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# Analyzing Pricing Strategies for Online-Services with Network Effect

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**Abstract**

*In this study, we model firms that sell a product and a complementary online service, where only the latter displays positive network effects. That is, the value each consumer derives from the service increases with the total number of consumers that subscribe to the service. In addition, the service is valuable only to consumers who buy the product. We consider two pricing strategies: 1) bundle pricing, in which the firm charges a single price for the product and the service; and 2) separate pricing, in which the firm sets the prices of the product and the service separately, and consumers self-select whether to buy both or only the product. We show that, in contrast to the common result in the bundling literature, often the monopolist chooses not to offer the bundle (he either sells the service separately or not at all) while bundling would increase consumer surplus and social welfare. Thus, under-provision of the service can be the market outcome. We also demonstrate that network effects may cause the under-provision of the service.*

**Keywords**: Bundling, Network Effects, Price Discrimination, Online Services, Online Game Industry

## 1. Introduction

Advances in information and communication technologies, such as Web 2.0 and 'Social Technologies' (Li and Bernoff 2008), enable firms to offer a variety of online services, such as blogs, peer-to-peer file sharing, social networking, and online gaming platforms, to their customers. Most of these online services promote relationship building and interactivity among the users, and thus exhibit positive network effects. That is, the value of the service to a user increases with the number of other consumers that subscribe to the service. Offering the service can increase a firm's revenue in two ways; it can generate direct revenue from service subscribers and, in addition, it can increase the demand for the firm's products. However, offering the service can be costly for the firm and thus not always profitable. In this paper, we model a firm that sells a product and can offer a complementary online service that displays positive network effects. We analyze the firm's optimal provision and pricing of the online service and derive strategic and welfare implications.

An example of a company that sells a product and offers a complementary online service is Ganz. Ganz released Webkinz, toy stuffed animals, on April 29 2005. The toys are similar to many other small plush toys (see an example in Figure 1); however, each Webkinz toy has an attached tag with a unique secret code printed on it that allows access to the Webkinz World website. On Webkinz World, the secret code allows the customer to own a virtual version of the pet (see Figure 2). The customer can then play games online, use KinzCash to buy virtual accessories for his virtual pet, and answer trivia questions to earn more KinzCash. Additional value is derived from interacting with other users of the Webkinz World website. A user can invite other kids' virtual pets to his online room, chat with them and play online games together. Thus, although the online service provides value to the customer even if no one else uses it, it often provides higher value when more kids do use it. In addition, the social site offers a more engaging experience than a mere stuffed animal does, thus allowing Ganz to charge a premium price for its stuffed animals (the price paid when buying the toy includes "free" access to Webkinz World online). In this example, the product and the online service are sold as a bundle, and a consumer who is interested only in a plush toy is less likely to become a customer due to the high price of the Webkinz plush toy.


**Figure 1. Webkinz beagle stuffed animal sold on Amazon for $18.95. The price includes the stuffed animal and access to Webkinz World website.**


**Figure 2. Webkinz World: each user has a virtual pet which looks like the stuffed animal purchased. The virtual pets can play with each other, visit each other's room, chat etc.**

Another example of a company that sells products and also offers a complementary online service to its customers is Blizzard, a leader in the computer games industry. A consumer who buys a computer game from Blizzard can play the game offline in the *single-player* mode. Playing offline, the consumer plays pre-defined scenarios, programmed in the game package, against hypothetical competitors with artificial intelligence. However, if the consumer connects to Blizzard's online gaming platform, *Battle.net*, he can also play in the *multi-player* mode, playing in real-time against other remote human players.

The set of pre-programmed game strategies included in the game package is narrower than the set of strategies that can be used by human players. Therefore, many Blizzard customers are willing to pay a premium in order to access the online service and play against remote human players. However, as is evident from several online forums, there are some consumers who prefer the simplicity and ease of playing alone in the single-play mode. Such consumers prefer to pay less and obtain only the basic game package.

There are many examples of firms who facilitate the creation of online communities for consumers who buy their products. Online communities are a typical online service that displays strong network effects. Dell, for example, successfully utilizes online communities in its business (Li and Bernoff 2008).

In Dell's online community[1], Dell's customers can share technical information and knowledge that they learn while using Dell's products. Its online community not only saves Dell millions of dollars in customer support, but it generates additional value for its customers from communicating with peers. Oracle[2] and IBM[3] also operate online communities for users and developers of their products. Microsoft operates an online community, Zune Social[4], where it's Zune MP3 player users can share music they purchase.

In this paper, we examine a firm that sells a product and may offer a complementary online service to its customers. We assume that the cost of operating the service increases with the number of users and that the service exhibits positive network effects. In addition, we assume that the service has no value to consumers who do not buy the product. The firm can either bundle the service with the product and sell both for a single price[5] or charge a separate service subscription fee, allowing some customers to pay a lower price to get only the product. The firm may also choose not to operate the service at all. To the best of our knowledge, this is the first paper to examine a bundling decision when one of the two components displays network effects.

We provide answers to the following research questions: 1) should the monopoly offer a bundle of product and service or sell the service separately? 2) are online services being over- or under-supplied by the monopoly compared to the socially optimal level?; and 3) does the fact that the service exhibits network effects affect the answers to the previous questions?

Both pricing strategies (bundling and selling the service separately) were observed in the online game industry in 2008. Blizzard Entertainment sold the games *Starcraft* and *Diablo II* each for $19.98. A customer could play these games either in the single-play mode, playing predefined scenarios, or in the multi-play mode, where he connects via the Internet to *Battle.net* and plays in real-time with other players

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[1] http://www.dell.com/community

[2] For example, Oracle E-Business Suite Application Community - http://www.oracle.com/applications/community/e-business-community.html

[3] IBM developerWorks http://www.ibm.com/developerworks/

[4] http://social.zune.net/

[5] This is the same as offering the service "free of charge", while increasing the price of the product.

from all over the world. The single payment included unlimited access to *Battle.net*. On the other hand, Blizzard sold the base package of *World of Warcraft*, which supports only the single play mode, for $36.99, and required players to pay a monthly subscription fee of $15 to play in the multi-play mode. In this paper, we examine when a firm should choose one strategy over the other and whether the strategy that maximizes the firm's profit also maximizes social welfare and consumer surplus.

We show that as the cost of providing the service decreases, the degree of network effects increases, or the number of potential customers increases, the bundling strategy may dominate selling the service separately. It is interesting to note that in the absence of network effects, the size of the market would have no effect on the optimal selling strategy. However, because the service exhibits network effects, as the number of potential customers increases (perhaps due to advertisements or increased popularity of the product) the seller might find it profitable to stop selling the service separately and start bundling it with the product. In addition, we find that when considering online services with little or no inherent value (i.e., the value of the service to a user comes mainly from the participation of other users), the seller should never sell the service separately. Instead, he should offer the service bundled with the product if the cost of offering the service is below a threshold value, and should not offer the service otherwise. Thus, as the nature of online services changes so that they rely less on inherent functionalities and rely more on interactions between users, we should expect to see less services sold separately.

Addressing our second research question, we show that the service may be supplied less than is socially optimal. Especially, we find that, under certain conditions, bundling is preferred by consumers and maximizes social welfare, but the monopoly chooses to sell the service separately. This result differs from the common findings reported in the bundling literature, according to which the monopoly uses bundling to extract surplus from consumers, who cannot benefit from bundling (Adams and Yellon 1976, Bakos and Brynjolfsson 1999). We also show that in some cases it is optimal for the firm not to offer the service, but social welfare and consumer surplus would be maximized when the service is offered. We demonstrate that network effects can be the reason why the service is often under-provided compared to the socially optimal level.

The paper structure is as follows. In Section 2, we review the related literature. In Section 3, we present our model and derive the optimal monopolist's strategy for different parameters values. In Section 4 we discuss welfare implications, and in Section 5 we examine how network effects drive the results. We conclude in Section 6.

## 2. Literature Review

Our study is related to the literature on network effects and on bundling. Under-provision of a single product with network effects has been previously shown in the literature. However, most of the bundling literature shows that consumers are better off when the monopoly does not offer the bundle but sell the components separately (Schmalensee 1984, McAfee et al (1989), Bakos and Brynjolfsson 200). Thus, in our setup, which involves both network effects and a bundling decision, it is not clear which effect would be stronger; one might expect to see over-provision of the service rather than under-provision (i.e., the monopoly sells a bundle while selling the service separately maximizes social welfare) due to the fact that bundling is usually more beneficial to the firm than to consumers. The question is thus whether network effects can make consumers prefer bundling more than the firm. This paper shows that this is indeed so.

### 2.1 Network Effects

Network effects arise when the utility that a user derives from a product increases with the number of other consumers that use the same or compatible product (Katz and Shapiro 1985). Therefore, a customer's utility from a product that displays network effects is usually modeled as a function of the product's inherent value and of the number of customers using the product (Ellison and Fudenberg 2000). In addition, most models consider the network effects to be linear in the size of the user-base (Katz and Shapiro 1986, Fudenberg and Tirole 2000, Jing 2007).

In the classic models of products with network effects, the value a consumer derives from the network is independent of the consumer type. Specifically, consumers have homogenous network valuations though they have heterogeneous product valuations (Cabral et al. 1999, Fudenberg and Tirole 2000, Jing 2007). In more recent models, however, a multiplicative function is used in which the consumer type determines both his product valuation and the benefit he obtains from the network (e.g., Ellison and Fudenberg 2000, Sundararajan 2003, Sundararajan 2004). In such models, a consumer with high (low) product valuation also has a high (low) marginal valuation for the network of product users. Following this recent approach, in our paper the consumer's type determines both his valuation for the inherent functionalities of the service and his valuation for the network of service users.

A large number of studies on network goods examine product compatibility and standardization (e.g., Farrell and Saloner 1986, Katz and Shapiro 1985), technology adoption (e.g., Choi 1994, Katz and Shapiro 1986) and entry deterrence (Cabral et al. 1999, Fudenberg and Tirole 2000). The concept of Fulfilled Expectation Equilibrium is often used in deriving the firm's optimal pricing strategy. In the presence of network effects, consumers purchasing decision is based on the expected network size, and under the Fulfilled Expectation Equilibrium requirement the realized demand indeed equals the expected network size (Katz and Shapiro 1985, Palma et al. 1999, Sundararajan 2004.)

The present paper contributes to the literature on network effects as it examines a monopolist's bundling decision when one of the components displays positive network effects. Specifically, we show that a service that displays positive network effects is often under-supplied by a monopoly. This finding is in accordance with arguments from the network externalities literature, according to which network goods may be adopted less than is socially optimum (Katz and Shapiro 1986, Farrell and Klemperer 2007). For example, Katz and Shapiro (1994) state that in the presence of network externalities, social marginal benefits from an increase of one unit in network size exceed private (i.e., the firm's) marginal benefits, and thus the equilibrium network size is smaller than the socially optimal network size. Farrell and Saloner (1985) suggest that excess inertia or excess momentum may be the explanation for inefficient adoptions of goods with network externalities. Sundararajan (2004) derives the monopolist's optimal nonlinear pricing strategy when consumers have heterogeneous network valuations and may purchase variable quantities of the good. He shows that the product might be under-supplied relative to the socially optimal level.

## 2.2 Bundling

Our work also contributes to the bundling literature. The seminal paper by Adams and Yellon (1976) provides a two-good bundling model and examines three pricing strategies: pure components, pure bundling, and mixed bundling. In the mixed bundling strategy each individual component as well as the bundle are sold to consumers, and the bundle price is lower than the sum of the two components' prices. While Adams and Yellen (1979) consider negative correlation between the valuations for the two goods, McAfee et al. (1989) allow for positive correlation or independence of valuations. Schmalensee (1984) considers a case in which one of two products is provided competitively while the firm is the sole vendor of the other product. Schmalensee (1984) and McAfee et al. (1989) show that mixed bundling always dominates offering only a bundle, and examine under which conditions the mixed bundling strategy yields higher profit than the pure component strategy. Both show that, although in some cases the mixed bundling strategy is optimal for the monopolist, the pure component strategy always yields higher consumer surplus.

Our model differs from those in the above papers in several ways. First, in our model one of the components, namely the service, has no value without the other, and thus consumers never subscribe to the service without buying the product; they either buy both or only the product. Therefore, in our setup, the seller needs to consider only two strategies when offering the service: pure bundling, forcing all customers who buy the product to also buy the service, or selling the two separately and thus allowing some consumers to buy only the product while others buy the product and the service. There is never a group of consumers that buys only the service and thus there is no difference between a strategy that sets two prices, one for the product and one for the service, and a strategy that sets a product price and a bundle price [6]. Second, we assume that the valuations for the two goods (the product and the service) are independent of each other. Third, we assume that one of the two components, the service, displays network effects. To the best of our knowledge this is the first paper to consider a bundling decision in this

[6] In our model, a strategy with price $f$ for the service, $p$ for the product and $p_b$ for a bundle of the two, where $p_b<p+f$, is identical to a strategy with prices $p$ for the product and $(p_b - p)$ for the service.

context. Lastly, unlike in Adams and Yellen (1976), in our model, the seller cannot execute first-degree price discrimination.

Bakos and Brynjolfsson (1999) model a monopolist selling a large number of information goods and show that bundling enables the firm to capture most of the consumer surplus, as it reduces the variance in consumers' valuations and makes the demand more elastic. In their following study (Bakos and Brynjolfsson 2000), they introduce a positive marginal production cost and a distribution cost for delivering the bundle to each individual customer. They show that these two cost components play a key role in the monopolist's decision whether or not to bundle the goods. In our model, the marginal cost of providing the service has a similar effect to the production cost from Bakos and Brynjolfsson (2000), i.e., the firm is more likely to offer a bundle of product and service if the marginal cost of the service decreases.

The literature generally agrees that bundling enables a seller to capture more value from consumers and thus it reduces the consumer surplus (Schmalensee (1984), McAfee et al. (1989), Bakos and Brynjolfsson 2000). However, several studies find that, under certain unique conditions, both the consumers and the seller may benefit from bundling. For instance, Salinger (1995) suggests that if producing a bundle of two goods costs less than producing the two goods separately, both the producer and the consumers may benefit from bundling. Dansby and Conrad (1984) show that if the value of a bundle is superadditive, i.e. the value of a bundle is greater than the sum of the values of the individual goods, bundling may increase both the seller's profit and consumer surplus.

Finally, Dewan and Freimer (2003) examine a monopolist who sells base software, such as an operating system, and an add-in, such as anti-virus software or Internet browser. The add-in has no value without the base software. In contrast to prior works, they show that often consumers prefer a bundle, while the monopoly chooses to sell the add-in separately. This result hangs on the assumption that some users incur a penalty when the add-in is bundled with the base software.

Our model is similar to that in Dewan and Freimer because both models examine a monopoly's bundling decision when one of the two components (the add-in in Dewan and Freimer 2003, and the online service in this paper) has no value without the other. Thus, the bundling problem is asymmetric. However, while in Dewan and Freimer (2003) both the base product and the add-in have zero marginal cost (both are information goods), in our model providing the service is costly for the firm. When more users register to the service, the firm has to invest in increasing communication capabilities, network capacity, and so on. In addition, in our model the component that has no value by itself displays network effects. Finally, unlike in Dewan and Freimer, in our model consumers never incur a penalty when the service is sold bundled with the product.

Though our setting and focus differ from those in Dewan and Freimer (2003), we reach a similar result. Consumers often prefer the service to be bundled with the product, while the monopoly does not find this pricing strategy optimal. In our setup, the reason for this under-provision of the service is the presence of network effects.

## 3. The Model

We consider a monopoly that sells a product and can provide a complementary online service to consumers who buy the product. We assume that the product has zero marginal production cost (a common assumption for most information goods), while the cost of providing the service increases with the number of subscribers. For instance, an online-game provider needs to operate a larger service system (e.g., servers, network facilities, and so forth) to serve a greater number of subscribers[7]. If the product has a positive marginal cost, then the model presented here still applies if consumers' valuations of the product are taken net of the marginal cost.

There are $N$ potential customers who are heterogeneous in terms of their valuations for the product and for the service. We assume that the valuation for the product, $\theta$, is uniformly distributed on $[0, \overline{\theta}]$. In addition, a consumer's valuation for the service is given by $\alpha(s+n_s)$, where $\alpha$ represents the consumer type, $n_s$ is the total number of customers who subscribe to the service, and $s$ is the intrinsic value of the service independent on the number of service users (for example, $s$ can be the value of the product's

[7] Blizzard reports that it invested more than $200 million in its service infrastructure for *World of Warcraft* online users.

functionalities which are activated only when a user subscribes to the service). This functional form is based on the assumption that a consumer who has a higher intrinsic value for the service also assigns more importance to the size of the network of service subscribes (Ellison and Fudenberg 2000). Here we assume that the value of the network increases linearly with the number of users. We discuss the case of concave network effects in Appendix 2.

If all consumers have the same valuation for the service and for the network of service users, i.e. if all consumers have the same $\alpha$, then the seller cannot segment the market. That is, the seller cannot set two prices, one for the product and one for the service, so that some consumers choose to get only the product while others choose to get both product and service. Even if the service is sold separately, the sum of the two prices would be the same as the price of a corresponding bundle, and the two strategies (bundling and selling the service separately) would yield the same profit. Given that we are interested in exploring when the seller should bundle the service with the product and when he should offer the service separately, we model consumers with heterogeneous network valuations. Our model thus differs from standard models of network effects in which consumers have homogenous network valuations (e.g., Katz and Shapiro, 1985) and resembles more recent models in which consumers may have different (marginal) valuations for the network (e.g., Sundararajan 2004).

The heterogeneity in the valuations for the service is modeled as follows. To simplify the analysis, we divide customers into two groups: $\beta N$ customers (Group 1) have $\alpha=\alpha_1$, while the remaining $(1-\beta)N$ customers (Group 2) have $\alpha=\alpha_2$. We assume that $0 < \beta \leq 1$ and $\alpha_1 > \alpha_2 \geq 0$, so that Group 1's customers have a higher valuation for the service and for the network.[8] We assume that the vendor cannot identify a consumer's valuation for the product or for the service. We also assume that $\theta$ and $\alpha$ are independent, so that some consumers can have a low product valuation but a high service valuation, and vice versa. This latter assumption well represents reality in many cases. For example, some kids buy a Webkinz plush toy (the product) mainly in order to have a virtual pet to play with on Webkinz World online (the service) and

[8] The main results presented in this paper still hold when there are $m$ consumer groups with parameters $\alpha_i$ , $i$=1,…,$m$ where $\alpha_i > \alpha_{i+1}$ for $i$=1, 2, …, $m$-1, and consumers from group $i$ have service valuation $\alpha_i(s+n_s)$. We discuss this extension in Appendix 2.

interact with other kids' virtual pets. They don't value the plush toy (i.e., they have low product valuation), but they do value the service and the social network. In contrast, younger kids, who cannot yet use a computer, might like the soft plush toy, but have little value for the related online service. Finally, the monopolist has to incur a marginal cost $c$ to serve each service subscriber. Table 1 summarizes the notation used in the model.

| | |
|---|---|
| $N$ | The number of potential customers |
| $\theta$ | Consumers' valuation of the product, drawn from the uniform distribution on $[0,\bar{\theta}]$ |
| $\alpha_i$ | The marginal service/network valuation for Group $i$'s consumers, where $\alpha_1 > \alpha_2 \geq 0$ |
| $\beta$ | The proportion of Group 1 consumers in the market |
| $\alpha_0$ | The average marginal service valuation (i.e. $\beta\alpha_1+(1-\beta)\alpha_2$ ) |
| $s$ | The intrinsic value of the complementary service |
| $c$ | The marginal cost for providing the service |
| $p_N$ | The product price when the service is not offered |
| $p_B$ | The bundle price |
| $p_S$ | The product price when the service is sold separately |
| $f$ | The service subscription fee when the service is sold separately |
| $n_p$ | Demand for the product |
| $n_S$ | Demand for the service (also the size of the network) |
| $\pi_B$ | Profit from selling a bundle |
| $\pi_N$ | Profit from selling only the product |
| $\pi_S$ | Profit when selling the service separately |

**Table 1. Notation**

The monopoly has three options as follows. He can sell only the product; this is likely to be his choice when consumers do not value the service highly enough to justify its provision cost. Alternatively, he can choose to offer the product and the service as a bundle, in which case all customers that buy the product also gain access to the service. This strategy can also be executed by selling the product at a higher price, while offering "free" access to the service. Finally, he may sell the service separately, in which case there can be consumers who buy the product but do not subscribe to the service. Table 2 lists the consumers' utility and the seller's profit, as functions of prices, for each of these three strategies.

In this section, we derive the monopolist's optimal strategy, which clearly depends on the parameters values. To do so, we first find the optimal prices for each of the three strategies listed in Table 2, and then compare the maximum profits from the three strategies.

| Firm's Strategy | Consumers' Utility | | Firm's Profit |
|---|---|---|---|
| Sell only product | $u_1=u_2=\theta-p_N$ | | $\pi_N=n_p\, p_N$ |
| Sell a bundle | $u_1=\theta+\alpha_1(s+n_S)-p_B$ | $u_2=\theta+\alpha_2(s+n_S)-p_B$ | $\pi_B=n_S\,(p_B-c)$ |
| Sell service separately | $u_1=\theta+\alpha_1(s+n_S)-p_S-f$ | $u_2=\theta-p_S$ | $\pi_S=n_p\, p_S+n_S\,(f-c)$ |

**Table 2. The consumer's utility and the monopolist's profit. Here, $u_i$ is the utility of a Group $i$'s customer and is a function of his product valuation $\theta$ and his type $\alpha_i$. The demand for the product and for the service, $n_p$ and $n_s$, are functions of the prices.**

When the monopoly sells only the product it is easy to show that the profit function is concave in the product price, the profit maximizing price is $p_N = \overline{\theta}/2$ , and half of the consumers choose to buy the product. Next, we analyze the profit from selling a bundle of product and service.

When the firm sells a bundle of product and service and the bundle price, $p_B$, does not exceed $\alpha_2(s+N)$, there is an equilibrium in which all consumers from both groups buy the bundle and thus $n_s=N$. The seller's profit is given by $\pi_B(p_B)=N(p_B-c)$ and is linearly increasing in the bundle price. Thus, it is never optimal for the seller to set a bundle price lower than $\alpha_2\,(s+N)$. When the bundle price is such that $p_B>\alpha_2\,(s+N)$, depending on the values of the parameters and the value of $p_B$, different equilibriums may prevail.

When the bundle price is such that $\alpha_2\,(s+N)<p_B<p_1$[9], there is an equilibrium in which all consumers from Group 1 buy the bundle, but only some consumers from Group 2 (those with high product valuations) choose to buy the bundle. As the bundle price increases in this range, the demand from Group 2 decreases but the demand from Group 1 remains constant at $\beta N$. In this range of prices, the seller's profit is given by $\pi_B(p_B)=(\beta N+n_{s2})(p_B-c)$, where $n_{s2}$ is the demand from Group 2 and is decreasing in the bundle price. When $p_1<p_B<p_2$ ,[10] in equilibrium some consumers from Group 1 with

[9] $p_1=\frac{(N+S)\alpha_1\overline{\theta}}{\overline{\theta}+N(\alpha_1-\alpha_2)(1-\beta)}$ and is derived by solving the two equations, $n_{s2}=(1-\beta)N\,(\overline{\theta}-p_B+\alpha_2(s+\beta N+n_{s2}))/\overline{\theta}$ and $\alpha_1(s+\beta N+n_{s2})=p_B$, for $p_B$. The first equation is the condition for fulfilled expectation equilibrium demand from Group 2. The second equation states that a consumer from Group 1 with zero product valuation is indifferent if to buy the bundle or not.

[10] $p_2=\overline{\theta}+\frac{s\alpha_2\overline{\theta}}{\overline{\theta}-M(\alpha_1-\alpha_2)\beta}$ , and is derived by solving $n_{s2}=(1-\beta)N\,(\overline{\theta}-p_B+\alpha_2(s+n_{s2}+n_{s2}))/\overline{\theta}$ and $n_{s1}=\beta N\,(\overline{\theta}-p_B+\alpha_1(s+n_{s2}+n_{s1}))/\overline{\theta}$ simultaneously, to find the fulfilled expectation demand from Group 2

low product valuations do not buy the bundle while there is still positive demand from Group 2. Here, the profit is given by $\pi_B(p_B) = (n_{s1} + n_{s2})(p_B - c)$, where $n_{si}$ is the bundle demand from Group $i$, and is decreasing in the price. Finally, when $p_2 < p_B < \overline{\theta} + \alpha_1 s$ , there is an equilibrium in which consumers from Group 2 do not buy the bundle, i.e., $n_{s2}$=0, and only some consumers from Group 1 buy the bundle.

The profit from selling the bundle as described above is a continuous and concave function of the bundle price, $p_B$. However, to have a non- empty range of $p_B$ values for which there is positive but partial demand from each group, we need to impose parameters conditions so that $\alpha_2(s + N) < p_1 < p_2$. Since we observe that for many bundles of product and service there is positive demand even from consumers who have low service valuation (consider the Webkinz example – some kids buy the plush toy and hardly use the Webkinz World website), and, in addition, we believe that often there are consumers with high service valuation who don't buy the bundle because their product valuation is too low, Assumption 1- (i) and (ii) follow. Specifically, conditions (i) and (ii) in Assumption 1 are necessary and sufficient for an equilibrium in which there is positive but partial demand from each group to prevail for a non-empty range of $p_B$ values. Finally, in this paper we focus on cases in which the profit from selling the bundle is maximized in the range $p_B > p_1$ (i.e., the profit function is increasing in $p_B$ at $p_B$=$p_1$). Thus, when the seller prices the bundle optimally, not all Group 1 consumers buy the bundle. The required condition on the parameters values for this to be the case is stated as Assumption 1 – (iii).

**Assumption 1**. *i)* $\overline{\theta} > \alpha_0 N$

ii) $\overline{\theta} > (\alpha_1 - \alpha_2)(s + \beta N)$

*iii)* $c > -s\alpha_0 - \overline{\theta} + \frac{2(N+s)\alpha_1\overline{\theta}}{N(\alpha_1-\alpha_2)(1-\beta)+\overline{\theta}}$

It is interesting to understand when Assumption 1 is likely to be satisfied. Conditions (i) and (ii) in Assumption 1 are satisfied as long as the upper bound on product valuations is reasonably high. Specifically, according to Assumption 1-(i) the highest product valuation, $\overline{\theta}$, should be larger than the average value of the network of service users when everyone uses the service. According to Assumption

as a function of $p_B$, and then solving for the $p_B$ value at which this equilibrium demand equals 0.

1-(ii) the highest product valuation should be larger than the difference between a Group 1's consumer service valuation and a Group 2's consumer service valuation when all Group 1 consumers use the service. Our paper thus examines services which complement a product with a reasonably large valuations range. In the case of a product that is never highly valued by consumers (compared to the service), the results would be different. Specifically, if Assumption 1-(ii) does not hold, then as the bundle price, $p_B$, increases, the demand from Group 2 becomes zero before anyone from Group 1 finds it optimal not to buy the bundle. We believe that this situation rarely represents reality. Finally, the right hand side of Assumption 1-(iii) is decreasing in $\overline{\theta}$, (the RHS is concave in $\overline{\theta}$, and the first derivative evaluated at $\overline{\theta} = 0$ is negative for any positive value of $N$). Thus, again, as long as $\overline{\theta}$ is reasonably large, this condition will be satisfied, and at the optimal bundle price not all Group 1's consumers buy the bundle.

We next derive the optimal bundle price and resulting profit when the profit from selling the bundle is maximized when some consumers from Group 1 do not buy the bundle and there is positive demand from Group 2. Letting $n_{s1}$ and $n_{s2}$ denote the number of customers from Group 1 and Group 2, respectively, and $n_s^e$ denote the total expected number of service users, we have :

$$n_{s1} = \frac{\beta N(\overline{\theta} + \alpha_1(s + n_s^e) - p_B)}{\overline{\theta}} \quad \text{and} \quad n_{s2} = \frac{(1-\beta)N(\overline{\theta} + \alpha_2(s + n_s^e) - p_B)}{\overline{\theta}}. \tag{1}$$

Substituting from the above two equations in $n_s^e = n_{s1} + n_{s2}$ and solving for $n_s^e$, we find the Fulfilled Expectations Equilibrium value of the demand, $n_s^*$, as a function of the price $p_B$:

$$n_s^* = \frac{N(\overline{\theta} + \alpha_0 s - p_B)}{\overline{\theta} - \alpha_0 N} \tag{3}$$

$$\text{where} \qquad \alpha_0 \triangleq \beta\alpha_1 + (1-\beta)\alpha_2 \quad . \tag{4}$$

The profit function is then given by $\pi_B = n_s^*(p_B - c)$, and the second-order condition is satisfied when $\bar{\theta} > \alpha_0 N$, which holds by Assumption 1-(i) . Using FOC, we find the optimal bundle price and the resulting profit:

$$p_B^* = \frac{\overline{\theta} + \alpha_0 s + c}{2} \ , \tag{5}$$

$$\pi_B^* = \frac{N(\overline{\theta} + \alpha_0 s - c)^2}{4(\overline{\theta} - \alpha_0 N)} \quad . \tag{6}$$

When Assumption 1-(iii) holds, at the price given by Equation 5 not all consumers from Group 1 buy the bundle. In addition, when the bundle price is $p_B^*$ as given in Equation 5, the demand from Group 2 is indeed positive only when $c$ is lower than the following value:

$$c_B = \overline{\theta} + s\left(\alpha_2\left(1 + \beta + \frac{2\beta N(\alpha_1 - \alpha_2)}{\overline{\theta} - \beta N(\alpha_1 - \alpha_2)}\right) - \alpha_1 \beta\right) . \quad (7)$$

If $c > c_B$ the seller would maximize the profit from selling the bundle only to Group 1 consumers and the profit maximizing price in this case would be $p_B^{**} = \frac{\overline{\theta} + \alpha_1 s + c}{2}$. However, as is shown in the Proof of Proposition 1 (in Appendix 1), the profit from selling the service separately is higher than the profit from selling the bundle at price $p_B^{**}$ so that only consumers from Group 1 buy it. Thus, this latter strategy will never prevail in equilibrium. If $c > c_B$, the seller would either sell the service separately or not sell it at all.

Next we consider the strategy of selling the service separately. If users from both groups subscribe to the service when the monopoly sells it separately, then there is no difference between the two strategies of selling the service separately and selling a bundle. Thus, when the monopoly sells the service separately, we maximize the profit gained when only customers from Group 1, who have higher valuation for the service, subscribe to the service. For this to be true, the two following incentive compatibility conditions should be met.

$$\alpha_1(s + n_s) \geq f \quad (8)$$

$$\alpha_2(s + n_s + 1) \leq f \quad (9)$$

Eq. 8 specifies the condition required so that no single Group 1's consumer who buys the product and the service has an incentive to unsubscribe from the service. Eq. 9 specifies the condition required so that a Group 2's customer has no incentive to deviate and subscribe to the service.

The demand for the service as a function of prices and expected network size is given by:

$$n_{s1} = \frac{\beta N(\overline{\theta} + \alpha_1(s + n_s^e) - p_s - f)}{\overline{\theta}} \quad (10)$$

Substituting from the above equation in $n_s^e = n_{s1}$ and solving for $n_s^e$, we find the Fulfilled Expectations Equilibrium value of the demand, $n_s^*$, as a function of the prices:

$$n_s^* = \frac{N\beta(\overline{\theta} + s\alpha_1 - p_s - f)}{\overline{\theta} - N\beta\alpha_1} \quad (11)$$

The profit function is then given by

$$\pi_s = (p_s + f - c)n_s^* + N(1-\beta)p_s\frac{\bar{\theta}-p_s}{\bar{\theta}}, \tag{12}$$

and the second-order condition is satisfied when $\bar{\theta} > \beta\alpha_1 N$, which holds by Assumption 1-(i). Using FOCs, we find the optimal prices:

$$p_s^* = \frac{\bar{\theta}}{2}, \qquad f^* = \frac{c+\alpha_1 s}{2} \tag{13}$$

We also make the following assumption, which guarantees that if the service is sold separately at above prices, some Group 1 consumers do not buy the product and the service.

**Assumption 2.** $c > \alpha_1(s + 2\beta N) - \bar{\theta}$.

Notice that as long as the upper bound on product valuations is sufficiently high, Assumption 2 would be satisfied. Table 3 summarizes the optimal prices and the resulting demand and profit for the three pricing strategies.

| | | | |
|---|---|---|---|
| **Selling only the Product** | | | |
| Profit | $\pi_N^* = \bar{\theta}N/4$ | | |
| Product Price | $p_N = \bar{\theta}/2$ | Demand | $n_p^* = N/2$ |
| **Selling the Bundle (when $c < c_B$)** | | | |
| Profit | $\pi_B^* = \frac{N(\bar{\theta}+\alpha_0 s - c)^2}{4(\bar{\theta}-\alpha_0 N)}$ | | |
| Bundle Price | $p_B^* = \frac{(\bar{\theta}+\alpha_0 s + c)}{2}$ | Demand | $n_S^* = \frac{N(\bar{\theta}+\alpha_0 s - c)}{2(\bar{\theta}-\alpha_0 N)}$ |
| **Selling the Service Separately** | | | |
| Profit | $\pi_S^* = \frac{N}{4}\left(\frac{\beta(\bar{\theta}+\alpha_1 s - c)^2}{\bar{\theta}-\beta\alpha_1 N} + (1-\beta)\bar{\theta}\right)$ | | |
| Product Price | $p_S^* = \frac{\bar{\theta}}{2}$ | Product Demand | $n_P^* = \frac{N}{2}\left(\frac{\beta(\bar{\theta}+\alpha_1 s - c)}{(\bar{\theta}-\beta\alpha_1 N)} + (1-\beta)\right)$ |
| Service Fee | $f^* = \frac{c+\alpha_1 s}{2}$ | Service Demand | $n_S^* = \frac{\beta N(\bar{\theta}+\alpha_1 s - c)}{2(\bar{\theta}-\beta\alpha_1 N)}$ |

**Table 3. The optimal prices and the resulting demand and profit**

Next, we present Proposition 1 which states the monopoly's optimal strategy. In Proposition 1, $c_{i,j}$ represents the value of $c$ at which $\pi_i^* > \pi_j^*$ if and only if $c < c_{i,j}$ (where the profit expressions, $\pi_i^*$ and $\pi_j^*$, are given in Table 3).

***Proposition 1.*** *The monopoly optimal strategy.*

*i)* *When $s \leq s_0$ , the monopoly sells a bundle of product and service if and only if $c<c_{B,N}$ and does not offer the service otherwise.*

*ii)* *When $s > s_0$ , the monopoly sells a bundle if and only if $c<c_{B,S}$, sells the service separately if and only if $c_{B,S} < c < c_{S,N}$, and does not operate the service otherwise,*

*where*

$$s_0 = \frac{\sqrt{\bar{\theta}}\left(\sqrt{\bar{\theta}-\alpha_1\beta N}-\sqrt{\bar{\theta}-\alpha_0 N}\right)}{(\alpha_1-\alpha_2)(1-\beta)} \ ,$$

$$c_{B,N} = \bar{\theta}\left(1-\frac{\sqrt{\bar{\theta}-\alpha_0 N}}{\sqrt{\bar{\theta}}}\right)+\alpha_0 s,$$

$$c_{S,N} = \bar{\theta}\left(1-\frac{\sqrt{\bar{\theta}-\alpha_1\beta N}}{\sqrt{\bar{\theta}}}\right)+\alpha_1 s; \text{ and}$$

$$c_{B,S} = \bar{\theta}-\frac{\alpha_2\bar{\theta}s-\sqrt{(\bar{\theta}-\alpha_1\beta N)(\bar{\theta}-\alpha_0 N)(\beta s^2(\alpha_1-\alpha_2)^2+\bar{\theta}\left(\bar{\theta}-\beta N(\alpha_1-\alpha_2)\right)}}{\bar{\theta}-\beta N(\alpha_1-\alpha_2)} \ .$$

Proofs of all lemmas and propositions are given in Appendix 1.

Figure 3 describes the monopolist strategy choice according to Proposition 1.

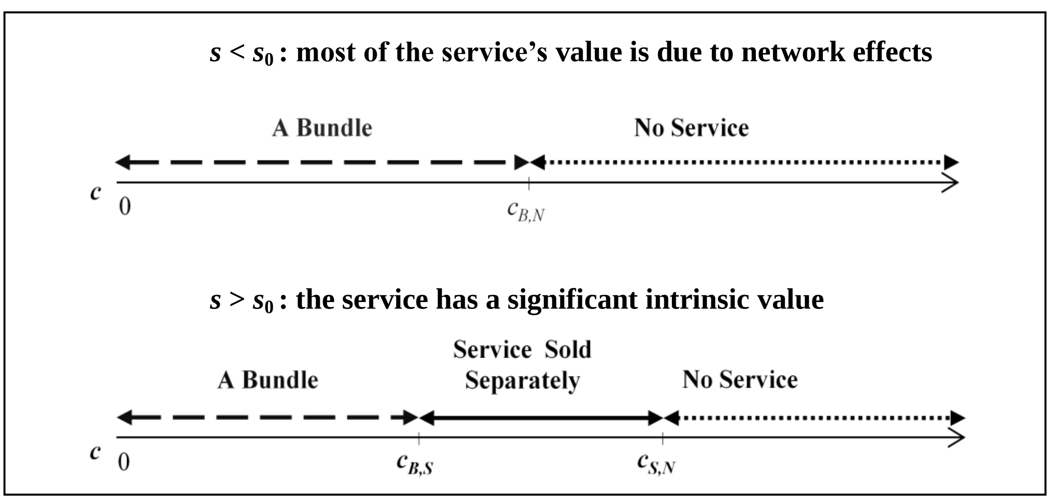


**Figure 3. The monopoly's optimal pricing strategy**

An important insight from Proposition 1 is that when considering online services with little or no intrinsic value, i.e., $s < s_0$, the seller should never sell the service separately. Instead, he should offer the service bundled with the product (or offer the service "free of charge" but increase the price of the product) if the cost of offering the service is below $c_{B,N}$. Notice that $c_{B,N}$ is always positive and is increasing in $s$. Thus as $s$ increases (but is still below $s_0$), the range in which the seller offers the bundle increases. Also, it is interesting to note that the threshold value, $c_{B,N}$ , is increasing in $N$, the size of the market. Thus, as the number of potential customers increases, the seller is more likely to offer the service. When the service displays network effects, there are economics of scale even if the marginal cost of offering the service is fixed. This would not be the case if the service did not exhibit network effects.

When $s > s_0$, if the marginal cost of offering the service is lower than a threshold value given by $c_{B,S}$, it is optimal for the vendor to charge customers a single price for the product and its complementary service. If the marginal cost is between $c_{B,S}$ and $c_{S,N}$, the monopolist should sell the service separately, and if the marginal cost is higher than $c_{S,N}$, the service should not be offered. When $s > s_0$ , there is always a range of $c$ values for which it is optimal for the seller to sell the service separately, though his range decreases as $s$ approaches $s_0$. Thus, as the nature of online services changes so that they rely less on inherent functionalities and rely more on the value created from network effects, we should expect to see less services sold separately.

According to Proposition 1, the intrinsic value of the service, $s$, has to reach a threshold value for separate pricing to be chosen by the monopoly. This finding is consistent with the literature on price discrimination (Mussa and Rosen 1978, Deneckere and McAfee 1996). Specifically, only when the two consumer groups are differentiated enough in their service valuations (the difference between the valuations is a function of $s(\alpha_1 - \alpha_2)$ and is thus increasing in $s$), the monopoly chooses to use second-degree price discrimination where consumers self-select what to buy, both product and service or only the product.

In addition, according to Proposition 1, as the marginal cost of service, $c$, increases, the separate pricing strategy might outperform the bundling strategy. This finding is consistent with a real-life example.

According to a 2004 survey, users of *Starcraft*, *Warcraft III*, and *Diablo II*, spent on average 10.5 minutes a day in *Battle.net*, Blizzard's online gaming system; for all three games, online access was bundled with the game package[11]. On the other hand, the average playing-time per day of a user of *World of Warcraft*, was 3.24 hours[12]. This shows that Blizzard has to incur a higher cost to serve a *World of Warcraft* player than to serve a player of *Starcraft, Warcraft III*, or *Diablio II*. As predicted by our model, access to the online platform of *World of Warcraft* was sold separately from the game package.

While in 2008 access to the online gaming platform of *World of Warcraft* was offered for a separate subscription fee, as of 2010 access to the online multi-player platform of *World of Warcraft* is offered free of charge to those who purchase the game package. That is, the seller switched from selling the service separately to selling a bundle. According to Proposition 1, the reason can be a decrease in the marginal cost of providing the service, $c$, or a decrease in the inherent value of the service, $s$. Another reason may be an increase in the size of the market. Notice that $s_0$ is increasing in $N$, the potential number of customers. Thus if the potential market increases, it might be profitable for the seller to switch from selling the service separately to selling it bundled with the product, even if $c$ and $s$ have not changed. In contrast, an increase in the market size will have no effect on the optimal selling strategy when there are no network effects.

## 4. Social Welfare

In this section we examine how the monopolist's choice affects social welfare and consumer surplus. We provide two typical numerical examples, presented in Figure 4.a and 4.b, which demonstrate that the complementary service may be supplied less than is socially optimal. Table 4 provides a legend for the different regions in Figures 4.a and 4.b, and summarizes the different cases that can prevail in the market in terms of the strategy that maximizes the seller's profit and the strategy that maximizes social welfare.

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[11] http://www.eff.org/files/filenode/Blizzard_v_bnetd/20040930BNETDOrder.pdf. According to this article (page 4), as of September 2004, the number of active users is approximately 12 million, and they spend more than 2.1 million hours a day. By dividing 2.1 million hours by 12 million, we get 10.5 minutes.

[12] http://www.nickyee.com/daedalus/archives/001365.php. This page reports that on average, a player spends 22.7 hours per week, equivalent to 3.24 hours per day.

| Region | Monopoly's Choice | Social Surplus maximized with: | Inefficiency |
|---|---|---|---|
| 1 | Bundle | Bundle | N/A |
| 2 | **Separate** | **Bundle** | **Under-provision** |
| 3 | Separate | Separate | N/A |
| 4 | **No Service** | **Bundle** | **Under-provision** |
| 5 | **No Service** | **Separate** | **Under-provision** |
| 6 | No Service | No Service | N/A |

**Table 4. Comparison of the profit's and the Social Surplus' maximizing strategies**

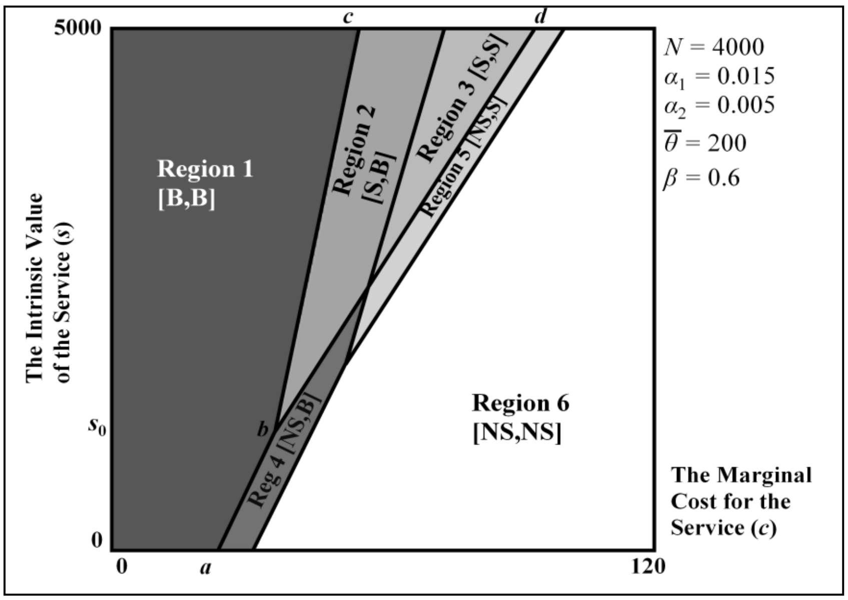


**Figure 4(a). The monopoly's profit maximizing strategy vs. the strategy that maximizes social welfare, for *β*=0.6**

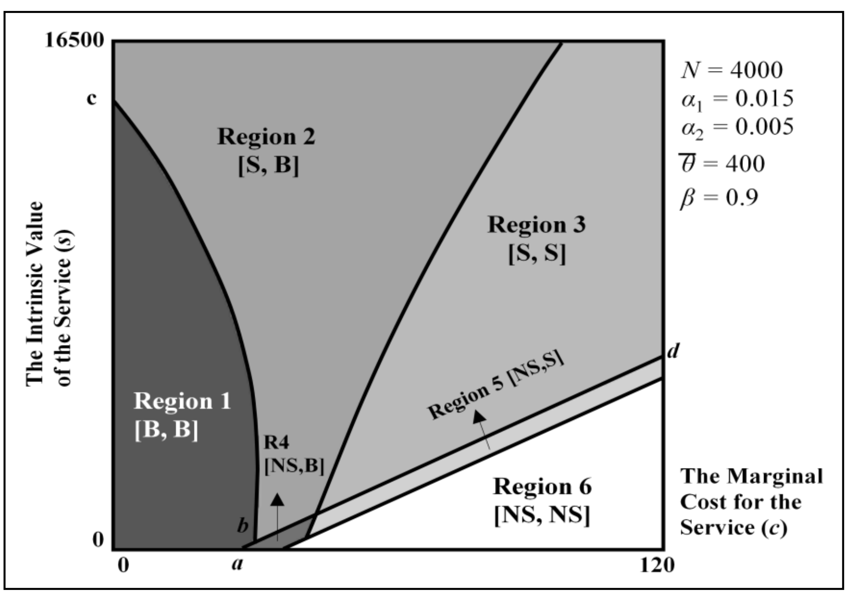


**Figure 4(b). The monopoly's profit maximizing strategy vs. the strategy that maximizes social welfare for *β*=0.9**

Figure 4.a and 4.b display the strategy that maximizes the monopoly's profit and the strategy that maximizes social welfare for different values of the marginal cost ($c$) and the intrinsic value of the service ($s$) for two possible sets of parameter values. The range of $c$-$s$ values for each figure was determined so that the conditions stated in Assumptions 1 and 2 are satisfied. Notice that $s_0$ from Proposition 1 corresponds to the value of $s$ at point $b$ in Figures 4.a and 4.b. In addition, the curves $a$-$b$, $b$-$c$, and $b$-$d$ in the two figures correspond to the thresholds $c_{B,N}$, $c_{B,S}$, and $c_{S,N}$ from Proposition 1, respectively.

Most of the bundling literature shows that bundling enables the monopoly to extract higher surplus from the consumers (e.g., Bakos and Brynjolfsson 1999, 2000). In contrast, our finding of Region 2 in Figures 4.a and 4.b in which the monopolist sells the service separately, but a bundle is socially optimal (which implies that consumer surplus is maximized with a bundle), is similar to the main finding from Dewan and Freimer (2003). They examine a monopolist that sells a base product and an add-in. As in our model, the monopoly can either bundle the two or sell the add-in separately allowing some consumers to buy only the base product. In their model there are no network effects, and both products have zero marginal cost. The main result in Dewan and Freimer (2003), that the monopolist might sell the add-in separately while consumers prefer a bundle, is driven by the existence of a group of consumers that incur a penalty when they are forced to buy the add-in bundled with the base product. We show that network effects can be another reason that makes consumers prefer bundling more than the monopoly, causing under-provision. The intuition behind this result is as follows. If the service is only available in a bundle with the product, then more people buy it compared to when the service is sold separately from the product. Clearly, the utility to customers from the service is higher in the bundling case because of the positive network effects. On the other hand, bundling reduces the firm's revenue **per** Group 1's customer ($p_B < p_S + f$). Hence, due to the monopolist's profit maximization incentive, it may sometimes choose to sell the service separately instead of selling a bundle, reducing the social welfare. When the service does not exhibit network effects, the fact that more consumers buy the service when it is sold bundled with the product does not increase social welfare. Thus, when there are no network effects, it is less likely that bundling would maximize social welfare when the monopoly chooses to sell the service separately.

The cases presented in Regions 4 and 5 in Figures 4.a and 4.b are not reported in Dewan and Freimer (2003). In their paper, the add-in has no marginal cost and thus it is always being offered, either bundled with the base product or separately. In our model, on the other hand, there is a cost for providing the service which increases with the number of subscribers. If the cost is high enough, the vendor may choose not to provide the service. Thus, in some cases the monopoly does not offer the service while it is socially optimal to offer the service either in a bundle or separately. Proposition 2 states that there is always a range of marginal costs (*c* values) for which this latter type of under-provision of service would be observed.

***Proposition 2.***

i) *When $s<s_0$, there exists a none-empty range of c values in which the service is under-provided. Specifically, the monopoly does not offer the service, but a bundle would increase consumer surplus and social surplus.*

ii) *When $s>s_0$, there exists a none-empty range of c values in which the service is under-provided. Specifically, the monopoly does not offer the service, but offering the service separately would increase consumer surplus and social welfare.*

Notice that in case (ii) of Propsition 2, we do not argue that selling the service separately maximizes social welfare or consumer surplus. It might be that selling a bundle maximizes social welfare. However, regardless, the seller does not offer the service, and thus the service is under-provided.

While clearly under-provision of the service is feasible (Regions 2, 4, and 5 in Figures 4.a and 4.b), we could not find any numerical examples in which the service is over-provided compared to the socially optimal level[13]. Numerical investigation was necessary because the inequalites that need to be satisfied for over-provision of service to be the market outcome are too complex to analtyically verify whether they can all be met simultenously.

[13] By comparing social welfare from the startegy that maximizes the seller's profit to the social welfare from other strategies, we searched for cases in which sevice is over-supplied. We searched over the following range of parameter valeus: $\beta$ in [0, 1] in increments of 0.1; $N$ in [0, 10000] in increments of 500; $\alpha_1$ in [0, 1], with increments of 0.05; $\alpha_2$ in [0 , $\alpha_1$] in increments of 0.05; $\bar{\theta}$ in [$\alpha_0 N$, 10000] with increments of 100; $s$ in [0, 20000] in increments of 500.

## 5. Network Effects and the Under-Provision of the Service

Under-provision of the service can be a result of the monopoly's incentive to price-discriminate. In such cases, the monopoly chooses to sell the service separately and price it so that only customers who value it highly, i.e., only Group 1's customers, buy it; however, consumer surplus and social welfare would be higher if the service was sold to consumers from both groups (i.e., bundled with the product). Under-provision of service can also happen when the seller chooses not to offer the service while social welfare would be higher if the service was offered, either bundled with the product or sold separately.

In this section we examine the relationship between the existence of network effects and the fact that under-provision of service prevails. To do so, we focus on two special cases.

1. The case in which consumers are homogeneous in terms of their service valuation, so that price discrimination cannot be the cause of under-provision of the service. Here, we compare the model in which the service displays network effects to a model without network effects, showing that the service can be under-provided in the former, but this does not happen in the latter.
2. The case in which the service has no inherent value, so that the value derived by service users relies solely on interactions with other users. Here, we show that, under some general conditions, an increase in consumer surplus caused by a decrease in the marginal cost of the service or by an increase in the degree of network effects cannot be fully captured by the monopoly. This can explain the under-provision of the service.

### 5.1 Homogenous Service Valuations

When consumers have homogeneous service valuations, given by $\alpha(s+n_s)$, the seller either sells the service to everyone that buys the product (sells a bundle) or does not offer the service. When the monopolist does not sell the service, the profit and consumer surplus at the optimal product price are given by:

$$\pi_N^* = N\overline{\theta}^2/4, \qquad CS_N^* = N\overline{\theta}^2/8 \quad . \tag{14}$$

When the monopolist sells a bundle, his profit and consumer surplus are given by

$$\pi_B^* = \frac{N(\overline{\theta}+\alpha s-c)^2}{4(\overline{\theta}-\alpha N)} \ , \qquad CS_B^* = \frac{N\overline{\theta}(\overline{\theta}+\alpha s-c)^2}{8(\overline{\theta}-\alpha N)^2} \quad . \tag{15}$$

Because $\pi_B^* + CS_B^*$ is decreasing in $c$, while $\pi_N^* + CS_N^*$ is constant in $c$, $\pi_B^* + CS_B^* < \pi_N^* + CS_N^*$ if and only if $c > d_{B,N}$, where

$$d_{B,N} = \overline{\theta}\left(1 - \frac{(\overline{\theta}-\alpha N)\sqrt{3}}{\sqrt{3\overline{\theta}-2\alpha N}}\right) + \alpha s \quad . \tag{17}$$

In addition, $\pi_B^* < \pi_N^*$ if and only if $c > c_{B,N}$, where

$$c_{B,N} = \overline{\theta}\left(1 - \sqrt{\overline{\theta} - \alpha N}\right) + \alpha s \tag{18}$$

Thus, the monopoly does not sell the service although the service would increase social welfare when $c_{B,N} < c < d_{B,N}$. Finally it can be shown that there is always a range of $c$ values in which the service is under-provided. Specifically**:**

$$d_{B,N} - c_{B,N} = \overline{\theta}\sqrt{\overline{\theta} - \alpha N}\left(1 - \frac{\sqrt{3\overline{\theta}-3N\alpha}}{\sqrt{3\overline{\theta}-2N\alpha}}\right) > 0, \tag{19}$$

where the last inequality holds because $\overline{\theta} > 0$, $3\overline{\theta} > 3N\alpha$ due to Assumption 1 and $\frac{\sqrt{3\overline{\theta}-3N\alpha}}{\sqrt{3\overline{\theta}-2N\alpha}} < 1$.

Figure 5 demonstrates this result. For $c < c_{B,N}$, the monopoly offers a bundle of service and product, and this strategy also maximizes social welfare. For $c_{B,N} < c < d_{B,N}$, the monopoly does not offer the service, although social welfare and consumer surplus would be maximized with a bundle, and for $c > d_{B,N}$, the monopoly does not offer the service, a choice which also maximizes social welfare.

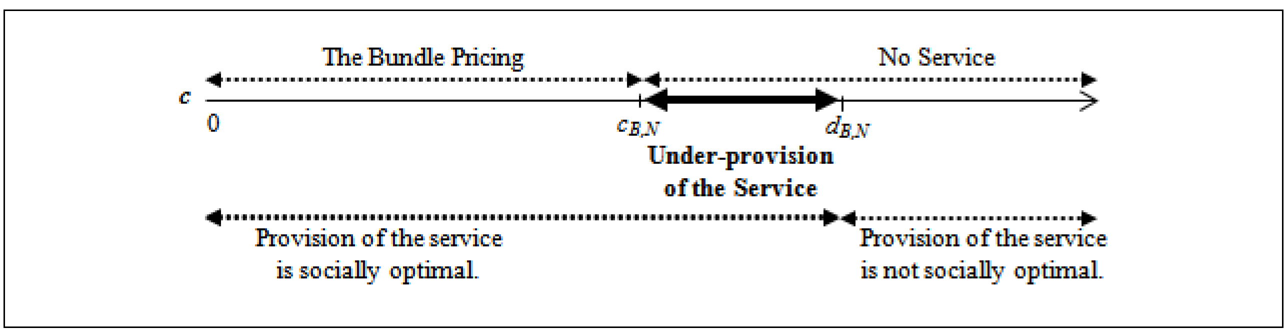


**Figure 5. The under-provision of the service when consumers have homogenous service valuation**,

We now analyze an alternative model in which the complementary service does not display network effects. In this model, the utility from the service is $\alpha s$ and it does not depend on the number of service subscribers. In this case, the monopoly's optimal strategy is to offer the bundle at a price of $(\bar{\theta} + c + \alpha s)/2$ when $c < \alpha s$ and does not offer the service otherwise, a decision which is socially optimal. Thus, when the service does not display network effects, there is no under-provision of the service.

We note that although there is no under-provision of service in the setting with no network effects, the range of $c$ values where the service is provided is smaller in this case (i.e., $c_{B,N} > \alpha s$). The reason is that if we hold $s$ fixed, the total value of the service to consumers is lower in the case with no network effects.

### 5.2 Social Networking Services

Here we consider the case in which the service has no inherent value, i.e. $s$ =0, and consumers use the service for the mere purpose of social networking and interaction with peers. According to Proposition 1, when $s = 0$ the seller's optimal strategy is to sell a bundle if the cost of providing the service is sufficiently low, and he would never find it optimal to sell the service separately. The following Lemma gives additional insights as to why such services may be supplied less than is socially optimal

***Lemma 1.*** *When $\bar{\theta} > c$ :*

- *i) If $\alpha_0 > \bar{\theta}/(2N)$ then a decrease in the marginal cost of the service, c, has a larger positive effect on consumer surplus from a bundle than on the monopolist's profit from selling a bundle*
- *ii) An incremental increase in $\alpha_1$ and $\alpha_2$ has a larger positive effect on consumer surplus from a bundle than on the monopolist's profit from selling a bundle*
- *iii) If $\alpha_0 > \bar{\theta}/(3N)$ then an increase in the market size, N, has a larger positive effect on consumer surplus from a bundle than on the monopolist's profit from selling a bundle*

Lemma 1 explains why there is always a range of parameter values for which the monopoly chooses not to offer the service while consumer surplus is maximized when the seller offers a bundle (See Proposition 2). Technological developments can cause a reduction in the operating cost of the service ($c$) or enable the vendor to offer a service that provides greater degree of network effects ($\alpha$). According to Proposition 1, as $c$ decreases or as $\alpha_1$ and $\alpha_2$ both increase, consumer surplus from a bundle may exceed their surplus from only the product (the latter is constant in $c$ and in $\alpha$ ) *before* the vendor's profit from offering the bundle exceeds his profit from selling only the product. This result explains the under-provision of service as stated in Proposition 2-(i). Notice that $\alpha_0 > \bar{\theta}/(2N)$ is a sufficient condition which holds when $\alpha_0$ or $N$ are large enough. Interestingly, an increase in either of these values represents an increase in the degree of network effects.

## 6. Conclusion

As online services proliferate and become strategically important for firms, it is important for researchers to investigate how firms should manage the offerings of such services. In this paper we provide an economic model and examine whether firms should bundle online services with their products or sell them separately, and whether the firm's choice maximizes social welfare.

We show that as the cost of providing the service decreases or the number of potential customers increases, the bundling strategy may dominate selling the service separately. In addition, we find that when considering online services with little or no inherent value (the value of the service to a user comes mainly from the participation of other users), the seller should never sell the service separately. Instead, he should offer the service bundled with the product if the cost of offering the service is below a threshold value. Thus, as the nature of online services changes so that they rely less on inherent functionalities and more on network effects, we should expect to see less services sold separately.

We show that often consumer surplus is maximized by a bundle of product and service, but the monopoly chooses to sell the service separately or not at all. This finding is in contrast to common contentions in the bundling literature, according to which bundling allows the monopoly to extract more consumer surplus, but it is consistent with previous work showing that a product with network effects maybe under-supplied. Thus, our paper shows that network effects can cause consumers to prefer bundling more than the firm. In addition, we show that as technology progresses, lowering the cost of the service and increasing the value of the service and its network, the adoption of online services by firms may lag behind the socially optimal level. Our findings imply that social planners or policy makers may have to provide the monopolist with incentives to offer online services.

We examined the robustness of the results to several model assumptions; specifically: the number of consumer types, the linearity of network effects and the fact that a consumer's product valuations is independent of his service and network valuations. We find that our results robust to these assumptions. The analysis is presented in Appendix 2.

An interesting extension to the present work would be to examine the duopoly case, where firms sell differentiated products and each firm may offer a complementary service to its customers. Our initial results show that in a duopoly, both under- provision and over-provision of the service can prevail. In addition, firms may be caught in a Prisoner's Dilemma, choosing to offer the service while not offering it would increase the industry's profit.

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## Appendix 1. Analytical Proofs

**Lemma 2.** (The results of Lemma 2 are used in the proof of Proposition 1)

***The proof of Lemma 2 follows the proof of Proposiiton 1***

*Let* $c_{i,j}$ denote the value of $c$ at which $\pi_i^* > \pi_j^*$ if and only if $c < c_{i,j}$ , where the profit expressions are given in Table 3, $i, j \in \{N, B, S\}$, and $i \neq j$,. Then :

*(i)* $\frac{\partial}{\partial c}\pi_B^* < \frac{\partial}{\partial c}\pi_S^*$ *if* $c < \overline{c_B}$

*(ii)* $c_{S,N} < \overline{c_S}$

*(iii)* $c_{B,S} < \overline{c_B}$

*(iv)* $\underline{c_S} < c_{B,S}$

*(v)* $\underline{c_S} < c_E$ *and* $\underline{c_S} < \overline{c_B}$

**Proof of Proposition 1**

For each of the three strategies, the optimal prices and the resulting profit and demand are given in Table 3. For the case in which the monopoly sells the service separately, Table 3 lists the results of solving the *unconstrained* maximization problem (maximizing the profit without considering the constraints given by Eqs. 8 and 9 in the paper). We will show that if the prices and demand derived by solving the *unconstrained* problem do not satisfy the conditions given by Equations 8 and 9, then the strategy of selling the service separately at a price so that only customers from Group 1 buy it, is anyway not optimal.

The optimal prices and resulting demand listed in Table 3 for selling the service separately satisfy the incentive compatibility constraints given in Equations 8 and 9 (that is $\alpha_1\left(s+n_s^*\right) \geq f^*$ and $\alpha_2\left(s+n_s^*+1\right) \leq f^*$) only when $\underline{c_S} \leq c \leq \overline{c_S}$ [14], where

[14] The range boundaries, $\underline{c_S}$ and $\overline{c_S}$ , are derived by solving for the $c$ values at which Eq. 8 and Eq. 9 are binding after substituting prices and demand from Table 3.

$$\underline{c_S} = \frac{\alpha_2\left(2s\bar{\theta}+\beta N\left(\bar{\theta}-2\alpha_2\right)\right)}{\bar{\theta}-\beta N(\alpha_1-\alpha_2)} - \alpha_1 s + 2\alpha_2 \text{ , and} \tag{A1}$$

$$\overline{c_S} = \alpha_1\left(\beta N + s\right) \text{ .} \tag{A2}$$

In addition, the optimal bundle price, and the resulting demand and profit expressions, given in Table 3 for selling a bundle were derived assuming positive demand from Group 2 are thus valid only when $c \le \overline{c_B}$ , where

$$\overline{c_B} = \bar{\theta} + s\left(\alpha_2\left(1+\beta+\frac{2\beta N(\alpha_1-\alpha_2)}{\bar{\theta}-\beta N(\alpha_1-\alpha_2)}\right) - \alpha_1\beta\right). \tag{A3}$$

When $c > \overline{c_B}$ , if the bundle price is $p_B^*$ from Table 3, Group 2's consumers will not buy the bundle.

We first show that selling the service separately, at a price so that only Group 1's customers buy it, is never optimal if $c > \overline{c_S}$ or $c < \underline{c_S}$ . When $c > \overline{c_S}$ , if the monopoly were to set prices ($p_S^*$, $f^*$ ) according to Table 3, customers from Group 1 would not subscribe to the service. In this range of $c$ values, the profit from selling the service only to Group 1 customers can be derived by solving the *constrained* maximization problem (maximizing the profit $\pi_S = n_p\, p_S + n_s\,(f - c)$ subject to $\alpha_1\left(s+n_s(p_S, f)\right) = f$ ), and this profit is clearly lower than the value given by $\pi_S^*$ in Table 3 (the solution of the *unconstrained* problem). Since $\pi_N^* > \pi_S^*(c = \overline{c_S})$ (See Lemma 2-(ii), showing that $c_{S,N} < \overline{c_S}$ ), $\pi_N^*$ is constant in $c$, and $\pi_S^*$ is decreasing in $c$, we conclude that for $c$ values such that $c > \overline{c_S}$ , not selling the service yields higher profit than selling the service only to Group 1 while maintaining $\alpha_1\left(s+n_s(p_S, f)\right) = f$ . Specifically, we have shown $\pi_N^* > \pi_S^*(\textit{unconstrained problem}) > \pi_S^*(\textit{constrained problem})$ when $c > \overline{c_S}$ .

When $c < \underline{c_S}$ , if the monopoly were to set prices ($p_S^*$, $f^*$) according to Table 3, Group 2 customers would subscribe to the service. In this range of $c$ values, the profit from selling the service separately at a price so that only consumers from Group 1 buy it, derived by solving the *constrained* maximization problem (maximizing the profit $\pi_S = n_p\, p_S + n_s(f - c)$, subject to $\alpha_2\left(s + n_s(p_S, f) + 1\right) = f$ ), is lower than the value given by $\pi_S^*$ in Table 3. Since $\pi_B^*(\underline{c_S}) > \pi_S^*(\underline{c_S})$ (see Lemma 2-(*iv*) showing that $\underline{c_S} < c_{B,S}$ ), both expressions are decreasing in $c$ but $\pi_B^*$ is decreasing faster than $\pi_S^*$ when $c < \overline{c_B}$ (see Lemma 1-(*i*)), and $\underline{c_S} < \overline{c_B}$ (see Lemma 1-(*v*)), we conclude that for $c$ values such that $c < \underline{c_S}$ , selling a bundle to both groups yields higher profit than selling the service separately at a price so that only consumers from Group 1 buy it. Specifically, we have shown that $\pi_B^* > \pi_S^*$ (*unconstrained problem*) > $\pi_S^*$ (*constrained problem*) when $c < \underline{c_S}$ .

Thus, selling the service separately so that only customers from Group 1 buy it might be optimal only for some $c$ values such that $\underline{c_S} \le c \le \overline{c_S}$ (if this range is not empty). Outside this range, we need only consider the profit from selling only the product, which is constant in $c$, and the profit from selling a bundle, which is decreasing in $c$.

Next, we show that selling a bundle only to Group 1 consumers (i.e., the bundle price is too high for any Group 2 consumers to buy it) can never be optimal. When $c < \overline{c}_B$ ,if the monopoly sells a bundle then the optimal bundle price is given by $p_B^*$ , and there is positive demand from both groups. When $c > \overline{c}_B$, if the bundle price is $p_B^*$ from Table 3, Group 2 consumers will not buy the bundle[15]. In this range of $c$ values, at price $p_B^*$ the seller can sell the bundle only to Group 1 consumers. However, selling the bundle only to Group 1, the optimal bundle price would be $p_B^{**} = \left(\overline{\theta} + \alpha_1 s + c\right)/2$, which is larger than $p_B^*$. Thus, when $c > \overline{c}_B$ the profit from selling a bundle at price $p_B^{**}$ is larger than the profit from selling the bundle at price $p_B^*$, and $\pi_B^*(c) = \pi_B^{**}(c)$ (the RHS is the profit from selling a bundle at price $p_B^{**}$) at a $c$ value smaller than $\overline{c}_B$, denoted by $c_E$. In addition, the strategy of selling a bundle only to Group 1 consumers at price $p_B^{**}$ is feasible only for $c > c_0$; for lower $c$ values some Group 2 consumers will buy the bundle at this price. Finally, because $p_B^* < p_B^{**}$ clearly $c_0 < \overline{c}_B$. Figure A1 demonstrates the profit from selling the

[15] Since Group 1 has a higher value for the service than Group 2, when the bundle price is $p_B$*, the demand from Group 1 becomes zero at a value of $c$ greater than $\overline{c}_B$

bundle at price $p_B^*$ and the profit from selling it at price $p_B^{**}$ .

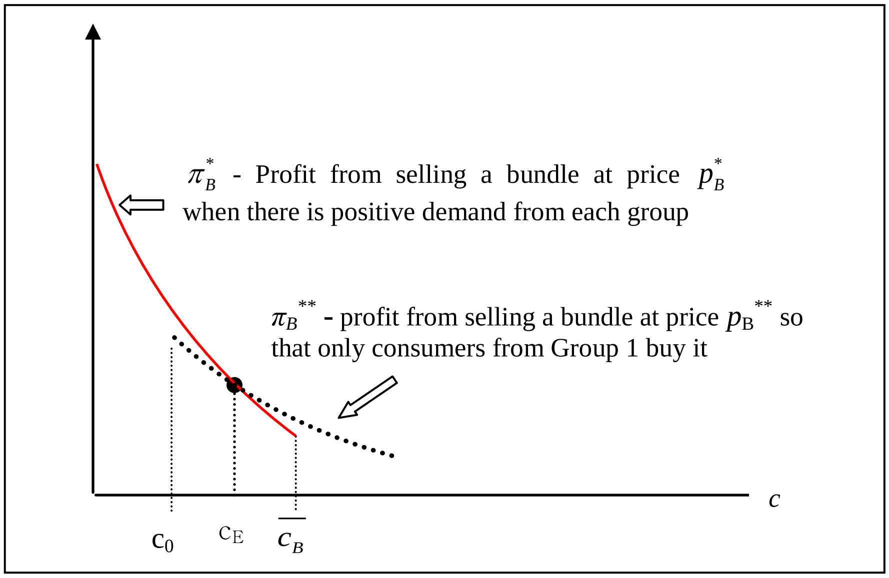


**Figure A1. Profit from selling the bundle to both groups vs. profit from selling it only to Group 1**

Next we show that the strategy of selling the bundle at price $p_B^{**}$, so that only consumers from Group 1 buy it, can never be optimal.

- When $c$ is such that $\underline{c_S} \le c \le \overline{c_S}$ , selling the service separately so that only consumers from Group 1 buy it (and consumers from Group 2 buy only the product) would yield larger profit than selling the bundle only to Group 1 at price $p_B^{**}$ (i.e., $\pi_S^* > \pi_B^{**}$ ). This is because the profit from Group 1 stays the same (under both strategies, Group 1 customers buy both product and service for total price of $\left(\overline{\theta} + \alpha_1 s + c\right)/2$), but when service is sold separately, the monopoly generates additional profit from sales of the product to Group 2.
- When $c > \overline{c_S}$ , the profit from not offering the service, $\pi_N^*$ , is larger than the expression given by $\pi_S^*$ (see Lemma 1-(ii) showing $c_{S,N} < \overline{c_S}$ ) and thus it is clearly larger than the profit from selling the bundle only to Group 1 at price $p_B^{**}$. That is, in this range $\pi_N^* > \pi_S^* > \pi_B^{**}$.
- When $c < \underline{c_S}$ , since $c < c_E$ (see Lemma 1-($v$) showing $\underline{c_S} < c_E$), selling the bundle to both groups for price $p_B^*$ as listed in Table 3 dominates selling the bundle only to Group 1 .

To summarize, when $c < \underline{c_S}$ or $c > \overline{c_S}$, we should consider selling a bundle to both groups with profit $\pi_B^*$, or not selling service at all with profit $\pi_N^*$. For $\underline{c_S} \le c \le \overline{c_S}$, all three strategies listed in Table 3 should be considered. We also note that $\pi_N^*$ is constant in *c*, while $\pi_B^*$ and $\pi_S^*$ are monotonically decreasing in *c* in the range of *c* values where the corresponding strategy is feasible. In addition, $\pi_B^*$ decreases in *c* faster than $\pi_S^*$ when $c < \overline{c_B}$ (see Lemma 1-(i)). Thus,

1) Each pair of functions out of the three ($\pi_N^*$, $\pi_B^*$, and $\pi_S^*$) might intersect at most once at the range where the relative strategies are feasible.
2) If $\pi_B^*$ and $\pi_S^*$ do intersect at *c* value smaller than $\overline{c_B}$, then $\pi_B^* > \pi_S^*$ only to the left of the intersection

To determine the optimal strategy for each *c* value, we next find the following values:

- $c_{B,S}$ : the *c* value at which $\pi_B^* = \pi_S^*$
- $c_{B,N}$ : the *c* value at which $\pi_B^* = \pi_N^*$
- $c_{S,N}$ : the *c* value at which $\pi_S^* = \pi_N^*$

The three thresholds are given by

$$c_{B,N} = \bar{\theta}\left(1 - \frac{\sqrt{\bar{\theta} - \alpha_0 N}}{\sqrt{\bar{\theta}}}\right) + \alpha_0 s$$

$$c_{S,N} = \bar{\theta}\left(1 - \frac{\sqrt{\bar{\theta} - \alpha_1 \beta N}}{\sqrt{\bar{\theta}}}\right) + \alpha_1 s \text{ , and}$$

$$c_{B,S} = \bar{\theta} - \frac{\alpha_2 \bar{\theta} s - \sqrt{(\bar{\theta} - \alpha_1 \beta N)(\bar{\theta} - \alpha_0 N)(\beta s^2 (\alpha_1 - \alpha_2)^2 + \bar{\theta}\left(\bar{\theta} - \beta N(\alpha_1 - \alpha_2)\right)}}{\bar{\theta} - \beta N(\alpha_1 - \alpha_2)}.$$

Notice that $c_{B,N} > 0$ and $c_{S,N} > 0$, and thus for low values of *c*, the service is always offered. In addition, Lemma 2 shows that $c_{S,N} < \overline{c_S}$, $c_{B,S} < \overline{c_B}$, and $\underline{c_S} < c_{B,S}$.Therefore, if the separate pricing is optimal in some range of *c* values, it must be that the bundling pricing is optimal for $c < c_{B,S}$, the separate pricing is optimal for $c_{B,S} < c < c_{S,N}$, and selling only the product is optimal for $c > c_{S,N}$.

Next, we show that $c_{B,N} > c_{S,N}$ if and only if $s < s_0$ where

$$s_0 = \frac{\sqrt{\bar{\theta}}\left(\sqrt{\bar{\theta}-\alpha_1\beta N} - \sqrt{\bar{\theta}-\alpha_0 N}\right)}{\left(\alpha_1-\alpha_2\right)(1-\beta)} \ .$$

When $s < s_0$ we have:

$$s(\alpha_1-\alpha_0) = s(\alpha_1-\beta\alpha_1-(1-\beta)\alpha_2) = s(\alpha_1-\alpha_2)(1-\beta) < \sqrt{\bar{\theta}}\left(\sqrt{\bar{\theta}-\alpha_1\beta N} - \sqrt{\bar{\theta}-\alpha_0 N}\right) \tag{A4}$$

Therefore,

$$\begin{aligned}
\pi_S^*(c=c_{B,N}) &= \frac{N}{4}\left(\frac{\beta\left(\sqrt{\bar{\theta}}\sqrt{\bar{\theta}-\alpha_0 N}+s(\alpha_1-\alpha_0)\right)^2}{\bar{\theta}-\beta\alpha_1 N}+(1-\beta)\bar{\theta}\right) \\
&= \frac{N}{4}\left(\frac{\beta}{\bar{\theta}}\left(\frac{\bar{\theta}\sqrt{\bar{\theta}-\alpha_0 N}+s(\alpha_1-\alpha_2)(1-\beta)\sqrt{\bar{\theta}}}{\sqrt{\bar{\theta}-\beta\alpha_1 N}}\right)^2+(1-\beta)\bar{\theta}\right) \\
&< \frac{N}{4}\left(\frac{\beta}{\bar{\theta}}\left(\frac{\bar{\theta}\sqrt{\bar{\theta}-\alpha_0 N}+\sqrt{\bar{\theta}}\left(\sqrt{\bar{\theta}-\alpha_1\beta N}-\sqrt{\bar{\theta}-\alpha_0 N}\right)\sqrt{\bar{\theta}}}{\sqrt{\bar{\theta}-\beta\alpha_1 N}}\right)^2+(1-\beta)\bar{\theta}\right) \\
&= \frac{N}{4}\left(\frac{\beta\bar{\theta}^2}{\bar{\theta}}+(1-\beta)\bar{\theta}\right) = \pi_N^* = \pi_B^*(c=c_{B,N})
\end{aligned} \tag{A5}$$

Since $\pi_S^*(c=c_{B,N}) < \pi_N^* = \pi_B^*(c=c_{B,N})$, it must be that $c_{S,N} < c_{B,N} < c_{B,S}$. More specifically, when $s < s_0$,

(i) $\pi_B^* > \pi_S^* > \pi_N^*$ if $c < c_{S,N}$,

(ii) $\pi_B^* > \pi_N^* > \pi_S^*$ if $c_{S,N} < c < c_{B,N}$,

(iii) $\pi_N^* > \pi_B^* > \pi_S^*$ if $c_{B,N} < c < c_{B,S}$, and

(iv) $\pi_N^* > \pi_S^* > \pi_B^*$ if $c > c_{B,S}$.

In addition, as shown in Lemma 2-(iii), $c_{B,S} < \overline{c_B}$, and thus $c_{B,N} < c_{B,S} < \overline{c_B}$ and selling a bundle at price $p_B^*$ so that both groups buy it is feasible for all $c$ value to the left of $c_{B,N}$. To summarize, when $s < s_0$, the monopoly chooses to sell a bundle when $0 < c < c_{B,N}$, and does not sell the service otherwise. Figure A2 demonstrates this case.

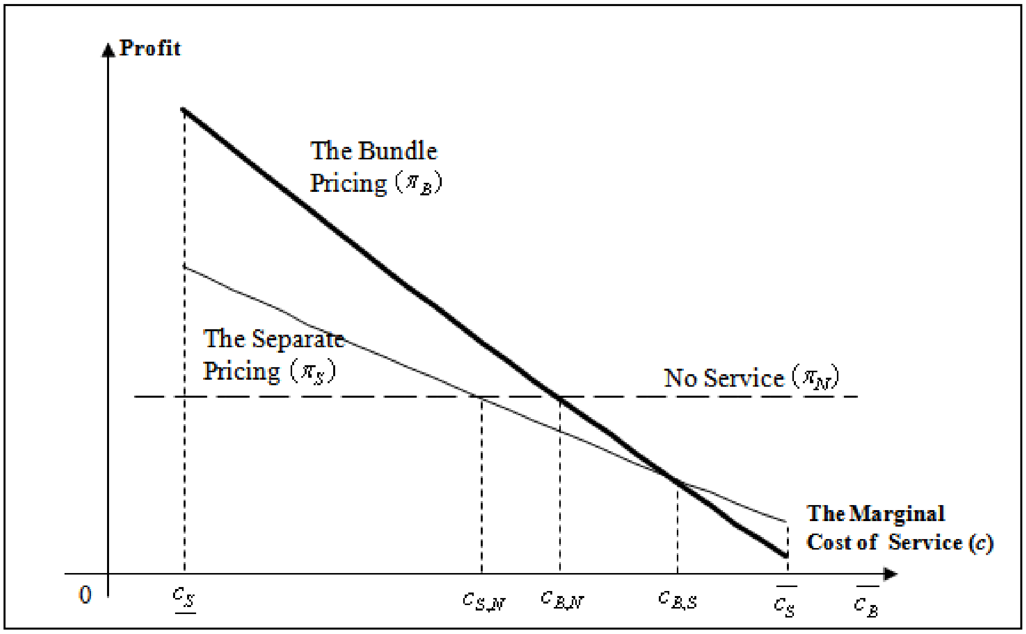


**Figure A2. Profit from the different strategies as function of the marginal cost of service when $s < s_0$ for** $(N = 4000, \alpha_1 = 0.015, \alpha_2 = 0.005, \overline{\theta} = 200, \beta = 0.6, s = 0)$

When $s > s_0$ we have $\pi_S^*(c = c_{B,N}) > \pi_N^* = \pi_B^*(c = c_{B,N})$ and therefore $c_{B,N} < c_{S,N}$. Thus, when $s > s_0$, it must be that $c_{B,S} < c_{B,N} < c_{S,N}$. More specifically, when $s > s_0$:

*(i)* $\pi_B^* > \pi_S^* > \pi_N^*$ if $c < c_{B,S}$,

*(ii)* $\pi_S^* > \pi_B^* > \pi_N^*$ if $c_{B,S} < c < c_{B,N}$,

*(iii)* $\pi_S^* > \pi_N^* > \pi_B^*$ if $c_{B,N} < c < c_{S,N}$,

*(iv)* $\pi_N^* > \pi_S^* > \pi_B^*$ if $c > c_{S,N}$.

In addition, from Lemma 2, $\underline{c_S} < c_{B,S}$, $c_{B,S} < \overline{c_B}$, and $c_{S,N} < \overline{c_S}$, so that $\underline{c_S} < c_{B,S} < c_{B,N} < c_{S,N} < \overline{c_S}$. We conclude that when $s > s_0$, bundling is optimal for $c < c_{B,S}$, selling the service separately is optimal for $c_{B,S} < c < c_{S,N}$ (this range is not empty), and not offering the service is optimal for $c > c_{S,N}$. Notice that if $c_{B,S} < 0$ then the bundle pricing is never optimal. Figure A3 demonstrate a typical case when $s > s_0$.

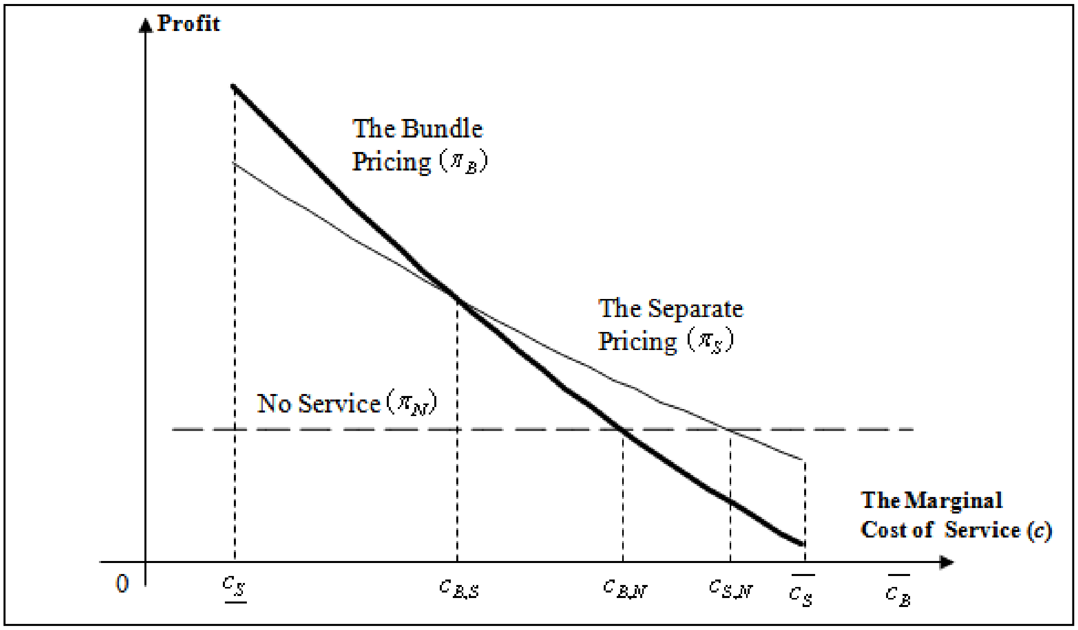


**Figure A3. Profit from the different strategies as function of the marginal cost of service when $s > s_0$, for** $(N = 4000, \alpha_1 = 0.015, \alpha_2 = 0.005, \bar{\theta} = 200, \beta = 0.6, s = 5000)$

**Proof of Lemma 2**

***(i)*** Here we show that $\frac{\partial}{\partial c}\pi_B^* < \frac{\partial}{\partial c}\pi_S^*$ *if* $c < \overline{c_B}$ . Taking derivatives of the profits from Table 3 we get:

$$\frac{\partial}{\partial c}\pi_B^* = -\frac{N\left(\bar{\theta} + \alpha_0 s - c\right)}{2\bar{\theta} - 2\alpha_0 N}, \qquad \frac{\partial}{\partial c}\pi_S^* = -\frac{\beta N\left(\bar{\theta} + \alpha_1 s - c\right)}{2\bar{\theta} - 2\alpha_1 N} \qquad . \tag{A6}$$

Thus, $\frac{\partial}{\partial c}\pi_B^* < \frac{\partial}{\partial c}\pi_S^*$ when

$$c < \bar{\theta} + \frac{\alpha_2 s \bar{\theta}}{\bar{\theta} - \beta N(\alpha_1 - \alpha_2)} \qquad . \tag{A7}$$

When Assumption 1 holds we have:

$$\left(\bar{\theta} + \frac{\alpha_2 s \bar{\theta}}{\bar{\theta} - \beta N(\alpha_1 - \alpha_2)}\right) - \overline{c_B} = \frac{\beta s(\alpha_1 - \alpha_2)\left(\bar{\theta} - \alpha_0 N\right)}{\bar{\theta} - \beta N(\alpha_1 - \alpha_2)} > 0 \qquad . \tag{A8}$$

Thus

$$\frac{\partial}{\partial c}\pi_B^* < \frac{\partial}{\partial c}\pi_S^* \text{ when } c < \overline{c_B} \qquad .$$

***(ii)*** Here we show that $c_{S,N} < \overline{c_S}$ . From the profit expressions in Table 3, we have:

$$\pi_S^*\left(c=\overline{c_S}\right)-\pi_N^* = -\frac{\alpha_1\beta^2N^2}{4}<0 \tag{A9}$$

In addition, $\pi_S^*$ is decreasing in *c* and $\pi_N^*$ is a constant in *c*. Thus, $\pi_S^*$ equals $\pi_N^*$ at a value of *c* that is smaller than $\overline{c_S}$ . That is, $c_{S,N} < \overline{c_S}$

*(iii)* Here we show that $c_{B,S} < \overline{c_B}$ . We have:

$$\pi_S^*\left(c=\overline{c_B}\right)-\pi_B^*\left(c=\overline{c_B}\right)=\frac{N(1-\beta)\left(\overline{\theta}\left(\overline{\theta}-\alpha_1\beta N\right)+\beta s^2(1-\beta)(\alpha_1-\alpha_2)^2\right)}{4\left(\overline{\theta}-\alpha_1\beta N\right)}>0$$

The last inequality holds under Assumption 1. Therefore, $\pi_S^*\left(c=\overline{c_B}\right)>\pi_B^*\left(c=\overline{c_B}\right)$ and in addition, as *c* decreases $\pi_B^*$ increases to a greater extent than $\pi_S^*$ (see Lemma 2-(i)). This shows that $\pi_S^*$ equals to $\pi_B^*$ at a value of *c* smaller than $\overline{c_B}$ , i.e., $c_{B,S} < \overline{c_B}$ .

*(iv)* Here we show that $\underline{c_S} < c_{B,S}$. The difference between the two values is given by:

$$c_{B,S}-\underline{c_S}=\frac{\sqrt{\overline{\theta}-\alpha_1\beta N}}{\left(\overline{\theta}-\beta N\left(\alpha_1-\alpha_2\right)\right)}\times \tag{A10}$$
$$\left(\sqrt{\overline{\theta}-\alpha_1\beta N}\left(\overline{\theta}-2\alpha_2+s\left(\alpha_1-\alpha_2\right)\right)-\sqrt{\left(\overline{\theta}-\alpha_0N\right)\left(\beta s^2\left(\alpha_1-\alpha_2\right)^2+\overline{\theta}\left(\overline{\theta}-\beta N\left(\alpha_1-\alpha_2\right)\right)\right)}\right)$$

We will show that under Assumptions 1 this difference is positive.

According to Assumptions 1, the following three inequalities hold:

$$\overline{\theta}-\alpha_1\beta N>\overline{\theta}-\alpha_0N>0 \tag{A11}$$

$$\overline{\theta}-\beta N\left(\alpha_1-\alpha_2\right)>\overline{\theta}-\alpha_0N>0\text{, and} \tag{A12}$$

$$\overline{\theta}-2\alpha_2>0 \tag{A13}$$

The latter inequality holds because by Assumption 1 and the fact that *N* >2 (or else there is no network effect) we have $\overline{\theta}-2\left(\beta\alpha_1+(1-\beta)\alpha_2\right)>0$, and $2\left(\beta\alpha_1+(1-\beta)\alpha_2\right)>2\alpha_2$ because $\alpha_1>\alpha_2$.

From inequalities A11 and A12 it is clear that the first term in A10 is positive.

Next we show that the second term in A10 is positive, i.e., we show that:

$$\left(\sqrt{\bar{\theta}-\alpha_1\beta N}\left(\bar{\theta}-2\alpha_2+s(\alpha_1-\alpha_2)\right)-\sqrt{\left(\bar{\theta}-\alpha_0 N\right)\left(\beta s^2(\alpha_1-\alpha_2)^2+\bar{\theta}\left(\bar{\theta}-\beta N(\alpha_1-\alpha_2)\right)\right)}\right)>0$$

From Equation (A13) we have:

$$\bar{\theta}-2\alpha_2+s(\alpha_1-\alpha_2)>0,\ \text{and} \qquad \text{(A14)}$$

from Equation (A12) we have:

$$\sqrt{\beta s^2(\alpha_1-\alpha_2)^2+\bar{\theta}\left(\bar{\theta}-\beta N(\alpha_1-\alpha_2)\right)}>0. \qquad \text{(A15)}$$

In addition, we have:

$$\begin{aligned}&\left(\bar{\theta}-2\alpha_2+s(\alpha_1-\alpha_2)\right)^2-\left(\sqrt{\beta s^2(\alpha_1-\alpha_2)^2+\bar{\theta}\left(\bar{\theta}-\beta N(\alpha_1-\alpha_2)\right)}\right)^2\\&=s^2(\alpha_1-\alpha_2)^2(1-\beta)+4\alpha_2\left(\bar{\theta}-\alpha_2\right)+2s(\alpha_1-\alpha_2)\left(\bar{\theta}-2\alpha_2\right)+\beta N\bar{\theta}(\alpha_1-\alpha_2)>0\end{aligned} \qquad \text{(A16)}$$

For any two expressions, $a$ and $b$, if $a^2$-$b^2$ >0 and ($a$+$b$) >0 then ($a$-$b$)>0. Thus, Equations A14, A15 and A16 show that:

$$\left(\bar{\theta}-2\alpha_2+s(\alpha_1-\alpha_2)\right)-\sqrt{\beta s^2(\alpha_1-\alpha_2)^2+\bar{\theta}\left(\bar{\theta}-\beta N(\alpha_1-\alpha_2)\right)}>0. \qquad \text{(A17)}$$

Finally, from A11 and A17 we conclude that

$$\sqrt{\left(\bar{\theta}-\alpha_1\beta N\right)}\left(\bar{\theta}-2\alpha_2+s(\alpha_1-\alpha_2)\right)-\sqrt{\left(\bar{\theta}-\alpha_0 N\right)\left(\beta s^2(\alpha_1-\alpha_2)^2+\bar{\theta}\left(\bar{\theta}-\beta N(\alpha_1-\alpha_2)\right)\right)}>0 \quad \text{(A18)}$$

Thus, $c_{B,S}-\underline{c_S}>0$.

*(v)* Here we show that $\underline{c_S}<c_E$ and $\underline{c_S}<\overline{c_B}$ .

When the vendor sells the bundle only to Group 1 at price $\left(\bar{\theta}+\alpha_1 s+c\right)/2$, the profit $\pi_B^{**}$ is given by

$$\pi_B^{**}=\frac{\beta N(\bar{\theta}+\alpha_1 s-c)^2}{4(\bar{\theta}-\beta\alpha_1 N)} \qquad \text{(A19)}$$

It can be easily shown that

$$\pi_B^{**} - \pi_S^* = -\frac{(1-\beta)N\overline{\theta}}{4} < 0. \tag{A20}$$

Therefore, the difference between the two is $\frac{(1-\beta)N\overline{\theta}}{4}$ which is constant in $c$. This implies that if $c_E$ denotes to the $c$ value at which $\pi_B^{**} = \pi_B^*$, and thus $\pi_B^{**} < \pi_B^*$ for $c< c_E$ , then $c_E > c_{B,S}$, and by Lemma 2-(iv),

$$\underline{c_S} < c_{B,S} < c_E \tag{A21}$$

In addition, because $c_E < \overline{c_B}$ we get $\underline{c_S} < \overline{c_B}$ .

**Proof of Proposition 2**

The profit from selling a bundle and the profit from selling only the product are given in Table 3.

The social welfare when the monopoly does not sell the service is not a function of $s$ or $c$ and is given by:

$$\pi_N^* + CS_N^* = \frac{3N\overline{\theta}}{8}. \tag{A22}$$

When the monopoly sells the bundle, the utility of a Group $j$' consumer is given by

$$u_j^* = \theta + \alpha_j(n_s^* + s) - p_B^* = \theta + \alpha_j\left(\frac{N(\overline{\theta}+\alpha_0 s - c)}{2(\overline{\theta}-\alpha_0 N)} + s\right) - \frac{\overline{\theta}+\alpha_0 s + c}{2} \tag{A23}$$

From $u_j^* = 0$, the marginal customer from Group $j$ is given by

$$\theta_j^* = \frac{\overline{\theta}+\alpha_0 s + c}{2} - \alpha_j\left(\frac{N(\overline{\theta}+\alpha_0 s - c)}{2(\overline{\theta}-\alpha_0 N)} + s\right) \tag{A24}$$

$$= \frac{2(\overline{\theta}-\alpha_0 N)\left(\overline{\theta}+s(\alpha_0-\alpha_j)\right) - (\overline{\theta}+\alpha_0 s - c)\left(\overline{\theta}-N(\alpha_0-\alpha_j)\right)}{2(\overline{\theta}-\alpha_0 N)} \qquad .$$

Given the above indifferent consumer, the surplus of all Group 1 customers is given by

$$\frac{\beta N}{\overline{\theta}}\int_{\theta_1^*}^{\overline{\theta}} u_1^* d\theta = \frac{\beta N}{\overline{\theta}}\int_{\theta_j^*}^{\overline{\theta}}\left(\theta + \alpha_1\left(\frac{N(\overline{\theta}+\alpha_0 s - c)}{2(\overline{\theta}-\alpha_0 N)} + s\right) - \frac{\overline{\theta}+\alpha_0 s + c}{2}\right) d\theta \qquad , \tag{A25}$$

and the surplus of all Group 2 customers is given by:

$$\frac{(1-\beta)N}{\overline{\theta}}\int_{\theta_2^*}^{\overline{\theta}} u_2^* d\theta = \frac{(1-\beta)N}{\overline{\theta}}\int_{\theta_2^*}^{\overline{\theta}}\left(\theta + \alpha_2\left(\frac{N(\overline{\theta}+\alpha_0 s - c)}{2(\overline{\theta}-\alpha_0 N)} + s\right) - \frac{\overline{\theta}+\alpha_0 s + c}{2}\right) d\theta. \tag{A26}$$

Thus, when the monopoly sells the bundle at the profit maximizing bundle price, total consumer surplus is given by

$$CS_B^* = \frac{\beta N\left(\left(\bar{\theta}+\alpha_0 s-c\right)\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_1)\right)^2}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2} + \frac{(1-\beta)N\left(\left(\bar{\theta}+\alpha_0 s-c\right)\left(\bar{\theta}-N(\alpha_0-\alpha_2)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_2)\right)^2}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}$$

We denote by $d_{B,N}$ the solution (c value) of the following equation $\pi_B^* + CS_B^* = \pi_N^* + CS_N^*$, and by $c_{B,N}$ the solution (c value) of $\pi_B^* = \pi_N^*$. However, after deriving the general expressions for $d_{B,N}$ and $c_{B,N}$ we could not directly show which expression is larger. Thus we take a different approach in this proof. Instead of comparing the two expressions for a general s value, we first show that when $s = 0$ $c_{B,N} < d_{B,N}$, and then we show that $\frac{\partial}{\partial s} d_{B,N} > \frac{\partial}{\partial s} c_{B,N}$.

When $s = 0$, the social welfare when the monopoly sells the bundle is given by:

$$\pi_B^* + CS_B^* = \frac{N\left(\bar{\theta}-c\right)^2}{4(\bar{\theta}-\alpha_0 N)} + \frac{\beta N\left(\bar{\theta}-c\right)^2\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)^2}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2} + \frac{N(1-\beta)\left(\bar{\theta}-c\right)^2\left(\bar{\theta}-N(\alpha_0-\alpha_2)\right)^2}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2} \quad \text{(A27)}$$

$$= \frac{N\left(\bar{\theta}-c\right)^2\left(\beta\left(3\bar{\theta}^2+N^2(\alpha_0-\alpha_1)^2-2N\bar{\theta}(2\alpha_0-\alpha_1)\right)+(1-\beta)\left(3\bar{\theta}^2+N^2(\alpha_0-\alpha_2)^2-2N\bar{\theta}(2\alpha_0-\alpha_2)\right)\right)}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2} =$$

$$\frac{N\left(\bar{\theta}-c\right)^2\left(\left(\bar{\theta}-\alpha_0 N\right)\left(3\bar{\theta}+\alpha_0 N\right)+N^2\left(\beta\alpha_1^2+(1-\beta)\alpha_2^2\right)\right)}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}$$

By solving $\pi_B^* + CS_B^* = \pi_N^* + CS_N^*$ with s=0 (the RHS of this equation is given in A22, and the LHS in A27) we find that social welfare under bundling is greater than under no service if and only if $c < d_{B,N}$, where

$$d_{B,N} = \bar{\theta} - \frac{\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)\sqrt{3}}{\sqrt{\left(\bar{\theta}-\alpha_0 N\right)\left(3\bar{\theta}+\alpha_0 N\right)+N^2\left(\beta\alpha_1^2+(1-\beta)\alpha_2^2\right)}} \quad \text{(A28)}$$

In addition, according to Proposition 1, when $s = 0$ the *c* value at which the seller would start offering the bundle is :

$$c_{B,N} = \bar{\theta} - \sqrt{\bar{\theta}}\sqrt{\bar{\theta} - \alpha_0 N}. \quad \text{(A29)}$$

Comparing the expressions for $c_{B,N}$ and $d_{B,N}$ when *s*=0 we get:

$$d_{B,N} - c_{B,N} = \sqrt{\bar{\theta}}\sqrt{\bar{\theta} - \alpha_0 N} - \frac{\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)\sqrt{3}}{\sqrt{3\left(\bar{\theta}-\alpha_0 N\right)\left(3\bar{\theta}+\alpha_0 N\right)+N^2\left(\beta\alpha_1^2+(1-\beta)\alpha_2^2\right)}}$$

$$= \sqrt{\bar{\theta}}\sqrt{\bar{\theta} - \alpha_0 N}\left(1 - \sqrt{\frac{3\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)}{\left(\bar{\theta}-\alpha_0 N\right)\left(3\bar{\theta}+\alpha_0 N\right)+N^2\left(\beta\alpha_1^2+(1-\beta)\alpha_2^2\right)}}\right) \quad \text{(A30)}$$

In addition, due to Assumption 1, according to which $\bar{\theta} - \alpha_0 N > 0$, we have :

$$\left((\bar{\theta} - \alpha_0 N)(3\bar{\theta} + \alpha_0 N) + N^2(\beta\alpha_1^2 + (1-\beta)\alpha_2^2)\right) - 3\bar{\theta}(\bar{\theta} - \alpha_0 N)$$

$$= (\bar{\theta} - \alpha_0 N)(3\bar{\theta} + \alpha_0 N - 3\bar{\theta}) + N^2(\beta\alpha_1^2 + (1-\beta)\alpha_2^2)$$

$$= N\left(\alpha_0(\bar{\theta} - \alpha_0 N) + N(\beta\alpha_1^2 + (1-\beta)\alpha_2^2)\right) > 0$$

As a result, $\sqrt{\frac{3\bar{\theta}(\bar{\theta}-\alpha_0 N)}{(\bar{\theta}-\alpha_0 N)(3\bar{\theta}+\alpha_0 N)+N^2(\beta\alpha_1^2+(1-\beta)\alpha_2^2)}} < 1$ and $d_{B,N} > c_{B,N}$.

Now, suppose $s > 0$. We use the following notation:

$F(c,s) = \pi_B^*(c,s) + CS_B^*(c,s) - \pi_N^* - CS_N^*$ and

$G(c,s) = \pi_B^*(c,s) - \pi_N^*$.

Thus, $d_{B,N}$ and $c_{B,N}$ are the $c$ values at which of $F(c,s) = 0$ and $G(c,s) = 0$, respectively. Then, by the Implicit Function Theorem, we have:

$$\frac{\partial}{\partial s} d_{B,N} = -\frac{\partial F/\partial s}{\partial F/\partial c}\bigg|_{c=d_{B,N}} \tag{A31}$$

$$\frac{\partial}{\partial s} c_{B,N} = -\frac{\partial G/\partial s}{\partial G/\partial c}\bigg|_{c=c_{B,N}} \tag{A32}$$

Next we show that $\frac{\partial}{\partial s} d_{B,N} > \frac{\partial}{\partial s} c_{B,N}$, and since $d_{B,N} > c_{B,N}$ when $s=0$ (shown above), this is also the case for larger $s$ values. The two derivatives are :

$$\frac{\partial}{\partial s} c_{B,N} = -\frac{\partial G/\partial s}{\partial G/\partial c}\bigg|_{c=c_{B,N}} = -\frac{\partial \pi_B^*/\partial s}{\partial \pi_B^*/\partial c}\bigg|_{c=c_{B,N}} = -\left(\frac{\alpha_0(\bar{\theta}+\alpha_0 s-c_{B,N})}{2(\bar{\theta}-\alpha_0 N)}\right)\Big/\left(\frac{-(\bar{\theta}+\alpha_0 s-c_{B,N})}{2(\bar{\theta}-\alpha_0 N)}\right) = \alpha_0 \tag{A33}$$

$$\frac{\partial}{\partial s} d_{B,N} = -\frac{\partial F/\partial s}{\partial F/\partial c}\bigg|_{c=d_{B,N}} = -\frac{\frac{\partial \pi_B^*}{\partial s}+\frac{\partial CS_B^*}{\partial s}}{\frac{\partial \pi_B^*}{\partial c}+\frac{\partial CS_B^*}{\partial c}}\Bigg|_{c=d_{B,N}} \tag{A34}$$

Comparing $\frac{\partial}{\partial s} d_{B,N}$ and $\frac{\partial}{\partial s} c_{B,N}$ we get:

$$\frac{\partial}{\partial s} d_{B,N} - \frac{\partial}{\partial s} c_{B,N} = -\frac{\frac{\partial \pi_B^*}{\partial s}+\frac{\partial CS_B^*}{\partial s}}{\frac{\partial \pi_B^*}{\partial c}+\frac{\partial CS_B^*}{\partial c}}\Bigg|_{c=d_{B,N}} - \alpha_0 = \frac{-\frac{\partial \pi_B^*}{\partial s}-\frac{\partial CS_B^*}{\partial s}-\alpha_0\left(\frac{\partial \pi_B^*}{\partial c}+\frac{\partial CS_B^*}{\partial c}\right)}{\frac{\partial \pi_B^*}{\partial c}+\frac{\partial CS_B^*}{\partial c}}\Bigg|_{c=d_{B,N}}$$

$$= -\frac{\alpha_0\frac{\partial CS_B^*}{\partial c}+\frac{\partial CS_B^*}{\partial s}}{\frac{\partial \pi_B^*}{\partial c}+\frac{\partial CS_B^*}{\partial c}}\Bigg|_{c=d_{B,N}} \tag{A35}$$

Where the last equality above was derived using the fact that $-\partial\pi_B^*/\partial s\ = \alpha_0\partial\pi_B^*/\partial c$ (see A33). Next, we derive $\frac{\partial CS_B^*}{\partial c}$ and $\frac{\partial CS_B^*}{\partial s}$ to be substituted in the numerator of above expression:

$$\frac{\partial}{\partial s}CS_B^* =$$

$$\frac{\partial}{\partial s}\left(\frac{\beta N\left((\bar{\theta}+\alpha_0 s-c)\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_1)\right)^2}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}+\right.$$

*1−βNθ+α0s−cθ−Nα0−α2−2sθ−α0Nα0−α228θθ−α0N2*=

$$\frac{\beta N\left(N\alpha_0(\alpha_0-\alpha_1)-\bar{\theta}(\alpha_0-2\alpha_1)\right)\left((\bar{\theta}+\alpha_0 s-c)\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_1)\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}+$$

$$\frac{(1-\beta)N\left(N\alpha_0(\alpha_0-\alpha_2)-\bar{\theta}(\alpha_0-2\alpha_2)\right)\left((\bar{\theta}+\alpha_0 s-c)\left(\bar{\theta}-N(\alpha_0-\alpha_2)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_2)\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}\quad .$$

$$\frac{\partial}{\partial c}CS_B^* =$$

$$-\frac{\beta N\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)\left((\bar{\theta}+\alpha_0 s-c)\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_1)\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}-$$

$$\frac{(1-\beta)N\left(\bar{\theta}-N(\alpha_0-\alpha_2)\right)\left((\bar{\theta}+\alpha_0 s-c)\left(\bar{\theta}-N(\alpha_0-\alpha_2)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_2)\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}$$

Thus, the numerator of (A35) , before substituting $c = d_{B,N}$, is given by:

$$\alpha_0\frac{\partial CS_B^*}{\partial c}+\frac{\partial CS_B^*}{\partial s} =$$

$$\frac{\beta N\left(N\alpha_0(\alpha_0-\alpha_1)-\bar{\theta}(\alpha_0-2\alpha_1)-\alpha_0\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)\right)\left((\bar{\theta}+\alpha_0 s-c)\left(\bar{\theta}-N(\alpha_0-\alpha_1)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_1)\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}+$$

$$\frac{(1-\beta)N\left(N\alpha_0(\alpha_0-\alpha_2)-\bar{\theta}(\alpha_0-2\alpha_2)-\alpha_0\left(\bar{\theta}-N(\alpha_0-\alpha_2)\right)\right)\left((\bar{\theta}+\alpha_0 s-c)\left(\bar{\theta}-N(\alpha_0-\alpha_2)\right)-2s(\bar{\theta}-\alpha_0 N)(\alpha_0-\alpha_2)\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2}$$

$$=\frac{-N\bar{\theta}(\bar{\theta}+\alpha_0 s-c)}{2\bar{\theta}(\bar{\theta}-\alpha_0 N)}\left(\beta(\alpha_0-\alpha_1)+(1-\beta)(\alpha_0-\alpha_2)\right)+$$

$$\frac{N\left(N(\bar{\theta}+\alpha_0 s-c)+2s(\bar{\theta}-\alpha_0 N)\right)}{2\bar{\theta}(\bar{\theta}-\alpha_0 N)}\left(\beta(\alpha_0-\alpha_1)^2+(1-\beta)(\alpha_0-\alpha_2)^2\right)$$

$$=\frac{N\left(N(\bar{\theta}+\alpha_0 s-c)+2s(\bar{\theta}-\alpha_0 N)\right)}{2\bar{\theta}(\bar{\theta}-\alpha_0 N)}\left(\beta(\alpha_0-\alpha_1)^2+(1-\beta)(\alpha_0-\alpha_2)^2\right) \qquad \text{(A36)}$$

Equation A(36) evaluated at $c = d_{B,N}$ is positive because $d_{B,N}$ cannot be greater than $\bar{\theta} + \alpha_0 s$ (the equilibrium bundle demand becomes zero at $c = \bar{\theta} + \alpha_0 s$). Thus, given Assumption 1, the numerator of (A35) is positive and in addition the denominator is negative, i.e.,$\frac{\partial \pi_B^*}{\partial c} + \frac{\partial CS_B^*}{\partial c} < 0$, because total social surplus decreases in $c$. Therefore,

$$\frac{\partial}{\partial s} d_{B,N} - \frac{\partial}{\partial s} c_{B,N} = -\frac{\alpha_0^m \frac{\partial CS_B^*}{\partial c} + \frac{\partial CS_B^*}{\partial s}}{\frac{\partial \pi_B^*}{\partial c} + \frac{\partial CS_B^*}{\partial c}} > 0$$

indicating that $d_{B,N}$ increases faster than $c_{B,N}$ in $s$. Thus, we conclude that $d_{B,N} > c_{B,N}$ when $s \geq 0$.

When s <$s_0$, the monopoly chooses only between bundling and not selling the service and thus, for $c$ values such that $c_{B,N} < c < d_{B,N}$, he will not sell the service while social welfare would be higher if a bundle was offered

*The proof of part (ii) follows a similar procedure and can be obtained from the authors.*

**Proof of Lemma 1**

***(i)***. When $s = 0$,

$$\pi_B = \frac{N(\bar{\theta} - c)^2}{4(\bar{\theta} - \alpha_0 N)}$$

$$CS_B = \frac{\beta N\Big((\bar{\theta} - c)\big(\bar{\theta} - N(\alpha_0 - \alpha_1)\big)\Big)^2 + N(1-\beta)\Big((\bar{\theta} - c)\big(\bar{\theta} - N(\alpha_0 - \alpha_2)\big)\Big)^2}{8\bar{\theta}(\bar{\theta} - \alpha_0 N)^2} = \frac{N(\bar{\theta} - c)^2(\bar{\theta}^2 + \beta N^2(1-\beta)(\alpha_1 - \alpha_2)^2)}{8\bar{\theta}(\bar{\theta} - \alpha_0 N)^2}$$

$$\frac{\partial}{\partial c}\pi_B = -\frac{N(\bar{\theta} - c)}{2(\bar{\theta} - \alpha_0 N)}$$

$$\frac{\partial}{\partial c} CS_B = -\frac{N(\bar{\theta} - c)(\bar{\theta}^2 + \beta N^2(1-\beta)(\alpha_1 - \alpha_2)^2)}{4\bar{\theta}(\bar{\theta} - \alpha_0 N)^2}$$

$$\frac{\partial}{\partial c}\pi_B - \frac{\partial}{\partial c} CS_B = \frac{N(\bar{\theta} - c)(-\bar{\theta}^2 + 2\alpha_0 N\bar{\theta} + \beta N^2(1-\beta)(\alpha_1 - \alpha_2)^2)}{4\bar{\theta}(\bar{\theta} - \alpha_0 N)^2} = \frac{N(\bar{\theta} - c)(\bar{\theta}(2\alpha_0 N - \bar{\theta}) + \beta N^2(1-\beta)(\alpha_1 - \alpha_2)^2)}{4\bar{\theta}(\bar{\theta} - \alpha_0 N)^2}$$

The above is positive if $\bar{\theta} < 2\alpha_0 N$ and $\bar{\theta} > c$ (these are sufficient , but not necessary conditions).

It should be noted that when $s$ =0, if $\bar{\theta} < c$ the seller would not sell a bundle anyway.

***(ii).*** Suppose that all $\alpha_j$ increase by same incremental amount, $\varepsilon$. After the increase, $\alpha_0$ (the average marginal network valuations) becomes $\alpha_0 + \varepsilon$. Denote the consumer surplus and the monopoly profit after the increase by $CS_B^{'}$ and $\pi_B^{'}$. What we show here is that

$$\lim_{\varepsilon\to 0}\frac{CS_B^{'}-CS_B}{\varepsilon} > \lim_{\varepsilon\to 0}\frac{\pi_B^{'}-\pi_B}{\varepsilon}$$

The right-hand side of the above inequality is equivalent to the derivative of $\pi_B$ with respect to $\alpha_0$.

$$\lim_{\varepsilon\to 0}\frac{\pi_B^{'}-\pi_B}{\varepsilon} = \frac{\partial \pi_B}{\partial \alpha_0} = \frac{N^2(\bar{\theta}-c)^2}{4(\bar{\theta}-\alpha_0 N)^2}$$

Using the expression for consumer surplus from the proof of part *(i)* we have:

$$\frac{CS_B^{'}-CS_B}{\varepsilon} = \frac{1}{\varepsilon}\left(\frac{N\left(\bar{\theta}-c\right)^2\left(\bar{\theta}^2+\beta N^2(1-\beta)(\alpha_1-\alpha_2)^2\right)}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^2} - \frac{N\left(\bar{\theta}-c\right)^2\left(\bar{\theta}^2+\beta N^2(1-\beta)(\alpha_1-\alpha_2)^2\right)}{8\bar{\theta}\left(\bar{\theta}-N(\alpha_0+\varepsilon)\right)^2}\right).$$

$$\lim_{\varepsilon\to 0}\frac{CS_B^{'}-CS_B}{\varepsilon} = \frac{N^2\left(\bar{\theta}-c\right)^2\left(\bar{\theta}^2+\beta N^2(1-\beta)(\alpha_1-\alpha_2)^2\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^3}$$

Finally, due to Assumption 1, it is easy to see that

$$\lim_{\varepsilon\to 0}\frac{CS_B^{'}-CS_B}{\varepsilon} - \frac{\partial \pi_B}{\partial \alpha_0} = \frac{N^3\left(\bar{\theta}-c\right)^2\left(\alpha_0\bar{\theta}+\beta N^2(1-\beta)(\alpha_1-\alpha_2)^2\right)}{4\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^3} > 0\ .$$

***(iii)*** Here we simply compare the derivatives:

$$\frac{\partial}{\partial N}\pi_B = \frac{\bar{\theta}\left(\bar{\theta}-c\right)^2}{4\left(\bar{\theta}-\alpha_0 N\right)^2}$$

$$\frac{\partial}{\partial N}CS_B = \frac{\left(\bar{\theta}-c\right)^2\left(\bar{\theta}^3+\alpha_0 N\bar{\theta}^2-3\beta N^2\bar{\theta}(1-\beta)(\alpha_1-\alpha_2)^2+\alpha_0\beta N^3(1-\beta)(\alpha_1-\alpha_2)^2\right)}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^3}$$

$$\frac{\partial}{\partial N}\pi_B - \frac{\partial}{\partial N}CS_B = \frac{\left(\bar{\theta}-c\right)^2\left(-\bar{\theta}^3+3\alpha_0 N\bar{\theta}^2+3\beta N^2\bar{\theta}(1-\beta)(\alpha_1-\alpha_2)^2-\alpha_0\beta N^3(1-\beta)(\alpha_1-\alpha_2)^2\right)}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^3}$$

$$= \frac{\left(\bar{\theta}-c\right)^2\left(\alpha_0\beta N^2(1-\beta)(\alpha_1-\alpha_2)^2\left(3\bar{\theta}-\alpha_0 N\right)+\bar{\theta}\left(3\alpha_0 N-\bar{\theta}\right)\right)}{8\bar{\theta}\left(\bar{\theta}-\alpha_0 N\right)^3}$$

The above is positive if $\bar{\theta} < 3\alpha_0 N$ (a sufficient condition)

## Appendix 2. Three Extensions

In this Appendix we discuss some of the assumptions from our model and show that the results are generally robust to changes in those assumptions.

### 1. The Number of Groups

In the model presented in Section 3 we assumed that there are only two possible consumer types in respect to service and network valuations. However, the results presented in this paper still hold for the more general case in which consumers are categorized into *m* groups according to their service valuations.

In the generalized model, consumers from Group *i* (*i*= 1, 2, … ,*m*) have marginal service and network valuation $\alpha_i$ , and, without loss of generality, we assume $\alpha_i > \alpha_{i+1} > 0$ for $i$= 1,…, *m*-1. In addition, the proportion of Group *i* of the entire population is $\beta_i$ (where $\beta_i \geq 0$ and $\sum_{i=1}^{m} \beta_i = 1$) .

As in the case of two groups, it can be shown that the profit from selling the bundle is concave in the bundle price $p_B$. As the bundle price increases, demand from each group decreases, and gradually demand from groups with lower service valuations may become zero. The maximum profit is achieved at some value of $p_B$ such that consumers from $k_B$ groups buy the bundle (i.e., there is positive demand from groups $i$=1 to $k_B$ and zero demand from groups $i=k_B+1, ..,m$.) It can be shown that if the profit is maximized when demand from groups $i$=1 to $k_B$ is positive and not all consumers from Group 1 buy the bundle (the equivalent of Assumption 1) then the optimal bundle price is:

$$p_B^* = \frac{\beta_0^{k_B}\,\bar{\theta} + \alpha_0^{k_B} s + \beta_0^{k_B} c}{2\beta_0^{k_B}} \qquad \text{(B1)}$$

and the resulting profit is given by:

$$\pi_B^* = \frac{N\left(\beta_0^{k_B}\,(\bar{\theta} - c) + \alpha_0^{k_B} s\right)^2}{4\,\beta_0^{k_B}\left(\bar{\theta} - \alpha_0^{k_B} N\right)} \qquad \text{(B2)}$$

where $\alpha_0^k = \sum_{i=1}^{k} \alpha_i \beta_i$, and $\beta_0^k = \sum_{i=1}^{k} \beta_i$ .

The profit from selling the service separately is concave in the service price *f* and in the product price $p_S$. Holding the product price fixed, as *f* increases demand for the service from each group decreases, and gradually the service demand from groups with low service valuations becomes zero. Holding the service fee fixed, as $p_s$ increases demand from groups that buy only the product decreases, as well as demand from groups that buy both the service and the product. The maximum profit is achieved at prices $p_s$ and $f$ such that $k_S$ groups buy the service (i.e., customers from groups $i$=1 to $k_S$ buy the service, while customers from groups $i=k_S+1, .. ,m$ buy only the product). Finally, it can be shown that when the profit from selling the service separately is maximized at prices such that only $k_S$ groups buy the service, customers from group $k_s$ have strictly larger utility from buying the service than from not buying it, and there are some consumers from Group 1 that do not buy the product and the service (the equivalent of Assumption 2), then the optimal service fee and product price are given by:

$$p_S^* = \frac{\overline{\theta}}{2} \qquad f^* = \frac{\alpha_0^{k_s} s + \beta_0^{k_s} c}{2\beta_0^{k_s}}, \tag{B3}$$

and the resulting profit is given by:

$$\pi_S^* = \frac{N\left(\beta_0^{k_s}(\overline{\theta}-c)+\alpha_0^{k_s}s\right)^2}{4\beta_0^{k_s}\left(\overline{\theta}-\alpha_0^{k_s}N\right)} + \frac{N\overline{\theta}\left(1-\beta_0^{k_s}\right)}{4}. \tag{B4}$$

Notice that if $k_s = k_B = m$, then the two strategies become equivalent. If $k_s = k_B < m$, then selling the service separately dominates selling the bundle. Thus for $m$=2 we needed to compare only the profit from selling a bundle at a price such that consumers from both groups buy it (i.e., $k_B$=2) to the profit from selling the service separately at a price such that only consumers from Group 1 buy it ($k_s$=1). The profit from selling a bundle at a price such that only consumers from Group 1 would buy it was dominated by the profit from selling the service separately.

Deriving analytically the optimal strategy for each set of parameters values (a generalized version of Lemma 1) becomes significantly more complex as it would depend on the specific values of $k_b$ and $k_s$ which also depend on the parameters values. However, for each set of parameters' values it is easy to numerically determine which strategy maximizes the seller's profit. In addition, we can show numerically cases of under provision of the service when $m$>2.

For simplicity of the model and the presentation of the results, we keep $m$ = 2 throughout the paper. It is important to note though that our results do not depend on the assumption that $m$ = 2, and under provision of the service as described in Sections 4 and 5 occurs even with $m$ >2. In fact, there are more possible types of under provision as $m$ increases. For example, it can be that the monopoly chooses to sell the service separately at a price so that only customers from Groups 1 and 2 buy it, but it is socially optimal to sell the service separately at a price such that customers from Groups 1, 2 and 3 buy it.

As in the case of $m$ = 2, searching a large space of parameters values we did not find any numerical examples of over provision of the service.

**2. Linear Network Effects**

In the model presented in this paper, we follow the common modeling approach according to which the value of the network to a user is linearly increasing in the number of network users (e.g., Katz and Shapiro 1986 and 1992, Fudenberg and Tirole 2000, Jing 2007). However, in some cases it might be argued that the value of the network to a user is a concave function of the number of users. That is, the marginal value derived from each additional service-user is decreasing.

When service valuations are a concave function of the size of the network, intuition predicts that there would be less incentive for the monopoly to offer the service and in addition the service might be less socially desirable (compared to the linear valuations case). However, we still expect to see under provision of the service, because, as with linear network effects, the value generated to consumers in the presence of network effects cannot be fully captured by the monopoly.

When the benefits of the network are a concave function of the number of users, the utility a user from Group $i$ derives from buying a bundle at price $p_B$ is given by

$$u = \theta + \alpha_i\left(s + n_s^{\rho}\right) - p_B, \qquad \text{(B5)}$$

and the utility the user derives from buying the product and the service separately is given by

$$u = \theta + \alpha_i\left(s + n_s^{\rho}\right) - p_s - f \qquad , \qquad \text{(B6)}$$

where $0 < \rho < 1$. When $\rho = 1$ we get the model presented in the previous sections, in which the utility is linear in the network size. When $\rho = 0$, we have a service that does not display network effects.

Unlike in the case of $\rho = 1$, when the utility is a concave function of the network size it is impossible to derive a closed form expression for the Fulfilled Expectations Equilibrium demand as a function of price(s). Thus it is also impossible to derive closed form expressions for the optimal price or the seller's profit. However, the monopolist's profit at different given prices and from different strategies, and thus the monopolist's optimal strategy, can be found numerically for each set of parameter values.

Figure 8 provides a numerical example that demonstrates the optimal strategy and the strategy that maximizes social welfare when there are two groups of consumers and service valuations are a concave function of the network size (specifically, $\rho = 0.5$). From Figure B1 we see that the results are very similar to those derived when the valuations are linearly increasing in the network size. Specifically, as the marginal cost of providing the service decreases, the optimal monopoly's strategy changes from not selling the service, to selling it separately to bundling it with the product. We also see that, as in the case of linear network valuations, there is a range of $c$ values where the service is under provided. Specifically, when $14< c <16$ the monopoly sells the service separately while offering a bundle would maximize social welfare, and when $44< c <45$ the monopoly does not offer the service, while selling it separately would maximize social welfare.

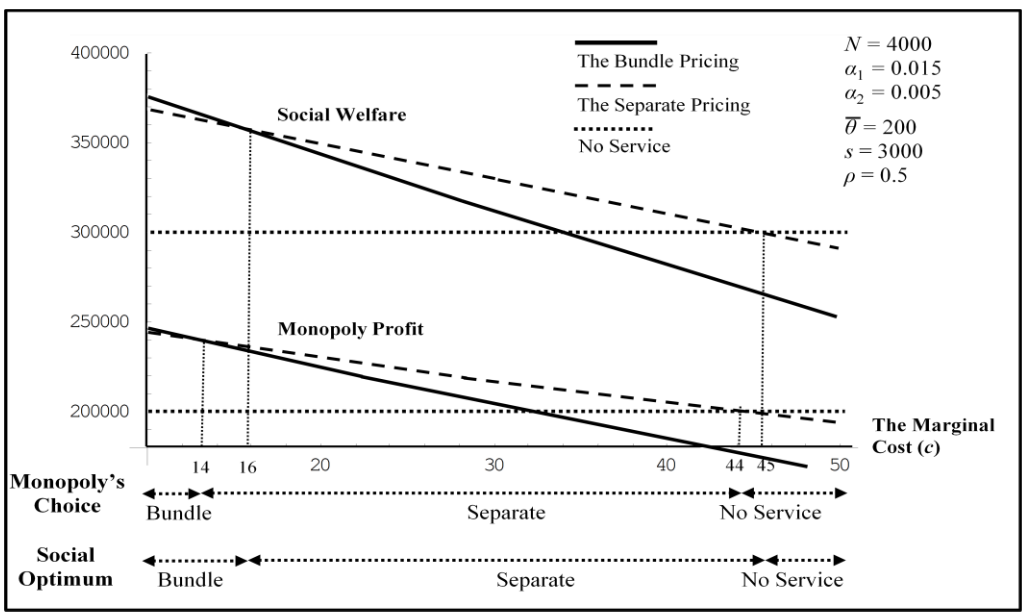


**Figure B1. Under provision of the service when service valuations are a concave function of the network size**

Holding all parameter values as in the numerical example displayed in Figure B1, but setting $\rho$ =1 (so that network valuations are linearly increasing in the network size) we find that the monopoly sells a bundle for $c$ < 45.24, offers the service separately for 45.24 < $c$ < 63.89 and does not sell the service otherwise. In addition, when $c$ < 59.87 social welfare is maximized by a bundle, and when 59.87< $c$ < 70.18 social welfare is maximized when service is sold separately. Thus, as predicted, everything else being equal, when valuations are a concave function of the size of the network, the service is less likely to be offered (i.e., it is offered at a smaller range of $c$ values), and in addition is less likely to be socially desirable. Thus, the range of parameter values for which under provision would be observed is smaller.

Finally, even though it is impossible to derive closed form expressions for the equilibrium network size, the optimal bundle price and the resulting profit, the following proposition demonstrates the existence of under-provision of the service when consumer utility is a concave function of the network size, and consumers have homogenous network valuations. Specifically, it shows that there is always a range of $c$ values for which the service would be under provided.

***Proposition 3.*** *When network valuations are given by a concave function of network size and consumers have homogenous network valuations, there exists a none-empty range of marginal costs in which service is under provided.*

The Proof can be obtained from the authors.

**3. Independency between Product and Service Valuations.**

In the model presented in Section 3, a consumer's product valuation and service valuation are independent of each other. This is a reasonable assumption for cases in which the product and service differ in nature and in use (for example, in the case of the Webkinz plush toy and the related Webkinz World website). However, in other cases it might be reasonable to assume that product and service valuations are positively correlated.

If consumers who value the product highly also value the service highly, and consumers with low product valuation also have low service valuation, then it is likely that selling a bundle would become less profitable, because now the seller has to set a lower bundle price to attract consumers with low service valuations (as most of them would also have low product valuations), or forgo sells to such consumers all together. Another important point to notice is that with positive correlation between product and service valuations we have fewer consumers with high product valuation but low service valuation. That is, there are less consumers who would buy a bundle but will not buy the service when it is sold separately. Thus when service and product valuations are positively correlated, we expect the seller to sell the service separately for a larger range of parameter values than when valuations are independent

To validate these predictions, we consider the following modification of our model. We assume that the valuation for the product, $\theta$, is uniformly distributed on $[0, \overline{\theta}]$. In addition, as before, a consumer's valuation for the service is given by $\alpha(s+n_s)$, however, here we assume that a consumer with product valuation $\theta$, has high marginal valuation for the service, i.e., $\alpha = \alpha_1$, with probability $\theta/\overline{\theta}$ and low marginal valuation, $\alpha = \alpha_2$, otherwise. Clearly, in this model product and service valuations are positively correlated (for example, a consumer with the highest product valuation will always have a high service valuation, and a consumer who does not value the product, would have low service valuation with certainty.)

Though this latter model cannot be solved analytically, for any set of parameters values we can numerically find: 1) the seller's optimal bundle price and the resulting profit and consumer surplus, and 2) the optimal product and service prices, when service is sold separately, and the resulting profit and consumer surplus. Figure B2 presents the profit and the social welfare for these two strategies, which were found numerically, as well as profit and the social welfare from selling only the product, as functions of the marginal cost, $c$. Figure B3 presents the monopolist's profit and the social welfare for the same parameters values as in Figure B2, but when product and service valuations are independent.

Comparing the two figures we see that, as predicted, when valuations are correlated, selling the service separately would become more profitable then selling a bundle for a larger range of parameters values. We also learn from this numerical example that with positive correlation, selling the service separately might dominate not offering the service for a larger range of parameters values. In addition, the profit from selling the bundle is lower in the case of positive correlation.

Finally, as is evident from Figure B2, even with positive correlation the service is often under provided. Specifically, the seller might choose to sell the service separately when social welfare is maximized by a bundle, or not offer the service while social welfare is maximized when service is sold separately. In fact, the first type of under provision is even more likely to prevail when valuations are positively correlated. We can provide many additional numerical examples that demonstrate the same results.

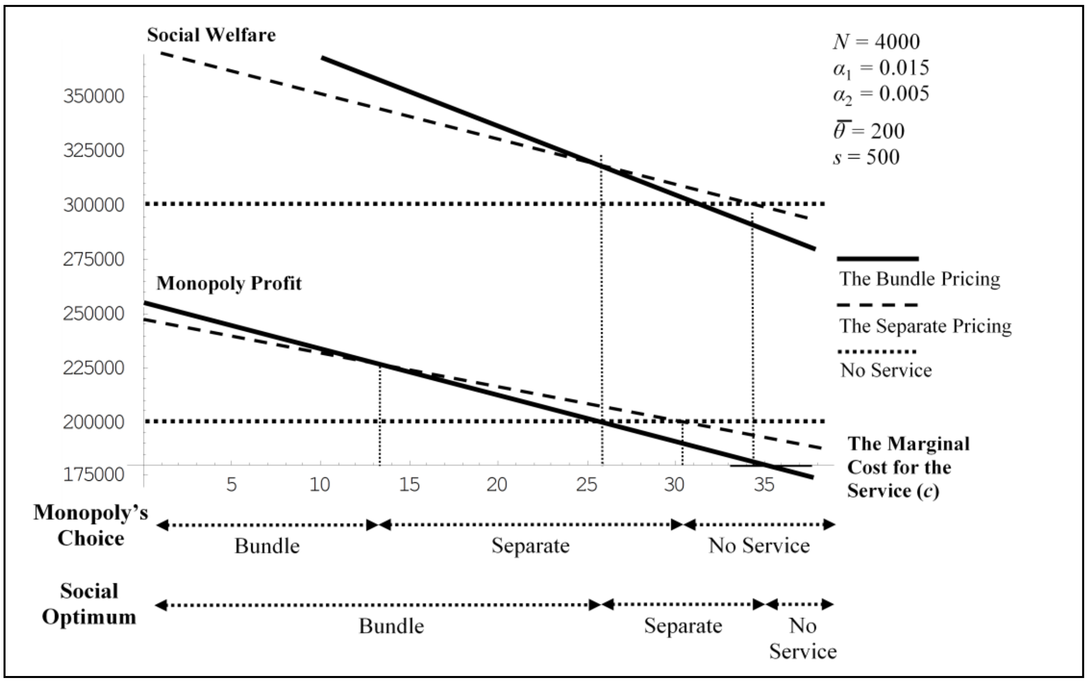


**Figure B2. The seller's profit maximizing strategy and the strategy that maximizes social welfare when product and service valuations are positively correlated.**

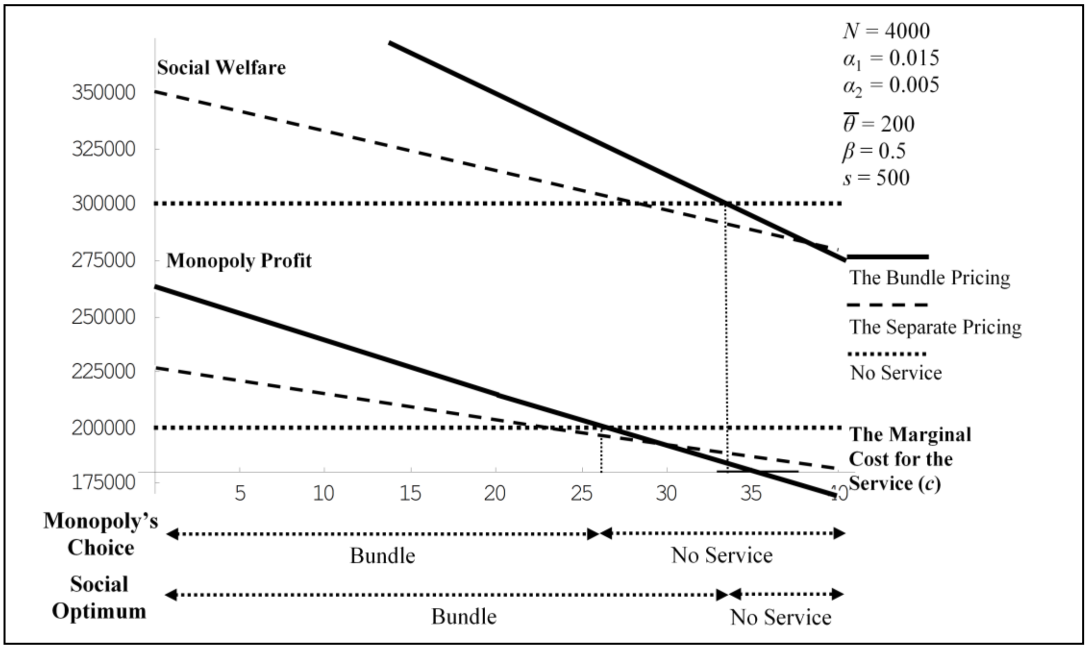


**Figure B3. The seller's profit maximizing strategy and the strategy that maximizes social welfare when product and service valuations are independent**