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Firms Should Adjust Their Strategies Due to Network Effects

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Complementary Online Services in Competitive Markets: How Firms Should Adjust Their Strategies Due to Network Effects

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Abstract

A growing number of firms are strategically utilizing IT and the Internet to provide online services to consumers who buy their products. Online services differ from traditional services, such as maintenance services, because they often promote interactivity among the firm's customers and exhibit positive network effects. In this paper, we model the competition between two firms that sell a differentiated product, when each firm can offer a complementary online service to its customers. We examine how the market equilibrium changes when the service exhibits network effects, and determine how firms should adjust their strategies to account for such effects. Specifically, we find that when the service exhibits network effects, a firm's decision whether to offer the service depends on the competitor's decision as well as on the competitor's service quality. When the service does not exhibit network effects, this is not the case. In addition, if the service exhibits network effects the two firms may be caught in a Prisoner's Dilemma; a situation that does not arise in the absence of network effect. We also show that technological progress that enhances the value of the network to consumers can reduce firms' profits when both offer the service. An increase in the size of the market can also have a negative effect on profits when the service exhibits network effects, but not otherwise. Finally, unlike previous works on products with network effects, showing under-provision in the context of a monopoly, we show that both under- and over-provision of services can arise in a competitive setting.

Keywords: Online Services, Network Effects, e-Commerce, Analytical Modeling

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1. Introduction

In various industries, ranging from traditional manufacturing to high-tech, the locus of competition has shifted from selling products to providing value-adding services. As many products are commoditized, firms have been focusing on developing complementary smart services that offer greater value to customers who buy their products. Such services can increase the firm's revenues and allow it to achieve a competitive advantage (Allmendinger and Lombreglia 2005, Reinartz and Ulaga 2008).

Recent advances in information and communication technologies, such as Web 2.0 and "Social Technologies" (Li and Bernoff 2008), enable firms to provide new types of value-adding services (such as blogs, peer-to-peer file sharing, social networking sites, and online gaming platforms), to their customers. These online services differ from traditional services (e.g., maintenance, repair, and training services) because most of them promote relationship building and interactivity among the users, and thus exhibit positive network effects. That is, the value of the service to a user increases with the number of other consumers that subscribe to the service.

One notable type of an online services that can add value to consumers that buy a firm's product, and exhibits positive network effects, is online communities. Dell operates Dell Community (www.dell.com/community), which offers valuable complementary services to Dell's PC users. In this online community, Dell customers can share information and knowledge that they have acquired while using Dell products. Oracle and IBM also operate online communities for users of their products (e.g., www.ibm.com/developerworks/). Microsoft operates an online

community, Zune Social (<http://social.zune.net/>), where it's Zune MP3 player users can share music they purchase.

Another type of online services that exhibit network effects are online gaming platforms which enable consumers who purchase a packaged computer game to play it online with remote human players. For example, a consumer who buys *Starcraft* or *World of Warcraft* from Blizzard (a leader in the computer games industry), can play the game offline in the *single-player* mode. Playing offline, the consumer plays pre-defined scenarios against hypothetical competitors with artificial intelligence. However, if the consumer connects to Blizzard's online gaming platform, *Battle.net*, he can also play the game in the *multi-player* mode, playing in real-time against other remote human players. The set of pre-programmed game strategies included in the game package is narrower than the set of strategies that can be used by human players. Therefore, many Blizzard customers are willing to pay a premium when buying the game package, knowing they can also access the online service. While some online games can be accessed and used without owning a game package, other games, such as *Starcraft* and *World of Warcraft*, do need client software (the product) installed on the user's computer. Such games have sophisticated and complex graphics and features, and thus a simple Web browser is not enough to access them; Players need proprietary "fat client programs" installed on their computers.

In this paper, we consider an industry in which competing firms sell a differentiated product and each firm can choose to offer a complementary online service to consumers that buy its product. We examine how the fact that the online service exhibits positive network effects may change the competitive outcome, and whether sellers and society always benefit from stronger positive network effects. To do so, we compare the competitive outcome when the service does not exhibit network effects (e.g., a maintenance service, a periodic newsletter, etc.)

with the competitive outcome when the service does exhibit positive network effects (e.g., an online forum, an online game platform, etc.). Specifically, we address the following research questions: 1) Under what conditions should each firm offer the complementary service to its customers? 2) Should firms adjust their strategies when the service exhibits network effects? 3) Do firms and society benefit from an increase in the degree of network effects?

Our investigation generates several interesting results. First, we find that when the service exhibits network effects, a firm's decision whether to offer the service may depend on the competitor's decision, as well as on the competitor's service quality. In contrast, when the service does not exhibit network effects, each firm bases the decision whether to offer the service only on its own service quality. In addition, we show that when the service exhibits network effects, in some cases two equilibriums may prevail. In these cases, there is a first mover advantage in the sense that each firm would offer the service if the competitor does not offer it, but would choose not to offer the service when the competitor does offer it. This situation does not happen when the service does not exhibit network effect.

We show that if both firms offer the service in equilibrium, and the firms' service qualities are similar, then the two firms are caught in a Prisoner's Dilemma. Specifically, both firms offer the service, but they would be better off when neither offers the service. Thus, advances in communication technologies, which enable the firms to offer online services with positive network effects, can reduce the industry's overall profitability, while benefiting consumers. We show that this does not happen in the absence of network effects. We also identify the existence of a Bertrand supertrap (Cabral and Villas-Boas, 2005), in which a common exogenous change that seems favorable to sellers (such as an increase in the degree of network effects or in the size of the market) may in fact lower the sellers' profits because it intensifies price competition.

Lastly, we show that while the literature on network effects often argues that a network good is underprovided by a monopoly, in the duopoly setting both under- and over-provision of the service may prevail. Specifically, whether the service exhibits network effects or not, price competition may lead the firms to excessively offer the service compared to the social optimum.

The paper structure is as follows. In Section 2, we review the related literature on products with network effects. In Section 3, we present our model. In Section 4, we derive the market equilibrium, examine how the firms' strategies change when the service exhibits network effects, and discuss social welfare implications. In Section 5, we examine how network effects and the size of the market influence the firms' profitability. In Sections 6 and 7, we discuss two possible extensions, and we conclude in Section 8.

2. Literature Review

Our study is related mainly to the literature on network effects. Network effects arise when the utility that a user derives from a product increases with the number of other consumers that use the same or compatible product (Katz and Shapiro, 1985). Therefore, a customer's utility from a product that displays network effects is usually modeled as a function of the product's inherent value and of the number of customers using the product (Ellison and Fudenberg, 2000). In addition, most models consider the network effects to be linear in the size of the user-base (Katz and Shapiro 1986, Fudenberg and Tirole 2000, Jing 2007). In this paper, we adopt a similar modeling approach and model the value a consumer obtains from the service as an additive function of the value derived from the inherent functionalities of the service and the value derived from the network of service users.

In the classic models of products with network effects, the value a consumer derives from the network is independent of the consumer type. Specifically, consumers have homogenous

network valuations though they may have heterogeneous product valuations (Cabral et al. 1999, Fudenberg and Tirole 2000, Jing 2007). In some recent models, however, a multiplicative function is used in which the consumer type determines both his product valuation and the benefit he obtains from the network (e.g., Ellison and Fudenberg 2000, Sundararajan 2003, Sundararajan 2004). In such models, a consumer with high (low) product valuation also has a high (low) marginal valuation for the network of product users. In the base model presented in this paper, we assume consumers have homogenous service valuations. However, we discuss the implications of heterogeneous service valuations in Section 7.

A large number of studies on network goods examine product compatibility and standardization (e.g., Farrell and Saloner 1986, Katz and Shapiro 1985), technology adoption (e.g., Choi 1994, Katz and Shapiro 1986) and entry deterrence (Cabral et al. 1999, Fudenberg and Tirole 2000). The present paper contributes to the literature on network effects as it examines a duopoly in which firms can offer a service that exhibits network effects to consumers that buy their product. The current paper focuses on how the presence of network effects changes the resulting market equilibrium and the profitability of the firms.

The concept of Fulfilled Expectation Equilibrium is often used in deriving the firm's optimal pricing strategy when the product exhibits network effects. In the presence of network effects, consumers purchasing decision is based on the expected network size, and under the Fulfilled Expectation Equilibrium requirement the realized demand indeed equals to the expected network size (Katz and Shapiro 1985, Palma et al. 1999, Sundararajan 2004).

Under-provision of a product with network effects has been shown in the literature. For example, Katz and Shapiro (1994) state that in the presence of network externalities, social marginal benefits from an increase of one unit in network size exceed private (i.e., the firm's)

marginal benefits, and thus the equilibrium network size is smaller than the socially optimal network size. Sundararajan (2004) derives the monopolist's optimal nonlinear pricing strategy when consumers have heterogeneous network valuations and may purchase variable quantities of the good. He shows that the product might be under-supplied relative to the socially optimal level. In this paper, we show that in a competitive setting, a service with network effects might be over-provided by firms.

Finally, our work is also related to Matutes and Regibeau (1988, 1992), who consider a duopoly in which each firm sells two components, and a consumer has to purchase both components to form a system; either component in isolation does not provide value. In Matutes and Regibeau (1988), a seller may choose to make its components compatible with the competitor's, so that a consumer is able to purchase the two components from different sellers. They analyze this compatibility decision and find that making the components compatible with the competitor's may increase the equilibrium price and profit. Matutes and Regibeau (1992) analyze a mixed-bundling strategy and find that the two vendors may end up in a Prisoner's Dilemma situation. Specifically, even though a pure-component strategy generates greater profit, the firms may choose the mixed-bundling strategy in equilibrium. They also find that the bundles may be provided more than the social optimum. Our paper differs in three major ways. First, in our paper, the two components, namely product and service, are not symmetric. That is, the product can be valuable in the absence of the service, but not the opposite. For that reason, a firm might choose to sell only the product. Second, we consider the case in which the online service is valuable only when buying the product from the firm that provides the service. Third, and most importantly, the aforementioned models do not consider network effects, while our paper focuses on how network effects change the market equilibrium and the firms' profitability.

3. The Model

We consider a market with two competing firms, Firm A and Firm B, selling a differentiated durable product. There are M consumers in the market who are heterogeneous in terms of their product preferences, and each consumer is interested in purchasing at most one unit of the product. We assume that consumers' product preferences are uniformly distributed along a unit line (Hotelling 1929), with Firm A's product located at 0 and Firm B's product located at 1.

When a consumer buys a product that differs from his ideal product, he incurs a misfit cost which is increasing in the distance between his ideal product and the product he buys. Thus, a consumer located at point x on the unit line obtains utility of $V - tx$ when purchasing the product from Firm A, and utility of $V - t(1 - x)$ when purchasing the product from Firm B, where V is the maximum utility from the product sold by either firm, and t is the per-unit disutility cost from the misfit between the consumer's ideal product and the product he considers to purchase. Without loss of generality, we also assume that the product's unit production cost is the same for both firms, and it is normalized to 0 (in case there is a positive product production cost, the model can still apply if consumers valuations of the product are taken net of the marginal cost).

Each of the two firms has the ability to offer a complementary online service to its customers, and needs to choose if to do so. The inherent value of the service (that is, the value that does not depend on how many consumers are using the service) offered by Firm i is γs_i , and the two firms may differ in the quality of the service that they offer (i.e., s_A may be higher or lower than s_B). Thus, the services can be vertically differentiated due to the difference in their quality, but, as seen later, also due to the difference in the number of service users. The parameter γ represents the increase in the consumer's utility caused by a marginal improvement

in the intrinsic functionalities of the service. Here, we assume that s_A and s_B are given and are common knowledge, and model the firms' decisions whether to offer the service assuming any development costs are sunk. Later, in Section 6, we discuss the case in which developing the service is costly and each firm determines the quality of the inherent functionalities of its service.

If the service exhibits positive network effects then, in addition to the inherent value described above, a consumer obtains utility of αN_i when using the service offered by Firm i . Here, N_i is the number of consumers using the service offered by Firm i , and α is the marginal network benefit obtained when an additional consumer uses the service. Thus, the parameter α represents the strength of the network effects. Specifically, when $\alpha = 0$ the service does not exhibit network effects, and as α increases the service exhibits stronger network effects. As in many papers on products with network effects (e.g., Katz and Shapiro 1985), we assume consumers have homogenous marginal valuation for the network. That is, all consumers have the same α .

We consider the case in which a consumer obtains positive utility from using the service offered by Firm i only when he purchases the product sold by Firm i . That is, the service offered by Firm i is not beneficial to consumers who buy a product from Firm j or to those who do not buy a product at all. Though we acknowledge that this assumption might not hold for all types of product-related online services, our paper focuses on services for which it does. Future research can consider services that can be valuable to consumers who do not buy the firm's product. To summarize, the added utility a Firm i 's customer (i.e., a consumer who purchase the product sold by Firm i) obtains from using the service offered by Firm i is given by:

$$\gamma s_i + \alpha N_i \tag{1}$$

The focus of our paper is to examine how the fact that many types of online services

exhibit positive network effects changes the competitive outcome, and whether sellers benefit from stronger network effects. We thus compare the competitive outcome when firms offer a service that does not exhibit network effects (e.g., a maintenance service, a periodic newsletter etc.) with the competitive outcome when the service considered does exhibit positive network effects (e.g., an online forum, an online game platform, etc.).

Finally, as the number of service users increases, the firm needs to invest in upgrading its hardware and network infrastructure. Thus the cost of offering the service increases with the number of service users. We assume that the marginal cost of offering the service, c , is the same for both firms. This assumption is reasonable due to the fact that hardware, bandwidth, processors, communication technology etc. are commodities and available for all firms for the same cost. Table 1 summarizes the notation used in the paper.

M	The number of consumers
V	The inherent value of the product
t	Misfit cost per unit distance between product purchased and one desired
γ	Marginal valuation of intrinsic functionalities of the service
α	The degree of network effects
s_i	The intrinsic value of the service of Firm i ($i = A$ or B)
c	The marginal cost for providing the service
p_k^i	The price of the product sold by Firm i when the firms choices regarding service provision are given by k , where $k \in \{NN, SN, NS, SS\}$
D_k^i	The demand for Firm i product when choices regarding service provision are given by k .
π_k^i	The profit of Firm i when choices regarding service provision are k .

Table 1. Notation

The timeline of the game is as follows. First, the firms, which know their service qualities (s_A and s_B), simultaneously choose whether to offer the service to their customers or not. Four market configurations are possible as a result of this first stage of the game. In the first configuration, labeled Case NN , both firms sell only the product. In the second configuration,

Case *SN*, and the third, Case *NS*, only Firm A or only Firm B, respectively, offer the service while the other firm sells only the product. Finally, in the fourth configuration, Case *SS*, both firms offer the service to consumers who buy their product.

Next, after observing the choices made in the first stage of the game (firms make their service offerings public), the two firms simultaneously set their prices. Notice that due to the fact that in our model all consumers have the same valuation for the service offered by Firm *i*, given by $\gamma s_i + \alpha N_i$, if Firm *i* were to sell the product for a price p_i and the service for a fee f_i , then in equilibrium either all the consumers that buy its product would also buy its service (this happens if the firm sets f_i such that $\gamma s_i + \alpha N_i \geq f_i$), or none of them would (when $\gamma s_i + \alpha N_i < f_i$). Thus, selling the product for a price p_i and the service for a fee, f_i , such that $\gamma s_i + \alpha N_i > f_i$, yields the same profit as selling a bundle of product and service for a single price of $p_i + f_i$, or, alternatively, selling the product for a price $p_i + f_i$ while offering the service “free of charge” to consumers who buy the product. These three strategies lead to the same demand and to the same profit. This would not be true if consumers had heterogeneous service valuations (an extension we discuss in Section 7). Since the latter strategy (offering the service free of charge to consumers who buy the product) is the one most commonly observed in reality (the reader is referred to the examples in the Introduction), in our model each firm sets its product price and offers the service free of charge to consumers who buy its product. Clearly, the price of the product depends on whether the service is offered. Finally, in the third stage of the game, consumers observe the prices set by the firms and the service offerings, and choose whether to buy a product and from which firm. Figure 1 describes the timeline of the game.

The consumer’s surplus functions and the firms’ profits, as function of prices, for the four possible outcomes of the first stage of the game are given in Table 2.

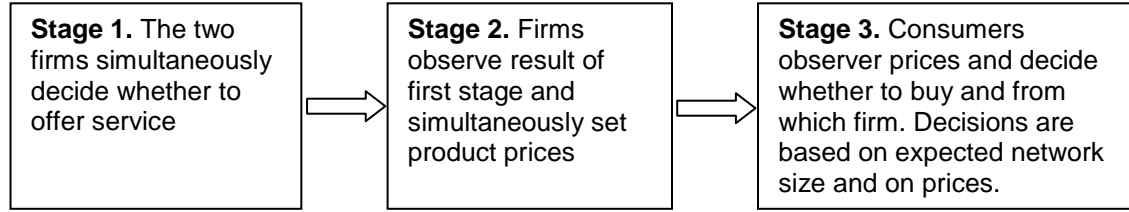


Figure 1. The timeline of the game

Firms Choices in 1 st Stage	Customer's Surplus From Firm A	Customer's Surplus From Firm B	Firm's Profit
Case NN. Both sell only the product	$u_{NN}^A = V - tx - p_{NN}^A$	$u_{NN}^B = V - t(1 - x) - p_{NN}^B$	$\pi_{NN}^i = p_{NN}^i D_{NN}^i$
Case SS. Both offer the service	$u_{SS}^A = V - tx + \gamma s_A + \alpha D_{SS}^A - p_{SS}^A$	$u_{SS}^B = V - t(1 - x) + \gamma s_B + \alpha D_{SS}^B - p_{SS}^B$	$\pi_{SS}^i = p_{SS}^i (D_{SS}^i - c)$
Case SN. Only Firm A offers a service	$u_{SN}^A = V - tx + \gamma s_A + \alpha D_{SN}^A - p_{SN}^A$	$u_{SN}^B = V - t(1 - x) - p_{SN}^B$	$\pi_{SN}^A = p_{SN}^A (D_{SN}^A - c)$ $\pi_{SN}^B = p_{SN}^B D_{SN}^B$

Table 2. The customer's surplus and the firms' profits as functions of prices and demands. Here, u_k^i refers to the surplus a consumer derives when buying from Firm i under market configuration k . The demand, D_k^i , is always a function of prices. (Case NS is symmetric to Case SN)

It is important to note that in this paper we chose to consider only parameters values such that, for each of the four possible outcomes of the first stage of the game, in equilibrium: *i*) each firm has positive demand for its product, and *ii*) firms face spatial competition in the product market. We believe that such cases are of the highest interest as they represent real competition between the two firms. When only one firm has positive demand for the product, the other firm is in fact inactive. When the product market is not covered, each firm behaves as a local monopoly. The specific conditions on the parameters values for the above two requirements to hold are derived in Appendix 1, and are summarized in Assumption 1 in Appendix 2.

In order to determine which market configuration (NN, SS, NS, or SN) prevails in equilibrium, we first find the equilibrium prices and profits for each such possible configuration. Specifically, using the consumer surplus functions given in Table 2, we derived the demand and the profit as functions of the prices (i.e., the result of the third stage of the game for a given

market configuration and a given pair of prices). For the cases in which at least one firm offers a service (i.e., Cases *SN*, *NS* and *SS*), we used the concept of Fulfilled Expectation Equilibrium (Katz and Shapiro 1985, Sundararajan 2003), in which the number of service users in equilibrium equals the expected number of service users which is used by consumers to make a purchase decision. After deriving the firms' profits as functions of prices, we find the Nash equilibrium prices for the second stage of the game for each of the four configurations. Tables 3 and 4 present these equilibrium prices and profits, respectively. The detailed derivations of the equilibriums, in the manner described above, are provided in Appendix 1.

Firm A \ Firm B	Only Product	Product + Service
Only Product	$p_{NN}^A = t$ $p_{NN}^B = t$	$p_{NS}^A = \frac{3t + c - 2\alpha M - \gamma s_B}{3}$ $p_{NS}^B = \frac{3t + 2c - \alpha M + \gamma s_B}{3}$
Product + Service	$p_{SN}^A = \frac{3t + 2c - \alpha M + \gamma s_A}{3}$ $p_{SN}^B = \frac{3t + c - 2\alpha M - \gamma s_A}{3}$	$p_{SS}^A = t + c - \alpha M + \frac{\gamma(s_A - s_B)}{3}$ $p_{SS}^B = t + c - \alpha M + \frac{\gamma(s_B - s_A)}{3}$

Table 3. The equilibrium prices for the four possible market configurations

Firm A \ Firm B	Only Product	Product + Service
Only Product	$\pi_{NN}^A = 0.5Mt$ $\pi_{NN}^B = 0.5Mt$	$\pi_{NS}^A = \frac{M(3t + c - 2\alpha M - \gamma s_B)^2}{9(2t - \alpha N)}$ $\pi_{NS}^B = \frac{M(3t - c - \alpha M + \gamma s_B)^2}{9(2t - \alpha N)}$
Product + Service	$\pi_{SN}^A = \frac{M(3t - c - \alpha M + \gamma s_A)^2}{9(2t - \alpha M)}$ $\pi_{SN}^B = \frac{M(3t + c - 2\alpha M - \gamma s_A)^2}{9(2t - \alpha M)}$	$\pi_{SS}^A = \frac{M(3(t - \alpha M) + \gamma(s_A - s_B))^2}{18(t - \alpha M)}$ $\pi_{SS}^B = \frac{M(3(t - \alpha M) + \gamma(s_B - s_A))^2}{18(t - \alpha M)}$

Table 4. The equilibrium profits for the four possible market configurations

In the next section we determine which market configuration prevails in equilibrium.

4. The Market Equilibrium

In this section, we derive the market equilibrium and compare the results when the service exhibits network effects and when it does not exhibit network effects, in order to determine whether firms should adjust their strategies when offering online services.

From the payoff matrix given in Table 4, we derive the conditions for the different possible pure-strategy Nash equilibriums, as specified in Proposition 1

Proposition 1. (Market Equilibrium)

i) Both firms offer the service. Case SS is an equilibrium iff (i.e., if and only if)

$$s_A > s_B X + Y \text{ and } s_B > s_A X + Y.$$

ii) Both firms offer only product. Case NN is an equilibrium iff $s_A < \bar{s}$ and $s_B < \bar{s}$.

iii) Only firm A offers a service. Case SN is an equilibrium iff $s_A > \bar{s}$ and $s_B < s_A X + Y$.

iv) Only firm B offers a service. Case NS is an equilibrium iff $s_B > \bar{s}$ and $s_A < s_B X + Y$.

where $X = 1 - \frac{\sqrt{2t-2\alpha M}}{\sqrt{2t-\alpha M}}$, $Y = \frac{(3t+c-2\alpha M)\sqrt{2t-2\alpha M}}{\gamma\sqrt{2t-\alpha M}} - \frac{3(t-\alpha M)}{\gamma}$, and $\bar{s} = \frac{2\alpha M+2c-6t+3\sqrt{2t(2t-\alpha M)}}{2\gamma}$.

Proofs of all propositions are given in Appendix 2. We note that X and \bar{s} are always positive when parameters values are such that both firms have positive demand.

Figure 2 exhibits the resulting market equilibrium in the s_A - s_B space. The lines d - e - f and g - e - h in Figure 2, indicate $s_A = s_B X + Y$ and $s_B = s_A X + Y$, respectively. In the region northeast of f - e - h , both $s_A > s_B X + Y$ and $s_B > s_A X + Y$ (i.e., Proposition 1-(i) holds). In this region, the intrinsic quality of both online services (s_A and s_B) is sufficiently high that both firms operate the service in equilibrium. On the other hand, in the region 0 - a - b - c , both s_A and s_B are less than \bar{s} (Proposition 1-(ii)), and thus neither Firm A nor Firm B offer the service. Southeast of the lines c - b - g - e - h , the conditions from Proposition 1-(iii) hold. In this area, s_A is relatively high (i.e., higher than \bar{s}), and s_B is either lower than s_A or not much higher than it (i.e., $s_B < s_A X + Y$), and

therefore there is an equilibrium in which only Firm A offers the service. Similarly, northwest of the lines $a-b-d-e-f$, there is an equilibrium in which only Firm B offers the service.

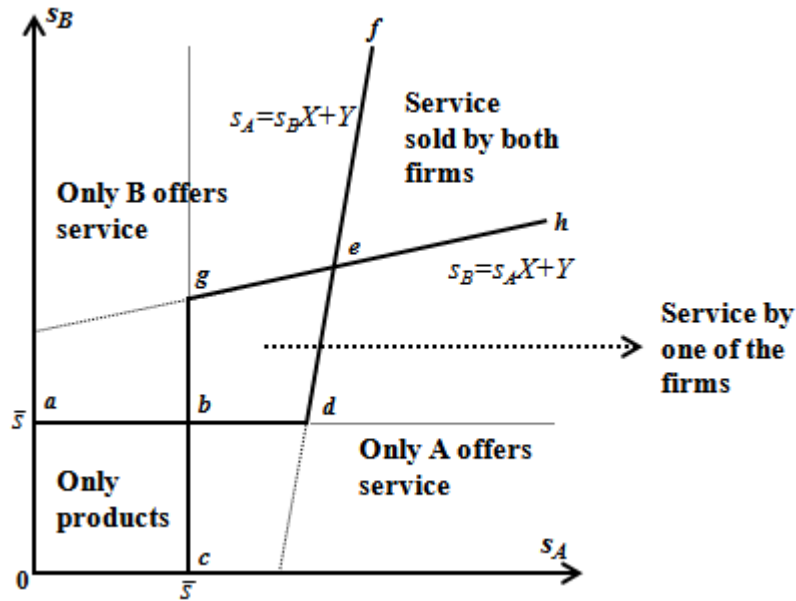


Figure 2. The market equilibrium in the s_A - s_B space

Interestingly, in the region $b-d-e-g$, two equilibriums are feasible: an equilibrium in which only Firm A offers the service and an equilibrium in which only Firm B offers the service. In this range of parameter values, each of the firms does not have an incentive to offer the service when the competitor offers it; however, each firm finds it optimal to offer the service when it is the only one doing so. In other words, when the service exhibits positive network effects, for a subset of the parameters values (in the region $b-g-e-d$) there is a “first mover advantage”, and thus two equilibriums are feasible. Notice, that there can be an equilibrium in which a firm with a lower quality of service offers the service, while the firm with the higher quality does not. Finally, we note that when there are positive network effects, there is always a none-empty range of s_A - s_B values in which two equilibriums are feasible. That is the value of s_B at point g (and value of s_A at point d), given by $\bar{s}X + Y$, is always larger than \bar{s} (this is shown in the end of the proof of Proposition 1).

Examining Proposition 1, we see that as the degree of network effect, α , increases, X always increases. In addition, as long as $14t > 16\alpha N$ (a condition which is often satisfied given Assumption 1), \bar{s} increases, and, finally, Y can either increase or decrease. Thus, in many cases, as the degree of network effects increases, the range of parameters values for which firms do not offer the service increases, and the range of parameters values for which both firms offer the service decreases. This result may seem surprising, but as we show later in Section 5, when both firms offer the service their profits decrease as the degree of network effects increases. This reduction in profit is due to the intensification of price competition. Thus, an increase in the degree of network effects may reduce the firms' incentive to offer the service to begin with.

Next, substituting $\alpha = 0$ (no network effects) in the expressions for X , Y , and \bar{s} from Proposition 1, we get: $X = 0$, $Y = c/\gamma$ and $\bar{s} = c/\gamma$. Corollary 1 describes which equilibrium prevails if the service considered does not exhibit network effects.

Corollary 1. *When service does not exhibit network effects, in equilibrium Firm i offers the service if $s_i > c/\gamma$ and does not offer it otherwise.*

From Corollary 1, we learn that if the service does not exhibit network effects, then a firm's decision whether to offer the service does not depend on whether the competitor is offering a service, or on the quality of the competitor's service. It depends only on whether the value a consumer derives from that firm's service (γs_i) exceeds the marginal service provision cost (c). For that reason, when the service does not exhibit network effects, we do not have parameters values for which two equilibriums may prevail, and there is no first mover advantage. In addition, when the service does not exhibit network effects, whenever $s_A = s_B$, either both firms offer the service or neither does. However, when the service exhibits network effects, if $\bar{s} < s_A = s_B < Y/(1 - X)$, in equilibrium only one firm offers the service, even though the firms

are symmetric.

The rationale for the differences in results is as follows. When the service exhibits network effects, the value a consumer derives from the service of Firm i depends not only on Firm i 's inherent service quality (s_i) but also on the number of service subscribers. Thus, the utility a consumer would obtain from the service of Firm i depends on whether the competing firm, Firm j , offers a service or not. Specifically, when the competing firm is offering the service, Firm i 's network of service users would be smaller than when the competing firm does not offer the service. In addition, as the competing firm's service quality increases, the resulting network of Firm i would be smaller. Thus, whether the competitor offers a service, and its service quality, affect the value a consumer would derive from Firm i 's service, and thus may affect Firm i 's decision whether to offer the service. This is not true when the service does not exhibit network effects.

Tables 5 and 6 provide two numerical examples. The profits in these tables are the equilibrium profits given the firms choices in the first stage of the game. For the parameters values presented in Table 5, both firms provide the service in equilibrium, while for the parameters values in Table 6, two equilibriums exist, and only one firm provides the service.

<table border="1"> <tr> <td>Firm A \ Firm B</td> <td>Only Product</td> <td>Product + Service</td> </tr> <tr> <td>Only Product</td> <td>500, 500</td> <td>145, <u>645</u></td> </tr> <tr> <td>Product+ Service</td> <td><u>788</u>, 88</td> <td><u>312</u>, <u>112</u></td> </tr> </table>		Firm A \ Firm B	Only Product	Product + Service	Only Product	500, 500	145, <u>645</u>	Product+ Service	<u>788</u> , 88	<u>312</u>, <u>112</u>	$M = 200, v = 50$ $t = 5, c = 3$ $\alpha = \gamma = 0.015$ $s_A = 450, s_B = 350$
		Firm A \ Firm B	Only Product	Product + Service							
Only Product	500, 500	145, <u>645</u>									
Product+ Service	<u>788</u> , 88	<u>312</u>, <u>112</u>									

Table 5. A numerical example of Prisoner's Dilemma

<table border="1"> <tr> <td>Firm A \ Firm B</td> <td>No Service</td> <td>Service</td> </tr> <tr> <td>Only Product</td> <td>500, 500</td> <td><u>125</u>, <u>551</u></td> </tr> <tr> <td>Product + Service</td> <td><u>516</u>, <u>142</u></td> <td>75, 128</td> </tr> </table>		Firm A \ Firm B	No Service	Service	Only Product	500, 500	<u>125</u>, <u>551</u>	Product + Service	<u>516</u>, <u>142</u>	75, 128	$M = 200, v = 50$ $t = 5, c = 3$ $\alpha = \gamma = 0.02$ $s_A = 190, s_B = 210$
		Firm A \ Firm B	No Service	Service							
Only Product	500, 500	<u>125</u>, <u>551</u>									
Product + Service	<u>516</u>, <u>142</u>	75, 128									

Table 6. A numerical example for two equilibriums

Interestingly, the example in Table 5 also illustrates a situation of Prisoner's Dilemma. The two firms' profits in equilibrium ($\pi_{SS}^A = 312$, $\pi_{SS}^B = 112$) are less than the profits when neither firm offers the service (500). Due to the high service quality of the two firms ($s_A > s_B X + Y$ and $s_B > s_A X + Y$), offering the service is a dominant strategy for both. That is, offering the service maximizes a firm's profit regardless of the strategy chosen by the other firm. However, when both firms offer the service, they are both worse off compared to when neither does. Proposition 2 specifies when a Prisoner Dilemma arises.

Proposition 2. *(Prisoner's Dilemma) both firms offer the service in equilibrium, even though both would enjoy higher profits when both do not operate the service, if and only if: $s_A > s_B X + Y$, $s_B > s_A X + Y$, and*

$$|s_A - s_B| < \frac{3(\alpha M - t + \sqrt{t(t - \alpha M)})}{\gamma} \quad (2)$$

According to Proposition 2, if both firms offer the service in equilibrium, then as long as neither firm has a significant quality advantage, both firms would be better off when neither offers the service. If one firm has a significant quality advantage over the other firm (Inequality 2 is not satisfied), then the firm with the higher quality is better off when both firms offer the service than when neither does, while the other firm is worse off. It is important to note that the right-hand-side of Inequality 2 is always positive when parameters values are such that both firms offer the service in equilibrium with positive product demand (see proof of Proposition 2).

Corollary 2 states that a Prisoner's Dilemma does not take place when the service does not exhibit network effects.

Corollary 2. *When the service does not exhibit network effects, a Prisoner's Dilemma does not take place. At least one of the firms is better off when both offer the service compared to when both sell only product.*

When there are no network effects, i.e., when $\alpha = 0$, the right hand side of Inequality (2) becomes 0, and thus the inequality is never satisfied. In addition, if $\alpha = 0$ then $\pi_{SS}^i - \pi_{NN}^i = \frac{\gamma M (s_i - s_j)(6t + \gamma(s_i - s_j))}{18t}$ and the latter is positive when $s_i > s_j$. Thus, at least the firm that has the higher service quality is better off when both offer the service compared to when neither does. In particular, in some cases, both firms are better off.

The managerial implications of Propositions 1 and 2 are significant. In the past, when firms considered offering services to consumers who buy their products, and the services did not exhibit network effects, the decision whether to offer the service depended solely on the capabilities of the firm; specifically on the quality of the service it can provide and on the marginal provision cost. Nowadays, however, when considering to offer an online service with network effects, firms have to also take into consideration whether the competitor is offering a similar service or not, and the quality of the competitor's service. In addition, competing firms might want to coordinate their service offering decisions, because not doing so might reduce both firms' profits. Such coordination was not necessary (or possible) when the service considered did not exhibit network effects, because at least the firm with the higher service quality would be better off when both firms offer the service.

Next, we examine social welfare implications. Proposition 3 lists the conditions under which the service is under-provided or over-provided in the market

Proposition 3.

- i) *The service is under-provided if $F^A(s_A, s_B) > 0$ and $F^B(s_B, s_A) > 0$, and at least one of the following conditions hold: $\frac{2c - N\alpha}{2\gamma} < s_A < s_B X + Y$ or $\frac{2c - N\alpha}{2\gamma} < s_B < s_A X + Y$.*
- ii) *The service is over-provided if $s_A > s_B X + Y$, $s_B > s_A X + Y$ and at least one of the following conditions hold: $F^A(s_A, s_B) < 0$, or $F^B(s_B, s_A) < 0$.*

where

$$\begin{aligned}
F^i(s_i, s_j) = & SW_{SS} - SW_i = \\
& \frac{\alpha M^2(12t^2 - 15\alpha Mt + 4\alpha^2 M^2)}{36(2t - \alpha M)^2(t - \alpha M)^2} \gamma^2 s_i^2 + \frac{\gamma M s_i}{18} \left(\frac{\gamma(4\alpha M - 5t)}{(t - \alpha M)^2} s_j + \frac{20ct - 8\alpha M(2t + c) + 7\alpha^2 M^2}{(2t - \alpha M)^2} \right) + \\
& \frac{M\gamma^2(5t - 4\alpha M)}{36(t - \alpha M)^2} s_j^2 + \frac{\gamma M s_j}{2} + \frac{M(\alpha M - 2c)(36t^2 + 10ct + \alpha M(14\alpha M - 47t - 4c))}{36(2t - \alpha M)^2}
\end{aligned} \tag{3}$$

SW_{SS} is social welfare when both firms offer service, and SW_i is social welfare when only Firm i offers the service.

Figure 3 exhibits the results from Proposition 2 when $2c < N\alpha$. Specifically it shows which strategy maximizes social welfare, and when the service is under or over provided. If $2c > \alpha N$, the only change to Figure 3 would be the addition of a rectangular range of s_A - s_B values such that $s_A < \frac{2c - N\alpha}{2\gamma}$ and $s_B < \frac{2c - N\alpha}{2\gamma}$, where $0 < \frac{2c - N\alpha}{2\gamma} < \bar{s}$ (see Proof of Proposition 3), in which it is socially optimal that neither firm offer the service, and in equilibrium indeed neither offer it.

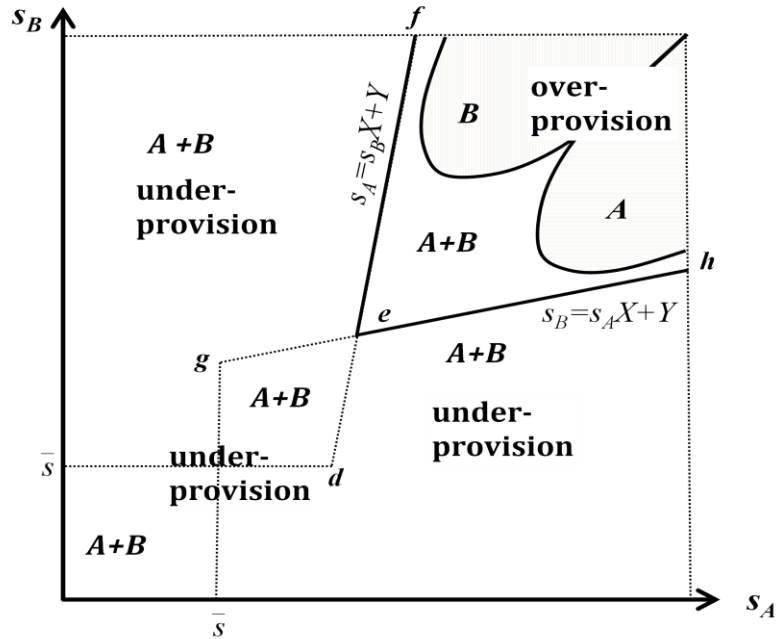


Figure 3. Firms' strategies that maximize social welfare in the s_A - s_B space, when $2c < \alpha N$. In the range labeled A (B) social welfare is maximized when only Firm A (B) offers service. In the ranges labeled A+B: social welfare is maximizes when both firms offer the service.

In Region A in Figure 3, $F^A(s_A, s_B) < 0$, and social welfare is maximized when only Firm A offers the service; in Region B in Figure 3, $F^B(s_B, s_A) < 0$, and social welfare is maximized when only Firm B offers the service; and in all the regions labeled A+B in Figure 3, social welfare is maximized when both firms offer the service. Thus, in Regions A and B we observe over-provision of the service. This seems surprising, given the high values of s_A and s_B in these regions. In these two regions, both firms offer the service in equilibrium (see Figure 2), but price competition reduces the firms' profits compared to the case in which only one firm offers the service. Price competition raises consumer surplus, but the reduction in profits outweighs the increase in consumer surplus. On the other hand, when $s_A < s_B X + Y$ or $s_B < s_A X + Y$, under-provision of the service is witnessed; it is socially optimal for both firms to operate the service, but in equilibrium, there is at most one service provider.

Next, we examine what happens when the service does not exhibit network effects. If $s_A > \bar{s} = c/\gamma$ and $s_B > \bar{s} = c/\gamma$, then in equilibrium both firms offer the service; specifically the line $g-h$ converges with the horizontal line of \bar{s} in Figure 3, while the line $d-f$ converges with the vertical \bar{s} (as $Y = \bar{s} = c/\gamma$, and $X=0$). Thus, when $s_A > \bar{s}$ and $s_B > \bar{s}$, we never observe under-provision of the service. The range of parapets values for which we observe under-provision is smaller than when there are no network effects.¹

We conclude that, supporting previous results regarding under-provision of products with network effects in a monopoly setup, social planners should be concerned with under-provision when the service exhibits network effects. However, unlike in a monopoly setup, due to the competitive setting, they should also be aware that online services might be over-provided. This can happen, surprisingly, when the inherent value of the services is high.

¹ When $s_A < \bar{s}$ or $s_B < \bar{s}$, the only inefficiently possible is under-provision (which is true also when $\alpha > 0$).

5. Network Effects and Profitability

In this section, we examine whether firms benefit from an increase in the degree of network effects, α , and from an increase in the market size, M . Both of these changes seem intuitively positive for the firms as they can lead to higher service valuation by consumers (larger market size, M , leads to larger network size, and α is the marginal valuation for the network). However, we find that as the degree of network effects (α) increases or as the market size (M) increases, the profits of both firms may in fact decrease. These results are presented in Propositions 4 and 5 respectively.

Proposition 4. *When both firms have positive product demand*

- i) *If both firms offer the service in equilibrium, the profit of each of the firms decreases as the degree of network effects increases.*
- ii) *When only Firm i offers the service in equilibrium, its profit increases as the degree of network effects increases if and only if*

$$s_i > \frac{t - \alpha M + c}{\gamma}. \quad (4)$$

The profit of the other firm always decreases as the degree of network effects increases.

According to Proposition 4, firms often do not benefit from an increase in the degree of network effects. When both firms offer the service, the benefit from an increase in the degree of network effects goes to consumers, and, although the demand for the product increases, the intensified price competition (as can be seen from Table 3, both prices decrease as α increases) reduces both firms' profits. In contrast, if γ , the marginal valuation for the inherent functionalities of the service, increases, then the profit of the firm with the higher service quality increases, while the profit of the other firm decreases.

The implications are again important. While the firm that offers the higher service quality

always benefits from an increase in γ , it might not benefit from an increase in α . Only if its service quality is high enough (see Equation 4) and the competing firm does not offer the service, a firm can benefit from an increase in the degree of network effects.

Proposition 5. *When both firms have positive product demand*

i) *If both firms offer the service in equilibrium, then the profit of Firm i increases in the size of the market (M) if and only if*

$$s_i - s_j > \frac{3(2\alpha M - t)(t - \alpha M)}{\gamma t}. \quad (5)$$

ii) *If only firm i offers the service, then Firm i 's profit increases in the size of the market (M) if and only if*

$$s_i > \frac{(c-3t)t+3Mt\alpha-M^2\alpha^2}{t\gamma} = \frac{c}{\gamma} - \frac{t(2t-M\alpha)+(t-M\alpha)^2}{\gamma} \quad (6)$$

According to Proposition 5-(i), when both firms offer the service and the service exhibits network effects, in some cases even the profit of the firm with the higher service quality decreases as M increases. This happens if $M > t/2\alpha$ (so that the RHS of (5) is positive) and the difference between the two service qualities (s_i and s_j) is not large enough. In contrast, when the service does not exhibit network effects, the firm with the higher service quality always benefits from an increase in the market size, M (as the RHS of Inequality 5 is negative when $\alpha = 0$).

Similarly, from Proposition 5-(ii), we see that if only Firm i offers the service in equilibrium, there are cases in which Firm i profit decreases as M increases, a situation that does not happen when the service does not exhibit network effects.

The above propositions show that, when both firms offer the service, an increase in α always leads to a reduction in the profit of both firms, while an increase in the market size M can cause a reduction in both profits if the difference in qualities ($s_A - s_B$) is small (else, it causes a reduction only in the profit of the firm with the lower quality). Thus, a seemingly favorable

exogenous change (an increase in α or in M) may undermine the profitability of the entire industry. What drives this counterintuitive result?

Without loss of generality, suppose $s_A > s_B$. When both firms offer the service, the reaction functions of Firm A and Firm B (in the second stage of the game) are given by

$$p_{ss}^A(p_{ss}^B) = \frac{t - \alpha M + \gamma(s_A - s_B) + c + p_{ss}^B}{2}, \quad p_{ss}^B(p_{ss}^A) = \frac{t - \alpha M + \gamma(s_B - s_A) + c + p_{ss}^A}{2} \quad (7)$$

Examining how the best response price of Firm A (B) changes with the degree of network effects, we get:

$$\frac{\partial}{\partial \alpha} p_{ss}^A(p_{ss}^B) = \frac{\partial}{\partial \alpha} p_{ss}^B(p_{ss}^A) = -\frac{M}{2}, \quad (8)$$

Equation 8 demonstrates that as α becomes greater, Firm A has to lower its price (p_{ss}^A) for a given price of Firm B. The same holds for Firm B.

Why does the price competition become more intense as the degree of network effects increases? From the demand functions, we can show that given the prices of two firms, the price elasticity of demand ($e_{D,p}$) for Firm A is decreasing in α ($\frac{\partial}{\partial \alpha} e_{D,p} < 0$), implying that the higher α , the more elastic the demand becomes. With a higher α , marginal customers who are indifferent to either firm react to a change in price more sensitively. The firms need to lower their price to accommodate such elastic demand. This explains why the optimal price decreases in α . We can show that this is also the case with respect to an increase in the market size (M). Specifically, while an increase in M increases the demand, the prices of both firms always decrease in the market size M .

This finding illustrates an instance of a Bertrand supertrap. Cabral and Villas-Boas (2005) define a Bertrand supertrap as a situation in which a common, industry-wide change in some exogenous parameter leads to an increase in profits when prices are held constant, but ultimately to a decrease in firms' *equilibrium* profits. Specifically, the direct effect of an

exogenous change (i.e., the effect on profits while the firms do not adjust the prices) is positive, whereas the total effect on profits after price adjustments is negative due to a strong negative strategic effect (i.e., intensification of price competition). In our context, the common change in some exogenous parameter is the increase in α or M . It can be shown that if the two prices (p_{ss}^A and p_{ss}^B) of two firms are unchanged, the profit of the firm with the higher s_i always increases in α or M . However, as Propositions 4 and 5 indicate, the strategic effect can reduce the equilibrium profits of the two firms.

6. The Service Development Decision

This section extends our base model to consider the case in which each firm incurs a development cost, and can endogenously choose the quality of its service (s_i). We examine whether a Prisoner's Dilemma and a Bertrand supertrap can emerge even when the service quality is endogenous, or perhaps firms can break out of such jeopardy by strategically choosing their service quality.

The game described in Section 3 is modified as follows. In the first stage of the game, each firm decides whether to develop the service at a cost of $C(s_i)$, where $C'(s_i) > 0$ and $C''(s_i) > 0$. In this extension, we assume that $C(s_i) = c_i s_i^2$ where $c_i > 0$, and c_i indicates the development capability of Firm i . Firms can differ in their development cost as some may have an internal department of developers while others outsource the project. The firm with the lower c_i has the cost advantage. Next, in the second stage of the game, s_A and s_B are observed, and both firms simultaneously decide whether to offer service to consumers who buy their product. In the next stage, prices are set, and, finally, in the last stage of the game, consumers choose whether to buy a product and from which firm. For a given result of the first stage of the game, i.e., for given values of s_A and s_B , the following three stages are the same as the game described in Section 3.

Thus, for each (s_A, s_B) pair that can be chosen in the first stage of the game described here², the equilibrium is determined by Proposition 1, and the profit of Firm i is:

$$\pi^i(s_A, s_B) = G^i(s_A, s_B) - c_i s_i^2$$

Where

$$G^i(s_A, s_B) = \begin{cases} \pi_{SS}^i(s_A, s_B) & \text{If both choose to offer the service in the 2nd stage} \\ \pi_{SN}^i(s_A, s_B) & \text{If only A chooses to offer the service in the 2nd stage} \\ \pi_{NS}^i(s_A, s_B) & \text{If only B chooses to offer the service in the 2nd stage} \\ \pi_{NN}^i(s_A, s_B) & \text{If neither chooses to offer the service in the 2nd stage} \end{cases}$$

The expressions for π_k^i are given in Table 4. Notice that in general, a firm might invest in developing the service in the first stage of the game, incurring the fixed development cost, but then after observing the competitor's service quality in the second stage of the game, it might decide not to offer its service (in the second stage of the game any development cost is sunk, and the firm might expect higher profit when not offering the service). However, this situation would not happen in equilibrium.

Table 7 exhibits the Firms' optimal service quality choices, for the four possible outcomes of the second stage of the game (the stage in which firms chooses if to offer their service). For example, Firm A's optimal service quality choice if it predicts that Firm B will choose not to offer service in the 2nd stage of the game, is $s_A = \frac{\gamma M(3t - \alpha M - c)}{9c_A(2t - \alpha M) - \gamma^2 M}$, while if Firm A predicts that

Firm B will offer service, its optimal service quality is $s_A = \frac{\gamma M(9c_B(t - \alpha M) - \gamma^2 M)}{54c_A c_B(t - \alpha M) - 3\gamma^2 M(c_A + c_B)}$. The

resulting payoff matrix of profits, as given in Table 8, was obtained by substituting the values of s_A and s_B from Table 7 in the profit expressions from Table 4.

With the payoff matrix of profits, Table 8, we can numerically determine the equilibrium

² Where a choice not to develop the service can be represented by $s_i = 0$.

(s_A, s_B) values for any set of parameters values ($M, c, t, \alpha, \gamma, c_A$ and c_B). We cannot derive an equivalent proposition to Proposition 1, stating which equilibrium prevails under given conditions on parameters values, as the inequalities are too complex.

Firm A \ Firm B	Only Product	Product+ Service
Only Product	N/A	$S_B = \frac{\gamma M(3t-\alpha M-c)}{9c_B(2t-\alpha M)-\gamma^2 M}$
Product +Service	$S_A = \frac{\gamma M(3t-\alpha M-c)}{9c_A(2t-\alpha M)-\gamma^2 M}$	$S_A = \frac{\gamma M(9c_B(t-\alpha M)-\gamma^2 M)}{54c_A c_B(t-\alpha M)-3\gamma^2 M(c_A+c_B)}$ $S_B = \frac{\gamma M(9c_A(t-\alpha M)-\gamma^2 M)}{54c_A c_B(t-\alpha M)-3\gamma^2 M(c_A+c_B)}$

Table 7. The equilibrium service qualities

Firm A \ Firm B	Only Product	Product + Service
Only Product	$\pi_{NN}^A = 0.5Mt$ $\pi_{NN}^B = 0.5Mt$	$\pi_{NS}^A = \frac{M(2t-\alpha M)(3c_B(3t-2\alpha M+c)-\gamma^2 M)^2}{(9c_B(2t-\alpha M)-\gamma^2 M)^2}$ $\pi_{NS}^B = \frac{c_B M(3t-\alpha M-c)^2}{9c_B(2t-\alpha M)-\gamma^2 M}$
Product +Service	$\pi_{SN}^A = \frac{c_A M(3t-\alpha M-c)^2}{9c_A(2t-\alpha M)-\gamma^2 M}$ $\pi_{SN}^B = \frac{M(2t-\alpha M)(3c_A(3t-2\alpha M+c)-\gamma^2 M)^2}{(9c_A(2t-\alpha M)-\gamma^2 M)^2}$	$\pi_{SS}^A = \frac{c_A M(18c_A(t-\alpha M)-\gamma^2 M)(9c_B(t-\alpha M)-\gamma^2 M)^2}{9(18c_A c_B(t-\alpha M)-\gamma^2 M(c_A+c_B))^2}$ $\pi_{SS}^B = \frac{c_B M(18c_B(t-\alpha M)-\gamma^2 M)(9c_A(t-\alpha M)-\gamma^2 M)^2}{9(18c_A c_B(t-\alpha M)-\gamma^2 M(c_A+c_B))^2}$

Table 8. The equilibrium profits when quality is endogenous

Figure 4 display the resulting equilibrium in the c_A - c_B space for a given instance of parameters values. We find that when service qualities are endogenous, both firms offer the service in equilibrium when c_A and c_B are sufficiently low and close enough to each other. If both firms incur a high enough development cost then in equilibrium neither offers the service. In all other cases, in equilibrium only one firm offer the service

We also see from Figure 4 that, as before, there is a range of parameters values in which two equilibriums may prevail. Specifically, in the range labeled SN|NS, each firm would offer the service only if the competitor does not. Thus, there can be a first mover advantage, and an

equilibrium in which the firm with the higher development cost is the only one offering a service may prevail.

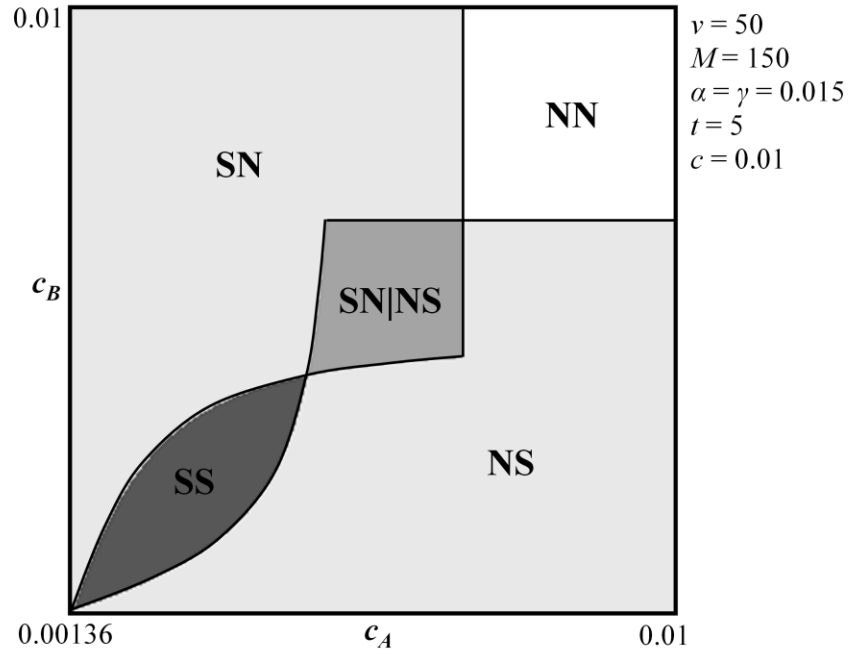


Figure 4. The market equilibrium in the c_A - c_B space.

We also find that a Prisoner’s Dilemma can arise when both firms offer the service in equilibrium, as shown in the numerical example in Table 9.

Firm A \ Firm B	Only Product	Product + Service	$M = 150, v = 50$ $t = 5, c = 0.01$ $\alpha = \gamma = 0.015$ $c_A = c_B = 0.004$
Only Product	375, 375	127.04, <u>379.16</u>	
Product+ Service	<u>379.16</u> , 127.04	<u>127.15, 127.15</u>	

Table 9. A numerical example of Prisoner’s Dilemma when service qualities are endogenous

Next we examine how an increase in the degree of network effects changes profitability.

The results are given in Proposition 6.

Proposition 6. *When both firms offer the service with positive demand, as the degree of network effects increases:*

- i) *The equilibrium service quality of Firm i increases if $c_i < c_j$, unchanged if $c_i = c_j$, and*

decreases otherwise.

- ii) *The profit of firm i increases if $c_i < U(c_j)$, and decreases if $U(c_j) < c_i < c_j$, where both ranges are none-empty as long as $c_i < c_j$, and*

$$U(c_j) = \frac{2\gamma^2 c_j M}{27c_j(t-\alpha M) - \sqrt{81c_j^2(t-\alpha M)^2 + 36\gamma^2 c_j M(t-\alpha M) - 4\gamma^2 M^2}} \quad (9)$$

From Proposition 6, we learn that when both firms choose to offer the service in equilibrium, as the degree of network effects increases, the firm with the higher development cost is always worse off; it will choose an even lower service quality, and its profit will decrease. In contrast, the firm with the lower development cost would choose a higher service quality and it may be better off; that is its profit may increase. Thus, when service qualities were held fixed (exogenous), we found that both firms' profits necessarily decrease as the degree of network effects increases. However, when service qualities are endogenous, the firm with the lower development cost might be able (if its cost is low enough) to benefit from the increase in the degree of network effects.

Since the profit of the firm with the cost advantage might also decrease as the degree of network effects increases, Proposition 6 shows that even when the firms can adjust the level of service quality, they may still be caught in a Bertrand Supertrap. Similar results hold when examining how a change in the market size, M , affects profitability. In particular, when both firms offer the service in equilibrium and $c_A = c_B$ it can be shown that the equilibrium quality level always increases in the size of the market (M), but under general conditions on parameters values, both firms' profit decreases in M , (for example, $t < 2\alpha M$ is a sufficient condition for such a profit reduction) .

Proposition 7 introduces another type of a Bertrand Supertrap. It considers a case in which technological development reduces the development costs of both firms and examines

how such an exogenous change shapes the industry's profitability.

Proposition 7. *Suppose $c_A = kd_A$, $c_B = kd_B$ and $d_A < d_B$. When both firms offer the service in equilibrium, the profit of Firm A increases in k if and only if*

$$d_A^2(96d_Bk^2(t - \alpha M)^2) - d_A(24d_B^2k^2(t - \alpha M)^2 + 30\gamma^2d_BkM(t - \alpha M) - \gamma^2M^2) + (\gamma^2d_BM(6d_Bk(t - \alpha M) + \gamma^2M)) > 0.$$

The profit of Firm B always increases in k .

Proposition 7 describes an interesting phenomenon. It shows that as k decreases, i.e. as service development becomes less costly for both firms, even the firm with the stronger development capability ($d_A < d_B$) may experience a profit reduction. The profit of the other firm always decreases as k decreases.

7. Heterogeneous Service Valuations

For simplicity, in the base model we assume that consumers are homogenous in terms of their service valuations. Specifically, all consumers have the same marginal valuation for an improvement in the quality of the service (γ) or for an increase in the network size (α). Thus, whether the seller offers the service separately for a fee, f , and the product for a price p , or whether he offers the two bundled for a single price, p_B ³, demand and profit would be the same. This simplification allowed us to focus on how firms should adjust their strategies when offering a service that exhibits network effects.

In contrast, when consumers have heterogeneous service valuations (either because they have heterogeneous γ values, or heterogeneous α values, or both), the profit and demand when selling the service separately for a fee might be different from the profit and demand when the service is sold bundled with the product. If consumers have heterogeneous service valuations,

³ Which is the same as selling the product for price p_B , and offering the service “free of charge”

when the seller offers the service separately for a fee, some consumers might choose to buy only the product, while others buy both product and service. This situation is not possible when the service is offered bundled with the product. Thus, the two strategies can lead to different profits.

We analyzed an alternative model, in which there are two types of consumers in terms of service valuations. Consumers in Group 1 have marginal valuation for the inherent functionalities of the service γ_1 , and for the network of service users α_1 , while consumers in Group 2 have marginal valuation for inherent service quality, γ_2 , and for network of service users α_2 . Proportion β of the population belong to Group 1 while the rest are in Group 2, and without loss of generality we assume $\alpha_1 > \alpha_2$ and $\gamma_1 > \gamma_2$. That is, we assume that consumers with higher valuations for the inherent functionalities of the service also have higher marginal valuation for the network related functionalities.

In the first stage of the game, each firm chooses whether to offer the service bundled with the product, sell it separately for a fee, or not sell it at all. In the second stage of the game, firms observe the choices made, and set prices accordingly. In the last stage, consumers decide from which firm to obtain the product and, if service is sold separately, decide whether to buy the service. In this model there are more possible market configurations than in the base model presented in Section 3. Specifically, there are 9 possible outcomes for the first stage of the game. Thus, for this model, the conditions that need to be satisfied for each possible equilibrium to prevail are very complex, and rather than deriving a proposition similar to Proposition 1, we derived the equilibrium numerically for different sets of parameters values. Table 10 presents the resulting equilibrium in the s_A - s_B space for one set of parameters values (profits expressions for each possible configuration can be obtained from the authors). The results presents in Table 10 are representative of any set of parameters values we have examined.

From Table 10, we learn that Firm i offers a bundle for high s_i values, sells the service separately for mid-range s_i values and does not sell the service for small s_i values. We note that, though this is not exhibited in Table 10, we found that the boundary values (for s_i) may increase as the competitor's service quality, s_j , decrease.

$s_B \backslash s_A$	160	200	240	280	320	360	400	440	480
160	NN	NN	Ns	Ns	Ns	Ns	NS	NS	NS
200	NN	NN	Ns	Ns	Ns	Ns	NS	NS	NS
240	sN	sN	ss	ss	ss	sS	sS	sS	sS
280	sN	sN	ss	ss	ss	sS	sS	sS	sS
320	sN	sN	ss	ss	ss	sS	sS	sS	sS
360	sN	sN	ss	Ss	Ss	Ss sS	sS	sS	sS
400	SN	SN	Ss	Ss	Ss	Ss	SS	SS	SS
440	SN	SN	Ss	Ss	Ss	Ss	SS	SS	SS
480	SN	SN	Ss	Ss	Ss	Ss	SS	SS	SS

$M = 200, v = 50, t = 5, c = 3, \alpha_1 = 0.015, \alpha_2 = 0.01, \gamma_1 = 0.015, \gamma_2 = 0.01, \beta = 0.3$

- S: sells a bundle. (e.g., **Ss**- Firm A sells a bundle)
- s: sells the service separately. (e.g., **Ss**, Firm B sells service separately)
- N: the firm sells only the product.
- Several equilibriums for the same cell are separated by |.
- NA – No equilibrium exists.

Table 10. Market equilibrium when consumers have heterogeneous service valuations

All the results from Section 5 still hold when parameters values are such that both firms sell a bundle in equilibrium, or when one firm sells a bundle while the other does not offer the service. That is, we can still show the existence of a Prisoner's Dilemma, and that the firms profit decreases as the degree of network effects increase. However, with heterogeneous service valuations, there are additional market configurations that need to be studied (such as when both firms sell the service separately). We leave such examination, which would have to be done numerically, for future work.

8. Conclusions

An increasing number of firms have been realizing that in order to remain competitive they can no longer only sell products, but they have to provide their customers with value adding services. The nature of such services has been changing though. While in the past, services were mainly limited to maintenance, installation, repair, and periodic updates, nowadays many services can be delivered online. Especially, advances in technology allow firms to deliver services that create value by enabling interactions between the firms' customers, who often benefit from such connectivity as they have common interests and goals.

While the profitability of offering product related services has been studied before, this is the first paper to examine how the switch from traditional services to online services with network effects changes the competitive outcomes and firms profitability. We find that, unlike when considering to offer a service without network effects, when considering an online service with network effects, firms have to also take into consideration whether the competitor is offering a similar service or not, and the quality of the competitor's service. In addition, competing firms might want to coordinate their service offering decisions, because not doing so might reduce both firms' profits. We also show that in many cases an increase in the degree of network effects actually reduces both firms profit. Thus, advances in technology which may benefit a monopoly, actually intensify competition and cause a reduction in profits.

Future work can examine a model with heterogeneous service valuations to determine how network effects change profitability when firms sell the service separately for a fee. When service is sold separately, some consumers can buy only the product and pay less than consumers who buy both product and service. In addition, this alternative model can be studied to determine how network effects can influence the decision whether to sell a bundle or sell the service

separately. Intuition predicts that as network effects intensify the incentive to sell the service bundled with the product increases, as this increases the size of the network of service users.

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Appendix 1.

Derivations of Equilibrium in the Second and Third Stages of the Game

We derive the equilibrium in prices and demands given the choices of the two firms in the first stage of the game. We consider only cases in which i) each firm has positive demand for its product, and ii) the market for the product is covered and firms face spatial competition.

1. Case NN: Both Firms Sell Only Product

When neither firm operates the service, the surplus a consumer obtains when buying the product sold by Firm A, and the surplus he obtains when buying the product sold by Firm B, are given respectively by

$$u_{NN}^A = V - tx - p_{NN}^A \quad (\text{A1})$$

$$u_{NN}^B = V - t(1 - x) - p_{NN}^B, \quad (\text{A2})$$

For spatial competition (the market is covered and the marginal customer has positive utility) to be the equilibrium outcome, it must be that $V > 1.5t$. It is easy to show that when this condition holds, in equilibrium the product price is

$$p_{NN}^{A*} = p_{NN}^{B*} = t. \quad (\text{A3})$$

The market share of each firm is $0.5N$, and the profits are given by

$$\pi_{NN}^{A*} = \pi_{NN}^{B*} = 0.5 Nt. \quad (\text{A4})$$

If $t < V < 1.5t$, then the market is still covered, but the indifferent customer obtains zero utility. In this case, $p_{NN}^{A*} = p_{NN}^{B*} = V - 0.5t$. Finally, if $V < t$, then in equilibrium market is not covered. In this paper we limit our attention to cases of spatial competition, that is, we assume $V > 1.5t$ (see Assumption 1 in Appendix 2).

2. Cases SN and NS: Only One Firm Offers a Service

W.L.O.G we assume that only Firm A decided to offer a service to its customers. The solution when only firm B offers the service can be derived in a similar manner.

When expected network size (i.e., the expected number of consumers that buy from Firm A) is N_A , consumer's surplus when buying from firm A, u_{SN}^A , and when buying from Firm B, u_{SN}^B are given by:

$$u_{SN}^A = V - tx + \gamma S_A + \alpha N_A - p_{SN}^A \quad (\text{A5})$$

$$u_{SN}^B = V - t(1 - x) - p_{SN}^B \quad (\text{A6})$$

The location of the customer who is indifferent between the two firms, denoted by \hat{x} , is

$$\hat{x}(N_A) = \frac{t + \alpha N_A + \gamma S_A - p_{SN}^A + p_{SN}^B}{2t} \quad (\text{A7})$$

The demand for the product and service of Firm A, D_{SN}^A , given that consumers expect the number of service uses to be N_A , is thus given by $M\hat{x}(N_A)$. In a Fulfilled Expectation Equilibrium, we require that

$$D_{SN}^A = M\hat{x}(D_{SN}^A). \quad (\text{A8})$$

Solving the above equation for D_{SN}^A we get

$$D_{SN}^A = \frac{M(t + \gamma S_A - p_{SN}^A + p_{SN}^B)}{2t - \alpha M}. \quad (\text{A9})$$

Given our assumption that the market is covered, the demand for product B is given by $D_{SN}^B = M - D_{SN}^A$. Finally, the profit functions of two firms are given by $\pi_{SN}^A = D_{SN}^A(p_{SN}^A - c)$, and $\pi_{SN}^B = D_{SN}^B p_{SN}^B$.

Solving the first-order conditions simultaneously (S.O.C is satisfied when $2t > \alpha M$, where the latter condition must hold in order for both firms to have positive demand when both offer service), we find that in equilibrium prices and profits are as follows:

$$p_{SN}^A * = \frac{3t - \alpha M + \gamma S_A + 2c}{3}, \quad p_{SN}^B * = \frac{3t - 2\alpha M - \gamma S_A + c}{3} \quad (\text{A10})$$

$$\pi_{SN}^A * = \frac{M(3t-c-\alpha M+\gamma S_A)^2}{9(2t-\alpha M)}, \quad \pi_{SN}^B * = \frac{M(3t+c-2\alpha M-\gamma S_A)^2}{9(2t-\alpha M)}. \quad (\text{A11})$$

At above prices, the condition for both firms to have positive demand (i.e., $0 < D_{SN}^i < M$ for $i = A, B$) is

$$\frac{c-3t+\alpha M}{\gamma} < S_A < \frac{3t-2\alpha M+c}{\gamma}, \quad (\text{A12})$$

To insure spatial competition at above prices, we need to find the surplus of the customer indifferent between the two products and require it to be positive. Doing so we get the following condition

$$V > \frac{(3t-\alpha M)(3t+c-2\alpha M-\gamma S_A)}{6t-3\alpha M}. \quad (\text{A13})$$

3. Case SS: Both Firms offer a Service

When both firms offer the service, the utility functions are given by

$$u_{SS}^A = V - tx + \gamma S_A + \alpha N_A - p_{SS}^A \quad (\text{A14})$$

$$u_{SS}^B = V - t(1-x) + \gamma S_B + \alpha N_B - p_{SS}^B \quad (\text{A15})$$

The location of indifferent customer \hat{x} is found by solving $u_{SS}^A = u_{SS}^B$ and is given by

$$\hat{x}(N_A, N_B) = \frac{t+\gamma(S_A-S_B)+\alpha(N_A-N_B)-p_{SS}^A+p_{SS}^B}{2t} \quad (\text{A16})$$

The demand for the product and service of Firm A, D_{SS}^A , given consumers expectations regarding networks sizes, is $M\hat{x}(N_A, N_B)$, and the demand for the product and service of Firm B, D_{SS}^B , given the assumption that market is covered is $M - D_{SS}^A$. In the Fulfilled Expectation equilibrium, we require that

$$D_{SS}^A = M\hat{x}(D_{SS}^A, D_{SS}^B) \quad \text{and} \quad D_{SS}^B = M\left(1 - \hat{x}(D_{SS}^A, D_{SS}^B)\right). \quad (\text{A17})$$

Solving the above two equations simultaneously for D_{SS}^A and D_{SS}^B , we get

$$D_B^A = \frac{M(t-\alpha M+\gamma(S_A-S_B)-p_{SS}^A+p_{SS}^B)}{2(t-\alpha M)}, \quad D_B^B = \frac{N(t-\alpha N+\gamma(S_B-S_A)+p_{SS}^A-p_{SS}^B)}{2(t-\alpha M)}. \quad (\text{A18})$$

The profit functions of the two firms are given by

$$\pi_{SS}^i = D_{SS}^i(p_{SS}^i - c) \quad (i = A \text{ and } B). \quad (\text{A19})$$

Solving the first order conditions simultaneously (S.O.C are satisfied when $\alpha M > t$, a condition which we shortly show is satisfied when both firms have positive demand in equilibrium), we find the equilibrium prices:

$$p_{SS}^A = t + c - \alpha M + \gamma \frac{(s_A - s_B)}{3}, \quad p_{SS}^B = t + c - \alpha M + \gamma \frac{(s_B - s_A)}{3} \quad (\text{A20})$$

The profits at the optimal prices are given by

$$\pi_{SS}^A = \frac{M(3(t - \alpha M) + \gamma(s_A - s_B))^2}{18(t - \alpha M)}, \quad \pi_{SS}^B = \frac{M(3(t - \alpha M) + \gamma(s_B - s_A))^2}{18(t - \alpha M)}. \quad (\text{A21})$$

The condition for both firms to have positive demand (i.e., the marginal customer's location is interior) is

$$|s_A - s_B| < \frac{3(t - \alpha M)}{\gamma}, \quad (\text{A22})$$

which also require that

$$t > \alpha M. \quad (\text{A23})$$

Finally, with above prices there is spatial competition if and only if:

$$V > \frac{3(t - \alpha N) + 2c - \gamma(s_A + s_B)}{2} \quad (\text{A24})$$

Appendix 2 – Proofs

Before we provide the proofs of all propositions, we introduce the following parameter assumptions, which guarantee that regardless of which strategy each firm chooses, the market for the product is covered and each firm has a positive demand.

Assumption 1.

$$(i) \quad t > \alpha M \quad (A25)$$

$$(ii) \quad \frac{c-3t+\alpha M}{\gamma} < s_i < \frac{3t-2\alpha M+c}{\gamma} \quad (i = A \text{ and } B) \quad (A26)$$

$$(iii) \quad |s_A - s_B| < \frac{3(t-\alpha M)}{\gamma} \quad (A27)$$

$$(iv) \quad v > \max\left(\frac{3t}{2}, \frac{(3t-\alpha M)(3t+c-2\alpha M-\gamma s_A)}{6t-3\alpha M}, \frac{(3t-\alpha M)(3t+c-2\alpha M-\gamma s_B)}{6t-3\alpha M}, \frac{3(t-\alpha M)+2c-\gamma(s_A+s_B)}{2}\right)$$

Assumption 1-(ii) ensures that, an equilibrium in which firms face spatial competition and both firms have positive product demand is possible when only one of the firms offers the service (it was derived by requiring $0 < D_{SN}^A < M$ and $0 < D_{SN}^B < M$); while Assumption 1-(iii) ensures an equilibrium in which both firms have positive demand is possible when both firms offer the service (it was derived by requiring $0 < D_{SS}^A < M$ and $0 < D_{SS}^B < M$).

Assumption 1-(i) is necessary for the condition given in Assumption 1-(iii) to be satisfied for some parameter values. If Condition (i) is not satisfied, then t , the per unit misfit cost, is small relative to the potential network effects, and thus in equilibrium all consumers will choose to buy the product and the service from one firm. That is, the strong network effects will dominate the cost of misfit for those consumers who buy a product that does not match their ideal product. Assumption 1-(i) also implies that the range given in Assumption 1-(ii) is not empty (i.e. $2t > \alpha M$, and thus $c - 3t + \alpha M < 3t - 2\alpha M + c$).

Finally, Assumption 1-(iv) ensures that the inherent value of the product, V , is sufficiently high so that the market for the product is covered by the two firms, whether both, neither or only one firm offer the service.

Proof of Proposition 1

Having obtained the optimal prices and profits in Appendix 1 (see also Tables 3 and 4 in the paper) for each of the four possible market configurations, we now derive the conditions for each possible market equilibrium. The conditions are derived as follows:

- i) Both firms offer the service in equilibrium if and only if $\pi_{SS}^A > \pi_{NS}^A$ and $\pi_{SS}^B > \pi_{NS}^B$.
- ii) Both firms sell only product in equilibrium if and only if $\pi_{NN}^A > \pi_{SN}^A$ and $\pi_{NN}^B > \pi_{NS}^B$.
- iii) Only firm A offers a service in equilibrium if and only if $\pi_{SN}^A > \pi_{NN}^A$ and $\pi_{SN}^B > \pi_{SS}^B$.
- iv) Only firm B offers a service in equilibrium if and only if $\pi_{NS}^A > \pi_{SS}^A$ and $\pi_{NS}^B > \pi_{NN}^B$.

Equilibrium in which neither firm offers the service

An equilibrium in which neither firm provides the service exists if and only if $\pi_{NN}^A > \pi_{SN}^A$ and $\pi_{NN}^B > \pi_{NS}^B$, so that neither firm has incentive to deviate and offer the service.

From the profit equations in Table 4, we find that $\pi_{NN}^A > \pi_{SN}^A$ and $\pi_{NN}^B > \pi_{NS}^B$ if and only if

$$s_i < \frac{2\alpha M + 2c - 6t + 3\sqrt{2t(2t - \alpha M)}}{2\gamma} \quad (i = A \text{ and } B) \quad (\text{A29})$$

We denote this upper bound by \bar{s} .

Equilibrium in which both firms offer the service

In order for both Firm A and Firm B to offer the service in equilibrium, it must be that $\pi_{SS}^A > \pi_{NS}^A$ and $\pi_{SS}^B > \pi_{NS}^B$, so that neither firm has incentive to deviate and not sell the service. These two conditions are given by

$$s_i > s_j X + Y \quad \text{for } (i = A, j = B) \text{ and for } (i = B, j = A),$$

$$\text{where } X = 1 - \frac{\sqrt{2t-2\alpha M}}{\sqrt{2t-\alpha M}} \text{ and } Y = \frac{(3t+c-2\alpha M)\sqrt{2t-2\alpha M}}{\gamma\sqrt{2t-\alpha M}} - \frac{3(t-\alpha N)}{\gamma}. \quad (\text{A30})$$

Equilibrium in which only one firm offers the service

The conditions under which there is an equilibrium in which only Firm A offers the service are:

(i) $\pi_{SN}^B > \pi_{SS}^B$ and (ii) $\pi_{SN}^A > \pi_{NN}^A$. Condition (i) implies that Firm B does not have an incentive to deviate and start offering the service. Condition (ii) indicates that Firm A does not have an incentive to deviate and not offer the service. Conditions (i) and (ii) translate to $s_B < s_A X + Y$ and $s_A > \bar{s}$, respectively. The conditions under which an equilibrium in which only Firm B sells the service is feasible can be derived in a similar manner.

Showing that $(\bar{s}X + Y) - \bar{s} > 0$:

$$(\bar{s}X + Y) - \bar{s} = \frac{3\sqrt{t-\alpha M}(\sqrt{2(2t-\alpha M)} - \sqrt{t-\alpha M} - \sqrt{t})}{\gamma}. \quad (\text{A31})$$

Given Assumption 1, $t > \alpha M$ and thus the nominator is positive iff $\sqrt{2(2t-\alpha M)} - \sqrt{t-\alpha M} - \sqrt{t} > 0$. Since $a-b=(a-b)^2/(a+b)$, we can rewrite the latter expression as:

$$\sqrt{2(2t-\alpha M)} - \sqrt{t-\alpha M} - \sqrt{t} = \frac{(\sqrt{2(2t-\alpha M)})^2 - (\sqrt{t-\alpha M} + \sqrt{t})^2}{\sqrt{2(2t-\alpha M)} + \sqrt{t-\alpha M} + \sqrt{t}}. \quad (\text{A32})$$

Finally, $(\sqrt{2(2t-\alpha M)})^2 - (\sqrt{t-\alpha M} + \sqrt{t})^2 = 2t - \alpha M - 2\sqrt{t}\sqrt{t-\alpha M} > 0$, where the last

inequality holds because $2t - \alpha M - 2\sqrt{t}\sqrt{t-\alpha M} = \frac{(2t-\alpha M)^2 - (2\sqrt{t}\sqrt{t-\alpha M})^2}{(2t-\alpha M) + 2\sqrt{t}\sqrt{t-\alpha M}} = \frac{\alpha^2 M^2}{(2t-\alpha M) + 2\sqrt{t}\sqrt{t-\alpha M}}$

> 0

Proof of Proposition 2

The inequalities $s_A > s_B X + Y$ and $s_B > s_A X + Y$ represent the conditions under which both

firms offer the service in equilibrium. The condition $|s_A - s_B| < \frac{3(\alpha M - t + \sqrt{t(t-\alpha M)})}{\gamma}$ is derived

from $\pi_{SS}^A < \pi_{NN}^A$ and $\pi_{SS}^B < \pi_{NN}^B$.

Finally we show that $(\alpha M - t + \sqrt{t(t - \alpha M)})$ is always positive when there is positive demand for both firms (which implies $t > \alpha M$, as stated in Assumption 1):

$$\sqrt{t(t - \alpha M)} - (t - \alpha M) = \frac{t(t - \alpha M) - (t - \alpha M)^2}{\sqrt{t(t - \alpha M)} + (t - \alpha M)} = \frac{t\alpha M + (\alpha M)^2 > 0}{\sqrt{t(t - \alpha M)} + (t - \alpha M)} > 0 \quad (\text{A33})$$

Proof of Proposition 3

Define x_{indif} as the location of the consumer indifferent between buying the product from Firm A and buying from Firm B. Then, when both firms offer the service in equilibrium we have:

$$x_{\text{indif}} = \frac{1}{6} \left(3 + \frac{(s_A - s_B)\gamma}{t - M\alpha} \right). \quad (\text{A34})$$

When only Firm i sells the service, in equilibrium we have

$$x_{\text{indif}} = \frac{3t - c - M\alpha + \gamma s_i}{6t - 3M\alpha}. \quad (\text{A35})$$

Consumer surplus when Firm A sells the service and Firm B does not is given by:

$$\begin{aligned} CS_A &= M \int_{x=0}^{x_{\text{indif}}} (V - tx - p_{SN}^A + \gamma s_A + \alpha M x_{\text{indif}}) dx + N \int_{x_{\text{indif}}}^1 (V - t(1-x) - p_{SN}^B) dx = \\ &= M \int_{x=0}^{x_{\text{indif}}} \left(V - tx - \left(\frac{3t + 2c - \alpha M + \gamma s_A}{3} \right) + \gamma s_A + \alpha M x_{\text{indif}} \right) dx + \\ &= M \int_{x_{\text{indif}}}^1 \left(V - t(1-x) - \left(\frac{3t + c - 2\alpha M - \gamma s_A}{3} \right) \right) dx = \\ &= MV - \frac{M(5t - 3\alpha M - 2\gamma s_A + 2c)}{4} + \frac{M(\alpha M + 2\gamma s_A - 2c)(2\gamma s_A t - 2ct + \alpha M(7t - 3\alpha M))}{36(2t - \alpha M)^2}. \end{aligned} \quad (\text{A36})$$

Similarly, consumer surplus when only Firm B sells the service is given by:

$$CS_B = MV - \frac{M(5t - 3\alpha M - 2\gamma s_B + 2c)}{4} + \frac{M(\alpha M + 2\gamma s_B - 2c)(2\gamma s_B t - 2ct + \alpha M(7t - 3\alpha M))}{36(2t - \alpha M)^2} \quad (\text{A37})$$

Consumer surplus when both firms offer the service is given by

$$CS_{ss} = M \int_{x=0}^{x_{\text{indif}}} \left(V - tx - \left(c + t - N\alpha + \frac{1}{3}(s_A - s_B)\gamma \right) + \gamma s_A + \alpha M x_{\text{indif}} \right) dx +$$

$$M \int_{x_{\text{indif}}}^1 \left(V - t(1-x) - \left(c + t - N\alpha + \frac{1}{3}(s_B - s_A)\gamma \right) + \gamma s_B + \alpha M(1 - x_{\text{indif}}) \right) dx =$$

$$MV + \frac{N}{4} (6\alpha M + 2\gamma(s_A + s_B) - 4c - 5t) + \frac{\gamma^2 M t (s_A - s_B)^2}{36(t - \alpha M)^2}. \quad (\text{A38})$$

Finally, consumer surplus when neither firm offers the service is given by:

$$CS_{NN} = M \int_{x=0}^{0.5} (V - tx - t) dx + M \int_{0.5}^1 (V - t(1-x) - t) dx = MV - 0.25Mt \quad (\text{A39})$$

We denote the social welfare when both firms offer service, $\pi_{SS}^A + \pi_{SS}^B + CS_{SS}$, by SW_{SS} , the social welfare when neither firm offers service by, $\pi_{NN}^A + \pi_{NN}^B + CS_{NN}$, by SW_{NN} , and the social welfare when only Firm i offers service by SW_i . The profit expressions are given in Table 4, and were derived in Appendix 1.

It is easy to show that $SW_{SS} > SW_{NN}$ if and only if

$$c < \frac{\alpha M}{2} + \frac{1}{36} (18(s_A + s_B)\gamma + \frac{(s_A - s_B)^2 (5t - 4M\alpha)\gamma^2}{(t - M\alpha)^2}) \quad (\text{A40})$$

Note that above is always satisfied if $c < \frac{\alpha M}{2}$, and is satisfied for large enough values of s_A and s_B otherwise. In addition, $SW_i > SW_j$ if and only if $s_i > s_j$.

$F^i(s_i, s_j)$ is defined as the difference between social welfare when both firms offer service to social welfare when only Firm i offers service, specifically:

$$F^i(s_i, s_j) = SW_{SS} - SW_i, \quad i=A, \text{ or } B \quad (\text{A41})$$

Thus, when $F^i(s_i, s_j) < 0$, social welfare when only Firm i offers the service exceeds social welfare when both firms offer the service.

Finally, given the conditions on s_i specified in Assumption 1, $SW_A > SW_{NN}$ iff $s_A > \frac{2c - M\alpha}{2\gamma}$.

Similarly, $SW_B > SW_{NN}$ iff $s_B > \frac{2c - M\alpha}{2\gamma}$. In addition it is easy to show that $\frac{2c - M\alpha}{2\gamma} < \bar{s}$. Thus, as

long as $s_A > \frac{2c - M\alpha}{2\gamma}$ or $s_B > \frac{2c - M\alpha}{2\gamma}$ (or both), social welfare when one firm offers service

exceeds social welfare when neither offers it, and social welfare is maximized when both offer service if and only if $F^A(s_A, s_B) > 0$ and $F^B(s_B, s_A) > 0$.

Finally, when $s_A < \frac{2c-N\alpha}{2\gamma}$ and $s_B < \frac{2c-N\alpha}{2\gamma}$, social welfare when neither firm offers service is larger than social welfare when only Firm A or only Firm B offers service. In addition, when $s_A < \frac{2c-M\alpha}{2\gamma}$ and $s_B < \frac{2c-M\alpha}{2\gamma}$, and $c > \frac{\alpha M}{2}$ we find that $SW_{SS} < SW_{NN}$, because condition A40 is not satisfied (notice that the RHS of condition A40 evaluated at $s_A = \frac{2c-M\alpha}{2\gamma}$ and $s_B = \frac{2c-M\alpha}{2\gamma}$ is c)

Notice that when $s_A < \frac{2c-N\alpha}{2\gamma}$ and $s_B < \frac{2c-N\alpha}{2\gamma}$ in equilibrium neither firm offers service (as $\frac{2c-N\alpha}{2\gamma} < \bar{s}$).

The rest is trivial based on the results from Proposition 1.

Proof of Proposition 4

(i) We examine the derivate of the profit of Firm A, when both firms offer the service, with respect to the degree of network effects:

$$\frac{\partial}{\partial \alpha} \pi_{SS}^A = \frac{\partial}{\partial \alpha} \frac{M(3(t-\alpha M) + \gamma(s_A - s_B))^2}{18(t-\alpha M)} = \frac{M^2}{18} \left(\frac{\gamma^2(s_A - s_B)^2}{(t-\alpha M)^2} - 9 \right) \quad (\text{A42})$$

In equilibrium, when both firms have positive demand, we have $D_{SS}^A = \frac{1}{6} M \left(3 + \frac{(s_A - s_B)\gamma}{t - M\alpha} \right)$ and $D_{SS}^B = \frac{1}{6} M \left(3 + \frac{(s_B - s_A)\gamma}{t - M\alpha} \right)$. Under our assumption that both firms have positive demand, it must

be that $\gamma|s_A - s_B| < 3(t - \alpha M)$. Thus,

$$\frac{\partial}{\partial \alpha} \pi_{SS}^A = \frac{M^2}{18} \left(\frac{\gamma^2(s_A - s_B)^2}{(t-\alpha M)^2} - 9 \right) < 0 . \quad (\text{A43})$$

By the same token, $\frac{\partial}{\partial \alpha} \pi_{SS}^B$ is negative when both firms have positive product demand.

(ii) Suppose that in equilibrium Firm A offers the service and Firm B does not. Then the derivative of Firm A's profit with respect to α is:

$$\begin{aligned}\frac{\partial}{\partial \alpha} \pi_{SN}^A &= \frac{\partial}{\partial \alpha} \frac{M(3t-c-\alpha M+\gamma s_A)^2}{9(2t-\alpha M)} = \frac{M^2(-t+\alpha M-c+\gamma s_A)(3t-\alpha M-c+\gamma s_A)}{9(2t-\alpha M)^2} \\ &= D_{SN}^A \left(\frac{M(-t+\alpha M-c+\gamma s_A)}{3(2t-\alpha M)} \right)\end{aligned}\quad (\text{A44})$$

Given our assumption that both firms have positive product demand, which also requires $t > \alpha M$

we see that $\frac{\partial}{\partial \alpha} \pi_{SN}^A$ is positive if and only if $\frac{M(-t+\alpha M-c+\gamma s_A)}{3(2t-\alpha M)} > 0$, which is equivalent to $s_A >$

$$\frac{t-\alpha M+c}{\gamma}.$$

It should be noted that $\frac{t-\alpha M+c}{\gamma} > \bar{s}$ if and only if $7t > 8M\alpha$.

Next we examine the derivative of the profit of Firm B:

$$\begin{aligned}\frac{\partial}{\partial \alpha} \pi_{SN}^B &= \frac{\partial}{\partial \alpha} \frac{M(3t+c-2\alpha M-\gamma s_A)^2}{9(2t-\alpha M)} = \frac{M^2(-5t+2\alpha M+c-\gamma s_A)(3t+c-2\alpha M-\gamma s_A)}{9(2t-\alpha M)^2} \\ &= D_{SN}^B \left(\frac{M(-5t+2\alpha M+c-\gamma s_A)}{3(2t-\alpha M)} \right)\end{aligned}\quad (\text{A45})$$

We see that $\frac{\partial}{\partial \alpha} \pi_{SN}^B$ is negative if and only if $\frac{M(-5t+2\alpha M+c-\gamma s_A)}{3(2t-\alpha M)} < 0$. Given that $2t > \alpha M$

(this conditions is required for an equilibrium in which only one firm sells service and both firms have positive product demand), we find that Firm's B profit is decreasing in α if and only if

$s_A > \frac{-5t+2\alpha M+c}{\gamma}$. Furthermore,

$$\frac{-5t+2\alpha M+c}{\gamma} - \bar{s} = \frac{-5t+2\alpha M+c}{\gamma} - \frac{2\alpha M+2c-6t+3\sqrt{2t(2t-\alpha M)}}{2\gamma} = -\frac{2(2t-\alpha M)+3\sqrt{2t(2t-\alpha M)}}{2\gamma} < 0$$

Thus, $\bar{s} > \frac{-5t+2\alpha M+c}{\gamma}$. We conclude that when Firm A offers the service in equilibrium (which

implies $s_A > \bar{s}$), it must be that $s_A > \frac{-5t+2\alpha M+c}{\gamma}$, and thus $\frac{\partial}{\partial \alpha} \pi_{SN}^B < 0$.

Proof of Proposition 5

(i) We examine the derivative of Firm A's profit with respect to M .

$$\begin{aligned}\frac{\partial}{\partial M} \pi_{SS}^i &= \frac{\partial}{\partial M} \frac{M(3(t-\alpha M)+\gamma(s_i-s_j))^2}{18(t-\alpha M)} = \frac{1}{18} \left(9(t-2\alpha M) + 6\gamma(s_i-s_j) + \frac{\gamma^2 t (s_i-s_j)^2}{(t-\alpha M)^2} \right) \\ &= D_{SS}^A \left(\frac{t}{3M} \left(\frac{\gamma(s_i-s_j)}{t-\alpha M} + 3 \right) - 2\alpha \right)\end{aligned}\quad (\text{A46})$$

$\frac{\partial}{\partial M} \pi_{SS}^i$ is positive if and only if $\frac{t}{3M} \left(\frac{\gamma(s_i-s_j)}{t-\alpha M} + 3 \right) - 2\alpha$ is positive, which is equivalent to

$$s_i - s_j > \frac{3(2\alpha M - t)(t - \alpha M)}{\gamma t} \quad (\text{A47})$$

The RHS of A47 can be either negative or positive.

(ii) Suppose only Firm A offers the service.

$$\begin{aligned}\frac{\partial}{\partial M} \pi_{SN}^A &= \frac{\partial}{\partial M} \frac{M(3t-\alpha M+\gamma s_A-c)^2}{9(2t-\alpha M)} = \left(\frac{M(3t-\alpha M+\gamma s_A-c)}{6t-3\alpha M} \right) \left(\frac{2(\alpha^2 M^2 - t(c-3t+3\alpha M-\gamma s_A))}{3M(2t-\alpha M)} \right) \\ &= D_{SN}^A \left(\frac{2(\alpha^2 M^2 - t(c-3t+3\alpha M-\gamma s_A))}{3M(2t-\alpha M)} \right)\end{aligned}\quad (\text{A48})$$

$\frac{\partial}{\partial M} \pi_{SN}^A$ is positive if and only if $\frac{2(\alpha^2 M^2 - t(c-3t+3\alpha M-\gamma s_A))}{3M(2t-\alpha M)}$ is positive, which, given the

assumption that $t > \alpha M$, is equivalent to

$$s_A > \frac{t(c-3t+3\alpha M) - \alpha^2 M^2}{\gamma t}$$

Proof of Proposition 6

In proving Proposition 6 and 7, we make the following parameters assumptions:

- i) $t > \alpha M$
- ii) $c_i > \frac{\gamma^2 M}{9(t-\alpha M)}$ ($i = A$ and B)
- iii) $c < 3t - \alpha M$ and $c > -3t + 2M\alpha + \frac{M\gamma^2}{3c_A}$ (but the latter is always satisfied given ii)

Above conditions are necessary to guarantee that both firms have positive product demand under both the

SS and the SN/NS market configurations.

We define R_i and Q as follows:

$$R_i = 9c_i(t - \alpha M) - \gamma^2 M > 0 \quad (i = A \text{ and } B) \quad (\text{A49})$$

$$\begin{aligned} Q &= 18c_A c_B (t - \alpha M) - \gamma^2 M (c_A + c_B) \quad (\text{A50}) \\ &= c_A (9c_B (t - \alpha M) - \gamma^2 M) + c_B (9c_A (t - \alpha M) - \gamma^2 M) = c_A R_B + c_B R_A > 0 \end{aligned}$$

The inequalities hold due to the above parameters assumptions.

Next we examine how the service quality and profit of Firm A change with the degree of network effects (due to symmetry, no need to do so for Firm B)

$$\begin{aligned} \frac{\partial}{\partial \alpha} S_A &= \frac{\partial}{\partial \alpha} \frac{\gamma M (9c_B (t - \alpha M) - \gamma^2 M)}{54c_A c_B (t - \alpha M) - 3\gamma^2 M (c_A + c_B)} = \frac{3\gamma^3 M^3 c_B (c_B - c_A)}{Q^2} > 0 \quad \text{if and only if } c_B > c_A \\ \frac{\partial}{\partial \alpha} \pi_{SS}^A &= \frac{\partial}{\partial \alpha} \frac{c_A M (18c_A (t - \alpha M) - \gamma^2 M) (9c_B (t - \alpha M) - \gamma^2 M)^2}{9(18c_A c_B (t - \alpha M) - \gamma^2 M (c_A + c_B))^2} \\ &= \left(\frac{c_A M R_B}{Q} \right) \left(\frac{2M \left(-((18c_B (t - \alpha M) + \gamma^2 M) R_B + 2\gamma^4 M^2) c_A^2 + 27\gamma^2 c_A c_B^2 M (t - \alpha M) - \gamma^4 c_B^2 M^2 \right)}{Q^2} \right) \end{aligned}$$

The sign of $\frac{\partial}{\partial \alpha} \pi_{SS}^A$ is equal to that of the following expression:

$$Z = -((18c_B (t - \alpha M) + \gamma^2 M) R_B + 2\gamma^4 M^2) c_A^2 + 27\gamma^2 c_A c_B^2 M (t - \alpha M) - \gamma^4 c_B^2 M^2$$

Note that the coefficient of c_A^2 is negative. Thus, Z is a concave function of c_A .

It can be show that Z is positive if and only if

$$\frac{2\gamma^2 c_B M}{27c_B (t - \alpha M) + \sqrt{81c_B^2 (t - \alpha M)^2 + 36\gamma^2 c_B M (t - \alpha M) - 4\gamma^4 M^2}} < c_A < \frac{2\gamma^2 c_B M}{27c_B (t - \alpha M) - \sqrt{81c_B^2 (t - \alpha M)^2 + 36\gamma^2 c_B M (t - \alpha M) - 4\gamma^4 M^2}}$$

$$\text{However, when } c_A = \frac{\gamma^2 M}{9(t - \alpha M)}, Z = \frac{\gamma^6 M^3 R_B}{81(t - \alpha M)^2} > 0. \text{ And when } c_A = c_B, Z = -2c_B^2 R_B^2 < 0.$$

Thus, in the range of parameters values considered, Z is positive if

$$\frac{\gamma^2 M}{9(t - \alpha M)} < c_A < \frac{2\gamma^2 c_B M}{27c_B (t - \alpha M) - \sqrt{81c_B^2 (t - \alpha M)^2 + 36\gamma^2 c_B M (t - \alpha M) - 4\gamma^2 M^2}} \text{ and negative if}$$

$$\frac{2\gamma^2 c_B M}{27c_B (t - \alpha M) - \sqrt{81c_B^2 (t - \alpha M)^2 + 36\gamma^2 c_B M (t - \alpha M) - 4\gamma^2 M^2}} < c_A < c_B, \text{ and so is } \frac{\partial}{\partial \alpha} \pi_{SS}^A.$$

Proof of Proposition 7

In Case SS, if $c_A = d_A k$ and $c_B = d_B k$.

$$R_i = 9d_i k(t - \alpha M) - \gamma^2 M > 0 \quad (i = A \text{ and } B)$$

$$Q = 18d_A d_B k^2(t - \alpha M) - \gamma^2 k M(d_A + d_B) = c_A R_B + c_B R_A > 0$$

$$\frac{\partial}{\partial k} \pi_{SS}^A = \frac{\gamma^2 M \left(d_A^2 (324 d_B k^2 (t - \alpha M)^2) - d_A (162 d_B^2 k^2 (t - \alpha M)^2 - 45 \gamma^2 d_B k M (t - \alpha M) - \gamma^2 M^2) + (\gamma^2 d_B M (9 d_B k (t - \alpha M) + \gamma^2 M)) \right)}{9Q^2}$$

This is positive if and only if the numerator $d_A^2 (324 d_B k^2 (t - \alpha M)^2) - d_A (162 d_B^2 k^2 (t - \alpha M)^2 - 45 \gamma^2 d_B k M (t - \alpha M) - \gamma^2 M^2) + (\gamma^2 d_B M (9 d_B k (t - \alpha M) + \gamma^2 M))$ is positive.

$$\begin{aligned} \frac{\partial}{\partial k} \pi_{SS}^B &= \frac{\gamma^2 M (162 d_A d_B k^2 (2d_B - d_A)(t - \alpha M)^2 + 9\gamma^2 d_A k M (d_A - 5d_B) + \gamma^4 M^2 (d_A + d_B))}{9Q^2} \\ &= \frac{\gamma^2 M \left((k(9d_A + 18d_B)(t - \alpha M) + R_A + 2R_B) R_A + 2\gamma^2 M (9d_B k (t - \alpha M) + R_B) \right) (d_B - d_A) + 2d_A R_A^2}{9Q^2} > 0 \text{ as } d_B > d_A. \end{aligned}$$