The stimulated Stern-Gerlach effect in charged particle storage rings

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ABSTRACT

The results of the study of possibilities to use spin-orbital force in order to split a circulating beam to two polarized beams are presented in this report. It is shown that the original spin-splitter[1,2] idea which is based on using intrinsic spin-orbital resonance is not sufficient for splitting, in principle, because the spin's long lasting effect on particle betatron oscillations is reduced just to a small tune shift. The theorem on the conservation of the sum or difference of quantum orbital and spin numbers, i.e. the combined spin-orbital invariance, is established for this case. The resonant RF magnetic field parallel to the plane of splitting is introduced in order to stabilize spin in the plane of its precession and remove the combined invariancy. The new double-resonance invariants are established, which describe the spin dynamics and the splitting process.

In addition, a method of spin-splitting a beam is considered using gradient RF magnetic field resonance to particle betatron oscillations. The field has a component along the axis of the equilibrium polarization in the storage ring. No resonance with spin precession around this axis is required.

The necessary conditions of beam splitting are discussed.

1. The neo-classical RF Stern-Gerlach method

We describe the principal aspects of this possibility. The spin-dependent part of the Hamiltonian corresponding to the resonant spin-orbit force can be written in the following form:

$$H_{sp} = S_n \vec{n} \cdot \vec{w}(\vec{r}, \vec{v}, t), \tag{1}$$

where S_n is the spin projection on $\vec{n}$, and vector $\vec{w}$ as a function of RF field can be written as follows:

$$\vec{w} \approx -(\frac{1}{\gamma} + G)\frac{\vec{B}(\vec{r}, t)}{B_o}, \tag{2}$$

where B_o is the average vertical field of the ring, and $\vec{B}(\vec{r}, t)$ is the gradient

RF field:

$$\vec{B}(\vec{r},t) = \frac{1}{2}\vec{B}_\omega(\vec{r})e^{-i\omega t} + \frac{1}{2}\vec{B}^*_\omega(\vec{r})e^{i\omega t}.$$

We assume that $\vec{B}_\omega$ linearly depends on x. Averaging of the Hamiltonian (1) for fast-oscillating terms, we get an effective Hamiltonian in the form:

$$H_{eff} = \frac{1}{2}S_n(g_\omega a \cdot e^{i\epsilon\theta} + g^*_\omega a^* e^{-i\epsilon\theta}), \tag{3}$$

where θ is the generalized azimuth of the particle,

$$g_\omega = \frac{1}{2B_o}(\frac{1}{\gamma} + G) \cdot < \vec{n}\frac{\partial\vec{B}_\omega(\vec{r})}{\partial x} \cdot f_x \cdot e^{-i(k_x+\nu_x)\theta} >_{x,y=0} \tag{4}$$
$$\epsilon = k_x + \nu_x - \frac{\omega}{\omega_o},$$

ω_o is the revolution frequency, and a is the complex amplitude of the betatron x-oscillations:

$$x = \frac{1}{2}\left[af_x(\theta) + a^*f^*_x(\theta)\right].$$

Using the Hamiltonian equations we get:

$$a' = i\frac{R}{p}g^*_\omega e^{i\epsilon\theta} \cdot S_n, \tag{5}$$
$$S'_n = 0;$$

here p is the particle total momentum and $2\pi R$ is the circumference. So, the spin projection on $\vec{n}$ is constant, and we have the resonance spin-splitting of the betatron oscillations:

$$a_+ - a_- = i\frac{R}{p} \cdot \hbar g^*_\omega \int_0^\theta e^{i\epsilon\theta}d\theta = -\frac{\hbar R}{p}g^*_\omega\frac{e^{-i\epsilon\omega_o t} - 1}{\epsilon}. \tag{6}$$

To conclude this section, we note that the method described above can be considered as an immediate extension of the classical Stern-Gerlach method to the case of circulating charged particles: the spin is stabilized by the basic conservative field, and the stable spin component is responding to the regular spin-orbit force; but, in order to enhance the splitting effect during many particle revolutions, we must apply the RF alternating gradient field with the frequency which is resonant to particle free oscillations.

From the practical point of view, one disadvantage of this method is the smallness of the value of the RF field strength compared to that of the intrinsic gradient field.

2. The RF-driven Stern-Gerlach effect near an intrinsic resonance

Now we add a dipole or solenoidal RF field to the intrinsic constant field of the ring:

$$\tilde{\vec{B}}(\vec{r}, t) = \mathrm{Re}\left[\vec{B}^{\circ}(\vec{r}) \cdot e^{-i\omega t}\right].$$

The frequency ω of this field must be in resonance with particle betatron ν_x-value and with spin precession frequency ν_{sp}. So, we have the double resonance condition:

$$\frac{\omega}{\omega_o} \approx \nu_{sp} + k_{sp}, \qquad \frac{\omega}{\omega_o} \approx \nu_x + k_x, \tag{7}$$

where k_{sp} and k_x are integers; note that $k_x - k_{sp} = k$, where k is the integer in resonance condition $\nu_{sp} \approx \nu_x + k$.

Let us observe the spin motions with respect to the system of base vectors

$$\vec{n}(\theta), \vec{n}_1(\theta), \vec{n}_2(\theta),$$

where

$$\vec{n}_1 + i\vec{n}_2 = \vec{e} \cdot e^{-i(\frac{\omega}{\omega_o} - k_{sp})\theta} \equiv \hat{\vec{n}}. \tag{8}$$

Complex vector $\vec{e}$ is defined by the relation

$$\vec{e} \;=\; \vec{\eta} \cdot e^{i\nu_{sp}\theta},$$

where $\vec{\eta}$ is the solution of spin equations on the closed orbit, with the feature $\vec{\eta}(\theta + 2\pi) \;=\; \vec{\eta}(\theta) \cdot e^{-2\pi i \nu_{sp}}$.

We should pay some attention to a peculiar effect which arises when using the RF field. In order to provide a small value for the phase difference between the particle revolution and RF field oscillation, we need a bunched beam, i.e. we have to supply a longitudinal electric RF field with the frequency ω_o or $q\omega_o$, where q is an integer. In such a regime particle energy and relative phase $(\theta - \omega_o t)$ will oscillate near equilibrium values with some low frequency $\nu_\gamma \omega_o$, $\nu_\gamma << 1$. Also the betatron and spin ν-values will oscillate due to chromaticity parameters $\partial\nu_x/\partial\gamma$ and $\partial\nu_{sp}/\partial\gamma$. It is not difficult to extend the consideration taking these synchrotron oscillations into account. But, for simplicity, we assume here that the amplitudes of all of these oscillations are small enough to neglect them and also that all of the frequencies are constant and equal to the averaged values. We also assume that we can neglect phase "mismatching" $(\theta - \omega_o t)$ since the beam is short enough. There is also an advantage of using bunched beams, in that the contribution of energy spread to betatron and spin frequency becomes a value of the second order. Note that the parameter $\frac{\partial\nu_x}{\partial\gamma}$ could be canceled by the introduction of sextupoles and the parameter $\partial\nu_{sp}/\partial\gamma$ vanishes by using Siberian Snakes.

Then we have the equations of motion as follows:

$$\begin{aligned} S_n' &= i\hat{w}\hat{S}_- - i\hat{w}^*\hat{S}_+, & & \\ \hat{S}_+' &= i\hat{w}S_n - i\epsilon_{sp}\hat{S}_+, & \epsilon_{sp} &= \nu_{sp} + k_{sp} - \frac{\omega}{\omega_o} \\ \hat{a}' - i\epsilon_x\hat{a} &= i\frac{R}{p}g\hat{S}_+; & \epsilon_x &= \nu_x + k_x - \frac{\omega}{\omega_o} \end{aligned} \tag{9}$$

we define here

$$\vec{w} \equiv \tilde{w} + ga, \qquad \tilde{w} = \frac{<\vec{W}_{RF}\cdot\hat{\vec{n}}>}{\omega_o}, \qquad ga = \frac{<\vec{W}_i\cdot\hat{\vec{n}}>}{\omega_o},$$

where $\vec{W}_{RF}$ and $\vec{W}_i$ are the angle speed of the spin precession in the RF field, and in the intrinsic gradient field, respectively, according to the BMT equation.

Now we assume the relation

$$|\epsilon_x| << |\hat{w}|, \tag{10}$$

which is not too precise a requirement from the practical point of view. With this condition, we can write the solution for spin motion in the base system (8) as:

$$\vec{S}(\theta) = \hat{S}_{||}\cdot\frac{\hat{\vec{w}}}{|\hat{\vec{w}}|} + S_n(\theta)\cdot\vec{n} + \frac{\vec{n}x\hat{\vec{w}}}{|\hat{\vec{w}}|}\hat{S}_\perp(\theta), \tag{11}$$

$$S_n(\theta) + i\hat{S}_\perp(\theta) = \left[S_n(0) + i\hat{S}_\perp(0)\right]\exp(-i\int|\hat{\vec{w}}|d\theta),$$

$$\hat{S}_{||} = \text{const}$$

where vector $\hat{\vec{w}}$ has the components:

$$\hat{\vec{w}} = (\hat{w}_1,\ \hat{w}_2,\ \epsilon_{sp}), \quad \hat{w}_1 + i\hat{w}_2 = \hat{w}(\hat{a}).$$

Further, we assume for simplicity, that

$$|\epsilon_{sp}| << |\hat{w}|; \tag{12}$$

then the axis of spin precession $|\hat{\vec{w}}|$ is close to the plane transverse to $\vec{n}$.

The evolution of amplitude a averaged for spin precession around $\hat{\vec{w}}$ is described by the equation:

$$a' + i\epsilon_x a = i\frac{R}{p}g^* \frac{\hat{w}}{|\hat{w}|}\hat{S}_{||}. \tag{13}$$

With taking adiabatic invariancy of $\hat{S}_|$ into account, the invariant of this equation is the Hamiltonian

$$\hat{H}_{eff} \to H_{split} = \frac{p}{2R}\int \epsilon_x d|a^2| + \hat{S}_{||} \cdot |\tilde{w} + ga|. \tag{14}$$

Thus, this Hamiltonian describes the splitting process, after using the eigenvalues of the operator $\hat{S}_{||}$:

$$\hat{S}_{||} \to \pm\frac{\hbar}{2}.$$

Let us assume that ϵ_x = const, i.e. x-oscillations are linear; then we obtain the trajectories of the splitting process in variables $|a|$ and ψ, i.e., $a = |a|e^{i\psi}$:

$$\frac{p}{2R}\epsilon_x|a|^2 \pm \frac{\hbar}{2}\sqrt{|\tilde{w}|^2 + 2|\tilde{w}ga|\cos\psi + |ga|^2} = \text{const}. \tag{15}$$

Let us focus our attention on the case of precise resonance, i.e. $\epsilon_x = 0$, when we can expect the biggest value of splitting. In this case, we have

$$|\hat{w}| \equiv |\tilde{w} + ga| = \text{const}, \tag{16}$$

i.e. total effective field $\hat{w}$, which is the vector sum of $\tilde{w}$ and the intrinsic field ga, just rotates in the plane transverse to $\vec{n}$. We can get the angle speed $\Delta\omega$ of this rotation (with respect to system $\hat{\vec{n}} = \vec{n}_1, \vec{n}_2$) using the equation (13):

$$\hat{w}' = \pm i\frac{R\hbar}{2p}|g|^2\frac{\hat{w}}{|\hat{w}|}, \tag{17}$$

and then we have

$$\Delta\omega = \pm\frac{R\hbar}{2p}\frac{|g|^2}{|\tilde{w} + ga_o|}\omega_o, \tag{18}$$

where a_o is the initial value of a.

The time progress of the splitting process depends on the evolution of $\hat{w}$:

$$\hat{w} = \hat{w}(0) \cdot \exp(i\Delta\omega \cdot t), \tag{19}$$

or

$$\tilde{w} + ga = (\tilde{w} + ga_o)\exp\Big\{\pm iR\hbar|g|^2\omega_o t/2p|\tilde{w} + ga_o|\Big\}. \tag{20}$$

Particularly, at $a_o = 0$ (i.e., $|ga_o| << |\tilde{w}|$) we have

$$a(t) = \frac{\tilde{w}}{g}\Big[\exp(\pm i\frac{R\hbar}{2p}\frac{|g|^2}{|\tilde{w}|}\omega_o t) - 1\Big]. \tag{21}$$

Thus, we have a process with a beat period of $2\pi/\Delta\omega$. The maximum splitting arises at the moments

$$t_n = (n + \frac{1}{2})\frac{\pi}{\Delta\omega}, \quad n = 0, 1, 2\ldots \tag{22}$$

when it is equal to

$$|a_+ - a_-|_{max} = 2|\frac{\tilde{w}}{g}|. \tag{23}$$

It is also important to note that for the condition $|ga_o| << |\tilde{w}|$, the value $|a|$ does not depend on $|\tilde{w}|$ in the initial stage of the splitting process, when $t << 1/\Delta\omega$:

$$|a_+ - a_-| \approx \frac{R\hbar|g|}{p}\omega_o t, \qquad t << \frac{1}{\Delta\omega}, \tag{24}$$

but the phase of $a \pm$ depends on the phase of $\tilde{w}$, which gives rise to a spin resonance motion:

$$a_\pm \approx \pm\, i\frac{R\hbar g^*}{2p}\frac{\tilde{w}}{|\tilde{w}|}\omega_o t, \quad t << 1/\Delta\omega. \tag{25}$$

3. The conditions of beam splitting

Now, we have to discuss and formulate the conditions which must be satisfied in order to realize the perfect splitting of a beam. First of all, note that after splitting the beam to the value of $|a_+ - a_-|$ above the beam size σ, we can frequently accelerate the process by engaging in some kind of instability. It could be quadrupole instability as a result of the beam interaction with some dissipative elements of the chamber, but, possibly, the simplest and most efficient way to accelerate the splitting process is to create the parametric instability by switching on the RF gradient magnetic field with the frequency

$$\omega = (k \pm 2\nu_x)\omega_o;$$

the field must be switched off when the amplitudes a_+ and a_- achieve the perfect value that is necessary for separation of two polarized parts of the beam.

So, in practice, the time of splitting would be limited by the value

$$\tau_{sp\ell} \sim \frac{\gamma M\sigma}{\hbar|g|}\sqrt{\nu_x}. \tag{26}$$

Correspondingly, the necessary strength of the spin-driving RF field has the order of value:

$$|\tilde{w}| \gtrsim |g|\sigma\sqrt{\nu_x}. \tag{27}$$

The betatron tune spread $\Delta\nu_x$ must be small enough to satisfy the condition

$$\Delta\nu_x < 1/\omega_o\tau_{sp\ell}. \tag{28}$$

The spin-driving RF field must be parallel to the plane of splitting, in order not to excite the dipole x-oscillation:

$$\vec{B}_{RF} \parallel (\vec{x}, \vec{v}). \tag{29}$$

Taking into account the admissible value of the dipole amplitude x_d, we obtain the limitation on the deviation angle α of the field $\vec{B}_{RF}$:

$$\alpha \lesssim \frac{x_d}{\beta} \cdot \frac{< B >}{< B_{RF} >} \cdot \Delta\nu_d, \tag{30}$$

where β is the β-function, B is the bending field, B_{RF} is the amplitude of the RF field, the brackets $< \ldots >$ mean averaging along the closed orbit, and $\Delta\nu_d$ is the detune between frequencies of dipole and quadrupole beam oscillations.

Note also that $\vec{B}_{RF}$ should not be parallel to the direction of the periodic polarization $\vec{n}$ in the storage ring. At low energies this condition can be satisfied together with (29) by using the RF solenoid. At high energies we should use the transverse RF field; in this case, the directions of $\vec{n}$ and x must be different.

Conclusion

Thus, the conclusions are as follows:

- the Stern-Gerlach method can work in storage rings, in principle, only with application of resonant RF magnetic field;

- the necessary time for splitting is defined by the beam size; therefore, it is especially important to provide a beam size as small as possible, using some cooling technique;
- the question of how to monochromize the betatron particle motion during the splitting process is still the most crucial problem from a practical point of view.

REFERENCES

1. R. Rossmanith, Proceedings of the 8th International Symposium on High Energy Spin Physics, Minneapolis, Minnesota (September 1988), AIP Conf. Proc., V. 2, p. 1085-1092.
2. T. O. Niinikoski and R. Rossmanith, AIP Conf. Proc. **N145**, Particles and Fields Series 34.