Representations of mathematics teaching and their use in teacher education: What do we need in a pedagogy for the 21st century?

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First discussion document for PMENA's RMT Working Group:
"Facilitating sessions where teachers
interact with and discuss representations of teaching" [v]

(a) Brief history of the Working Group

This is the first meeting at PMENA of this RMT working group. The idea of this working group emerged during a series of three-day conferences on representations of mathematics teaching held in Ann Arbor, Michigan in August 2009 and June 2010. These conferences were organized by project ThEMaT (Thought Experiments in Mathematics Teaching), an NSF-funded research and development project directed by Herbst and Chazan. ThEMaT originally created animated representations of teaching using cartoon characters to be used for research, specifically to prompt experienced teachers to relay the rationality they draw upon to justify or indict actions in teaching. The project also aimed to disseminate those animations to be used in teacher development and for that purpose held summer workshops in 2007 and 2008. The workshops evolved into the RMT conferences in 2009 and 2010, whose purpose was to gather developers and users of all kinds of representations of teaching to present their work and discuss issues that might be common to them. RMT conference participants included users of video, written cases, dialogues, photographs, comic strips, and animations. An outcome of the 2009 RMT conference was a special double issue of the journal *ZDM--The International Journal of Mathematics Education*, guest edited by Herbst and Chazan; the issue will appear in 2011 and contains 15 papers, currently at various stages of post-review production. Discussions at the 2010 conference created agendas for working groups and sessions in other, more prominent conferences. One of those agendas, based on a discussion about the facilitation of sessions with representations of teaching and the tools available to facilitate those sessions online and face-to-face, and for different clienteles[vi] (practicing teachers, preservice teachers, teacher leaders, others), stimulated this working group.

(b) Issues in the psychology of mathematics education that will be the focus of the work

The working group is focused on elaborating a pedagogy of mathematics teacher education assisted by representations of teaching. Like other technological innovations, representations of teaching not only offer opportunities for teachers' learning but also call for specialized pedagogical practices from teacher developers. The working group will engage in such elaboration in two ways. First, the work will consist of using some conceptual and technological tools (described below) to design some teacher development experiences. Second, reflections on such design work are expected to call attention for more and better tools.

For some years now, teacher educators worldwide have used classroom video records, samples of student work, narrative cases, and other artifacts to engage teachers in discussions about teaching (Fishman, 2003; Lampert & Ball, 1998; Merseth, 2003; Sherin & Han, 2004; Smith, Silver, & Stein, 2004; Tochon, 1999). These artifacts afford opportunities for teachers to learn from practice, whether this learning focuses on pedagogical or mathematical aspects of the work of teaching or on understanding students and their thinking. More recently, animations and

comic books using cartoon characters have been created and used for similar purposes (Herbst, Chazan, Chen, Chieu, and Weiss, in press). In parallel, advances in information technologies have made it easy to create and manipulate rich media objects (graphics, photo, video) and share them in the Internet where they can be tagged, commented, and repurposed. This technology enables collaborative work across geographic boundaries. More importantly, it enables a different kind of work with records of teaching, particularly work that, by enabling more detailed and active experiences with the media, has the potential to increase learning opportunities for clients of teacher education.

Materials exist describing how to use some of these artifacts in teacher learning contexts (e.g., Merseth, 2003; Seago, Mumme, & Branca, 2004). The scholarly literature has also addressed facilitation in the context of describing teacher learning from professional development (e.g., Borko, 2004) and even the learning of the facilitators (e.g., Stein, Smith, & Silver, 1999). We choose not to review this literature here.

Our present purpose is to stimulate the development of pedagogical practices attuned to the possibilities that novel media--particularly cartoon-based representations of teaching--and new technologies offer for teacher development. Herbst, Chazan, Chen, Chieu, and Weiss (in press) have argued that cartoon-based representations of teaching can have virtues similar to video (e.g., the possibility of an animation to immerse the viewer in a timeline and cadence of events comparable to that of real action) as well as some of the virtues of written cases (e.g., the possibility of cartoons to represent selected facets of the individuality of people and settings rather than show by default as many of those facets as the recording technology allows). These characteristics add to available technologies that permit to create, annotate, and reuse computer graphics and interact with others about them. These tools make cartoon-based representations of teaching a malleable medium for learning in, from, and for practice (Lampert, 2010). In calling for the development of a pedagogy adapted to the use of representations of teaching we operate on the assumption that representations of teaching can do more than support usual teacher development activities. They can also create new spaces for the development of professional knowledge and skills. In a way this is analogous to how the availability of new technological artifacts (calculators, computers) not only permits the emergence of new ways of knowing but also requires novel pedagogical practices to fulfill their promise.

In this document we contribute to a discussion on a pedagogy of mathematics teacher education assisted by representations of teaching by proposing some basic categories for such pedagogy. We also provide examples that can get the working group started in the work of fleshing out such pedagogy. We start this discussion, however, with a more basic conceptualization of representations of teaching that can underscore the important role that cartoon-based representations can play in teacher development practice.

What is a Representation of Teaching

The expression "representation of mathematics teaching" suggests a semiotic mediation—a *sign,* or *representamen* (Peirce, 1955) pointing to mathematics teaching as the *referent*, the *object* represented. That expression might trigger associations with the notion of representation in the teaching of mathematics. At first blush those associations could be dispelled just by noting that the preposition "of" refers not to the role that representations may play in teaching mathematics but to the representation of the practice of teaching mathematics itself and the role that these can play in the learning of teaching. But on second thought, the extant literature on mathematical representations might be of use in understanding what a representation is and what

role it could play in the learning of teaching. We come back to it below, after we consider the object or referent in representations of teaching.

The question of what is mathematics teaching has been vastly addressed in the literature. The teacher's work used to be seen as administering an unproblematic body of subject matter to children and, or alternatively, cultivating a personal relationship with children. But successive improvements in conceiving the subject of studies, students' cognition and learning, and eventually the work of teaching itself, have contributed to portray a rather complex profession (see Doyle, 2006; Fenstermacher, 1994; Lampert, 2001). The conception of teaching proposed by Cohen (in press) asserting that teaching is a practice that deliberately attends to students' learning of disciplinary subject matter by attending to the representations of disciplinary knowledge, the cognitions of students, and the instructional medium in which teacher and student interact, seems useful as a starting point in describing the object or referent in representations of teaching.

Representations of teaching could help the work of teacher developers by pointing to the many tasks of teaching that derive from the three domains of teacher work that Cohen describes. The notion that the work of the teacher includes attending to the representation of disciplinary knowledge includes tasks of teaching such as selecting or designing embodiments of mathematical ideas, formulating mathematical statements that are true, crafting mathematically compelling explanations, identifying errors, choosing problems for students that give opportunity to use target mathematical ideas, etc. The notion that the work of the teacher includes attending to the cognitions of students includes tasks of teaching such as eliciting students' thinking, interpreting students' conceptions, creating and issuing specific challenges to students' conceptions, etc. The notion that the work of the teacher includes attending to the instructional medium includes a number of diverse tasks of teaching associated with shepherding interpersonal dynamics, communication through personal relationships, and the affordances and constraints of the institution where the work is done (e.g., using well the time allotted for class or the space allotted for public displays; see also Ball, Thames, and Phelps, 2008; Lampert, 2001). The work of learning to teach includes learning to attend to those three commonplaces, eventually acting in such a way as to exercise attentiveness to those different foci simultaneously and over relatively long expanses of time. From such a brief, initial description of mathematics teaching as the object of learning it should be apparent that we are talking about an object of study that potentially makes high demands on intellectual and performance capacity. Experiences that scaffold that learning, enabling learners of teaching to engage with and learn about some features of that complex practice while keeping others simple, may succeed in creating those capacities incrementally. Representations of teaching can display particular enactments of those tasks; cartoon-based representations of teaching, particularly those realized with non-descript characters, can help focus on the enactments and the tasks enacted rather than on the actors themselves (McCloud, 1994). Along those lines an analogy with representations of mathematical ideas can be quite productive.

Mathematical representations can help make some aspects of mathematical ideas salient by embodying the structure or function of those ideas in the structure and function of the sign system that performs the representation (see also Goldin, 2003; Resnick and Omanson, 1986; Schoenfeld, 1987). Peirce (1955) addresses this with the notion of the *interpretant*, a decodification of the sign into another sign (what some have called an internal representation). A representation of a mathematical object by a sign is thus realized through the creation of an interpretant that basically establishes how the sign points to the object (and what aspects of the

sign predicate about the object).

Various kinds of mathematical signs (such as those in figure 2) can perform a representation of a mathematical idea (e.g., circle); they do so by creating an interpretant that makes the mapping between sign and object operational, by noting how the sign calls forth the object. Cartoon-based animations and comic strips (and for that matter also video clips of images of real people and written words and sentences) can do the same with respect to teaching. The role of any of those representations of teaching is to build the interpretant–to enable ways of thinking about teaching called forth by the sign that might progressively grasp the complexities of the practice itself. While the things represented, mathematical ideas on the one hand and the practice of mathematics teaching on the other, are quite dissimilar, we might learn something about what representations do by working through a metaphor between them. Like any metaphor one could push this one to limits where it would stop making sense, but like any metaphor, this metaphor can, within boundaries, help us understand the power of some representations of teaching in regard to the development of the interpretant.

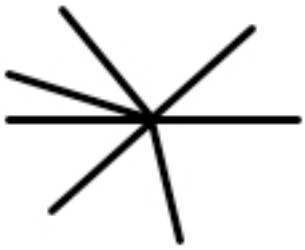

Figure 1. Several positions of a spinner

$\{(x,y) \in \Re^2 / (x-a)^2 + (y-b)^2 = r^2\}$

The intersection of a sphere with a plane

Figure 2. Other representations of circle realized with different semiotic resources (diagram, symbols, language)

We'd like to propose an analogy between a particular representation of circle (the spinner alluded to in Figure 1) and the representation of the practice of mathematics teaching made possible through comic strips or animations of cartoons characters. Of course there are other representations of circle, such as those shown in Figure 2. The spinner in Figure 1 contrasts with all of those in Figure 2 in its dynamic nature. One could think of the spinner as creating a message over time that says, "this is a circle." At any one moment in time the spinner only points at one point, as if writing one letter of that message. One has to look at it over some length of time to gather what the aggregate is doing. The spinner also contrasts with the other representations in regard to how it shows not just what the circle is but also how the points in the circle are obtained. Finally, this spinner also represents the circle without actually depicting any of its points—we see the circle by putting into the image a response of sorts to the stimulus provided by the rotating segment, but neither the segment is part of the circle nor the circle is really drawn there. In all these regards cartoon-based representations are like the spinner.

The spinner representation of the circle reminds us of Walt Whitman's[vii] words "all music is what awakes from you when you are reminded by the instruments." In Peirce's terms, the representamen (or sign) is the collection of strokes at different locations, sorted over time; the referent or object is the mathematical notion of circle, say, the set of points on a plane at a fixed, given distance from a given point on that plane; the interpretant notes that the strokes are the same length and start from one common point at each moment while the other point makes over

time a familiar figure. Indeed, through the interpretant, the moving strokes remind us where the circle's points should be, and over time they remind us where the complete circle should be; finally the interpretant elides the moving strokes–they are not the circle. The moving strokes are like the instruments in Whitman's quote, the notion of circle could be the orchestral arrangement handled by the orchestra director, while the points becoming the circle are the music we hear. It can also be noted that we see the circle through the work of the spinner partly because we have an idea of what the circle should be—the interpretant has some prior associations such as for example one between the sign in Figure 2a and a synthetic idea of the circle (e.g., an equivalence class of all the diagrams that look like that one).

Just as the spinner shows over time how the circle is made to someone who has a prior idea of what a circle is supposed to be like and without actually creating a physical sign for the circle itself but by evoking it from the viewer, a representation of teaching can point us to how teaching is done even if the signs themselves don't show each of its actions but just point to them. A comic strip including a sequence of photographs of the teacher writing at the board can portray an explanation or the setting up of a task (see also Crespo, 2010). So can a comic strip containing snapshots of action in response to a problem such as those shown in Figure 3. The interpretant associates those signs with the actual events that could happen in practice, say when a student solves a problem about linear functions, and can not only fill the gaps in between frames but also predict the actions that might come after.

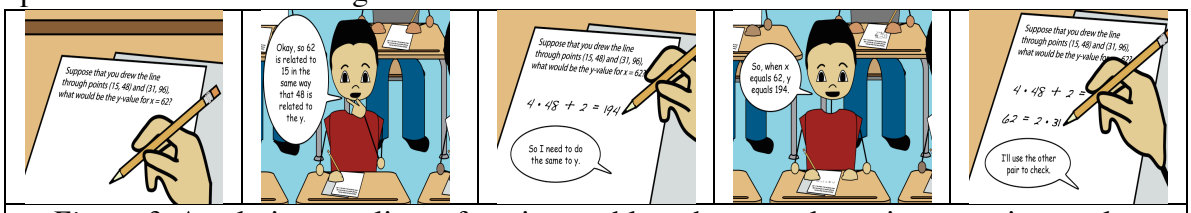

Figure 3. A solution to a linear function problem that reveals a misconception at play

Building the Interpretant

All communication about ideas, be that in mathematics instruction or in teacher education, happens through transacting representations; at the very least because language is also a semiotic system. In regard to transacting about the practice of teaching, we'd say that language can be quite an abstract semiotic system. Words such as *constructivist* or *traditional* may be useful to name chunks of action in the manner that the English word "circle" names the mathematical object; but those words are not like the spinner that tells you how a circle comes to be. For teacher education we need representations that help connect *theory* of what teaching is with *practice* on how to do it. Note, along those lines, that the spinner allows one to interact with the notion of circle, for example by permitting the construction of a different, static representation of the circle (such as the one on Figure 2a) by plotting a point at the mobile end of the spinner at various moments in time and then by using that representation to make conjectures about properties of the circle other than the constant distance from a center that defines it. Clearly a symbolic representation of circle, such as the one on Figure 2b also enables one to do things with the circle, and notice different properties. A diagrammatic representation of circle has been a preferred one for beginners for ages. In the case of teaching we ask what kind of sign can produce a representation of teaching that can be so generative of knowledge about teaching as the spinner is for the circle?

Why are these semiotic considerations of any use in thinking about a representations-based pedagogy of mathematics teacher education? Grossman et al. (2009) note the role that

representations of practice play in professional education; while they distinguish between *representations* as those artifacts that display practice (e.g., video records), *decompositions* as those artifacts that unpack and elaborate on aspects of practice (e.g., a rubric that outlines the characteristics of an instructional explanation), and *approximations* as those artifacts that create opportunities to engage in practice (e.g., a simulation software that invites teacher candidates to act the teacher in front of virtual students), it seems to us that all of those are to some extent based on a representation of teaching. We prefer the words in their gerund form as *representing*, *decomposing*, and *approximating* practice and use them to designate the activity systems in which the interpretant employs the signs to transact and think about the object of consideration.

The work of mathematics teacher education, the pedagogy of teaching a practice assisted by representations of practice thus involves building the interpretant through activities of representing, decomposing, and approximating practice with different kinds of semiotic resources. It includes, in particular, developing teacher capacity for using signs to describe practice, for 'reading' a practice from signs, and for creating enactments of that practice within the grammar of a semiotic system. Such activities include exercises like "Ms. Shackleforth's lesson," (see Ghousseini, 2008) where teacher candidates are asked to supply the lines spoken by the teacher in a written dialogue where only student lines are provided. Clearly such approximation of practice does not preserve all the complexities of practice; but rather than discarding such approximations for what they miss, we could see them as building some opportunity for practice. In generating the teacher's lines in a hypothetical classroom dialogue, clients have the chance to design the precise words that a teacher would say (rather than just the informational content of their response), offering an opportunity for them to consider the informational and relational entailments of teacher talk in the context of a specific mathematical discussion. While engaging with representations of teaching (doing things with signs) does not substitute engagement with actual teaching, the former scaffolds the latter much in the same way as engaging diagrammatically with a representation of a circle (such as drawing a stroke between a point on the circle and the center) enables the user to think about aspects of the circle (e.g., the notion of radius) without loss of generality.

What kinds of tasks can elaborate the three activities noted above (representing, decomposing, and approximating)? How can facilitation with such tasks engage teachers in grappling with important aspects of practice that they need to learn? This is where we want the working group to get started. A preliminary framework to guide the work of the group is provided below.

A Pedagogical Framework for Facilitation

The framework we propose includes four kinds of elements. One of those can be described at face value as *open-ended expressions* or boundary objects that can be used in transactions between teacher developers and their clients without needing to be completely defined. One of those expressions is "mathematical action" which we observed being used in a geometry class for future teachers and in the context of having the students watch an animation of geometry instruction. The college instructor's goal was to have her clients flag moments that had mathematical significance hoping to get them to bring up things like "making a claim," "offering a counterexample," "extending a claim to a larger set of objects," etc. Clearly, it would have defeated the purpose of the activity to give such a list (and its vocabulary) before watching the movie; and the work could have been stifled or miscued if the instructor had used better understood words (e.g. if she had said something like "flag moments where they are using a

mathematical concept," the students might have restricted themselves to uses of known mathematical vocabulary). Other open-ended expressions that can serve comparable purposes are "student thinking" and "teaching move." Their usefulness in mobilizing the work with representations depends on their ambiguous nature—the work set up will then involve discovering or inventing their meaning.

The first proposal of this framework is that a pedagogy for mathematics teacher education assisted by representations of teaching needs a set of usable open-ended expressions or boundary objects that the teacher developer can use to formulate, in rather non-descript ways (i.e., not calling attention to the words), tasks that the learner of teaching can do with representations. For example, a client could be offered the student solution to a problem sketched in Figure 4 and then asked to represent, using the cartoon character set, an answer to the question "What would be your next move?" This question is ambiguous in comparison with the more concrete "What would you say next?" and for that reason it might allow the client to make the cartoon teacher do other things than talking. A client could have the teacher erase the board, while another might have the teacher correct the construction, and another might have the teacher ask Alpha what they know about the relationship between radii and tangents. A discussion of those options and the possible justification for these moves could ensue without ever dwelling on the word *move*.

Figure 4. A student's response to a problem inviting the teacher to make a move

A second element of this framework is relatively expected and documented in earlier literature about facilitation. It consists of having a taxonomy of activity structures or activity types for mathematics teacher education. Clearly these activity types could have elements in common with those found in K-12 classrooms, such as triadic dialogue, homework review, etc. (see Lemke, 1990). But there are other activity types that are particular to the work of mathematics teacher education assisted by representations of teaching. A quite common activity type could be described as "working on the math." Quite often, mathematics teacher developers who intend to show a video that displays students working on a mathematical task will first have their clients work on the mathematics that will be featured in the video. The goal of this activity is not necessarily to teach the mathematics at stake in the problem to the clients, and for that reason the clients may or may not be expected to complete the problem. Yet this activity is done for the clients to develop some familiarity with the mathematics at play in the representation so that they can attend to other things, such as student thinking, when they interact with the representation.

Another activity type we have used in the context of teacher education is a form of review of homework in which clients enact scripts of action that they conceive outside of class in response to practical problems, such as shown in Figure 5, in the context of learning to explain procedures. Usually those enactments give clients some practice in delivering a teaching move that had been planned; they also get other clients involved in giving feedback, and help raise

more substantive questions about the task of teaching being learned. In the context of the teacher study groups organized by ThEMaT, an activity structure involved asking participants to watch animations and tap the table to stop the animation so they could make a comment. Stopping the video not only enabled the person who tapped the table to make a comment, but it also opened the floor to other participants to chime in. The video would only continue when the participants were no longer interested in following up. We bring these examples to illustrate the more general contention that a pedagogy for mathematics teacher education that uses representations of teaching needs to include a taxonomy of activity structures. These activity structures could specify in particular how the clients might interact with (manipulate, annotate, etc.) the representations being used.

You are going to teach a lesson on tangents to circles. The lesson includes teaching the procedure for constructing a tangent to a given circle. In your plan you have sketched the procedure as follows.

(1) Given, a circle with center O, and a point P
(2) Draw segment OP, find its midpoint D
(3) With compass centered at D draw a circle of radius DO, intersect the circle at points A and B
(4) With straightedge, draw lines PA and PB, which are the required tangents

Your field instructor indicates that the procedure is okay but that it is not clear how your students will know that the lines are indeed tangents to the given circle. Script a few lines where you explain to the students why is it that the lines they constructed are the tangents to the given circle.

Figure 5. A homework problem for clients to practice a task of teaching

A third element of the framework consist of problem types. By this we mean specific intellectual work that participants do within an activity type involving representations. An initial list of types derives from the three words used by Grossman et al. (2009): representing (the client might view), decomposing (the client might study), and approximating (the client might do). Clearly more is hinted—one could expect that participants would not just view a representation and be impressed by it; they might also describe a segment of it or point to special moments. In fact the wording of the tasks do matter (Morris, 2006). Likewise, a representation of teaching useful for decomposing teaching could look like a rubric that unpacks the characteristics of a task of teaching (e.g., how to explain a concept). The study of a decomposition of teaching could involve work such as judging a performance on that task of teaching using the decomposition. As noted above, completing dialogues or comic strips could be used as activity types for approximating teaching. Approximating teaching might include not only problems for clients to conceive the actions they might do by writing lines of dialogue but also rehearsing those actions by working on aspects of professional performance such as voice, use of interjections, posture, board writing, etc. Table 1 provides an initial list of problem types based on what Herbst & Chazan (2006) identified as attributes of cartoon-based representations of teaching.

Table 1: Problem types to engage participants with representations

Problem Type Name	Sample Problem Statement
Alternativity problem	What else could one do at this moment?
Generality problem	What would you call that teaching move?
Normativity problem	What should be done in these circumstances?
Projectiveness problem	How would you have felt if those were your students?
Reflectiveness problem	What do you think about this episode?
Temporality problem	At what moment would you say [such thing] happened?

All of the elements in Table 1 unpack the work that the viewer could do when confronting what Grossman et al. (2009) called a representation of teaching (and what we would call a representation used in representing and viewing teaching). But some of them can also be used in the context of decomposing and studying teaching—for example a normativity problem could be used along with a video record or animation to give learners a chance to learn a rubric about an activity of teaching. For example, they could be given a rubric that describes the components of an instructional explanation, and a video clip that purportedly contains one such explanation (not necessarily a good one); and they could be asked a normativity problem so that they apply the rubric. The problem types listed in Table 1 are only some examples. More generally, the question to the members of the working group is what are other types of problems that could be used along with representations of teaching and in activities such as representing, decomposing, or approximating teaching? Our contention in this paper is that a pedagogy of mathematics teacher education assisted by representations of teaching needs a taxonomy of problem types.

The final element of this emerging framework addresses the technological affordances needed to realize this pedagogy of mathematics teacher education. Clearly one could do many of these activities having only a video projector and playing media off a single computer. But there are important pedagogical considerations associated with more technology-intensive environments. For example, if every client could play the media at their leisure but contribute comments to a common conversation, in face-to-face, chat, or forum, this could have the beneficial effect of diversifying the discussions, particularly in regard to temporality problems. Along those lines a recent study by Chieu, Herbst, and Weiss (in press) shows evidence that clients' comments in forum or chat benefitted from having an embedded screen for the animation being discussed, which they could access at the same time as they interacted with peers in a forum or chat. This media-enabled-forum is one of several functionalities available in ThEMaT Online, a resource for teacher developers to use with their clients.[viii] Another functionality present in ThEMaT Online is a lesson sketching software that permits users to develop or contribute to a classroom episode using cartoon characters (Herbst et al., in press). Such lesson sketching software is the first step of what a virtual setting for teacher education could look like; Chieu and Herbst (in review) offer a longer term prospect with their design of a teaching simulator. An authoring tool, which is part of ThEMaT Online, allows teacher developers to create lessons and assessments by juxtaposing different functionalities like those in a sequence of activities. We expect the working group will be able to explore these functionalities, and the potential combinations that could be made with them; we also expect the working group to have suggestions of new functionalities to add.

More generally we think that the development of a pedagogy assisted by representations of teaching requires a menu of tools that facilitates the creation and delivery of experiences for clients as well as the collection of their work. It requires more than a course management system because the representations are not atomic objects. In order to pose temporality problems (see Table 1) for example, it is important to be able to find, tag, and communicate specific moments in the timeline of a representation of teaching. In general, we believe that the continuous development of the pedagogical functionalities of the ThEMaT Online system can be a scaffold in the process of developing a pedagogy of teacher development assisted by representations of teaching.

(c) Plan for active engagement of participants in productive reflection on the issues

The plan includes starting with a brief exposition by the authors of the contents of the paper

followed by a collective discussion framed around the following questions:

- On the *facilitation* of discussions or investigations assisted by representations of teaching: What are the possibilities and demands of a pedagogy assisted by representations? What considerations are needed to make when framing the encounter of an audience with an artifact, depending on who the audience is and the specifics of the artifact? What else can be done beyond the general "watch and we'll talk"? How does the multidimensionality of representations (the fact that they involve representations of mathematics and of students' thinking as much as those of teacher action) feature in the organization of encounters with representations? In particular, does it help to organize an earlier encounter with the mathematics of the representation?
- On *learning technologies* to navigate representations of teaching: What is available and what is needed? Web 2.0 technologies have brought up the possibility for users to do more than read or view media artifacts—they can record, share, tag, comment, rate, index, and mix media artifacts. Are those capabilities useful in enabling the work with representations of teaching? What other capabilities are desirable?

We estimate that the first meeting of the working group will be consumed by the exposition and the discussion. The second meeting will be organized around pairs of participants involved in the creation of exemplar sessions for a chosen clientele and around a particular representation of teaching. Participants are invited to bring a representation of teaching they would want to work with, and accompanying notes about facilitation. The organizers will provide some scaffolds in the form of virtual index cards (realized as Power point slides) that contain some of the elements of the taxonomy. People will divide in groups and design a lesson, session, tutorial, or assessment that implements elements of the taxonomy with the representation chosen. The purpose of this task is to push for the development of the taxonomies in a concrete context. Inasmuch as possible these materials will be shared among the members of the group. The third, and final, meeting will provide a forum for subgroups to share these emerging products and the extensions of the taxonomies that these products require. Participants will also discuss opportunities for follow-up activities.

(d) Anticipated follow-up activities

We have been allotted a pre session slot at the AMTE Annual Meeting in 2011. We plan to use that slot to display the exemplars developed at the PMENA meeting and to engage in further work on (1) improving the exemplars and (2) using the exemplars to improve the taxonomies. We hope we will be able to use those products to propose a second iteration of this working group at next year's PMENA.

The characteristics of the working group are such that people may expect collaboration and sharing of materials. We will strive to acknowledge all authors in these materials as well as in the development of the ideas.

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[vi] We use the word "client" to refer to the students of teaching. Among other things, this choice prevents ambiguity with our references to students in the representations of teaching that those clients might be exposed in their teacher development opportunities.

[vii] A song for occupations, 4. In Leaves of Grass.

[viii] ThEMaT Online is a set of representations of teaching and tools to manipulate them. The system is currently being designed for teacher educators to use with their clients. It can be previewed at http://grip.umich.edu/themat.