RARE K and μ DECAYS: THEORETICAL AND EXPERIMENTAL STATUS AND PROSPECTS

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I. THEORETICAL CONSIDERATIONS

At present there is a remarkably successful standard model of electroweak and strong interactions. There are no contradictions between experiment and theory! Nevertheless, there are many questions which the standard model does not answer, questions of the "why" kind. The origin of quark and lepton masses is not understood, nor is the reason for generations , or how many of these there are. In the most fundamental sense, it is hoped that rare decays will occur that are not required by the standard model (that is, their rates will be nonzero if predicted to be zero in the standard model, or will differ significantly from a nonzero standard model prediction). If indeed this happens, one may expect that it will provide important clues to help answer the open questions.

There are two classes of decays. Some are expected to occur at some level in the standard model (henceforth denoted as SM), and provide tests of this model. Some examples of these are given in Table 1. New clues could arise from finding rates or other observables which differ from those predicted by the SM. For the second type of decays, the SM prediction is zero--they do not occur at any level. Finding them at all would demonstrate the existence of new interactions. For the various processes to be discussed below, these SM predictions will be mentioned; they are summarized in the first line of Table 1.

Some of the decays might give results different from the SM because of the existence of new interactions, or equivalently, the exchange of new particles. Others might be of interest as a way to discover neutrino masses because of modified kinematical distributions, or a decay into a new kind of particle.

In this section we shall provide a brief description of some ideas that might lead to observable new effects. While many of the ideas suggest that new effects should occur, they do not require that they occur at any specified level. Sometimes particular models may predict an effect, but at the present level of understanding, one can usually find another model which predicts a smaller or larger effect. Rather than trying to provide complete and detailed references in the proceedings of a workshop such as this, we shall only mention a few reviews from which the literature can be traced.[1] Partly this is because of the perspective that the ideas discussed do provide good motivation that some effects should occur somewhere, but no particular idea should be interpreted to imply that a specific prediction (beyond the SM) is more than suggestive.

TABLE I

Decay or Process / Motivation & Physics Issue & Process	$K^+\to\pi^+\mu^+e^+$ $K^o_L\to\mu^\pm e^\mp$ (II)	$K^o_L\to\left\{\begin{matrix}\mu^+\mu^-\\ e^+e^-\end{matrix}\right.$ (II)	$K^+\to\left\{\begin{matrix}\pi^+e^+e^-\\ \pi^+\mu^+\mu^-\end{matrix}\right.$ (II)	$K^+\to\pi^+$ + Missing Neutral(s) (III)	$K^+\to\left\{\begin{matrix}\mu^+\nu_i\\ e^+\nu_i\end{matrix}\right.$ $\pi\to\left\{\begin{matrix}\mu^+\nu_i\\ e^+\nu_i\end{matrix}\right.$ ν_i Heavy, Subdominantly coupled
1) Predicted by Standard Model	N	Y	Y	$\nu\bar{\nu}$: Y other: N	N
2) ν Masses, Heavy Leptons	negligible unless very heavy	N	N	Y, P	Y
3) Lepton family number violation	Y	N	N	N	N
Total Lepton number violation	N	N	N	N	N
4) e–μ Universality violation	N	Y	Y	N	Y
5) Horizontal Interactions	Y	Y	Y	Y, P	N
6) Leptoquark Interactions	Y	Y	Y	Y	N
7) Flavor-changing Higgs	Y	Y, P	Y,P	Y,P	Y
8) Technicolor	Y	Y			
9) Supersymmetry	Y	Y, P	Y,P	Y,P	N
10) Axions,	N	N	N	Y	N
11) Composite Models process	Y	N	N	N	N

Y(N): Physics of type (1)-(11) causes or influences (does not cause or significantly influence) the given process

P : While caused or influenced by the physics of type (1)-(11), process constitutes only a poor probe for it

TABLE I (continued)

$\pi^+\to\mu^+\nu_2$ $K^o_L\to\pi^\pm\mu^\mp\overset{(-)}{\nu}_2$ $K^+\to\pi^o\mu^+\nu_2$ (V)	$K^o_L\to K^\pm e^\mp\overset{(-)}{\nu}_\ell$ (VII)	$K^+\to\ell^+\nu_\ell\gamma$ $\pi^+\to\ell^+\nu_\ell\gamma$ ℓ + e, μ (VII)	$K^+\to\pi^+\gamma\gamma$ $K^o_L\to\gamma\gamma$ (VII)	a $\pi^+\to\pi^o e^+\nu_e$ b $\pi^o\to e^+e^-$ c $\pi^o\to\gamma\gamma\gamma$ d $\pi^o\to$ Missing Neutrals (VII)	Regular decay ρ,η,ξ,δ (VI), (VIII)	$\mu^+\to e^+\gamma$ $\mu^+\to e^+e^+e^-$ $\mu^+\to e^+\gamma\gamma$ $\mu^-N\to e^\pm N'$ (VIII)
1) Y, for $m(\nu_2)=0$	Y	Y	Y	a,b :Y; c,d: N	Y	N
2) Y	N	N	N	a,b,c: N; d: Y	Y	Y if very heavy
3) N	N	N	N	N	N	Y
4) Y	N	N	N	N	Y	Y
5) N	N	N	N	a,c: N, b,d: Y	Y	Y
6) N	N	N	N	N	N	N
7) Y, P	N	N	N	a,Y,P; b,c,d:N	Y	Y
8) N	N	N	N	N	N	Y
9) N	N	N	N	a,b,c:N:d:Y,P	Y	Y
10) N	N	N	N	N	N	N
11) N	N	N	N	N	N	Y

At the most fundamental level, one must recognize that there is no understanding of the fermion mass spectrum. Such an understanding would, for example, entail an ab initio calculation of the mass ratio m_e/m_μ , ideally to an accuracy comparable with the accuracy of 3 parts per million with which it is known or, at least, to a few per cent. Instead, all one has is results such as the relation[2] $m_e/m_\mu = m_d/m_s$, which is not based on any calculation, but rather on an ad hoc choice of Higgs representations within a specific GUT group, SU(5), and in any case is wrong by an order of magnitude! In the absence of such an understanding, it is not valid to infer that neutrinos have zero masses, just because their upper mass limits are small compared to the masses of their corresponding charged leptons. Indeed, for at least one "known" neutrino, ν_τ , the present upper limit on its mass, viz., 250 MeV, is not even small on a particle physics scale. There are two points of view that would actually lead one to suspect that neutrino masses may be nonzero. First, in the context of gauge theories, it is expected that a symmetry is needed to have a zero-mass particle. Such a symmetry could be a gauge or chiral symmetry, or a spontaneously broken (continuous) global symmetry. None of these necessarily applies to neutrinos. Secondly, sometimes models suggest that neutrino masses should be nonzero-- one gets such effects in some grand unified theories, some horizontal symmetry theories, and in various other models. Finally, of course, one never measures any mass to be exactly zero, and, as noted above, the upper limit on at least one neutrino mass is not very small.

Proceeding to another area of theoretical model building, we note that there is no understanding at all of the meaning of flavor. If the lesson of the past decade that one should try to interpret particle physics in terms of gauge theories is relevant, then a local (or conceivably, a global) horizontal symmetry may be applicable. Then, horizontal gauge bosons or Higgs bosons, or possibly Goldstone bosons, may cause flavor transitions such as $s \leftrightarrow d$, $c \leftrightarrow u$, $b \leftrightarrow s$, $t \leftrightarrow c$, $\mu \leftrightarrow e$, $\tau \leftrightarrow e$, $\tau \leftrightarrow \mu$, etc. These could induce decays that are zero in the SM, or modify the rates for other decays.

In the SM, the SU(2)×U(1) symmetry is broken, and masses are introduced, by coupling new scalar bosons (the Higgs bosons) to gauge bosons and fermions. The ad hoc nature of this procedure, and the difficulty of dealing with fundamental scalar bosons, whose masses are normally sensitive to the highest mass scale present in the theory, has led to several new approaches, particularly to technicolor and supersymmetry models.(Supersymmetry is a possibility independent of this problem and, in its local realization as a gauge symmetry, may help one to understand quantum gravity, but some of the present interest in it, at least among particle physicists, derives from the above-mentioned connection.) Technicolor basically assumes that rather than having fundamental scalar bosons, one should introduce new fundamental fermions and a new gauge interaction (viz., techicolor) analogous to color QCD. Then the bound state "pions" of the new force are the "Higgs bosons" of the SM. The mechanism by which gauge bosons get mass is different from that for fermions in this approach, the latter requiring yet another interaction with massive bosons, namely the so-called "extended technicolor" or ETC interaction.

These new bosons have masses which are constrained to give the known fermion masses. At the same time, the new bosons will couple different flavors together, unless symmetry constraints are imposed, thus potentially yielding flavor-changing decays. Since masses of the new bosons are not free parameters, one can estimate the expected rates for decays such as $K_L^o \to \mu^\pm e^\mp$, and typically, one gets rates of the order of (or, indeed, larger than) present limits. Technicolor theories include as bound states both neutral pseudoscalar bosons and leptoquark bosons that can also mediate neutral flavor-changing transitions. The rates expected from both of these sources are again at about the level of the experimental limits. While it could happen that symmetries reduce the expected rates considerably, the general interpretation is that substantial motivation for anticipating rare decays is provided by technicolor ideas, particularly since the mass scales are not very flexible in such theories.

A different approach to dealing with scalar (Higgs) bosons is that of supersymmetry, where the scalars are just as fundamental as the fermions. One should recognize that, despite some attempts to hybridize supersymmetry and technicolor (e.g., supercolor), the basic philosophies underlying the two theories are fundamentally antithetical. The uneasy coexistence of both approaches in the present mélange of theoretical speculations affords some evidence of just to what extent these speculations are simply groping in the dark. From the point of view of rare decays, there are two kinds of possible implications. In supersymmetric theories, every known particle has a partner differing in spin by ½ unit. Needless to say, none of these additional particles has been observed in any existing data. Some of the particles might be light (e.g., the photino, $\tilde{\gamma}$, the partner of the photon) and occur as final state particles in various decays such as $K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}$. Alternatively, supersymmetric partners might occur internally in Feynman diagrams and thereby generate additions to several of the rare decays. As always, this may be avoided or highly suppressed by imposing certain relations among masses and/or couplings, but given the mass scales expected in supersymmetric models, it would not be surprising if rare decays were induced.

Supersymmetric and technicolor approaches are, as emphasized above, quite different ones since the former treats scalars as fundamental while the latter treats them as composite. If rare decays are found, it may be possible to distinguish between these approaches (and others to be mentioned) by studying the pattern of induced decays, the form of the induced decay currents, and other observables.

If axions or Goldstone bosons are produced in any model, they can occur as external particles X in the decay $K^+ \to \pi^+X$. This decay would give a two-body peak in what would otherwise be a three-body decay distribution (aside from background from $K^+ \to \pi^+\pi^o$). The normal axion, originally introduced as part of an effort to eliminate strong CP violation, would already have been observed in this decay if it had the standard Peccei-Quinn-Weinberg-Wilczek couplings and mass. Its coupling can be reduced in various models, such as the "invisible" axion models. Speculative possibilities such as these, or Goldstone bosons that couple one flavor to another (see the talk by Wilczek) could occur.

The proliferation of supposedly fundamental quarks and leptons, and the inability of current theory to provide any explanation or understanding of the masses of these particles, has led many physicists to question whether indeed these fermions are fundamental, and to investigate the implications of their possible composite structure. This line of reasoning is very natural in view of the historical development of physics, in which, as one probed to higher and higher energies and shorter and shorter distances, level after level of purportedly fundamental matter was found to be composite. The false conclusions of generations long dead are even enshrined in common terminology -- classic examples are Democritus' "atom" from the Greek "ατομος", meaning indivisible, and "proton" from "προτος" meaning first or most fundamental. It is a staggering hubris indeed (although presently consistent with all available data) to assume that somehow we are more fortunate than all our ancestors, that we have reached the final, most basic, interactions and constituents of matter, beyond which there are no others. This attitude is exemplified by the famous desert hypothesis, according to which there is no new physics between $\sim 10^2$ GeV and the grand unification scale of $\sim 10^{14}$ GeV. There are several mechanisms in composite models which can induce rare decays. In such models one can rearrange constituents and get bosons with leptoquark quantum numbers.[3] These can include vector bosons whose couplings are not suppressed by factors of the form $(m_{fermion}/m_{vector\ boson})$ and whose exchange could induce rare decays at interesting levels, unless certain symmetries are imposed. Excited q^* or ℓ^* states can occur in loop diagrams with non-diagonal couplings and hence produce rare decays. Constituent rearrangement can occur and yield effective four-fermion neutral, flavor-changing interactions.

In the above remarks we have provided a short introduction to some of the motivations for searching for rare decays. It should be remembered that the basic motivation is simply to find new interactions in nature, to help integrate masses and flavors into the theory, and to understand why the standard model takes the form that it does. We next proceed to give more detailed discussions of specific decays.

II. THE DECAYS $K^+ \to \pi^+\mu^+e^-$ and $K_L^o \to \mu^\pm e^\mp$

These decays are of interest because they violate lepton family number and can probe possible new physics in the multi-TeV mass range. The present upper bounds on their branching ratios are[4]

$$B(K^+ \to \pi^+\mu^+e^-) < 5 \times 10^{-9} \quad (90\%\ CL) \tag{2.1}$$

and

$$B(K_L^o \to \mu^\pm e^\mp) < 6 \times 10^{-6} \quad (90\%\ CL) \tag{2.2}$$

(In the latter case a more stringent bound $B(K_L^o \to \mu^\pm e^\mp) < 2 \times 10^{-9}$ was reported by an old LBL experiment[4] but is not included in the Particle Data Tables because of uncertain systematic errors in the experiment.) The limit

$$B(K^+ \to \pi^+\mu^-e^+) < 7 \times 10^{-9} \text{ (90\% CL)} \tag{2.3}$$

has also been achieved.

There are now two experiments approved to search for the decays (2.1) and (2.2) at the Brookhaven AGS. A Yale-BNL-Seattle collaboration (E777, M. Zeller, spokesman) will search for the decay $K^+ \to \pi^+\mu^-e^+$ with an estimated sensitivity of $\sim 10^{-11}$ in branching ratio.[5] It will use a 6 GeV/c beam of K^+'s and a detector with a gas Čerenkov counter, e^+ calorimeter, μ^+ range counter, and very good particle identification. The main background is from $K^+ \to \pi^+\pi^+\pi^-$ where one of the π^+'s decays to $\mu^+\nu_\mu$ and the π^- is misidentified as an e^-. This background is expected to produce spurious events at the level of a few $\sim 10^{-12}$ in branching ratio. Secondly, a Yale-BNL collaboration (E780, M. Schmidt and W. Morse, cospokesmen) will search for the decay $K_L^o \to \mu^+ e^-$ down to an anticipated level of 10^{-10} in branching ratio.[6,7] This group utilizes minidrift chambers with $\sim 200\,\mu$ resolution. The main background is from $K_L^o \to \pi^+e^-\bar{\nu}_e$ where the π^+ decays to $\mu^+\nu_\mu$ in the spectrometer magnet and comes in at a level $\sim 2 \times 10^{-11}$ in branching ratio. Both of these experiments will be on line in early 1985, or possibly earlier, and should have first results by early 1986. They would both benefit from the cleaner environments which would be possible with more intense beams, and it has been roughly estimated[7] that with requisite improvements in detectors, one might be able to achieve sensitivities of 10^{-12} for $B(K^+ \to \pi^+\mu^+e^-)$ and 10^{-11} for $B(K_L^o \to \mu^+e^-)$ if such experiments were performed with better beams.

There are a number of models that could give rise to effective leptonic flavor-changing neutral current (LFCNC) decays such as these. These include:

(1) an extended electroweak theory with flavor-changing Higgs bosons; (2) (extended) technicolor; (3) horizontal generation-changing interactions; (4) baryon - and lepton - violating low-mass Higgs in grand unified theories (GUT's); (5) some supersymmetric GUT's; (6) preon models; and (7) theories with very heavy neutral leptons. The theoretical motivations for these various models have been briefly described above. The contributions of the decay mechanisms are, of course, constrained by the requirement that they do not produce unacceptably large $K^o - \bar{K}^o$ mixing, $K_L^o \to \mu^+\mu^-$ decay rate, and so forth for known processes. As a rough indication of the type of limit that one might set, consider a theory with horizontal gauge interactions, involving V or V-A couplings to fermions. Let $m(V_h)$ and g_{Vh} denote the mass and gauge coupling constant in such a theory. Further, denote the gauge coupling of the SU(2) factor in the standard model as g. Then in the absence of GIM-type cancellations, one obtains the lower limits[8]

$$m(V_h) \gtrsim 20 \text{ TeV} \left(\frac{5 \times 10^{-9}}{B(K^+ \to \pi^+\mu^+e^-)} \right)^{1/4} \left| \frac{g_{V_h}}{g} \right| \tag{2.4}$$

and

$$m(V_h) \gtrsim 38 \text{ TeV} \left(\frac{2 \times 10^{-9}}{B(K_L^o \to \mu^+e^-)} \right)^{1/4} \left| \frac{g_{V_h}}{g} \right| \tag{2.5}$$

Thus, if the two current experiments at BNL do not detect any signals down to their anticipated levels of sensitivity, $B(K^+ \to \pi^+\mu^+ e^-) \sim 10^{-11}$ and $B(K_L^o \to \mu^+e^-) \sim 10^{-10}$, one may infer the bounds $m(V_h) \gtrsim 90$ TeV and $\gtrsim 80$ TeV, respectively. It should be stressed that these limits are highly model-dependent and are given only as a rough indication of the scale of vector boson masses that may be probed within the next few years. For example, if the horizontal gauge bosons coupled in a purely vectorial manner to quarks, then they would not contribute to the decay $K_L^o \to \mu^+e^-$, which involves only an axial-vector hadronic current. On the other hand, if these horizontal gauge bosons coupled in a purely axial-vector manner to the quarks, then they would not contribute to the decay $K^+ \to \pi^+\mu^+e^-$, which proceeds via the vector hadronic current. In passing, we note that in this class of theories, the $|\Delta G| = 2$ decay $K^+ \to \pi^+e^+\mu^-$ would be suppressed relative to the $\Delta G = 0$ decay $K^+ \to \pi^+\mu^+e^-$, (Here, G denotes generation, with $G = 1$ for $\{u,d,\nu_e, e\}$, $G = 2$ for $\{c,s,\nu_\mu,\mu\}$, etc.) Both of the decays $K_L^o \to \mu^+e^-$ and $K_L^o \to \mu^-e^+$ contain $\Delta G = 0$ parts as well as $|\Delta G| = 2$ parts. The possibility that there may be such horizontal gauge interactions mediated by vector bosons with masses in the range accessible to the current BNL experiments, viz., ~ 100 TeV (for typical theories with $g \sim g_{Vh}$), is an intriguing one. However, there is no particular reason to expect that this situation actually obtains in nature. Similarly, there is no particular reason why effective LFCNC interactions arising in SSGUT's or preon models should be characterized by a mass scale of order 1-100 TeV. Very heavy neutral leptons with masses greater than $\sim$ 1GeV, which might couple virtually in loop diagrams contributing to these rare K decays are certainly possible, but again, there is no strong motivation for supposing that they actually exist. (Of course, before the discovery of the muon there would not have been any theoretical motivation for supposing that it existed either, and there is still no theoretical understanding of why it and the other fermions of the second and third generations exist.) Moreover, heavy neutrinos with masses $\lesssim$ 350 MeV are constrained to have very small couplings to e and μ.[9,10] There are other indirect constraints on the couplings of heavy neutrinos with masses above m_K which again severely limit their contribution to LFC K decays.[11,12] Thus, it is unlikely (although possible) that category (7) would yield leptonic flavor-changing K decays at levels which are experimentally accessible at a foreseeable kaon factory.

In contrast, the TeV mass scale is probably a natural one for Higgs bosons in electroweak theories, and, in particular, flavor-changing Higgs. Similarly, the effective FCNC interactions resulting from technicolor models are typically characterized by mass scales of order 10-100 TeV.[13-15] We suspect however, that specific claims such as the claim[15] that "If at the quark vertex the ETC interactions are not purely vectorial, as is expected because of the u-d mass splittings, then the decay $K_L^0 \to \mu^+e^-$ should probably show up within an order of magnitude below the present experimental limit. If the ETC interactions are not purely axial at the quark vertex, then the decay $K^+ \to \pi^+\mu^+e^-$ should probably show up within two orders of magnitude of the present upper limit," are somewhat too strong because of the severe model dependence involved, and the very speculative nature of the models themselves. Unfortunately, the models being discussed here are considerably less well-formed, definite, and predictive than the standard electroweak theory. Thus, there are not yet firm predictions that such-and-such a decay will occur at a given level in branching ratio. The situation is, rather, that there are theoretical motivations for considering a number of various models which go beyond the standard "low" -energy SU(3)×SU(2)×U(1) theory; and within certain of these models, FCNC decays such as the two possible K decays under discussion here <u>may</u> occur at rates which are within range of planned or future experiments.

An experiment that searches for these rare decays can also measure and/or search for other related conventional decays. Specifically, an experiment looking for $K^+ \to \pi^+\mu^+e^-$ should obtain a good sample of $K^+ \to \pi^+e^+e^-$ events, since $B(K^+ \to \pi^+e^+e^-) = (2.7 \pm 0.5) \times 10^{-7}$, four orders of magnitude greater than the expected sensitivity of the experiment. This large sample would enable one to study the e^+e^- invariant mass spectrum and search for peaks such as would be due to $K^+ \to \pi^+ H$; $H \to e^+e^-$, where H denotes a generic Higgs boson. It is true that the "natural" value for the Higgs mass is one comparable to its vacuum expectation value, $\sim$ 250 GeV, but light Higgs are still possible, at a phenomenological level. This study of the e^+e^- spectrum could also yield some further information concerning the decay mechanism.[16] In addition, one could search for the decay mode $K^+ \to \pi^+\mu^+\mu^-$, for which there is only the upper limit $B(K^+ \to \pi^+\mu^+\mu^-) < 2.4 \times 10^{-6}$. This decay will certainly occur via one-loop electroweak diagrams involving emission of a virtual photon which creates the $\mu^+\mu^-$ pair.[16] The branching ratio is suppressed significantly relative to that for the $K^+ \to \pi^+e^+e^-$ mode because of the reduced phase space available. (However, there is no photon propagator suppression, because the $1/q^2$ from this propagator is cancelled by the q^2 from the quark part of the diagram, which consists dominantly of a nondiagonal charge-radius term. A priori, one would also be interested in examining the $\mu^+\mu^-$ invariant mass distribution to search for a Higgs boson which was massive enough to decay to a $\mu^+\mu^-$ pair. However, the existence of a massive Higgs boson with standard couplings to fermions has been ruled out for $m_H < 409$ MeV by a recent experiment on the decay $\eta' \to \eta\mu^+\mu^-$ which made the analogous search for a peak in the $\mu^+\mu^-$

invariant mass distribution.[18] This result forbids the decay $K^+ \to \pi^+ H$; $H \to \mu^+\mu^-$. Concerning present experiments, the Yale-BNL group does plan to study the $K^+ \to \pi^+ e^+ e^-$ decay mode and measure the e^+e^- invariant mass spectrum from 140 MeV to 350 MeV. (It does not plan to search for the decay $K^+ \to \pi^+\mu^+\mu^-$ because of the very severe background from the decay $K^+ \to \pi^+\pi^+\pi^-$, where a $\pi^+\pi^-$ pair is misidentified as $\mu^+\mu^-$.)[5]

Similarly, an experiment which searches for the decay $K_L^o \to \mu^\pm e^\mp$ can amass a sizeable sample of $K_L^o \to \mu^+\mu^-$ events. This decay has long been of interest as a higher-order induced neutral $|\Delta S| \neq 0$ weak process.[19] Like K^o-$\bar{K}^o$ mixing and other one-loop induced rare K decays, it is sensitive to the couplings of heavy quarks. In particular it provides constraints on the quark mixing matrix coefficients that determine the strength of the charged current couplings of these heavy quarks to d and s.[20] The present data on this decay comes from the three experiments by Columbia-BNL[21] , Princeton-BNL[22], and Chicago-FNAL[23] groups and consists of 27 events. A sample of $\sim 10^3$ events ought to be obtainable in a current experiment. This would obviously yield a commensurately more accurate measurement of the branching ratio and, in addition, could render feasible a measurement of the muon polarization. The latter might be of some interest as a probe of the decay mechanism: if the conventional two-photon intermediate state and similar long-distance contributions really dominate the amplitude,[24] then this polarization, $P_\mu = 0$. This is also true for the part of the amplitude arising from the short-distance, quasi-free quark diagrams.[25] However, it is conceivable that FCNC interactions with effective S,P Lorentz structure could contribute significantly to $K_L^o \to \mu^+\mu^-$ while not giving too large a rate for $K^o - K^o$ mixing. This would require rather artificial fine tuning of certain couplings (the coupling to eμ would have to be much larger than the coupling to sd), but is nonetheless possible. Through interference effects, one can then obtain a nonzero muon polarization.[26] Although unlikely, it is possible that this might be large enough to measure in a dedicated experiment.

An experiment on $K_L^o \to \mu^+e^-$ could also search for the decay $K_L^o \to e^+e^-$, for which the present limit upper limit is $B(K_L^o \to e^+e^-) < 2.0 \times 10^{-7}$ (90% CL). To estimate a lower limit on the branching ratio for this mode, one may simply take the contribution of the two-photon intermediate state to the absorptive part of the amplitude, which gives[24]

$$\frac{B(K_L^o \to e^+e^-)_{2\gamma\ \mathrm{abs}}}{B(K_L^o \to \mu^+\mu^-)_{2\gamma\ \mathrm{abs}}} \simeq \frac{3 \times 10^{-12}}{6 \times 10^{-9}} \qquad (2.6)$$

The real part of the amplitude is harder to estimate. The short distance quasi-free quark contributions to both the real and imaginary parts of the amplitude, in the standard model, give rates which scale like

$$\frac{B(K_L^o \to e^+e^-)_{f.q.}}{B(K_L^o \to \mu^+\mu^-)_{f.q.}} = \frac{m_e^2}{m_\mu^2} \left(\frac{1 - \frac{4m_e^2}{m_\pi^2}}{1 - \frac{4m_\mu^2}{m_\pi^2}} \right)^2 = 3.5 \times 10^{-5} \tag{2.7}$$

Long-distance contributions to the real part are also present at some level, although arguments have been given that they are small.[25] The suppression factor (2.6) is due to the vector nature of the electromagnetic couplings in the $2\gamma_{virtual}$ diagram, while that in (2.7) is due to the fact that the quasi-free quark amplitude takes the form

$$\{<0|\bar{s}_L\gamma_\lambda d_L|K^o> + <0|\bar{d}_L\gamma_\lambda s_L|\bar{K}^o> \} \; [\bar{u}_\mu\gamma_\lambda \; (a + b\gamma_5)v_\mu]$$

$$\sim \text{const.} \quad 2m_\mu \; b \quad [\bar{u}_\mu\gamma_5 v_\mu] \quad . \tag{2.8}$$

Here the s-channel Z-exchange and WW box graphs contribute to both a and b in the quark amplitude, but the s-channel γ-exchange contributes only to a. As the a term does not enter in the physical amplitude, the effective lepton current is purely axial-vector. If one assumes that this effective (V,A) leptonic current also applies to the real part, then

$$B(K_L^o \to e^+e^-) = \frac{B(K_L^o \to \mu^+\mu^-)}{B(K_L^o \to \mu^+\mu^-)_{2\gamma\ abs}} \times B(K_L^o \to e^+e^-)_{2\gamma\ abs}$$

$$\simeq 5 \times 10^{-12} \tag{2.9}$$

This is below the level of 10^{-10} in branching ratio to which the present Yale-BNL experiment is statistically sensitive. However, precisely because of this standard-model suppression of the ee mode, it serves as a good probe for nonstandard contributions. For example, if there is an effective FCNC interaction with (S,P) Lorentz structure, then the size of the $e\bar{e}$ mode might be increased, relative to that of the $\mu\bar{\mu}$ mode. It should be noted, however, that if this interaction arises directly from Higgs exchange, then the $(m_e/m_\mu)^2$ suppression factor might very well be reinstated, since Higgs bosons typically (although not necessarily) couple to fermions with strengths $\sim (m_f/m_V)$, where m_V denotes a generic vector boson mass in the theory. Thus, even if such additional interactions were present, they might well be suppressed for the $e\bar{e}$ mode in a manner much like that in the standard model case. Nevertheless, it is clearly possible and very worthwhile to search for this decay; a positive signal at a level significantly higher than that expected in the standard model would indicate new physics, while a null result down to $B(K_L^o \to e^+e^-) \sim 10^{-10}$ would still constitute a great improvement over the existing upper limit.

III. THE DECAY $K^+ \to \pi^+$ + MISSING NEUTRAL(S)

(A) The Case of One Missing Neutral Particle: $K^+ \to \pi^+X^o$

This subsumes the actual decays $K^+ \to \pi^+H$, π^+a, or π^+f, where H, a, and f denote a Higgs boson, axion,[27] and familon,[28] respectively, as well as any other generic neutral spin-0 boson. It is necessary that the X^o not decay in a visible manner in order for the observed signal to be of this type. A recent experiment at KEK has set the very stringent upper limit[29]

$$B(K^+ \to \pi^+ + \text{axion}) < 3.8 \times 10^{-8} \quad (90\%\ \text{CL}) \tag{3.1}$$

for $m_a << m_\pi$. This upper limit is substantially smaller than most calculations of the branching ratio due to the standard (light) axion and almost certainly rules it out.[30] The mass of the standard axion is[27]

$$m_a \sim 150\ \text{KeV}\ \left(\frac{n}{3}\right)\left(\frac{x + x^{-1}}{2}\right) \tag{3.2}$$

where n denotes the number of quark generations and x denotes the ratio of vacuum expectation values of the two Higgs fields in the Peccei-Quinn model. Thus, if, as might be considered natural, x is not too far from unity, then $m_a \sim 150$ KeV. However, if $x << 1$ or $x >> 1$, then m_a might be large enough not to have produced a peak in the π^+ momentum spectrum at the position $(m_K/2)(1 - m_\pi^2/m_K^2) \simeq$ 229 MeV looked for by the experiment of Ref. 29. It might still have been detected if it decayed sufficiently rapidly to $\gamma\gamma$, since this experiment also compiled data on the decay mode $K^+ \to \pi^+\gamma\gamma$.[31]. Obviously, if $m_a \simeq m_{\pi^0}$ then the $\gamma\gamma$ decay signal would be swamped by far more numerous events from $K^+ \to \pi^+\pi^o$; $\pi^o \to \gamma\gamma$. In any case, theorists have long since invented the "invisible" axion whose coupling to matter is $\sim g(m_f/M)$ where $M \sim 10^{12}$ GeV, rather than $g(m_f/m_W)$ as for the original axion, and hence is strongly suppressed.[32]

In addition to the possibility of heavy axions, there is also the possibility of a Higgs boson in the mass range where it could be emitted in a decay of the form $K^+ \to \pi^+H$. A more recent KEK experiment[33] has searched over a wide mass range for such a light Higgs, or more generally X^o, with no visible decay, by means of looking for the peak which it would cause in the π^+ momentum spectrum. This experiment has established a correlated upper limit on $B(K^+ \to \pi^+X^o)$, as a function of m_{X^o}, which varies from $\sim 10^{-6}$ for $m_{X^o} \lesssim 75$ MeV to

$\sim 10^{-5}$ for 170 MeV < m_{X^o} < 260 MeV. For $m_{X^o} \sim m_{\pi^o}$ the limit obtained is less stringent because of background from the decay $K^+ \to \pi^+\pi^o$, where the two photons from the π^o decay are not detected. A Higgs boson with a mass in the range < $2m_\mu$ would normally decay dominantly to e^+e^- with a lifetime short enough to be detectable. However, one should keep in mind the possibility that for an interesting range of masses, the Higgs boson could decay primarily in an invisible way, viz., $H \to aa$, hh, MM, or $\nu\bar{\nu}$, where h and M denote a lighter Higgs and a Majoron, respectively, and ν denotes a massive neutrino.[34] Similar comments apply to a heavy axion or familon.

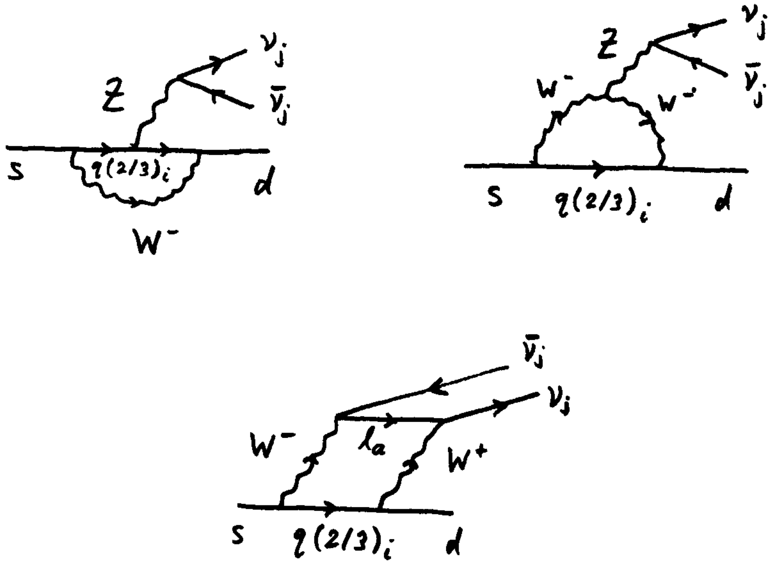
As with the other decays surveyed herein, the limits achieved by these KEK experiments on the decay $K^+ \to \pi^+ X^o$ involving a light or massive X^o could be improved by further detector developments and greater statistics.

(B) The Case of Several Missing Neutral Particles

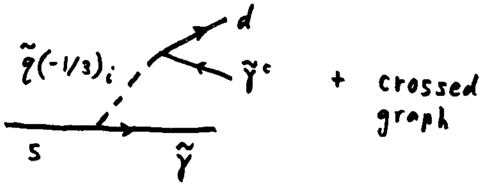
The decay $K^+ \to \pi^+$ + missing neutrals would be the observed signal resulting from one definite standard-model source,[16]

$\sum_{i=1}^{3} K^+ \to \pi^+ \nu_i\bar{\nu}_i$, and from a number of possible sources involving new physics. Like some other rare K decays such as $K_L^o \to K^{\pm}e^{\mp}(\bar{\nu})_e$, this mode has the advantage that since it occurs in the standard model, if one can perform an experiment with the required sensitivity, one can expect to see a signal rather than just obtaining a null result, as might happen for the lepton flavor-violating decay modes. However, it has two important disadvantages. First, given the uncertainties in the standard-model prediction, even if a signal is seen, it is not clear at what level one will be able to claim that one has seen new physics. This is in sharp contrast to the situation for the decays $K^+ \to \pi^+\mu^+e^-$ and $K_L^o \to \mu^+e^-$, for which the observation of a nonzero signal by itself constitutes new physics beyond the standard model. Secondly, even if the observed rate differs enough from the standard model so that one can convincingly make a case for new physics, it will not in general be possible to determine _what_ this new physics is! These are significant disadvantages to weigh against the possibility of not seeing any signal in the lepton flavor-violating decay modes.

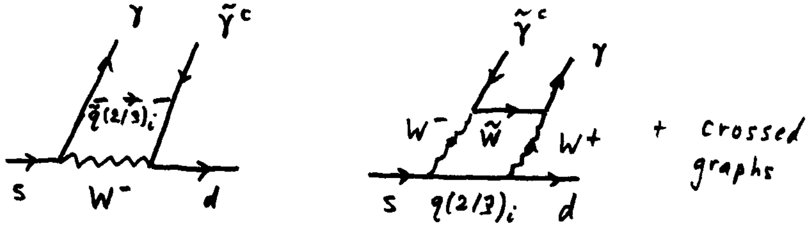
The conventional source $\sum_{i=1}^{3} K^+ \to \pi^+\nu_i\bar{\nu}_i$, gives[16,35-38] $B(K^+ \to \pi^+ + \text{missing neutrals}) \sim \text{few} \times 10^{-9}$ to 10^{-10}. There are uncertainties in the calculation having to do with the mass of the t-quark, which is currently unknown, and the associated weak couplings V_{td} and V_{ts}, for which approximate bounds have been derived. (Of course, the existence and mass of the t-quark may be established directly at PETRA or inferred indirectly from W and/or Z decay data from the CERN $\bar{p}p$ collider in the near future.) Equally important, there is an uncertainty arising from the parametrization of the q^2-dependence in the form factors for the hadronic matrix element.



(a) Graphs for the decay $K^+ \to \pi^+ \nu_j \bar{\nu}_j$



(b) Tree-level graph for $K^+ \to \pi^+ \tilde{\gamma}\tilde{\gamma}$



(c) Representative one-loop graphs for $K^+ \to \pi^+ \tilde{\gamma}\tilde{\gamma}$

Figure 1

$$<\pi^+(p_\pi)|\bar{s}_L\gamma_\lambda u_L|K^+(p_K)> \sim f_+(q^2)(p_K+p_\pi)_\lambda + f_-(q^2)q_\lambda \tag{3.3}$$

where $q = p_K - p_\pi$. The decay depends, like the decay $Z \to \Sigma\, \nu_i\bar{\nu}_i$ and the reaction $e^+e^- \to \nu\bar{\nu}\gamma$, on the number of neutrino types. However, since m_t is not known at present, one does not, strictly speaking, have any prediction for the branching ratio, even for the case of n = 3 generations. This is significant for groups that are considering doing this experiment in the near future. Assuming that the t-quark is observed and its mass determined before a possible experiment is run, one would at least have a standard-model calculation for $\sum_{i=1}^{3} B(K^+ \to \pi^+ \nu_i\bar{\nu}_i)$, which could be expected to be accurate to within perhaps a factor of 3 either way. The problem is that, given this uncertainty, one could not perform any convincing test of the standard model prediction or test for $n > 3$ generations. First, as noted at the beginning, since one does not observe what the missing neutrals are, one obviously cannot claim that they are additional neutrinos of higher generations, nor can one claim that they are photinos, higgsinos, etc. Second, unfortunately, one cannot even test a more limited hypothesis such as the hypothesis that there are $n > 3$ generations in the standard model. The reason is that the test is circular: if n is assumed to be greater than 3, then the amplitude depends on the unknown masses and couplings of the additional higher generation quarks which enter in all the Feynman diagrams of Fig. 1 and in addition, on the masses of the corresponding charged leptons which enter in the WW box diagram. It is true that there are approximate constraints on the mixings of heavy quarks with light ones arising from the requirement that they do not produce an unacceptably large effect in other one-loop induced kaon processes such as $K^0 \leftrightarrow \bar{K}^0$, $K_L^0 \to \mu\bar{\mu}$, etc. However, since the functional forms of the amplitudes are not the same, these constraints cannot be taken over directly and applied to $K^+ \to \pi^+ \nu\bar{\nu}$. Even worse, the present process (with $n > 3$) also depends on the additional charged heavy leptons, the masses of which are unknown, except that they must be greater than ~ 18 GeV from PETRA data. Compounding the problems, there can be destructive interference, so that the rate for a decay $K^+ \to \pi^+ \nu_i\bar{\nu}_i$ can actually vanish at the one-loop level! Other things being equal, the values of the lepton masses at which such a vanishing would occur are in the range $\sim 10^2$ GeV, quite possible for higher generation fermions being considered.

In summary, the decay $K^+ \to \pi^+$ + missing neutrals can, logically, never used to make any specific claim about a particular channel, $\sum_i \nu_i\bar{\nu}_i$, since by assumption, one does not actually <u>observe</u> the $\nu_i\bar{\nu}_i$, and there could well be other decays which yield exactly the same experimental signature.[38] However, if and when the t-quark is "observed," and an effective value of m_t determined, and if the relevant quark mixing angles can be measured to sufficient accuracy, then a SM prediction will be possible for n = 3 generations, albeit with some intrinsic theoretical uncertainties, such as those connected with unknown q^2 - dependence of the hadronic form factors. A strong deviation from this prediction would demonstrate the existence of some new physics.

We proceed to consider specific contributions to $K^+ \to \pi^+$ + missing neutrals from sources beyond the standard model. One interesting source is $K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}$, where $\tilde{\gamma}$ denotes the supersymmetric partner to the photon, viz., the photino.[39,36,37,40] In all the known cases of states that have the same charge, spin, and color (if any), the eigenstates of broken symmetry groups are not actually mass eigenstates but rather linear combinations thereof. Examples include the Cabibbo-Kobayashi-Maskawa quark mixing, the mixing of the gauge eigenstates A_3 and B of the standard SU(2)×U(1) theory to form the mass eigenstates Z and γ, and the mixing of ω_8 and ω_1 to form ω and and ϕ in flavor SU(3). In the case of supersymmetry, one thus is led in general to expect that there will be a similar mixing of the mass eigenstates of the scalar supersymmetric partners of the quark, viz., the quarks, to form the gauge group, and hence interaction,eigenstates. If this is the case, then there will be a tree-level decay diagram contributing to the decay $K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}$, as shown in Fig. 1. The resulting branching ratio depends on (a) the CKM-type mixing angles of the squarks; (b) the squark masses; and (c) the degrees of suppression due to the super-GIM mechanism, which is operative. Consistently with other constraints such as $K^0 - \bar{K}^0$ mixing and $K_L^0 \to \mu\bar{\mu}$, this tree-level diagram could produce a branching ratio not far below the present experimental upper limit

$$B(\sum_i K^+ \to \pi^+ \nu_i\bar{\nu}_i) < 1.4 \times 10^{-7} \quad (90\%\ \mathrm{CL}) \tag{3.4}$$

established by the KEK experiment. (The possibility of the such a tree-level decay was neglected in Ref. 37.) If the squark mixing is fine-tuned to be extremely small, then the tree-level decay could be suppressed so much that the main contributions would arise from various one-loop diagrams. Again, the resulting branching ratio is uncertain, since it depends on unknown masses of squarks and fermionic partners of gauge bosons. Estimates based on current ideas about where these masses might lie give, from the one-loop graphs,[37,40] $B(K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}) \sim 10^{-10}\text{-}10^{-11}$. There could also be other contributions from decays into fermionic partners of Higgs, called shiggses or Higgsinos: $K^+ \to \pi^+ + \tilde{H}\tilde{H}$.[37]

Another possibility is that the the photino is sufficiently massive that it would decay in a suitably designed detector.[36] There have been several analyses of astrophysical and cosmological bounds on photino masses, including one which yielded the constraints $m_{\tilde{\gamma}} < 30$ eV or $m_{\tilde{\gamma}} \gtrsim 0.3$ MeV [41] and more recently another yielding (with somewhat different theoretical inputs) the lower limit

$m_{\tilde{\gamma}} \gtrsim$ 2 -30 GeV, where the range depends on the values of squark masses. If one accepts the latter analysis, then if the $K^+ \to \pi^+ \tilde{\gamma}\tilde{\gamma}$ decay occurs at all, the photino must be so light that it will not decay visibly.[43]

IV SEARCHES FOR MASSIVE NEUTRINOS AND LEPTON MIXING IN $K_{\ell 2}$ AND $\pi_{\ell 2}$ DECAYS

In 1980 a new class of correlated tests for neutrino masses and mixing was pointed out.[12] The basic observation underlying this new class of tests was that, in general, a decay such as $\pi^+ \to \mu^+ \nu_\mu$ does not just yield a μ^+ and possibly massive ν_μ, as had previously been assumed in searches for a nonzero ν_μ "mass." Rather, if one entertains the possibility of massive neutrinos at all, as, of course, one must in testing for them, then it is not justified to assume that the weak eigenstates ν_e ν_μ ν_τ, etc. coincide with the mass eigenstates ν_1, ν_2, ν_3, etc; instead, the former are linear combinations of the latter as given by Eq. 1.2, of reference 12. It follows that a decay such as $\pi^+ \to \mu^+ \nu_\mu$, which had been used to set the upper bound on "$m(\nu_\mu)$", does not yield just a μ^+ and ν_μ, but rather consists of the sum of separate decays $\pi^+ \to \mu^+ \nu_i$ into all the mass eigenstates ν_i comprising the weak eigenstate ν_μ and allowed phase space to occur in the decay. Thus, the conventional test for "$m(\nu_\mu)$" $\neq 0$, viz., a shift in the peak in $dN/d|\vec{p}_\mu|$ downward slightly from the "$m(\nu_\mu)$" = 0 value, might very well have missed a positive signal; in the dominantly coupled mode, $\pi^+ \to \mu^+ \nu_2$, $m(\nu_2)$ might be sufficiently small that there would be no observable shift in the main peak, while a heavy, subdominantly coupled ν_i might be emitted, in the decay mode $\pi^+ \to \mu^+ \nu_i$ and might produce a peak substantially below the main one. Since previous experiments had regarded ν_μ as a mass eigenstate, they never searched for a multitude of peaks and, indeed, set cuts which would have excluded such peaks from their data even if the latter had been present. It was thus proposed[12] that new experiments be performed with $\pi_{\ell 2}$ and $K_{\ell 2}$ decays to search for possible additional peaks in the charged lepton momentum or energy spectra which would accompany the emission of heavy neutrino(s)

The peak search test is very sensitive because the signal, if it exists at all, is monochromatic and can be distinguished well from various backgrounds, which are continuous. If an additionalpeak is discovered, one can, of course, immediately determine $m(\nu_i)$ for the corresponding decay mode $M^+ \to \ell^+ \nu_i$, where $M = \pi$ or K, and, using the assumption of standard V-A couplings, one can determine $|U_{ai}|^2$, where $a = 1$ or 2 for $\ell = e$ or μ, respectively. Moreover, by carrying out polarization measurements in a second-generation experiment, one can test the assumption of V-A couplings, which thus does not have to be made blindly. Alternatively, if no additional peak is observed, to a given accuracy, then one can set a correlated upper bound on $|U_{ai}|^2$ for fixed $m(\nu_i)$. Because of the slow falloff of two-body phase space, there is little phase-space suppression of $M^+ \to \ell^+ \nu_i$ modes until $m(\nu_i)$ approaches the maximum allowed value. Indeed, for $\ell = e$, there is very drastic helicity enhancement of the rate

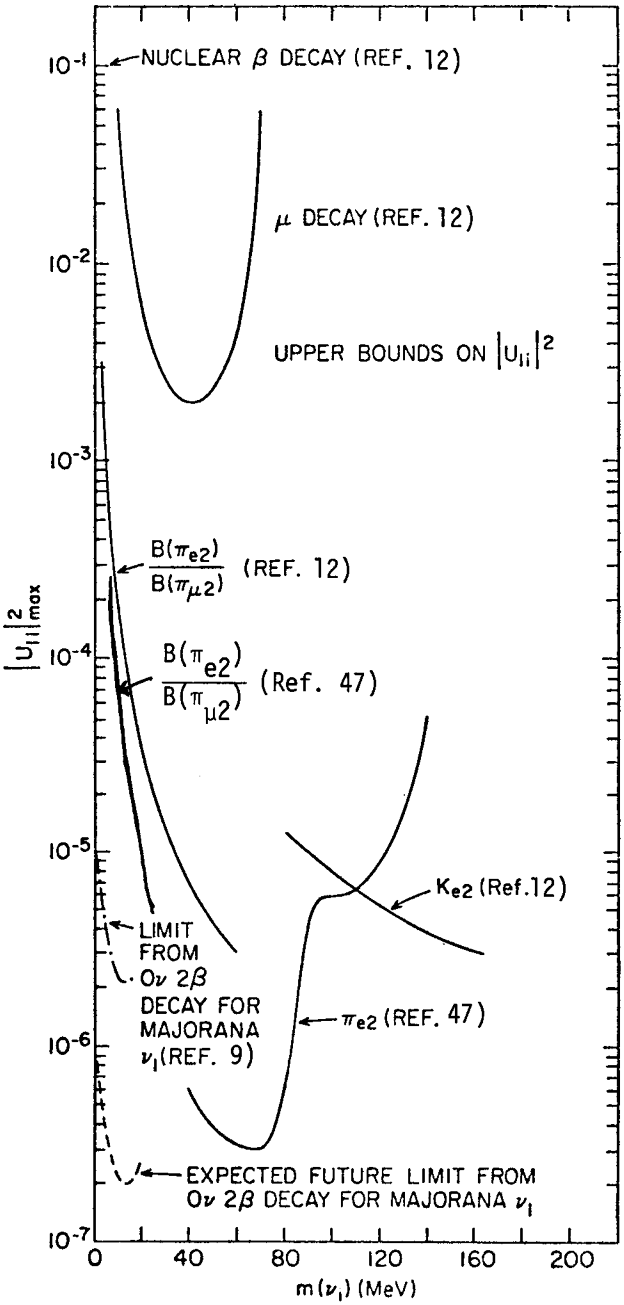

NUCLEAR β DECAY (REF. 12)
μ DECAY (REF. 12)
UPPER BOUNDS ON $|U_{li}|^2$
$\frac{B(\pi_{e2})}{B(\pi_{\mu 2})}$ (REF. 12)
$\frac{B(\pi_{e2})}{B(\pi_{\mu 2})}$ (Ref. 47)
K_{e2} (Ref.12)
LIMIT FROM 0ν 2β DECAY FOR MAJORANA ν_l (REF. 9)
π_{e2} (REF. 47)
EXPECTED FUTURE LIMIT FROM 0ν 2β DECAY FOR MAJORANA ν_l
$|U_{li}|^2_{max}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
40
80
120
160
200
$m(\nu_l)$ (MeV)

Figure 2

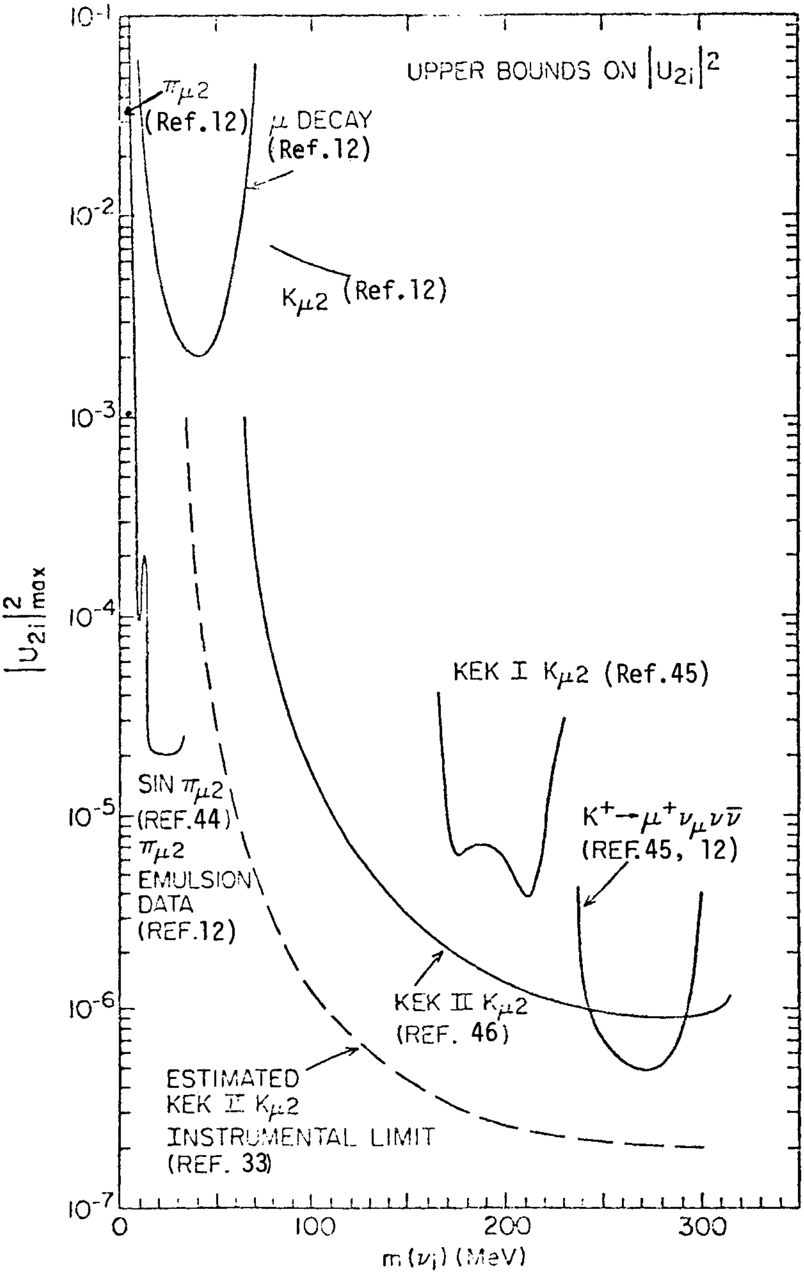

UPPER BOUNDS ON $|U_{2i}|^2$
$\pi_{\mu 2}$ (Ref.12)
μ DECAY (Ref.12)
$K_{\mu 2}$ (Ref.12)
KEK I $K_{\mu 2}$ (Ref.45)
SIN $\pi_{\mu 2}$ (REF.44)
$\pi_{\mu 2}$ EMULSION DATA (REF.12)
$K^+ \to \mu^+ \nu_\mu \nu \bar{\nu}$ (REF.45, 12)
KEK II $K_{\mu 2}$ (REF. 46)
ESTIMATED KEK II $K_{\mu 2}$ INSTRUMENTAL LIMIT (REF. 33)
$|U_{2i}|^2_{max}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
100
200
300
$m(\nu_i)$ (MeV)

Figure 3

factor for massive neutrino modes relative to that for massless neutrinos.

After the test was proposed, it was applied to existing data on $\pi_{\mu 2}$, $K_{\mu 2}$, and K_{e2} decays to search for any additional peaks within the cuts. (Data on π_{e2} decay could not be used for technical reasons; see Ref. 12). No such peaks were found, and correlated constraints on $|U_{1i}|^2$ and $|U_{2i}|^2$ were set as functions of $m(\nu_i)$. These upper bounds are shown in Figs. 2 and 3. The most stringent of the limits reached the 10^{-5} level for $|U_{2i}|^2$ (from $\pi_{\mu 2}$ emulsion data) and $\sim 3 \times 10^{-6}$ for $|U_{1i}|^2$ (from K_{e2} data). An additional constraint, primarily on $|U_{1i}|^2$, arose from the ratio of branching ratios $B(\pi_{e2})/B(\pi_{\mu 2})$ (see Fig. 2).

We proceed to discuss the experiments that have applied the peak search test. A group at SIN[44] performed the search in $\pi_{\mu 2}$ decay in 1981 and obtained the 90% CL upper limits on $|U_{2i}|^2$ for a heavy ν_i shown in Fig. 3. These improve upon the limits established from the analysis of data for 4 MeV $\lesssim m(\nu_i) \lesssim$ 12 MeV and are comparable to the limits which we obtained from $\pi_{\mu 2}$ emulsion data for 12 MeV $\lesssim m(\nu_i) \lesssim$ 34 MeV. The great sensitivity of the peak search test is demonstrated by these bounds, which extend down to the 10^{-5} level and apply for a decay mode in which massive neutrinos have so significant helicity enhancement. Two peak search experiments have been carried out by T. Yamazaki and collaborators at KEK on $K_{\mu 2}$ decay.[45,46] The first used an already existing apparatus and determined $|\vec{p}_\mu|$ via a range measurement technique.[45] It achieved a good upper limit on $|U_{2i}|^2$ in the range 160 MeV $\lesssim m(\nu_i) \lesssim$ 230 MeV, as indicated in Fig. 3. The Yamazaki group then proceeded to perform a beautiful, dedicated high-precision magnetic spectrometer experiment in early spring, 1982.[46] The apparatus for this experiment is shown in Fig. 1 of Ref. 46; there is a substantial quantity of NaI(Tl) crystals to veto events from the decays $K^+ \to \pi^o \mu^+ \nu_\mu$; $\pi^o \to \gamma\gamma$ and $K^+ \to \mu^+ \nu_\mu \gamma$. No definite additional peak was observed in the range of muon momenta corresponding to $m(\nu_i) \epsilon$ (60,320)MeV and an extremely good correlated upper bound was thus set on mixing strength $|U_{2i}|^2$ in this range, extending down to $\sim 10^{-6}$ (see Fig. 3). This group will take data again in 1983. It is anticipated that this further work at KEK will enable one to widen the range of the peak search down to $m(\nu_i) \sim$ 50 MeV and up to $m(\nu_i) \sim$ 400 MeV.[46] Beyond that, there are two possibilities for further work. First, consideration is being given to a TPC type of detector which would have greater efficiency for detecting photons and thereby vetoing background events. It is estimated that such a third-generation $K_{\mu 2}$ peak search experiment might be able to push the 90% CL limit on $|U_{2i}|^2$ down to the 10^{-7} level for $m(\nu_i)$ in the range from $\sim$ 200 to $\sim$ 300 MeV, and achieve commensurate improvements for lower masses. Second, there is perhaps some possibility of a peak search in the K_{e2} decay mode. Although a massive neutrino signal would be kinematically enhanced by a factor as large as $\sim 10^5$ relative to a massless neutrino mode, there is the challenge of a very small overall branching ratio with which one must contend.

A peak search has also been conducted recently by a group at TRIUMF using π_{e2} decay.[47] The upper limits on $|U_{1i}|^2$ from this experiment are shown in Fig. 2. The reason that the bound becomes much weaker for $m(\nu_i) \gtrsim 80$ MeV is that the background from the dominant decay chain $\pi^+ \to \mu^+ \to e^+$ sets in at a serious level in this lower e^+ momentum region, thereby reducing the sensitivity of the experiment to a small peak. Nevertheless, the power of the peak search method is again demonstrated by the extremely small limit of $|U_{1i}|^2 \leq 3 \times 10^{-7}$ which was established for 60 MeV $\leq m(\nu_i) \leq$ 75 MeV. In addition, a new measurement of $B(\pi_{e2})/B(\pi_{\mu 2})$ has been performed at TRIUMF which yields an improved upper limit on $|U_{1i}|^2$ for $m(\nu_i) \leq$ 40 MeV.

This, then, is the present state of experiments that have applied the peak search test for heavy neutrinos. In Europe there is a μ-capture experiment being performed by J. Deutsch and collaborators at SIN.[48] Since the final state is two-body, this group will again perform a search for anomalous peaks in the kinetic energy spectra of the recoiling nucleus. This experiment will directly probe the ν_i mass range below 100 MeV, and will be especially useful in the interval 34 MeV $< m(\nu_i) \leq$ 60 MeV, where only the previous analysis[12] of old $K_{\mu 2}$ data and μ decay data set upper limits on $|U_{2i}|^2$.

Concerning the future, as with other experiments searching for possible rare decays, peak searches in $K_{\ell 2}$ (and $\pi_{\ell 2}$) decays would profit from higher intensity beams. However, in order to utilize such higher intensities, they would need corresponding improvements in detectors, including, in particular, better photon detection and vetoing capabilities.

It is obvious that the flurry of excitement about nonzero neutrino masses which was generated by the reactor experiment of Reines and collaborators[49] has dissipated, now that their reported positive effect has been refuted by the Grenoble and Gösgen experiments.[50] The nonzero value of "$m(\nu_e)$" claimed by the ITEF experiment on ^{3}H beta decay also still awaits confirmation (or refutation) by independent experiments.[51] Nevertheless, it is certainly true that the issues of possible nonzero neutrino masses of Dirac or Majorana type, and associated lepton mixing, are quite general and will remain of continuing interest.

V SEARCHES FOR MASSES OF DOMINANTLY COUPLED NEUTRINOS IN $\pi_{\mu 2}$ AND $K_{\mu 3}$ DECAYS

The dominantly coupled neutrino mass eigenstate that can be studied usefully in K and π decays is ν_2, corresponding to the gauge group eigenstate ν_μ. Upper bounds on "$m(\nu_\mu)$", or more precisely, $m(\nu_2)$, have come historically from three sources, (and predominantly from the third): (1) measurement of the endpoint of the e^+ momentum spectrum in μ^+ decay[52]; (2) measurement of the $\pi\mu$ invariant mass distribution in $K^0_{\mu 3}$ decay;[53] and (3) measurement of $|\vec{p}_\mu|$ in $\pi_{\mu 2}$ decay.[54,55] The most recent and best such limit was obtained by Daum et al.[54] in a precision magnetic spectrometer experiment at SIN on $\pi_{\mu 2}$ decay and was reported as "$m(\nu_\mu)$" $<$ 0.57 MeV (90% CL). This limit relied upon the value of m_{π^+} which was then available.

Subsequent to this experiment, a new measurement of m_{π^-} was carried out; if one combines this with previous measurments to obtain a new world average and then uses that quantity in conjunction with the SIN data from Ref. 54, one obtains the slightly lower limit, $m(\nu_2) < 0.52$ MeV (90% CL).[55]

A new $\pi_{\mu 2}$ experiment is now under development by P. Nemethy and collaborators at LBL.[56] It will utilize a ring-imaging Čerenkov counter and anticipates achieving an ultimate sensitivity of 50 KeV in $m(\nu_2)$. Thus, in the near future, one may look forward to a dramatic reduction in the upper limit on $m(\nu_2)$ (or, of course, possibly an observation of a nonzero value of this mass).

In addition to $\pi_{\mu 2}$ decays, one might consider the possibility of using $K_{\mu 3}$ decays to get an improved limit on $m(\nu_2)$. The last such limit was obtained in 1974 by an LBL group using $K^o_{\mu 3}$ decay and is "$m(\nu_\mu)$" < 1.5 MeV (90% CL). Some features of such an experiment were reviewed at the DPF Snowmass Summer Study.[9,57] However, assuming that the LBL-BNL experiment of Nemethy et al. does achieve a level of sensitivity roughly equal to its expectations, it is not clear that a $K_{\mu 3}$ experiment would be competitive. Thus, summarizing, it would be of interest to consider extensions of the dedicated $\pi_{\mu 2}$ experiment of Ref. 56 which might be designed for a pion port at a high luminosity kaon factory or kaon beam at an upgraded AGS.

VI OTHER BOUNDS ON NEUTRINO MASSES AND MIXING FROM MUON AND NEUTRINO DECAYS

In order to assess possible future limits on neutrino masses and mixing which might be obtained from K and π decays, one must also take account of bounds from other sources. One such bound was derived in 1980-1981 from an analysis of muon decay data.[12] The basic starting point of this analysis was the realization that in general "the" decay $\mu^- \to \nu_\mu e \bar{\nu}_e$ really consists of a subset of all of the modes $\mu \to \nu_i e \bar{\nu}_j$ which are allowed by phase space, where ν_i, etc. are the neutrino mass eigenstates. Now, if one or more neutrinos with non-negligible masses is emitted, it will alter the overall, observed distribution in a calculable manner. Hence, the data analyses that were carried out to test for anomalous Lorentz structure in leptonic weak interactions would have derived apparent non-(V-A) values for the various spectral parameters in muon decay, given the fact that they fit the data to the well-known zero neutrino mass distributions depending on ρ, η, ξ, and δ. It was thus possible to use the agreement between the measured values of these spectral parameters and the V-A values (with radiative corrections taken into account to the requisite degree of accuracy) to place upper limits on any admixture, via lepton mixing, of heavy neutrino decay modes. It was found that the parameter ρ was the most sensitive to massive neutrino effects, and this was utilized to set an upper bound on $|U_{ri}|^2$, r = 1 and 2, for $m(\nu_i)$ in the range 10 MeV < $m(\nu_i)$ < 70 MeV. This bound is shown in Figs. 2 and 3 for the respective cases r = 1 and r = 2. For the latter case, i.e., the coupling of a heavy neutrino to the muon, it provides the primary constraint on this coupling and the associated lepton mixing in the region of $m(\nu_i)$ covered. In contrast to the peak search experiments discussed above, which do not directly probe the charge conjugation properties of the neutrinos, muon decay does. A general analysis of the decay distribution for the case of massive neutrinos of both Dirac and Majorana types, lepton mixing, and arbitrary Lorentz structure has been given.[58] The distribution in the case of Majorana neutrinos and V ± A couplings was also studied by the Osaka group of Doi et al.[59] Finally, a special case of the decay into Dirac neutrinos analyzed in Ref. 12 was later considered by Kalyniak and Ng, who also attempted to calculate the decay distribution for the Majorana case with V-A couplings.[60] However, the results of Kalyniak and Ng are incorrect, owing to their failure to take proper account of the self-conjugacy of Majorana neutrinos, as was noted in Ref. 58.

There are currently several very high-precision experiments on regular muon decay at TRIUMF, LAMPF, and SIN. These will be discussed further in Section VIII below. The increased accuracy which these experiments will achieve in the measurement of the muon decay distribution, and thus the associated spectral parameters (especially ρ) will make possible a commensurate improvement of the bounds on leptonic mixing angles via the method of Ref. 12. These experiments will be especially valuable since, together with the ongoing SIN muon-capture experiment of J. Deutsch and collaborators, they will constrain $|U_{2i}|$ over a range of $m(\nu_i)$ between ∿ 34 MeV and ∿ 60 MeV not covered at all in $\pi_{\mu 2}$ and below the optimally sensitive region of neutrino masses covered in $K_{\mu 2}$ peak search experiments. The TRIUMF experiment of M. Strovink and collaborators[61] is now reporting results; the others should have results within a few years. As will be stressed in Section VIII, tests of lepton mixing are but one of many reasons for guaranteeing a high intensity muon beam as an auxiliary part of a future kaon factory.

The future prospects for further neutrino oscillation experiments have been discussed in detail in the LANL Proposal for a National Facility to Provide a High Intensity Neutrino Source[62], by R. Lanou in the DPF Snowmass Summer Study[63], and for this workshop by the Kayser-Rosen subgroup.[64] Neutrino oscillation experiments nicely complement searches for effects of neutrino masses and lepton mixing in particle decays since they are sensitive to much smaller masses (more precisely, mass differences), but cannot yet probe for such small mixing angles as peak search experiments. An experiment to look for the decays of massive neutrinos is currently under consideration by the CERN PS. The revised version[65] of the proposal for this experiment envisions a search for the decay $\nu_i \to \nu_j e^+e^-$; the ν_i thus cannot be the primary mass eigenstate in either ν_e or ν_μ and hence would be produced only subdominantly in π or K decays.[12] Clearly, given the upper limits that have been established on lepton mixing for such massive neutrinos, the production of the initial ν_i's would be very strongly suppressed. Since the decay $\nu_i \to \nu_j e^+e^-$ itself only only proceeds via lepton mixing, it is also suppressed by a lepton mixing matrix coefficient squared (for the dominant mode, in which j = 1). However, assuming this experiment is approved to run, it will be interesting to see what results it obtains.

VII OTHER K DECAYS, AND SOME π DECAYS, OF INTEREST

In this section we shall mention some further K decays which might be of interest for a dedicated kaon facility. Although the primary purpose of this workshop is to assess the physics uses of a high-intensity source of strangeness, it does seem worthwhile to list several pion decays which could be searched for or studied via an auxiliary pion beam at such a facility. Obviously, experimental programs on pion and muon decays do not require K beams, but if one is envisioning a high-intensity, low-energy machine, it is important to retain a strong continuing effort in these two other areas. We proceed with several conventional kaon decays:

(1) $K^+ \to e^+\nu_e\gamma$

This radiative decay is useful for studying structure-dependent electromagnetic corrections to two-body leptonic K decays. Such information is valuable in order to make a precise comparison between the V-A predictions and experimental measurements for the ratio $B(K_{e2})/B(K_{\mu 2})$. Since structure-dependent corrections are expected to be substantially larger for this ratio than for the corresponding pion ratio $B(\pi_{e2})/B(\pi_{\mu 2})$,[66] and since they are hard to calculate reliably, further experimental input is helpful. The $K^+ \to e^+\nu_e\gamma$ decay was studied recently in a CERN-Heidelberg experiment[67]; the current value of the branching ratio for the structure-dependent contribution involving photons of positive helicity is [55] $(1.52 \pm 0.23) \times 10^{-5}$

(2) $K_L^0 \to \gamma\gamma$; $K^+ \to \pi^+\gamma\gamma$

In the quasi-free quark model approach, the decay $K_L^0 \to \gamma\gamma$ proceeds via one-loop electroweak diagrams. It has the interesting feature that the GIM suppression mechanism operates differently than in the case of $K^0 - \bar{K}^0$ mixing or the decays $K_L^0 \to \mu^+\mu^-$ and $K^+ \to \pi^+\nu\bar{\nu}$. Rather than a multiplicative suppression factor $\propto (m_c^2 - m_u^2)/m_W^2$ in the amplitude (for two generations), Gaillard and Lee found a different dependence, involving m_q^2/m_K^2 but not additional m_W^{-2} suppression.[16] This absence of a severe GIM suppression was seen to be in agreement with the measured value of the branching ratio for this decay, the present value of which[55] is $B(K_L^0 \to \gamma\gamma) = (4.9 \pm 0.4) \times 10^{-4}$. A similar one-loop electroweak K decay is $K^+ \to \pi^+\gamma\gamma$, for which Gaillard and Lee estimated a branching ratio, in the two-generation case, $\sim 10^{-6}$-10^{-7}. As with other rare K decays, the free-quark contribution to this decay could be somewhat larger for the present three-generation theory due to the t-quark terms. The current upper limit, established recently by a KEK experiment,[29] is $B(K^+ \to \pi^+\gamma\gamma) < 0.84 \times 10^{-5}$ (90% CL). It would certainly be worthwhile to perform an experiment with improved sensitivity in order actually to observe this decay.

(3) $K_L^o \rightarrow K^{\pm} e^{\mp} \overset{(-)}{\nu}_e$

The branching ratio for kaon beta decay is calculated to be extremely small: $\sim 3 \times 10^{-9}$. The potential value of an experimental measurement of the branching ratio for this decay is that the latter depends quite sensitively on the mass difference between the neutral and charged kaons, as $(m_{K^o} - m_{K^+})^5$, approximately. Hence, a sufficiently precise measurement of the decay branching ratio could provide a more accurate determination of this mass difference than the one now available, viz., $m_{K^o} - m_{K^+} = 4.01 \pm 0.13$ MeV.

Rare K decays which yield information concerning CP violation are discussed in detail in the corresponding section of these Proceedings, chaired by L. Wolfenstein. Accordingly, we shall not analyze such decays here. We proceed to consider briefly some interesting pion decays which might deserve further study at a pion port in a future high-intensity, low-energy facility:

(4) $\pi^+ \rightarrow \pi^o e^+ \nu_e$

Pion beta decay has provided one of the important tests of the conserved vector current hypothesis. The current value of the branching ratio for this decay is $1.02 \pm 0.07 \times 10^{-8}$. It would clearly be worthwhile to improve the precision with which this number can be measured, to carry further the comparison with theory.

(5) $\pi^o \rightarrow e^+ e^-$

Until recently, the only observation of this rare decay came from a CERN-Geneva-Saclay experiment.[68] A LAMPF experiment has now reported[69] a branching ratio $B(\pi^o \rightarrow e^+e^-) = (1.8 \pm 0.6) \times 10^{-7}$. Although the rate for this decay cannot be calculated with quite the precision of pion beta decay, more data would certainly be helpful in understanding the mechanisms responsible for it.

(6) $\pi^o \rightarrow \gamma\gamma\gamma$

This decay mode provides a test of charge conjugation invariance in the electromagnetic interactions. The present upper limit on the branching ratio is 3.8×10^{-7}, established by a recent LAMPF experiment.[70] Because of the additional factor of α (relative to the rate for the regular decay $\pi^o \rightarrow \gamma\gamma$), the suppression due to three-body phase space, and other factors, the branching ratio would be expected to be very small even if charge conjugation invariance were violated in electromagnetic interactions. Therefore, it is worthwhile to push this upper bound down further in dedicated future experiments.

(7) $\pi^o \to$ missing neutrals

A specific exmaple of this decay would be $\pi^o \to \nu\bar{\nu}$, either involving anomalous Lorentz structure[71] or massive neutrinos.[72] The obvious problems are (1) how to tag the π^o with sufficiently high reliability, and (2) how to detect and veto the regular $\gamma\gamma$ decay with the requisite extremely high efficiency. The decay $K^+ \to \pi^+\pi^o$ might be used to provide the tagging.

VIII MUON DECAYS AND REACTIONS

Regular and rare muon decays serve as probes for much of the same new physics that rare K decays do. We shall briefly mention some muon decays of interest in this section:

(1) "Regular" Muon Decay: $\mu^+ \to e^+ +$ missing neutrals

The conventional source for the experimentally observed final state in this decay is $\mu^+ \to \bar{\nu}_\mu e^+ \nu_e$, where the neutrinos are massless, and the weak couplings are of V-A type. Exotic possibilities include possible right-handed current contributions, or more generally, effective interactions with anomalous Lorentz structure, and effects of massive neutrinos and associated lepton mixing. Indeed, if the latter are present, then "the" decay $\mu^+ \to \nu_\mu e^+ \nu_e$ is not one decay at all, but rather a sum of all the separate modes $\mu^+ \to \bar{\nu}_i e^+ \nu_j$, where the ν_i's are neutrino mass eigenstates. The observed e^+ momentum spectrum is thus the sum due to all the actual (i,j) modes and thus differs from the spectrum predicted by the standard model. Consequently, the values of the spectral parameters ρ, η, ξ, and δ which were inferred from the data by fitting it to an assumed standard-model form with generalized Lorentz structure would differ from the V-A values even if the couplings were purely of V-A type! These effects were pointed out in Ref. 12 and were used to derive correlated bounds on neutrino masses and lepton mixing, as was discussed in Section VI. The upper bounds on $|U_{ri}|^2$, r = 1 and 2, which were obtained are shown in Figs. 2 and 3, as functions of $m(\nu_i)$. Moreover, in the context of supersymmetric theories, the decay $\mu^+ \to e^+\tilde{\gamma}\tilde{\gamma}$ could occur, if the photinos are light enough, and would yield a generic final state of the form e^+ + missing neutrals if neither of the photinos decayed in the detector. The momentum spectrum of the positrons would again differ from the standard model prediction, both because of generally different effective Lorentz structure and because of possible nonzero photino masses.

There are currently three new experiments on regular muon decay, by M. Strovink and collaborators at TRIUMF,[61], K. Crowe and collaborators, also at TRIUMF,[73] and H. Anderson and collaborators, at LAMPF.[74] The first of these has obtained the result $\xi P_\mu \delta/\rho > 0.9959$ (90% CL).[61] The second will measure η, while the LAMPF experiment plans to measure each of the four spectral parameters. One should also mention the recent measurement of the longitudinal polarization of the positron in muon decay at SIN, and the fact that this group plans to carry out an experiment measuring the spectral parameters in the near future.[75] The new measurement by Strovink et al. provides a sensitive limit on possible contributions due to right-handed currents. The results from the other experiments listed should be forthcoming in the near future and will provide similarly useful constraints on possible new physics.

(2) $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$, and $\mu^+ \to e^+\gamma\gamma$

These decays are all forbidden by lepton family number conservation. Many of the new physics possibilities which would give rise to rare K decays such as $K^+ \to \pi^+\mu^+e^-$, $K^0_L \to \mu^+e^-$, etc., also would cause these decays to occur at some level. The decays $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ were analyzed in the standard $SU(2)_L \times U(1)$ electroweak theory, extended to include massive neutrinos and lepton mixing, in Refs. 11 and 76; and in theories with right-handed currents in Ref. 77. A discussion of how these decays proceed in theories with horizontal gauge interactions was given in Ref. 8. Effects expected in (extended) technicolor models have been considered in Ref. 15.

The current upper limit $B(\mu^+ \to e^+\gamma) < 1.9\times 10^{-10}$ was achieved by a LAMPF experiment,[78] while the bound $B(\mu^+ \to e^+e^+e^-) < 1.9 \times 10^{-10}$ was reported by a Dubna group.[79] The present upper limit on the two-photon mode is[80] $B(\mu^+ \to e^+\gamma\gamma) < 5 \times 10^{-8}$. There is now a dedicated experiment running at LAMPF searching for each of these three decays.[81] It anticipates a sensitivity in the 10^{-11}-10^{-12} range for the branching ratios in all three decays. Results from this experiment should be available on a time scale of 1-2 years. It is further anticipated that with the requisite improvements in detectors, an experiment using a muon beam at a high-intensity, low-energy facility in the early 1990's might be able to achieve a sensitivity of 10^{-13} or better in branching ratio.[82]

(3) $\mu^-N \to e^{\pm}N'$

Like the muon decays discussed above in category (2), muon conversion in the field of a nucleus (N) violates lepton family number. The second reaction, yielding an e^+, also violates total lepton number. The first reaction was analyzed in the extended standard model in Ref. 83; other sources for both reactions have been discussed in several of the works cited above. The current upper limit $\sigma(\mu^- + {}^{32}S \to e^- + {}^{32}S)/\sigma(\mu^- + {}^{32}S \to \nu_\mu + {}^{32}P) < 7 \times 10^{-11}$ has been achieved in an experiment at SIN.[84] A current experiment at TRIUMF anticipates reducing this limit to[85] 10^{-12}. It is further estimated that the limit might be reduced to 10^{-13} or better at the high-intensity facility under consideration here, but only if one could achieve sufficient improvements in the necessary detectors.[85,86]

Thus, in summary, the study of regular muon decay and the search for possible rare decays are worthwhile activities to pursue at a future high-intensity facility. We would like to thank T.P. Cheng and L.F. Li for helpful input for this section.

IX. CONCLUSION

The overall conclusions are summarized in Table 1 presented earlier. In ending this report, we would like to stress that experimentalists should search for as wide a range of new physics as possible and should not be deterred from considering a particular decay mode just because some theorist claims that it is predicted not to occur in his set of fashionable models. New physics has sometimes been anticipated by theorists, but at least as often, it has not. Moreover, theorists may not correctly assess the experimental implications of a given model. It is instructive to recall, for example, that $|\varepsilon'/\varepsilon|$ was once claimed to be negligibly small in the standard electroweak theory,[86] so that there was no reason, supposedly, for experimentalists to search for nonzero ε'. This claim was shown to be incorrect by Gilman and Wise,[87] and subsequently we have seen a great resurgence of interest in CP-violation experiments, as was discussed at this workshop by Winstein and Wolfenstein. The moral of this story should be clear. It is also clear that, despite some theorists' possible negative views, it is worthwhile to search for new physics via rare K (and π and μ) decays to as low levels as allowed by beam intensities, backgrounds, and detector capabilities. An upgraded AGS and dedicated kaon factory would constitute very powerful research tools for this purpose.

REFERENCES

The following references are not to be taken as an exhaustive compilation. They are only intended as an initial guide to the theoretical literature, especially for those who are unfamiliar with certain topics.

1. Some reviews which may be of help to the reader are listed below: Elementary Particle Physics and Future Facilities, Proceedings of the APS Division of Particles and Fields Workshop, Snowmass, June-July, 1982 R. Donaldson, R. Gustafson, and F. Paige, eds. (in particular, the discussions of standard model predictions and tests of theoretical developments beyond the standard model); Proceedings of the Second LAMPF II Workshop, H. A. Thiessen, et al., eds. Los Alamos Pub. LA-9572-C. LAMPF II Proposal, LANL preprint. For a review of technicolor, see, e.g., E. Farhi and L. Susskind, Phys. Rep. 74C, 277 (1981). For supersymmetry, see, for example, the discussion of theoretical rudiments in P. Fayet and S. Ferrara, Phys. Rep. 32C, 249 (1977) and, for supergravity, P. van Nieuwenhuizen, Phys. Rep., 68C, 189 (1981). For up-to-date reviews of the implications for particle physics, see, for example, G. Kane and J. Polchinski in the Proceedings of the Fourth Workshop on Grand Unification, to be published.
2. The possibility of such a mass relation (based on using just one or more 5's of Higgs, but no 45's) was first noticed in the classic SU(5) paper of H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974) and later rejected in H. Georgi and C. Jarlskog, Phys. Lett. 86B, 297 (1979). The statement that one should use only a 5 of Higgs and the claim that the generic mass relation is correct is due to A. Buras, J. Ellis, M. Gaillard, and D. Nanopoulos, Nucl. Phys. B135, 66 (1978).
3. See e.g. C. Burges and H. Schnitzer, Brandeis preprint (1983); and E. Eichten, K. Lane, and M. Peskin, Phys. Rev. Lett. 50, 811 (1983); and references therein.
4. A. R. Clark et al. Phys. Rev. Lett. 26, 1667 (1971).
5. M. E. Zeller, spokesman, "Search for the Rare Decay Mode $K^+ \to \pi^+\mu^+e^-$", BNL AGS Experiment E777.
6. W. Morse and M. P. Schmidt, co-spokesmen, "A Search for the Flavor-Changing Neutral Currents $K_L^o \to \mu + e$ and $K_L^o \to e^+e^-$.
7. M. Schmidt, private communication.
8. R. Cahn and H. Harari, Nucl. Phys. B176, 135 (1980). Similar limits are given in G. Kane and R. Thun, Phys. Lett. 94B, 513 (1980) and D. R. T. Jones, G. Kane, and J. Leveille, Nucl. Phys. B198, 45 (1982). See also P. Herczeg, in Proceedings of the Kaon Factory Workshop, TRIUMF (1979), M. Craddock, ed., and O. Shanker, Nucl. Phys. B185, 382 (1981). A number of the limits given by Shanker involve unconventional assumptions and calculations.
9. For a review, see R. Shrock, in the Proceedings of the DPF Summer Study on Elementary Particle Physics and Future Facilities,

eds., R. Donaldson, R. Gustafson, and F. Paige, June-July, 1982, p. 264; and Ref. 10.

10. T. Yamazaki, in the Proceedings of the International Conference on Neutrino Physics and Astrophysics, 1982.
11. B. W. Lee and R. Shrock, Phys. Rev. D16, 1444 (1977).
12. R. Shrock, Phys. Lett. 96B, 159 (1980); Phys. Rev. D24, 1232, 1275 (1981).
13. E. Eichten and K. Lane, Phys. Lett. 90B, 125 (1980).
14. S. Dimopoulos, G. Kane, and S. Raby, Nucl. Phys. B182, 77 (1981).
15. J. Ellis et al., Phys. Lett. 101B, 387 (1981); S. Dimopoulos and J. Ellis, Nucl. Phys. B182, 505 (1981).
16. M. K. Gaillard and B. W. Lee, Phys. Rev. D10, 897 (1974).
17. B. W. Lee, J. Primack, and S. B. Treiman, Phys. Rev. D7, 510 (1973) (which considered rare induced K decays in the old Georgi-Glashow O(3) model); A. Vainstein and I. Khriplovich, Pis'ma Zh. Eksp. Teor. Fiz. 18, 41 (1973) (JETP Lett. 18, 83 (1973)).
18. R. Dzhelyadin et al., Phys. Lett., 105B, 239 (1981).
19. For early reviews, see M. K. Gaillard and H. Stern, Ann. Phys. 76, 580 (1973); A. Dolgov, V. Zakharov, and L. Okun, Usp. Fiz. Nauk 107, 537 (Sov. Phys. Usp. 15 404 (1973)).
20. R. Shrock and M. Voloshin, Phys. Lett., 87B, 375 (1979); T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981) Concerning the long-distance contribution to Re $\{Amp(K^o_L \to \mu\bar{\mu})\}$, see M. B. Voloshin and E. P. Shabalin, Pis'ma Zh. Eksp. Teor. Fiz. 23 (1976) 123 [JETP Lett. 23 (1976) 107].
21. R. Carithers et al., Phys. Rev. Lett. 31, 1025 (1973)
22. Y. Fukashima et al., Phys. Rev. Lett. 36, 348 (1976).
23. M. Shochet et al., Phys. Rev. Lett. 39, 59 (1977); Phys. Rev. D19, 1965 (1979).
24. C. Quigg and J. D. Jackson, UCRL Report 18487 (1968); L. Sehgal, Phys. Rev. 183, 1511 (1969); B. Martin, E. deRafael, and J. Smith, Phys. Rev. D2, 179 (1970); M. K. Gaillard, Phys. Lett. 35B, 431 (1971); G. Farrar and S. B. Treiman, Phys. Rev. D4, 257 (1971); M. Pratap, J. Smith, and Z. Uy, Phys. Rev. D5, 269 (1972); S. Adler, G. Farrar, and S. B. Treiman, Phys. Rev. D5, 770 (1972); L. Bergstrom, Zeit. Phys. C14, 129 (1982).
25. M. K. Gaillard, B. W. Lee, and R. Shrock, Phys. Rev. D13, 2674 (1976).
26. P. Herczeg, Phys. Rev. D27, 1512 (1983).
27. R. Peccei and H. Quinn, Phys. Rev. Lett. 38, 1440 (1977); Phys. Rev. D16, 1791 (1977); S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, ibid., 40, 279 (1978).
28. F. Wilczek, Phys. Rev. Lett. 49, 1549 (1982).
29. Y. Asano et al., Phys. Lett. 107B, 159 (1981).
30. S. Weinberg, op cit., Ref. 27; T. Goldman and C.M. Hoffman, Phys. Rev. Lett., 40, 220 (1978); W. Bardeen and S.-H. H. Tye, Phys. Lett. 74B, 229 (1978); J. Kandaswamy et al., Phys. Lett. 74B , 377 (1978); J. Frere et al., Phys. Lett. 103B, 129 (1981).
31. The KEK experiment of Ref. 29 obtained the limits $B(K^+ \to \pi^+\gamma\gamma) < 0.84 \times 10^{-5}$ and $B(K^+ \to \pi^+a;\ a \to n\gamma's) < 1.4 \times 10^{-6}$ (both at the

90% CL) for m_a < 100 MeV and $\tau_{a \to n\gamma's} < 10^{-9}$ sec. See also Ref. 33.

32. J. Kim, Phys. Rev. Lett. 43, 103 (1979); M. Dine et al., Phys. Lett. 104B, 199 (1981); H.P. Nilles and S. Raby, Nucl. Phys. B198, 102 (1982); M.B. Wise et al., Phys. Rev. Lett. 47, 402 (1981).
33. T. Yamazaki et al., University of Tokyo preprint UTMSL-35, and T. Yamazaki, private communication.
34. R. Shrock and M. Suzuki, Phys. Lett. 110B, 250 (1982).
35. E. Ma and J. Okada, Phys. Rev. D18, 4219 (1978).
36. R. Shrock, in the Proceedings of the DPF Summer Study on Elementary Particle Physics and Future Facilities, Snowmass, June-July, 1982, R. Donaldson et al., eds., p. 291.
37. J. Ellis and J. Hagelin, Nucl. Phys. B217, 189 (1983).
38. This negative conclusion was reached in the first work, that of Ma and Okada, which investigated the possibility of counting neutrinos via this decay. It was also reached in Ref. 36 upon a reexamination of the issue.
39. M. Suzuki, UC Berkeley-LBL preprint UCB-PTH-82/8.
40. M. K. Gaillard, Y.-C. Kao, I.-H. Lee, and M. Suzuki, Phys. Lett. 123B, 241 (1983).
41. G. Farrar and P. Fayet, Phys. Lett. 105B, 155 (1981) and references therein.
42. H. Goldberg, Phys. Rev. Lett. 50, 1419 (1983).
43. An up-to-date review of mass limits pertaining to supersymmetric partners of known particles is given in G. Kane, in the Proceedings of the Fourth Workshop on Grand Unification, 1983, to be published (op. cit., Ref. 1).
44. R. Abela et al., Phys. Lett. 105B, 263 (1981).
45. Y. Asano et al., Phys. Lett. 104B, 84 (1981).
46. R. Hayano et al., Phys. Rev. Lett. 82, 1305 (1982); T. Yamazaki, private communication.
47. D. Bryman et al., Phys. Rev. Lett. 50, 7 (1983); and TRIUMF preprint.
48. J. Deutsch, private communication and to be published.
49. F. Reines, H. Sobel, and E. Pasierb, Phys. Rev. Lett. 45, 1307 (1980).
50. J. Vuilleumier et al., Phys. Lett. 114B, 298 (1982); and to be published.
51. A review of ongoing ^{3}H beta decay experiments is given in Ref.9.
52. M. Bardon et al., Phys. Rev. Lett. 14, 449 (1965).
53. A. Clark et al., Phys. Rev. D9, 533 (1979).
54. M. Daum et al., Phys. Lett. 74B, 126 (1978); Phys. Rev. D20, 2692 (1979).
55. Particle Data Group, Review of Particle Properties, Phys. Lett. 111B, 1 (1982).
56. P. Nemethy, spokesman, letter of intent submitted to Brookhaven National Laboratory (September, 1982), and private communication.

57. C. Hoffman and V. Sandberg, in Proceedings of the 1982 DPF Summer Study on Elementary Particle Physics and Future Facilities, Snowmass, eds., R. Donaldson, R. Gustafson, and F. Paige, p. 552.
58. R. Shrock, Phys. Lett. 112B, 382 (1982).
59. M. Doi et al., Prog. Theor. Phys. 67, 281 (1982).
60. P. Kalyniak and J. Ng, Phys. Rev. D25, 1305 (1982).
61. J. Carr et al., LBL preprint in preparation, and M. Strovink, private communication.
62. LANL Working Group, G. J. Stephenson, chairman, Proposal to the Department of Energy for a National Facility to Provide a High Intensity Neutrino Source (Los Alamos National Laboratory, 1982).
63. R. Lanou, in the Proceedings of the 1982 DPF Summer Study, op. cit., p. 538.
64. B. Kayser and S. P. Rosen, in these Proceedings.
65. F. Vannuci, spokesman, CERN Experimental Proposal (1983), revised version.
66. T. Goldman and W. J. Wilson, Phys. Rev. D15, 709 (1977).
67. J. Heintze et al., Nucl. Phys. B149, 365 (1979).
68. J. Fischer et al., Phys. Lett. 73B, 364 (1978).
69. R. E. Mischke et al., Phys. Rev. Lett. 48, 1153 (1982).
70. V. L. Highland et al., Phys. Rev. Lett. 44, 628 (1980).
71. E. Fischbach, B. Kayser, G. Garvey, and S. P. Rosen, Phys. Lett. 52B, 385 (1974).
72. P. Herczeg and C. M. Hoffman, Phys. Lett. 100B, 347 (1981).
73. K. Crowe et al., TRIUMF Experimental Proposal.
74. H. Anderson et al., LAMPF Experimental Proposal.
75. F. Corriveau et al., Phys. Rev. D24, 2004 (1981); and F. Scheck, private communication. (For a review of previous experimental and theoretical results on muon decay, see F. Scheck, Phys. Rep. 44, 187 (1978).)
76. W. J. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977).
77. T. P. Cheng and L. F. Li, Phys. Rev. D16, 1425 (1977); J.D. Bjorken, K. Lane, and S. Weinberg, ibid., D16, 1474 (1977); S. B. Treiman, F. Wilczek, and A. Zee, ibid., D16, 152 (1977).
78. J. Bowman et al., Phys. Rev. Lett. 42, 556 (1979).
79. M. Korenchenko et al., JETP 43, 1 (1976).
80. J. Bowman et al., Phys. Rev. Lett. 41, 442 (1978).
81. J. Bowman et al., LAMPF Experiment.
82. G. Sanders and C. M. Hoffman, private communications.
83. W. J. Marciano and A. I. Sanda, Phys. Rev. Lett. 38, 1512 (1977).
84. A. Badertscher et al., Lett. Nuovo Cim. 28, 401 (1980).
85. D. Bryman et al., TRIUMF Experiment; D. Bryman, private communication.
86. J. Ellis, M.K. Gaillard, and D.V. Nanopoulos, Nucl. Phys. B109, 213 (1976).
87. F. Gilman and M. Wise, Phys. Lett. 83B, 83 (1979).