Superlattice in an interminiband resonance ac field

V. I. Litvinov[a)]
WaveBand Corporation, 17152 Armstrong Avenue, Irvine, California 92614

A. Manasson
Department of Applied Physics, University of Michigan, 1301 Beal Avenue, Ann Arbor, Michigan 48109

(Received 31 January 2005; accepted 1 July 2005; published online 18 August 2005)

We discuss the properties of miniband electrons in a superlattice illuminated by a strong electromagnetic field. If the ac field is in resonance with a two-miniband superlattice, gaps in the quasienergy spectrum appear. The gap is proportional to the Bessel function and oscillates with the ac electric field. The resonant ac driving force makes the quasienergy spectrum tunable and may decrease the critical dc field for electrical domains to form. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2010619]

INTRODUCTION

The coherent state of an ac-driven solid can be described in terms of quasienergy: the electron energy spectrum in a temporarily periodic external electric field.[1] This concept allows a treatment of the nondissipative state of equilibrium between the solid and the electromagnetic field. In this paper, we consider the quasienergy states of two-miniband superlattice (SL) driven by an ac field, $E=E_1\cos(\omega t)$. The problem has been considered in Refs. 2–5. There are two distinct regimes that have been described: band collapse and interband resonances. The band collapse appears in a narrow-band SL where a strong enough ac field modulates the weak electron tunneling between the adjacent quantum wells. It is known that the exact band collapse is absent if the next to nearest neighbors contribute to the SL band structure.[6] The approach to the quasienergy spectrum, developed in Ref. 2, exploits the approximation $\hbar\omega \ll D$, with D being the interminiband energy gap. Similar calculation in a two-miniband SL (Ref. 3) was performed using the perturbation method, assuming that $\hbar\omega \gg D$ and the Bloch frequency ω_B is a multiple of ω.

The purpose of this paper is to present a calculation of the quasienergy spectrum in a two-miniband SL, which does not rely on the use of the perturbation method and the assumptions mentioned above. Instead, we derive the quasienergy bands by treating the resonant inter- and nonresonant intraminiband electron-photon interactions exactly. The result shows that, if the ac field is in resonance with a two-miniband SL, gaps in quasienergy spectrum appear. The resonance gap is proportional to the Bessel function and oscillates with the ac electric field. The lowest quasienergy branch has an admixture of the excited miniband states, making the dispersion relation strongly different from that in the simple cosine-type miniband. The dc conductivity in this quasienergy branch may reveal favorable conditions for the electrical domains to form: a negative differential conductivity (NDC) occurs at a lower dc field compared with that in a simple cosine-type miniband.

a)Electronic mail: vladimir.litvinov@waveband.com

MODEL AND HAMILTONIAN

We consider superlattice minibands that appear as a result of the long-periodic (for example, Kronig-Penney) superlattice potential $V_{SL}(z)$ applied to a semiconductor conduction band. The Hamiltonian in Eq. (1) describes the effective-mass-approximation conduction band subjected to the external field $V_{SL}(z)$ and electromagnetic field $\mathbf{A}$:

$$H=H_0+H_1; \quad H_0=\frac{p_\parallel^2}{2m_\parallel}+\frac{p_z^2}{2m_z}+V_{SL}(z), \quad H_1=-\frac{ep_zA_z}{m_z}. \tag{1}$$

In the Hamiltonian Eq. (1) we neglected the A^2 term that implies $e|A|$ is small as compared with the typical electron momentum, which is true at least for short-period SLs. Besides, the A^2 term does not induce the electron transitions; it gives energy correction to the ground state and does not contribute to the spectrum of collective excitations. We also assumed that the ac electric field is spatially independent over the SL length scale ($\nabla\mathbf{A}=0$) and that maximum electron-photon coupling occurs when an ac electric field is parallel to the SL axis $\mathbf{A}\parallel z$. The eigenfunctions $|n,\mathbf{p}\rangle$ of the Hamiltonian H_0 are the SL Bloch functions in the miniband n. They form the basis set for the second quantization representation: $|\Psi\rangle=\sum_{n,\mathbf{p}}a_{n,\mathbf{p}}\exp(i\varepsilon_{np}t/\hbar)|n,\mathbf{p}\rangle$, where $a_{n\mathbf{p}}(a_{n\mathbf{p}}^+)$ is the annihilation (creation) operator of an electron in the SL state $\exp(i\varepsilon_{np}t/\hbar)|n,\mathbf{p}\rangle$. This representation being used to account for the near-resonance transitions between minibands 1 and 2 gives the Hamiltonian in the form:

$$H=\sum_{n=1,2,\mathbf{p}}[\varepsilon_{n\mathbf{p}}+\mu_{n\mathbf{p}}\sin(\omega t)]a_{n\mathbf{p}}^+a_{n\mathbf{p}} + \sum_{\mathbf{p}}\lambda_{12}\sin(i\omega t)a_{1\mathbf{p}}^+a_{2\mathbf{p}}+\text{c.c.},$$
$$\mu_{n\mathbf{p}}=-\frac{eE_1}{m_z\omega}\langle n,\mathbf{p}|\hat{p}_z|n,\mathbf{p}\rangle, \quad \lambda_{12}=-\frac{eE_1}{m_z\omega}\langle 1,\mathbf{p}|\hat{p}_z|2,\mathbf{p}\rangle. \tag{2}$$

In the time-dependent Hamiltonian Eq. (2) we keep the nonresonance intraminiband μ terms and both resonant and nonresonant λ terms.

The superlattice potential in H_0 transformed almost free (effective-mass-approximation) conduction electrons into particles with a SL dispersion relation $\varepsilon_{1,2\mathbf{p}}=p_{\parallel}^2/2m_{\parallel}+\varepsilon_{1,2}(k_z)$. The dispersion relation in the excited miniband $\varepsilon_2(k_z)$ is more complicated than that in a simple cosine-type miniband.

QUASIENERGY SPECTRUM

In what follows we treat the Hamiltonian [Eq. (2)] by making a unitary transformation $U(t)|n\mathbf{p}\rangle$, where the operator $U(t)$ given below, is chosen to remove the explicit time dependence from the Hamiltonian Eq. (2) to a possible extent:

$$U(t)=\exp\left\{i\sum_{\mathbf{p}}\left[\frac{\cos(\omega t)}{\hbar\omega}\mu_{1p}+\frac{\omega t}{2}\right]a_{1\mathbf{p}}^{+}a_{1\mathbf{p}}+\left[\frac{\cos(\omega t)}{\hbar\omega}\mu_{2p}-\frac{\omega t}{2}\right]a_{2\mathbf{p}}^{+}a_{2\mathbf{p}}\right\}. \quad (3)$$

The transformed Hamiltonian is given as

$$\begin{aligned}\tilde{H}&=U^{+}HU-i\hbar U^{+}\frac{\partial}{\partial t}U\\&=\sum_{\mathbf{p}}\left(\varepsilon_{1\mathbf{p}}+\frac{\hbar\omega}{2}\right)a_{1\mathbf{p}}^{+}a_{1\mathbf{p}}+\left(\varepsilon_{2\mathbf{p}}-\frac{\hbar\omega}{2}\right)a_{2\mathbf{p}}^{+}a_{2\mathbf{p}}\\&\quad+\sum_{\mathbf{p}}\tilde{\lambda}_{1\mathbf{p}}a_{1\mathbf{p}}^{+}a_{2\mathbf{p}}+\text{c.c.}\\&\quad+\sum_{\substack{m=-\infty\\m\neq1}}^{\infty}\sum_{\mathbf{p}}\tilde{\lambda}_{m\mathbf{p}}\exp(im\omega t)a_{1\mathbf{p}}^{+}a_{2\mathbf{p}},\end{aligned} \quad (4)$$

where $\tilde{\lambda}_{m\mathbf{p}}=\left(\frac{(-i)^m\lambda_{12}(\mathbf{p})m\hbar\omega}{\mu_{1p}-\mu_{2p}}\right)J_m\left(\frac{\mu_{1p}-\mu_{2p}}{\hbar\omega}\right)$.

The intraminiband terms in Eq. (4) constitute a quasienergy-band-defined modulo $\hbar\omega$. As shown in Eq. (4), we removed the time-dependent μ terms[7] and the time dependence from the resonant λ terms.[8] The first three terms in Eq. (4) describe a SL under resonant conditions where time-independent coupling appears between two lowest quasienergy branches. The remaining terms ($m\neq1$) include interference contributions from intra- and interband terms and describe m-photon transitions which are nonresonant. These nonresonant terms give rise to the fast oscillating corrections to the wave function. The corrections being averaged over time are small and can be neglected. Assuming $|\lambda_{1\mathbf{p}}|\gg\hbar\tau_{12}^{-1}$ (τ_{12} is the interminiband recombination time), we obtain two branches of the quasienergy spectrum from Eq. (4) as

$$E_{\pm}(\mathbf{p})=\frac{\varepsilon_{1\mathbf{p}}+\varepsilon_{2\mathbf{p}}}{2}\pm\left[\left(\frac{\varepsilon_{2\mathbf{p}}-\varepsilon_{1\mathbf{p}}-\hbar\omega}{2}\right)^2+|\lambda_{1\mathbf{p}}|^2\right]^{1/2}. \quad (5)$$

The energy gap in a quasienergy spectrum $2|\lambda_{1\mathbf{p}}|$ is an oscillating function of frequency and intensity of the electron-photon interaction. It becomes zero at the zeros of $J_1[(\mu_{1p}-\mu_{2p})/\hbar\omega]$.

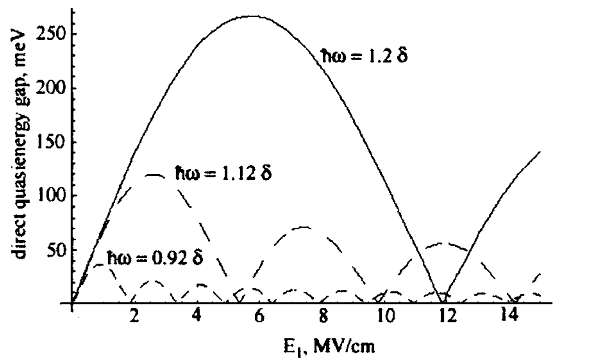

FIG. 1. Direct gap between quasienergy bands at various off-resonance conditions. δ is the difference between centers of the ground and excited minibands.

The dependence of the direct gap between quasienergy bands on the intensity of the interminiband resonance field is shown in Fig. 1.

An important feature of the quasiparticle spectrum Eq. (5) is that the branches contain a mixture of the initial minibands, making the dispersion relation tunable by an external resonance source. In the coherent state the resonance light renormalizes the miniband spectrum, allowing the energy gaps to appear at the quasi-Fermi levels in the minibands 1 and 2. As we have made a transition from the initial SL electrons to the quasiparticle electrons plus light, the result means that the quasiparticle spectrum $E_{\pm}(p)$ has an energy gap, and the Fermi level lies in the gap.

The complex dispersion relation of quasiparticles [Eq. (5)] may influence the conditions for the onset of NDC in a dc-biased SL. The role of the miniband dispersion relation in NDC and Bloch oscillations has been discussed in Refs. 9 and 10. Below we estimate the critical dc field for NDC to occur in a dc-biased SL.

If only the lower initial miniband is partially filled by electrons, one may assume that only $E_{-}(p)$ branch is filled by quasiparticles as long as the temperature is less than the energy gap λ_{1p}. To make the estimations we represent the interminiband coupling matrix elements λ_{12} and the intraminiband velocity matrix elements $\mu_{1,2}$ as follows:

$$\lambda_{12}=\frac{eE_1}{\omega}\mathbf{v}_{1,2},\quad \mu_{1,2}=\frac{eE_1}{\omega}\mathbf{v}_{1,2}, \quad (6)$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are velocities of the electrons in the first and second minibands $\mathbf{v}_{1,2}\approx\hbar k_{\text{res}}/m_{1,2}(k_{\text{res}})$, $\mathbf{v}_{12}\approx\sqrt{\hbar\omega/m_1(k_{\text{res}})}$ is the interminiband velocity,[7] where $\hbar^2m_n^{-1}(k_{\text{res}})=\partial^2\varepsilon_n(k_z)/\partial k_z^2|_{k=k_{\text{res}}}$. For a given $\hbar\omega$ the momentum $\hbar k_{\text{res}}$ satisfies the resonance condition $\hbar\omega=\varepsilon_2(k_{\text{res}})-\varepsilon_1(k_{\text{res}})$.

Numerically calculated SL quasienergy dispersion [Eq. (5)] is shown in Fig. 2.

As seen from Fig. 2, a strong ac field has a significant effect on the dispersion of the SL minibands. Both the quasienergy gap and the dispersion relation of an illuminated SL can be influenced by an external ac field.

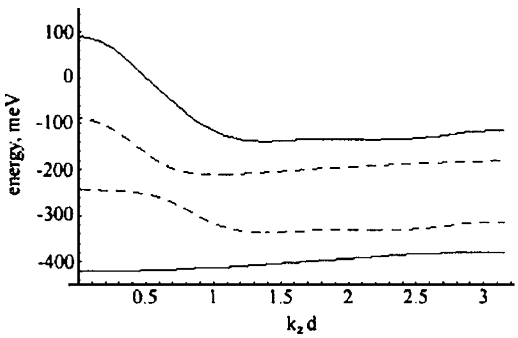

FIG. 2. Quasienergy dispersion of an $Al_{0.5}Ga_{0.5}N$/GaN SL in a strong ac resonance field. $d_{well}=25A$, $d_{barrier}=10A$, $\hbar\omega=1.2\delta$, $E_1=1$ MV/cm. Solid line: unperturbed energy spectrum; dashed line: energy spectrum in a strong resonance ac field.

DC CONDUCTIVITY IN A QUASIENERGY BAND

The effect of the ac field on the lowest quasienergy band dispersion relation is calculated from Eq. (7) and is shown in Fig. 3.

Quasiclassical dc conductivity in an SL with the complicated dispersion follows from the Boltzmann equation:[10]

$$\text{Re}\,\sigma_{dc} = \sigma_0[W_0(1) - 2\eta W_0(2)],$$
$$W_0(\nu) = \frac{1-(\nu\Omega_0\tau + k\omega\tau)^2}{[1+(\nu\Omega_0\tau+k\omega\tau)^2]^2}, \qquad (7)$$

where

$$\sigma_0 = \frac{e^2d^2n\tau b}{\hbar^2}\frac{R(1)}{Q}, \quad \eta = \frac{2cR(2)}{bR(1)},$$

$$Q = \int_0^\pi \exp[b\cos(x)/k_BT - c\cos(2x)/k_BT]dx, \qquad (8)$$

$$R(\nu) = \int_0^\pi \cos(\upsilon x)\exp[b\cos(x)/k_BT - c\cos(2x)/k_BT],$$

and b and c are the coefficients in the simple cosine and next k_z harmonic in a SL dispersion relation, respectively. The parameter $\Omega_0=edE_0/\hbar$ and contains a dc electric field; τ is the intraband momentum relaxation time; and n is the volume electron density.

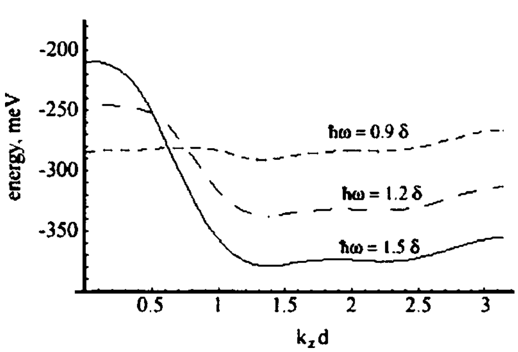

FIG. 3. Lowest quasienergy band dispersion relation for different frequencies of the external ac field. $E_1=1$ MV/cm.

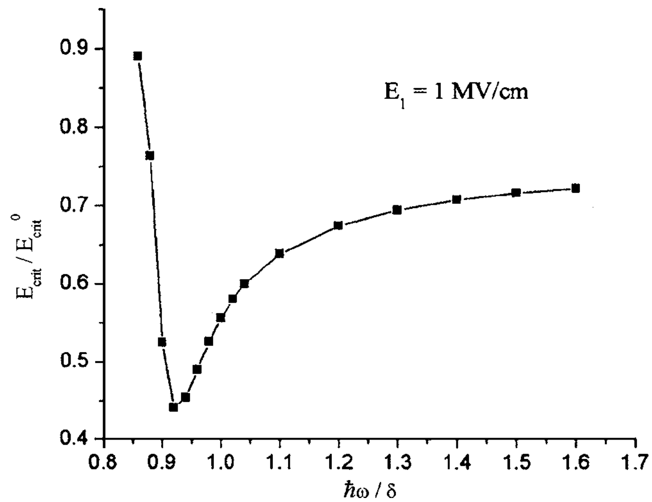

FIG. 4. Critical dc field as a function of frequency of the resonant field: $E^0_{crit}=\hbar/ed\tau$ is the critical field of the unperturbed SL.

Negative differential resistance appears if $E>E_{crit}$. Figure 4 illustrates the dependence of E_{crit} on the frequency of the resonance ac field.

As seen from Fig. 4, the critical voltage necessary for the onset of NDC changes considerably with the frequency, and a sharp change appears when the effective mass changes its sign.

CONCLUSIONS

Under an interminiband resonant illumination the dispersion relation in the first quasienergy branch $E_{-}(\mathbf{p})$ may significantly differ from that in the original lowest SL miniband (simple cosine-type band) due to an admixture of the excited miniband states. The dispersion relation is tunable since it depends on the characteristics of an electron-photon interaction. Tunable quasienergy bands may provide a range of conditions necessary for NDC to occur, thus making the ac-driven SL device suitable for submillimeter wave generation. Reduction of the critical dc voltage in a resonance ac field makes a SL less sensitive to the dc-power dissipation that is important for large band-gap structures such as GaN/AlGaN SLs.

The light-induced energy gaps discussed in this paper appear as a response of the medium to an intense temporarily periodic electric field. The gaps or, more generally the renormalized joint density of states affect the absorption of the probe's weak-electromagnetic perturbation. In this respect the phenomenon refers to a dynamical Franz-Keldysh effect (DFKE) discussed in Refs. 11–13. The difference is that we consider a strong resonant electromagnetic field that is generally not a case in DFKE.

[1]Y. B. Zeldovich, Sov. Phys. JETP **24**, 1006 (1966); V. I. Ritus, *ibid.* **24**, 1544 (1966).
[2]M. Holthaus and D. Hone, Phys. Rev. B **47**, 6499 (1993).
[3]K. Drese and M. Holthaus, J. Phys.: Condens. Matter **8**, 1193 (1996).
[4]J. Rotvig, A. P. Jauho, and H. Smith, Phys. Rev. Lett. **74**, 1831 (1995).
[5]X. G. Zhao, D. Suqing, and W. Zhang, Physica E (Amsterdam) **9**, 674 (2001).
[6]M. M. Dignam and C. Martijn de Sterke, Phys. Rev. Lett. **88**, 046806 (2002).

[7]V. D. Blazhin, Sov. Phys. Solid State **17**, 1537 (1975).
[8]V. M. Galitskii, S. P. Goreslavskii, and V. F. Elesin, Sov. Phys. JETP **30**, 117 (1970).
[9]Y. Romanov, L. G. Murokh, and N. J. M. Horing, J. Appl. Phys. **93**, 4696 (2003).
[10]V. I. Litvinov and A. Manasson, Phys. Rev. B **70**, 195323 (2004).
[11]A. P. Jauho and K. Johnsen, Phys. Rev. Lett. **76**, 4576 (1996).
[12]A. Srivastava, R. Srivastava, J. Wang, and J. Kono, Phys. Rev. Lett. **93**, 157401 (2004).
[13]K. Johnsen and A. P. Jauho, Phys. Rev. B **57**, 8860 (1998).