Robust and Reliable Multidiscipline Ship Design

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Mathematical formulations of reliability-based design and robust design have been developed for addressing the presence of uncertainty during optimization. Reliability addresses the effects of uncertainty on the constraints; constraints that are influenced by uncertainty are converted into new probabilistic constraints to ensure that the optimal solution will satisfy the constraints within a prescribed reliability level. Robustness is introduced by including the effects of the mean and variance of the response of the objective function in the optimization. This paper presents a formulation for applying reliability-based design and robust design within a network of parallel optimizations to account for uncertainty in multidiscipline design optimization. The method for multidiscipline design optimization under uncertainty is applied to a conceptual ship design with a model of a bulk carrier from the literature. The optimization results demonstrate that reliable and robust optimization can be applied effectively to ship design, and that it is important to consider uncertainty during the multidiscipline ship design process.

Nomenclature

α	=	weighting parameter for robust objective function
β_i	=	target reliability index for constraint i
$\mathbf{d}$	=	vector of deterministic design variables
$\boldsymbol{\sigma}$	=	vector of standard deviations for design variables and parameters with uncertainty
$\boldsymbol{\delta}$	=	vector of variation in design variables due to uncertainty
f	=	generic objective function
F_i	=	generic helpful function evaluated in discipline i
G_i	=	generic constraint
$\mathbf{g}_i$	=	inequality constraints for the ith discipline
$\mathbf{g}_T$	=	inequality constraints for the top level
μ_f	=	mean of objective function f under uncertainty
$\boldsymbol{\mu}_p$	=	vector of mean values of parameters with uncertainty
$\boldsymbol{\mu}_x$	=	vector of mean values of design variables
N_j	=	number of disciplines that share the jth design variable
$O_i^{optimal}$	=	optimal value for the objective function of the ith discipline
O_i^{start}	=	starting value for the objective function of the ith discipline
O_T	=	original top level objective function
P	=	probability distribution function
$\mathbf{p}$	=	vector of parameters with uncertainty
σ_f^2	=	variance of objective function f under uncertainty
$\mathbf{x}$	=	generic vector of design variables
$\mathbf{x}_i^{current}$	=	vector of current values of design variables for the ith discipline
$\mathbf{x}_i^{start}$	=	vector of starting values of design variables for the ith discipline
$x_{j,i}^{optimal}$	=	the optimal value of design variable x_j in the ith discipline
x_j^{target}	=	target value for the jth design variable
$\mathbf{x}_i$	=	vector of design variables for the ith discipline
$\mathbf{x}_T$	=	vector of design variables for the top level

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I. Introduction

IN design optimization, the optimal solution is a single point that minimizes (or maximizes) a function of interest while satisfying a prescribed set of constraints. Often, the effects of uncertainty are not considered during the ship design optimization process. However, in real life, design variables or environmental parameters contain uncertainty; for example, manufacturing tolerances or environmental conditions are beyond the control of the designer, but they do affect the performance of the design. Further, the simulation tools or the regression models which provide performance information for a ship during the decision-making process are additional sources of uncertainty because of their inherent variability in the predictions. Due to the variations in the uncertain design variables and parameters, or due to the uncertainty in the simulation models, the response of the optimal design will differ from the deterministic expectation. This can lead to violation of the active constraints and/or deterioration of the expected optimal performance. Capturing such effects within the early stages of the ship design process will eliminate expensive design modifications at later stages.

Multidisciplinary design optimization (MDO) describes the process of dividing the design of a system into disciplines and seeking a solution which satisfies all discipline-level constraints, improves the discipline level objectives as much as possible, and improves the overall system level objective. MDO can be a valuable tool for the design of any complex system which features competing disciplines; MDO has previously been applied to the design of an aircraft wing, an undersea vehicle, and to preliminary ship design[1-3], among many examples in the literature.

In reliability-based design, the effects of uncertainties in design variables and parameters on the constraints are considered. When reliability is introduced in a design process, the constraints which are influenced by uncertainty are converted into probabilistic constraints. This ensures that the optimal solution will satisfy the constraints in the presence of uncertainty within a prescribed reliability level. Methods for accounting for uncertainty in the constraints have been shown in References 4 and 5. Many applications of reliability-based design can be found in the literature. References 6 and 7 apply reliability-based design to vehicle design for crashworthiness; Reference 8 applies reliability-based design to an airplane wing structure; Reference 9 uses reliability-based design for structural design subject to earthquake loading; and Reference 10 applies reliability-based design to the design of an exhaust system. Additionally, reliability-based design optimization has been applied to multidisciplinary design optimization in References 11-13.

In robust design optimization, the goal is to minimize not only the objective function but also the variation in the objective function about the optimum point. Then a new robust objective function is formulated as a function of the mean and variance of the response of the objective function at a selected design point. Reference 14 provides a thorough summary of methods for applying robust optimization. Many applications of robust design optimization can be found in the literature. In Reference 15, robust optimization is applied to the design of layered plate bonding; Reference16 uses a robust design strategy with a snap-fit device; Reference 17 applies robust optimization to an airplane component; and Reference 18 introduces robustness in optimizing the design of a transistor device. Robust optimization has also been applied to multidisciplinary design problems. Reference 19 uses robust optimization with a multi-objective design application to a V6 engine; Reference 20 applies a multi-objective evolutionary algorithm for robust design of a welded beam and a bulk carrier; and Reference 21 uses multidisciplinary robust design for chemical filtration optimization.

This paper introduces a formulation for multidisciplinary design optimization under uncertainty that includes principles of reliability-based design and robust design within a network of parallel optimizations. The theoretical background is presented first, including the multidisciplinary design optimization method for accounting for the effect of uncertain design variables and parameters on the constraints. Methods for introducing a robust objective function to the multidisciplinary design optimization are also discussed.

The main new element introduced by this paper is the combination of multidisciplinary design optimization which takes into account both reliability and robustness in the solution. Additionally, this new approach is applied to the concept design of a bulk carrier to demonstrate how uncertainty can be accounted for in multidisciplinary ship design.

II. Multidiscipline Design Optimization

The mathematical background of the method which is employed in this work for driving the network of optimizations and coordinating the exchange of information is presented in this section. The optimization of several disciplines is coordinated through a top level optimization, which is typically selected as a discipline of a global nature (such as cost). Discipline level constraints are also enforced in the top level, ensuring that the final solution satisfies all of the constraints. Discipline optimizations explicitly influence the overall direction of the system

solution based on the improvement encountered at the discipline level. An additional feature of this algorithm is that different optimization methods can be used for the system and discipline level optimizations.

At the beginning of each iteration of the top level optimization, individual discipline-level optimizations are performed using the values of the design variables from the previous step of the top level optimization as the starting values. After the discipline level optimizations are complete, a group of target values for the design variables are determined based on the improvement in the discipline level objectives:

$$x_j^{\ target} = \frac{\sum_{i=1}^{N_j}\left(x_{j,i}^{\ optimal}\left|O_i^{\ start} - O_i^{\ optimal}\right|\right)}{\sum_{i=1}^{N_j}\left|O_i^{\ start} - O_i^{\ optimal}\right|}$$

(1)

In Eq. (1), $x_j^{\ target}$ is the target value for the jth design variable; $x_{j,i}^{\ optimal}$ indicates the optimal value of design variable x_j in the ith discipline. The objective function of the ith discipline is O_i where the superscripts *start* and *optimal* indicate starting and optimal values the objective functions. N_j is the total number of disciplines that share the jth design variable.

The overall top level objective function can be stated as

$$\min\left(O_T + \sum_{j=1}^{J}\left|x_j^{\ target} - x_j\right|\right)$$

(2)

where O_T is the original top level objective function and J is the total number of design variables in the top level. The effects of the discipline objective functions are included by requiring that the top level design variables match the target values, and at the same time the top level objective function is improved.

All of the discipline-level constraints are included in the top level optimization along with the top level constraints; this ensures that the optimal point will also be a feasible point for all of the disciplines. The constraints are expressed mathematically as

$$\mathbf{g}_T(\mathbf{x}_T) \leq 0$$
$$\mathbf{g}_i(\mathbf{x}_i) \leq 0$$

(3)

where $\mathbf{g}_T$ are the inequality constraints for the top level and $\mathbf{g}_i$ are the inequality constraints for the ith discipline. The vector of design variables for the top level objective is $\mathbf{x}_T$ and the vector of design variables for the ith discipline is $\mathbf{x}_i$.

The MDO algorithm is illustrated in Fig. 1. The top level coordinates information about the top level objective, the top and discipline level design variables, and the top and discipline level constraints with the information from the discipline level optimizations. The algorithm also includes the capability to use compute (optional) additional useful functions F_i that may be used in the evaluation of the objective functions.

III.   Reliability-Based Design

The multidisciplinary optimization framework used in this work is also capable including the effect of uncertainty in design variables and/or parameters on the constraints. The framework uses a single-loop reliability-based design optimization (RBDO) algorithm[22,23].

The single-loop RBDO algorithm converts the probabilistic optimization problem into an equivalent deterministic problem. Then the single-loop RBDO problem for one objective function f and the accompanying set of constraints can be stated as:

$$\min_{\mathbf{x}, \boldsymbol{\mu}_x} f(\mathbf{d}, \boldsymbol{\mu}_x, \boldsymbol{\mu}_p)$$

(4)

subject to: $G_i(\mathbf{d}, \mathbf{X}_i, \mathbf{p}_i) \leq 0 \qquad i = 1, \ldots, n$
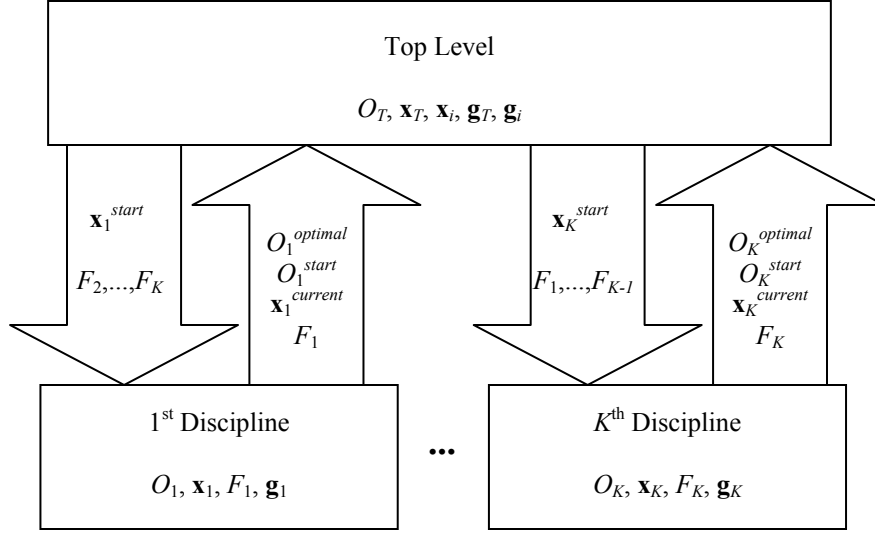
(5)

Figure 1. Flow chart of the multidisciplinary design optimization algorithm.

where f is the objective function, $\mathbf{x}$ is the vector of deterministic design variables, $\boldsymbol{\mu}_x$ is the vector of the mean values of the random design variables, and $\boldsymbol{\mu}_p$ is the vector of the mean values of the parameters. The n constraints G_i are functions of $\mathbf{d}$ and the quantities:

$$\mathbf{X}_i = \boldsymbol{\mu}_x - \boldsymbol{\sigma}\beta_i\alpha_i \tag{6}$$

$$\mathbf{p}_i = \boldsymbol{\mu}_p - \boldsymbol{\sigma}\beta_i\alpha_i \tag{7}$$

$$\alpha_i = \frac{\boldsymbol{\sigma}\cdot\nabla G_i(\mathbf{d},\mathbf{X}_i,\mathbf{p})}{\left\|\boldsymbol{\sigma}\cdot\nabla G_i(\mathbf{d},\mathbf{X}_i,\mathbf{p})\right\|} \tag{8}$$

$$\mathbf{d}^l \leq \mathbf{d} \leq \mathbf{d}^u \tag{9}$$

$$\boldsymbol{\mu}_x^l \leq \boldsymbol{\mu}_x \leq \boldsymbol{\mu}_x^u \tag{10}$$

The target reliability index for the ith constraint is β_i, which describes the desired confidence that a constraint will be satisfied. α_i is the normalized gradient of the ith constraint, and $\boldsymbol{\sigma}$ is the vector of standard deviations for the design variables and parameters. Equations (9) and (10) describe the upper and lower bounds on the design variables and the mean values.

The objective function is evaluated at the mean point ($\mathbf{d}$, $\boldsymbol{\mu}_x$, $\boldsymbol{\mu}_p$), and the constraints are evaluated at the deterministic optimum. This introduces a safety margin in each constraint determined by the target reliability index and the rate of change of the constraint function.

IV. Robust Optimization

In this work, the robust objective function R_f for an objective function f is defined as in Refs. 14 and 19, using a weighted-sum approach:

$$R_f(\mathbf{x}) = \alpha\mu_f(\mathbf{x}) + (1-\alpha)\sigma_f^2(\mathbf{x}) \tag{11}$$

where $\mu_f(\mathbf{x})$ and $\sigma_f^2(\mathbf{x})$ are the mean and variance of the response of f at the design point $\mathbf{x}$, where α is a weighting parameter, $0 \leq \alpha \leq 1$, that can be adjusted for the relative importance of the mean and variance for the particular application.

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The following equations (from Ref. 14) can be used to calculate the mean and variance of the objective function f that are needed for Eq. (11):

$$\mu_f(\mathbf{x}) = \int f(\mathbf{x} + \boldsymbol{\delta}, \mathbf{p}) P(\boldsymbol{\delta}, \mathbf{p}) d\boldsymbol{\delta} d\mathbf{p} \tag{12}$$

$$\sigma_f^2(\mathbf{x}) = \int (f(\mathbf{x} + \boldsymbol{\delta}, \mathbf{p}))^2 P(\boldsymbol{\delta}, \mathbf{p}) d\boldsymbol{\delta} d\mathbf{p} - (\mu_f(\mathbf{x}))^2 \tag{13}$$

In Eqs. (12) and (13), the design variables $\mathbf{x}$ are assumed to be deterministic, but with a random variation term $\boldsymbol{\delta}$; that is, the probabilistic design variables can be described by $\mathbf{x} + \boldsymbol{\delta}$. $\mathbf{p}$ is the vector of random parameters, and $P(\boldsymbol{\delta}, \mathbf{p})$ is the joint probability distribution function for $\boldsymbol{\delta}$ and $\mathbf{p}$.

V. Bulk Carrier Model

Reference 24 presents the preliminary design of a bulk carrier using simple empirical models. The optimization of this preliminary design has been studied in the literature, including using a multi-objective design optimization approach[25]; implementing a particle swarm optimizer[1]; and applying neural networks to reduce computational effort[20].

The ship is described by six design variables: length L, beam B, depth D, draft T, block coefficient C_B, and speed V_k. The design variables are used to evaluate the three objective functions: lightship weight, annual cargo, and transportation cost. Details for the calculation of the objective functions are included in Table 1. Table 1 shows the objective functions, design, variables, constraints, and other formulas used in the calculations.

The notation in Table 1 has been modified slightly from Ref. 24. It is worth noting that the empirical constraint for stability listed in Table 1 comes from the empirical formula and constraint in Ref. 24 for GM; the empirical expression for GM is given by

$$GM = 0.53T + \frac{(0.085C_B - 0.002)B^2}{T \cdot C_B} - 1.0 - 0.52D \tag{14}$$

and the requirement on GM for stability is

$$GM \geq 0.07B \tag{15}$$

VI. Application to the Bulk Carrier Example

The new formulation that integrates algorithms for multidiscipline design, reliability-based design, and robust design is applied to the preliminary design for the bulk carrier. The transportation cost was the system level optimization, while the lightship weight and annual cargo comprise the discipline level optimizations. The goal is to minimize the transportation cost and the lightship weight while maximizing the annual cargo. (Note that the optimizer only seeks to minimize each objective function, so the negative of the annual cargo is minimized.)

The initial point for the optimization analysis was $(L, B, D, T, C_B, V_k) = (195, 32.31, 20, 10.5, 0.7, 16)$, because this point satisfies all of the constraints and has been used in previous work with the bulk carrier model. The objective functions were scaled by their values at this initial point to prevent the very large values (order 10^5) of the annual cargo from dominating over the smaller (order 10^0) values of the transportation cost. The values of the objective functions at the initial point are: 10,303 t lightship weight; 547,860 t/yr annual cargo; and 9.926 £/t transportation cost. Multiple analyses were performed in order to compare deterministic and probabilistic results, and for investigating the effects of alternative robustness definitions on the results.

Objective Functions:		Design Variables:	
Lightship Weight (t)	$LS = W_S + W_O + W_M$	Length, L (m)	$150 \leq L \leq 274.32$
Annual Cargo (t/yr)	$AC = DW_C \times RTPA$	Beam, B (m)	$20 \leq B \leq 32.31$
Transportation Cost (£/yr)	$TC = \dfrac{C_A}{AC}$	Depth, D (m)	$13 \leq D \leq 25$
		Draft, T (m)	$10 \leq T \leq 11.71$
		Block Coefficient, C_B	$0.63 \leq C_B \leq 0.75$
		Ship Speed, V_k (knots)	$14 \leq V_k \leq 20$

Constraints:	
Length-to-beam ratio	$\dfrac{L}{B} \geq 6$
Length-to-depth ratio	$\dfrac{L}{D} \leq 15$
Length-to-draft ratio	$\dfrac{L}{T} \leq 19$
Froude number	$Fn \leq 0.32$
Deadweight	$25{,}000 \leq DW \leq 500{,}000$
Empirical constraint on T and DW	$T - 0.45DW^{0.31} \leq 0$
Empirical constraint on T and D	$T - 0.7D - 0.7 \leq 0$
Empirical constraint for stability	$0.07B - 0.53T - \dfrac{(0.085C_B - 0.002)B^2}{TC_B} + 1 + 0.52D \leq 0$

Calculations for Model:	
Steel Weight	$W_S = 0.034L^{1.7}B^{0.7}D^{0.4}C_B^{0.5}$
Outfit Weight	$W_O = L^{0.8}B^{0.6}D^{0.3}C_B^{0.1}$
Coefficient for P Calculation	$a = 4977.06C_B^2 - 8105.61C_B + 4456.51$
Coefficient for P Calculation	$b = -10847.2C_B^2 + 12817C_B - 6960.32$
Froude Number	$Fn = \dfrac{0.5144V_k}{\sqrt{9.8065L}}$
Displacement	$\Delta = 1.025LBTC_B$
Power	$P = \dfrac{\Delta^{2/3}V_k^3}{a + b \cdot Fn}$
Machinery Weight	$W_M = 0.17P^{0.9}$
Deadweight	$DW = \Delta - LS$
Daily Fuel Consumption	$DC = 0.2 + \dfrac{0.19 \times 24}{1000}P$
Sea Days	$D_S = \dfrac{5000}{24V_k}$
Fuel Carried	$FC = DC(D_S + 5)$
Crew, Stores, and Water	$CSW = 2DW^{0.5}$
Cargo Deadweight	$DW_C = DW - FC - CSW$
Port Days	$D_P = 2\dfrac{DW_C}{8000} + 1$
Round Trips per Year	$RTPA = \dfrac{350}{D_S + D_P}$
Ship Cost	$C_S = 1.3(2000W_S^{0.85} + 3500W_O + 2400P^{0.8})$
Capital	$C = 0.2C_S$
Running Cost	$C_R = 40000DW^{0.3}$
Voyage Cost	$C_V = 1.05DC \times D_S \times 100 + 6.3DW^{0.8}$
Annual Cost	$C_A = C + C_R + C_V \times RTPA$

Table 1. Bulk carrier model definition.

A. MDO without Uncertainty

The multidisciplinary optimization was run for the bulk carrier example using the objective functions and constraints listed in Table 1, without including uncertainty in any of the design variables or parameters. This was done to give a baseline for comparison to later results where uncertainty is considered. The results are listed in Table 2 in the column labeled Deterministic Optimum.

	Deterministic Optimum	Reliable Optimum	Reliable & Robust Optimum
Transportation Cost	8.424	8.635	8.838
Annual Cargo	506,320	550,140	566,690
Lightship Weight	8,198	8,803	8,462
L	188.90	193.86	182.61
B	31.485	32.310	30.435
D	15.729	15.364	15.729
T	11.710	11.455	11.710
C_B	0.630	0.657	0.750
V_k	14.000	15.027	15.027

Table 2.   Summary of results from multiple MDO analyses.

B. MDO with Reliability-Based Design

Uncertainty was introduced into the bulk carrier model in two ways: uncertainty in a design variable and uncertainty in a parameter. Uncertainty in a design variable was introduced for the ship speed. For a selected design ship speed V_{design}, the actual ship speed follows a normal distribution about V_{design}; that is,

$$V_{actual} = V_{design} + v \tag{16}$$

where V_{actual} is the actual ship speed and v is a normally distributed random variable with mean zero; the uncertainty in the ship speed was defined in this manner to agree with the form of Eqs. (12) and (13). Equation (15) describes the physical situation that the intended ship speed is V_{design}, but due to sea conditions the actual ship speed varies by the amount v. For this analysis, v follows a normal distribution with mean zero and standard deviation of 0.5 knots.

Next, uncertainty in a parameter was introduced in the model. The model defined by Reference 24 as given in Table 1 uses the following regression equation to calculate the steel weight, W_S, of the ship:

$$W_S = 0.034L^{1.7}B^{0.7}D^{0.4}C_B^{0.5} \tag{17}$$

To apply uncertainty in a parameter to this model, the exponent on the ship length L is considered to be a random parameter, ε:

$$W_S = 0.034L^{\varepsilon}B^{0.7}D^{0.4}C_B^{0.5} \tag{18}$$

This uncertain parameter can be used to describe uncertainty in the model itself and to illustrate the effects of any modeling inaccuracies. For this analysis, the parameter ε follows a normal distribution with mean 1.7 and standard deviation 0.05.

There are five total constraints that are affected by these uncertainties. The simplest constraints are the upper and lower limits on the ship speed. The other constraints affected by the uncertainties (repeated from Table 1) are the constraints on Froude number Fn and deadweight DW:

$$Fn \leq 0.32 \tag{19}$$

$$25,000 \leq DW \leq 500,000 \tag{20}$$

$$T - 0.45DW^{0.31} \leq 0 \tag{21}$$

The multidisciplinary optimization was run including the uncertainty in the constraints (where applicable) and the results are included in Table 2. A 98% reliability level was prescribed for the probabilistic constraints. It is immediately clear that the performance in all three objectives at this selected optimum is inferior to the performance at the deterministic optimum. This is expected because a safety margin has been introduced in order to satisfy the constraints in the presence of uncertainty. The constraints were evaluated at the optimum point and the values are shown in Table 3; note that the constraints that are affected by uncertainty are indicated with a check mark in the column labeled Uncertainty. Two constraints are active, but the three constraints which are affected by the uncertainty have an appropriate margin to account for the variability in the ship speed V_k and exponent ε.

Constraint	Constraint Value	Uncertainty
$\dfrac{L}{B} \geq 6$	6.000	
$\dfrac{L}{D} \leq 15$	12.6180	
$\dfrac{L}{T} \leq 19$	16.9241	
$Fn \leq 0.32$	0.1773	✓
$25{,}000 \leq DW \leq 500{,}000$	39,435	✓
$T - 0.45DW^{0.31} \leq 0$	-0.5110	✓
$T - 0.7D - 0.7 \leq 0$	-2.0206×10^{-14}	
$0.07B - 0.53T - \dfrac{(0.085C_B - 0.002)B^2}{TC_B} + 1 + 0.52D \leq 0$	-2.2892	

Table 3. Constraint evaluation using reliability-based design optimization.

C. MDO with Uncertainty in Constraints and Objective Functions

A slightly modified form of Eq. (11) (similar to that of Reference 19) is used to apply robustness to the bulk carrier example for each objective function f:

$$R_f(L,B,D,T,C_B,V_k) = \alpha \frac{\mu(L,B,D,T,C_B,V_k)}{f_0} + (1-\alpha)\frac{\sigma(L,B,D,T,C_B,V_k)}{\sigma_0} \tag{22}$$

The mean μ and variance σ are evaluated at the design point (L, B, D, T, C_B, V_k). The variance is scaled by σ_0, the value of the variance at the initial point. The mean is scaled by f_0, the value of the objective function f at the initial point, instead of the mean value at the initial point in order to allow comparison between the robust optimization results and the other results.

To calculate the necessary mean and standard deviation, Equations (12) and (13) are used with the two random variables v and ε. Then the equations become:

$$\mu(L,B,D,T,C_B,V_k) = \iint f(L,B,D,T,C_B,V_k + v, \varepsilon)p(v,\varepsilon)dvd\varepsilon \tag{23}$$

$$\sigma^2(L,B,D,T,C_B,V_k) = \iint (f(L,B,D,T,C_B,V_k + v, \varepsilon))^2 p(v,\varepsilon)dvd\varepsilon - (\mu(L,B,D,T,C_B,V_k))^2 \tag{24}$$

where f is any of the objective functions and $p(v, \varepsilon)$ is the joint probability distribution function for v and ε. The ship speed variation and the exponent are assumed to be independent, so the equations become

$$\mu(L,B,D,T,C_B,V_k) = \iint f(L,B,D,T,C_B,V_k + v, \varepsilon)p_v(v)p_\varepsilon(\varepsilon)pdvd\varepsilon \tag{25}$$

$$\sigma^2(L,B,D,T,C_B,V_k) = \iint \{f(L,B,D,T,C_B,V_k + v, \varepsilon)\}^2 p_v(v)p_\varepsilon(\varepsilon)dvd\varepsilon - \{\mu(L,B,D,T,C_B,V_k)\}^2 \tag{26}$$

where $p_v(v)$ and $p_\varepsilon(\varepsilon)$ are the probability distribution functions for v and ε, respectively.

The integrals are evaluated numerically during the optimization. Each PDF is integrated over ±4σ to cover approximately 99.99% of the area under the normal curve. The numerical integration is performed using 200 points in each random variable, or 40,000 points total over the integrand. The effect of increasing or decreasing the number of points was investigated and it was found that 200 points led to a satisfactory compromise of computing time and accuracy. Additionally, the results of the integration were compared to the results of a simple Monte Carlo simulation, which randomly placed 1000 points in the design space according to the distributions of v and ε. The mean and variance resulting from the Monte Carlo simulations confirmed the results of the numerical integration.

The robust and reliable optimization was performed using the parameter value $\alpha = 0.5$; the results are included in Table 2. The constraints at the robust optimum were evaluated and are included in Table 4. All of the constraints are satisfied, with two of the constraints active: the constraint on the length-to-beam ratio and the empirical constraint on the draft and depth. The safety margins introduced in the constraints are comparable to the margins in the probabilistic constraints of the MDO reliability analysis. However, the margins differ because in the current case the optimum has shifted due to the robust considerations.

Constraint	Constraint Value	Uncertainty
$\dfrac{L}{B} \geq 6$	4.7071×10^{-6}	
$\dfrac{L}{D} \leq 15$	11.6102	
$\dfrac{L}{T} \leq 19$	15.5946	
$Fn \leq 0.32$	0.1827	✓
$25{,}000 \leq DW \leq 500{,}000$	$41{,}569$	✓
$T - 0.45DW^{0.31} \leq 0$	-0.4528	✓
$T - 0.7D - 0.7 \leq 0$	2.2427×10^{-14}	
$0.07B - 0.53T - \dfrac{(0.085C_B - 0.002)B^2}{TC_B} + 1 + 0.52D \leq 0$	-1.4099	

Table 4. Constraint evaluation using reliable and robust optimization.

To study the effects of the robust optimization on the optimum design, the statistics at the three different optima were computed; Table 5 summarizes the statistics for the three optimization approaches. The deterministic optimum shows the best performance for the transportation cost (the top level objective) and the lightship weight. However, for the deterministic case, without the consideration of uncertainty in the constraints, it is unlikely that the constraints would be satisfied when considering the uncertainty of the ship speed V_k and exponent ε. For the robust and reliable case, the standard deviation has been reduced from the reliable case for all three of the objective functions.

	Deterministic Optimum	Reliable Optimum	Robust & Reliable Optimum ($\alpha = 0.5$)
Transportation Cost	8.4240	8.6351	8.8380
Transportation Cost Mean	8.5401	8.7523	8.9472
Transportation Cost Standard Deviation	0.7174	0.7241	0.6792
Annual Cargo	506,320	550,140	566,690
Annual Cargo Mean	503,650	547,300	564,160
Annual Cargo Standard Deviation	20,504	21,022	19,266
Lightship Weight	8,198	8,803	8,462
Lightship Weight Mean	8,434	9,137	8,698
Lightship Weight Standard Deviation	1,858	2,012	1,857

Table 5. Statistics for the optimization results.

To illustrate the effect of selecting a reliable optimum, a Monte Carlo simulation was performed to generate random realizations of the ship design about the optimum design values. For the deterministic optimum design (188.90, 31.485, 15.729, 11.710, 0.630, 14.000), only 2424 out of 10,000 randomly generated points satisfied all of

the constraints in the presence of uncertainty. For the reliable optimum design (193.86, 32.310, 15.364, 11.455, 0.657, 15.027), 9574 out of 10,000 randomly generated points satisfied all of the constraints in the presence of uncertainty; this is a very significant improvement over the deterministic case. For the reliable and robust optimum design (182.61, 30.435, 15.729, 11.710, 0.750, 15.027), 9595 out of 10,000 randomly generated points satisfied all of the constraints in the presence of uncertainty. The results for the robust and robust and reliable optima are reasonable for the 98% reliability used in the analysis.

To visualize the results, the values of the objective functions at the optima can be plotted. A simple Monte Carlo technique was applied to approximate the Pareto front in the three-dimensional plot (this method was selected for convenience, since the purpose of this work was not to find the Pareto front). The optima are shown in two two-dimensional plots in Fig. 2: the deterministic optimum is shown in blue, the reliable optimum is shown in red, and the robust and reliable optimum is shown in green.
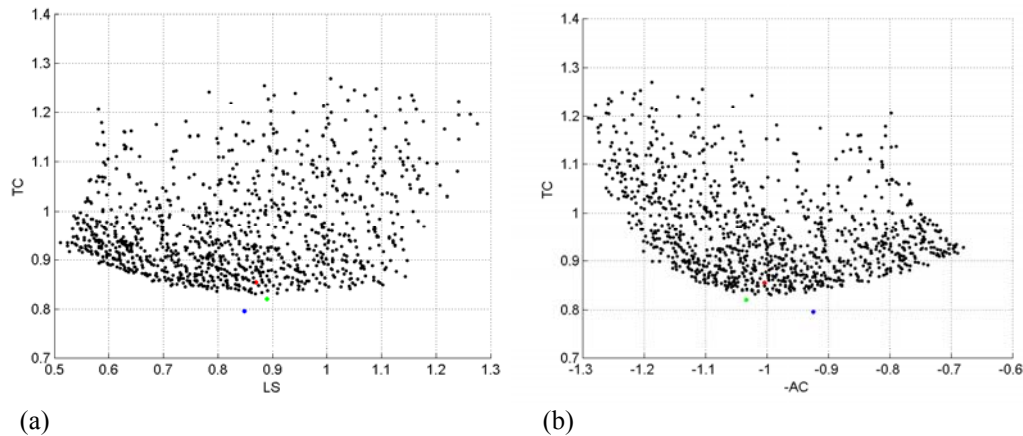

(a)   (b)

Figure 2. Approximated Pareto front plots with optima.

In Fig. 2, the objective functions are scaled so that all of the objective functions have a magnitude of one at the initial point. Fig. 2(a) shows the scaled lightship weight (LS) versus the scaled transportation cost (TC), and Fig. 2(b) shows the negative of the scaled annual cargo (-AC) versus the scaled transportation cost. Note that for this problem which has three objective functions, the Pareto front would be represented by a surface; the two-dimensional plots in Fig. 2 effectively show a projection of an approximation of the surface.

Figure 2 shows not only that the new MDO algorithm is selecting points that perform relatively well, but also that the algorithm seeks to reduce the top level objective as much as possible (transportation cost) while the discipline level objectives are not as important.

VII.   Conclusion

Methods of reliability-based design and robust optimization have been applied to a multidiscipline design optimization framework. The multidiscipline design optimization is applied to the conceptual design of a bulk carrier using a model from the literature. The results for the optimum ship design are compared for cases which do not consider uncertainty and for cases which consider reliable optimization and reliable and robust optimization.

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