account for the low intensities of the observed lines. More work on these levels should be done to clear up this point.

The results of the electric excitation experiments which have been carried out by many workers in this field all serve as an excellent confirmation of the general features of the collective model. The existence of a great number of fast $E2$ transitions, the existence of well-defined rotational spectra, the probable existence of vibrational spectra, and the trends exhibited by the energies and the transition probabilities are all in agreement with the predictions of the model (see Fig. 8). There are, however, several important experimental results for which the model does not as yet provide any explanation. The effective moments of inertia of the rotational levels which have been identified are all larger by factors ranging from three to seven than those which are calculated from intrinsic nuclear deformations measured in other ways.[8,17,34] (That is, the energies of the observed rotational levels are too low by factors of ten to fifty.) The reason for the apparently sharp break between the regions in the periodic table where the "strong coupling" and the "weak coupling" approximations are applicable is also not well understood. Finally, if the energy levels which are observed in the "weak coupling" region are indeed "vibrational" levels, then their energies are also much lower[9,33] (factors of roughly ten) than those which would be expected from the crude hydrodynamical estimates worked out in Bohr and Mottelson.[5] The fact that both the "rotational" and "vibrational" levels apparently have energies which are smaller by roughly the same order of magnitude than those expected from the model suggests that a revision of the fundamental assumptions about either the nature of the nuclear "fluid" or its distribution in the nuclear volume may be necessary.

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ACKNOWLEDGMENTS

The authors are indebted to Dr. C. P. Keim of the Oak Ridge National Laboratory for his very kind help in procuring the enriched isotopes, and to Professor L. C. Bradley of MIT for providing some of the samples of rare-earth oxides used in this work. We would like to thank Professor V. F. Weisskopf, Professor M. Deutsch, Professor S. D. Drell, Dr. Gertrude Scharff-Goldhaber, and Dr. G. Goldring for many useful conversations and for their continued interest in these experiments. We are grateful to many of our colleagues in this field for keeping us informed of their latest results, some of which were invaluable in the preparation of this report. Finally it is a pleasure to acknowledge the help of Mr. Matti K. Salomaa, who designed and built much of the electronic equipment and Mr. Allen Lenchek who performed many of the calculations.

PHYSICAL REVIEW VOLUME 100, NUMBER 3 NOVEMBER 1, 1955

Empirical Correlation of Nuclear Magnetic Moments

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(Received April 18, 1955)

A simple generalization of the extreme shell model, based upon the kinematics of the collective model, is proposed for the correlation of nuclear magnetic moments. It is shown that, if the concept of a rotating core is adjoined to that of the single-particle model, largely in disregard of dynamical considerations, then by the aid of three simple empirical rules ground-state wave functions may be easily constructed which correctly express the parities, spins, and magnetic moments of all nuclei for which $A \geqslant 7$, with the exception of W^{183}.

The choice of a particular set of empirical rules was dictated primarily by the twofold desire to keep their number to a minimum and at the same time restrict the consequent wave function to but two components; hence, considerable oversimplification of the true state of affairs is inevitable. However, the internal consistency of the results does point up strikingly the previously observed, but not explicitly investigated, possibility that the nature of the variable degrees of freedom required for generalization of the single-particle model may differ fundamentally for $j=l+1/2$ in contrast to $j=l-1/2$ single-particle configurations—being predominantly those of the core in the former instance and those of the single particle in the latter.

I. INTRODUCTION

THE single-particle model of the nucleus, which assumes that the relevant degrees of freedom required for a description of the nuclear ground state are those of the last odd particle (for even-even and odd-even nuclei, and hence no degrees of freedom for the former), leads to the well-known Schmidt values for magnetic moments. Presumably the deviations of the experimental moments from the Schmidt limits are to be accounted for through the interplay of some other degrees of freedom of the nuclear system than those of the last odd particle. One of three main lines of endeavor have usually been followed in the attempt to uncover the nature and significance of these extra degrees of freedom required for an understanding of nuclear ground-state properties.

One approach has been predicated upon the assumption that these extra degrees of freedom are representable as appropriately restricted sets of single-particle degrees of freedom. According to this view, a wave function for the ground state is presumed to be a superposition of properly symmetrized products of single-particle wave functions which satisfy the requirement of fixed parity and spin. So far as magnetic moments are concerned, a reasonably tractable scheme for configurational mixing has had considerable success.[1,2]

A second approach is based upon fundamental considerations concerning the nature of nucleon-nucleon interactions. Upon the assumption that these interactions include charge-exchange forces, one is led immediately to the expectation of contributions to nuclear magnetic moments arising from the exchange currents.[3] This concept was exploited successfully by Villars[4] in accounting for the moments of H^3-He^3 in terms of a pseudoscalar meson theory employing a charge exchange potential. Such an approach, of course, exhibits explicitly the mechanism of charge exchange in terms of charged meson currents. The same concept was employed in a phenomenological sense[5,6] by Russek and Spruch[7] to odd-A nuclei in general. Their results indicated that deviations from the Schmidt limits might well be largely accounted for in terms of exchange current effects.

The third main line of attack has proceeded from the opposite extreme, namely on the idea that the motions of all the particles with the exception of the last odd one are characterized by some small set of collective degrees of freedom. Various investigators have associated these degrees of freedom with one or the other or both of two quite different types of collective core motions—oscillations or rotations. Foldy and Milford[8,9] have examined the magnetic moments to be expected on the basis of an oscillator-plus-particle model, in which it was assumed that the single-particle quantum numbers were good quantum numbers. The model appears qualitatively successful for nuclei with total spin $I\geqslant 3/2$ and for which the single-particle configuration is characterized by $j=l+1/2$. The remaining nuclei are, however, not satisfactorily accounted for.

Employing the same model but including the possibility of configurational mixing as well as surfon mixing, Kerman[10] computed magnetic moments and found the situation somewhat improved, especially for spin-1/2 nuclei, but still not completely satisfactory, particularly in the regions of high spin.

Conversely, Bohr and Mottelson[11] and Davidson and Feenberg[12] have extensively investigated nuclear magnetic moments on the assumption that the collective degrees of freedom relevant for the ground state are essentially rotational in character. These investigators also took into account the possibility that more than one single-particle configuration might participate in the ground state. Such a model appears to be potentially adequate to correlate all nuclear magnetic moments within a relatively simple scheme. However, restrictions imposed upon the allowable ranges for the degrees of freedom chosen to characterize the system, primarily dictated by considerations of energetics, somewhat obscure this potentiality.

II. KINEMATICS OF A SIMPLE MODEL

As there appears to be considerable evidence[11] that degrees of freedom characteristic of an asymmetric rotator play an important role in the description of nuclear ground and low-lying excited states, we felt that it would be worthwhile to introduce them kinematically, rather than dynamically, as a simple, but purely empirical, generalization of the single-particle model. Such an approach is suggested by the fact that the best-known properties of nuclear ground states—parity, spin, and magnetic moment—are predominantly determined by the kinematics of the system; whereas, conversely, those properties which appear to depend to a greater extent upon details of interactions are the least well known.

Thus we reject at the outset any detailed attempt to tie conclusions obtainable from the model to energy considerations. We simply postulate that a nucleus can, for some purposes at least, be regarded as a permanently deformed core (if even-even and not doubly magic); a permanently deformed core plus a single particle (if odd-even); or a permanently deformed core plus two particles (if odd-odd). We seek a small set of empirical rules to guide us in the determination of whatever deformation parameters are needed in order to correlate the measurements of nuclear ground-state properties.

In the first place, the shell model is to be invoked for the principal ordering of the nuclear ground states, and for the spin and parity assignments for the single-particle configurations. We further assume that the nuclear parity is that of the single particle, and that the core deformation is axially symmetric. The assumption that the shell configuration of the last odd particle should determine the parity of the nucleus requires evenness of the core parity. Such a rotator exhibits generally three degrees of freedom which may conveniently be characterized by the projection of the

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total angular momentum upon the space z-axis, the magnitude of the total angular momentum, and its projection on the symmetry axis fixed in the rotating body.

The appropriate operator representatives of these degrees of freedom are $\mathfrak{L}_z$, $\mathfrak{L}^2$, and $\mathfrak{L}_z'$, respectively. Primes will be used whenever quantities referred to body axes are to be indicated. The usual commutation rules for the components of angular momentum of a "rigid body" are invoked; i.e.,

$$[\mathfrak{L}_i,\mathfrak{L}_j]=i\epsilon_{ijk}\mathfrak{L}_k \tag{1a}$$

and

$$[\mathfrak{L}_i',\mathfrak{L}_j']=-i\epsilon_{ijk}\mathfrak{L}_k'. \tag{1b}$$

Operators characterized by these commutation rules possess the usual diagonalizing representation[11] of the axially symmetric top,

$$D_{MK}{}^{\lambda *}(\theta_i)$$

where the $\theta_i=(\alpha,\beta,\gamma)$ are the Euler angles employed in the specification of the orientation of the deformed core relative to a space-fixed set of axes. In this representation,

$$\mathfrak{L}_z D_{MK}{}^{\lambda *}=MD_{MK}{}^{\lambda *}, \tag{2a}$$

$$\mathfrak{L}^2 D_{MK}{}^{\lambda *}=\lambda(\lambda+1)D_{MK}{}^{\lambda *}, \tag{2b}$$

$$\mathfrak{L}_z' D_{MK}{}^{\lambda *}=KD_{MK}{}^{\lambda *}. \tag{2c}$$

It is useful also to note that

$$\mathfrak{L}_\mu D_{MK}{}^{\lambda *}=[\lambda(\lambda+1)]^{\frac{1}{2}}C(\lambda 1\lambda;M,\mu)D_{M+\mu,K}{}^{\lambda *}, \tag{3a}$$

$$\mathfrak{L}_\mu' D_{MK}{}^{\lambda *}=(-1)^\mu[\lambda(\lambda+1)]^{\frac{1}{2}} \times C(\lambda 1\lambda;K,-\mu)D_{M,K-\mu}{}^{\lambda *}, \tag{3b}$$

where $\mathfrak{L}_{\pm 1}=\mp(1/\sqrt{2})(\mathfrak{L}_x\pm i\mathfrak{L}_y)$ and $\mathfrak{L}_0=\mathfrak{L}_z$, and where the C's are Clebsch-Gordan coefficients. The parity operation on these wave functions leads to

$$\begin{aligned}\Pi D_{MK}{}^{\lambda *}(\gamma\beta\alpha)&=D_{MK}{}^{\lambda *}(\pi-\gamma,\pi-\beta,\pi+\alpha)\\ &=(-1)^{\lambda+K}D_{M,-K}{}^{\lambda *}(\gamma\beta\alpha)\end{aligned} \tag{4}$$

and consequently the requirement that the core functions be of even parity demands that they appear only in the combination

$$D_{MK}{}^{\lambda *}+(-1)^{\lambda+K}D_{M,-K}{}^*.$$

We here invoke another greatly simplifying assumption, namely that $K=0$.[11] This provides immediately the additional consequence that λ must always be even.

The relevant degrees of freedom for the description of the odd nucleon (if there is one) are presumed to be characterized by the operators $\mathbf{L}^2$, orbital angular momentum, $\mathbf{S}^2$, intrinsic spin, $\mathbf{J}^2$, total particle angular momentum, and $\mathbf{J}_z$ (or $\mathbf{J}_z'$), projection of total particle angular momentum on the space (or body) z-axis. In the representation we have chosen in which to investigate the consequences of the model, it does not matter whether the particle motions are quantized in the body system—strong coupling—or in the space system—weak coupling. The choice of a system of quantization must ultimately be determined by the form of the Hamiltonian for the system, but in the present work such considerations are avoided.

The appropriate diagonalizing representation for these operators is

$$\chi_{jl}{}^m=\sum_\tau C(\tfrac{1}{2}lj;\tau,m-\tau)\chi_{\frac{1}{2}}{}^\tau Y_l{}^{m-\tau}. \tag{5}$$

Both representations have been chosen so that they transform under rotations of the coordinate system according to

$$\psi_j{}^m(e')=\sum_{m'}\psi_j{}^{m'}(e)D_{m'm}{}^j(R_{e\to e'}). \tag{6}$$

We now simply assert that the appropriate nuclear wave function shall transform under rotations according to D^I, where I is the magnitude of the total nuclear angular momentum, consisting in general of a sum of that for the odd particle, or particles, and the core. This leads us directly to at least three possible representations for the system as a whole:

$$\Psi_{I\lambda jl}{}^M(e,\theta_i)=\sqrt{2}N_\lambda\sum_m C(j\lambda I;m,M-m) \times\chi_{jl}{}^m(e)D_{M-m,0}{}^{\lambda *}(\theta_i) \tag{7a}$$

if the particle is quantized in space,

$$\Psi_{I\lambda jl}{}^M(e',\theta_i)=N_\lambda\sum_m C(j\lambda I;m0)[\chi_{jl}{}^m(e')D_{M,m}{}^{I*}(\theta_i) +(-1)^{j-I}\chi_{jl}{}^{-m}(e')D_{M,-m}{}^{I*}(\theta_i)] \tag{7b}$$

if the particle is quantized in the body;

$$B_{Ijl}{}^{Mm}(e',\theta_i)=N_I[\chi_{jl}{}^m(e')D_{M,m}{}^{I*}(\theta_i) +(-1)^{j-I}\chi_{jl}{}^{-m}(e')D_{M,-m}{}^{I*}(\theta_i)] \tag{7c}$$

and

$$\Phi_{I\lambda\rho l}{}^M(e,\theta_i)=\sum_j(-1)^{l-\rho-j+I}[(2\rho+1)(2j+1)]^{\frac{1}{2}} \times W(\lambda lI\tfrac{1}{2};\rho j)\Psi_{I\lambda jl}{}^M(e,\theta_i) \tag{7d}$$

where the W-function is a Racah coefficient.

As stated above, the representations (7a) and (7b) in which the normalizing coefficient N_λ is defined by $N_\lambda=[(2\lambda+1)/(16\pi^2)]^{\frac{1}{2}}$, are essentially the same for our present purpose. They are the representations in which $\mathfrak{J}^2=(\mathfrak{L}+\mathbf{J})^2$, $\mathfrak{L}^2$, $\mathbf{J}^2$, $\mathbf{L}^2$ and $\mathfrak{J}_z$ are the constants of the motion. Hereafter we shall designate them as the λ representation. The simple, but important, connection between the two λ representations for particle quantization in space and body coordinate systems should be pointed out here. As is to be expected, they are related by a rotation of *particle* wave functions. Since

$$\chi_{jl}{}^m(e')=\sum_{m'}\chi_{jl}{}^{m'}(e)D_{m'm}{}^j(\theta_i),$$

we have

$$\sum_m C(j\lambda I;m,0)\chi_{jl}{}^m(e')D_{M,m}{}^{I*}(\theta_i)$$
$$=\sum_{mm'} C(j\lambda I;m,0)\chi_{jl}{}^{m'}(e)D_{m',m}{}^{j}(\theta_i)D_{M,m}{}^{I*}(\theta_i)$$
$$=\sum_{m'L} C(jIL;m',-M)\chi_{jl}{}^{m'}(e)D_{M-m'0}{}^{L*}(\theta_i)$$
$$\times\sum_m(-1)^{m'-m}C(j\lambda I;m,0)C(jIL;m,-m)$$
$$=\sum_m C(j\lambda I;m,M-m)\chi_{jl}{}^m(e)D_{M-m,0}{}^{\lambda*}(\theta_i); \qquad \text{(8a)}$$

and similarly for the symmetrization term. The identity of the two representations is a simple consequence of the fact that regardless of the quantization coordinate system, both $\mathbf{J}$ and $\mathfrak{L}$ are presumed to precess about the same fixed vector, $\mathfrak{J}$.

The representation (7c) is the one employed by Bohr and Mottelson[11] and by Davidson and Feenberg[12] and is distinguished from the λ representation in that it exhibits the z component (m) of the angular momentum of the particle in the body system as a constant of the motion. We shall call this the m representation. Both of these representations employ the same angular momentum coupling schemes; i.e.,

$$\mathbf{J}=\mathbf{L}+\mathbf{S},$$

and $\mathfrak{J}=\mathbf{J}+\mathfrak{L}$. As is to be expected, the λ and m-representations in the body system are connected by a unitary transformation; explicitly,

$$\Psi_{I\lambda jl}{}^M(e',\theta_i)=\sum_m(-1)^{j-m}C(jI\lambda;m,-m)B_{Ijl}{}^{Mm}(e',\theta_i). \qquad (9)$$

The representation (7d) is given little consideration herein, but is listed simply for completeness. It is the representation for the particle quantized in the space coordinate system which employs the alternative coupling scheme

$$\mathbf{R}=\mathfrak{L}+\mathbf{L},\quad \mathfrak{J}=\mathbf{R}+\mathbf{S},$$

where

$$\mathbf{R}^2\Phi_{I\lambda\rho l}{}^M=\rho(\rho+1)\Phi_{I\lambda\rho l}{}^M. \qquad (10)$$

This would be a natural representation if it were assumed that the particle-core coupling was very strong compared to the spin-orbit coupling. However, such an assumption somewhat obscures the significance of shell theory, and since we wish to build upon shell theory as a foundation, we choose to ignore the implications of this coupling scheme.

We have found from empirical considerations that the λ representation is more satisfactory than any of the others listed above for the correlation of nuclear magnetic moments. Primarily for this reason, but supplemented by some other considerations to be discussed below, we have settled upon the representation in which $\mathfrak{J}^2$, $\mathfrak{L}^2$, $\mathbf{J}^2$, $\mathbf{L}^2$, and $\mathfrak{J}_z$ are commuting, diagonal operators as the one with which, by the aid of a few rules of superposition, to attempt to describe nuclear ground states.

At this point we postulate the empirical rules which guide us in the choice of a wave function:

(1) The ground states of nuclei are predominantly determined by the degrees of freedom associated with the last odd particle, or particles, in the nucleus and those associated with the rotation of an axially symmetric core consisting of all but the last odd particle, or particles.

(2) The predominant single-particle configuration in the ground state is that assigned by the single-particle model.

(3) The quantum numbers required to specify these degrees of freedom are taken to be: I, magnitude of total angular momentum; M, space z-component of total angular momentum; l, magnitude of orbital angular momentum of odd particle; s, magnitude of intrinsic spin of odd particle; $j=l\pm s$, magnitude of total angular momentum of odd particle; λ, magnitude of core angular momentum; and $K=0$, projection of core angular momentum upon body system of axes. It is further assumed that the core states shall be of even parity—hence in connection with $K=0$, λ must be even.

(4) a. All nuclei with spin greater than 1/2 and for which $j=l+1/2$ are characterized by the good quantum numbers I, M, l, and j; and by a superposition of at most two interacting core states.

b. All nuclei with spin greater than 1/2 and for which $j=l-1/2$ are characterized by the good quantum numbers I, M, and λ; and by a superposition of at most two interacting single-particle configurations within the same major shell.

c. Nuclei with spin 1/2 are characterized by the single-particle configuration assigned by shell theory and the nearest interacting core-plus-particle configuration.

The concept of interacting states implies, of course, some consideration of the nature of the particle-core interaction. Assuming that the predominant part of this intereaction is independent of velocities, we may express it as

$$\sum_{Lmm'} T_L{}^{m'}D_{m'm}{}^L(\theta_i)\varphi_m,$$

where $T_L{}^{m'}$ is the m' component of an Lth rank irreducible tensor in the space of the particle and φ_m is an empirical scalar coefficient. The matrix elements of such an interaction operator in the representation characterized by the quantum numbers indicated above are

$$E_L(I;\lambda jl;\lambda'j'l')$$
$$=(2L+1/4\pi)(-1)^{I+l+\lambda'-\frac{1}{2}}[(2l'+1)(2\lambda'+1)(2j+1)$$
$$\times(2j'+1)]^{\frac{1}{2}}C(l'Ll;0,0)C(\lambda'L\lambda;0,0)W(ljl'j';\tfrac{1}{2}L)$$
$$\times W(jj'\lambda\lambda';LI)\varphi_0. \qquad (11)$$

If we now take as a convenient, though hardly critical, assumption that the interaction is characterized primarily by $L=2$ (suggested by the usual form of the

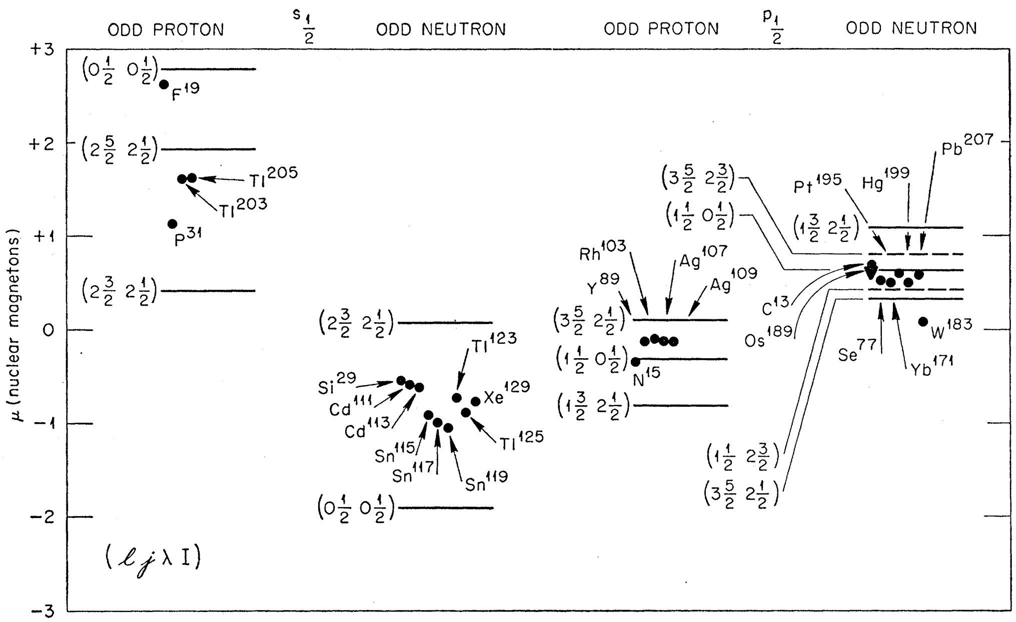

FIG. 1. Experimentally observed and calculated magnetic moments for $j=1/2$ nuclei. The solic circles and triangle represent the measured values and the solid and dashed lines represent the calculated moments for the indicated values of (l, j, λ, I) as defined in the text.

particle-core interaction in the hydrodynamical model[11]), we see immediately that the concept of interacting states contains the selection rules $\Delta(jj'2)$, $\Delta(ll'2)$, and $\Delta(\lambda\lambda'2)$, where the symbol Δ implies the usual restriction on the magnitudes of three vectors which form a triangle.

It is a consequence of assumption 4(b) above that particle configurations which mix are usually characterized by $l\neq l'$; hence the contributions to the magnetic moment from the assumed mixtures of states will rarely include cross terms. However, for the sake of completeness, we give all of the matrix elements of the magnetic moment operator; i.e.,

$$\mu(Ij\lambda l)=\left[g_{jl}+(g_c-g_{jl})\frac{I(I+1)-j(j+1)+\lambda(\lambda+1)}{2I(I+1)}\right]I \quad (12a)$$

for the diagonal elements, and

$$\mu(I\lambda l;\, j=l+\tfrac{1}{2},\, j'=l-\tfrac{1}{2})=\frac{(g_l-g_s)[(I+l+\lambda+\frac{3}{2})(I+l-\lambda+\frac{1}{2})\times(l-I+\lambda+\frac{1}{2})(I-l+\lambda+\frac{1}{2})]^{\frac{1}{2}}}{2(I+1)(2l+1)},\quad \lambda=\lambda'\neq 0 \quad (12b)$$

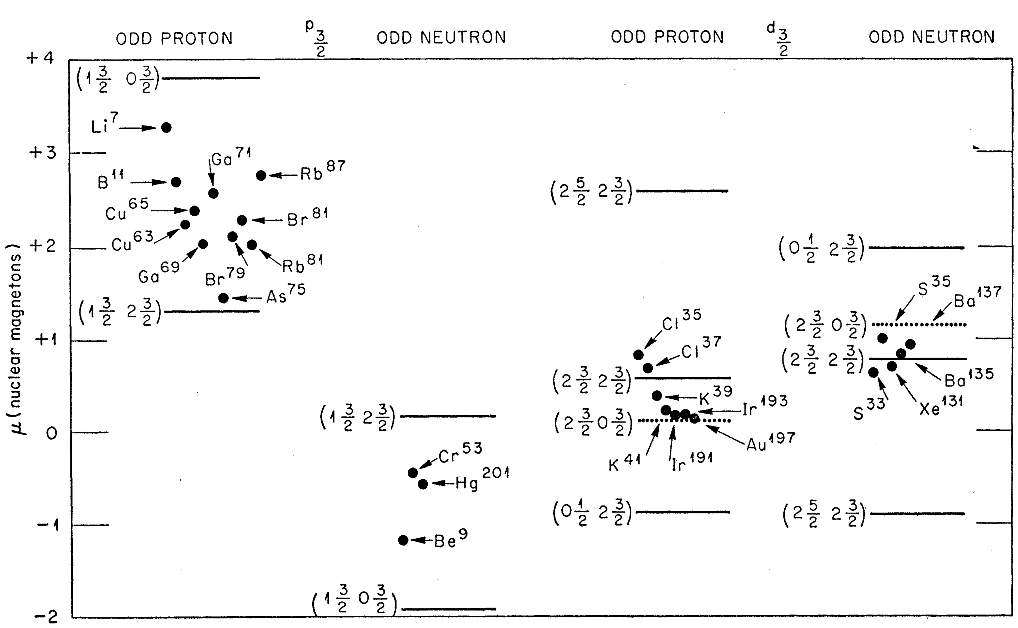

FIG. 2. Experimentally observed and calculated magnetic moments for $j=3/2$ nuclei. The solid circles represent the measured values and the solid and dotted lines represent the calculated moments for the indicated values of (l, j, λ, I) as defined in the text.

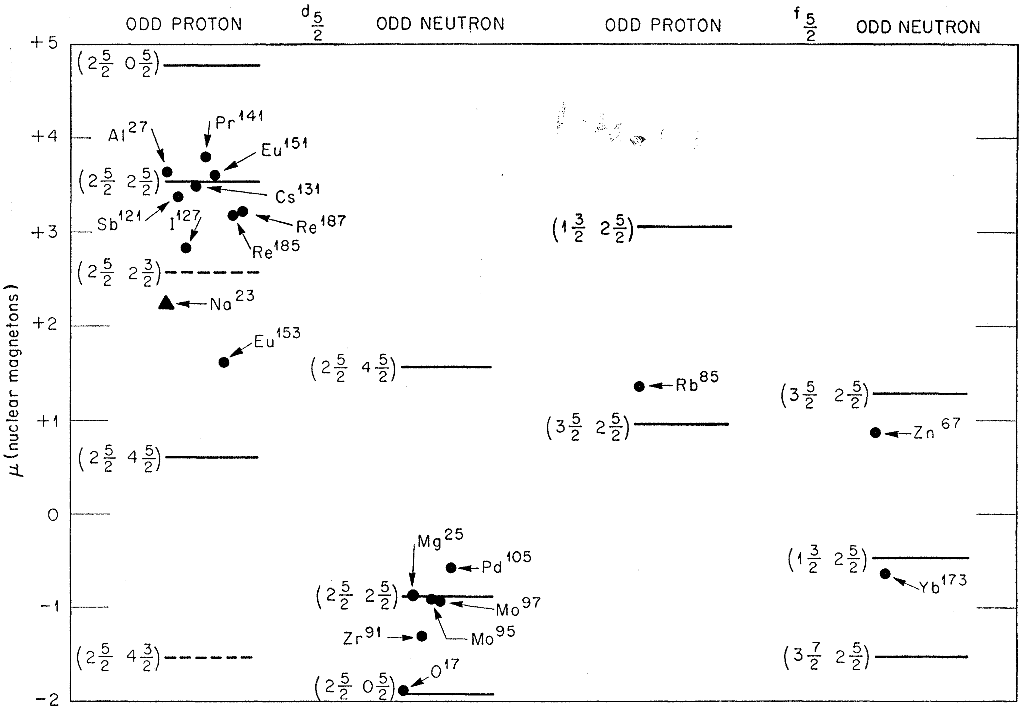

FIG. 3. Experimentally observed and calculated magnetic moments for $j=5/2$ nuclei. The solid circles and triangle represent the measured values and the solid and dashed lines represent the calculated moments for the indicated values of (l, j, λ, I) as defined in the text.

for the nondiagonal elements. We have taken throughout $g_c=0.45\sim Z/A$. The quantity g_{jl} is the usual single-particle theory gyromagnetic ratio; thus the Schmidt limiting values for the magnetic moments follow immediately upon setting $j=I$ and $\lambda=0$.

The contribution to the magnetic moment from a particular state in the m-representation is given by

$$\mu(Ijlm)=\left[g_{jl}+(g_c-g_{jl})\times\frac{2I(I+1)-2m^2-(-1)^{j-I}(j+\frac{1}{2})(I+\frac{1}{2})\delta_{m\frac{1}{2}}}{2I(I+1)}\right]I. \quad (13)$$

For $I=m>1/2$, this becomes $\mu(IjlI)=[Ig_{jl}+g_c]I/(I+1)$, and for $I=m=1/2$ we have $\mu(\frac{1}{2}jl\frac{1}{2})=\frac{1}{6}[3g_c+(g_{jl}-g_c)\{1+(-1)^{j-I}(2j+1)\}]$ as noted previously by Davidson and Feenberg.[12]

In accordance with the ideas presented above, the ground-state wave function can be taken to be (to within a single-particle radial factor)

$$(1-a^2)^{\frac{1}{2}}\Psi_{I\lambda jl}{}^M+a\Psi_{I\lambda' j'l'}{}^M$$

where $(jl)=(j'l')$ and $\lambda\neq\lambda'$ for $l+1/2$ nuclei, and $\lambda=\lambda'$ but $(jl)\neq(j'l')$ for $l-1/2$ nuclei. The mixing coefficient a is to be determined by the relation

$$a^2=[\mu(\text{obs})-\mu(Ij\lambda l)]/[\mu(Ij'\lambda'l')-\mu(Ij\lambda l)], \quad (14)$$

except in the rare cases in which $l-1/2$ nuclei admix with the spin-orbit partner. For such cases, of course, the cross terms in the magnetic moment relation must be considered.

It is not our intention here to suggest that the extremely simple scheme outlined above should supplant more sophisticated calculations whenever they have been successfully carried through. In particular, the detailed examinations of the many-particle model for light nuclei conducted by Mizushima and Umezawa[13] and Flowers[14] probably render the extension of at least the dynamical aspects of the deformed-core model into the region of light nuclei superfluous, if not dubious. Furthermore, so far as heavy nuclei are concerned, the scheme should be regarded simply as an alternative explanation to that proposed by Blin-Stoyle and Perks[1,2]—an alternative which hitherto has not been explicitly recognized as adequate. Demonstration of the alternative is pertinent since for heavy nuclei at least it is expected that a collective model will be far more tractible for correlating quadrupole moments. Thus, though in many special instances and for light nuclei generally, it may seem objectionable to do so, we shall nevertheless extend consideration of the scheme practically without restriction.

All odd-even nuclei from Li^7 on whose spins and magnetic moments have been measured have been considered from the standpoint of the model described above. The results are presented in graphical form in Figs. 1 to 5. On these figures the nuclei are grouped according to the single-particle assignment of the last odd particle, and the values of the calculated magnetic moments for states characterized by the quantum numbers (l, j, λ, I) are shown as either solid or dashed

[13] M. Mizushima and M. Umezawa, Phys. Rev. 85, 37 (1952).
[14] B. H. Flowers (private communication).

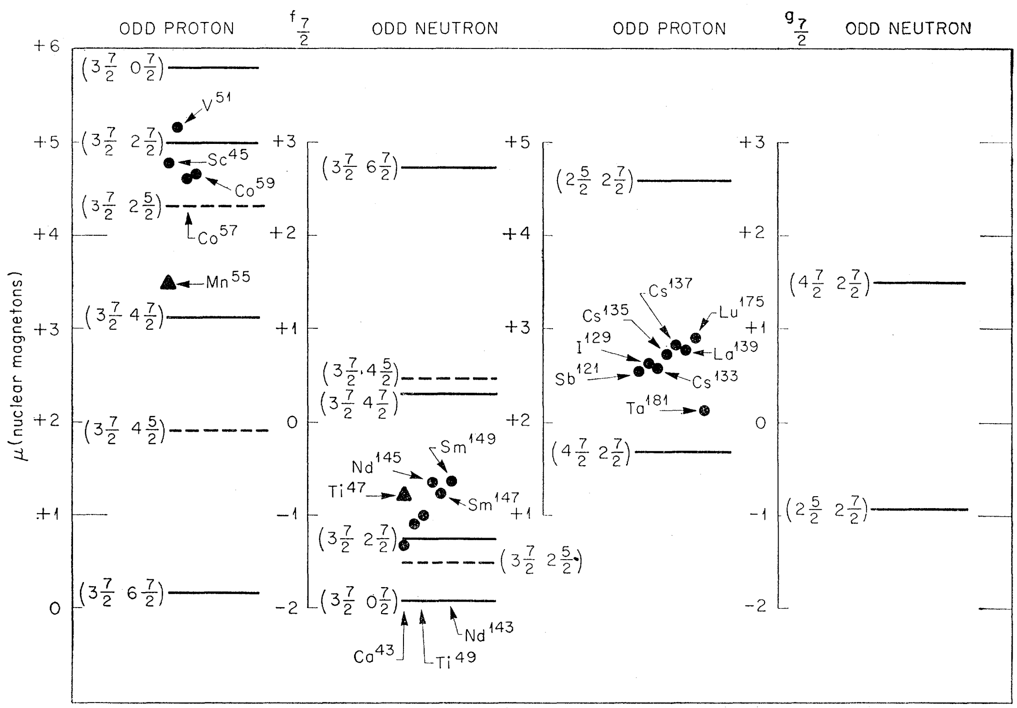

FIG. 4. Experimentally observed and calculated magnetic moments for $j=7/2$ nuclei. The solid circles and triangles represent the measured values and the solid and dashed lines represent the calculated moments for the indicated values of (l, j, λ, I) as defined in the text.

lines. The experimentally measured magnetic moments are indicated by solid circles or triangles.

The results are also given numerically in the corresponding Tables I to X. The sources of the data are Klinkenberg's compilation,[15] designated by K; Bohr and Mottelson's compilation,[11] designated by B-M; or other sources, designated by the reference to the published paper describing the measurement. These tables give the calculated values of the magnetic moments of the pure states, the experimentally measured moments, and the experimentally measured spins of the ground states of the nuclei in question. In those cases in which the spin listed is not that given in Klinkenberg's paper,[15] the reference to the later measurement of the spin has been given.

An examination of Figs. 1 to 5 reveals that the magnetic moments of nearly all the odd-even nuclei are correlated by the present model in a reasonably consistent fashion. The results are discussed in detail below.

TABLE I. Experimentally observed and calculated magnetic moments for $s_{1/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron		
(0 1/2 0 1/2) $\mu=2.79$			(0 1/2 0 1/2) $\mu=-1.91$		
(2 5/2 2 1/2) $\mu=1.94$			(2 5/2 2 1/2) $\mu=-1.19$		
(2 3/2 2 1/2) $\mu=0.41$			(2 3/2 2 1/2) $\mu=$ 0.067		
Nucleus	Observed I	Observed[a] μ	Nucleus	Observed I	Observed[a] μ
F^{19}	1/2	2.63 (K)	Si^{29}	1/2	−0.56 (B-M)
P^{31}	1/2	1.13 (B-M)	Cd^{111}	1/2	−0.59 (K)
Tl^{203}	1/2	1.61 (K)	Cd^{113}	1/2	−0.62 (K)
Tl^{205}	1/2	1.63 (K)	Sn^{115}	1/2	−0.92 (K)
			Sn^{117}	1/2	−1.00 (K)
			Sn^{119}	1/2	−1.05 (K)
			Te^{123}	1/2	−0.74 (K)
			Te^{125}	1/2	−0.89 (K)
			Xe^{129}	1/2	−0.78 (K)

[a] Values labeled (K) are taken from Klinkenberg's compilation (reference 15). Values labeled (B-M) are taken from Bohr and Mottelson's compilation (reference 11).

The $s_{1/2}$ Nuclei

The pattern of configurational mixing exhibited by these nuclei is particularly interesting. The three nuclei ${}_{9}F^{19}$, ${}_{15}P^{31}$, and ${}_{14}Si^{29}$, which appear in the early $2s$ shell, seem to behave quite differently from the other members of this group. In the case of ${}_{9}F^{19}$, an admixture of either $d_{3/2}$ or $d_{5/2}$ will account for the observed moment, though considerably less of the $d_{3/2}$ is needed, a fact which might well be a criterion of preference. However, both ${}_{15}P^{31}$ and ${}_{14}Si^{29}$ must be admixed with the $d_{3/2}$. But it is to be noted that in both of these instances the preceding $d_{5/2}$ subshell has been filled.[15] (Unless explicitly stated to the contrary, we have accepted Klinkenberg's single-particle configuration assignments.) Hence for $d_{5/2}$ admixture one must postulate the configurations $(d_{5/2})^6(s_{1/2})^1+(d_{5/2})^5(s_{1/2})^2$. But because the pairing energy[16,17] for nucleons paired in a subshell is proportional to $(2j+1)$, it is quite possible that there is an energy discrimination against the $d_{5/2}$ admixture. It is noteworthy, in fact, that this phenomenon of an apparent pairing energy discrimination against certain

[15] P. F. A. Klinkenberg, Revs. Modern Phys. 24, 63 (1952).

[16] M. G. Mayer, Phys. Rev. 78, 16 (1950).

[17] G. Racah and I. Talmi, Physica 18, 1097 (1952).

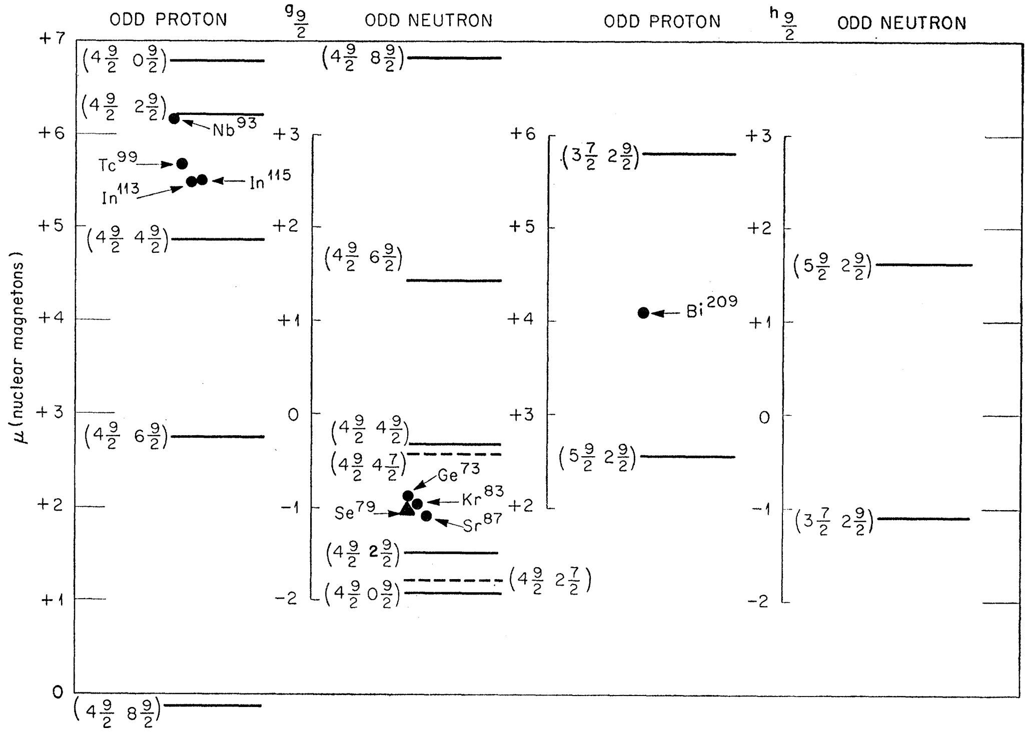

FIG. 5. Experimentally observed and calculated magnetic moments for $j=9/2$ nuclei. The solid circles and triangle represent the measured values and the solid and dashed lines represent the calculated moments for the indicated values of (l, j, λ, I) as defined in the text.

configurational admixtures is encountered frequently. The remaining $s_{1/2}$ nuclei all appear in the later $3s$ subshell which is presumably preceded by the $4d_{3/2}$ subshell. In all of these cases $d_{3/2}$ admixture is required. This s-d mixing was also noted by Davidson and Feenberg[12] and Bohr and Mottelson.[11]

The $p_{1/2}$ Nuclei

It is expected that ${}_7N^{15}$ and ${}_6C^{13}$ would both require $p_{3/2}$ admixture, as indeed they do. But it is most interesting to observe that none of the moments of the remaining nuclei—all of which apparently have an $f_{5/2}$ subshell as a nearer neighbor than $p_{3/2}$—could be accounted for by $p_{3/2}$ admixture; whereas the $f_{5/2}$ does very well. The nucleus ${}_{76}Os^{189}$ is the one case found so far of configurational mixing in which the observed spin is not that of the single particle. Its moment is accounted for by mixing with the $f_{5/2}$, just as in the case with the other members of this group. In Fig. 1 the moments for the two pure states required for this nucleus are shown by the dashed lines and the experimentally observed moment is indicated by the solid triangle. The only true exception found so far on the basis of the present model, ${}_{74}W^{183}$, appears in this group. Its observed moment cannot be fitted.

The $p_{3/2}$ Nuclei

The assumption of but one single-particle state and two core states is very satisfactory here. It is to be observed that configurational mixing with fixed λ would not adequately account for these moments.

The $d_{3/2}$ Nuclei

Four nuclei in this group, ${}_{16}S^{33}$, ${}_{17}Cl^{35}$, ${}_{17}Cl^{37}$, and ${}_{54}Xe^{131}$, exhibit anomalous, but not exceptional, behavior. Their moments require admixture of the spin-orbit partner instead of the s state as might be expected. It is to be noted that in all four cases the amount of admixture is small. The moments of the remaining members of the group are satisfactorily accounted for by s-state mixing. It should be pointed out that the latter cases could also be accounted for by λ mixing.

TABLE II. Experimentally observed and calculated magnetic moments for $p_{1/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton	Odd neutron
(1 1/2 0 1/2) $\mu=-0.26$	(1 1/2 0 1/2) $\mu=0.64$
(3 5/2 2 1/2) $\mu=\ 0.10$	(3 5/2 2 1/2) $\mu=0.34$
(1 3/2 2 1/2) $\mu=-0.81$	(1 3/2 2 1/2) $\mu=1.09$
	(3 5/2 2 3/2) $\mu=0.80$
	(1 1/2 2 3/2) $\mu=0.43$

Nucleus	Observed I	Observed μ	Nucleus	Observed I	Observed μ
N^{15}	1/2	−0.28 (K)	C^{13}	1/2	0.70 (K)
Y^{89}	1/2	−0.14 (K)	Se^{77}	1/2	0.53 (B-M)
Rh^{103}	1/2	−0.10 (B-M)	Yb^{171}	1/2	0.5 (B-M)
Ag^{107}	1/2	−0.11 (K)	W^{183}	1/2	0.08[b]
Ag^{109}	1/2	−0.13 (K)	Os^{189}	3/2[a]	0.65[a]
			Pt^{195}	1/2	0.60 (K)
			Hg^{199}	1/2	0.50 (K)
			Pb^{207}	1/2	0.59 (K)

[a] H. R. Loeliger and L. R. Sarles, Phys. Rev. **95**, 291 (1954).
[b] J. A. Vreeland and K. Murakawa, Phys. Rev. **83**, 229(A) (1951).

TABLE III. Experimentally observed and calculated magnetic moments for $p_{3/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron		
(1 3/2 0 3/2) $\mu=3.79$			(1 3/2 0 3/2) $\mu=-1.91$		
(1 3/2 2 3/2) $\mu=1.30$			(1 3/2 2 3/2) $\mu=0.16$		
Nucleus	Observed I	Observed μ	Nucleus	Observed I	Observed μ
Li^7	3/2	3.26 (K)	Be^9	3/2	−1.18 (K)
B^{11}	3/2	2.69 (K)	Cr^{53}	3/2	−0.47 (B-M)
Cu^{63}	3/2	2.23 (K)	Hg^{201}	3/2	−0.56 (K)
Cu^{65}	3/2	2.38 (K)			
Ga^{69}	3/2	2.02 (K)			
Ga^{71}	3/2	2.56 (K)			
As^{75}	3/2	1.44 (K)			
Br^{79}	3/2	2.10 (K)			
Br^{81}	3/2	2.27 (K)			
Rb^{81}	3/2[a]	2.00[a]			
Rb^{87}	3/2	2.75 (K)			

[a] Hobson, Hubbs, Nierenberg, and Silsbee, Phys. Rev. **96**, 1450 (1954).

The $d_{5/2}$ Nuclei

Again in this group there is a case, $_{11}Na^{23}$, in which the total spin is not the single-particle spin. The $d_{5/2}$ assignment for the single-particle configuration is the natural one and, because of the active core, the fact that the total spin is not equal to the single-particle spin presents no difficulties. Again the calculated moments for this case have been shown as dashed lines and the observed moment as a solid triangle in Fig. 3. It is seen that this magnetic moment is quite consistent with the general pattern.

It is clear that the moments of all of the nuclei in this group are satisfactorily accommodated by one single-particle configuration and two core states. If one invokes the selection rule $\Delta(\lambda\lambda'2)$ consequent upon the assumption of predominantly quadrupole interaction between particle and core, one notices there is some correlation between a preference for higher spin core-state admixtures ($\lambda=2,4$), rather than the lower ($\lambda=0,2$), and increasing mass.

TABLE IV. Experimentally observed and calculated magnetic moments for $d_{3/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron		
(2 3/2 2 3/2) $\mu=0.56$			(2 3/2 2 3/2) $\mu=0.77$		
(0 1/2 2 3/2) $\mu=-0.87$			(0 1/2 2 3/2) $\mu=1.96$		
(2 5/2 2 3/2) $\mu=2.58$			(2 5/2 2 3/2) $\mu=-0.90$		
(2 3/2 0 3/2) $\mu=0.12$			(2 3/2 0 3/2) $\mu=1.15$		
Nucleus	Observed I	Observed μ	Nucleus	Observed I	Observed μ
Cl^{35}	3/2	0.82 (K)	S^{33}	3/2	0.64 (K)
Cl^{37}	3/2	0.68 (K)	S^{35}	3/2	1.00[a]
K^{39}	3/2	0.39 (K)	Xe^{131}	3/2	0.68 (B-M)
K^{41}	3/2	0.22 (K)	Ba^{135}	3/2	0.83 (K)
Ir^{191}	3/2	0.16 (B-M)	Ba^{137}	3/2	0.94 (K)
Ir^{193}	3/2	0.17 (B-M)			
Au^{197}	3/2	0.14 (B-M)			

[a] Burke, Strandberg, Cohen, and Koski, Phys. Rev. **93**, 193 (1954).

Configurational mixing with but a single core state would be unsatisfactory here.

The $f_{5/2}$ Nuclei

There are only three examples in this group, two of which, $_{37}Rb^{85}$ and $_{30}Zn^{67}$, behave quite regularly, the mixing configuration being the nearby $p_{3/2}$. In both cases this particular type of admixture is favored as a consequence of pairing in the subshell of higher spin. Conversely, the nucleus $_{70}Yb^{173}$ appears here as somewhat of an anomaly (though hardly as an exception), since the spin-orbit partner must be admixed in order to account for its moment. One would expect little or no mixing of $f_{7/2}$ (in contrast to $p_{3/2}$) with the $f_{5/2}$, because of the relatively large spin-orbit separation, made still larger when a pair in the 7/2 subshell is broken up in favor of a pair in the 5/2 subshell.

TABLE V. Experimentally observed and calculated magnetic moments for $d_{5/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron		
(2 5/2 0 5/2) $\mu=4.79$			(2 5/2 0 5/2) $\mu=-1.91$		
(2 5/2 2 5/2) $\mu=3.54$			(2 5/2 2 5/2) $\mu=-0.87$		
(2 5/2 4 5/2) $\mu=0.60$			(2 5/2 4 5/2) $\mu=1.56$		
(2 5/2 2 3/2) $\mu=2.58$					
(2 5/2 4 3/2) $\mu=-1.53$					
Nucleus	Observed I	Observed μ	Nucleus	Observed I	Observed μ
Na^{23}	3/2	2.22 (K)	O^{17}	5/2	−1.89 (K)
Al^{27}	5/2	3.64 (K)	Mg^{25}	5/2	−0.86 (K)
Sb^{121}	5/2	3.36 (K)	Zr^{91}	5/2	−1.3[b]
I^{127}	5/2	2.81 (K)	Mo^{95}	5/2	−0.91 (K)
Cs^{131}	5/2[a]	3.48 (B-M)	Mo^{97}	5/2	−0.93 (K)
Pr^{141}	5/2	3.9 (B-M)	Pd^{105}	5/2	−0.57[c]
Eu^{151}	5/2	3.6 (K)			
Eu^{153}	5/2	1.6 (K)			
Re^{185}	5/2	3.17 (K)			
Re^{187}	5/2	3.20 (K)			

[a] E. H. Bellamy and K. F. Smith, Phil. Mag. **44**, 33 (1953).
[b] S. Suwa, J. Phys. Soc. Japan **8**, 734 (1953).
[c] A. Steudel, Z. Physik **132**, 429 (1952).

The $f_{7/2}$ Nuclei

All of the odd-proton nuclei in this group and three of the odd-neutron members, $_{20}Ca^{43}$, $_{22}Ti^{47}$, and $_{22}Ti^{49}$, appear between the major-shell closings at 20 and 28. Thus, in terms of a rotating core-plus-single-particle model, one might expect little likelihood of configurational mixing here, as the degrees of freedom associated with single-particle configurations lying below closed shells should presumably exhibit little or no independence. However, admixture of core states quite satisfactorily accounts for all of the magnetic moments in this group.

It is especially to be noted that we have here two more examples ($_{25}Mn^{55}$ and $_{22}Ti^{47}$) of the situation in which the single-particle spin and nuclear spin are not the same, but that again core activity provides a simple accounting for boths pins and moments. If the selection rule $\Delta(\lambda\lambda'2)$ is to be invoked for the core-state mixing for the members of this group, it is seen that in all cases except ${}_{23}V^{51}$ and ${}_{20}Ca^{43}$ the core states ($\lambda=2,4$) are required.

Again for some members of this group configurational mixing to account for the moments is possible only if core-state admixture is simultaneously invoked.

The $g_{7/2}$ Nuclei

All of the single-particle configurations for this group lie between the major shells at 50 and 82. The natural interacting configuration is the nearby $d_{5/2}$, and it is apparent from Fig. 4 that admixture of this configuration satisfactorily accommodates all of the magnetic moments of these nuclei.

The $g_{9/2}$ Nuclei

The $g_{9/2}$ level lies between the major-shell closings at 28 and 50, and is the only subshell of even parity in this major shell. Thus here, as in the case of some of the $f_{7/2}$ nuclei, the model exhibits marked correspondence to the expectation of little or no configurational mixing, in that it demands core admixture for the ground state. It is clear from Fig. 5 that such admixture successfully accounts for all of the magnetic moments of this group. On the assumption that the ground state consists of but two core states, it is seen that in all cases the core states involved are those for $\lambda=2$ and $\lambda=4$.

TABLE VI. Experimentally observed and calculated magnetic moments for $f_{5/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron		
(3 5/2 2 5/2) $\mu=0.95$			(3 5/2 2 5/2) $\mu=$ 1.28		
(1 3/2 2 5/2) $\mu=3.05$			(1 3/2 2 5/2) $\mu=-0.48$		
(3 7/2 2 5/2) $\mu=4.31$			(3 7/2 2 5/2) $\mu=-1.51$		
(3 5/2 0 5/2) $\mu=0.86$			(3 5/2 0 5/2) $\mu=$ 1.37		
Nucleus	Observed I	Observed μ	Nucleus	Observed I	Observed μ
Rb^{85}	5/2	1.35 (K)	Zn^{67}	5/2	0.88 (B-M)
			Yb^{173}	5/2	-0.65 (K)

It is to be noted that in this group there appears another nucleus, ${}_{34}Se^{79}$, whose measured spin does not correspond to the spin of the single-particle configuration to which it has been assigned.

The $h_{9/2}$ Nuclei

This group is important because of the presence in it of ${}_{83}Bi^{209}$. Though Blin-Stoyle and Perks[2] have been able to account for the magnetic moment of this nucleus by their procedure of pure configurational mixing, it has remained anomalously large within the context of the collective model according to the investigations of Bohr and Mottelson[11] and Kerman.[10] However, the latter authors predicated their assumptions as to the character of the ground state upon energy considerations. Thus if the collective degrees of freedom associated with the core are presumed to be predominantly rotational, and if it is further presumed that the motion of the single particle relative to the core is such that the projection of its total angular momentum on the body symmetry axis is maximal and a constant of the motion, then one is led to selection rules which forbid the admixture of the nearby $f_{7/2}$ single-particle configuration. Alternatively, if the core degrees of freedom are presumed to be representable as the normal modes of surface oscillations, the admixture of $f_{7/2}$ with $h_{9/2}$ to form the ground state of ${}_{83}Bi^{209}$ occurs naturally, but quite inadequately. However, if the appeal to an explicit model for the energy is abandoned, it is seen that $h_{9/2}-f_{7/2}$ admixture for $\lambda=2$ successfully accounts for the magnetic moment of ${}_{83}Bi^{209}$.

TABLE VII. Experimentally observed and calculated magnetic moments for $f_{7/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron		
(3 7/2 0 7/2) $\mu=5.79$			(3 7/2 0 7/2) $\mu=-1.91$		
(3 7/2 2 7/2) $\mu=4.99$			(3 7/2 2 7/2) $\mu=-1.25$		
(3 7/2 4 7/2) $\mu=3.12$			(3 7/2 4 7/2) $\mu=$ 0.30		
(3 7/2 6 7/2) $\mu=0.17$			(3 7/2 6 7/2) $\mu=$ 2.74		
(3 7/2 2 5/2) $\mu=4.31$			(3 7/2 2 5/2) $\mu=-1.51$		
(3 7/2 4 5/2) $\mu=1.90$			(3 7/2 4 5/2) $\mu=$ 0.48		
Nucleus	Observed I	Observed μ	Nucleus	Observed I	Observed μ
Sc^{45}	7/2	4.76 (K)	Ca^{43}	7/2[b]	-1.32 (B-M)
V^{51}	7/2	5.15 (K)	Ti^{47}	5/2[c]	-0.79[c]
Mn^{55}	5/2	3.47 (K)	Ti^{49}	7/2[c]	-1.10 (B-M)
Co^{57}	7/2[a]	4.6 (B-M)	Nd^{143}	7/2	-1.0 (K)
Co^{59}	7/2	4.65 (K)	Nd^{145}	7/2	-0.65 (K)
			Sm^{147}	7/2[d]	-0.76[d]
			Sm^{149}	7/2[d]	-0.64[d]

[a] Baker, Bleaney, Bowers, Shaw, and Trenham, Proc. Phys. Soc. (London) **A66**, 305 (1953).
[b] C. D. Jeffries, Phys. Rev. **90**, 1130 (1953).
[c] C. D. Jeffries, Phys. Rev. **92**, 1096(A) (1953).
[d] K. Murakawa, Phys. Rev. **93**, 1232 (1954).

III. DISCUSSION

It is clear that, though no explicit model was employed to define the interdependence of the variables

TABLE VIII. Experimentally observed and calculated magnetic moments for $g_{7/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron
(4 7/2 2 7/2) $\mu=1.69$			(4 7/2 2 7/2) $\mu=$ 1.50
(2 5/2 2 7/2) $\mu=4.59$			(2 5/2 2 7/2) $\mu=-0.92$
(4 7/2 0 7/2) $\mu=1.72$			(4 7/2 0 7/2) $\mu=$ 1.49
Nucleus	Observed I	Observed μ	
Sb^{123}	7/2	2.55 (K)	
I^{129}	7/2	2.62 (B-M)	
Cs^{133}	7/2	2.58 (K)	
Cs^{135}	7/2	2.73 (K)	
Cs^{137}	7/2	2.84 (K)	
La^{139}	7/2	2.78 (K)	
Lu^{175}	7/2	2.9 (K)	
Ta^{181}	7/2	2.1 (K)	

TABLE IX. Experimentally observed and calculated magnetic moments for $g_{9/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron		
$(4\ 9/2\ 0\ 9/2)$ $\mu=6.79$			$(4\ 9/2\ 0\ 9/2)$ $\mu=-1.91$		
$(4\ 9/2\ 2\ 9/2)$ $\mu=6.21$			$(4\ 9/2\ 2\ 9/2)$ $\mu=-1.48$		
$(4\ 9/2\ 4\ 9/2)$ $\mu=4.87$			$(4\ 9/2\ 4\ 9/2)$ $\mu=-0.32$		
$(4\ 9/2\ 6\ 9/2)$ $\mu=2.75$			$(4\ 9/2\ 6\ 9/2)$ $\mu=1.43$		
$(4\ 9/2\ 8\ 9/2)$ $\mu=-0.14$			$(4\ 9/2\ 8\ 9/2)$ $\mu=3.82$		
			$(4\ 9/2\ 2\ 7/2)$ $\mu=-1.78$		
			$(4\ 9/2\ 4\ 7/2)$ $\mu=-0.42$		
Nucleus	Observed I	Observed μ	Nucleus	Observed I	Observed μ
Nb^{93}	9/2	6.16 (K)	Ge^{73}	9/2	-0.88[a]
Tc^{99}	9/2	5.68 (B-M)	Se^{79}	7/2[b]	-1.02[b]
In^{113}	9/2	5.49 (K)	Kr^{83}	9/2	-0.97 (K)
In^{115}	9/2	5.50 (K)	Sr^{87}	9/2	-1.1 (K)

[a] C. D. Jeffries, Phys. Rev. **92**, 1262 (1953).
[b] Hardy, Silvey, Townes, Burke, Strandberg, Parker, and Cohen, Phys. Rev. **92**, 1532 (1953).

chosen to describe the nuclear system (which would in principle completely determine the relative importance of these degrees of freedom for a particular ground state), considerable appeal to somewhat gross energy considerations was made for guidance in the construction and application of the rules discussed above.

First of all, heavy reliance was placed upon the extreme shell model for the specification of single-particle configurations. Such reliance contains the implication that nuclear ground-state energies are predominantly determined by the dynamics of the single particle. Thus though it is assumed that core degrees of freedom play a critical role in the determination of some ground state properties—spins in a few cases and magnetic moments in all cases—their role so far as the energy is concerned is considerably less significant.

Secondly, it has been assumed that the single-particle degrees of freedom associated with configurations lying below closed shells are so tightly bound into the core that these levels are no longer available for configurational mixing. Accordingly the $4f_{7/2}$ and $5g_{9/2}$ ground states must be characterized by an admixture of different core states rather than of different single-particle states.

The first of these considerations was employed primarily as a guide in the selection of single-particle parameters to characterize the ground state. The second was invoked as an empirical rule. There is thirdly a consideration of major interest and importance which, however, was ignored in the construction of the rules and arises rather as a consequence of the application of the model than as a guide to its application. This is the question of the relative energy separation of the pair of levels employed in the construction of the ground-state wave function to that of other possible pairs. In accordance with the assumption that the ground-state energies are predominantly determined by the dynamics of the single particle, one would expect configurational mixing to be characterized by a relatively small admixture of that configuration which lies nearest to the single-particle-state assignment for a particular nucleus. Since the core must be active ($\lambda\neq 0$) in either the case of configurational admixture—in which case it is the mechanism that makes possible the combination of different single-particle-spin states into a state of fixed total spin—or the case of core-state mixing, by definition; the question of the relative energy separation of pure particle states to that of pure core states is not of dominant importance. Thus, the fact that the single-particle quantum numbers are good in the case of $l+1/2$ particle assignments, whereas conversely the core quantum numbers are good for $l-1/2$ particle assignments, is apparently primarily a consequence of the presence or absence of interacting single-particle states lying close to a given $l+1/2$ or $l-1/2$ configuration, respectively.

The situation is most strikingly illustrated by the high-spin nuclei. The configurations $5g_{9/2}$ and $4f_{7/2}$ lie between major-shell closings which enclose no interacting single-particle configurations. Conversely the levels $6h_{9/2}$, $5g_{7/2}$, and $4f_{5/2}$ all have interacting nearby levels. The odd neutrons in the $5f_{7/2}$ level do lie close to the $6h_{9/2}$, and indeed an appropriate admixture of the $h_{9/2}$ with the $f_{7/2}$ will account for the magnetic moments of this group; but if the $h_{9/2}-f_{7/2}$ separation is sufficiently great, core-state admixing in the ground states of these nuclei would be preferred.

The assumption of core-state admixture for the $d_{5/2}$ and $p_{3/2}$ nuclei and configurational mixing for the $d_{3/2}$ acquires less clear-cut *a posteriori* justification from attempts to estimate the energy contributions from admixed single-particle configurations, the rearrangement of pairs, and admixed core states. It is often possible, by treating each nucleus in these groups as a special case, to choose a different characterization of particular ground states than was employed herein and still account for the magnetic moment while simultaneously improving the situation in the light of the crude energy considerations indicated above. However, this investigation was predicated upon the assumption that present knowledge of nuclear ground-state energies was the least valid basis for determining ground-state wave functions. Hence, since the proposed rules are simple, relatively unambiguous in application, and extremely successful in correlating nuclear magnetic moments, we have chosen to ignore the alternatives mentioned above.

Of course, there is no question of pure core-state admixture in the $p_{1/2}$ and $s_{1/2}$ ground states. For these

TABLE X. Experimentally observed and calculated magnetic moments for $h_{9/2}$ nuclei.

Calculated magnetic moments, (l, j, λ, I) Odd proton			Odd neutron
$(5\ 9/2\ 2\ 9/2)$ $\mu=2.55$			$(5\ 9/2\ 2\ 9/2)$ $\mu=1.62$
$(3\ 7/2\ 2\ 9/2)$ $\mu=5.80$			$(3\ 7/2\ 2\ 9/2)$ $\mu=-1.10$
$(5\ 9/2\ 0\ 9/2)$ $\mu=2.62$			$(5\ 9/2\ 0\ 9/2)$ $\mu=1.56$
Nucleus	Observed I	Observed μ	
Bi^{209}	9/2	4.08 (K)	

nuclei the major question is the choice of admixing single-particle configurations. Correlation of the moments of these nuclei permits very little latitude as regards the above choice, and one notes that usually the state required corresponds to the one that would be expected to admix from simple energy considerations.

It is interesting to observe that in nearly all cases the state of minimal core activity characterized by $\lambda=2$ is of major importance in the correlation of the moments. The core admixture required for $l+1/2$ nuclei usually consists of either the pair $\lambda=0$ and 2, or the pair $\lambda=2$ and 4. In either event it is almost always the case that the core state $\lambda=2$ predominates. Furthermore, the core good quantum number for all of the $l-1/2$ nuclei is $\lambda=2$. It is further noteworthy that for nearly all of the $l-1/2$ cases the amount of admixed single-particle configuration is small. However, the assumption that the core quantum number is good for $l-1/2$ nuclei may well be indefensible; and, as inspection of the tables reveals, is certainly not necessary. For all $l-1/2$ nuclei with $I>3/2$ it is seen that, in fact, it makes practically no difference empirically whether the shell theory single-particle configuration is admixed with $\lambda=0$ or 2. However, considerable difference is indicated for the $d_{3/2}$ cases, as is shown by the dotted lines in Fig. 2.

The application of these rules to the odd-odd nuclei can be carried out in a straightforward manner and their magnetic moments satisfactorily accounted for, but in general the situation is quite nonunique. If one takes as the coupling scheme

$$\mathbf{J}_p+\mathbf{J}_n=\mathbf{J}, \quad \mathbf{J}+\mathfrak{L}=\mathfrak{J},$$

where $\mathbf{J}_p$ and $\mathbf{J}_n$ are the total angular momentum operators for the proton and neutron, respectively, it is apparent that there is still an unspecified quantum number; i.e., j, the eigenvalue of $\mathbf{J}$ representing the resultant angular momentum of the two single-particle configurations. Furthermore there does not appear to be any simple criterion by means of which one might even restrict the range of values available to this quantum number—other than the not very restrictive requirements that

$$|I-\lambda|\leqslant j\leqslant |I+\lambda|, \quad |j_p-j_n|\leqslant j\leqslant |j_p+j_n|.$$

Noting, however, the consistent predominance of the core state $\lambda=2$ in the ground states of the odd-even nuclei, we have taken as starting assumptions that $j\sim I$ and $\lambda=2$ will be of major importance in the construction of the ground states here also. The particle quantum numbers are again taken to be those associated with the single-particle configuration assignments of the extreme shell model. Although configurational mixing might be expected in those cases in which one or both of the single-particle assignments is $l-1/2$, it is found that pure core-state admixture successfully accounts for all of the moments of the odd-odd nuclei. The significance of this apparently simpler situation's obtaining for these nuclei in comparison to the odd-even is not clear.

TABLE XI. Experimentally observed and calculated magnetic moments for odd-odd nuclei.

Nucleus	Observed I	Observed μ	$(l_p, j_p, l_n, j_n, j, \lambda, I)$	Calculated μ
B^{10}	3	1.80 (K)[b]	(1 3/2 1 3/2 3 0 3)	1.88
			(1 3/2 1 3/2 3 2 3)	1.75
N^{14}	1	0.40 (K)	(1 1/2 1 1/2 1 2 1)	0.49
			(1 3/2 1 3/2 1 2 1)	0.36
Na^{22}	3	1.74 (K)	(2 5/2 2 5/2 3 0 3)	1.73
Na^{24}	4	1.69 (B-M)	(2 5/2 2 5/2 2 4 4)	1.88
			(2 5/2 2 5/2 2 6 4)	1.60
K^{40}	4	−1.29 (K)	(2 3/2 3 7/2 4 0 4)	−1.68
			(2 3/2 3 7/2 4 2 4)	−1.16
K^{42}	2[a]	−1.14 (B-M)	(2 3/2 3 7/2 2 0 2)	−1.72
			(2 3/2 3 7/2 2 2 2)	−0.41
V^{50}	6 (B-M)[b]	3.35 (B-M)	(3 7/2 3 7/2 6 0 6)	3.32
Rb^{86}	2	−1.69 (B-M)	(3 5/2 4 9/2 2 0 2)	−2.13
			(3 5/2 4 9/2 2 2 2)	−0.62
Cs^{134}	4[a]	2.96 (B-M)	(2 5/2 2 3/2 2 2 4)	4.54
			(2 5/2 2 3/2 2 4 4)	2.62

[a] E. H. Bellamy and K. F. Smith, Phil. Mag. 44, 33 (1953).
[b] See reference a of Table I.

As a consequence of the above considerations, we shall not enter into any detailed discussion of these nuclei. Table XI lists the members of this group for which the spins and magnetic moments have been measured and indicates for each a reasonable set of quantum numbers defining ground states which satisfactorily correlate these moments. The diagonal elements of the magnetic moment operator are given by

$$\mu(Ij\lambda j_pj_n)=\Bigg\{\Bigg[g_{j_pl_p}\frac{j(j+1)+j_p(j_p+1)-j_n(j_n+1)}{2j(j+1)} +g_{j_nl_n}\frac{j(j+1)+j_n(j_n+1)-j_p(j_p+1)}{2j(j+1)}\Bigg] \times\frac{I(I+1)+j(j+1)-\lambda(\lambda+1)}{2I(I+1)} +g_c\frac{I(I+1)-j(j+1)+\lambda(\lambda+1)}{2I(I+1)}\Bigg\}I. \quad (15)$$

For $\lambda=0$ and $j=I$, this reduces to

$$\mu(Ij_pj_n)=\Bigg[\tfrac{1}{2}(g_{j_pl_p}+g_{j_nl_n}) +(g_{j_pl_p}-g_{j_nl_n})\frac{j_p(j_p+1)-j_n(j_n+1)}{2I(I+1)}\Bigg]I. \quad (16)$$

It is finally to be noted that in the main a somewhat analogous scheme can be developed in the m-representation (strong-coupling representation), in which admixture of different single-particle projection states replaces pure core-state admixture, and configurational mixing occurs in principle the same as in the λ representation. It is found, however, that the development of this scheme demands somewhat greater complication in order to accommodate the $I=3/2$ nuclei. As it is one of our principal motives to attempt to find the scheme with the minimal set of arbitrary rules, we have chosen the λ representation, in which the $I=3/2$ nuclei present unexceptional behavior. The fact that the m- and λ representations differ unambiguously when examined in the light of the empirical model discussed herein may possibly be significant. The principal difference lies, so far as magnetic moments are concerned, in the fact that in the m-representation $p_{3/2}$ ground states require an admixture of other single-particle configurations, usually $f_{5/2}$. Thus, if there is independent evidence for strong admixture of this type for these nuclei,[18] a preference for the m-representation might well be indicated in spite of its greater complexity empirically.

Though the investigations necessary for a detailed discussion of the significance of the model relative to electric quadrupole moments are not completed, a few comments on the matter are relevant here. It is clear, of course, that quadrupole moments cannot be completely determined within the context of the present model, as they depend sensitively upon the magnitude of the distortion of the core which in turn depends upon nuclear degrees of freedom which have been explicitly excluded from consideration. However, in those cases for which the quadrupole moments have been experimentally determined, it is possible to estimate the magnitude and sign of the quantity which contains the dependence of the theoretical quadrupole moment upon the dynamics of the core shape.

The appropriate quadrupole moment operator for a system consisting of a single particle plus a deformed core is

$$Q_{\text{op}}=\sum_{m'} D_{mm'}{}^{2*}(\theta_i)Q_{m'}+2(\tfrac{4}{5}\pi)^{\frac{1}{2}}e_p r_p{}^2 Y_2{}^m(\hat{r}_p), \qquad (17\text{a})$$

where $D(\theta_i)$ is as previously defined and $Y(\hat{r}_p)$ is a spherical harmonic in the space of the particle. For axially symmetric deformations this reduces to

$$Q_{\text{op}}=(\tfrac{4}{5}\pi)^{\frac{1}{2}}Q_m Y_2{}^m(\theta_i)+2(\tfrac{4}{5}\pi)^{\frac{1}{2}}e_p r_p{}^2 Y_2{}^m(\hat{r}_p). \qquad (17\text{b})$$

The pertinent matrix elements of this operator are

$$\begin{aligned}Q(I,\lambda jl,\lambda' j'l')&=(\Psi_{I\lambda jl}{}^I,Q_{\text{op}}\Psi_{I\lambda' j'l'}{}^I)\\&=\delta_{jj'}\delta_{ll'}Q_0P_s(Ij;\lambda\lambda')\\&\quad+\delta_{\lambda\lambda'}\langle e_pr_p{}^2\rangle P_p(I\lambda;jlj'l') \qquad (18)\end{aligned}$$

where

$$Q_0=e\int \rho r^2(3\cos^2\theta-1)d\tau_c, \qquad (19)$$

$$P_c(Ij;\lambda\lambda')=\left[\frac{(2\lambda'+1)I(2I-1)(2I+1)}{(I+1)(2I+3)}\right]^{\frac{1}{2}}\times C(\lambda'2\lambda;0,0)W(jI\lambda 2;\lambda' I), \qquad (20)$$

$$P_p(I\lambda;jlj'l')=2(-1)^{l+j'-\frac{1}{2}}\times\left[\frac{(2l'+1)(2j+1)(2j'+1)I(2I-1)(2I+1)}{(I+1)(2I+3)}\right]^{\frac{1}{2}}\times C(l'2l;0,0)W(ljl'j';\tfrac{1}{2}2)W(2j'I\lambda;jI). \qquad (21)$$

P_c is a projection factor for the core and P_p a projection factor for the particle, both of which vanish for $I=0$ or $1/2$. For $\lambda=\lambda'=0$, it follows that $j=I$, $P_c=0$, and $P_p=-(2j-1)/2(j+1)$.

The quantity Q_0 contains all of the dependence upon the dynamics of the deformation. For the hydrodynamical model[11] one has

$$Q_0=\frac{3}{(5\pi)^{\frac{1}{2}}}ZeR_0{}^2\langle\beta\cos\gamma\rangle,$$

where R_0 is the nuclear radius; and for a rigid rotator it may conveniently be expressed as

$$Q_0=(6/5)ZeR_0{}^2(I_s-I_z)/I_s,$$

where I_s and I_z are the spherical moment of inertia and the moment of inertia of the deformed object about the symmetry axis, respectively. It is obvious from the latter expression that Q_0 is positive for a prolate deformation and negative for an oblate deformation.

The first point relevant to the present discussion is that the quantity $\langle\beta\cos\gamma\rangle$, which for axial symmetry is simply $\pm\langle\beta\rangle$, has been computed for the nucleus $_{83}Bi^{209}$ employing the wave function determined by its magnetic moment and found to be ~-0.06. Thus both sign and magnitude correspond reasonably to expectation. Secondly, because of the not-inconsiderable nondiagonal contribution to the quadrupole moment for the $l+1/2$ nuclei whose ground states contain core-state admixtures, calculation of nuclear distortions from measured quadrupole moments provides the possibility of a determination of the sign of the mixing coefficient. Thirdly, the calculation of the core distortions may provide a test of the selection rule $\Delta(\lambda\lambda'2)$ in the construction of the $l+1/2$ ground-state wave functions. It will be remembered that the admixing core states for the heavier nuclei were generally characterized by $\lambda=2$ and $\lambda=4$. But in all of these cases, as far as fitting magnetic moments is concerned, the states with $\lambda=0$ and 4 would have been equally satisfactory. However, the core contribution to the quadrupole moment changes sign in going from the admixture (0,4) to (2,4) for $I=5/2$, and changes substantially in magnitude but not in sign for $I>5/2$.

IV. ACKNOWLEDGMENTS

The authors wish to express their gratitude to Dr. A. H. Snell for his sustained interest and encouragement throughout the course of this work and to Professor L. W. Nordheim for reading and commenting on this manuscript. They wish also to thank Virginia Klema for her assistance in carrying out and checking calculations.

[18] The existence of such evidence has been suggested to us by L. W. Nordheim (private communication).