A Preconditioner for Hybrid Matrices Arising in RF MEMS Switch Analysis

Z. Wang(1*), B. Jensen(1), J. L. Volakis(1,2), K. Saitou(1) and K. Kurabayashi(1)

The University of Michigan(1), The Ohio State University(2)

INTRODUCTION

RF MEMS switches have demonstrated low on-state insertion loss, high off-state isolation, and very linear behavior [1-6]. Despite these excellent characteristics, they generally suffer from low power-handling capability, with most switches operating well below 1 W [4]. This limitation is due to the complex interactions among electromagnetic losses, heat transfer, and mechanical deformations associated with the switches. To better understand these failure mechanisms, we proposed a multiphysics model [7]. This model is based on an extended finite element-boundary integral model (EFE-BI) that allows efficient modeling of the boundary (MEMS beam for our case) exterior to the volumetric region modeled by the standard FE-BI method (see Fig. 1). The resulting EFE-BI matrix system consists of two types of boundary integral sub-matrices: (1) one associated with the volume enclosing the switch substrate and (2) another from the MEMS beam itself. The second sub-matrix incorporates junction conditions to handle field and current continuity between the two domains shown in Fig. 1 (b), which illustrates a simplified structure of the actual MEMS switch.

The multi-physics analysis and experimentation of RF MEMS switches require our model to be interfaced with other models in the thermal and mechanical domains. However, the EFE-BI modeling of the micrometer-scale MEMS structures results in an ill-conditioned matrix system, making difficult the multiphysics modeling of these structures. Typically, the entire size of a practical MEMS switch is less than $\lambda/250$. Characterization of certain geometrical dimensions requires edge lengths on the order of $\lambda/150{,}000$ to $\lambda/50{,}000$.

Fig. 2(b) shows how the condition number deteriorates with frequency (size reduction). Specifically, the matrix condition number increases by 4 orders of magnitude as the frequency decreases from 40GHz to 5GHz. A matrix condition number with an additional order (10^{15}) is required at 2 GHz, which warrants computational accuracy beyond the capability of normal CPUs. For this reason, our EFE-BI analysis and validation of the code that was performed using commercially available package were only limited to high frequency cases (see Fig 2(a)). The low-frequency analysis and model validation await availability of high-performance computational capacity and experimental data in our future work.

Below we propose a preconditioning approach that lowers the condition number of the system. The proposed approach allows for fast, efficient analysis of RF MEMS switches at practical RF frequencies as low as 500 MHz, which will enable the aimed multiphysics modeling. The reader is referred to [7] and [8] for details related to the formulation of the EFE-BI and the element evaluations. Here, we focus only on the preconditioning approach and the relevant results. The reader is also referred to [9] and [10] for a review of iterative solvers and pre-conditioners available.

MATHEMATICAL FORMULATION

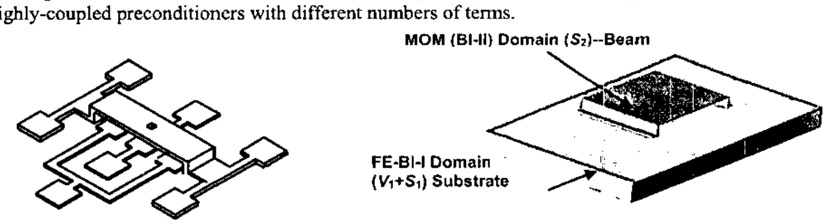
The EFE-BI formulation for RF-MEMS switch shown in Fig.1(b) can be written as [8]:

This work is supported by the National Science Foundation under grant no. ECS-01152222 and ECS-0330963

$$\begin{Bmatrix} \mathbf{FE+BI1} & \mathbf{BI12} \\ \mathbf{BI21} & \mathbf{BI2} \end{Bmatrix} \mathbf{x} = \mathbf{b} \tag{1}$$

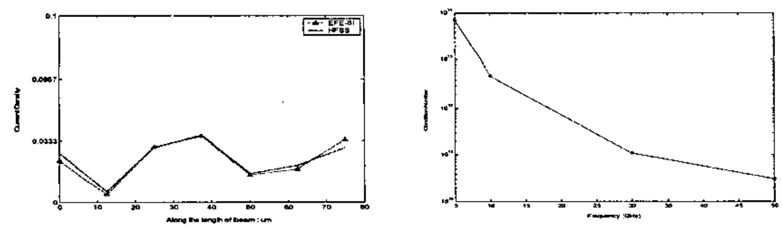
where $\mathbf{FE+BI1}$ represent the FE-BI system for the fixed volume enclosed by S_1 shown in Fig. 1, $\mathbf{BI12}$ and $\mathbf{BI21}$ are dense matrices representing the interaction between the beam and the BI enclosing the substrate, $\mathbf{BI2}$ is a dense submatrix representing the discrete MOM system, $\mathbf{b}$ is the excitation vector, and $\mathbf{x}$ is the unknown vector. In general, $\mathbf{FE}$ is a very sparse matrix whereas $\mathbf{BI1}$ is dense. The micrometer-scale size of the modeled structure leads to nearly singular integrals in some of the submatrices of (1). Although these integrals can be efficiently evaluated using semi-analytic integration [11], the resulting matrices are still ill-conditioned. Moreover, effectively solving the matrix equation in (1) is difficult as it consists of both partially full and partially sparse submatrices. The best approach is to use an iterative solver, which requires evaluation of the matrix-vector product for each iteration process. The evaluation process is complicated because the entire EFE-BI matrix needs to accommodate additional BI-related submatrices besides those found in the common FE-BI method. In addition, the analysis accounts for the electromagnetic coupling of the beam and substrate boundaries, which creates another mathematical challenge.

In this paper, GMRES is selected as a good solver for our problem. This solver converges monotonically and gives the smallest residual errors among the Krylov subspace method in each iteration process [12] by storing the direction vectors. Using GMRES, we compared the convergence rates for the same system with a diagonal preconditioner, no preconditioner, and highly-coupled preconditioners with different numbers of terms.



(a) (b)

Fig.1 (a) A typical contact RF MEMS switch; (b) Simplified RF-MEMS contact switch



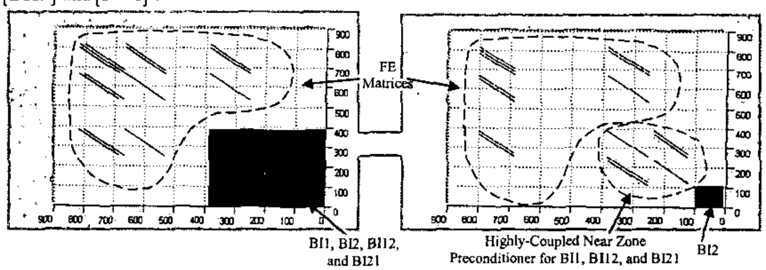
(a) (b)

Fig.2 (a)Comparison of current density along the beam length at 3000 GHz
(b) Matrix condition number increases with decreasing frequency

It is well known that a good preconditioner is sparse and well approximated such that it yields eignvalues similar to the larger eigenvalues of the original matrix. To meet these conditions in generating our preconditioner, we substitute the BI and coupling matrices with highly-coupled sparse elements, with the sparse FE part unchanged. Based on the above discussion, the preconditioner for (1) can be expressed as

$$\begin{Bmatrix} \mathbf{FE}+(\mathbf{BI1})_{NZ1} & (\mathbf{BI12})_{NZ12} \\ (\mathbf{BI21})_{NZ21} & (\mathbf{BI2})_{NZ2} \end{Bmatrix}^{-1} \begin{Bmatrix} \mathbf{FE}+\mathbf{BI1} & \mathbf{BI12} \\ \mathbf{BI21} & \mathbf{BI2} \end{Bmatrix} \mathbf{x} = \begin{Bmatrix} \mathbf{FE}+(\mathbf{BI1})_{NZ1} & (\mathbf{BI12})_{NZ12} \\ (\mathbf{BI21})_{NZ21} & (\mathbf{BI2})_{NZ2} \end{Bmatrix}^{-1} \mathbf{b} \quad (2)$$

where $\{\mathbf{BI1}\}_{NZ1}$ contains a pre-specified number of the strongest coupling matrix elements in each row of $\{\mathbf{BI1}\}$. To generate $\{\mathbf{BI1}\}_{NZ1}$, the matrix elements within each row of $\{\mathbf{BI1}\}$ are sorted with respect to its modulus, and the n_{NZ1} elements with the largest modulus are included into matrix $\{\mathbf{BI1}\}_{NZ1}$. The remaining elements belong to $\{\mathbf{BI1}\}_{NF1}$. Typically, most elements of $\{\mathbf{BI1}\}_{NZ1}$ are located in a band around the main diagonal. But edge numbering can make some of the elements arbitrarily distributed over the whole extent of the square matrix, which cannot be accommodated into the conventional preconditioner. Our preconditioner can include these matrix elements. The procedures described above are also applied to matrices $\{\mathbf{BI2}\}$, $\{\mathbf{BI12}\}$ and $\{\mathbf{BI21}\}$.



(a) Original EFE-BI matrix profile (b) Our preconditioner profile

Fig.3 Fill profile for EFE-BI and preconditioner matrix

Fig. 3 compares the profile of the original EFE-BI matrix and of the highly-coupled preconditioner. The elements in the beam are all in the near zone with respect to each other and strongly coupled to the source to model the current flowing through the beam. These conditions require us to include the entire BI matrix for the beam (marked in black) directly into the preconditioner. This process was later found to ensure convergence in all of our calculations.

RESULTS

Fig. 5 shows the results of our analysis for different preconditioning cases. The analysis

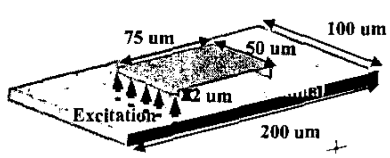


Fig. 4 RF-MEMS switch for our modeling

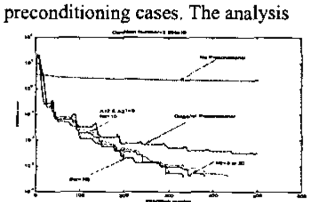


Fig.5 Convergence vs. iteration number

used a very high matrix condition number (3.694×10^{10}), which required the iterative preconditioner. It is clearly shown that the highly-coupled preconditioner allows for faster convergence. To better understand the preconditioner's influence on convergence, we show in Fig 6 the computed eigenvalue spectrum before preconditioning. We show the spectrum when

NZ for the preconditioner is set to 1 (same as the diagonal preconditioner) and 10 at 50 GHz. It is seen that with NZ=10 we get a large number of eigenvalues closer to those of the original matrix. This explains why the proposed preconditioner is a better choice after preconditioning. Also, the case with NZ=10 has smaller eigenvalue spectrum cluster radius in comparison with NZ=1 case, thus the convergence rate becomes faster.

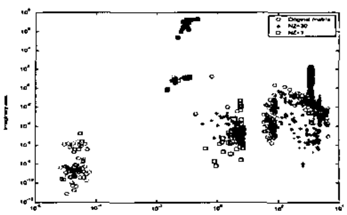


Fig.6 Eigenvalue spectrum distribution

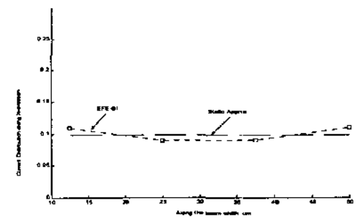


Fig.7 Current density vs. beam width (f=5 GHz)

Fig. 7 shows the current distribution across the beam of the switch (Fig. 4) at 5 GHz. The computation was performed using a preconditioner with a larger number of NZ (=30). This result is in good agreement with the predicted using a static approximation. Most importantly, convergence was achieved rather rapidly.

REFERENCES

[1] Yao, J.J., "RF-MEMS from device perspective", *J. Micromech. and Microeng.*, Vol. 10 (2000), pp. R8-R38.

[2] Pacheco, S., Nguyen, C.T., and Katehi, L.P.B., "Micromechanical Electrostatic K-Band Switches," *1998 IEEE MTT-S Digest*, 1998, pp. 1569-1572.

[3] Zavracky, P.M., McGruer, N.E., Morrison, R.H., and Potter, D., "Microswitches and Microrelays with a View Toward Microwave Applications," *Intl. J. RF and Microwave Theory and Techniques*, Vol. 9, No. 4 (1999), pp. 338-347.

[4] Rebeiz, G.M. and Muldavin, J.B., "RF MEMS Switches and Switch Circuits," *IEEE Microwave Mag.*, Vol. 2, No. 4 (Dec. 2001), pp.59-71.

[5] Duffy, S., Bozler, C., Rabe, S., Knecht, J., Travis, L., Wyatt, P., Keast, C., and Gouker, M., "MEMS Microswitches for Reconfigurable Microwave Circuitry," *IEEE Microwave and Wireless Components Letters*, Vol. 11, No. 3 (Mar. 2001), pp. 106-108.

[6] Muldavin, J.B. and Rebeiz, G.M., "Inline Capacitive and DC-Contact MEMS Shunt Switches," *IEEE Microwave and Wireless Components Letters*, Vol. 11, No. 8 (Aug. 2001), pp. 334-336.

[7] B.D. Jensen, K. Saitou, J.L. Volakis, K. Kurabayashi, "Fully Integrated Electrothermal Multi-Domain Modeling of RF MEMS Switches," *IEEE Microwave and Wireless Components Letters*, vol. 13, no. 9, pp. 364-366, 2003

[8] Z. Wang, B. Jensen, J. Volakis, K. Saitou, and K. Kurabayashi, "Analysis of RF-MEMS switches using finite element-boundary integration with moment method," *Proc. IEEE Society International Conference on Antennas and Propagation*, vol. 2, pp. 173 -176, June 22-27, 2003

[9] T.F. Eibert, "Iterative Near-Zone Preconditioning of Iterative Method of Moments Electric Field Integral Equation Solutions," *IEEE Antennas and Wireless Propagation Letters*, vol.2, pp.101-102, 2003

[10] Volakis, J.L., Chatterjee, A., and Kempel, L.C., *Finite Element Method of Electromagnetics*, New York, IEEE Press, 1998

[11] Z. Wang, J. Volakis, K. Saitou, and K. Kurabayashi, "Comparison of semi-analytical formulations and Gaussian quadrature rules for quasi-static double surface potential integrals," IEEE Antenna Prop. Magazine, in press

[12] J.L. Volakis "Iterative Algorithms for Sparse Systems," IEEE Antennas and Propagation Magazine, Vol. 37, No.6, Dec. 1995, pp. 94-96

[13] Y. Saad, *Iterative Methods for Sparse Linear Systems*. Boston, MA: PWS, 1996