative six.[22]" She begins solving it, multiplying pairs of the sixes and then mentions to the students that she may have made an error. She turns the problem over to the students, labeling it a challenge problem and suggesting that those who are interested may take it home and do it for homework (Wave 1, Lesson 6). In this way, based on the questions that Rebecca's students ask in response to her teaching, it appears that, although Rebecca's teaching seems to help students develop some computational fluency skills, her teaching may create misunderstandings by her students. Rebecca's instruction illustrates the importance of subject matter knowledge because its need becomes evident from the consequences of its absence.

[22] In Wave 1, Lesson 6, Rebecca asks the students to solve the problem (-6) (-6) (-6) (-6) (-6) (-6) (-6). The "mathematical" answer (as Rebecca calls it in class) is – 279936.

Emphasis on Developing Basic and Procedural Skills

Rebecca's teaching matches the description of " 'typical' mathematics classrooms described in the literature more than 20 years ago where students' work consists almost entirely of memorizing presented facts or applying formulas, algorithms or procedures without attention to why or when it makes sense to do so" (Stodolsky, 1988). The tasks used in Rebecca's lessons represent what Henningsen and Stein (1997) call "lower level tasks". In each of her lessons, the assigned tasks require procedures without connections to concepts, meaning, and understanding. Rebecca seems to focus on the completeness and accuracy of answers, rather than focusing on meaning and understanding. Because the "greatest gains in student achievement occurred in classrooms where high levels of cognitive demands were consistently maintained," this type of mathematics instruction works against the interests of equity because students are not provided with opportunities to learn that focus on developing mathematical proficiency (Stein, et al., 2007).

One example of Rebecca's emphasis on basic and procedural skills comes from a lesson on adding positive and negative numbers. This segment is in the first five minutes of the second consecutive lesson on learning about adding and subtracting negative numbers.

Rebecca: Alright, so … like always we have to make it harder. We're gonna have … let's make a bigger number.

We're gonna have four positive cubes and we're gonna add two negative cubes. So, we're gonna have four positive cubes and we're gonna add two negative cubes. So, you have four cubes in front of you that are positive. Okay, you have four cubes in front of you that are positive. You now need to add two negative cubes. But there's a trick to this.

Student: A what?

Rebecca:	There’s a trick to this. So you’re gonna add two negative cubes, right? So you’re gonna add two negative cubes and you’re gonna add them. Well guess what? The positive and negative they don’t like each other. So they’re gonna become zero and you’re gonna move them out of the way.
Students:	Which one?
Rebecca:	Your positive and your negative, they don’t like each other … so they’re gonna cancel each other out and they’re gonna now become a zero. Another positive and negative, right. They don’t like each other, they’re gonna become a zero.
Students:	Okay.
Rebecca:	So this is what you call your field of zeros. Your field of zero’s. What do you now have left with? Two what?
Students:	Negative …
Rebecca:	Negative or positive?
Students:	Positive.
Rebecca:	Positive. Positive.

The students in this example are simply following directions and learning to do a procedure. The “field of zeros” is an almost magical place where the zeros can go and where one can go later and “get” them if needed for solving a problem.

Another example of Rebecca’s emphasis on procedural and basic skills comes from the lesson on areas of circles, triangles, and parallelograms (Wave 2, Lesson 26). This segment comes from the end of the middle part of the lesson.

Rebecca:	All right. So we got over the triangles. We went over the circle. Let's go to the parallelogram. And then, we're gonna have class competition, where you're gonna have to tell me what the answer is, 'cause I can't tell you anymore. It's not on those ones. I forgot, you guys are gonna get that paper. I'm so glad you did it, at least you guys know what you did. Okay, so four centimeters, nine centimeters, nine centimeters, four centimeters
Rebecca:	Rashi, how do I find the area?
Rashi:	Base times height.
Rebecca:	Base times height?

Rashi: Yeah.

Rebecca: So, what is my base?

Rashi: Nine.

Rebecca: Nine what?

Rashi: Centimeters.

Rebecca: And what is my height?

Students: Four.

Rebecca: So what do I do?

Class: Nine times four.

Rebecca: And what's nine times four?

Students: Thirty-six centimeters.

Rebecca: Thirty-six centimeters, what?

Rashi: Squared.

Rebecca: Squared. So, if I gave you a quiz on just those three things, how many of you guys think you would have passed it?

This lesson shows what is typical of Rebecca's teaching, the frequent use of one word answers and little focus on conceptual understanding. More specifically, Rebecca's lessons exhibited a common pattern: Rebecca gives information, asks questions, giving directions, making assignments, monitoring seat work, reviewing assignments, and assigning homework" are all typical characteristics of Rebecca's lessons. Recall the lesson described earlier in this case on multiplying binomials (in Rebecca's class as "making the F.O.I.L 'happy face'") where students are directed to focus on copying her examples from the board. Rebecca's teaching represents what Haberman (1991) calls "the pedagogy of poverty" (p. 291). Haberman claims that this style of teaching appeals to those who do not know the full range of pedagogical options available and who also believe that basic skills are a prerequisite for learning and living (p. 291). He cautions that the “pedagogy of poverty” does not work because “youngsters achieve neither minimum levels of life skills nor what they are capable of learning and that it does not work because it is a pedagogy in which learners can ‘succeed’ without becoming either involved or thoughtful” (p. 291-292). Instead, Haberman advocates for teaching acts that are aligned with the description of equitable instruction used in this dissertation. He suggests that students benefit from teaching strategies that use “heterogeneous grouping to promote divergent questioning strategies, multiple learning assignments in the same class, and activities that allow for alternative responses and solutions” (p. 294). He further advocates for teaching acts that provide opportunities for students to “compare, analyze, synthesize, evaluate, and generalize (claiming that) the acquisition of information and skills, without the ability to think is an insufficient foundation for later life” (p. 294).

Use of Mathematical Contexts

Mathematical contexts is advocated because it can motivate students to work on the mathematics or help to make “real world connections” between the mathematics being learned and students’ everyday lives. However, as described by Heaton (1992), effective use of context requires pedagogical and mathematical knowledge. Below, I offer two examples of Rebecca’s ineffective use of contexts.

Rebecca begins a lesson on rules for products of positive and negative numbers by reciting a ‘(mean) poem’:

Rebecca: All right. Yesterday we talked about adding integers, adding positive and negative integers and today were going to talk about integers again. So I'm gonna tell you a story. You have to listen very, very closely. Hopefully I don't mess up the story. Okay? It's not a story. It's a poem. Okay.

When a good thing happens to a good person- wait.
When a good thing happens to a good person, that's a good thing.
When a good thing happens to a bad person, that's a bad thing.
When a bad thing happens to a good person, that's a bad thing.

Okay. I'll say it one more time. (Repeats poem.)

Students work on a few problems, about ten minutes while Rebecca is still leading the students through example problems on the front board, the following exchange takes place:

Rebecca: Is my answer gonna be negative or positive, Sahan?

Sahan: Negative.

Rebecca: It's gonna be negative. Why?

Sahan: Because the sign always represents what it'll equal?

Rebecca: The sign always represents what it equals. Can you go more into detail with that? No idea, can somebody help him out?

Student: Because a bad, uh, a good thing happened to a bad thing- a bad person?

Rebecca: Okay or a bad thing happened to a-

Student: Yeah.

Rebecca: A good person? Okay. So you know it's gonna be negative. Right?

Although the mathematical purpose of this lesson is to understand the rules for multiplication of integers, Rebecca's students focus more on learning the 'poem' than on interpreting the meaning she intended for them to master. In this lesson, Rebecca spends sixty-one minutes on two tasks: helping students memorize this poem, and practicing the procedure for multiplying integers. After her initial introduction of the poem, Rebecca makes up random problems for the students to use as practice. Many of these problems do not have appropriate solutions, for example, an answer that is in the hundreds of thousands when the focus is simply on being able to identify if the answer is going to be positive or negative. In another lesson (Wave 2, Lesson 26), Rebecca makes up a "real world context" claiming that she has a house and she is "tired of the black…So I wanna change the color of my house to brown, so I need to know how much of an area it is to buy all of the stuff I need. So I have a pretty big house… the base of the house is twenty-five yards and the height of my house if seventy-two yards…. so tell me how much is the area?" The numbers Rebecca is using are not even in the realm of reasonable for her house! In these examples, and on several other occasions, Rebecca made decisions that influenced her teaching without sufficient knowledge or concern for the meaning of the mathematics of the lesson.

In Rebecca's lessons, contexts (often in the form of "tricks", like the 'mean poem' describe above) are treated as a theme that run throughout a lesson; there is no increasing complexity (as admitted by Rebecca), or connection to conceptual understanding or strategic competence. As such, Rebecca's use of context often misses the mathematical point of the lesson and even clouds the students' abilities to obtain a mathematical focus on the procedure (Sleep, 2009). Her interview data reveals that she is highly committed to making mathematics enjoyable and wants her students to find mathematics engaging. Furthermore, she believes that she is teaching for understanding. In her post observation interviews, she often comments that the students' work on the activity went well, but she is often puzzled that she feels as if her students aren't really "grasping the concepts" (Wave 1, Lesson 6, post-observation interview). Similar to the teacher in (1992)

Heaton's case, Rebecca appears to struggle with understanding the difference between "teaching for understanding" and "meaningful" activities for students.

Summary

Because presence of common mathematical errors, the lack of conceptual emphasis in Rebecca's teaching, her inappropriate use of contexts it suggests that Rebecca's limited knowledge of mathematics may significantly impede the mathematical understanding that her students are able to gain from her teaching. In this way, Rebecca's mathematical teaching would be characterized as inequitable.

Summary: What Does Rebecca's Case Illustrate?

Rebecca's mathematics teaching illustrates how mathematical knowledge for teaching is a pre-requisite of equitable mathematics instruction. I selected Rebecca because she earned the second highest score on the equity video codes, but earned the lowest scores on the LMT paper-and-pencil MKT measures. Rebecca appears to struggle in her teaching as evidenced by frequent subtle and blatant mathematical errors. According to state and national standards, she would not qualify as a highly qualified teacher. However, Rebecca shows remarkable sensitivity to the needs of students in her classroom. Her interviews suggest that she is very committed to teaching in under-resourced classrooms with non-white students who have struggled in school and wants her students to learn the content needed to improve their academic performance. I defined equitable instruction as instruction that provides equitable access to challenging and meaningful opportunities for learning mathematics (through instruction) that increasing participation and academic success, especially for marginalized students.

Equitable teaching only works if students develop mathematical proficiency as what the National Research Council (2001) has defined as: "Mathematical proficiency is composed of five interwoven, interdependent strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition." Because Rebecca's teaching only focuses on developing procedural knowledge and does not provide opportunities for students to learn the other four strands, her practice does not represent equitable or high quality mathematics instruction. In this way, analyses of Rebecca's teaching help to substantiate the argument that a teacher who is very sensitive to issues of equity and diversity but has inadequate mathematical knowledge can produce inequitable mathematics instruction.

Karen: A Case of High Mathematical Knowledge and Low Mathematical Quality and Equity Video Code Scores

The case below has four sections. The first section offers a general description of Karen as a teacher, including a description of her classroom and a general overview of her teaching. The second section analyses the mathematical quality of Karen's instruction and the third section explores how Karen's teaching promotes or impedes equity.

Karen's Students and Classroom

The students in Karen's classroom represent a wide range, academic abilities, facility with English, and racial and cultural backgrounds. Karen views herself as a mathematics resource to her students and her colleagues. In more than half of her general and post observation interviews, she discusses at length the conversations she's had with other teachers at her grade level about what content to cover, how long to spend on certain mathematical topics, which resources are best for teaching particular mathematical content, and ideas about state standards and assessments.

Her classroom is neat and well organized. It appears to be purposefully arranged based on her teaching style, as we will see below. Karen's desk is in the front corner of the room and on it, she organizes the worksheets and transparencies she uses through her lessons. The overhead projector positioned at the front of the class, seems to be the central feature of the room enabling every student to focus their attention to the front of the room. In each round of observations, Karen had a different room arrangement for the students' desks. However, in each arrangement the overhead projector was central. For example, in the second round of taping, with fifth grade students, Karen arranged three parallel rows of students' desks facing the overhead. At the back of the room, the students' desks were arranged in groups of four or five, facing each other. Karen had purposefully arranged the room this way based on her perceptions and beliefs about the students' abilities and needs. In her post observation interview (Wave 2, Lesson 16) she states, "because these groups at the back can pretty much solve a problem among themselves, whereas the kids at the front need more (guidance and support)… the ones that are more reluctant are closer to me…" In other interviews, Karen refers to the students who she believes need additional support for learning as "low" and "resource" and one of her strategies for supporting these students was to arrange a room so that these students sit closer to the source of academic knowledge—the teacher and the overhead projector.

Planning

Karen uses the district-adopted curriculum published by, Harcourt-Brace, but primarily as a reference and a source of "homework drill." In her Wave 1 general interview she indicated that as an experienced teacher who enjoys mathematics and teaching mathematics, she has a wide variety of visual aids, manipulatives, and curriculum materials and resources, including "various books from the teacher store, Dale Seymour, Singapore math, Spectrum math," and materials from many years of mathematics professional development.

In her Wave 3 general interview, Karen was prompted to discuss how she decides when she is going to use standard lessons from the district-adopted textbook, when she chooses to modify lessons from the textbook, or when she supplements lessons using other curriculum materials or designs unique lessons. She responded by describing how she combines these options by first evaluating the usefulness of the structure of the lessons, problems, and representations from the district textbook. She explained that she uses the lessons that she believes are appropriate from the district textbook series and supplements with other materials as she sees fit. Later in the interview, Karen justified this strategy by claiming that she uses her knowledge of what she thinks her students know and what they are ready to learn, along with her mathematical goal for the lesson for choosing tasks. She offers an example of how she looks to see if the representations in the book are accurate, familiar for students, and useful in helping students solve problems. If the representations in the book do not meet these criteria, she either doesn't use those lessons or explains the representation or model on the over-head to the class before having students work in their textbooks.

In this and other interviews, Karen describes how she, like many other experienced teachers, doubts the usefulness of the district-adopted curriculum materials. While she believes the district materials are useful for homework "drill," she questions the sequencing and length of the lessons, the emphasis on the computational tasks, and the mathematical representations used in particular lessons. Karen's use of her mathematical knowledge to substitute the district curriculum demonstrates her confidence in her high level of knowledge for teaching mathematics. Karen claims that she designs lessons and selects problems and tasks that have proven to be more useful in teaching students mathematics based on her teaching experiences instead of simply using (and unmodified version of) the textbook series provided by the district (Wave 1 & 2, general interviews).

General Overview of Karen's Teaching

Karen's mathematics instruction is teacher directed and looks in many ways rather traditional. Mathematical explanations are, however, an important feature of her mathematics instruction. During many of her mathematics lessons, she requests and creates opportunities for students to offer mathematical explanations for their mostly computational answers. Additionally, she emphasizes that students need to learn to offer explanations that demonstrate that they know why an algorithm works and comments on the usefulness of mathematics in "the real world." Karen makes few mathematical errors; the mathematical topics and skills she teaches are critical for students to develop mathematical proficiency.

A typical lesson in Karen's classroom begins with mental math, which she prefers to conduct in silence, unless she calls on a particular student. She calls her mental math activities “short, quick, fast thinking” (Wave 1, Lesson 14). This “warm up” phase of the lesson, also includes sometimes word problems done on the overhead. These problems are selected to help reinforce “basic” skills and are only sometimes connected with the mathematical work of the day’s lesson.

Next, she typically launches into the work for the day, offering a brief explanation about the skill or topic they will be working on, and she often links this topic with a concept they have previously learned. Although Karen’s teaching is teacher directed, during her lessons she frequently calls on students to solve problems on the overhead, or show their picture or diagram, and she often requests that students share their strategies and solutions for solving problems. At times, she will ask either for the whole class to indicate agreement or disagreement with a particular strategy or solution or will call on individual students to respond to another student’s idea.

Throughout the lesson she often navigates between whole group work and individual work. At times, she strategically uses small groups to separate students based on ability levels for work on varied levels of mathematical tasks. That she is able to readily diagnose difficulties students are having with the content is additional evidence of her knowledge of the mathematics content and of her skill with using mathematics.

The above section paints a portrait of Karen’s classroom setting and general teaching style. The following section analyzes the mathematical quality of Karen’s instruction, and the last section explores how Karen’s teaching promotes or impedes equity.

Exploring Karen's Teaching: What is the Mathematical Quality of Her Instruction?

There are three features of Karen's teaching that illustrate the high mathematical quality of her teaching. The first is her ability to diagnose quickly student difficulties. In many instances, Karen was able to diagnose appropriately what difficulties students are having and to develop mathematically sensible responses. A second characteristic is careful selection of tasks. Karen uses appropriate mathematical tasks to help students learn complex mathematical concepts and skillfully uses numerical examples for modeling problems and computations. A third characteristic is her emphasis on conceptual understanding throughout her lessons. Karen deepens her students' understanding of the mathematics by helping students offer mathematical explanations and encouraging students to mathematically prove their solutions.

(Quickly) Diagnosing Students' Difficulties

As described by Ball, Hill and Bass (2005), teachers must go beyond being able to see and size up a typical wrong answer. They argue that effective teaching also entails analyzing the source of the error, which requires mathematical knowledge and skill (p.17). Skillful teaching requires not only that teachers be able to recognize common and unique errors students may make; it also requires carefully using prompts, questions, and tasks to address student difficulties to help students move toward mastery and proficiency.

In the following segment, Karen is teaching a review lesson in preparation for a test (Wave 2, Lesson 21). The primary mathematical concept being reviewed during this time focuses on whole number division and how to interpret the remainder. The lesson begins with a teacher-directed review problem posed on the overhead projector from a homework assigned a few days before. The first problem asks the students to divide a ten-foot piece of wire equally into three pieces. The second problem asks students to find how many tables they will need if a total of three hundred twenty-five people will be attending an event where eight people sit at each table.

Karen spends the first ten minutes of class reviewing two homework problems, Karen next offers a summary of ways to interpret the remainder. Referring to the problems reviewed, Karen details how the interpretations are different:

> Okay, sometimes we have to keep the whole thing like this (referring to the table problem). And sometimes we can't give an extra one so we just drop the remainder to answer the question (referring to the wire). It's important that we think about which one of those things is necessary because just doing the division problem is not enough. (Wave 2, Lesson 21)

After the first ten minutes of review, the students transition into working in small group or individually at their desks. Karen often divides the class for small group work based on "ability" or past academic performance. In Karen's post observation interview she comments that, "The children I sat with at the front to work with, were the only ones that has had some trouble with solving for the 'n' in the unknown expression. And basically I think it was a question of not understanding the directions, because they all seemed to be able to do it up there. But Kiesha has a little more trouble with the long division, and so I wanted to do some work with her on long division, and I'm not sure that she's still there…she was the only one I found really struggling continually with the long division." (Wave 2, Lesson 21)

In this example, she separates one small group she will work with at the front of the class, while the remainder of the students work individually on a task from their textbook. Papers in hand, she identifies the students who will work with her in small group by calling their names.

> Okay, today I'm gonna have to call some of you up here to work on--on the algebra and the rest of you I want to turn to page one hundred ninety-six and see how much you can get done before lunch. If you have a question you need to quietly ask somebody close to you and I would like to have Priscilla, and Maria, (looking through papers), Maria, and Kiesha, and Julio, and Tou, and Cassandra.

Karen works with six students who did not perform well on the assignment. She believes that without intervention these students would most likely perform poorly on the upcoming test (Wave 1, general interview). Karen directs the students to get focused and settled on the floor. She then asks Maria to read the directions for the homework. After Maria reads directions the problem, Karen questions the students about their interpretations of the work "evaluate", to which Maria offers, "Find the answer." Karen repeats her response, clarifying this term for the students, "(repeating Maria) Find the answer. Tell the value of. That's what evaluate means." Karen encourages Maria to finish reading the problem and then guides the group in recognizing that the problem asks them to "evaluate the expression for each value of 'n'," and "to think about how many 'n' values they were given." In this discussion, Karen explicitly directs the students to notice that they are given three values and therefore must have three answers. She also provides a brief example of solving for 'n', "Solve for 'n', if 'n' is ninety-six. So 'n' divided by six means ninety-six divided by six." Karen asks the students to write this example on their individual white boards. Additionally, Karen points out to one student, that he had written all of the problems, but simply forgot to solve them.

This first exchange already provides examples of Karen's substantial mathematical knowledge. Based on student performance and student work, Karen has predicted that language might be a barrier for some of the students in solving the problems and starts their work by defining and explaining the term "evaluate." Karen not only describes what the term means, ("find the answer; tell the value of; that's what evaluate means"), but she extends her explanation by continuing to talk through the problem with the students. Additionally, the use of the term "evaluate" for this task required students to find multiple answers for one problem. Students who were not able to solve the problem because they did not know how to use the term "evaluate" can begin work on the mathematics with new understanding. Anticipating students difficulties with particular terms or mathematical task, helping students to understand the meaning of a particular term (i.e., that "evaluate" means "to find the answer") and then supporting students' use of that term to solve mathematical problems, (i.e., for this task, knowing that it means you need to have three answers) requires mathematical knowledge.

In this case, explaining the term "evaluate" was enough support for some students to work autonomously on correcting their work. Working with students who are still struggling in the same small group, Karen continues to walk students through individual problems, helping them solve and "check" their responses with the opposite operation.

Karen observes as students work independently on their problems. After glancing quickly at Keisha's paper, Karen comments that her answers are out of order. Karen evaluates her answer and checks for understanding by asking, "So you understand how that works? Okay, could you do this problem for me?" While Keisha is working, she turns her attention to a different student, asking him to solve the next problem, "The next ones says thirty-eight divided by six…can you show me that one please, Tou?" As Tou continues to work, Karen turns her attention to another student. "Priscilla, do you remember when we talked about multiplying and dividing being opposites and that we could use (the) inverse operation? I think it would be easier if you did that." She continues to prompt Priscilla, "No, twelve times something makes forty-eight…(pause) forty-eight's gonna be your answer. Put the forty-eight down here so you can see." After working with Pricilla, Karen turns her attention back to Keisha. "Which one is that? Okay, you went to the next problem and see, there is three of these, Keisha. First you have to divide fifty-four divided by nine. Then you have to do ninety-six divided by nine. And then you have to do one hundred thirty eight divided by nine." Keisha indicates that she understands and Karen begins releasing students from the small group. "You take that back to you seat and you can work on the same thing the other people are doing."

This segment further illustrates Karen's sound mathematical knowledge. The most fascinating aspect in the segment is that Karen is managing various conversations simultaneously. She is tracking students' progress as the students work to correct their papers and solve problems. At each turn, Karen is deciding which students need what kind of support to work through additional problems. For example, with some students she starts new practice problems (e.g. "So now you understand how that works? Okay, could you do this problem for me?"); helps student who appear to be stuck in the middle of solving a problem (e.g. "You don't want to be confused by that. This one right here. What does it say?"), and judges which students have had sufficient support and mediation to join the other students in the class who are working on an individual assignment.

("You know exactly what you're doing, Tou. You take that back to your seat and you can work on the same thing the other people are doing.") In each response, Karen has used her knowledge of mathematics to diagnose and "treat" students' difficulties as they solve the assigned mathematics problems

Another example of Karen diagnosing a student difficulty is her exchange with Tyrone. The following illustration is from the warm up portion of the lesson, immediately after the mental math problems. Karen reads the word problem aloud to the class, "Brian counted fourteen legs. He saw two pigs and how many chickens?" Karen tells the students that she wants to start their discussion by focusing on counting the legs. One student answers three chickens. Karen asks for another student's answer who offers the number seven. Next she asks Tyrone, who has answered five, to come and show how he found his answer. She asks the class how many legs the "two little piggies have", to which they respond four, which makes eight legs. Karen continues, explaining that there are "six more legs to go." They establish two legs per chicken, and Karen asks "how many two's make six?" Tyrone answers "three." Karen prompts Tyrone, "So, how many chicken's were there?" Tyrone answers again, "three." Karen states to class that Tyrone now agrees (with the class). In this example, instead of Karen continuing to ask students for responses, she uses Tyrone's error to teach the class by connecting the chicken's feet in the problem with the mathematical skill of doubling, or counting by two's, to help him find the correct answer.

Tracking and paying simultaneous attention to students' progress requires mathematical and pedagogical skill. Mathematical knowledge is also needed so teachers can listen and interpret students' mathematical ideas and strategies, and to differentiate between students' understanding of, or skill with, the content and students' understanding of the task (Ball & Bass, 2000). Karen is able to do this work quite skillfully as evidenced in the illustrations above. She relies on her solid understanding of mathematics and her thirty-seven years of teaching experience to navigate this terrain and support the mathematical work of these students.

Choice of Tasks with Students

As described by Ball and Bass (2000), one of the many tasks of teaching requires teachers to develop a best possible assignment or instructional segment in response to their analysis of students' work. To do this effectively teachers require content knowledge. Preparing and using mathematics tasks require several considerations. For example, as teachers construct or select a task, they must analyze the nature and territory of the task, consider the curricular learning goals it supports, and determine whether it better supports collective class work or is better suited to small group or individual work (p. 100). As illustrated in the following segment, Karen does this well.

Throughout many of her lessons, especially in her work with small groups, Karen demonstrates the strategic selection of numerical examples. At times, this involves evaluating the mathematical problems posed in the curriculum. However, in the following episode, Karen is not using a curriculum source. This segment is from a third grade class at the end of the year; the students are working on long division. She leads the whole class through mental math exercises during the first portion of the lesson. Next, Karen transitions to working with a small group of students, two boys and four girls. The remaining students in the class have been directed to work on a worksheet ("quietly") at their desks.

Karen leads the students in her small group to work on division of whole numbers. Using Unifix cubes,[23] the teacher directs one student in modeling the problem "72 ÷ 4" by dividing seventy-two cubes among four children. In the following episode, Karen directs one student to first divide the seven groups of ten Unifix cubes among the four girls. She then guides him in counting how many he has distributed (forty). Next he subtracts forty from seventy-two to see that he has thirty-two Unifix cubes remaining that need to be distributed among the four girls. The group is able to easily see that each of the four girls gets eight cubes, "…therefore, 72 ÷ 4 = 18". Karen directs the students to check their work by multiplying 18 × 4 = 72. She continues to work with children in this way on through a series of problems: 72 ÷ 2; 42 ÷ 3; and 78 ÷ 6. Once the students demonstrate that they can solve a problem independently, they are released to return to their seats and work on the worksheet with the rest of the class.

In this episode, Karen has modeled the division for the students and worked carefully through the division with the Unifix cubes. The numerical examples were carefully selected, for example, in the numbers selected by Karen, there were no reminders for the problems chosen; the divisor, dividend, and quotient are all reasonable numbers these students can easily use for these calculations.

Another strong point of this episode was Karen's work in connecting the work with the Unifix cubes with the standard long division algorithm. Additionally, helping the students learn to check their work by using the opposite operation helps illustrate the relationship between multiplication and division. Karen's skillful selection and use of

[23] Unifix cubes are colorful interlocking cubes that connect only in one direction. Creating a chain of cube by snapping them together, students can form long "sticks" or "trains." Unifix cubes are often used in elementary classrooms to work on concepts related to helping students develop one-to-one correspondence, counting, sorting, number and operations, and patterns.

numerical examples and her explicit connection between the modeling work with the algorithm are evidence of Karen using her understanding of mathematics to design and enact robust mathematics lessons.

"This is why this works": Karen's Emphasis on Conceptual Understanding

In many of Karen's lessons she asks her students to engage in in-depth reasoning, for example, by requesting that students explain their solutions and their thinking. As evidenced by her post-observation interview, Karen believes that in order for students to really understand mathematics, they have to explain their thinking. She does not simply accept their solution, she insists they explain how they solved the problem, and how they checked and verified their solution. As evidenced through her post-observation interview, Karen explains "They have to really understand the concept of mathematics—they're used to computation, computation rather than attaching it to meaning. So it's important that they get lots of experience doing it" (Wave 1, Lesson 8). Similarly, referring to the long division example above (Wave 1, Lesson 11), during her post-observation interview Karen stated that she was, "…trying to get them to see (that) they need to explain everything and I want to know what their thinking is and (for the students to know) that it's ok to have different thinking, as long as you get to the right solution."

In her general interview for Wave 2, Karen explained that a goal of her teaching was:

> I'm trying to get them to think about—to do things mentally more because I think it does help them improve their number sense and especially after all this time that they've been into these algorithms. Sometimes it just falls flat because they're just—they just don't want to think about math, but other times I get enlightening little statements from them here and there and as long as they share those things and I continue to do that with them, I think that it makes it easier for them.

She further explains that encouraging students to offer mathematical explanations is a challenge because students often don't understand the purpose of explaining and that, especially for her fifth grade students, offering explanations for their thinking and solutions is very unfamiliar to them.

> "…I think that a lot of kids, they just want to finish what you have assigned and then that's their whole intent and it doesn't matter how you do it, just so it's on the paper….But if they don't understand then that (assignment) isn't doing much. (As a teacher) you just hope that you can convince them to try to do some of that a-ha-ing. ..I want them to be able to explain what they're doing even if it's a simple computation, you know, 'Why does this work? Any other way to do this? Any other way to think about it? Can you show me?' That kind of stuff. Because some of them are really derailed in that (explaining) department, 'Because that's why my (former) teacher said I had to do.' That's as far as it goes and they don't care to go any farther…because they're not used to talking about any math. But I kept telling them they had to think math…I make my kids explain what they're doing when they come up to the board (to solve problems). When we do our group stuff, I have them (students) come to the overhead and explain or explain to each other from the chair or do something mentally and explain how they go it (the answer). And I am finding that I do a lot of questioning about everything they say because they don't give a complete explanation." (Wave 2, general interview)

Additionally, Karen recognizes and uses mathematical explanations as a useful teaching strategy to illuminate student errors and expose misconceptions, especially when students have a fragile understanding of a particular concept. She also appreciates how mathematical explanations can promote understanding. For example, Karen designs opportunities to provoke discussions about misconceptions through mathematical discussions. When responding to a question about what students seemed to struggle with during the lesson, Karen explains, "Well, I think that having to explain it to their partner was a good experience for them because some of them couldn't do it. And in many instances their partner knew that they were doing the wrong thing and that they weren't getting it yet" (Wave 3, Lesson 19). This response illustrates Karen's mathematical knowledge as used to anticipate student difficulties and in designing opportunities for students to use mathematical explanations to deepen their understanding of the mathematical content. Similarly, one interview question asks if she decided to use any part of her district-adopted textbook when planning the lesson. She responds,

> I didn't take anything out of the lesson, but I added a lot in because I believe that they need that in order to start thinking about the mathematics that when they just have a page or so to do or a worksheet to do, there's not much thought. They think everything's the same and so everyday I introduce something that they have done in the past that makes them explain how they're getting somewhere or what the math is (Wave 2, Lesson 16).

Karen's response implies that she believes more emphasis on mathematical explanation is needed which led her to modify the lesson in order to build connections between previous lessons and the day's topic and to add a layer mathematical explanation to their work as a strategy to emphasize conceptual understanding.

Furthermore, in her Wave 3 general interview, she connects this goal of understanding mathematics conceptually to skills they will need later in life.

> I tell them "you guys nobody's going to hand you a worksheet when you get to be eighteen. You've got to know how to apply this math. So think about it. If you have to draw pictures when you hear it….

In this way, Karen's use of mathematical explanations as an instructional tool, serve multiple purposes. It encourages her students to work on their conceptual understanding, it illuminates errors, it promotes understanding, and it prepares them for using mathematics later in life.

Summary

Each of the above examples illustrates the ways in which Karen is able to flexibly user her mathematical knowledge to carry out the tasks of teaching in skillful ways. From using her discretion in planning and designing lessons and activities, to selecting students and work for use in small group, Karen is obviously a skillful mathematics teacher. The next section in this case explores how these skills create or impede opportunities for diverse students to learn mathematics.

Exploring Karen's Instruction: How does Karen's Teaching Promote or Impede Equity in Her Classroom?

I focus next on Karen's inattention to equity issues in her classroom. In particular, her perspectives, manner and expectations with students and her use of a narrow definition of what it means to be successful in mathematics impede equitable participation. Her inattention to issues of equity leads Karen to distribute her mathematical knowledge unevenly, which may result in unequal access to the mathematics being taught.

Perspectives, Manner, and Expectations with Students

Observations of Karen's teaching reveal that she treats students unequally in her class. As evidenced through observations and interviews, her manner with students seems to be dependent on her perceptions of student's abilities and work ethic. For example, she uses a select group of students as exemplary, asking the other students to replicate their work, "Everyone in the room draw a picture of Jennifer's problem" (Wave 1, Lesson 8). Earlier in the same lesson, Karen commented, "I want you to listen to Jennifer's (explanation) again so you can see how to do it right." Observations of her teaching and detailed analyses of her interviews imply that some students are capable of the work, and others are not. For example, this is evident in her room arrangement, with the "capable" students at the back of the room grouped in tables to facilitate more group work, and the "low" and "resource" students at the front of the room in rows to focus only on Karen's direct instruction. Although Karen is skillful in listening and interpreting students' responses, she deploys these skills unevenly. Based on available data, it is difficult to discern between her interest and her ability to listen to different students. However, she displays clear and high expectations for some students and not for others, as seen in the following vignettes.

Insensitive and inattentive mathematics teaching: The case of Cho. Cho is a third grade student in Karen's classroom. He is new to the class, but at the time of videotaping it is not known how long he has been attending the school or been in the United States. Karen is inattentive to his needs and does very little to support Cho's mathematical work in her classroom. During one of the lessons, she walks over to the videographer and in a sarcastic tone says, "Can you tell that Cho doesn't speak English? He's learning" (Wave 1, Lesson 8). It is not clear why the videographer would know that he doesn't speak English as this would not be a visible characteristic, nor is it obvious why it would be important for the observer to know this piece of information. If Karen was taking additional steps to include him in the mathematics lesson, and she were explaining these features, then it would make sense to highlight this characteristic. That she is not doing either of these implies that her comment was designed to marginalize Cho as an English Language learner. In the math lesson the following day, she calls on Cho to share his answer aloud to the class. Instead of supporting him to find his error and honoring his effort to engage in the mathematics and the class, she dismisses his answer and quickly moves on to another student. Halfway through a different lesson on the following day, as the students are being separated into small groups, Karen loudly tells

Cho, who apparently is not following directions quickly enough, "Math, do you understand math? We are having math" (Wave 1, Lesson 11). As some students work in small groups while others work individually, Cho is assigned to work from his desk with another student (Jennifer) translating for him. This common act usually lasts the length of the work period and is generally used to help students who struggle with their fluency with English. However, Karen only allows this for a short period of time. Karen speaks in a louder voice to Cho as a way of making her directions in English easier to understand, which suggests insensitivity to differences. Moreover, because Karen takes no steps to make the mathematics more accessible to a student she knows needs additional support in order to engage in the mathematics, these actions represent inequitable mathematics instruction.

Narrow definition of What it Means to be Good in Mathematics. Although Karen explains frequently in her interviews that conceptual understanding is very important for her students, there are many examples of her classroom instruction where solving problems quickly and correctly is highly valued. Although this practice is not inherently inequitable, Karen privileges students who are fast and penalizes students who need more time. For example, when the "smart" students are finished quickly, Karen often highlights their work as exemplary, a practice that illustrates inequitable mathematics instruction. Additionally, some general comments she makes to the whole group, (e.g., "I'm waiting, but not forever…" [Wave 1, Lesson 8]) imply that being skillful in mathematics in Karen's class is primarily connected to speed. This comment was used as a prompt to help students know that there was a limit on her wait time, implying that success in mathematics in this class was related to speed. In Karen's class, students who know how to get the right answer the first time and know how to quickly offer mathematical descriptions of their steps are used as examples and references for other students.

Additionally, Karen scaffolds students into the work in uneven ways. Some students are given adequate wait time, support, and scaffolding to make their ideas clear while others are shut down. In many of her lessons, Karen uses questioning to press for mathematical explanation. Although mathematical explanation is an important skill for being mathematically proficient, this teaching practice becomes inequitable when a teacher leaves students without sufficient scaffolding for their public description of their work, as in the case of Latoya. Latoya's episode comes from the end part of the lesson. Students have been writing word problems to match the given mathematical expressions. Working with the whole class, Karen has written a multiplication expression (i.e. $7 \times 4 = 28$) on the board and the students have been working on writing their own word problems while Karen circulates the classroom, reading and appraising students' work.

Karen	Okay. Stop for a moment. Look at your problem you wrote. Does your problem that you wrote have seven groups of four somewhere in it?
Students	Yes.
Karen	Okay. Read your problem to us then, Latoya. You said it has seven groups of four in it. Just tell me what it's going to say then? (Pause) How did you start it, Latoya?
Latoya	Maya had seven cats in a pen. She-
Students	I can't hear you.
Karen	Okay. What's she gonna do now?
Students	Can't hear you, Latoya.
Karen	Shh! Have you got one Kimberley?

Although Latoya raised her hand to volunteer an answer, when Karen calls on her, Latoya seems insecure about her solution and about reading it aloud. As Karen stands at the front of the classroom, arms folded, Latoya begins to read her problem almost in a whisper. Almost immediately she is interrupted by students saying "I can't hear you." Karen asks one more follow up question and without waiting for an answer, moves on to Kimberley. Although the task of writing a matching word problem to the given mathematical expression is a productive and worthwhile mathematical task to help students focus on the meaning of particular operations, Latoya's opportunities for participating were interrupted as the teacher choose to move on instead of waiting or supporting Latoya. As if to coax Latoya into reading her problem aloud, Karen gave her the illusion her incomplete response was acceptable ("Just tell me what it was going to say then"), however Karen quickly moves to another student without allowing or supporting Latoya to present her solution.

Latoya is a student who struggles in this class, as evidenced by her frequent selection for working with Karen in small group. Karen could have promoted equitable participation for Latoya into the mathematics work by formulating additional probes to understand Latoya's thinking, offered a hint or allowed Latoya more time to complete her response. That Karen did none of these tasks is an additional example of how, although Karen has the mathematical insight and mathematical understanding needed to help Latoya "remobilize" from being stuck, she chose not to. She chose to move on to Kimberly who has already offered several "stellar" answers and publicly used and recognized explanations previously in this same lesson.

Although one of the important tasks of the work of leading class discussions is to make a decision about which (and whose) ideas to pick up and pursue, and which (and whose) to let drop (Ball, 2000) Karen's decisions seem to be based more on which students she feels are capable. Teachers who broadly define what it means to be mathematically successful in mathematics classrooms, find multiple ways for students to contribute. Teachers who teach in these ways would deliberately find a way to allow Latoya to contribute to the class instead of marginalizing her participation. In the examples above, Karen has used a narrow definition of what it means to be successful in her class and isolated Cho, Latoya, and other students who are depending on Karen for their access to complex mathematical ideas.

Another example of Karen's inequitable teaching happens with a student named Juan who is in her third grade class. In her post observation interview for this lesson, Karen explains that she works on mathematical explanation with Juan, "but I don't know if Juan is ever gonna (be able) to think about the mathematics." Doubting the long term abilities of a student who is eight or nine years old is an example of inequitable mathematics. Karen's doubts about Juan's mathematical abilities are further evidenced by an episode from the same lesson on the meanings of operations and using word problems. The teacher asks Juan if he has a word problem to match how many groups of four are in twelve.

Karen	You got one, Juan?
Juan	Yeah.
Karen	Okay. Let's hear it.
Juan	Twelve divided by three–
Karen	Oh, is that a word problem?

Immediately after Karen's comment, the entire class erupts in laughter, which lasts for quite a few seconds. Karen chuckles as well, then moves on to another student. That Karen not only used Juan for the amusement of the other students in the class, but also did not allow him sufficient time or support to offer his solution is an additional example of how Karen's mathematics instruction impedes equitable mathematics learning in her class. Additionally, allowing the students to laugh at Juan and allowing Juan to laugh at him self, only helps play out a self-fulfilling prophecy of low expectations (Rosenthal, et al., 1992).

If observations of Karen's instruction revealed that she was unable to help any student recognize their error or that she was unable to use whole group discussions for analyzing errors in ways that promote understanding, it might imply that Karen did not have the skills required to do this work. However, recall her work with the small group practicing long division, or the example of Tyrone counting chicken feet in the warm up problem. In that example, Karen took the time to use Tyrone's mistake as a teaching strategy to help other students recognize that they were making a similar error. In this way, observations of Karen's teaching indicate that she is able to support some students in using their error in productive ways and in attending to the needs of students who need more scaffolding and support for learning complex mathematical ideas. However, this skill is deployed quite unevenly to students throughout her mathematics lessons. This unequal distribution of her mathematical knowledge and skill produces uneven access into the mathematics, making her instruction inequitable.

Summary: What Does Karen's Case Illustrate?

Karen is a teacher who is mathematically skillful in her teaching. Karen was selected because she is a case of a teacher with high mathematical knowledge for teaching (MKT) who earned low scores on the equity video codes. Observations of Karen's teaching and analysis of her interviews indicates that Karen does have a significant understanding of the mathematics she is teaching, and is able to hear students and to select high quality mathematical tasks. However, the ways she deploys practices associated with equitable instruction, namely deliberately working to help all of her students learn, is uneven. Based on my analyses, I conclude that this is because she lacks sensitivities about equity issues in her classroom. Karen clearly knows the mathematics of the curriculum she is teaching and her teaching provides many illustrations of solid mathematics instruction. For example, she uses mathematical terms precisely and correctly and she can represent and explain core mathematical ideas. Her interviews suggest that she has a significant amount of content knowledge, knowledge about the curriculum and students. However, what is interesting in Karen's classroom, and central to the case, is that she distributes her knowledge and skill differentially among her students. This dissertation asserts that robust equitable teaching depends on <u>both</u> robust mathematical knowledge and practices for engaging in equitable instruction.

Karen's instruction illustrates how a teacher with very high degrees of explicit mathematical knowledge can create inequitable instruction by a disparate treatment of her students. Her case helps to substantiate the argument that content knowledge per se is not sufficient on its own to produce equitable instruction for all students.

Lauren: A Case of High Mathematical Knowledge and High Mathematical Quality and Equity Video Code Scores

The case below has three major sections. The first section offers a general description of Lauren as a teacher, including a description of her classroom and a general overview of her teaching and planning. The second major section analyses the quality of Lauren's instruction and the third section explores how Lauren's teaching promotes or impedes equity.

Lauren's Students and Classroom

Lauren's classroom is neat and well organized. It creates a learning space where it is easier for her students to be focused on learning. The classroom has several posters, which appear to serve an instructional, rather than aesthetic purpose, such as common measurement conversions, and posters of data generated in previous lessons. Displaying posters of students' work is one visible strategy Lauren uses for valuing students' mathematical ideas and work to promote equity within the classroom. Additionally, the students' desks are organized into three paired rows, because student group work and collaboration are expected norms for working on mathematics in this class. The aisles between the rows allow space for Lauren to kneel and join groups of students working on mathematical tasks, also a frequent occurrence. When she walks over to join a group of students, she kneels down to their level, looks them in the eye and listens and responds to their questions and ideas. She listens, and often times waits, before interrupting the students as they are working. The care in which she engages in this type of intimate interaction indicates a deep level of respect of students and of their ideas, which in turn, supports work on rich and rigorous mathematical tasks.

The students in Lauren's class represent a wide variety of cultural and ethnic backgrounds. Her approaches to teaching these students is both respectful and deliberate, in that she tailors her teaching to the needs of the students in her class. Her responses to students and their mathematical ideas, are both kind and respectful.

Planning

When designing lessons, Lauren draws from a variety of sources, starting with the district curriculum, Harcourt Brace. She also uses other curriculum materials including Marilyn Burns and Dale Seymour, both of which are commonly used in the professional development workshops she attends. Lauren starts each lesson with prepared lesson ideas, activities, and materials. As evidenced through her post-observation interviews, she spends a great deal of time, thought, and deliberate study into planning each of her lessons. Lauren articulates a clear mathematical focus for each lesson, including describing the mathematical skills central to the mathematical purpose of the lesson, such as how this lesson relates to a state or national learning standard. Additionally, she is quite skillful in her ability to plan for using mathematical representations when illustrating a mathematical concept or skill. For example, in her post observation interview about a lesson on multiplication and arrays (Wave 2, Lesson 20), Lauren describes the great effort she invests in planning and preparing lessons. In the excerpts below, she articulates the purpose of the lesson and describes how it connects to broader mathematical requirements (i.e. state standards), and details how she strategically choose to use particular representations based on what she knew about the lesson, the mathematical concept, and what the students needed to see and understand. Additionally, while many teachers struggle with how to work with students who come to their class without the required mathematical skills for taking the next steps in the mathematics standards for their grade level, Lauren plans to deliberately mediate these issues.

> .. .One of the key standards for fourth grade is understanding that many whole numbers break down in different ways,. And this is this hopefully the whole idea was a review for them. Arrays are supposed to be covered fully, extensively in third grade, but this is our, what you basically saw was an introduction to our multiplication and division unit… I'm trying to make sure they have a fundamental understanding of multiplication, not that they just have their tables memorized before we move on and move into division and move into other things.

She continues to describe that a central goal of this lesson is to make sure that students have a "real" understanding of what multiplication is, and even though all of the students don't have their multiplication facts memorized, that this lesson is designed to make sure that all of the students are "on the same page in our understanding." She further explains how she deliberately planned to track students' mathematical productions as evidence of (or lack of) students' understanding. Lauren describes how she has to "pick a place to start" for her fourth grade students, but in her planning she carefully considered what the shape of the arrays might look like if students had a solid understanding of multiplication and what shape arrays might look like if students were struggling with using arrays or seeing the relationship between multiplication and arrays.

> You know, one of the things I was looking for was if they were just totally going to cut weird shapes out, you know with the arrays and because you know, I could have really given them a lot of guidance on how to do it step by step, but, I really- this is you know, you have to pick a place to start and this is fourth grade, so I had my radar open for that, but, I didn't see that happening. So, and then in looking at their chocolate box papers that they turned in, they all seemed to, I think have some- some prior knowledge that helped them with this.

As she reflects on the lesson during her interview, Lauren continues to discuss concerns she had during the planning process about using particular representations.

> I had a lot of angst about whether to show all of the arrays. Like to show, you know- you know and we addressed that at the beginning of the lesson, you know, five times two and then taping up the, you know, two times five, in a you know, vertical position. And I thought about that an awful lot before I did this…And so, I wanted to make sure that the kids understood that the arrays were different even though they represented the same number. But, I- you know, made a conscious decision to put up one array...You know it's an important concept the three times five equals five times three, but you know, the book refers to as the order of property and the commutative property. And I- I hope I didn't do that a disservice by having them only do the one array for each factor group. My reason for doing that was because today I knew I was going to get into talking about prime numbers. I knew I was going there and I thought I could illustrate prime numbers in a much more powerful way if there was only one array up for the prime numbers. I was just concerned about those visual learners. Even though it would've been one times thirteen and then we could have put it horizontally and put thirteen times one. I was afraid that some kids would still think four factors and I didn't want that to happen with this lesson. So, that's why I made the decision that I did.

Lauren's response provides additional evidence of her mathematical knowledge when she grapples with representing the arrays. She uses her pedagogical knowledge and her mathematical knowledge to anticipate using a particular representation (showing arrays for both 1×13 and 13×1) might be misleading to some students and lead to a misconception about factors and prime numbers, and therefore decided to alter the lesson plan to mediate this issue. Said another way, Lauren anticipated that students might not as easily distinguish between the horizontal representation (1×13) and the vertical representation (13×1). To mediate this, she decided to use one array and simply show the students that it could be positioned either way on the poster, but that it was the same number. Additionally, her mathematical purpose for deciding this was because she wanted students to more readily see that composite numbers had many more arrays than the prime numbers. This foresight and careful consideration about how to use representations for mathematical purposes, requires a sophisticated level of mathematical knowledge that Lauren demonstrates throughout each of her lessons. This detailed investment in preparing for teaching her students mathematics allows her to spend most of the time during a lesson engaged in tasks of teaching such as leading a mathematical discussion, reviewing homework, or helping students prepare for a quiz, instead of administrative tasks, such as passing out or making materials during the teaching time, which may distract students from the mathematical task at hand.

General Overview of Lauren's Teaching

Overall, Lauren's teaching is mathematically solid and precise; she rarely makes mathematical errors or slips, and her use of mathematical language and explanation have strong mathematical integrity. A typical lesson in Lauren's classroom begins with a clear statement of the objective for the day's lesson, and by providing the context for this lesson in terms of what they have previously studied or what they will be working on next. For example, in the first of a series of three lessons on probability Lauren offered the following introduction:

> Lauren: Let's see, last week we … made a likelihood line together as a class. I wanted everybody to just take a look at it and familiarize yourself with what we did last week. Okay, we're talking about probability and I told you that we would be spending the next couple of weeks on probability. Probability helps us to figure out how likely it is that an event will happen. That something will happen. So what I'd like is for a couple of volunteers just to help us review what we did last Friday cause I know have had a long weekend. Who can tell me an event that we listed up here on our chart that … having to do with something that's impossible. (Wave 1, Lesson 7)

In this example, Lauren has framed this day's work by reminding the students of previous work that is similar to the day's mathematical work and by reminding them of a definition for probability, the central mathematics topic for this day's lesson. Another common feature of the launching of Lauren's lessons was the review of homework. In many of her lessons, she dedicates time at the beginning of the lesson to review work done by the students.

> Lauren: Yeah, you can go get both of your homework's out. Ok actually we're going to talk about this one, the one we had from two nights ago. That's the only one I'm going to go over today. But go ahead and get them both out because you will be turning them both in and I also want you to have out on your desk the likelihood line. I gave you the worksheet that looks like this that you guys filled out on Tuesday. I just want you to have this out on your desk today. Okay? Your two math assignments … And the likelihood drawings yes. Okay (student's name) also has her math book opened because she knows that she has any questions about her homework she can look inside her math book (Wave 1, Lesson 13).

Lauren's lesson structure include time for whole group work, small group work, partner work, and almost always include time for the students to work independently or in small groups. During transitions from whole group work to small group work she often explicitly reminds students of their guidelines for listening and responding to each other's mathematical ideas. For concepts that are being extended, such as day 2 of a 3 days series of lessons on a probability, she begins by reviewing the previous days' work, either through reviewing homework or warm up problems and discussions, about the salient mathematical ideas. Throughout each lesson, Lauren's teaching style is marked by respect and close attention to each of her students. She circulates while they are working to clarify her directions or fills in gaps where the student's current mathematical understanding is not sufficient for work on the problem or tasks. Additionally, Lauren's lessons are mathematically dense in that she covers a great deal of substantial content in a lesson and the content is taught in deep and meaningful ways. Her lessons consistently conclude by summarizing both the tasks students worked on, such as sharing and discussing results from an experiment, and summarizing the mathematical ideas from the lesson, such as key ideas about probability or prime numbers.

Exploring Lauren's Teaching: What is the Mathematical Quality of Her Instruction?

The mathematical quality of Lauren's instruction is marked by two themes of high quality instruction: instructional time spent on mathematics and precision with language, tasks and representations during instruction. In this section, I will provide glimpses into Lauren's teaching that illustrates how her instruction represents each of these themes.

Instructional Time Spent on Mathematics

The length of an elementary mathematics lesson varies widely from teacher to teacher. Although some school and district policies may mandate a particular length of time set aside for teaching mathematics, what happens during the allocated time will vary. In this way, time is a resource for teaching mathematics[24]. More specifically, the degree to which teachers wisely use the time allocated for teaching mathematics plays a significant role in the quality of the instruction. In Lauren's classroom, students spend all of their instructional time on learning mathematics. As described in greater detail below, Lauren assigns tasks where students are engaged in problem solving, proving a mathematical solution, or interpreting alternative approaches for solving problems. She

[24] Lampert (2001) notes that "Time, space, materials, and social arrangements can become resources if they can be arranged in concert to contribute positively to students' engagement with the content. They can be distractions if they are arranged to contribute negatively to such engagement." (p. 99)

helps the students navigate between reviewing homework or solving warm up problems, to working in small groups or pairs to solve problems. Students spend time evaluating each other's solutions, building connections between mathematical representations and models, and using key mathematical terminology to describe their work. Lauren assigns mathematically focused tasks, circulates while the students are working to answer clarifying questions, probe students' ideas and refocus students' work time. She summarizes lessons by leading mathematical discussions that have students engage in public demonstrations of solutions and listen to student explanations of solutions and review key terms, phrases and procedures. As such, Lauren's students have more opportunities to learn challenging and complex mathematics than students who spend a great deal of their mathematics time cutting, gluing, or pasting.

Care and Precision with Mathematical Language During Instruction

Another strong point of Lauren's teaching is her consistency in using correct mathematical language with students. While students may struggle with articulating their ideas and solution using mathematical language, she deliberately models and reinforces the correct mathematical terms. More specifically, Lauren is quite skillful in her "technical" language; her use if "general" language; directing students in their tasks and work; and using models and representations. She uses mathematical terminology correctly, precisely, and appropriately. Mathematical vocabulary is taught explicitly, completely, and precisely. Her lessons provide high quality examples of defining key terms central to the mathematical concepts of a particular lesson. In addition to defining key mathematical terms and concepts, she generally asks questions that push students to describe these mathematical terms, or find ways to represent their mathematical productions.

The following three illustrations are examples of Lauren's use of "technical" mathematical language, that is, "the terms in the elementary curriculum that derive from the discipline of mathematics that must be used with mathematical accuracy" (Hill et.al, 2008). Three segments are described below. In the first example, Lauren is starting a series of three lessons on probability and the excerpt is from her introduction of the first lesson. The second example is drawn from the second probability lesson where the focus is on using spinners to record outcomes, and the third example is from a lesson on working on multiplication using arrays. Each of these three examples will illustrate the care and precision with which Lauren uses mathematical language.

In this first example (Wave 1, Lesson 7), Lauren is launching the day's work on probability. She refers to work completed on previous days where students made a likelihood line to more precisely quantify probability.

> Lauren: Let's see, last week we … made a likelihood line together as a class. I wanted everybody to just take a look at it and familiarize yourself with what we did last week. Okay, we're talking about probability and I told you that we would be spending the next couple of weeks on probability. Probability helps us to figure out how likely it is that an event will happen. That something will happen. So what I'd like is for a couple of volunteers just to help us review what we did last Friday cause I know have had a long weekend. Who can tell me an event that we listed up here on our chart that … having to do with something that's impossible.

In her introduction, Lauren has referenced previous work and offered a definition of probability that is both mathematically precise, and usable and understandable by her fourth grade students. By defining key words in the launch of a lesson, Lauren helps to create access into the mathematics for each of her students.

A second example of Lauren's use of technical mathematical language is drawn from a lesson on using arrays to work on understanding multiplication. This lesson (Wave 2, Lesson 23) begins with students looking for "discoveries" in examining arrays they have built using graph paper. Although the directions seem unclear, the students seem to set about finding interesting features of the arrays e.g., that some numbers have multiple arrays and others have only one. The students first work alone writing down their discoveries, and then share them with a partner. As the lesson progresses, the students begin to comment on patterns that they notice by examining the arrays they have produced earlier in this lesson that are posted on the board. The following segment is drawn from the end and summarizing part of the lesson. Here, prime and other relevant mathematical terms are being defined based on classroom-generated arrays for a select group of numbers.

Lauren: Okay. Well, these ones- these numbers, that only have one array have a name. Does anybody know what they're called? (Pause) It's a number that only has two factors. (Pause) One and itself. (Pause) Only two factors. Thirteen is an example, eleven is an example, three would be an example.

Student: That is what I was just going to say.

Lauren: Three would be an example. Okay, and it's called a prime number.

Student: Oh yeah

Lauren: Okay. So a prime number only has, well, it only has two factors. Remember, it could have two arrays, right? If- if we took the one in thirteen and put it the other way?

Students: Mm-hm.

Lauren: So you want to think about it as having two factors. Okay? Then all the other numbers have another name. The ones that have more than two factors….And they're called-

Students: Composite Numbers.

Lauren: Composite numbers. Repeat after me. Composite. So any number that only has two factors is called a prime number. It has a special name. *(Referencing the arrays on the chart)* So eleven would be a prime number 'cause it only has two factors. Okay? And thirteen would be a prime number 'cause it only has two factors. The other ones all have more than two factors.

Student: So composite number has more than two factors.

Lauren: Okay. Great. I guess we're ready to go.

The MQI coding glossary states, that when coding for technical language (mathematical terms and concepts), look for "use of mathematical terms, "such as 'angle', 'equation', 'perimeter', and 'capacity'. Appropriate use of terms includes care in distinguishing everyday meanings from their mathematical meanings." (See Appendix A) In the above segment, Lauren's teaching represents "present-appropriate" based on her defining the terms "prime" and "composite". Additionally, this example of Lauren's teaching creates access to mathematics content by explicitly teaching students what a prime number is, showing examples from their work, discussing composite numbers in relation to the discussion on prime numbers, and defining prime numbers as only having two factors (two arrays), one and itself.

Summary

The examples above illustrate the ways in which the quality of Lauren's teaching is precise, engaging, and rigorous. In particular, her use of mathematical language both maintains the integrity of the content being taught, but it also represents for students correct use of mathematical language, both generally and technically. Additionally, her effective and efficient use of class time to focus on mathematics is also representative of her high quality teaching. The next section in this case explores Lauren's attention to issues of equity within her class.

Exploring Lauren's Teaching: How does Lauren's Instruction Promote Equity in her Classroom?

In the previous section, this chapter has focused on the mathematical quality of Lauren's instruction. I now turn to analyze how her teaching promotes equitable access for learning the mathematics content. Lauren works to deliberately provide access into the mathematics for each of her students. In her teaching she leverages her knowledge of individual students to create opportunities for each student to contribute and participate in the mathematics of the lesson. For example, by seeking multiple solutions or ways of thinking about mathematical problems for students Lauren is able to increase participation of the students in her class. Lauren's teaching illustrates equitable instruction throughout each of the observed lessons in the data set. Lauren prepares and teaches mathematics in ways that are deliberately designed to provide access into the mathematics for every student in her class. The equitable nature of Lauren's instruction is marked by four themes of equitable instruction: soliciting and supporting access into the mathematics; generously interpreting students' mathematical productions; attending to the needs of struggling students; and creating a "safe" learning environment. In this section, I will provide glimpses into Lauren's teaching that illustrates how her instruction represents each of these themes.

Solicits and Supports Broad Access to the Mathematical Content of the Lesson

One strategy for creating access into the mathematics is to broaden what it means to be "good" at mathematics. Lauren's teaching has several examples of strategies for doing this work. In the following section, I will draw examples from three lessons on multiplication that involve building arrays and defining prime, composite, and square numbers.

During her post-observation interview about a lesson on multiplication and arrays using chocolate boxes, (Wave 2, Lesson 24), when prompted to share what the students need to know before this lesson in order to be successful with this lesson, Lauren responded that students need to be able to see beyond the individual arrays, because they needed to see how the arrays came together. She further explains that while she was circulating when the students were working, she noticed that her students had three different ways for seeing the number eighty-one. Some students were counting by nines (Haven's way), while others were adding all of the rows of the arrays (like repeated addition), yet one student was counting individually. Lauren adds that when she noticed the student counting individually, it indicated to her that this student would most likely need additional scaffolding and support to move on in their work with arrays. In this way, Lauren has used the student's explanations for instructional purposes, to better understand how the students are grappling with the assigned mathematics tasks, in particular how they are drawing on previous mathematical knowledge of multiplication and repeated addition, to solve problems. In this case, she was also able to discern a lack of knowledge and skill based on one student's mathematical explanation.

How does Lauren broaden student's opportunities in mathematics who are second language learners? Lauren often talks about how she values students' thinking. She comments that she finds it amazing to see what the kids are doing and how they are thinking and the she learns a great deal from the dialogue about what the students are thinking and from grading their written explanations. In her post-observation interview (Wave 3, Lesson 16), Lauren is responding to a question asking if she thought any of the students struggled with any part of the lesson. Lauren responds,

> Well, Uriah has language issues and I wish I would have checked in with her a little bit more. But she continues to surprise me with the output of her work in that often times when you talk with her you don't think she understands. But then when you actually see what she produces, she does. So that's kind of a struggle with me in how to work with her on that. Spanish is spoken strictly at the home. Pancho has similar issues in that his expressive language is sometimes really hard to comprehend. But again, his written work often reflects more understanding than his expressive language.

In this example, Lauren has clearly worked to deliberately broaden ways for these students to participate. Instead of her requiring that each student contribute in the same manner, like completing a written worksheet or handout, she deliberately creates opportunities to see how each of her students are trying to engage in learning mathematics. For both of these students, having an opportunity to share their ideas in written form allows them to demonstrate what they truly understand. By allowing students to participate both verbally, by offering mathematical explanations and solutions, and in written form, students with different needs are able to demonstrate their mathematical understanding.

How does Lauren Create Opportunities for Struggling Students: The case of Christopher. Lauren describes Christopher as a student who is usually "checked out". Although, according to Lauren, he generally isn't a behavior problem, he usually is not paying attention, participating, or tracking on the mathematical tasks at hand. Teachers generally respond to students like Christopher in a variety of ways, some choose to ignore his behavior while others may use his failures as a source of amusement for other students. Consider the way Karen responds to students who are off task, as was the case with Jose and Kimberly. These students were not able to respond to Karen's questions quickly so in one case she choose to move on and in the other case, the student's performance became a source of humor for the class to laugh at. Instead of responding in either of these ways, Lauren chooses to call on Christopher and allows him the opportunity to be involved. This gesture is even more commendable considering the class was being video-taped this particular day for our research project. In her post-observation interview, she recalls that she was "happy to see him involved and engaged" and that she was proud that he was "brave" enough to come up and volunteer, even offering a solution to one of the problems posed, drawing his representation on the board and explaining it with the class. In this case, Lauren used a broad definition for who could offer mathematically substantively solutions to the class and invited Christopher to participate. This example demonstrated to Christopher and to the other students in the class, that everyone is able to be successful in mathematics. In this case, Lauren was able to reiterate that being mathematically proficient, being "good" at mathematics, meant more than just getting the right answer quickly, but involved taking risk, offering mathematical explanations, and building a connection between a mathematical solution and the corresponding representation, even from a student who is usually "checked out".

In a different lesson (Wave 2, Lesson 24), Lauren reflects on a time in the lesson when responses were being compiled, she noticed that "Christopher had not turned in his paper and I didn't want to exclude him, so I wanted to write his responses in (to the collection of class responses)…" This deliberate attention to how Christopher is engaging in the tasks, and her efforts to finding an alternative way for him to participate by allowing him to read his responses orally because he had not submitted his work in written form, is an additional example of how Lauren broadens opportunities for students to participate in the mathematics tasks in their efforts to become mathematically proficient.

Generously Interpreting Students' Mathematical Productions and Ideas

Many scholars have studied ways of attending to students' ideas as a fruitful site for improving students participation and engagement with mathematics content (Ball and Bass, 2000; 2003; 2009; Ball, et al., 2003; Cohen & Lotan, 1997, 1999; Lampert, 2001). More specifically, these works have described how making student thinking a central component of mathematics teaching can lead to opportunities for soliciting broad participation into the mathematics work and create opportunities for valuing student contributions. In particular, the analyses below explore three specific ways for generously interpreting students' mathematical productions and ideas. They are: tracking on students; mathematical productions, creating new opportunities from errors and mistakes, and attending to the needs of struggling students.

Tracking on student's mathematical productions and creating multiple opportunities to be successful. Lauren often elicits multiple strategies from her students. She has them discuss their solution strategies and encourages other students to provide alternative strategies. In her post-observation interview, Lauren recalls evaluating students' responses and recognizing a range of ways they were working on the task. She describes:

> As I was moving around the room, kids seem to have strategies to solve this problem. They were different. Some kids were listing their multiplication facts out and- and just finding the factors and then cutting them out, but, based over there in the corner, it was so great to see her with the tiles. Very carefully, she was trying to find arrays for forty two and she was very carefully using the tiles to see if the fours would fit. And she needed that, you know. I didn't tell the kids they had to use the tiles, but I- was, you know, a lot of them went to use them to do that, so. So, they had strategies and they had ways to get there (Wave 2, Lesson 20).

This response from Lauren offers evidence that she anticipated that some students might need a smaller tactile representations to understand and solve problems to create arrays and strategically and deliberately offered the use of manipulatives so that each of the students would have access into the mathematical work. Additionally, Lauren tracked carefully on how students were managing the assigned tasks, maintaining a dual focus on both on helping students focus on the core mathematical concept of multiplication and ensuring that each student could be engaged with the task.

Creating New Opportunities From Mistakes

Many activities and tasks assigned in school mathematics curriculum often have two answers, right and wrong. However, as described earlier in this dissertation, equitable teaching broadens opportunities for students to learn. One example of broadening is by creating new opportunities from student mistakes and errors, as illustrated in Lauren's reflection. When reflecting on a lesson, Lauren describes how two of her students found solutions that were different from the given answer in the written curriculum and different from the other students' in the class. She describes that these two students, Hassan and Jessica, were actually correct based on how they were interpreting the task on the worksheet, however they didn't follow the directions. Instead of simply instructing them to mark their answers wrong, Lauren took the time to listen to their explanations and learned that they had in fact solved the problems correctly based on their interpretation of the problems. However, they did not follow the directions closely, so Lauren allowed them to make corrections. She further comments, "And that did surprise me, although, it shouldn't have, because once they explained what they were thinking I could totally see what, how they were looking at the problem." In this example, Lauren took the time to listen carefully to the students explanations and drew on her mathematical knowledge to discern that it the students knew the how to solve the problem. Instead of penalizing these students, Lauren created an opportunity for the students to participate by allowing them to correct their answers. Translating penalties into opportunities is another example of equitable instruction.

Attending to the needs of struggling students

Another strategy for teaching in equitable ways is by attending to the needs of struggling students, in particular, by anticipating student's needs and deliberately working to mediate them. Recall her comments about the student who choose to use tiles when the remainder of the class was primarily using multiplication facts (Wave 2, Lesson 20). Because Lauren is acutely aware of the range of mathematical knowledge and skills that her students have, she anticipated that they would need a variety of strategies for creating arrays. Some students were able to leverage their knowledge of multiplication to quickly cut arrays while others needed to literally build the arrays using individual cubes. By anticipating student's needs and making conscious instructional decisions about which manipulatives to offer, Lauren has illustrated equitable teaching by not only broadening how students can participate in the mathematics, but by attending to the needs of students who are depending heavily on their opportunities for learning in Lauren's class to understand mathematics.

Lauren is attuned to her students' strengths, weaknesses, and life experiences outside of class. She leverages this knowledge as a resource for promoting equity in her classroom, especially for her who came to her class with weaker mathematical skills. These decisions about broadening opportunities for learning and anticipating student's difficulties and planning to mediate them are conscious and deliberate choices, as evidenced in her post-observation interview (Wave 3, Lesson 20). "But in moving around the room and I really, I checked in with the kids that I was most concerned about and just with help of the language they really seemed to be understanding. And I was really pleased with that." She deliberately and purposefully attends to the needs of her most struggling students as an attempt to ensure that the mathematics is accessible and available for each of them. She doesn't eliminate or water down the cognitive demand for students who don't have solid mathematical knowledge of particular topics, as warned by Stein & Lane (1999).

Creating a "safe" learning environment

After introducing the task (Wave 2, Lesson 20), Lauren reviews how the students are expected to work together. She tells the class, "And I just want to remind you to please use polite and courteous words when talking about mistakes because we don't want to make anybody feel bad. We want them to be able to share what they did and if they made a mistake, you know, let 'em talk about it. 'Oops I forgot this or I did this instead' and talk about it. But remember to use polite words so we're not hurting anybody's feelings." In another lesson, she explicitly reminds students, "Remember we talked a lot in here about a safe environment being ok to share and ask question…"

(Wave 3, Lesson 16). She elaborates in her post-observation interview that this work is not always easy:

> This class has really struggled with speaking kindly to one another and we've been working really hard on making it a safe environment so the kids can feel that they can talk about their answers whether they're right or wrong. I'm trying to really get them comfortable with their sharing and their thinking and I wanted to broach that but in a gentle way (Wave 2, Lesson 20).

In her clinical interview (Wave 2) she recalls learning from professional development how important it is to be in a safe environment. She adds that she remembers wanting to feel comfortable asking for help with a math problem, thus inspiring her to "really foster a safe environment in my class and have the kids work together. Not necessarily all the time, but being able to have someone to talk about the math that you feel safe with. That's helped me, so I try to work on that with the kids" (Wave 2, general interview).

Additionally, Lauren teaches the students to respond to each other's mathematical solutions and explicitly models how to interpret each other's thinking. In many of her lessons she says, "If I am hearing you right, you are saying…" This deliberate strategy for re-voicing teaches students to respond to the mathematics of the explanation instead of arguing in small group and partner work about who is wrong. These discussions focus on the explanation so that students have opportunities to either learn an alternative strategy for finding the correct answer or have the opportunity to learn from an error that their partner may have made. Using group work in these strategic ways, not only creates a safe environment for equitable participation, but also increases the mathematical density of the lessons by deepening the mathematical understanding being developed.

Summary: What does Lauren's Case Illustrate?

Lauren's teaching represents an illustration of high scores of mathematical knowledge for teaching and high equity scores. Her teaching is mathematically accurate and challenging, coupled with being explicit in ways that allow all students access into the problems and tasks. In Lauren's interviews, she describes how much of her emphasis and deliberate work on making mathematics accessible is drawn from her desire to meet the needs of the diverse learners in her classroom (Wave 2, general interview). She appears to be motivated by the challenges her students present and she seems to use that motivation to learn more about the mathematical knowledge needed to provide rich and explicit instruction for her students. Lauren holds a wealth of mathematical knowledge as evidenced by her robust mathematical teaching. She uses her high level of mathematical knowledge to provide accurate mathematical definitions for general and technical terms, to walk students through the steps for providing a mathematical explanation, and to make mathematical connections between concepts previously learned.

Conclusions

This chapter used the MQE measure described in chapter 2 in an attempt to see if one could quantify a teacher's equity orientation. In addition to providing illustrations about how this tool works, the analyses in this chapter have also shown how the MQE measure works to differentiate practices of equitable mathematics instruction from practices of inequitable mathematics instruction. Leveraging these paradigmatic examples of teaching, one who focuses primarily on mathematics with little attention to issues of equity, one who focuses primarily on using teaching to mediate equity issues with little attention to the integrity of the discipline, and a third teacher who deliberately works to attend to both the mathematics and issues of equity, the next chapter offers a cross case comparison organized around themes of equitable mathematics instruction.

CHAPTER FOUR:

CROSS CASE ANALYSIS: EQUITABLE MATHEMATICS TEACHING

Overview

In this chapter, I compare the three cases of teaching presented in Chapter 3. The chapter is organized around three broad themes of equitable instruction: mathematically rich and rigorous instruction, uptake of and responses to students' mathematical ideas, and equitable participation. These three themes were selected through a review of the literature on equity and mathematics teaching and are based on the findings from the case studies. They represent three key commitments on which high quality instruction is based (Ball, 1993): the discipline, students' thinking, and the collective. Focused on these three themes, each section offers a brief review of the literature and a cross comparative summary that includes examples from all three teachers.

Theme 1: Mathematically Rich and Rigorous Instruction

Overview of Mathematically Rich and Rigorous Instruction

Over the last 20 years, teaching and learning mathematics has come to mean more than being proficient in basic skills. Studies have shown the significance of teaching for deeper understandings of mathematical ideas, relationships, and concepts (Lampert, 1991, National Council of Teachers of Mathematics, 2000). These standards call for students to do more than memorize rules, formulas, and algorithms and apply basic procedures to do mathematics. Moreover, students are expected to develop deep conceptual understanding of a broad range of mathematical content areas. The mathematics education community has worked to persuade teachers to shift their practices away from emphasizing procedural fluency and basic skills, to focus on conceptual meaning and developing deeper understandings of mathematical concepts and ideas (National Council of Teachers of Mathematics, 2000). Jamar and Pitts (2005) state, "As the vision of what it means to be mathematically literate in our society has shifted from a basic skills curriculum for some to a more demanding standard for all, the limits of past pedagogical practice have become increasingly apparent." (p. 127) Additionally, Perry and Dockett (2001), in their description of "powerful mathematical ideas", assert that mathematical connections are strongly related to other mathematical concepts such as numeracy and mathematical literacy. In particular, they describe the components of mathematical literacy as, "thinking, talking, connecting and problem solving" (, p. 13, see also Liedtke, 1997). Additionally, Lappan (1998) described how a shift in vision of mathematics requires new work of mathematics teachers, namely, "pushing student thinking while the exploration is proceeding; helping students to make the mathematics more explicit during whole-class and group interactions; using and responding to the diversity of the classroom to create environments in which all students feel empowered to learn mathematics; and engaging students in the (mathematics) tasks" (p. 135).

Literature Review: Mathematically Rich and Rigorous Instruction

The purpose of the following brief review is two-fold. One purpose is to describe concepts and theories that offer a rationale for how mathematically rich and rigorous instruction is related to equitable instruction, a second focuses on unpacking the demands of designing and enacting rich and rigorous mathematics teaching.

Defining and describing "rich mathematics." The first theme focuses on the rigor and rich quality of the mathematics represented through instruction. Examples of this theme include the use of multiple representations, linking among representations, mathematical explanation and justification, and explicitness around mathematical practices such as explanation, proof and reasoning (Fennema & Franke, 1992). An emphasis on explicit teaching (Adler, 2001; Hill, et. al., 2008) reveals the demands of doing this work, especially as it relates to the tasks of teaching and to issues of equity.

According to Fennema and Franke (1992) analyzing the richness of the mathematics is a way of claiming that classroom mathematics should be rigorous and focus on conceptual understanding, instead of focusing on algorithms, facts, and procedures (p. 151). They assert that students from these classrooms are better positioned to learn rich sets of interrelated mathematical ideas and develop positive belief systems about themselves and mathematics (p. 151). According to these authors, this instruction is primarily characterized by the use of multiple representations, linking among representations, mathematical explanation and justification and explicitness around mathematical practices such as proof and reasoning. It is important to note that the term "rich mathematics" in this dissertation is not meant to characterize a view of teaching. Instead, a narrow definition of "rich math" is used and focuses on characterizing mathematics instruction that uses multiple representations, makes explicit links between representations, and creates access to complex mathematics in ways that lead to developing proficiency. It is "rich" with respect to its depth and the multiplicity of well-connected pathways through the content. In particular, two teaching practices associated with rich mathematics are assigning mathematical tasks and the degree to which the teacher is explicit in his or her teaching. The practice of assigning tasks is paramount for designing "rich" mathematics instruction. Additionally, explicitness is also significant tool for providing equitable access to the complex mathematical tasks assigned.

Mathematical tasks. Mathematical tasks are important for students' learning because "tasks convey messages about what mathematics is and what doing mathematics entails" (NCTM, 2000). Doyle (1986) writes about teaching and academic tasks. He asserts that, "the curriculum exists in classrooms in the form of academic tasks that teachers assign for students to accomplish with subject matter (p. 365). Stein and Lane (1996) report that learning environments in which teacher encourage multiple strategies and ways of thinking and support students to make conjectures and explain their reasoning were associated with higher student performance on measures of thinking, reasoning, and problem solving. As Doyle (1988) states, "the work students do, defined in large measure by the tasks teachers assign, determines how they think about a curricular domain and come to understand it's meaning" (p.167). A mathematical task is defined "as a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (Stein, Grover, & Henningsen, 1996). Literature has shown that, "different kinds of tasks lead to different types of instruction, which subsequently lead to different opportunities for students' learning" (Doyle, 1988; Stein, Smith, Henningsen, & Silver, 2000). Said another way, mathematical tasks determine the mathematical learning opportunities available to students, shaping both the content they learn and their views around the subject matter (Kilpatrick, Swafford, & Findell, 2001). Because students have opportunities to learn concepts and skills from the tasks they are assigned, the opposite is also true; that when students are not assigned complex and rigorous mathematical tasks, they are denied opportunities for learning and this might hinder progress for developing skills that lead to mathematical proficiency, thus impeding equity.

Explicitness is a key feature of rich mathematics and equitable instruction. Recent research has claimed that explicitness is a key feature of rich mathematics and equitable instruction (Boaler, 1997; Cohen and Lotan, 1997; 1999). These scholars describe how being explicit creates broad access into the mathematical work, especially for linguistically diverse students. Explicit instruction provides clear directions about the assigned tasks for students. It is important to note that desirable explicitness should not water down or weaken the mathematical goal or purpose of a lesson. Explicit instruction should create opportunities to learn for all students. Ball and Bass (2003) note that, "opportunities that arise for learning to reason mathematically depend on how the problem is used" (p. 37). Furthermore, Henningsen and Stein (1997) caution against lowering the cognitive demand of tasks in ways that compromise the integrity of the mathematics at hand by allowing work on complex tasks to be reduced to using procedures without connection to concepts, meaning or understanding (p. 535). Researchers have argued that teachers may leave implicit key features of what students need to learn and do based on their own tacit understandings, which works against the interests of equity (Ball, et al., 2003). One counter-example of explicit instruction is when a teacher "facilitates" a classroom discussion. This practice often leads to students who have fewer opportunities for learning outside of school to have less access into the discussion others. More specifically, this practice may be more culturally congruent with some students' cultures and learning experiences, thus providing uneven access for fully participating in the mathematical discourse (Boaler, 2002; Delpit, 1988; Heath, 1983; Lubeinski, 2000). Similarly, Ball and her colleagues (Ball, Goffney, & Bass, 2005) argue that explicit guidance for learning complex skills and ideas is crucial if all students are to develop such capacities, and not leave the construction of these to chance or to cultural differences in discursive norms (p. 4).

In summary, explicit instruction can serve as a significant resource for students' access to complex mathematical concepts and ideas by providing students with mathematical resources, including skills and dispositions about mathematical problem solving and mathematical reasoning that are important for developing mathematical proficiency. For example, Lee (2007) explains that one goal of the Cultural Modeling framework is to make the norms for reasoning and participation in academic tasks explicit where explicitness is mean to describe methods of "guided practice" rather than "direct instruction." (p. 141) She elaborates offering examples of explicit instruction as "using guided questions, creating participation structures in which students know how to take part and value taking part,….and creating routines that socialize the kinds of practice required for complex problem solving in the discipline" (p. 142). More specifically, the degree to which teachers are explicit in their teaching can serve as a significant resource for students' access to complex mathematical concepts and ideas. By teaching in ways that do not rely on students' outside of school skills for being able to engage in learning the content, teachers who are able to provide explicit instruction create broad access to complex mathematical ideas.

The above sections have detailed the research and arguments for the importance of focusing on rich mathematics instruction and explicitness as critical resources for equitable instruction. Now I turn to looking at the three cases of teaching — Karen, Rebecca, and Lauren — to compare how they fare in providing rich mathematics and explicit instruction. The next section provides a summary of these themes and a comparative analysis between these teachers.

Cross Case Comparison: Mathematically Rich and Rigorous Instruction

Using the scale scores generated by the LMT project [25] I obtained an additional quantitative measure of the differences among these three teachers in terms of how they fare along the dimensions of providing "rich" mathematics instruction. These "meta-codes" were scales or clusters of related codes and they aggregated codes from several of the scales in the MQI tool. As such, these scales represent more than just the equity codes. These codes represent a way of quantifying the degree to which a teacher's teaching practices represent the different scales. For example, to consider how well Rebecca's teaching represents "rich mathematics", especially as compared to Karen and Lauren, a meta-code was calculated using the coding for each teacher. Rebecca's meta-code is .02, Karen's is .15, and Lauren's is .24. The "rich math" meta code was designed to represent a "composite code of teacher's appropriate use of representations and

[25] Blunk, 2007; Meta codes were calculated by averaging the number of clips in a lesson that were coded present or present appropriate to determine each teacher's scale score (p. 13)

manipulatives, use of explanation and justification, and explicitness about mathematical practices, reasoning, and language. The analysis below helps to illustrate the differences in these meta-code scores.

Explicitly teach students what it means to do good work in mathematics and sensitively choose tasks and examples. Examples from each of these three teachers in Chapter 3 illustrate the ways in which these three teachers vary on how they fare on offering rich mathematics instruction to their students. Karen and Lauren both offer rich mathematics by using tasks that are mathematically important and build connections between representations and conceptual meaning, and focus on developing practices associated with providing mathematical explanation (Fennema & Franke, 1992; Lappan, 1998; Stein & Lane, 1996). They also explicitly use mathematically precise language in explaining general and technical mathematical terms. In particular, Karen's emphasis on helping her students practice providing mathematical explanations for their solutions are examples of rich mathematics instruction. Additionally, her connections between helping students link representations (i.e. recall the example of using Unifix cubes to practice long division) are also examples of rich mathematics in Karen's classroom. Similarly, in each lesson in our data set Lauren provides explicit directions about the tasks and activities she expects her students to complete. For example, in one of the lessons described in the previous chapter, she provided opportunities for her students to engage in making predictions, explain their answers and consider why results might differ from predictions, all of which engage her students in mathematical reasoning and mathematical explanation. This work on mathematical reasoning gives meaning and purpose to the mathematics topics being studied thus providing an example of rich mathematics.

Conversely, Rebecca's instruction is clearly not a case of rich mathematics. Each of her lessons usually focuses on one procedure with little attention to conceptual understanding and she makes few connections between mathematical representations. Additionally, her lessons contain mathematical errors, several of them mathematically significant (i.e. in one lesson [Wave 2, Lesson 26] she repeatedly says that pi is "three point one fourteen [3.114]). Although Rebecca usually provides direct instruction focused on learning to do a procedure or solve a problem, this type of instruction would not be characterized as explicit. The lack of focus on conceptual understanding and frequent mathematical errors that characterize Rebecca's lessons do not promote equitable achievement because they do not provide students with the mathematical skills to become proficient. Because these lessons lack rigor and explicitness, Rebecca's teaching is not a case of rich mathematics.

Summary: Mathematically Rich and Rigorous Instruction

As evidenced from the above classroom characterizations and the cases described in Chapter 3, these three teachers varied greatly in to the degree to which they provide "rich" mathematics instruction. Access to rigorous and complex mathematics in ways that are deliberately attentive to students' needs and respectful of the discipline is important for equitable mathematics instruction.

Theme 2: Teachers' Uptake of and Responses to Students' Mathematical Ideas and Productions

Overview: Teachers' Responses to Students' Mathematical Ideas and Productions

The second major theme of equitable mathematics instruction focuses on the interactions between teachers and students, and how these interactions can either promote or hinder equitable participation in the mathematics class. Teachers vary in their skills for being able to listen and interpret student thinking. This theme focuses on the ways in which teachers elicit and use talk in mathematics classrooms. Characteristics of this theme range from requests by the teacher for students to provide descriptions of mathematical procedures, ideas, or processes to providing the support needed for students to do this work. For example, when the teacher tries to understand and appropriately interpret students' comments, questions, solutions, or ideas. Additionally, teachers must be skillful at attending to the mathematically significant elements of a student's explanation while respecting the differences students bring to learning mathematics. In this way, teachers are drawing from both their knowledge of mathematics and their appreciation of the diverse perspectives and experiences as leverage for making mathematical concepts more accessible to a broader population of students. In this way, responding to students' mathematical ideas and productions in ways that promote equity in the classroom require mathematical knowledge but also require sensitivities and knowledge about the students and outside of school experiences. More specifically, teachers' beliefs about students and their abilities to be able to do challenging mathematical work may support or hinder teachers' abilities to respond to students' mathematical ideas and productions and scaffold their work appropriately.

Literature Review: Teachers' Uptake of and Responses to Students' Mathematical Ideas and Productions

Assigning competence. Teachers' use of their mathematical knowledge to manage classrooms is tied to promoting equity for students' mathematical achievement. Studies suggest that teachers vary in their skills in being able to listen and interpret student thinking. As noted by Cohen and Lotan (1997; 1999), assigning competence by making mathematical thinking and work public includes labeling student work as important and valuable to the collective mathematical work of the class. Teachers who are skillful at being able to attend to the mathematically significant elements of a student's explanation is a critical way for teachers to attend to the integrity of the discipline of mathematics while also honoring the differences students bring to learning mathematics. Ball and Bass (2000) describe a similar practice as "naming." (p. 42) These authors argue that it is important to make records of mathematical knowledge and making knowledge public. They suggest that one efficient strategy is "naming" where teachers choose to name ideas for students and they caution teachers to exercise sensitivity when considering whose ideas are named. As such, "naming" is a strategy for assigning competence where each of these practices can promote equity.

Issues of language and equitable instruction. Teachers use language to help students make sense of mathematical concepts. Some scholars argue that this work becomes more complex, requiring more and different sets of skills when teaching with linguistically diverse students (Boaler, 2002; Cohen and Lotan, 1997; 1999; Khisty, 1995). Research described in the section below illustrates the important role language plays in equitable mathematics instruction.

Khisty's (1995) discussion on the development and use of a 'mathematical register' illuminates how mathematical meanings, by the nature of their use, create an added barrier to linguistically diverse students' learning mathematics (p. 282). She describes how the mathematical register, as a set of unique meanings and structures, is expressed through every day language and through the reinterpretation of words that carry different meanings, which is especially common in the English language. Her work with Hispanic students reveals that if students do not know the language or have a way to express mathematical ideas, then they cannot *do* the mathematical work and thus are eliminated from participation in those activities that develop and enhance mathematical meanings and comprehension (Khisty, 1995). Being eliminated from being able to participate and learn from instruction is counter-productive for working towards equity of the access to opportunities for learning complex mathematics. Zevenbergen (2000) adds that students who lack linguistic capital have an additional barrier for learning. She argues that because language is inextricably bound to learning and provides the medium through which communication of ideas is made possible, that the negotiation of ideas and concepts requires explicit and deliberate attention. Similarly, Gorgorio and Planas (2001) describe how language is a "social tool within the mathematics classroom and that language is the primary vehicle in the construction of mathematical knowledge" (p. 7). These authors claim that "minority" language learners are disadvantaged by the ways in which teachers and schools view students' language and advocate for instruction that "assumes that the students' languages and different cultural backgrounds can be seen as a resource in their learning" (2001). Additionally, Boaler (2002) notes that attending to issues of equity in mathematics instruction requires teachers to be able to explicitly attend to students' understanding of the ways they need to work and giving students a clear sense of the characteristics of high quality work (p. 2). In these classrooms, equitable mathematics instruction is teaching that successfully uses language as a tool to share ideas and thus providing equitable access for engaging in rigorous mathematical practices, including ways of reasoning and promoting conceptual understanding.

Mathematical explanations. Ball and Bass (2003) argue that explaining is a central component of effective mathematics instruction in elementary classrooms. Equitable teaching includes listening closely to students' ideas, being sensitively careful at the interface between mathematical and everyday language are especially important to recruit students into mathematics, and help them succeed there (Ball, Goffney, & Bass, 2005, p. 4). These authors argue that teachers in elementary classrooms have to be especially attentive to the needs of students who are working on mathematics in English where they are just learning to express mathematical ideas resulting in unusual phrases and responses at times. Additionally, the many varieties of English spoken in the classroom make it especially important to notice the ambiguities between technical and "everyday" uses of English. Equitable teaching demands that teachers are not only aware of work students are required to do when making these transitions, but to also build bridges for students to help them navigate between home language, school language, and technical language, by listening and responding to students' ideas and mathematical explanations with a sensitive and knowledgeable ear.

Instructional scaffolding. Ladson-Billings (1997) describes how providing instructional scaffolding for students allows them to move from what they know to what they do not know. She argues that effective pedagogical practice requires in-depth knowledge of students as well as of subject matter. Here the teacher must draw from the resources of her mathematical knowledge, and from her knowledge of students' culture, to teach equitably.

Cross Case Comparison: Uptake of and Response to Student' Mathematical Ideas and Productions

Meta-code scores. The scale for "responding to students" was designed to help characterize instruction as being appropriately reactive to students' questions, comments, ideas, including interpreting ambiguous student answers and addressing student errors. (Blunk, 2007, p.14) Along the dimension of "responding to students appropriately", Rebecca earned a .10, Karen's score was .41 and Lauren's scores was .46. Although it might be surprising for Rebecca's score to be so low, it is important to note that the video codes were designed to focus on the quality of mathematics instruction and as such, prioritizes the mathematics, which also explains why Karen's scores are higher than might be expected. A cross case analysis discussion on this theme is found below.

Develop explanations that are effectively accessible for a range of student in the class and that have mathematical integrity. Mathematical communication is an integral aspect of learning mathematics. One practice associated with equitable instruction that relates to responding to students is when teachers develop explanations that are mathematically sound and effectively accessible for the wide range of students in their class. Responses that gloss over the important mathematical points are inequitable by not helping students gain mathematical knowledge and understanding that leads to proficiency. Similarly, explanations that are inaccessible, especially for those students who most rely on school for their access to learning, are inequitable by only being a resource for students who have skills that enable to be easily successful in school. Recall the way in which Karen ignored Cho, passing on the responsibility of responding to his questions to another student in the class. Also recall when Juan tried to contribute his solution to the class discussion and Karen stopped him four words into his explanation, correcting him in a manner where the entire class, including Juan, laughed at Juan's contribution before Karen quickly moved on to the next student. In both of these illustrations, Karen did respond in a way that was effectively accessible to the wide range of students in her class. Juan and Cho grappled with different issues in learning the task Karen was teaching. Cho struggled with his limited understanding of English, while Juan either misunderstood the task or was unable to work on the assigned task. In both of these situations, Karen's mathematical knowledge was not available for her students as a resource for their learning mathematics, thus hindering equity instead of promoting it. Said another way, evidence from the case of Karen's teaching indicates that she does possess substantial mathematical knowledge and in other examples, she is able to use this knowledge as leverage for using mathematical activities and tasks with her students. However, this practice of Karen's teaching is problematic because she doesn't use her high levels of mathematical knowledge for teaching for all of her students, only a select few.

Conversely, Rebecca's passion for teaching the students in her class inspires her to respond with care to her student's mathematical ideas and productions. However, her thin and fragile understanding of mathematics and her purely procedural lessons compromise the mathematical integrity of her responses. Although she doesn't dismiss students or consider student's ideas a joke in the way the illustrations from Karen's classroom do, her responses are also inequitable. Her responses are primarily characterized by repeating the written directions from the curriculum or re-phrasing or repeating the algorithm or procedure assigned for students. Because Rebecca does not respond in ways that open up the content or create mathematical connections between procedures and concepts her responses, like Karen's, are not effectively accessible for her students in that they lack mathematical integrity. In contrast, Lauren's responses are equitable. Lauren plans for her responses to students. By first carefully establishing a mathematical goal, then deliberately planning the tasks and mathematical activities and then considering how she anticipates her students interacting with the activities, she is able to respond with mathematical integrity using responses that are effective and accessible to her students. Recall the representation dilemma with the arrays chart. Lauren deliberately decided to only display one array for each set of numbers instead of displaying both the vertical and horizontal representations. She explained that she didn't want to unnecessarily complicate her students' fragile understanding of arrays by displaying both arrangements. During the class discussion on the chart of arrays, students posed questions about the limited number of arrays for some numbers and the many arrays for other numbers. As Lauren responded to these questions and helped the students to see that prime numbers only had two factors (and one arrangement on the chart), while the other numbers had more than one arrangement. One student questions her about the arrangement, to which Lauren responds, "We could put it the other way" indicating that the 1×13 and 13×1 is the same array, just arranged differently. Additionally, Lauren's responses to students' non-mathematical questions are always gracious and kind. For example, "I am so glad you decided to share today" and "I would be happy to help." Conversely, in her post-observation interviews, Karen refers to her students as "low" or "high", labels some students as "resource" indicating that they receive additional learning services. These comparisons between the students she seems to enjoy teaching, and the students who make it difficult for her to teach result in some students being assigned as competent and others being ignored.

Summary: Teachers' Uptake of and Responses to Students' Mathematical Ideas and Productions

As evidenced from the above discussions of the themes, the teaching varied greatly in its attentiveness to and use of student's mathematical ideas and productions. Equitable instruction is directly linked to teachers' skills for listening and responding to students. These skills draw on teachers' mathematical knowledge in being able to attend to the salient mathematical purposes and connections, but also draw on their knowledge and sensitivities to the students in their class. In conclusion, the research reviewed earlier in this section have helped to illustrate the resources needed for teaching mathematics equitably. Taken together over these three themes of equitable instruction, teachers must deeply know the content they are teaching as well as have a cultural awareness of their students out-side of school lives and experiences to teach equitably. Said another way, this knowledge of students' culture and mathematical knowledge give the teacher the leverage needed to teach in equitable ways.

Theme 3: Equitable Participation in Mathematics Class

Overview: Equitable Participation in Mathematics Class

Recent scholarship regarding culturally relevant and responsive theories of instruction argue that high quality, equitable teaching incorporates and includes diverse ways for students to contribute and participate in mathematics classrooms. Here equitable mathematics teaching occurs when teachers are able to include all students in the classroom mathematical work (Irvine, 2003; Ladson-Billings, 1994; Malloy, 1997).

Literature Review: Equitable Participation in Mathematics Class

As described by Stein et al, (2007), the goal of teaching (and mathematical discussions and activities) is for the teacher to "move students collectively toward the development of a set of ideas and processes that are accountable to the discipline" (p. 332). However, designing for equitable participation is no easy feat. As described below, deliberately designing opportunities for all students to participate (with complex tasks) in the mathematics requires discipline (mathematical) knowledge sensitivities to issues of equity.

Creating access to mathematics through participation. Researchers have studied classrooms in an effort to understand how teachers can increase access for culturally and linguistically diverse students in the mathematics of the lesson (Boaler, 1998; Cohen & Lotan, 1997; Cohen *et al.*, 1999). They claim that teachers can provide access and opportunity for students by designing multiple forms of classroom participation and valuing different kinds of contributions from their students. Moreover, these kinds of instructional practices and considerations broaden what it means to be successful in mathematics, thus promoting equity by finding multiple ways for students to be cognitively engaged in the mathematical work of the class. Broadening here is meant to describe when teachers take deliberate steps to find ways for students to participate in the mathematics tasks and activities. For example, Cohen et al. (1999) described a "multiple-abilities treatment" claiming that one way to address issues of unequal access and learning for "low-status" students is to broaden the conception of what it means to be 'smart'….which involves the teacher's public recognition of a wealth of intellectual abilities that are relevant and valued into the classroom and daily life" (p. 84). Additionally, use of cooperative grouping in Complex Instruction involves students being assigned different roles for working on mathematics tasks. Because students are assigned tasks that require collective work, students who have weaker mathematical skills are still able to contribute to the collective work. In non cooperative learning setting, teachers who allow students to verbally provide their answers, write or draw pictures of their solutions, and work in partners or small groups are also broadening the traditional definition of what it means to be skillful in mathematics. Teaching in these ways requires both mathematical knowledge and sensitivities to and knowledge about their students to promote equitable outcomes. Further examples of these practices and considerations are: supporting students work for offering mathematical explanations; valuing diverse array of mathematical productions; encouraging and supporting students' development of conceptual understanding, adaptive reasoning, and strategic competence. Each of these practices enables teachers to promote their students' developing the skills needed for mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001).

Cultural considerations of equitable participation. Scholars have argued that teaching differently creates opportunities for children to learn a different mathematics (i.e. Wood, 2001). Therefore, to promote equity through participation, it is important to attend to issues of culture, diversity, and equity in the mathematics classroom. More specifically, some scholars have worked in a number of ways to better understand the relationship between students' cultural backgrounds and participation structures in classrooms, both within the discipline of mathematics (i.e. Cobb & Hodge, 2002; Khisty, 1995; Nasir, 2002) and outside the discipline of mathematics (i.e. Lee, 2001, 2007). Nasir (2002) examined how students' outside of school experiences (playing basketball and dominos) might provide students with resources needed to engage in working on complex mathematical tasks. Lee's (2001, 2007) work on Cultural Modeling, describes a program of research that leverages students' outside of school and cultural experiences as tools for deconstructing genres of "canonical" literature. Each of these studies offer insight into investigating the influence of students' cultural perspectives and experiences when considering what practices are associated with providing opportunities for a wide range of students to participate, especially in working on complex mathematical tasks.

Complex instruction, cooperative learning, and group work as a means of promoting equitable participation. This section offers evidence about participation and grouping as a means of teaching equitably. Scholars such as Cohen and Lotan, (1997; Cohen *et al.*, 2002) and Boaler (1998) have studied classrooms in an effort to understand how teachers can increase access for culturally and linguistically diverse students into the work of the classroom. More specifically, they consider how teachers can broaden what it means to be successful in mathematics classrooms. For example, teachers can provide space and opportunity for students by designing multiple forms of classroom participation and valuing different kinds of contributions in their classrooms. These studies support the need to focus on the teachers' efforts to broaden what it means to contribute and participate in classroom mathematics work. For instance, teachers who accept and encourage multiple solution strategies encourage students to work together and apply their mathematical reasoning strategies with one another, and also to respond in respectful ways to each other's mathematical thinking.

Tasks and equitable participation. Teaching students to work in this way requires careful design of rigorous tasks and explicit teaching about responding to each other's mathematical ideas. Because not all mathematics problems and tasks lend themselves for multiple solution paths, carefully choosing tasks and sensitive use of them with students is key for providing equitable instruction. For example, in Complex Instruction, teachers

> …use cooperative group work to teach at a high academic level in diverse classrooms. They assign open-ended interdependent groups tasks and organize classroom to maximize student interaction. In their small groups, students serve as linguistic and academic resources for one another…[these teachers also] pay particular attention to unequal participation of student and employ such strategies to address status problems. (Cohen et al., 1999, p. 80)

Similarly, Ball describes how selecting and adapting tasks can create opportunities for students to practice mathematical reasoning (Ball & Bass, 2003, p.41). Furthermore, teachers can promote equity by "knowing how to launch complex mathematical problems so that students can actually work on them fruitfully" (Ball, et al, 2003, p. 10). In this way, teachers' selection and use of mathematical tasks can create or impede equitable participation in learning substantive mathematics.

Cross Case Comparison: Equitable Participation in Mathematics Class

Meta-Code Measures. The scale for "equitable participation" was designed as the only equity metacode to characterize lessons where the teacher is actively involved in eliciting students' descriptions, explanations, and other contributions and valuing a range of mathematical work and competence (Blunk, 2007, p.14) Along the dimension of "equitable participation", Rebecca earned a .11, Karen's score was .24 and Lauren's score was .46. The substantial difference between Karen' score and Lauren's score is indicative of how their teaching practices differ in terms of supporting equitable participation and access for every student into the mathematics. Below is a discussion that compares all three teachers' practices.

Teachers' use of a variety of participation structures to ensure that various learning and participation styles are respected. In this section I compare Rebecca's, Karen's and Lauren's teaching practices in terms of their similarities and differences in providing opportunities for equitable participation for their students. These three teachers vary greatly in how they support students working together and in the ways they work for or against promoting equity in the classrooms. Similar to the other summary sections in this chapter, the subcategory described below is drawn from the General Principles and practice of equitable instruction developed by the University of Michigan Teacher Education Initiative Equitable Instruction working group.

Promoting equity means teaching in ways that actively support the learning of every student in ways that do not inadvertently reproduce inequality across social groups, especially in ways that marginalize students of color and students living in poverty, thus providing equitable access to challenging and meaningful mathematics content. It is widely believed that teaching can shape students' experiences, their sense of themselves as mathematical learners, and the development of their mathematical capacities. Teachers who are able design multiple forms of participation are able to provide access and opportunity for a wide range of students (Boaler, 1998; Cohen & Lotan, 1997; Cohen, et. al., 2002). These authors discuss how small group work enables students to serve as resources for one another as they talk and work together on rich mathematical tasks. The students in Karen's small group often work on routine tasks, where they follow simple directions to arrive at one correct answer. Similarly, Rebecca's students are successful by carefully following instructions, such as repeating memorized information or systematically and repeatedly applying an algorithm to similar problems. As Lotan describes, "Group-worthy tasks require students to share their experiences and justify their beliefs and opinions. In such activities, students analyze, synthesize, and evaluate; they discuss cause and effect, explore controversial issues, build consensus, and draw conclusions. By assigning such tasks, teachers delegate intellectual authority to their students and make their students' life experiences, opinions, and points of view legitimate components of the content to be learned." (Lotan, 1997) Similarly, these tasks have multiple ways to show competence by calling on multiple intellectual abilities for successful completion. Cohen (1994) further describe how "one-dimensional tasks require the same skills, resulting in uniform success or failure…leading students to conclude that some students are 'smart' and others are 'dumb'". Additionally, multiple-ability tasks allow more students to show their knowledge and intellectual competence, thus making the learning environment more equitable (Cohen & Lotan, 1997; Boaler, 1998). Lauren's classroom clearly provides this kind of learning environment. Different than the one dimensional tasks and strategies for working that are present in

Karen's and Rebecca's classrooms, Lauren deliberately distributes widely opportunities for different kinds of participation in class discussions, ensuring that every student participates in the mathematical work. By comparison, not only does Karen use many one-dimensional tasks, she also appears to use small groups for grouping students by her perception of their ability levels for working on mathematical tasks, assuming that students who are skillful and "able" can solve problems on their own, and those who are less "able" need her direct instruction to learn. However, in Lauren's classroom, some students contribute by sharing their solutions with their partners or small groups explaining how they solved the problems and showing their drawing or representation. Students also contribute by responding to the mathematical ideas of their classmates. They claim agreement or disagreement with their peer's solution applying their own explanation or reasoning during the discussion. Different from the mental math activities common in Karen's classroom or the fill-in-the-blank styled questions Rebecca poses during whole class discussion, Lauren offers different kinds of participation opportunities to be distributed within the class discussion, thus providing multiple and varied opportunities for competence and promoting equity.

Summary: Equitable Participation in Mathematics Class

Equitable instruction is directly linked to teachers' skills for listening and responding to students. These skills draw on teachers' mathematical knowledge in being able to attend to the salient mathematical purposes and connections, but also draw on their knowledge and sensitivities to the students in their class. Equitable instruction encourages equitable participation for all students, and provides the scaffolds to support students in their work on rigorous mathematical tasks. The practices typically used in many mathematics classrooms provide access and opportunities for a select group of students who share similar characteristics, usually white and middle-class. These practices have contributed to the differences academic achievement, graduation rates, access to higher education and social mobility. Equitable participation, in many ways, is necessary for equitable achievement. Teachers who design opportunities for and encourage equitable participation for all students works against these norms.

Summary

In conclusion, discussions around these three themes of equitable instruction have helped to illustrate the demands of teaching mathematics equitably. The analyses of each of these themes, mathematically rich and rigorous instruction, uptake of and responses to students' mathematical ideas and equitable participation illustrate how teachers must deeply know the content they are teaching as well as have a cultural awareness of their students out-side of school lives and experiences to teach equitably. Said another way, this knowledge of students' culture and mathematical knowledge give the teacher the leverage needed to teach in equitable ways.

CHAPTER FIVE:

CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

Summary of Dissertation

Multiple data points show persistent problems of inequity in opportunities to learn and in patterns of success in mathematics. Educational researchers and policy makers agree that there are often cited problems with inequities in mathematics learning in this country, from access, to instructional quality, to achievement. Many scholars in the last twenty years have studied classrooms as a context in which to intervene on patterns of equity. Although many factors beyond the classroom clearly impact student' opportunities to learn, widespread evidence suggest that teacher quality matters for student learning; teachers are a key component for academic success. My dissertation examined the interplay of equity and mathematics through a study of instruction. The analyses focused on identifying what constitutes equitable instruction and to what extent can equitable instruction be identified and measured. In this section I will review the purpose of this study, highlight some of the findings from the analyses, describe a few limitations of this study, offer suggestions about the contributions this study may offer to the field of education and describe what this implies for future research.

What was the purpose of this dissertation? Identifying and measuring equitable mathematics instruction

The overall purpose of this study was to probe what constitutes equitable mathematics instruction for students in elementary classrooms. In this dissertation I conducted a series of strategically-designed studies that analyzed specific aspects of teachers' work that examined the interplay of quality mathematics, quality instruction and attention to equity in the teaching of mathematics in elementary classrooms. Originally, there were three central research questions for the dissertation overall. They were:

1. What are the demands of teaching mathematics equitably in elementary classrooms?
2. What are the challenges of teaching elementary mathematics in ways that are deliberately attentive to equity and quality mathematics?
3. Can the mathematical quality of a teacher's instruction be measured simultaneously with attention to issues of equity?

Additionally, there were two additional sub-questions for chapter 2:

1. What is known about equitable mathematics instruction in elementary schools?
2. What is the relationship between a teacher's scores on the equity video codes and the quality of their mathematics instruction?

However, during the work of this dissertation, the focus shifted slightly to prioritize studying what constitutes equitable instruction and considering the extent to which it can be identified and measured. Studies of the challenges and demands to enact equitable instruction are a next step and a logical extension of the work in this dissertation. These ideas will be discussed later in this section under contributions and implications for future research.

The analyses in this dissertation focused on testing a hypothesis that teaching equitably would require attention to both equity and mathematics. In particular, this study sought to find and describe tools for measuring equitable mathematics instruction and for testing such a tool to determine if one could differentiate classrooms based on the degree to which the mathematics teaching is equitable. The lessons of teachers who modeled three critical contrasts for this study were used to develop detailed case studies to determine if patterns of instruction emerged that would help differentiate among these paradigms. These three paradigms represented teaching based on high disciplinary knowledge and high commitment to equity; teaching comprising high disciplinary knowledge and low equity scores, and teaching based on low disciplinary knowledge and high equity commitments. (It is important to note that it was not of interest in this dissertation to study the instruction based on low disciplinary knowledge and low equity scores.) The results from these analyses reveal that using such a tool (the Mathematical Quality and Equity video codes) does reveal patterns of instruction that distinguish among these paradigms. Highlights of these analyses and suggestions for future research are included below.

Findings

The analyses for this dissertation have two central features. First is a description of the (MQE) equity video codes. The analyses conducted in this chapter focused on determining how the video codes helped to differentiate teachers' instruction based on their attention to issues of equity and mathematics. The findings from these analyses were quite interesting, revealing dramatic differences between lessons that earn high equity video code scores and those that earn low scores. Lessons that earned high equity scores were not just those where the teacher was committed to working on issues of equity and diversity. Instead, because this tool was deliberately designed to attend to both issues of equity and the mathematics, lessons that earned high scores needed to be focused on substantive mathematics as well. These findings indicate that the equity video codes are a useful tool to help differentiate among teachers' instruction and have substantial implications for future research.

The second set of analyses focused on paradigmatic cases of teachers' instruction. These analyses focused on the different ways in which attention to mathematics and attention to issues of equity play out in the course of instruction. The design provided a structure for studying the extent to which equitable instruction could be identified and measured and subsequently comparing differences between lessons based on their relative attention to mathematics and attention to equity. Some scholars believe that the primary method for improving student achievement for diverse learners and those in urban schools is to focus attention and resources on equity by hiring teachers deeply committed to working with students in these classrooms. Rebecca's case is a paradigmatic example of the focus primarily on commitment and attending to students' culture and outside of school lives. Rebecca's teaching earned a fairly high score on the equity video scores, and she is committed to working with diverse learners, even preferring to teach them over "gifted and talented" students. However, as her case illustrates, her mathematics knowledge is extremely thin, as represented by common mathematical errors during her lessons. Furthermore, in her lessons, based primarily on learning procedures and little on conceptual understanding, she does little to help her students develop the skills needed for mathematical proficiency. In this way, her commitment to teaching particular students may not be enough to improve students' academic performances, especially for those students who are most in need of quality mathematics instruction. This case substantiates the claim that equitable instruction requires substantial mathematical knowledge in addition to attention and sensitivities to issues of equity.

Another perspective is scholars who believe that to improve student achievement in mathematics, efforts must solely focus on increasing teachers' mathematical knowledge. Advocating for increased mathematics course work in pre-service teacher education and requiring mathematics focused continuing education requirements, they believe that increasing teachers' mathematical knowledge would improve students' academic performance. Karen's case is a paradigmatic example of this focus primarily on the importance mathematical knowledge. Karen earned the highest score on the paper-and-pencil items and her lessons were mathematically precise. However, as her case illustrates, her attention to issues of equity is extremely thin. She at times marginalizes students, ignores some relational issues, and structures opportunities for learning based on their perceived ability and makes this publically visible. In this way, although her mathematical knowledge is an important resource for student learning, a lack of attention to issues of equity and diversity may create barriers for students' learning, thus possibly not increasing students' academic achievement. This case substantiates the hypothesis that equitable instruction requires some level of sensitivity and attention to issues of equity in addition to mathematical knowledge.

A third perspective, one that has been the focus of this dissertation, attempts to keep a dual emphasis on attending to issues of equity while helping students gain access to mathematical knowledge needed for complex problem solving, rigorous learning and high academic achievement. Lauren's case is a paradigmatic example of teaching that represents dual attention to both the mathematics and to issues of equity. Lauren earned the second highest score on the paper and pencil items and also earned the highest equity video code score. In each of her lessons, she is mathematically precise designing opportunities for her students to learn important mathematical skills such as explaining and justifying their solutions and reasoning with each other. Detailed analyses of her teaching provided quality illustrations of the equity video codes and substantiate the hypothesis that equitable teaching demands both substantial mathematical knowledge and sensitivity to issues of equity. Overall, the findings in this dissertation suggest that the overall hypothesis was correct; that equitable instruction appears likely to require both mathematical knowledge and attention to equity.

Limitations of this study

The most obvious limitation of this study is the missing connection to improved student achievement. Because this was a study of instruction, the analyses and findings focus on identifying instructional practices that are equitable, however what is missing is evidence about how the practices represented by the indicators in the equity video codes matter for student achievement and influence student learning. Further research can provide evidence, or lack thereof, regarding the ways in which these indicators influence students' mathematics achievement. Results from this dissertation indicate that equitable instruction can in fact be measured quantitatively. A next step that would promise to offer significant contributions to efforts to work toward equity in elementary classrooms would be to use the MQE measure to study the relationship between the patterns that emerged with data on student achievement.

One possible future study may involve using a value-added approach and examining teachers who have a consistent positive effect on learning gains of students from traditionally underperforming groups. Studying these teachers using the MKT and MQE measures would provide a means of investigating the relationship between student achievement and these MQE measures.

Contributions of this Study and Implications for Future Research

The limitations described above suggest fruitful opportunities for continued research that specifies the nature of ambitious instruction, studies the demands of equitable mathematics instruction, and create new tools to help pre-service and practicing teachers develop the knowledge and skills needed to teach in equitable ways.

One strategy may be to refine and revise the equity video codes for new uses for research purposes and for use in teacher education.

Equity video codes as a tool for research. One future use of the equity video codes may be to consider the relationship between teachers' equity video code scores and student achievement. Student achievement gain scores are a logical next step in connecting teachers' resources to equitable mathematics teaching.

Equity video codes for use for training teachers. An additional future use of the equity video codes may be for use in teacher education. In particular, the codes may be used in teacher preparation courses, such as mathematics methods courses or field experience courses, as a means to structure beginning teachers' opportunities to learn and their specific attention to mathematics and equity in their instruction. Additionally, professional development opportunities, such as those that help teachers learn to examine their own practice, can be designed for teachers to learn how to offer equitable instruction.

An additional future line of research would be to consider whether equitable mathematics instruction, as defined by the indicators in the equity video codes, produces gains in student achievement. For example, does a teacher with high mathematical knowledge and who earns high equity scores produce gains in student achievement? Does a teacher who represents high mathematics and low equity produce similar scores in student achievement as a teacher who represents low mathematics but high equity?

Another line of fruitful research would be to take up the demands aspect of the original research question and consider how a teacher preparation program might be built to produce novice teachers who are able to teach equitably? This line of work could build on the findings from this study to better understand the characteristics of equitable mathematics instruction and explore the challenges of deliberately teaching in these ways.

APPENDICES

Appendix A: Mathematical Quality and Equity Video Coding Glossary

Quality of Mathematics in Instruction: Video Coding Glossary

Section V: Use of Mathematics to Teach Equitably

In this section, we are coding for evidence of the teacher's use of mathematics in order to teach equitably. (We want to add that our focus on equity is not about ability differences, but focusing on race, social class and cultural differences/ issues.) Most of these codes are meant to capture *positive* instances of teaching moves that bridge the equity gap.

To code, first decide whether or not the teacher made the indicated move.

- If **present**, then:
  - Mark <u>appropriate</u> (A) if the teacher's deployment of mathematics in that move was, for the most part, mathematically accurate and appropriate; if it did not distort the mathematical content.
  - Mark <u>inappropriate</u> (I) if the teacher made the move but the teacher's deployment of mathematics in that move was, for the most part, inaccurate, distorted the mathematics, or was inappropriate for the grade level.
- If **not present**, then:
  - Mark <u>appropriate</u> (A) if absence of the move seems appropriate.
  - Mark <u>inappropriate</u> (I) if absence of the move seems problematic—i.e., the move should have happened.

Do not code both A and I. (These codes are higher inference. Allowing both is not reliable.)
Note: NP is the default; the teacher must actively do something to get a P.
For codes C-L in this section, only present (P) and not present (N) apply.

a) *Real world problems or examples:* 1. Code for whether real-world contexts and examples are present or not present (a1 and a2). 2. If present, code in a2 for whether they are sensitive (S) or insensitive (I) to students' experiences. We do this on evidence that real-world contexts and examples may motivate or afford students leverage into mathematics, as when problems are drawn from students' homes, neighborhoods, or schools. Contexts and materials may also exclude students from mathematics by using situations with which some students are not familiar or do not have access (e.g., spending an allowance at the pet store; analogies to games played by only some cultural groups). Money can be sensitive or insensitive dependent on its use in the lesson; if money is explicitly a focus of instruction, or if money is used in situations realistic to children of all social classes, it is sensitive. Insensitive cases may include allowances, discretionary income, or purchase of luxury items. If teacher works to make unfamiliar context interpretable to all students, code as sensitive. 3. If present, also code for whether the context is appropriate for lesson's mathematical goals, or inappropriate – that it significantly distorts or complicates mathematics. For all instances, code the context or material <u>as enacted</u> by the teacher, not for the potential it displays.

b) *Explicit student tasks and work.* Students are clear what they are supposed to do in this segment. May include solving mathematics problems, working on a representation or display, listening to a demonstration or directions, etc. Code P if the teacher has made the task explicit in a previous segment, and this carries over into the segment being coded now.

c) *Explicit talk about the meaning and use of mathematical language.* Teacher attends explicitly to language used (by the teacher or the students): defines terms, shows how to use them, points out specific labels or names. Code P if the teacher has made terms explicit in a previous segment, and this carries over into the segment being coded now.

d) *Explicit talk about ways of reasoning.* Teachers may highlight elements in an explanation, guide students toward more rigorous proof, or otherwise provide students opportunities to learn about mathematical reasoning itself. Explicit talk about ways of reasoning may ensure all students have access to features of mathematical reasoning and argument. Code NP if teacher simply asks students to reason, but does not make elements of that reasoning explicit. Code P when teacher points out parts of a mathematical argument, prods students to add to or change explanations, etc. Code P if the teacher has made ways of reasoning explicit in a previous segment, and this carries over into the segment being coded now.

e) *Explicit talk about mathematical practices.* Teacher is explicit about practices such as how to use representations, how to pose or use a definition, test an assertion, or respond to an argument. Code P if the teacher has made ways of mathematical practices explicit in a previous segment, and this carries over into the segment being coded now.

f) *Instructional time is spent on mathematics.* Most (over 3/4 of segment) instructional time is spent on mathematics, or setting up a mathematical task, rather than on administrative/organizational/disciplinary matters (e.g., excessive time spent passing out papers, preparing mathematical materials, disciplining students), or resolving student confusion over poorly presented tasks (i.e., answering the question "what do I color?").

g) *Teacher encourages diverse array of mathematical competence.* Teacher explicitly invites and supports broad participation by encouraging a wider range of mathematically relevant forms of work and talk. Mathematical work is broad and diverse and student interactions support that array of mathematical performance. What counts as "mathematics" includes more than speed and accuracy, but also representational skills; reasoning; questioning analysis, and critique; seeking precision and utility with language; and the like.

h) *Teacher emphasizes student effort and conveys message that effort will eventually pay off*. Teacher may praise student effort or encourage students to keep trying.

i) *Teacher encourages and gives opportunities for students to work autonomously.* Students make decisions about how to do the mathematical work; teacher might set up a problem, provide some support, then ask students to solve the problem on their own. Examples include pointing out resources in the room, encouraging students to engage in self-evaluation, encouraging students to work with one another. Simply working alone, such as when students practice a skill, does not count here.

j) *Expressed expectation that everyone will be able to do the work*. Teacher says something that conveys belief that mathematics or the mathematics task at hand is something everyone can do. Examples may include encouraging participation by pointing out that a student(s) has something to add, valuing a student answer or solution, or making a “go team” type comment. Warrant: Educational psychology literature on teacher expectation and student achievement.

Appendix B: (Excel) Mathematical Quality and Equity (MQE) Coding Document

	V. Use of mathematics to teach equitably																					
a1. Real-world problems or examples not present	a2. Real-world problems or examples present				b. Explicit student tasks and work		c. Explicit talk about themeaning and use of mathematical language		d. Explicit talk about ways of reasoning		e. Explicit talk about mathematical practices		f. Instructional time is spent on mathematics		g. Teacher encourages diverse array of mathematical competence		h. Teacher emphasizes student effort and conveys message that effort will eventually pay off		i. Teacher encourages and gives opportunities for students to work autonomously		j. Expressed expectation that everyone will be able to do the work	
NP*	S		I		P	NP	P	NP	P	NP	P	NP	P	NP	P	NP	P	NP	P	NP	P	NP
	a	i	a	i																		

*check if not present, otherwise leave blank and code a2.

Appendix C: Mathematical Quality of Instruction Video Coding Glossary

Section I: Instructional Formats and Content

This section is for recording the instructional format and content that predominated within each 5-minute segment of the lesson. These were included not because they capture teachers' mathematical knowledge for teaching, but so we could track on questions that might come up in analysis (e.g., is technical language a particular problem in geometry?)

A. Format for segment: Indicate the main format in which students worked. If a major shift occurs in the middle of the segment, please code both formats. If class splits into two or more groups, please code all applicable formats.

a) Whole group: Teacher leads discussion or presentation of mathematical material. May be moments where students work individually to solve a problem, but these are brief interludes before return to whole-group work. Whole group does not necessarily imply whole class; class can be split into halves or other fractional components. Key element is teacher presentation of mathematical material or posing of mathematical problems.

b) Small group/partner: Teacher divides students into small groups or pairs for work on mathematical problem or task. Typically, teacher circulates among groups or pairs, checking progress. However, teacher may be working on administrative issues, etc. during this time.

c) Individual work: Students working individually on mathematical problem or task.

B. Content: Mark the major topics that receive major or minor focus in this segment

a) *Number concepts.* Number concepts refers to all non-computational work on whole numbers, decimals, or fractions. This includes writing, reading, or naming numbers; counting; comparing or ordering quantities; understanding place value; relationships between fractions and decimals; and estimating. For whole numbers only, it also includes properties of numbers (such as odd and even, prime and composite, square numbers), and factors, multiples, or divisibility. For fractions, it also includes work on the meaning of a fraction, on equivalent fractions, and on simplifying fractions. Please do not mark work on computation, basic facts, or patterns here unless that work was accompanied by a significant piece of work on a number concept topic as well.

b) *Operations*: Addition, subtraction, multiplication, and division. Include any work on meanings of these operations, understanding and developing competency with basic facts, multi-digit computation with whole numbers, and any computation with decimals or fractions or integers.

c) *Geometry*: Mark this category any instruction about geometry—area and perimeter, shapes, angles, points and lines, and spatial reasoning. Also include instruction that covers geometric concepts, e.g.: parallel, perpendicular, or congruent. Also include here work on geometric designs (e.g., tessellations).

d) *Measurement*: Length, weight, volume or capacity, units of measurement, and systems of measurement (e.g., metric, English).

e) *Probability*: Concept of probability, methods of estimating or calculating the likelihood of different outcomes, or problems involving combinations and permutations to calculate probabilities.

f) *Data collection, representation & analysis*: Creating or using tallies, tables, graphs or charts to represent data; making inferences or drawing conclusions from data; and lessons on mean, median, or mode.

g) *Discrete mathematics*: Combinatorics and permutations, counting problems, networks, alternative routes.

h) *Patterns, functions, and algebra*: Organizing objects by size, number, or other properties into groups, categories, or lists; different types of patterns; generalizing patterns; using symbols to express unknown and variable quantities; and understanding and using formulas.

i) *Money, time, and calendar*: Include in this category only instruction about features of money, time, or the calendar, <u>not</u> instruction that merely uses these to help students practice facts or procedures. For instance, include instruction about the value of money, but do not include multi-digit computation problems that use money as a context—record the latter above in "operations." By "reading a calendar" we mean instruction that helps students understand the ideas of days, weeks, months, and the construction of the calendar, but not work which uses the calendar to help students explore or practice other mathematical concepts or procedures.

j) *Percent, ratio, and proportion*: Concepts of percent, ratio, or proportion, as well as computation that involves percent, ratio, or proportion, or applications of these concepts.

C. Lesson/segment type (our perception, not what is stated by teacher). If shift occurs in the segment, code both types. Or, if class splits into two or more groups, please code all applicable segments.

a) *Review, warm up or homework:* This segment typically occurs at the very beginning of a lesson and includes time spent in the following ways: doing mental math or other practice type activities indicated by the teacher to be something the students already know how to do, or going over correct answers for homework, or some other activity which is clearly not directly connected to the new ideas to be taught in this lesson. May also be the case that the entire lesson is review.

b) *Introducing major task or concept.* Code this segment type at when the teacher is introducing a major mathematical topic, task or activity. This typically occurs at the beginning (or after warm-up or review) of mathematics lesson for the day.

c) *Student work time:* This is the part of the lesson where the students are doing math (not just listening to the teacher). The work may be individual, small group or large group. It may be completing a worksheet, or it may be taking turns answering teacher questions. The students are (expected to be) actively engaged with the mathematics introduced in the previous segment. Teacher might be circulating around the room, answering student questions. Teacher may briefly address the whole group to remedy a common misunderstanding or problem.

d) *Synthesis, or closure:* Code this segment when the purpose of the lesson at this point is to summarize or in some way wrap up the mathematics.

Section II: Knowledge of Mathematical Terrain of Enacted Lesson

> In this section, we are coding the mathematical resources used by the teacher in the course of the lesson and the evidence available therein about the teacher's understanding of the mathematical content.
>
> Note: Code teachers' use of examples, representation, and language even if such material originates in the curriculum material.

To code, first decide whether the mathematical element is **present** (P) or **not present** (NP).

- If **present**, then:
  - Mark <u>appropriate</u> (A) if the teacher's use of the element was, for the most part, mathematically accurate and appropriate—it did not distort the mathematics.
  - Mark <u>inappropriate</u> (I) if the teacher's use of the element distorted the mathematics <u>or</u> was inappropriate for the grade level.
- If **not present**, then:
  - Mark <u>appropriate</u> (A) if absence of the element seems appropriate.
  - Mark <u>inappropriate</u> (I) if absence of the element seems problematic—i.e., the element should have happened.

Note: If a mathematical element is **not present** then the default code is **not present-appropriate**. Only code the segment as not present-inappropriate in extreme cases where the element's absence significantly hinders or obscures the mathematics, or significantly limits students' access to the mathematics in that moment. For example, if a student asks a question that calls for an explanation and an explanation is not offered, then this would be coded as not present-inappropriate. Most NP codes will be NP-A.

Note: Unless otherwise noted, count instances even if they occur in small group, pairwork, or one-on-one instruction – they are all instances of mathematical knowledge in teaching.

A. Knowledge of Mathematical Terrain

a) *Conventional notation (mathematical symbols)*: Use of conventional symbols and mathematical notation, such as +, -, =, or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents. Inappropriate or inaccurate uses of notation might include inaccurate use of the equals sign, parentheses, or division symbol. By "conventional notation," we do <u>not</u> mean use of numerals or mathematical terms.

b) *Technical language (mathematical terms and concepts)*: Use of mathematical terms, such as "angle," "equation," "perimeter," and "capacity." Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. When the focus is on a particular term or definition, code errors in spelling, pronunciation, or grammar related to that term as present-inappropriate.

c) *General language for expressing mathematical ideas (overall care and precision with language)*: Code general language <u>used to convey mathematical concepts</u>. This includes analogies, metaphors, and stories when used. Appropriate use of language includes sensitive use of everyday terms when used in mathematical ways (e.g., borrow).

d) *Selection of numbers, cases & contexts for mathematical ideas*: Selection of numbers for problems and examples, use of real-world or pretend contexts as the settings for developing ideas and procedures, and selection of figures, shapes and cases (e.g., in geometry). Code for the appropriateness of the teacher's selections in relation to working on specific mathematical content with these particular students. This includes attention to staging and sequencing, care that the numeric or contextual detail matches and does not obscure or confound the development of the mathematical content.

e) *Selection of correct manipulatives, and other visual and concrete models <u>to represent</u> mathematical ideas*: The teacher's selection of pictures, diagrams, and manipulatives or other models to represent a mathematical idea or procedure. Of interest is whether the teacher chooses models appropriate to the mathematics at hand. The degree of accuracy of a model should be calibrated to the requirements of the activity at hand. Do not code models that are solely stories (e.g., "numbers do not like each other" to represent the addition of positive and negative numbers of the same magnitude but different direction – – (-5) + (+5) = 0); instead, code these under *language*. Do not code objects that are merely used as tools (e.g., a ruler used to measure pencils). Instead, the object must be *representing* an idea (e.g., a ruler used to illustrate the idea of measurement, or the idea of a number line). A thing cannot represent itself (i.e. a Kleenex box does not represent a rectangular prism).

f) *Multiple models*: Whether or not the teacher uses more than one model for the mathematical content. By multiple models, we mainly mean models across "families" – e.g., graphs, equations and tables, or pictures, numeric procedures and stories. However, multiple models within a family can be coded here if they represent significantly different features of an idea. Do not count the model used in the problem statement unless it is also used in the solution in a significant way. For instance, if the problem is given symbolically or as a concrete scenario, but interpreted and solved using a graphical model, this does not count as using multiple models, but if the concrete situation is used to make sense of and manipulate the graphical model, this does count as using multiple models.

g) *Makes links among any combination of symbols, concrete pictures, diagrams, etc.*: When the teacher makes explicit links among any of the above. The links must be mathematically significant—for instance, pointing out connections between representations in ways that allow students to grasp how representations are alike or different, how pieces of one relate to pieces of another, or what one representation affords that another does not. Includes explicit links between <u>pairs</u> of representations: word problems and symbolic notation, manipulatives and symbolic notation, story problems and manipulatives, etc.

h) *Mathematical descriptions (of steps)*: Teacher's directing of mathematical descriptions (by self or co-produced with students) provides clear characterizations of the steps of a mathematical procedure or a process (e.g., a word problem). Does not necessarily address the meaning or reason for these steps. Code I for incomplete or unclear attempts.

i) *Mathematical explanations—giving mathematical meaning to ideas or procedures:* Teacher's directing of explanations (by self or co-produced with students) includes attention to the meaning of steps or ideas. Does not necessarily provide mathematical justification. Code I for incomplete or unclear attempts.

j) *Mathematical justifications*: Teacher's directing of explanations (by self or co-produced with students) include deductive reasoning about why a procedure works or why something is true or valid in general.

k) *Development of mathematical elements of the work (i.e., moving the mathematics along)*: Code here for whether the teacher's moves, questions, and statements keep the development of the mathematics moving along. Considering the content topics marked in Ib, does the content appropriately open up, develop, or solidify in this segment, or does the teacher seem to move the lesson off track (i.e., it lacks a sense of mathematical direction) without a plausible rationale about why this is happening? This is a global decision, hence, there are only two options: P-A or NP-I.

l) *Computational errors or other mathematical oversights:* Use this code when the teacher makes computational errors either in spoken or written language, or when the teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem). If present, computational errors or oversights are always inappropriate. If not present, the segment is appropriate. Hence there are only two options for this code: P-I or NP-A.

B. Overall level of teacher's knowledge of mathematics. The purpose of the videocoding study is to determine whether teacher's knowledge of mathematics is related to the quality of their mathematics instruction. Based on the mathematics demonstrated in this lesson, what level of mathematical knowledge would you rate this teacher on this lesson as compared to the other teachers in our video validation study? Code only one: Low, Medium, High.

Section III: Use of Mathematics with Students

In this section, we are coding for the presence of teaching moves with students and the mathematical quality of those moves.

To code, first decide whether or not the teacher made the indicated move.

- If **present**, then:
    - Mark appropriate (A) if the teacher's deployment of mathematics in that move was, for the most part, mathematically accurate and appropriate; if it did not distort the mathematical content.
    - Mark inappropriate (I) if the teacher made the move but the teacher's deployment of mathematics in that move was, for the most part, inaccurate, distorted the mathematics, or was inappropriate for the grade level.
- If **not present**, then:
    - Mark appropriate (A) if absence of the move seems appropriate.
    - Mark inappropriate (I) if absence of the move seems problematic—i.e., the move should have happened.

Note: If a mathematical element is **not present** then the default code is **not present-appropriate**. Only code the segment as not present-inappropriate in extreme cases where the element's absence significantly hinders or obscures the mathematics, or significantly limits students' access to the mathematics. For example, if a student asks a question that calls for an explanation and an explanation is not offered, then this would be coded as not present-inappropriate. Most NP codes will be NP-A.

Note: Unless otherwise noted, count instances even if they occur in small group, pairwork, or one-on-one instruction – they are instances of mathematical knowledge in teaching.

a) *Classroom work is connected to mathematical idea or procedure*: Activity, task, lecture or discussion is connected to mathematical procedure or idea. Rule of thumb: can students accurately answer "what mathematics were you working on in this segment?" Code P-A if work is connected to mathematical procedure or idea. Code P-I if such connections distort the mathematics. Code NP-A in cases where connection is apparent later in the tape, but not apparent here. Code NP-I if students/teacher are completing a task, but the ways that task illustrates, solidifies, or connects to a mathematical idea or procedure is not apparent throughout segment and tape. If explained in earlier segment, carry over code into current segment. This code is intended to pick up work that teachers might endorse as part of current reform efforts, but which instead of illustrating mathematical ideas turns into students following directions to cut, paste, draw, etc.

b) *Deploys manipulatives and other visual and concrete models to represent mathematical ideas:* Use this code to mark the teacher's use of manipulatives and other visual and concrete models to represent mathematical ideas in class. (Compare to code IIe.) Can be thought of as whether the teacher is pulling the representation off mathematically with the students, including making the materials seen and understood by students. Other present-appropriate examples might include times when the teacher uses language that supports their use of the representations, or when the teacher makes explicit links between the representations and the mathematical content.

c) *Elicits student description*: Mark whether the teacher requests that students provide descriptions of procedures, ideas, or processes, and supports, as needed, their efforts to do so. Requests for description include: (a) asking about the steps a student used; (b) asking additional what and how questions that correspond to the parts of a good description; (c) providing parts or pieces of a description; and (d) modeling good descriptions. Single-word student answers or fill-in-the-blank answers (as in inquiry/response/evaluation) will generally not qualify as a description.

d) *Elicits student explanation*: Mark whether the teacher requests that students provide explanations for solutions, and supports, as needed, their efforts to do so. If students need support in giving explanations, support should be given. Requests for explanation include: (a) asking additional "why" questions that contribute to building the parts of a good explanation; (b) providing parts or pieces of an explanation; and (c) modeling good explanations. This can include asking students to address the meaning of steps or ideas, or reasoning about why a procedure works or is valid. Do not code as eliciting explanation if the teacher asks only for description of steps.

e) *Records the mathematical work of the lesson*: The teacher writes or otherwise publicly records work in the course of the lesson. Attend to the teacher's choices about what to record, and the clarity with which the teacher records the ideas. Code here for the teacher's use of the board, chart paper, overhead, or other space, the clarity of record of what was said, done, agreed upon or discussed. Inaccurate use of mathematical terms or symbols should <u>not</u> be coded here. Do not count recording in one-on-one/pair settings.

f) *Interprets unusual/tentative/promising student productions*: Use this code to indicate whether or not the teacher tries to understand and appropriately interpret students' comments, questions, solutions, or ideas when those productions contain potentially significant mathematical understanding. For instance, a student might have the germ of an idea, but not be able to express it well. Teacher would clarify and address student production. Mark (P-A) if teacher appropriately interpreted student production; (P-I) if teacher inappropriately interpreted student production; (NP-A) if no student productions, or if student productions were so short or perfunctory as to not require significant interpretation (e.g., recitation of answer to computational problem, fill-in-the-blank answers). (NP-I) student production was present and required interpretation, but not interpreted by teacher.

g) *Uses students' errors:* Use this code to record when teachers respond to, use, or otherwise address student errors (errors from the teacher's perspective) in some way other than simply telling the student it is wrong or ignoring the error. One compelling instance in the segment is enough to code for appropriate use of errors. Mark (P-A) if teacher used a student error appropriately; (PI) if teacher used student error in a way that significantly distorted the mathematics or missed the point of the student error; (NP-A) for no significant student errors, no need for teacher interpretation; (NP-I) for cases when teachers should have used student errors in order for instruction to reasonably proceed.

h) *Launch of tasks/problems:* Indicate whether the teacher appropriately conveys mathematical tasks or problems to students. Code P-A if the launch of the task is mathematically appropriate and enables students to work productively. Code P-I if launch distorts central mathematical features of the problem, if students are not able to work productively, including cases when curriculum materials contain error teacher does not fix. Code NP-A if either tasks are not posed or tasks are carried over from previous segments. There is no NP-I for this code.

Section IV: Mathematical Features of the Curriculum and the Teacher's Guide

A. Mathematical features of the curriculum

In this section, we are coding for the presentation of mathematics in the curriculum.

To code, first decide whether the mathematical element is Present (P) or Not Present (NP).

- If **present**, then:
    - Mark <u>appropriate</u> (A) if the curriculum's use of the element was, for the most part, mathematically accurate and appropriate—it did not distort the mathematics.
    - Mark <u>inappropriate</u> (I) if the curriculum's use of the element distorted the mathematics or was inappropriate for the grade level.
- If **not present**, then mark (NP)

Do not code both A and I. (These codes are higher inference. Allowing both is not reliable.)

a) *Conventional notation (mathematical symbols)*: Use of conventional symbols and mathematical notation, such as +, -, =, or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents. Inappropriate or inaccurate uses of notation might include inaccurate use of the equals sign, parentheses, or division symbol. By "conventional notation," we do <u>not</u> mean use of numerals or mathematical terms.

b) *Technical language (mathematical terms and concepts)*: Use of mathematical terms, such as "angle," "equation," "perimeter," and "capacity." Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. When the focus is on a particular term or definition, code errors in spelling or grammar related to that term as present-inappropriate.

c) *General language for expressing mathematical ideas (overall care and precision with language)*: Code general language <u>used to convey mathematical concepts</u>. This includes analogies, metaphors, and stories when used. Appropriate use of language includes sensitive use of everyday terms when used in mathematical ways (e.g., borrow).

d) *Selection of numbers, cases & contexts for mathematical ideas*: Selection of numbers for problems and examples, use of real-world or pretend contexts as the settings for developing ideas and procedures, and selection of figures, shapes and cases (e.g., in geometry). Code for the appropriateness of the curriculum's selections in relation to working on specific mathematical content with these particular students. This includes attention to staging and sequencing, care that the numeric or contextual detail matches and does not obscure or confound the development of the mathematical content.

e) *Selection of correct manipulatives, and other visual and concrete models <u>to represent</u> mathematical ideas*: The curriculum's selection of pictures, diagrams, and manipulatives or other models to represent a mathematical idea or procedure. Of interest is whether the curriculum chooses models appropriate to the mathematics at hand. The degree of accuracy of a model should be calibrated to the requirements of the activity at hand. Do not code models that are solely stories (e.g., "numbers do not like each other" to represent the addition of positive and negative numbers of the same magnitude but different direction — (-5) + (+5) = 0); instead, code these under *language*. Do not code objects that are merely used as tools (e.g., a ruler used to measure pencils). Instead, the object must be *representing* an idea (e.g., a ruler used to illustrate the idea of measurement, or the idea of a number line). A thing cannot represent itself (i.e. a Kleenex box does not represent a rectangular prism).

f) *Multiple models*: Whether or not the curriculum uses more than one model for the mathematical content. By multiple models, we mainly mean models across "families" – e.g., graphs, equations and tables, or pictures, numeric procedures and stories. However, multiple models within a family can be coded here if they represent significantly different features of an idea. Do not count the model used in the problem statement unless it is also used in the solution in a significant way. For instance, if the problem is given symbolically or as a concrete scenario, but interpreted and solved using a graphical model, this does not count as using multiple models, but if the concrete situation is used to make sense of and manipulate the graphical model, this does count as using multiple models.

g) *Makes links among any combination of symbols, concrete pictures, diagrams, etc.*: When the curriculum makes explicit links among any of the above. The links must be mathematically significant—for instance, pointing out connections between representations in ways that allow students to grasp how representations are alike or different, how pieces of one relate to pieces of another, or what one representation affords that another does not. Includes explicit links between <u>pairs</u> of representations: word problems and symbolic notation, manipulatives and symbolic notation, story problems and manipulatives, etc.

h) *Mathematical descriptions (of steps)*: Curriculum's directing of mathematical descriptions (by teacher or co-produced with students) provides clear characterizations of the steps of a mathematical procedure or a process (e.g., a word problem). Does not necessarily address the meaning or reason for these steps. Code I for incomplete or unclear attempts.

i) *Mathematical explanations—giving mathematical meaning to ideas or procedures:* Curriculum's directing of explanations (by teacher or co-produced with students) includes attention to the meaning of steps or ideas. Does not necessarily provide mathematical justification. Code I for incomplete or unclear attempts.

j) *Mathematical justifications*: Curriculum's directing of explanations (by self or co-produced with students) include deductive reasoning about why a procedure works or why something is true or valid in general.

k) *Development of mathematical elements of the work (i.e., moving the mathematics along)*: Code here for whether the curriculum's moves, questions, and statements keep the development of the mathematics moving along. Considering the content topics marked in Ib, does the content appropriately open up, develop, or solidify in this segment, or does the curriculum seem to move the lesson off track (i.e., it lacks as sense of mathematical direction) without a plausible rationale about why this is happening? This is a global decision, hence, there are only two options: P-A or NP-I.

l) *Computational errors or other mathematical oversights:* Use this code when the teacher makes computational errors either in spoken or written language, or when the teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem). If present, computational errors or oversights are always inappropriate. If not present, the segment is appropriate. Hence there are only two options for this code: P-I or NP-A

B. Mathematical features of the teacher's guide

In this section, we are coding for the presentation of mathematics in the curriculum from the perspective of a teacher using the materials. What guidance is given to teachers about mathematically salient issues that arise in teaching the lesson?

To code, first decide whether the mathematical guidance to teachers is **present** (P) or **not present** (NP). (This decision should be based on the quantity of the guidance, not the quality or timeliness of it. Little or no significant guidance of the specified kind should be coded as NP.)

- If **present**, then:
  - Mark appropriate (A) if the curriculum's guidance is deployed, for the most part, in mathematically accurate and appropriate—does not distort the mathematics. Mark A if guidance is given at key places in the materials, without expecting that it is given at every turn.
  - Mark inappropriate (I) *either* if guidance is missing at key places in the materials (i.e., misplaced) *or* if the curriculum gives guidance but its deployment of mathematics in that guidance is, for the most part, inaccurate or distorts the mathematical content. Mark Inappropriate (I) if the curriculum's use of the element distorted the mathematics or was inappropriate for the grade level.
- If **not present**, then mark (NP)

Do not code both A and I. (These codes are higher inference. Allowing both is not reliable.)

a) *Mathematical point of the lesson*: Materials make clear to the teacher what the mathematical point is for the lesson. Beyond saying what the goal of the lesson is, the materials explain what is important about the goals of the lesson. Examples will include something more than a statement of what students will learn in the lesson. For example, do the materials help the teacher understand issues about why and how the goals are enacted in the lesson?

b) *Mathematical goal of the lesson*: The materials explicitly state objectives or goals, saying what students are to learn in the lesson, and the goals are clearly developed across the lesson.

c) *Choice/benefit of notation and recording:* Materials address issues regarding conventional symbols, mathematical notation, and the recording of these. This might include pointing out when the use of a symbol is a matter of convention, why or how a choice in notation will support students mathematical work and their learning, or caution points in using a specific notation to record particular ideas.

d) *Choice/benefit of language*: Materials address issues regarding mathematical language likely to arise for teachers. Why specific terms are used, what a particular analogy hides or makes visible, or problems that can arise with common terms such as "borrowing" or "adding a zero."

e) *Choice/benefit of examples or contexts:* Materials address issues regarding examples and contexts—which are used, why they are used, and pitfalls they create. For example, materials might draw teachers attention to the sequence and structure of a set of examples, might suggest alternatives depending on the population and interest of students, or might say something about the choice of particular numbers in the examples given.

f) *Additional problems to scale up or scale down as needed*: Materials provide additional problems meant to anticipate students' difficulties or students' potential facility with the content. The inclusion of either type of problems is adequate, but must be clearly, if not explicitly, noted for the teacher. For example, including a section of challenge problems in a problem set at the end of the lesson without tying those to the specific content at hand would not be adequate. Some indication should be given to what specific difficulties or extensions the scaled-up or scaled-down problems are designed to address.

g) *Choice/benefits of the representation*: Materials address issues regarding the representations used. For example, is what the representation is intended to accomplish explicit and clear to the teacher? Examples of yes would include more than a description of the representation, but more explicitly helping the teacher understand why a particular representation is used for a particular concept and how this representation is beneficial to the concept or mathematical topic being taught.

h) *How to get the model working*: Materials address issues regarding the deployment of the model. For example, is there guidance about making sure all students can see, about what language to use with the model, about important features to introduce to students?

i) *How to use multiple methods, explanations, or ways of thinking*: Materials suggest ways to introduce or elicit multiple strategies or thinking, when to do so, and what to do to make productive use of them. The materials might offer questions designed to elicit different approaches, they might suggest questions to pose that would get students to identify what is similar or different about approaches, or they might give advice about how to help students make sense of the alternatives—when they are in conflict and when not.

j) *What students might have difficulty with and how to respond*: Materials point out where students are likely to have difficulties, why they are likely to have them, and how to prepare to respond. For example, materials might describe common errors, might indicate points in the lesson to proceed with caution, or might situate specific difficulties in a larger picture of student development. Do not require that materials specify specific responses to difficulties, but that they orient and prepare teachers to respond.

k) *What students might say, along with what to say/ask/do if the student says a particular thing*: Materials indicate what students might say or how they might respond and they offer specific guidance, such as suggesting what to say/ask/do in response. Recording of the correct answers in scripted materials are not enough.

l) *Ways to be sensitive to issues of culture, diversity or language*: Do the materials explicitly raise issues of teaching for equity? For example, materials might suggest altering contexts or wordings to fit local students' backgrounds or interests, might offer caution points for second-language speakers, might describe mathematical contributions of disadvantaged groups that are more than token, or might raise sensitivity to the out-of-school lives of students when suggesting homework.

m) *Ways to check for student understanding*: Materials suggest ways of checking student understanding and strategic times during the lesson to do so. For example, randomly picking a group representative to report, asking specific questions designed to elicit student thinking, or walking around as students work to look at a specific feature of their work.

C. Content: Mark the major topics that receive major or minor focus in this segment (Same codes as in I B)

a) *Number concepts.* Number concepts refers to all <u>non-computational</u> work on whole numbers, decimals, or fractions. This includes writing, reading, or naming numbers; counting; comparing or ordering quantities; understanding place value; relationships between fractions and decimals; and estimating. For whole numbers only, it also includes properties of numbers (such as odd and even, prime and composite, square numbers), and factors, multiples, or divisibility. For fractions, it also includes work on the meaning of a fraction, on equivalent fractions, and on simplifying fractions. Please do <u>not</u> mark work on computation, basic facts, or patterns here unless that work was accompanied by a significant piece of work on a number concept topic as well.

b) *Operations*: Addition, subtraction, multiplication, and division. Include any work on meanings of these operations, understanding and developing competency with basic facts, multi-digit computation with whole numbers, and any computation with decimals or fractions or integers.

c) *Geometry*: Mark this category any instruction about geometry—area and perimeter, shapes, angles, points and lines, and spatial reasoning. Also include instruction that covers geometric concepts, e.g.: parallel, perpendicular, or congruent. Also include here work on geometric designs (e.g., tessellations).

d) *Measurement*: Length, weight, volume or capacity, units of measurement, and systems of measurement (e.g., metric, English).

e) *Probability*: Concept of probability, methods of estimating or calculating the likelihood of different outcomes, or problems involving combinations and permutations to calculate probabilities.

f) *Data collection, representation & analysis*: Creating or using tallies, tables, graphs or charts to represent data; making inferences or drawing conclusions from data; and lessons on mean, median, or mode.

g) *Discrete mathematics*: Combinatorics and permutations, counting problems, networks, alternative routes.

h) *Patterns, functions, and algebra*: Organizing objects by size, number, or other properties into groups, categories, or lists; different types of patterns; generalizing patterns; using symbols to express unknown and variable quantities; and understanding and using formulas.

i) *Money, time, and calendar*: Include in this category only instruction about features of money, time, or the calendar, <u>not</u> instruction that merely uses these to help students practice facts or procedures. For instance, include instruction about the value of money, but do not include multi-digit computation problems that use money as a context—record the latter above in "operations." By "reading a calendar" we mean instruction that helps students understand the ideas of days, weeks, months, and the construction of the calendar, but not work which uses the calendar to help students explore or practice other mathematical concepts or procedures.

j) *Percent, ratio, and proportion*: Concepts of percent, ratio, or proportion, as well as computation that involves percent, ratio, or proportion, or applications of these concepts.

In addition to the codes above, the lesson curriculum coding will include a narrative description including the following:

a) What the curricula for this lesson includes (e.g. textbook material, teacher handout, materials from a professional development activity)

b) What, if anything, stands out in this lesson as potentially difficult for the teacher to teach or manage (e.g. lots of materials, unusual activities for the students or teacher, etc.)

Appendix D: (Excel) Mathematical Quality of Instruction Coding Document

VIDEOCODES: MATHEMATICAL QUALITY OF INSTRUCTION

Directions: Stop tape every 5 minutes and check mathematical events.

I. Instructional formats and content

	A. Format for segment			B. Content topic									
Clips	a. whole group	b. small group/ partner	c. individual	a. Number concepts	b. Operations	c. Geometry	d. Measurement	e. Probability	f. Data	g. Discrete math/Combinatorics	h. Patterns, functions, and algebra	i. Money, time, and calendar	j. Percent, ratio, and proportion
1													
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4													
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12													
13													
14													

	C. Lesson/segment type			
Clips	a. review, warm up or homework	b. Introducing major task	c. Student work time	d. Synthesis or closure
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				

	II. Knowledge of mathematical terrain of enacted lesson																																															
	a. Conventional notation (mathematical symbols)				b. Technical language (mathematical terms and concepts)				c. General language for expressing mathematical ideas (overall care and precision with language)				d. Selection of numbers, cases & contexts for mathematical ideas				e. Selection of correct manipulatives, and other visual and concrete models to represent mathematical ideas				f. Multiple models				g. Makes links among any combinations of symbols, concrete pictures, diagrams, etc.				h. Mathematical descriptions (of steps)				i. Mathematical explanations (giving mathematical meaning to ideas or procedures)				j. Mathematical justifications				k. Development of mathematical elements of the work (i.e., moving the math along)				l. Computational errors or other mathematical oversights			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
Clips	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
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B. Overall level of teacher's knowledge of mathematics		
Low	Medium	High

	III. Use of mathematics with students																															
	a. Classroom work is connected to mathematical idea or procedure				b. Deploys manipulatives and other visual and concrete models to represent mathematical ideas				c. Elicits student description				d. Elicits student explanation				e. Records the mathematical work of the lesson				f. Interprets student productions				g. Uses students' errors				h. Launch of task/problems			
	P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP		P		NP	
Clips	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I	A	I
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IV. Mathematical features of the curriculum and the teacher's guide

A. Mathematical features of the curriculum

	a. Conventional notation (mathematical symbols)			b. Technical language (mathematical terms and concepts)			c. General language for expressing mathematical ideas (overall care and precision with language)			d. Selection of numbers, cases & contexts for mathematical ideas			e. Selection of manipulatives, and other visual and concrete models to represent mathematical ideas			f. Multiple models			g. Makes links among any combination of symbols, concrete pictures, diagrams, etc			h. Mathematical descriptions (of steps)			i. Mathematical explanations–– giving mathematical meaning to ideas or procedures			j. Mathematical justifications			k. Development of mathematical elements of the work (i.e., moving the mathematics along)			l. Computational errors or other mathematical oversights		
	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP
Name of curriculum	A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I	

B. Mathematical features of the teacher's guide

	a. Mathematical point of the lesson			b. Mathematical goal of the lesson			c. Choice/benefit of notation and recording			d. Choice/benefit of language			e. Choice/benefit of examples or contexts			f. Additional problems to scale up or scale down as needed			g. Choice/benefits of representation			h. How to get the model working			i. How to use multiple methods, explanations, or ways of thinking			j. What students might have difficulty with and how to respond			k. What students might say, along with what to say/ask/do if the student says a particular thing			l. Ways to be sensitive to issues of culture, diversity, or language			m. Ways to check for student understanding		
	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP	P		NP
	A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I		A	I	

C. Content topic of curriculum

	a. Number concepts	b. Operations	c. Geometry	d. Measurement	e. Probability	f. Data	g. Discrete math/ Combinatorics	h. Patterns, functions, and algebra	i. Money, time and calendar	j. Percent, ratio, and proportion

Appendix E: Mathematical Knowledge for Teaching Measures

(Released Items)

Sample Items[26]

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

		Yes	No	I'm not sure
a) 0 is an even number.		1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.		1	2	3
c) The number 8 can be written as 008.		1	2	3
d) You can't subtract a number from 0.		1	2	3

[26] Downloaded from: http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

7[27]. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 1 25 +7 5 8 75	3 5 x2 5 17 5 +70 0 87 5	35 x 25 25 1 50 1 00 +6 00 8 75

Which of these students is using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

[27] Downloaded from: http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

4[28]. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

[28]Downloaded from: http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

Appendix F[29]: Table of Teachers' MQI, MKT, and MQE scores

Name	Overall MQI Lesson Score*	Overall MKT Score*	*Lesson*	MQE Score	MQE avg. for 3 lessons
Robin	2.22	0.64	W2-025	.223	
			027	.421	
			029	.227	.290
Rebecca	1.40	-0.71	W2-026	.41	
			028	.40	
			030	.42	.41
Lauren	2.78	1.30	W2-020	.821	
			023	.506	
			024	.767	.698
Karen	2.56	1.50	W2-016	.23	
			018	.375	
			020	.307	.304
Mimi	1.44	-0.19	W2- 010	.21	
			012	.111	
			014	.303	.208
Noelle	2.11	1.21	W2-017	.23	
			019	.272	
			022	.327	.276
Anna	1.33	0.56	W2-011	.267	
			013	.26	
			015	.23	.252
Sally	1.33	0.09	W2-001	.22	
			002		
			006	.28	.25
Zoe	1.11	-0.43	W2-003	.38	
			007	.45	
			009	.28	.37
Lisa	2.00	-.023	W2-004	.244	
			005	.362	
			008	.42	.342

[29] See Hill, et al., 2008, p. 441 for a similar chart that details analyses from the LMT "meta-codes".

Appendix G: Case Study Teacher Profile and General Characteristics

Name	Credentialing information/ advanced degrees?	Grades taught?	Years of Teaching	Hours of Mathematics Professional Development (in the year before the study)	Attitudes towards mathematics
Rebecca	Pre-intern status (not certified); enrolled in an alternative certification program but has been teaching full time and taking classes in the evening.	6th and 8th	Four complete years of classroom experience, one of science, one of both, and three of math and science	"Over 600 (clock hours), including bi-weekly classes required for the alternative certification program, district required PD, and school based math professional development	Math would be my first choice to teach; math is my favorite subject to teach (Wave 1)
Karen	Life Credential for grades k-9; Masters' degree. At the time of the study, she had just completed training to become a credential instructor to teach math content courses. Has also previously been a math coach for the district.	Third grade, fifth grade	37 years of experience.	72 clock hours of mathematics professional development	"I always try to improve the way I teach mathematics because it's my favorite thing" (Wave 1 interview)
Lauren	"Clear", multiple subjects credential; working on her Masters' degree at the time of the study.	3rd grade and 4th grade	15 years of classroom experience	Over 75 clock hours of mathematics professional development in the year prior to the start of our study and an additional 60 hours in math courses at a local university.	Loves the challenges of teaching math and working find new and better strategies for teaching because "I don't want to lose any of them."

Appendix H: Post-Lesson Interview Questions

Videotaping Study

1. How did the lesson go? What went well? What didn't go so well? What would you change if you did this again?

2. Did anything surprise you today?

3. What mathematics topics or ideas were you teaching in this lesson?

- Where does that fit with other things you've been doing?
- What did students need to know in order to work on this lesson?

4. What were you hoping students would learn from this lesson?

5. What's your sense of whether the students learned the things you listed above?

6. Did the students seem to struggle with any part of the lesson? What part(s)? Why do you think this was?

7. Was there anything that <u>you</u> struggled with mathematically in this lesson? What was it? Why do you think you struggled with this?

8. Thinking back to getting ready for this lesson, how did you prepare for the class?

- Were you working from particular materials? *(if so, get specific title; publisher)*

9. Did you modify the lesson as presented in the materials? How? Why?

  - Did you have to come up with any new examples? Why?
  - Did you change any of the student activities or tasks? Why?
  - Did you take anything out of the lesson? Why?

Appendix I: Videotaping Study Wave 2 General/ Equity Interview

Questions and Protocol

Note: *In this interview, first we investigate teachers' experience of the summer professional development, and probe what they think they learned. We focus here on (a) mathematics, including topics and ways of doing mathematics; (b) students as learners of mathematics, including issues of diversity and equity; and (c) the teacher's own orientation to and knowledge of mathematics. We also ask about other possible learning outside of mathematics. These questions correspond to the domains in which we code the teachers' videotaped lessons.*

We also probe teachers' ideas about using mathematics in their teaching, and about working with students, including how they perceive and manage issues of equity.

The interview scripts opening statements, as well as probes and extensions. In order to get common, good data, please use the wording developed in this script. Use it flexibly so that the interview does not feel stilted, but please attend to covering the different elements of each question.

Outline:

1. **Introduction**
2. **Professional development**
    a. **Mathematics**
    b. **Students**
    c. **Teacher/teaching**
3. **Using and working with mathematics with students**
4. **Wrap up**

***Equity issues are distributed throughout.**

1. Introduction

First of all, I know you're really busy. Thank you for all the time you are giving us. It is very helpful. Even though I have many questions I want to ask, please keep in mind that what I am interested in is what you think, your experiences, your views. There aren't right answers to my questions!

Let me give you an overview. I'm going to be asking you about your experiences in the professional development you have been attending at *(name site)*. In this part, I'll ask about what stood out to you about the experience and what you think you took away – in terms of your own knowledge and skills with mathematics, students, and teaching. Throughout, I am also interested in whether issues of diversity, culture, or equity were important to any of your experience or your learning. I am also interested in your teaching, and how you work with students.

At the end, I'll also ask you whether there are some other things that seem important to you about your experiences with the professional development or in your teaching.

2. Professional development

Let's start out with your experience of the professional development program this summer. Was there something in particular you took away from it?

Let the teacher talk first, and listen carefully to what he/she brings up.

If the teacher mentions something that is not related to mathematics, students, or teaching, probe:

Can you say a little more about that?

What's an example?

Why did that stand out to you?

If the teacher says that the experience was really useless, try asking some of these probes:

This is important for me to understand. Can you say a little more?

Can you give me any examples of what you mean?

What's your sense of what the goals were?

Was there anything you felt <u>was</u> useful at all?

a. Mathematics

a.1. *(If not mentioned*) I noticed you have not mentioned mathematics. Did you feel you learned any <u>mathematics</u>?

(If mentioned) You've been talking some about mathematics. I'd like to talk a little more about the <u>mathematics</u> that stood out to you about what you learned?

- What did you think you learned?
- Can you give me an example?
- What was important to you about ___?
- Was there anything else about <u>mathematics</u> that stood out to you about what you learned?

(If no) Why was mathematics not a central part of what you learned in the institute/workshop/program?

a.2. Were there particular mathematical <u>topics</u> that you learned more about in the professional development?

(If yes) Can you tell me more about ______?

What stood out about your work on ____?

Were there other mathematical topics that you felt you learned more about? ***(Probe for examples and specifics)*** a.3. Were there ways of thinking or reasoning, or other aspects of <u>doing mathematics</u> that you learned more about in the professional development?

(If yes) Can you tell me more about ______? What stood out about your work on ____?

Were there other mathematical practices that you felt you learned more about? *Probe for examples and specifics. If the teacher doesn't seem to know what is being asked, you might refer to some of the NCTM process standards (e.g., problem solving, reasoning).*

We are interested in some specific aspects of mathematics and I am curious whether any of these might have come up in your work this summer in the professional development.

a.4. So, first, did you learn more about what is involved in <u>explaining</u> in mathematics? For example, did the professional development affect how you think about what counts as a "good" explanation for a mathematical idea or procedure?

(If yes) Could you give me an example?

How did this affect your thinking?

Was there anything new or different about this for you? *If yes*: How so?

Does this seem important to you? (*probe for why or why not*)

a.5. Here's another one. Did you learn new ways to <u>represent</u> or show particular mathematical ideas or procedures?

(If yes) Could you give me an example?

Was there anything new or different about this for you? *If yes*: How so?

Does this seem important to you? (*probe for why or why not*)

a.6. Did you learn anything about alternative methods or procedures in solving particular kinds of problems, or about alternative algorithms?

(If yes) Could you give me an example?

Was there anything new or different about this for you? *If yes*: How so?

Does this seem important to you? (*probe for why or why not*)

a.7. Did you learn anything about standard computational algorithms? *(if the teacher isn't sure what is meant by "standard algorithm," give the example of subtraction with "borrowing" or regrouping.)*

(If yes) Could you give me an example?

Was there anything new or different about this for you? *If yes*: How so?

Does this seem important to you? (*probe for why or why not*) a.8. Did you learn anything about <u>mathematics</u> that might help you address issues of diversity, culture, or equity — for example, in the examples that you use, or story problems you choose, or in your word choices, making mathematical connections? Or something about particular mathematical content?

If yes: Could you give me an example?

Was there anything new or different about this for you? *If yes*: How so?

Does this seem important to you? (*probe for why or why not*)

a.9. Is there anything else in mathematics you would have liked to have an opportunity to learn more about?

If yes: Could you give me an example?

Why would this be important to do?

a.10. Was there anything about mathematics that you learned in the professional development that was particularly <u>useful</u> to you in your teaching, or your work with students?

If yes: Could you give me an example?

If not: Why was that mathematics you learned not that useful?

b. Students

b.1. *(If not mentioned already)* Did this professional development contribute in particular ways to your knowledge of how students learn mathematics?

If yes: Could you give me an example?

Was this new to you?

Does this seem significant to you?

Has this been helpful in your work? *If so:* How? *If not:* Why not so much?

b.2. Did you develop any new skills in dealing with students' ideas in class — maybe something about dealing with common misconceptions, or responding to ideas that kids bring up in class?

If yes: Could you give me an example?

What was new about this for you?

Has this been helpful in your work? *If so:* How? *If not:* Why not so much?

b.3. Aside from how students learn mathematics, were there other things you learned about students more generally in the professional development?

If yes: Could you give me an example?

Has this been helpful in your work? *If so:* How? *If not:* Why not so much?

b.4. Did the professional development offer you some resources or ways of dealing with issues of student diversity, culture, or equity in learning mathematics?

If yes: Could you give me an example?

Was this useful to you? *If so:* How? *If not:* Why not so much?

Have you been able to make use of this in your own teaching?

Was there anything else you learned about dealing with issues of student diversity and equity?

c. Teacher/Teaching

c.1. Did you learn anything about yourself as a "do-er" of mathematics?

If yes: Could you give me an example?

How did you learn this?

Does this seem important to you? *If so*: How is it important?

Has this affected your teaching in any way? *If so*: How has it affected your teaching? *(probe for specific examples)*

c.2. Did you learn anything about yourself as a <u>teacher</u> of mathematics?

If yes: Could you give me an example?

How did you learn this?

Does this seem important to you? *If so*: How is it important?

Has this affected your teaching in any way? *If so*: How has it affected your teaching? *(probe for specific examples)*

c.3. Would you say that any of your beliefs or attitudes about <u>teaching mathematics</u> changed from attending this professional development?

If yes: Could you give me an example?

How did you learn this?

Does this seem important to you? *If so*: How is it important?

Has this affected your teaching in any way? *If so*: How has it affected your teaching? *(probe for specific examples)* c.4. I am also interested in whether the professional development offered you some skills or ways of dealing with issues of diversity, culture, or equity. Many teachers comment on the challenges of teaching in highly diverse environments, and they are also aware of some of the big gaps in mathematics achievement. Did the professional development focus on these problems at all?

If yes: Could you give me an example?

What did you take from this?

Has this had any impact on your thinking as a teacher? *(probe for specific examples)*

Has it affected what you are doing in your classroom? *(probe for specific examples)*

Was there anything else you learned that related to this?

c.5. What curriculum materials do you use? Are you required to use these materials? How much latitude do you have to use other materials? *(if the teacher reports having some latitude)* What else do you use?

c.6. How comfortable with teaching mathematics would you say you are? Did your comfort level change in any way as a result of your experience with the professional development?

If yes: Could you give me an example?

c.7. I'm interested in how any of what you did or learned in the professional development is showing up in your work this year. Is there anything you learned, or began to think about, that is shaping what you are doing this fall?

If yes: Could you give me an example?

How is this affecting your practice?

Why is this something that you wanted to try to do? *or* Why has this affected your thinking as it has?

Are there other examples of this?

3. Using/working with mathematics with students

Now I'd like to ask you a few questions about your work with students.

a. Are there some specific things that you really value in students' work in mathematics? For example, are there certain things you praise students for when you see them doing those things?

If yes: Could you give me an example?

Why is this important, in your view? *or* Why do you value this?

Do you explicitly teach students to do this? *If so:* How do you work on this with them?

How do you encourage this?

b. Are there some students who seem to have a great deal of difficulty learning mathematics?

If yes: What is it that causes them difficulty?

Is there anything particular you do when you have students like that?

c. Conversely, are there some students who seem to learn mathematics very easily?

If yes: What is it that makes it easy for them to learn mathematics?

Is there anything particular you do when you have students like that?

4. Wrap up

This has been really helpful – thank you. Before we end, though, I wonder whether there are other issues you'd like to bring up. For example, was there something else about the professional development, or about your teaching, or about your context, that you think is important for me to understand?

(as always, probe for examples)

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