KINETIC TREATMENT OF PARALLEL GRADIENT NONUNIFORMITY FOR ICRF HEATING - A FOURIER INTEGRAL APPROACH

D. Smithe, T. Kammash
University of Michigan, Ann Arbor, MI 48109

P. Colestock
Princeton Plasma Physics Laboratory, Princeton, NJ 08544

ABSTRACT

We study the generally nonlocal Vlasov-Maxwell wave propagation and absorption problem for an arbitrarily nonuniform plasma. The Fourier transform of the nonlocal dielectric response kernel, $\mathbf{K}(r,k)$, is constructed by integration along particle orbits in the nonuniform field. Although a finite Larmor radius expansion of the transverse particle motion still applies, the phase integrals which comprise the usual plasma dispersion function are altered, containing an additional parameter characterizing the parallel field gradient. The use of realistic phase decorrelation estimates over a single bounce orbit leads to a reduction of the phase integrals to a tractible form. We numerically solve a 1-D sheared field version of the resultant integral equation describing the mode conversion physics. Significant changes are found for small $k_{\|}$ values. In addition, local absorption in the resonance zone appears to be stratified in conjunction with the rf-particle phase correlation which occurs for particles passing through the localized resonance.

INTRODUCTION

This paper addresses two difficulties arising in the treatment of wave propagation in a nonuniform plasma possessing both parallel and perpendicular field gradients. The first, exact treatment of particle orbits in the lineraized Vlasov theory, is needed for an accurate description of the particle-wave phase integral near the ion cyclotron resonance and its harmonics. Our work furthers an endpoint expansion method previously employed by Itoh. *et al.*[1]; we analytically reduce the velocity-integrated phase to a single simple integral. In doing so it is found that collisional phase-diffusion plays an important role in preserving the analyticity of velocity-integrated phase in the uniform limit. We have adopted the methods of Kerbel and McCoy[2] and Cohen *et al.*[3] to estimate phase damping for uniform or weakly nonuniform conditions.

Second, we treat questions concerning formulation and solution of the wave equation, especially in what concerns the correct implemtation of the nonlocal plasma response. For a fixed frequcney oscillation, the linear response has a general form $\underline{J}_p(\underline{r})=\int d^3r\ e^{i\underline{k}\cdot\underline{r}}\ \boldsymbol{\sigma}(\underline{r},\underline{k})\cdot\underline{E}(\underline{k})$, where $\underline{J}_p$ is the plasma currnt, and $\underline{E}$ is the electric field in the Fourier domain. When there is shear the parallel wavenumber, $k_\parallel$, which appears in the phase integral, is no longer ignorable with respect to the nonlocal integration. To produce a differential formulation, one is forced to approximate the phase integral with a Taylor expansion about some fixed value of $k_\parallel$.[4] However, even for small values of shear, this expansion is inappropriate when the spectrum of $\mathbf{E}(\mathbf{k})$ is fairly wide, such as for rapid variation of the Bernstein wave's wavelength, and when the phase integral has significant structure. Both conditions occur when there is a parallel gradient.

We avoid this second difficulty entirely by solving the nonlocal integral equation directly. The wave equation is formulated as

$$\int d^3r\ e^{i\mathbf{k}\cdot\mathbf{r}}\ \mathbf{D}(\mathbf{r},\mathbf{k})\cdot\mathbf{E}(\mathbf{k}) = \mathbf{J}_{ant}(\mathbf{r}) \qquad (1)$$

where $\mathbf{D}(\mathbf{r},\mathbf{k})$ is the dispersion kernel, which in form resembles the dispersion tensor of uniform plasma theory. A nonsparse matrix equation is produced which is the discrete version of integral equation (1), and we invert directly for the quanities $\mathbf{E}(\mathbf{k})$.

PHASE INTEGRAL

The most important effect of the parallel gradient on the conductivity kernel, $\boldsymbol{\sigma}(\mathbf{r},\mathbf{k})$, is the alteration of the phase integral which arises from integration along the particle orbits in the nonuniform field. This integral is:

$$\int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} d\tau\ \exp\left(i \int_0^{\tau} d\tau'\,(\omega - n\Omega' - k_\parallel v_\parallel')\right)\ \exp(-v_\parallel^2/v_{Th}^2)$$

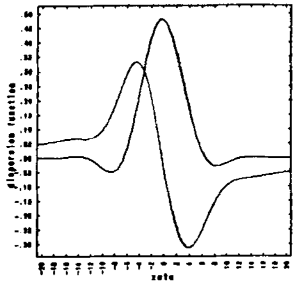
where primed quantities follow the unperturbed orbits. The values of $v_\perp$ and $v_\parallel$ change slowly, and are expanded around the endpoint, $\tau=0$, in a Taylor series, to give "quasilocal" particle orbits which depend on the local parallel gradient through the single parameter $L_\parallel^{-1} = \nabla_\parallel B/B$. The $v_\parallel$ integral can be done analytically, leaving just the integral over τ, which replaces the standard plasma dispersion function, $Z((\omega-n\Omega)/|k_\parallel|v_{Th})$. After inluding collisional phase-damping, we find its replacement to be:

$$Z(\zeta,\alpha;\gamma) \equiv i\int_0^{\infty} dx\ \exp(-x^2(1-\alpha x/2)^2/4 + i\zeta x - \gamma x^3/8),$$

where $\zeta = (\omega - n\Omega)/|k_\parallel|v_{Th}$, and $\alpha = n\Omega/k_\parallel|k_\parallel|L_\parallel v_{Th}$; and $\gamma = \nu/|k_\parallel|v_{Th}$ is the phase diffusion parameter, where ν is the ion-ion deflection frequency.

For comparison we note that the new parameter, α, measuring the parallel gradient, strongly resembles the quantity $(k_\parallel\lambda_c)^{-2}$, which Gambier and Samain[5] use to gauge parallel gradient effects in their nonlocal variational treatment. We also note analytical agreement in all limits with Itoh *et al.*'s phase integral.[1]

Plots of $Z(\zeta,\alpha;\gamma)$ versus ζ for a large negative and a small positive value of α are shown in Figure 1. In the $k_\parallel \to 0$ limit (large α) there is damping, with $Z \propto i/\sqrt{|\alpha|}$, in contrast to the uniform plasma result. For small positive α, there appears a modulation of integrated phase due to systematic particle-rf phase accumulation along the field gradient. The average value of this modulation is the uniform plasma value. As $\alpha \to 0^+$, the modulation becomes finer, and is reduced in magnitude by collisional phase-diffusion.

a) 

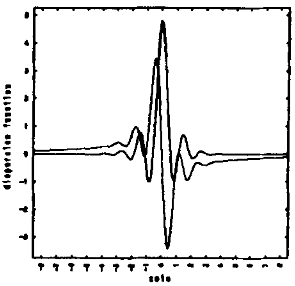
b) 

Figure 1. $Z(\zeta,\alpha;\gamma)$ vs. ζ for: a)α=-10. and b)α=.5; real part is solid line, imaginary part is dotted line.

SOLUTIONS OF THE INTEGRAL WAVE EQUATION

The phase integral is employed in wave equation (1), which in its discretized form requires $O(N^2)$ evalutions of the replacement Z-function, where N = # grid points = # Fourier components. In our 1-D calculations we use N≈100. For sheared field runs we used $B_p/B=1/10$. Several series of runs were made showing the wavenumber spectrum, the values $E_r(x)$, and local energy fluxes for TFTR-like conditions (f=60MHz, B_o=40kG, $n_o=4\times10^{13}cm^{-3}$, T=5keV, $k_y=0m^{-1}$, R_{maj}=3m, a_{wall}=1m, 95%D-5%H minority heating, or 100%D 2nd harmonic heating). Individual modes were easily identifiable on the wavenumber spectrum. When there was strong cyclotron damping at positive k_z, a highly damped local oscillation was barely visible in the E_r plot around the cyclotron resonance, with wavelength corresponding to the replacement Z-function's modulation for Hydrogen at the prominent $k_\parallel$'s of the fast wave. The local energy balances showed stratification of the energy deposition, indicative of a local standing wave at resonance. Our energy-like quantities derive from extrapolation of self-adjoint methods applied to the shear-free geometry. Here we point out that the correct distinction between reactive and dissipative power in nonlocal systems is still an active area of inquiry.

In any case, the form of local energy quantities does not affect the validity of the scattering coefficients which are calculated in the asymptotic regions. Figure 2 compares the scattering coeffients versus k_z of a minority heating outside launch scheme, with and without shear, for the above parameters. Transmission appears to be systematically shifted in k_z, caused simply by the relation $k_\parallel=b_Tk_z+b_pk_x$. Reflection and Absorption are altered in more complicated fashion, with significant absorption at $k_\parallel\approx0$ being clearly evident. Comparisons were also made for minority heating inside launches, and 2nd harmonic heating, both inside and outside launches. The implication from this study, for experiments, is that the ion cyclotron resonance is more absorptive than predicted by gradient-free theory, thus relaxing the constraints on $k_{\parallel}$ for adequate wave damping.

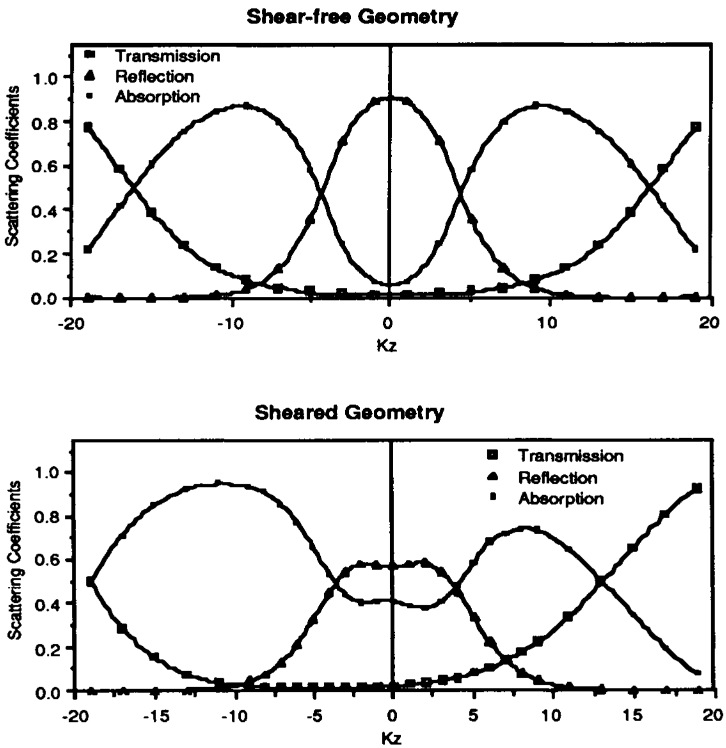


Figure 2. Scattering coefficients vs. k_z for a minority heating inside launch scheme, in shear-free and sheared geometries. Plasma parameters are given in text.

REFERENCES

1. S.-I.Itoh, A.Fukuyama, K.Itoh, and K.Nishikawa, Journal of the Physical Society of Japan 54, 1800 (1984).
2. G.D.Kerbel and M.G.McCoy, Comp. Phys. Comm. 40, 105 (1986).
3. B.I.Cohen, R.H.Cohen, and T.D.Rognlien, Phys. Fluids 26, 808 (1982).
4. A.Fukuyama, K.Itoh, S.-I.Itoh, Comp. Phys. Reports 4, 137 (1986).
5. D.J.Gambier and A.Samain, Nuclear Fusion 25, 283 (1985).