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Effect of wall conductivity on turbulent channel flow under spanwise magnetic field

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The effect of wall conductivity on turbulence in electrically conducting fluid in the presence of a constant magnetic field is considered. A channel flow with a spanwise magnetic field is analyzed using high-resolution direct numerical simulations performed for the case of low magnetic Reynolds number. It is found that the effect of suppression of wall-normal momentum transfer and reduction of wall friction identified earlier for the flow with perfectly insulating walls is enhanced if the walls are electrically conducting.

1 Introduction

This study continues our previous works [1, 2] of turbulent flow of an incompressible electrically conducting fluid in a channel with a steady uniform magnetic field imposed in the spanwise (parallel to the walls but normal to the mean flow) direction. Pertinently to the analysis of the present paper, it was found that the imposed magnetic field strongly suppresses the turbulent fluctuations in the buffer layer. This leads to significant suppression of wall-normal momentum transfer and to a decrease of wall friction drag. The mean flow profile demonstrates increased centerline velocity, decreased velocity gradient near the wall, and loss of the log-layer behaviour. In the present paper, we extend the investigation by considering the case of perfectly conducting channel walls. The goal is to understand how the wall conductivity affects the turbulence suppression and drag-reduction by the imposed magnetic field.

2 Equations and numerical method

We calculate the flow of an incompressible, electrically conducting fluid in a plane channel between two walls located at $z = \pm L$. The flow is driven by a streamwise pressure gradient, which is adjusted to keep constant mean flow velocity U_q. The magnetic field is constant and in spanwise direction: $\boldsymbol{B} = B\boldsymbol{e}_y$.

The dimensionless governing equations and the boundary conditions are:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\boldsymbol{u} + \frac{\mathrm{Ha}^2}{\mathrm{Re}}(\boldsymbol{j}\times\boldsymbol{e}_y), \tag{1}$$

$$\boldsymbol{j} = \nabla\phi + \boldsymbol{u}\times\boldsymbol{e}_y, \tag{2}$$

$$\nabla\cdot\boldsymbol{u} = \nabla\cdot\boldsymbol{j} = 0, \tag{3}$$

$$\boldsymbol{u} = 0 \text{ at } z = \pm 1, \tag{4}$$

$$\partial\phi/\partial z = 0 \text{ at } z = \pm 1 \quad \text{perfectly insulating walls}, \tag{5}$$

$$\phi = 0 \text{ at } z = \pm 1 \quad \text{perfectly conducting walls}. \tag{6}$$

The scales used for non-dimensionalization are the mean flow velocity U_q, half-channel width L, and ULB (for ϕ). The non-dimensional parameters are the Reynolds number $\mathrm{Re} \equiv LU_q/\nu$ and the Hartmann number $\mathrm{Ha} \equiv BL(\sigma/\rho\nu)^{1/2}$. Periodic boundary conditions are applied in the streamwise (x) and spanwise (y) directions, the periodicity lengths are $L_x = 2\pi$ and $L_y = \pi$. The numerical method (described in [1]) is pseudo-spectral based on Fourier expansion in the x and y directions and Chebyshev expansion in the wall-normal z-direction. The computations are performed for $Re = 6667$ and $Ha = 30$, the numerical resolution is $N_x \times N_y \times N_z = 256^3$.

3 Results and discussion

The integral characteristics of the computed flows are presented in table 1. We show the centerline mean velocity $U_{cl} = U(0)$, the friction Reynolds number $\mathrm{Re}_\tau \equiv u_\tau L/\nu$ and the friction coefficient $C_f \equiv \tau_w/(1/2U_q^2)$. In these formulas, $U(z) = \langle u_x \rangle$

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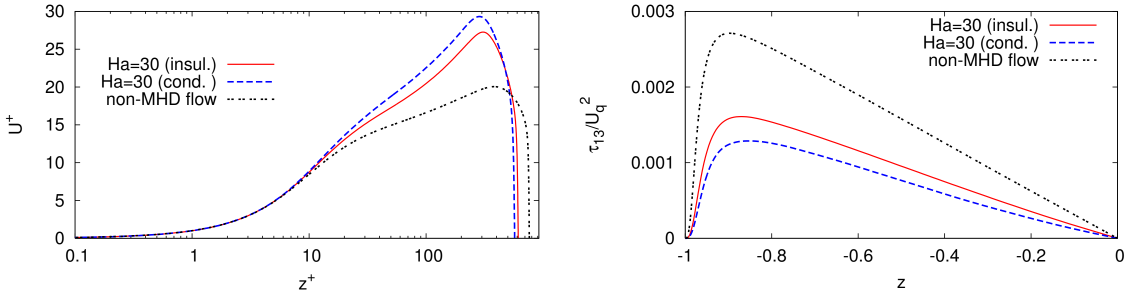

Fig. 1 Effect of the magnetic field and wall conductivity on turbulent flow. Mean flow profiles in wall units (left), horizontally and time-averaged turbulent shear stress τ_{13} (right).

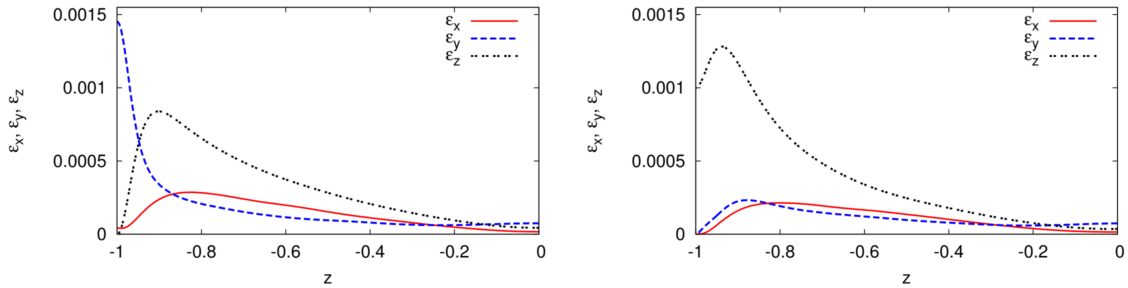

Fig. 2 Components of the time- and horizontally averaged Joule dissipation rate corresponding to different current components in the flows with perfectly insulating (left) and perfectly conducting (right) walls.

stands for the mean velocity obtained by horizontal and time-averaging of the computed fields. The wall friction velocity u_τ is related to the wall stress τ_w by $\rho u_\tau^2 = \tau_w$.

One can see from Table 1 that the magnetic field leads to significant increase of the centerline mean velocity, which is slightly more pronounced in the case of conducting walls. The transformation of the mean velocity profile is illustrated in Fig. 1(left). The profiles are shown in the wall units, i.e. $U/u_\tau = U^+(z^+)$, where $z^+ = zu_\tau/\nu$. Outside the viscous sublayer, the mean velocity profile in the MHD flows is characterized by larger slope. This effect is stronger in the case of conducting walls.

	U_{cl}	Re_τ	$C_f \times 10^3$
Insulating	1.2542	306.69	4.232
Conducting	1.2566	285.75	3.675
Ha=0	1.1469	381.34	6.543

Table 1 Computed integral characteristics. Results for a hydrodynamic flow at the same Reynolds number are included for comparison.

The spanwise magnetic field can only affect the mean flow via the suppression of turbulent fluctuations. This effect is illustrated in Fig. 1(right) for the wall normal turbulent stress $\tau_{13} = \langle u'w' \rangle$. We can also see that the reduction is stronger in case of conducting walls. Our further analysis is, therefore, focused on the mechanism of the suppression, i.e. on the Joule dissipation rate $\epsilon \equiv \frac{\mathrm{Ha}^2}{\mathrm{Re}} \boldsymbol{j}^2 = \epsilon_x + \epsilon_y + \epsilon_z$, where ϵ_x, ϵ_y, and ϵ_z are the components of the dissipation associated with the x, y, and z components of the electric current $\boldsymbol{j}$.

As shown in Fig. 2, the dissipation ϵ_x is larger in the flow with insulating walls and, particularly, in the layer immediately adjacent to the wall. In the flow with conducting walls, the dominating component of the dissipation rate is the component ϵ_z due to the non-zero wall-normal current and it is much larger than in the flow with insulating walls. The wall-normal component j_z creates the streamwise component of the Lorentz force $F_x = j_z \times B_y$. A plausible conclusion is that a stronger suppression of turbulence, found for electrically conducting walls, is associated with the streamwise Lorentz force F_x. The stronger force F_x could inhibit the streak formation, which is an essential mechanism to maintain the generation of turbulent eddies. A more detailed analysis of this near wall dynamics is the subject of ongoing work.

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References

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