## Search and Matching Models of Housing and Macroeconomic Activity

by

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To Abbajan and to Jinnah, my inspirations.

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**ABSTRACT** 

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This dissertation presents three papers on the macroeconomics of the housing

market. Motivated by the fact that U.S. housing data are characterized by highly

persistent and non-zero vacancies in both short run and long run, I develop dynamic

search and matching models of the housing market with flexible prices, that produce

coincidental steady state equilibrium vacancies and prices. The structural models

developed in this dissertation analyze the impact of macroeconomic shocks on demand

and on flow and stock supply of housing under differing assumptions about demand.

The basic model with homogeneous demand is successful in generating highly

persistent fluctuations in ownership vacancies at business cycle frequencies. For certain

shocks it can also reproduce the negative correlation between flow investment and

vacancies observed in the data.

In the second paper I apply the model to quantitatively analyze the impact of

Hurricane Katrina on the housing market in affected cities. I also present an empirical

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analysis using the hurricane's impact on local housing markets as a natural experiment. I posit that exogenous variation in housing stock due to the hurricane causes the demand curve for housing to shift out in affected cities. This yields an estimate of the elasticity of flow supply in housing. Model simulations successfully predict a rise in prices for disaster areas even though it overshoots in magnitude compared to actual data. The model also correctly predicts that residential investment increases after the hurricane and tracks actual data closely in magnitude.

In the final paper I present the matching model modified to include heterogeneous demand. Introducing heterogeneity provides the insight that sellers trade off vacancy duration and fit with buyer against price after a negative demand shock, thereby reducing prices less than they would be forced to in a Walrasian setting. I use this model to analyze the home buyer tax credit from the U.S. government in 2008-2010. I find that the credit raises total sales to include lower fit values that previously yielded a price with negative match surplus. The paper verifies that the policy is successful in reducing unsold vacancies while boosting average price above steady state.

# Chapter 1 Introduction

Despite the co-existence of searching buyers and available houses for sale, many houses for sale remain vacant for long periods of time, documented by the U.S. Census which reports the national average ownership vacancy rate at 1.56 percent of the housing stock over the period 1965 to 2009. This non-market clearing feature of housing contradicts Walrasian notions of equilibrium and seems to fit in with the idea of an equilibrium rate of vacancies, similar to equilibrium in search and matching models of the labor market. Given that long-run persistent vacancies are a readily observable feature of the housing market, their impact on price and on new investment in housing should be important but it is largely neglected in macroeconomic literature.

In this dissertation I focus on filling these gaps in the literature by exploring the role of search and matching frictions in housing markets. This dissertation contributes two original dynamic structural models of decentralized search and matching in the housing market that incorporate demand and supply of existing vacancies and new residential investment and relate them to observed price and time on market.

In the first, basic, search and matching model with homogeneous searchers, I show the propagating effect of persistent vacancies on new investment in response to aggregate shocks. In the second model, I introduce heterogeneous searchers and show the impact of both search-and- matching frictions and heterogeneous demand on price and sales.

I analyze two separate empirical episodes affecting U.S. housing markets to illustrate policy applications. The basic homogeneous searchers model developed in Chapter 2 is applied in an extensive case study of the impact of Hurricane Katrina on affected housing markets in Chapter 3. In Chapter 4, I use the search and matching model with heterogeneous searchers and use it to analyze the impact of the federal

housing tax credit scheme run by the U.S. government between 2008 and 2010, in the aftermath of the twin subprime mortgage and financial crises.

A dynamic matching framework is a natural approach to modeling a market with equilibrium vacancies because it introduces a real friction that impedes market clearing even with flexible prices. The model I present here is based on the labor search literature such as the classic search models by Mortensen (1970) and Pissarides (1985). A matching framework captures the time delay involved in searching for a vacant housing unit that exactly meets the requirements of a searcher, in the presence of informational and other frictions. The thrust of the matching friction is to impose 'stochastic rationing' (Pissarides, 2000) on searchers and sellers, whereby searchers are probabilistically matched with vacancies for sale. With a positive fraction of searchers and vacancies left unmatched each period, matching technology also gives rise to a so-called trading externality on the long side of the market. That is, if the number of vacancies rises relative to the number of searchers in the market, the hazard rate for vacancies will fall.

The search and matching model presented in this dissertation is able to concurrently produce a dynamic negative relationship between market price and time-to-sale and the negative effect of vacancy duration on new investment that are both commonly observed in the housing market. These features of our matching model are discussed in Chapter 2, in which I present the theoretical model.

The search and matching model constructed in Chapter 2 analyzes a market for owner-occupied housing. Searchers and sellers meet in a decentralized market to trade and the meetings are governed by an exogenous matching technology that imposes stochastic rationing and trade externalities on agents. Profit-maximizing firms construct new housing units for sale each period, facing upward sloping marginal costs. The model does not address the role of land in the housing market. Potential buyers do not distinguish between new and existing housing units so construction firms care about the expected duration of vacancy, which is increasing in the total number of vacant houses already on the market. Thus the existing stock of vacant housing units directly impacts the flow of new housing because the longer is the expected duration of vacancy the lower is equilibrium new investment.

The model assumes risk neutrality and perfect foresight across agents and assumes homogeneity of searchers and vacancies. Prices are negotiated through Nash bargaining by each matched searcher-seller pair. A match refers to the meeting of a searcher and seller that generates positive surplus for each. Given their outside options, each pair agrees on a mutually acceptable division of this surplus from a successful sale in the current period, rather than waiting to find another transaction opportunity. Therefore, when a searcher-seller pair meets it results in a sale.

In steady state equilibrium, a negative relationship exists between the vacancy rate and the search rate, which is the housing market equivalent of the Beveridge curve in labor matching models. The model is able to produce this relationship, which has also been documented to exist empirically in the housing market by Peterson (2009).

The structural relationship between vacancies and supply of new housing units in the model also captures a stock-and-flow characteristic of housing supply whereby a shock to the housing stock can result in a much larger and persistent adjustment of flow investment in new housing mediated by persistent and durable vacancies. This is particularly well illustrated in Chapter 3 where I use the model to analyze the shock to the housing market from Hurricane Katrina

Chapter 3 first undertakes an empirical difference-in-differences estimation exercise that uses Hurricane Katrina as a natural experiment. The treatment due to Katrina is the destruction of a large share of the existing housing stock and extensive outward migration from affected areas. Next, I simulate these shocks to housing stock and population in the model and compare the predictions of the model on price, investment and vacancies against my empirical findings.

I use longitudinal data on residential new building permits, net migration, ownership vacancy rates and the House Price Index (HPI) for a total of 57 MSAs that lie within a 550 mile radius of New Orleans, Louisiana, which was the epicenter of the hurricane. In this sample, 13 MSAs receive the treatment. I show that in metro areas closest to the epicenter of Hurricane Katrina, house prices rise by almost 7 percent after the shock and have continued to rise steadily till 2008Q4. Across MSAs in which one or more counties were declared disaster zones by FEMA, residential new building permits rise by 19 percent over the previous year after the shock. Moreover, the impact on prices

and investment is lower, the greater the distance at which a city lies from New Orleans. I also use the exogenous variation in prices from the shock to housing stock to obtain a post-disaster estimate of the elasticity of flow supply of housing in hurricane-affected areas of 2.3. I use this estimated elasticity in the model simulations to follow.

With a few modifications to the theoretical model, I include Hurricane Katrina as an unanticipated shock to the housing market in each affected city. The shock moves each market out of its steady state equilibrium with a sudden reduction in the housing stock and sudden change in population from net migration. Out-migration from affected cities is comprised of all steady-state renters and a fraction of previous owner-occupiers who are rendered homeless by the hurricane. The remaining owner-occupiers whose units are destroyed in the shock stay in the city and become active searchers for new housing. Since the inflow into the search population exceeds its steady state level, the hurricane shifts out net demand for new investment in affected cities. I use the hurricane to identify the slope of the flow supply curve, given that it causes a shift in demand for housing. I calibrate the size of the drop in housing stock and population to match what is actually observed. I match remaining moments of the model to moments of the data and simulate the model to generate artificial data on prices, flow investment and vacancies for 57 MSAs. Finally, I re-estimate the difference-in-differences regressions using these artificial time series and compare coefficient estimates based on the simulated data with estimates from the actual data.

The model successfully predicts a rise in prices for disaster areas but overshoots in magnitude compared to actual data. Across all affected cities, the average predicted quarterly price increase in simulated data is 39 percent from 2005Q3 to 2005Q4, compared to 3.4 percent in actual data. In actual data, prices in the affected metro areas continue to rise after the hurricane while prices decline towards steady state in the model, producing the typical shape of an impulse response in a perfect foresight model. That prices overshoot in the model relative to the data is unsurprising, given that the model abstracts from labor and financial markets.

The model predicts that residential investment increases after the hurricane and tracks actual data closely. For affected MSAs, permits rise by 33 percent in a year in simulations while they rise by 19 percent in the data. While in actual data, building

permits rise by 36 percentage points from 2005 to 2006 for MSAs within 100 miles of New Orleans, in the model the change in flow housing investment is about 49 percent.

Thus, the matching model provides a set of structural relationships in the housing market in cities affected by the hurricane that are demonstrably reasonable. It is able to capture the dynamic path of new construction and prices in the aftermath of migration and reduction in housing stock due to Hurricane Katrina, in sign and significance, though not necessarily in magnitude.

Other matching models of housing mostly dwell on the observed negative relationship between sales price and existing structures while my matching framework focuses on the real side of the housing market in a recursive dynamic environment, specifically on new housing supply and existing housing stock. My model's success in predicting the path of new real investment shows that it is a good starting point to analyze new housing supply when existing vacancies and price are both factored into the producer's decision. This chapter provides evidence to support the structural relationships that the matching model posits and illustrates the propagating effects of matching frictions on demand and supply of housing.

In the final chapter of this dissertation, I modify the basic search and matching model of housing to allow for heterogeneous matches. Matches are now heterogeneous in quality of fit, which is reflected in the size of the match surplus. I assume that the quality of housing units is homogeneous from the perspective of quality of materials, total living space and access to amenities. These are all aspects of quality that one would expect to be incorporated into house prices.

In this paper, the fit of a match captures instead the differences in how well a housing unit meets the searcher-buyer's preferences on aspects that are a matter of individual circumstance or taste. I assume that surplus is increasing in quality-of-fit, which is only realized after a match has been made. Hence, when paired with a low quality-of-fit match, the searcher-buyer may receive a bigger surplus by choosing to extend his search and wait for a better match. Therefore, only matches with a surplus greater than some threshold value will materialize as sales per period. The additional friction of searcher heterogeneity allows me to model the decision of searcher-buyers on match quality, when given the incentive of a tax credit. It also illustrates a composition

effect on price, whereby sellers mitigate downward adjustment of sales prices by accepting a longer wait for to match with a better fit if a tax incentive is offered to searcher-buyers, compared to the counterfactual.

In 2008, federal housing tax credits for new homebuyers were introduced as part of stimulus measures to revitalize the U.S. housing market in the aftermath of the subprime mortgage crisis. They were enacted as part of the 2008 American Housing Rescue and Foreclosure Prevention Act; 2009 Worker, Homeownership and Business Assistance Act; and 2009 American Recovery and Reinvestment Act.

The homebuyer's tax credit scheme poses a macroeconomic shock that affects the cost of buying a new house. I show the impact of the credit shock on the length of search for potential buyers and on the threshold quality of matches and match surplus. By distorting the effective surplus of a given match upwards independent of the fit, the credit lowers the threshold quality of matches in the market and consequently raises the number of sales albeit with higher variation in quality of fit. As people become more willing to accept lower quality matches to take advantage of the credit, the duration of search also falls. Hence, all else equal, the model predicts that the tax credit effectively greases the wheels of the housing market and raises both sales and new flow investment in the market.

Previous search models of housing have been preoccupied mainly with the behavior of house prices given a fixed housing stock. Wheaton (1990) presents a static unit-switching search model of housing vacancies to rationalize the existence of long run equilibrium vacancies. He shows that longer expected matching times are inversely related with final sales price given a fixed housing stock. Albrecht et al. (2007) attribute the negative relationship between price and time-on-market to increasing desperation on the part of both searchers and sellers, while ignoring new construction altogether.

Krainer (1999) and Novy-Marx (2009) focus on price behavior in hot and cold markets. Krainer (1999) shows that prices do not fluctuate as much as they should due to sellers' preoccupation with maintaining liquidity and their expectations of the aggregate state of the economy. Novy-Marx (2009) argues the same thing while emphasizing the role of self-reinforcing behavior by sellers in hot and cold markets that creates "feedback loops", and assuming finite elasticity of entry with respect to expected value of

participation. Ngai and Tenreyro (2008) develop a search and matching explanation for seasonal fluctuations in house prices.

Peterson (2009) uses a search and matching explanation for an empirical Beveridge curve relationship between the vacancy rate and housing demand growth rate as proxied by the rate of growth of owner-occupied housing units. Finally, Piazzesi and Schneider (2009) try to explain the correlation between buyer sentiment and observed prices by approximating momentum trading in a search and match framework.

Overall, the search and matching models presented in this dissertation present a dynamic framework with different types of demand heterogeneity. This allows me to analyze the impact of aggregate shocks to demand on key aspects of demand and supply, including time to sale, price and new investment. In this way, I am able to bring together different themes that have been explored in the housing literature so far in one cohesive framework that is, moreover, suitable for policy analysis.

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# Chapter 2 A Search and Matching Model of the Housing Market

#### I. Introduction

This paper relates vacancies for sale in the housing market to new residential investment in housing units in a search and matching framework. Persistent vacancies are a documented feature of the housing market and contradict the idea that flexible prices are sufficient to allocate housing units across potential buyers. A matching framework is perfectly suited to analyze the role of vacancies in the housing market, in which both price and the degree of matching friction mediate the allocation of vacant units across searchers. Since housing units are highly durable, the existence and persistence of vacancies naturally must have an impact on new construction. Indeed, empirical evidence shows that while new construction is positively correlated with price, duration of vacancy seems to matter as much or more (Di Pasquale and Wheaton, 1999).

In this paper I construct a dynamic macroeconomic search and matching model of housing that incorporates the existing stock of vacant units and investment in new units in the supply of housing for sale. This implies a structural relationship between vacancies and new construction that depends on the degree of friction in the market, and hence duration of vacancy. Interaction between vacancies and supply of new housing units reflects the stock-flow nature of housing supply whereby a shock to the housing stock can result in a much larger and persistent adjustment of flow investment in new housing. Firms care about future prices and the time to sale for vacancies in the model, while buyers are bound by the market technology in how frequently they can expect to match. Hence, the speed of adjustment towards the steady state after a shock depends on search and vacancy duration, which underpin the propagation role of vacancies in the market.

The model constructed here analyzes a decentralized market for owner-occupied housing governed by an exogenous matching technology. Profit-maximizing firms

construct new homes for sale each period. Searchers looking for vacant housing units to buy do not distinguish between new and existing homes so construction firms care about the expected duration of vacancy for houses, which is increasing in the total number of vacant houses already on the market. Thus the existing stock of vacant housing units directly impacts the flow of new housing because the longer is the expected duration of vacancy, the less responsive is flow housing supply to price changes.

I numerically solve the model by log-linearization around its steady state equilibrium, using the algorithm developed by Andersen and Moore (1985). Finally, I present impulse responses from the log-linearized model to shocks to population and firms' production costs to analyze the model's dynamic mechanisms.

#### II. Housing Demand

The model can be used to describe a single housing market, whether at the city level or at the level of the macroeconomy. The market for non-rental housing has a fixed population of N households, H total existing houses in stock of which a fraction depreciate completely each period, and a finite number of firms that produce new houses. New and existing houses are bought and sold in the market without distinction over their vintage.

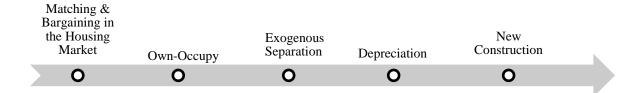
The housing market is mediated by a matching technology that dictates the rate at which searchers and sellers can meet and transact per period. Households are assumed to have homogeneous preferences and all agents are risk neutral. Once a searcher and seller match, the price at which they agree to transact is determined by Nash bargaining.

#### *Market Timing and Matching Technology*

The model is specified in discrete time, where a period can be thought of as lasting 13 weeks or a quarter and the sequence of events within a period is illustrated in the timeline below. The period begins with an existing total stock of housing H, of which there are V already lying vacant, and an existing body of searchers S.

The housing market meets at the start of the period and matches  $\mu S$  searchers with qV vacancies. All remaining searchers and sellers wait until the next period to try to match again. Because a match creates surplus for the searcher and seller, successfully

matched searchers will purchase vacancies and become owner-occupiers rather than bear the cost of waiting at least another period. Unmatched searchers are absorbed by exogenous temporary lodgings until next period, so the flow cost of searching can be thought of as period-by-period renting at a given rate *R*.



Total matches per period, M, are determined by a concave function, M(S,V), which is increasing in both arguments. To solve the model I assume that this matching technology takes the Cobb-Douglas functional form with constant returns to scale,  $M(S,V) = \overline{m}S^{\gamma}V^{1-\gamma}$ , where S is the number of households searching for a house, V is the number of houses for sale and  $\gamma$  is the elasticity of total matches with respect to the number of searchers.

Standard search and matching models of the labor market feature 'stochastic rationing' (see Pissarides, 2000), whereby there is a positive chance that an agent will simply not be able to find a counterparty. This paper also incorporates such rationing, which mechanically captures the time to search that must be taken by a household in the real world before it can find a match suitable for its needs. When a searcher does match with a vacancy, a surplus is created because the house fits the searcher's requirements and tastes and the seller finds a credible buyer.

When a house for sale does not match with a searcher in a particular period, that vacancy is again available for sale next period. The hazard rate for a typical vacancy is the ratio of total matches to the total number of vacancies for sale, which given the matching function is  $q(\theta) = \bar{m}\theta^{-\gamma}$ , where  $\theta = \frac{v}{s}$  is the relative supply of vacant houses. Hence, for the seller, the likelihood of matching with a searcher is decreasing in the total number of vacancies in the market.

Analogously, the representative searcher faces a positive probability of not matching with a vacancy in a given period. The hazard rate for a searcher is simply the ratio of total matches to the number of searchers in the market,  $\mu(\theta) = \frac{M}{S} = \overline{m}\theta^{1-\gamma}$ . As relative supply increases, the rate of successfully matching with a vacant house rises for the representative searcher.

This matching technology is standard in the literature. Both  $\gamma$  and  $\overline{m}$  determine the number of matches made, given the total number of searchers and sellers in the market. The matching elasticity with respect to searchers is assumed  $0 < \gamma < 1$  so both searchers and vacancies are necessary inputs for a match. Given some  $\gamma$ ,  $0 < \overline{m} < 1$  captures the efficiency of the matching technology, which ultimately determines the speed of adjustment after a shock given  $\theta$  and  $\gamma$ .

The matching function rations houses for sale across housing demand distinctly from the market price mechanism. In classical theory, if the number of sellers outweighs the number of searchers or vice versa,  $\theta \neq 1$ , market price will adjust to reach an equilibrium point where supply and demand are equal. In a matching framework, even if  $\theta = I$  the market mechanism imposes non-clearing. Moreover, the relative strength of supply of vacant houses to demand,  $\theta$ , imposes an additional so-called 'trading externality' on agents. For any  $\theta \neq 1$ , the matching technology will favor the short side of the market with a higher rate of matching.

Once matching is complete, a fraction  $\lambda$  of owner-occupiers are exogenously separated from their existing houses by a move shock. These households become searchers next period and their houses enter the pool of vacancies for sale. Notice that while in aggregate the number of households separated from their existing housing units each period is deterministic at a constant Poisson rate of separation  $\lambda$ , the move event is random at the individual level.

Following the move shock, a further fraction  $\delta$  of remaining owner-occupiers lose their houses to depreciation. These households must also search next period, but do not become sellers. The assumption of complete depreciation of a fraction of the housing stock each period has no effect on the results of the model since old and new houses are perfect substitutes in the model and the equilibrium market price is independent of vintage.

Finally, construction firms build new houses for sale *X* given expected future demand for housing. New houses are added to the pool of vacant housing available for sale next period.

#### Population and Searchers

Total population N is held fixed in the model but is subject to exogenous migration shocks that will affect demand for ownership vacancies. The law of motion for population is given by equation [1], where IM denotes in-migration and EM is outmigration and both are zero in steady state. New immigrants must wait one period before they can match in the market and similarly emigrants must wait one period to sell houses they are making vacant.

$$N_{t+1} = N_t + IM_t - EM_t \tag{1}$$

The evolution of searchers in the market is dictated by realized matches, separations and net migration each period, per equation [2].

$$S_{t+1} = (1 - \mu_t)S_t + IM_t + (\lambda + \delta(1 - \lambda))(N_t - (1 - \mu_t)S_t)$$
 [2]

The total number of searchers entering the market at the start of period t+1 sums unmatched searchers from the previous period, new immigrants into the city, and all matched owner-occupier households in the total population who are forced to move due to exogenous separation or depreciation. For simplicity, I assume that existing searchers do not emigrate or otherwise 'drop out' of searching in favor of their temporary accommodation. Hence, the only possible exit from search is into a state of ownership. I assume that separated homeowners cease to occupy their housing units and must become searchers when they create a vacancy. I thereby maintain homogeneity of all searchers and all sellers on each side of the market.

#### III. Housing Supply

The distinction between stock and flow supply is significant for housing given its durable nature. In this section I model the relationship between inventory of unsold vacancies, occupied housing and new construction. Chapter 3 analyzes the impact of a large negative shock to total housing stock on new construction, where I show that, all else equal, flow supply must adjust by several orders of magnitude more than fraction of housing stock destroyed given that the ratio of equilibrium vacancies to existing stock is relatively small.

In a matching framework vacancies behave as a propagation mechanism for aggregate shocks to housing, because of its durable nature. Hence, the effect of an aggregate shock on the market is not restricted to the jump response of prices and new investment alone. Instead, there is a stock effect that necessitates a protracted adjustment to return to steady state. For instance, when the market receives a negative demand shock, prices and investment jump down, but housing units that have already been produced or lie vacant for sale do not disappear. Instead, as vacancies increase after the shock, they dampen new investment for several periods until the excess inventory can be eliminated at the current rate of sales in the market. Because housing units have longer shelf lives than other products, one can expect new housing investment to experience a far slower return to normal production levels than for non-durable goods and in this way the shock is propagated over several periods after its actual occurrence. A matching model captures this by modulating the rate of matches and resulting sales according to changes in housing inventory relative to demand.

#### Vacancies & Housing Stock

Vacancies and total housing stock are predetermined variables in the model, like searchers and total population. The equation of motion [3] shows that vacancies evolve according to matches made per period, newly constructed housing and existing housing that is made vacant by a separation or out-migration shock, denoted by  $\lambda$  and EM, respectively.

$$V_{t+1} = (1 - q_t)(1 - \delta)V_t + \lambda(1 - \delta)(H_t - (1 - q_t)V_t - EM_t) + X_t + EM_t$$
[3]

Total vacancies at the start of period t+1 include unmatched vacancies from last period net of depreciation. The depreciation rate is denoted  $\delta$ .

The housing stock at t+1 is the sum of existing houses after depreciation and new construction, as in equation [4].

$$H_{t+1} = H_t(1 - \delta) + X_t$$
 [4]

I assume that the market does not distinguish between new and old housing units, so  $V_{t+1}$  is the total supply of vacant units available for purchase at t+1, for sale at a single market price.

#### New Construction

Flow supply of housing comprises of units that are entering the vacancy state because of a separation or out-migration shock and new construction (which I also call new investment). All new investment is undertaken by firms.

I assume that the representative investment firm faces an upward sloping marginal cost curve, implying convex costs of construction of new housing units. This is reasonable given that essential inputs to new construction are fixed in the short term, most notably land, by law or nature, which may be reclaimed in the long run. Moreover, while labor might be elastically supplied in residential construction, capital and material inputs may be inelastically supplied in the short run due to capacity constraints. Similar assumptions regarding the cost of construction are made by Glaeser, Gyourko and Saiz (2008), who assume that marginal costs rise with the amount of production linearly. Topel and Rosen (1988) provide empirical evidence for the U.S. showing that factor costs are positively correlated with construction levels.

In this paper, the stochastic nature of matching naturally forces optimizing firms to consider the asset value of an unsold vacancy because there is a positive probability that they will not be able to sell new housing units immediately. This asset value is simply the present discounted value of the sum of expected sales prices given expected hazard and depreciation rates. Firms thus care not only about the current price, but about the expected path of market prices when making the decision to invest in new housing

units today. At the optimal level of production marginal cost must equal the forward-looking expected value of a vacancy, denoted  $A_t$ . Letting  $X_t$  stand for investment in new housing, equation [5] is the firm's short run "supply" curve for new housing units.  $\xi$  is the inverse elasticity of supply and  $\alpha$  is a stochastic cost-shifter.

$$A_t = \alpha X_t^{\xi} \tag{5}$$

Units newly constructed in period t can only be sold when the market meets in t+1. If a newly built vacancy is not sold right away, firms may make positive profits on these units and the original cost of construction is effectively sunk. The interim cost incurred by waiting to sell a house is depreciation.

#### IV. Payoffs and Price

As illustrated in the timeline in Section II, a household can be in one of two states at any time: searcher or owner-occupier. Each searcher, owner-occupier and seller earns a flow payoff and a continuation value given expected hazard, separation and depreciation rates. Households and firms are homogenous, risk-neutral, forward-looking, and seek to maximize net payoffs. All payoffs in the model are expressed as dynamic recursive asset value equations in monetary units. These value equations are used to determine the surplus created for a buyer-seller pair by a successful match, which is divided between the two parties during price-setting.

Assuming that a household may only own one housing unit at a time, the present-value payoff,  $W_t$ , from being an owner-occupier is stated in equation [6].

$$W_t = R_t + b + \lambda (L_t + (1 - \delta)A_t) + \delta (1 - \lambda)L_t + \frac{(1 - \lambda)(1 - \delta)}{1 + r} E_t W_{t+1}$$
 [6]

An owner-occupier in period t earns a flow payoff equal to the sum of exogenous (imputed) rent  $R_t$  and an advantage of owning which is identical across households. This 'joy of ownership' is expressed in monetary terms by b, and can represent the ability to customize one's residence. It is the joy of ownership, in essence which creates match surplus, making ownership more desirable than renting. The continuation value of being an owner-occupier is the probability-weighted sum of all the events that may occur in the

interval [t,t+1). The owner-occupier experiences involuntary separation from his existing housing unit with probability  $\lambda$ , given which he will earn the present-value payoff of a searcher,  $L_t$  and if the house does not depreciate in the same time interval, he will also earn the present-value payoff of a seller,  $A_t$ . If the existing house depreciates, the owner-occupier only receives the searcher's payoff  $L_t$ . Finally, if the owner-occupier does not experience either the separation or depreciation events, he will earn the present discounted value of being an owner-occupier in t+1. Notice that the interest rate used to discount future returns is assumed exogenous and constant at r.

A household already in the search state will be matched by the market next period with likelihood  $\mu_{t+1}$ . Once matched, the searcher has a choice to buy the vacancy, or continue in the search state and will choose the higher of these two values. Hence, the payoff of searching,  $L_t$ , is the discounted probability-weighted maximum of the expected value earned by an owner-occupier  $E_tW_{t+1}$  net of the expected purchasing price  $E_tP_{t+1}$ , and the value of remaining in the search state next period if matched; plus the value of remaining a searcher if not matched by the market. This is written in equation [7].

$$L_{t} = \frac{1}{1+r} E_{t} [\mu_{t+1} \max\{(W_{t+1} - P_{t+1}), L_{t+1}\} + (1 - \mu_{t+1}) L_{t+1}]$$
 [7]

Since we ignore costs of home ownership other than depreciation for simplicity, the net value of ownership will always be higher than continued search. Thus the first term inside the parentheses can be rewritten to simply be the probability weighted net value of ownership  $\mu_{t+1}(W_{t+1} - P_{t+1})$ .

Whether an existing homeowner or a firm, a seller has three available choices at the start of period t+1 when the matching market meets. The first is to sell the vacancy at market price  $P_{t+1}$ . The second is to leave the unit vacant till the next period and receive a seller's present-value payoff in t+1 if it does not depreciate. Finally, sellers have an outside option in the rental market, analogous to searchers: the third choice for a seller is to convert the vacancy into a rental unit permanently and pay some z > 0 in one-off transaction costs at t, incurred in selling the unit to a rental management company or taking on rental management oneself. As expressed in equation [8], a seller faces these choices if his vacancy is matched at rate  $q_{t+1}$ . If the vacancy is not matched, with

probability  $(1 - q_{t+1})$ , the seller still has a choice between waiting to sell next period or converting to a rental unit permanently today.

$$A_{t} = \frac{1}{1+r} E_{t}[q_{t+1} \max\{P_{t+1}, (1-\delta)A_{t+1}, (\Gamma_{t+1}-z)\} + (1-q_{t+1}) \max\{(1-\delta)A_{t+1}, (\Gamma_{t+1}-z)\}]$$
[8]

The expected present discounted value from converting a vacancy into a rental unit is summarized by  $\Gamma_{t+1}=R_{t+1}+\frac{1-\delta^R}{1+r}E_{t+1}\Gamma_{t+2}$ . Rental units are assumed to depreciate at a higher rate than ownership units denoted  $\delta^R>\delta$ .

#### Price Determination

Once matched, a searcher in principle has a choice between purchasing a house now and remaining in the search state for a match next period, which is reflected in equation [7]. Since a positive surplus is created by matching with a vacancy and there is no heterogeneity in searchers or vacancies, a searcher will always make the purchase right away, if matched. The match surplus is simply the difference between the net payoff earned by an owner-occupier and that earned by a searcher. In other words, it is the net gain to a household from owning rather than renting.

Similarly, the seller's surplus from a match is his net gain from selling now over his other two choices. For current matches to proceed to sale there must be non-negative surplus for sellers. In this case, the seller's payoff equation [8] will include  $P_t \ge \max\{(1-\delta)A_t, (\Gamma_t - z)\}$  where  $(1-\delta)A_t > (\Gamma_t - z)$ .

Each searcher-seller pair will negotiate over how to divide total surplus generated from a match. I make the standard assumption that this negotiation takes the form of Nash bargaining, and solves the problem in equation [9].

$$P_{t} = \arg\max_{P_{t}} (W_{t} - P_{t} - L_{t})^{\phi} (P_{t} - (1 - \delta)A_{t})^{1 - \phi}$$
[9]

Equation [9] defines the bargained sales prices of a vacancy, where  $\phi$  denotes the searcher's bargaining weight. Since all searchers and sellers are homogeneous, there is a single market price at which all housing units are sold in period t.

#### V. Steady State Equilibrium

The model in steady state equilibrium is characterized by a constant level of searchers, S, constant level of vacancies, V, and consequently a constant ratio of vacancies to searchers,  $\theta$ . In steady state, population size is also constant, at N, with zero net migration, and there is a constant level of housing stock, H, that is maintained by flow investment in new units to replace depreciated units. Finally, W, L, and A denote the steady state values of owning, searching for and selling a house in the steady state. The equilibrium price, P, and relative supply of vacancies to searchers,  $\theta$ , satisfy equations [1] through [9] given the vector of parameters  $\Omega = [\alpha, \gamma, \overline{m}, \lambda, \delta, \phi, z, \xi]$ .

The Nash bargaining solution holds in steady state, so substituting steady state values for W, L and A yields the steady state bargained-price curve as a decreasing function of  $\theta$  in equation [10]. Notice that the bargained price curve does not represent the quantity of housing units demanded for a schedule of potential prices. Instead, it is the schedule of realized bargained prices for any given level of relative supply.

$$P = g(\theta; \Omega)(R + b)$$
 [10]

Let  $\psi = \lambda + \delta(1 - \lambda)$ . The function  $g(\theta; \Omega)$  is decreasing in  $\theta$ . It takes the form,

$$g(\theta;\Omega) = \frac{(1+r)(1-\phi)(r+\delta+\overline{m}\theta^{-\gamma}(1-\delta))}{(r+\delta)(r+\psi+\overline{m}\theta^{1-\gamma}\phi(1-\psi)+\overline{m}\theta^{-\gamma}(1-\phi)(1-\psi))}.$$

Notice that the steady state bargained sales price [10] in the matching model differs from the Walrasian free-market price of a housing unit by the portion of total match surplus that the searcher-buyer receives,  $\phi$ , and the degree of friction and trade externality imposed by the matching technology for any  $\mu \neq 1$  and  $q \neq 1$ . In the absence of these frictions and for the case where the full surplus is obtained by the seller, the steady state price reduces to the infinite sum of the present discounted stream of steady state rents and ownership benefits  $P = \frac{1+r}{r+\delta}(R+b)$ .

Substituting steady state values A and X into [5] yields the upward-sloping steady state new construction curve for new housing units in  $(P, \theta)$  space in equation [11]. This curve is the long-run equilibrium cost of construction of the marginal new unit, given the interest rate, rental return in the economy and steady state population.

$$P = h(\theta; \Omega) N^{\xi}$$
 [11]

Here, 
$$h(\theta;\Omega) = \frac{\alpha(\theta\delta)^{\xi}}{\bar{m}\theta^{-\gamma}} \Big( r + \delta + \bar{m}\theta^{-\gamma} (1 - \delta) \Big) \Big( \frac{\bar{m}\theta^{-\gamma} + (1 - \bar{m}\theta^{-\gamma})(\psi - \delta)}{\bar{m}\theta^{1-\gamma} + \psi(1 - \bar{m}\theta^{1-\gamma})} \Big)^{\xi} \quad \text{and} \quad \text{is increasing in } \theta.$$

Equations [10] and [11] are two equations in two unknowns P and  $\theta$  and the set of parameters  $\Omega$ . Their intersection yields the steady state values of P and  $\theta$ , given underlying profit-maximization by firms undertaking new construction. A lower bound on market price is given by the outcome where a seller is indifferent between selling today, waiting to sell and renting out the vacancy. This is the case where  $= (1 - \delta)A = \Gamma - z$ . By no arbitrage, the minimum possible sales price at which a market for ownership exists is one at which the steady state return to a seller from permanently renting out a housing unit is equal to that from selling it today. This is the price

$$P = A = \frac{1+r}{(1-\delta)(r+\delta^R)}R - \frac{z}{1-\delta}.$$

It is important to note that the flow supply curve is only the new construction curve, and signifies the profit-maximizing number of new units constructed given the expected future path of price. This model incorporates both stock and flow housing supply. The total number of units available for sale is a stock variable, given by the total number of vacancies in equilibrium which includes new construction by firms and previously owned housing units for sale.

Equilibrium is therefore described not merely by an equilibrium price, but also by the corresponding ratio of total vacancies to searchers in the market. A state of vacancy is essentially costly unemployment of the housing stock. Equilibrium P and  $\theta$  imply the equilibrium asset value of a vacancy A and the expected vacancy duration  $\frac{1}{m\theta^{-\gamma}}$ , which together dictate how many new units firms will optimally invest in.

#### Price and Vacancy Duration

The bargained price schedule [10] is downward sloping in  $(P, \theta)$  space. The meaning of this extends beyond the standard Walrasian law of demand. Here, a negative slope for the bargained price curve says that the higher is the equilibrium number of vacancies to searchers, the longer is vacancy duration for the representative house and the lower is the

equilibrium price received upon sale. In other words, there is a negative relationship between time-to-sale and sales price that is implied in the matching model. This relationship between price and vacancy duration is not restricted to steady state equilibrium, and will be demonstrated again in the dynamic context, discussed below.

#### Vacancy Duration and New Investment

The fact that the matching equilibrium is a combination of price and the number of vacancies per searcher has an important implication on the investment behavior of construction firms. As a consequence of the trading externality for sellers in the matching framework, the expected payoff of a vacancy and hence new investment is lower the higher is relative supply in the market.

illustrate, one can rewrite the new construction curve as  $=\frac{q(\theta)P}{(1+r)-(1-q(\theta))(1-\delta)}=\alpha X^{\xi} \ . \ \text{ As the relative supply of vacant housing rises, both } A \text{ and}$ new flow supply are lower since the hazard rate for vacancies is decreasing in  $\theta$ . Specifically,  $\frac{\partial X}{\partial a(\theta)} > 0$  and by the property of the matching technology,  $\frac{dq(\theta)}{d\theta} < 0$ . This occurs because the higher is the number of vacant units per searcher in the market, the lower is the likelihood of matching for an individual vacancy and the higher is expected waiting time for the individual seller, given by  $1/q(\theta)$ . Hence, because new investment depends on both P and  $\theta$ , the response of forward-looking firms to a higher price will be directly mitigated by the expected effect of a higher number of total vacancies on the time-to-sale for a new unit, both of which are summarized in A. This result holds both in steady state as well as in the dynamic context, discussed below. In the dynamic context, it is the dampening effect of total vacancies on new investment that lies at the heart of the propagation mechanism of vacancies in the housing market.

#### 'Beveridge' Curve Relationship

Labor matching models were formulated to rationalize a negative long run tradeoff between job vacancies and the rate of unemployment. The matching model developed here predicts a similar negative relationship between housing vacancies and search demand for housing in long run equilibrium. To derive this relationship, I start by rewriting equation [3] as the sum of flows out of and into total vacancies from t to t+1. Subtracting  $V_t$  from both sides of the equation and rearranging yields

$$\Delta V_{t+1} = -(\delta + q_t(1-\delta))V_t + \lambda(1-\delta)[H_t + (1-q_t)V_t] + X_t$$

The first term on the RHS is the outflow from vacancies due to depreciation and matching; the second and third terms are inflows into vacancies. In steady state equilibrium, the overall number of vacancies is constant, so period-by-period change (the LHS) equals zero. Simplifying and dividing both sides by the steady state housing stock results in an expression for the long run negative relationship between the vacancy rate and  $\theta$  in equation [12].

$$v = \frac{\lambda + \delta(1 - \lambda)}{1 - (1 - q(\theta))(1 - \lambda)(1 - \delta)}$$
 [12]

Next, we can express the steady state demand for housing as a percentage of the total housing stock, and define this as a new variable s. We can also rewrite =  $\left(\frac{v}{H}\right)/\left(\frac{s}{H}\right) = \frac{v}{s}$ . Then, it follows that because  $\frac{dv}{dq} < 0$  and  $\frac{dq}{ds} > 0$  from the matching technology, equation [12] represents a negatively sloped curve in (v,s) space. The curve captures the tradeoff between vacancies and searchers in a housing market, given matching technology and separation and depreciation rates. Figure 2.1 plots [12] in  $(v,\frac{1}{\theta})$  space, which I discuss further below.

For the equilibrium price and  $\theta$ , the housing Beveridge curve will yield the particular vacancy rate that prevails in the long run for a given rental return on housing units. From the intersection of [10] and [11], equilibrium  $\theta$  is given by

$$\frac{\overline{m}\theta^{-\gamma}(1-\phi)(1+r)(R+b)}{(r+\delta)\big(r+\psi+\overline{m}\theta^{1-\gamma}\phi(1-\psi)+\overline{m}\theta^{-\gamma}(1-\phi)(1-\psi)\big)}-\ \alpha(\theta\delta N)^\xi\left(\frac{\overline{m}\theta^{-\gamma}+(1-\overline{m}\theta^{-\gamma})(\psi-\delta)}{\overline{m}\theta^{1-\gamma}+\psi(1-\overline{m}\theta^{1-\gamma})}\right)^\xi=0.$$

In (v, s) space, this can be plotted as a straight line through the origin with constant slope  $\theta = \frac{v}{s} = (V/H)/(S/H)$ . The intersection of the Beveridge curve with equilibrium relative supply thus delivers the model's predicted vacancy rate in steady state.

An empirical investigation of whether a Beveridge curve exists for housing has been undertaken by Peterson (2009). He shows that a negative relationship exists between the vacancy rate and housing demand growth rate, using the rate of growth of owner-occupied housing units in the U.S. as a proxy for the latter. Peterson predicts a slope parameter between -0.4 to -1 for the Beveridge curve. <sup>1</sup> Notice that he is comparing the rate of change of housing demand with the stock rate of supply, rather than estimating a stable relationship between the levels of demand and supply of housing.

The matching model in this paper has static population, implying a constant level of search demand for housing in steady state equilibrium. Using the baseline calibration of the model in Table 2.1, I plot the matching model's prediction of the Beveridge relationship in Figure 2.1, where the vacancy rate is free to vary for a given range of values of  $\frac{1}{\theta}$ . The slope implied by the model for the baseline calibration for the log-linear version of [12] is -0.392, given  $\theta = 1$  in steady state equilibrium.

#### VI. Solution and Model Dynamics

This dynamic matching model has a total of 18 equations, including two shock processes (see below). Unlike labor matching models, there are two primary backward looking variables in the model: vacancies and searchers. To achieve the saddle point equilibrium, new housing investment and price play the role of corresponding jump variables. Prices, asset values and payoffs are all forward-looking and rationally determined.

I log-linearize this large system of forward and backward looking equations and solve it numerically using the Anderson-Moore algorithm (Andersen and Moore, 1985). Using given parameter values, the algorithm computes the reduced form VAR(1) system for a dynamic structural model and imposes restrictions on initial conditions as well as

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<sup>&</sup>lt;sup>1</sup> That is a vacancy rate that is lower by 0.4 percentage points corresponds to an increase in the growth rate of housing increases by 1 percentage point.

stability and historical conditions that rule out explosive solutions and solve for a saddlepoint steady state equilibrium. The reduced form system produced by the algorithm is then used to produce impulse responses of the model to shocks in the vicinity of the steady state.

We can admit multiple potential sources of aggregate uncertainty in the model by allowing several variables to stochastically vary over time: namely, the exogenous separation rate  $\lambda$ ; the rental rate R which can be interpreted as the productivity of a housing unit in monetary units; or the firm's cost of construction through the shifter  $\alpha$ . Explicitly included in the model equations already are net migration shocks, which I have discussed above.

In this section, I briefly discuss the calibration of parameter values to solve the system numerically. This is followed by a discussion of dynamic behavior of the matching model in response to in- and out-migration shocks and a cost shock. In that analysis I present impulse response functions of the log-linearized model and relate these to the analytically derived price, flow supply and Beveridge curves.

#### Calibration

The model has a total of 10 free parameters and 1 exogenous variable, R. I use two degrees of freedom to pick the steady state equilibrium ( $P = 1, \theta = 1$ ) for simplicity, which fixes total population N and the benefit of ownership b. The remaining free parameter values are calibrated to observed data, summarized in Table 2.1. I also specify the steady state values of the shock variables that I use.

Given  $\theta = 1$ ,  $\overline{m}$  is calibrated to yield a search duration for the representative household in the model that matches the median observed length of search for home buyers in the U.S. according to the National Association of Realtors of 8 weeks.

I choose  $\gamma$ , the elasticity of the matching function with respect to searchers, to be 0.5. This implies that searchers and vacancies are relatively substitutable in the matching function. As Shimer (2005) clarifies, substitutability between inputs in the matching function dictates the degree of flexibility of equilibrium  $\theta$ . In essence, the more substitutable are the inputs the steeper is the Beveridge curve. Shimer's example is for the labor market: suppose labor productivity rises relative to the unemployment benefits

and the cost of advertising a new vacancy for firms. This raises the return to vacancies, relative to unemployment in the market. Then, the more substitutable are unemployment and vacancies in the matching function, the more the market can switch away from unemployment towards vacancies and raise equilibrium matches which raises job creation. Thus the efficiency of matching depends not only on the  $\overline{m}$  factor, but also on  $\gamma$ .

In a centralized market the degree of substitutability depends only on the parameters of the matching function. However, Hosios (1990) shows that for a decentralized framework, both the Nash bargaining weight of searchers and their weight in the matching function will dictate the flexibility of the market to substitute between searchers and vacancies in equilibrium. To achieve the same solution as the social planner in a decentralized market, a necessary and sufficient condition is that the searchers' Nash bargaining weight should equal the Cobb Douglas searcher elasticity of the matching function (Hosios, 1990). I therefore choose  $\phi$  to equal  $\gamma = 0.5$ . The results of the model are invariant to changing  $\phi$  and  $\gamma$ .

The frequency of separation for owner-occupiers from their existing housing unit is set to match the 6-year median length of stay in one house in the U.S. reported by the National Association of Realtors.

The depreciation rate is fixed to 1.14% per year, which implies a service life of 80 years for the average house, as reported by the Bureau of Economic Analysis (February 2008).

The real risk-free rate is fixed to the time-average of the real federal funds rate. The steady state value of R is fixed to the mean of the average range for real rents observed in U.S. data. This is calculated by Davis et al (2008) as 3.5 percent to 5.5 percent of the value of a house.

Finally, a neutral steady state value  $\alpha = 1$  is fixed for the shifter in the firm's marginal cost. The firm's elasticity of supply for new units is fixed based on my own estimates using MSA-level building permits as an instrument for new investment, for a panel of 57 MSAs. <sup>2</sup>

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<sup>&</sup>lt;sup>2</sup> A detailed description of this estimation exercise is provided in Chapter 3.

#### Shocks to Demand: In- and Out-Migration

Consider an unanticipated non-transitory in-migration shock to the market of 1 percent of the steady state population. The shock moves the market out of its steady state and new entrants to the market enter the pool of searchers immediately. The bargained price jumps up on impact to reflect the higher demand for housing. This is effected by a shift of the 'demand' curve for housing [10] to the right, along the upward sloping new construction curve [11], as illustrated in Figure 2.2. The shift of the bargained price curve from  $D_0$  to  $D_1$  shows that desired vacancies per searcher  $\theta^*$  exceed the equilibrium total 'vacancy rate'  $\overline{\theta}$ , inducing firms to create new housing units.

In (v, s) space the increase in investment units is reflected in a pivot of the equilibrium new construction curve. Initially, the market is in equilibrium at point A. Once the shock occurs, the number of searchers jumps up and vacancies fall, moving the market to the southwest of A (marked by an asterisk). Since the increase in demand is permanent, firms respond by raising the number of units constructed and this raises the equilibrium  $\theta$ . With more vacancies per searcher, the hazard rate for searchers rises and for constant probabilities of separation and depreciation, the new equilibrium is at point B with a higher vacancy rate and a lower ratio of searchers to housing stock. The inmigration shock permanently raises the asset value of vacancies to firms and permanently lowers the value of searching. Just as in the labor example by Shimer (2005) mentioned above, the decentralized market moves to substitute the lower-value search activity for higher-value vacancies, to point B on the Beveridge curve. Since the model is calibrated to satisfy the Hosios (1990) condition, this is in fact the efficient outcome.

Turning to the dynamic predictions of the model, Figure 2.3 presents impulse response functions (IRFs) from the linearized model given an unanticipated permanent in-migration shock equal to 1 percent of the population. These allow us to trace out the adjustment paths of the key variables in the model after the shock. The solid lines in each sub-figure represent results for the benchmark calibration in Table 2.1; the dashed lines with an asterisk marker show results for a higher rate of matching, such that search duration is only 4 weeks per year (i.e. lower  $\overline{m}$  given  $\theta=I$ ); the dotted lines with square

markers show results for higher elasticity of investment,  $\frac{1}{\xi} = 10$ . The in-migration shock is not reversed, which is reflected in the population level being higher by 1 percent.

In Figure 2.3, firms see the asset value of vacant units jump up and immediately raise investment in housing by twice as much (reflecting the assumed elasticity of flow supply). In the meantime, as the relative number of vacancies to searchers falls, the hazard rate for vacancies rises and outstanding inventory in the market quickly dissipates so vacancies initially fall. Over time, as new housing units come on the market, vacancies rise, searchers fall, and prices also fall.

Notice that the inverse relationship between vacancy duration and price holds in the dynamic context as well as in comparative statics. The dynamic path of  $P_t$  is inverse to the path of  $\theta_t$ . Once  $\theta_t$  begins to rise,  $q_t$  starts to fall, vacancy duration is rising and  $P_t$  is falling towards long run equilibrium. Similarly, new investment does not adjust to the shock all at once on impact, given the expected path of  $\theta_t$ . Investment firms only adjust construction by as much as they can expect to sell in each period, depending on  $q_t(\theta)$  and ultimately the matching technology.

The total length of time that the market takes to adjust to steady state equilibrium after the shock depends on both the matching efficiency and the elasticity of investment. Altering the rate of matching through the matching efficiency parameter illustrates the role of vacancies in propagating the shock and affecting the path of flow investment. As the dashed lines with asterisk markers in Figure 2.3 show, if  $\overline{m}$  is doubled, the dynamic path of flow investment lies above the solid path depicting the benchmark calibration: if searchers and vacancies are matched faster, investment responds slightly more quickly at each point in time to the shock. The dynamic path of price, vacancies and  $\theta$  is steeper when search duration is lower. Notice, however, that some of the gain in speed of adjustment of investment to the demand shock is offset by a slightly higher percentage of households separated from their current units because of the move or depreciation shocks. Hence, the higher  $\overline{m}$  does not make itself apparent in a lower path for searchers compared to the baseline case in Figure 2.3.

The second experiment in Figure 2.3 (the dotted lines with square markers) illustrates that the degree of elasticity of flow investment can dramatically influence how many vacant units are available to absorb additional demand. An elasticity of 10 for flow

investment implies the IRF paths shown by the dotted line compared to the benchmark case. An extreme case of this is shown in Figure 2.4, which plots results for  $\frac{1}{\xi} = 1000$ , a good approximation for infinite elasticity. Here, the market adjusts in as little as 3 years because flow investment jumps up by 125 percent of steady state level on impact and thereby *raising* vacancies on impact, rather than running them down immediately when newly in-migrating searchers flood into the market.

Figure 2.5 shows IRFs from a permanent out-migration shock of 1 percent of the population. This helps to further illustrate the interaction between inventory of vacant units and flow supply. Notice that the response of flow investment is symmetric (for the benchmark calibration in Table 2.1) for positive and negative migration shocks. Investment rises and falls by 6 percent relative to steady state, respectively, which is a reflection of the forward-looking optimizing behavior of firms in the model. Since the expected path of prices is symmetric between the two shocks, and given the rate of matching in the market, firms respond symmetrically in each case.

Comparing Figure 2.3 (benchmark solid plots) with Figure 2.5, shows that vacancies perform the function of sponging up excess demand or supply, falling or rising in varying degree in response to in-migration or out-migration of 1 percent of the population, respectively. When there is an influx of new searchers in the housing market, inventory is depleted on impact and stays below long-run equilibrium in the near horizon while all new units are absorbed by searchers. When homeowners put their units up for sale and leave the market altogether, causing a negative shift of the housing bargained price "demand" curve, inventory rises on impact as expected and flow investment falls well below steady state given the expected path of searchers and  $\theta$ . Even though the initial increase in vacancies is roughly halved in the first quarter after the shock, vacancies remain above steady state level for several years after the shock, illustrating their propagation role. Because of the matching friction, price adjustments cannot assist in dissipating vacancies altogether. Instead, vacancies and  $\theta$  remain higher than steady state, though falling over time, and investment remains below steady state, though rising, in the near horizon.

The predominant feature of a matching framework is that there is a positive vacancy rate for housing units in both short and long-run equilibrium at the equilibrium

*price*. As the accelerated paths of investment, price and vacancies in Figure 2.4 shows, even when investment has fully adjusted to the positive demand shock, matching frictions entail a non-zero vacancy rate in steady state.

## Supply Shock (a)

I next consider the model's dynamic response to a change in productions conditions, reflected in a transitory but persistent increase in marginal cost through a 1 percent rise in the cost shifter  $\alpha$ . I assume that the shock decays at the rate of 5 percent per year.

When the shock occurs, new investment immediately contracts, which is reflected in a shift leftward of the static new construction curve in Figure 2.6. Prices rise to reflect the increase in marginal construction costs, raising also the value of a vacancy. In Figure 2.6, the static Beveridge curve diagram shows that the market moves to a point where vacancies are lower because fewer new units are produced and the ratio of vacancies to searchers falls. The shock is transitory, so price adjusts over time and the market moves slowly back to its original equilibrium at A.

Figure 2.7 shows the impulse response functions of the log-linearized model. As  $\theta$  falls, the hazard rate for vacancies rises. As new supply contracts when the shock occurs, the market runs down existing inventory of vacant units to adjust to the shock.

The main counterintuitive prediction of the model in this scenario is that the value of ownership falls when housing units are in relatively short supply. By rights, rent and hence the value of owning should also rise when new supply contracts. However, in Figure 2.7 the opposite is true. Thus, searchers rise in number as more households become indifferent between searching and owning.

This small example illustrates how a worsening of credit or other conditions affecting the cost of production for construction firms can cause the market for ownership housing units to contract. To compensate for the higher cost of production of housing, the return on housing must rise not only for sellers (through price) but also for buyers (through rent) in order for the market for ownership to exist.

#### VII. Conclusion

The chapter develops a new macroeconomic matching model of housing that is unique as it incorporates the supply of new units alongside the stock supply of housing. I analyze steady state and dynamic equilibrium and show that after a demand shock, it is both the degree of matching friction and the elasticity of supply that determine the speed of adjustment.

Persistent vacancies have a propagating effect on new construction given their durable nature, in the presence of a matching friction. That is, the market technology dictates that only a fraction of vacant units are matched with a fraction of searchers each period. Because housing units are durable, the remaining unmatched vacancies persist over time and remain on sale the next time the market meets. When a shock occurs, the speed of matching determines the pace at which changes in housing demand are satisfied. Not only do available vacancies adjust slowly to the shock at the rate determined by the market technology, but due to their durability they also impact how much new investment responds per period.

The model is presented with two particular assumptions that affect the scope of analysis. Firstly, I assume no population growth, which diminishes the role of a Beveridge curve relationship in the model's applications. Second, I assume fixed rent here. However, Chapter 3 of this dissertation reconsiders the role of rent in the model's underlying mechanisms, using data on median rents across the U.S. at the MSA level.

This model of ownership vacancies and investment has the potential to shed light on the decision between renting and buying once it is combined with a counterpart for rental vacancies. This is an important part of my research agenda stemming from this dissertation.

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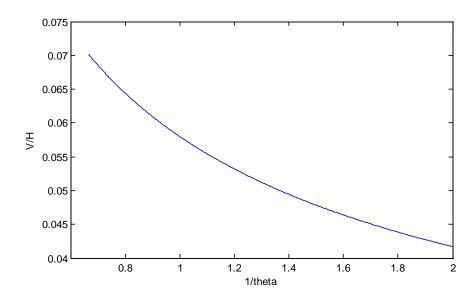
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**Table 2.1 - Calibrated Values for Parameters of the Matching Model** 

Symbol	Description	Value
$\overline{m}$	Matching efficiency	0.125/week; 0.999/year
γ	Searchers' elasticity in match function	0.5
α	Supply shifter	1
$1/\xi$	Elasticity of flow supply	2
λ	Separation rate	0.2 per year
δ	Depreciation rate	0.0114 per year
$\phi$	Nash bargaining weight of households	0.5
r	Real discount rate	0.02 per year
R	Steady state exogenous real rental rate	0.045 per year

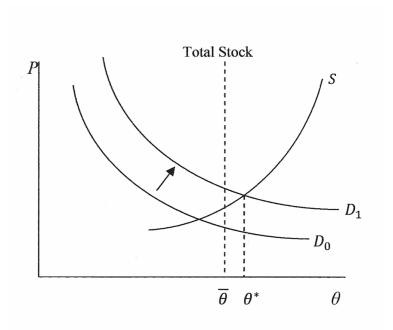
Note: This table shows the baseline values chosen for parameters and exogenous variables in the quantitative solution of the matching model.

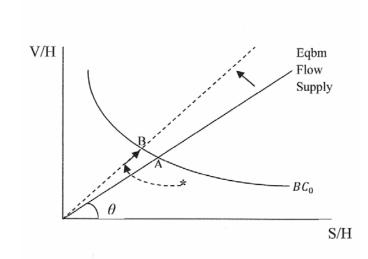
Figure 2.1 – Housing Beveridge Curve in Matching Model Using Baseline Calibration



**Notes:** This figure shows the equilibrium Beveridge curve generated by the market in the search and matching model. Allowing relative demand to vary independently, this graph shows equilibrium combinations of the number of searchers per vacancy in the market and percentage of the housing stock that is vacant for sale (vacancy rate). The housing Beveridge curve illustrates the static tradeoff between demand for housing and available housing units for sale. As equilibrium searchers per vacant unit increase, each vacancy is in higher demand and the market adjusts its equilibrium rate of vacancies out of total housing stock downwards.

Figure 2.2 – Static Analysis of In-Migration Shock



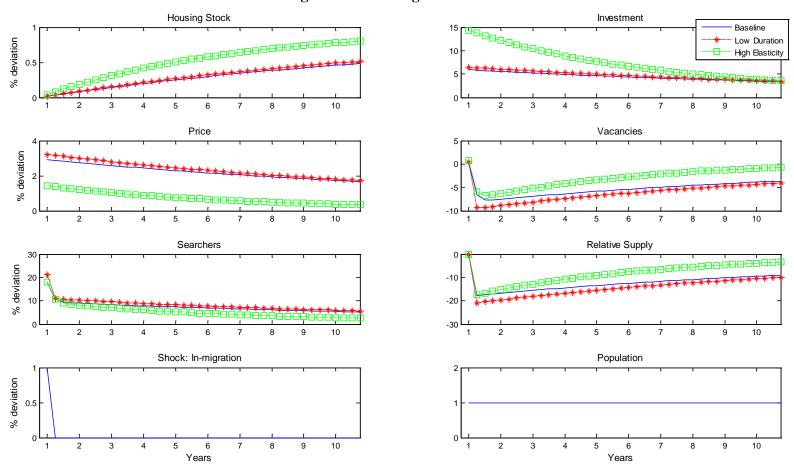


A. Bargained Price and Flow Supply

B. Housing Beveridge Curve

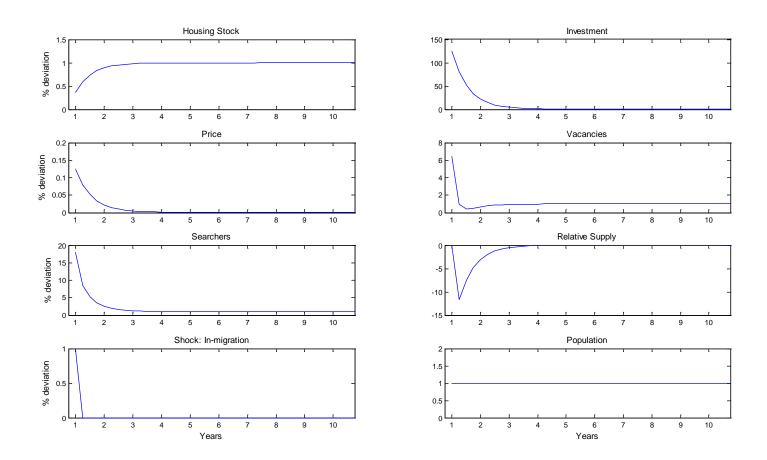
**Notes:** This figure shows the impact of positive shock to demand from an increase in the population size due to in-migration. In Figure A, in-migration shifts the bargained price curve out, reflecting an exogenous increase in housing demand. In Figure B, the increase in housing demand is represented in terms of the Beveridge curve. The higher rate of construction relative to searchers in the new equilibrium is represented as a pivot of the equilibrium new construction curve in (S/H, V/H) space. The impact effect of the in-migration shock can be visualized as an upward jump in the number of searchers and a decline in vacancies, which moves the market somewhere to the southwest of point A (marked by an asterisk). Since the price increase resulting from higher demand is permanent, firms respond by raising the number of units constructed and this raises the equilibrium  $\theta$ , given by the intersection of the Beveridge curve and the dotted new construction curve. This makes point B the new equilibrium point in Figure B.

**Figure 2.3 – In-Migration Shock** 



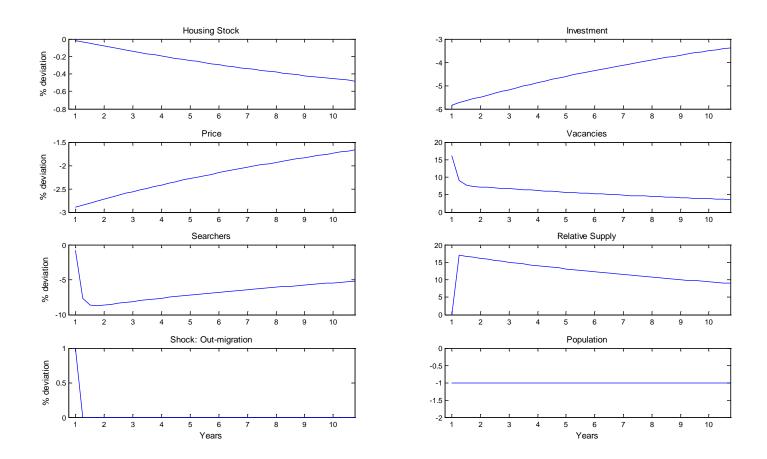
**Notes:** This figure shows impulse responses of key variables to a 1 percent in-migration shock to the housing market. The impulse responses are reported in percentage deviations from steady state levels for each variable. The benchmark calibration corresponds to the solid lines, marked "Baseline" in the legend. Two alternative calibrations are also presented for comparison. The "Low Duration" calibration shows adjustment paths when vacancy duration is only 4 weeks (half the length of the baseline calibration). The "High elasticity" calibration uses a flow supply elasticity of 10, compared to 2 in the baseline calibration.

Figure 2.4 – In-Migration Shock with Infinitely Elastic Investment



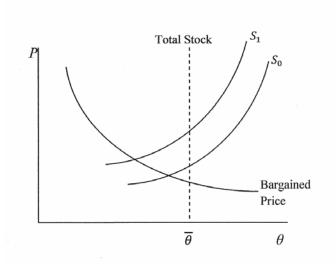
**Notes:** This figure shows impulse responses of key variables to a 1 percent in-migration shock to the housing market when the **supply** of vacancies can adjust quickly to the additional demand due to infinitely elastic investment. The impulse responses are reported in percentage deviations from steady state levels for each variable. The parameter  $\xi = 1/1000$ , implying an elasticity of supply of 1000.

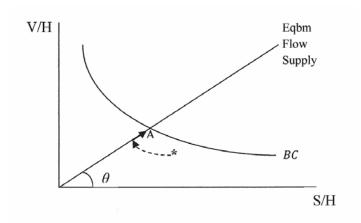
**Figure 2.5 – Out-Migration Shock** 



**Notes:** This figure shows impulse responses of key variables to a 1 percent out-migration shock to the housing market. The impulse responses are reported in percentage deviations from steady state levels for each variable.

Figure 2.6 – Static Analysis of Cost Shock



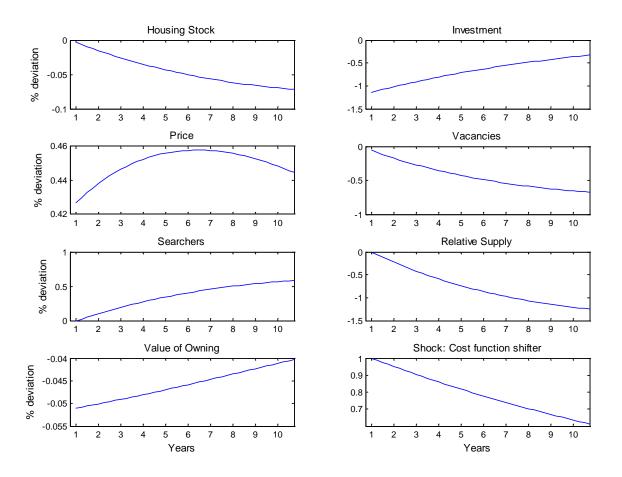


A. Bargained Price and Flow Supply

B. Housing Beveridge Curve

**Notes:** This figure shows the static equilibrium resulting from a higher marginal cost of construction. With a higher marginal cost, flow supply is lower for each price, resulting in a shift leftward of the static new construction curve in Figure A. The equilibrium price of a vacancy is higher which equivalently implies that the equilibrium value of a vacancy is higher. Thus, in Figure B, in the new short run equilibrium the market moves to a point where vacancies are lower because fewer new units are produced and the ratio of vacancies to searchers falls, relative to the original intersection point. Because the shock is transitory, when price returns to its original steady state level, the original intersection point will be reached with lower demand for each vacant unit and a higher vacancy rate.

Figure 2.7 – Cost Shock



**Notes:** This figure shows impulse responses of key variables to a 1 percent transitory increase in the marginal cost of construction. The impulse responses are reported in percentage deviations from steady state levels for each variable.

## **Chapter 3**

## House Prices, Investment and Vacancies after Hurricane Katrina: Empirical Analysis of the Search & Matching Model

#### I. Introduction

In this chapter, I make small modifications to the search and matching model constructed in Chapter 2, and use it to analyze the housing markets affected by Hurricane Katrina. In affected cities, Katrina caused a sharp reduction in the stock of housing and unanticipated changes in migration, providing a natural experiment suitable for analysis using the search and matching model of housing.

This chapter presents both an empirical analysis of the natural experiment and a theoretical investigation of it using the model developed in Chapter 2. I construct and use a panel data set for 57 Metropolitan Statistical Areas (MSAs) within roughly 500 miles of the epicenter of the hurricane. I use a difference-in-differences estimation to produce reduced-form empirical impulse responses of house prices and new residential investment to the change in housing stock and migration from the hurricane. Calibrating the log-linearized structural matching model for the shocks experienced in each MSA, I simulate the dynamic paths of price and investment predicted in response. In this way, I am able to compare the estimated responses of price, flow investment and vacancy rates with the predicted responses of the model. I find that the model successfully predicts a rise in prices for disaster areas even though it overshoots in magnitude compared to actual data. The model also correctly predicts that residential investment increases after the hurricane and tracks actual data closely in magnitude.

## II. Matching in the Housing Market

The model, its timing and notation remain mostly unchanged here from Chapter 2. As before, the market here represents a single MSA with *H* denoting the total housing stock and *N* being total population, which is constant except for migration shocks.

In steady state, demand for housing units is generated by existing homeowners who are forced to enter the search state due to a separation shock with Poisson probability  $\lambda$ , or complete depreciation of their unit with Poisson probability  $\delta$ . Those who are merely separated from their units may resell in the matching market, alongside firms constructing new units for sale. Searchers and sellers are matched according to the same market technology function as before, which is assumed to take the Cobb-Douglas form,  $M(S,V) = \overline{m}S^{\gamma}V^{1-\gamma}$ . As before, the likelihood of a seller being matched with a searcher,  $q(\theta)$  is decreasing in the ratio of total vacancies to searchers,  $\theta$ ; while the likelihood of a searcher matching with a seller,  $\mu(\theta)$  is increasing in  $\theta$ .

The sequence of events in a time interval is as before, illustrated below. Given a pre-determined stock of housing,  $H_t$ , vacancies,  $V_t$ , and searchers,  $S_t$ , the market meets at the start of the period and matches vacancies with searchers. Unmatched searchers and sellers must wait until the next period, paying rent and depreciation costs, respectively. Households who are successfully matched become owner-occupiers.

Matching & Bargaining in the Housing Market	Own & Occupy	Separation	Depreciation	New Construction	
0	0	0	0	0	

Following the payoffs to all parties, a fraction  $\lambda$  of owner-occupiers are exogenously separated from their existing housing units. A fraction  $\delta$  of owners that are not separated may still have to search next period because their unit has completely depreciated.

At the end of the period, construction firms make investment in new housing units for sale,  $X_t$ . New units augment the stock of vacant non-rental housing available for sale

next period,  $V_{t+1}$ , and add to total stock,  $H_{t+1}$ . Similarly, existing houses that are separated from their current owners enter the stock of vacancies for next period.

I model the event of Hurricane Katrina with a new variable  $\kappa_t$ , that takes value 1 if there is a hurricane and 0 otherwise. Then to capture the unanticipated shock to the housing stock from Hurricane Katrina, I allow the rate of depreciation to vary from its fixed value,  $\delta$ , when  $\kappa_t = 1$ . Hence depreciation may be rewritten as  $\delta_t = \delta + \eta \kappa_t$ , varying from its steady state value only on the date of the hurricane. The coefficient on  $\kappa_t$  captures the percentage of housing units destroyed by the shock.

In the model, the hurricane event also causes migration out of cities that lie in its path. Cities that are not in the hurricane's path receive immigrants from the affected areas. This migration is modeled as unanticipated shocks to population. While the destruction of housing occurs only once, the migration shocks occur both on the hurricane date itself and in each period after it, to mimic the migration patterns observed in actual data for my sample MSAs.

## Population, Searchers, Vacancies & Housing Stock

Since there are no births and deaths in the model, population varies only by migration, namely  $EM_t$ , which denotes out-migration in period t or  $IM_t$ , which is in-migration as per equation [1].

$$N_t = N_{t-1} + IM_t - EM_t \tag{1}$$

The evolution of searchers in the market depends on the match and separation probabilities as well as net migration, per equation [2].

$$\begin{split} S_{t+1} &= (1-\mu_t)S_t + IM_t + \left(\lambda_t + \delta_t(1-\lambda_t)\right)(N_t + EM_t - IM_t - (1-\mu_t)S_t) \\ &\quad - (\sigma + \rho(1-\sigma))EM_t \end{split}$$
 [2]

The total number of searchers entering the market at the start of period t+1 is the sum of unmatched searchers from the previous period and new immigrants into the city. Further, all matched owner-occupier households in the total population who are forced to move due to a separation or depreciation shock also enter the body of searchers.

The last term is new to the searcher evolution process, whereby emigration is an outflow from the total number of searchers. Specifically,  $\sigma EM$  emigrants are searchers

who leave the city instead of continuing to look for housing there. The remaining emigrants are all owner-occupiers, of whom the fraction  $\rho$  have experienced destruction of their housing units and would have entered search if they had remained in the city. That is, the remaining emigrating owner-occupiers  $(1-\rho)(1-\sigma)$  are those who have not experienced destruction of their housing units by the hurricane and become only sellers in this market when they emigrate.

This leads to the modification of the law of motion for vacancies in the model, to include the housing units sold by emigrating owner-occupiers in equation [3].

$$V_{t+1} = (1 - q_t)(1 - \delta_t)V_t + \lambda_t(1 - \delta_t)(H_t - (1 - q_t)V_t - (1 - \sigma)EM_t) + X_t + (1 - \sigma)(1 - \rho)EM_t$$
[3]

Total vacancies at the start of period t+1 include unmatched vacancies from last period net of depreciation and existing houses put up for sale by owner-occupiers who experience the move-shock but not depreciation in interval [t, t+1]. New construction,  $X_t$ , also raises vacancies. Finally, the last term in [3] is emigrating owner-occupiers who sell their units upon leaving the city.

The housing stock at t+1 is the sum of existing houses after depreciation and new construction, as before,  $H_{t+1} = H_t(1-\delta) + X_t$ . New construction is undertaken by firms and the market faces an upward sloping supply curve for new units as before,  $A_t = \alpha X_t^{\xi}$ .

## Asset Values, Price and Equilibrium

The remainder of the model equations are the same as in the basic matching framework and are reproduced here for convenience. The asset value of being in the search state is denoted L, given by the recursive equation [4].

$$L_{t} = \frac{1}{1+r} E_{t} [\mu_{t+1} (W_{t+1} - P_{t+1}) + (1 - \mu_{t+1}) L_{t+1}]$$
 [4]

The asset value of the owner-occupier state is given by W, as expressed in [5].

$$W_{t} = R_{t} + b + \lambda_{t}(1 - \delta)A_{t} + (\lambda_{t} + \delta(1 - \lambda_{t}))L_{t} + \frac{(1 - \lambda_{t})(1 - \delta)}{1 + r}E_{t}W_{t+1}$$
 [5]

The asset value of a vacancy, A is earned in the seller state, and given by equation [6].

$$A_t = \frac{1}{1+r} E_t[q_{t+1} P_{t+1} + (1 - q_{t+1})(1 - \delta) A_{t+1}]$$
 [6]

Finally, price is given by the Nash bargaining solution that divides the match surplus between searcher-buyer and seller, as in equation [7].

$$P_{t} = \arg\max_{P_{t}} (W_{t} - P_{t} - L_{t})^{\phi} (P_{t} - (1 - \delta)A_{t})^{1 - \phi}$$
 [7]

I solve for steady state equilibrium and log-linearize the model around the steady state as described in detail in Chapter 2.

## III. Data and Stylized Facts

In order to establish the impact of Hurricane Katrina on the housing market in affected cities, I build a panel data set comprising time series for house prices, new investment, ownership vacancy rates and net migration. The panel consists of 57 MSAs, of which 13 are cities comprise the treatment group that was directly hit by the hurricane. The sample period is 2000 to 2008.

My treatment group is not an exhaustive list of the cities that were in fact hit by the hurricane. A total of 38 MSAs can be identified with one or more hurricane-hit counties, but not all have data series on prices and vacancy rates. These data are typically restricted to the largest 75 MSAs in the U.S., which limits the number of in-FEMA MSAs that can be included in my sample. Therefore, the treatment group in my sample consists of the maximum number of cities for which price data are available: there are 13 of these.

The remainder of the panel forms a comparison group. Comparison cities lie within a 550 mile radius around New Orleans (excluding Florida<sup>1</sup>). As the analysis below will show, the comparison cities display no divergence in migration, price and investment trends from the treatment group prior to the hurricane date and are geographically similar to the treatment group. See Map 1 for a visual representation of all included MSAs. The full list of MSAs in the panel together with distance in miles from the hurricane's epicenter in New Orleans can be found in Appendix Table 3.8.

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<sup>&</sup>lt;sup>1</sup> Florida is not a homogeneous housing market to New Orleans. It regularly experiences large in-migration of retirees which likely influences its demand and supply in a way that is different from the rest of the cities in my sample.

## Housing Stock Damage Estimates

Hurricane Katrina occurred on August 29, 2005. The worst-hit cities were New Orleans, Louisiana; Gulfport-Biloxi and Pascagoula in Mississippi; and Beaumont-Port Arthur in Texas, where from 25 to 38 percent of the housing stock was severely damaged or destroyed. These cities also saw massive outmigration immediately following the hurricane, much of which remains unreversed in New Orleans to date.

To establish the size of the shock to the housing market from the hurricane, I use estimates of housing stock destroyed produced by the Federal Emergency Management Agency (FEMA) at the county level across Texas, Louisiana, Mississippi and Alabama. In Table 3.1, I aggregate county level estimates to show estimated loss of housing stock at the MSA level. Total destruction of housing stock is clearly the worst in New Orleans where the breaking of the levees submerged 80 percent of the city, with water rising up to 20 feet high in places. The estimates do not include damaged vacant or seasonal housing and also exclude second homes.

Table 3.1 also provides data on total housing stock, number of occupied units, population in occupied units and occupancy by tenure for affected MSAs, from the Census Bureau's American Community Survey and population estimates.

For the reader interested in further breakdowns, Table 3.6 in the Appendix shows the breakdown of damage by county in these MSAs and Table 3.7 shows damage by tenure. This is helpful in illustrating geographical focus of damage and clarifying that in general, the owner-occupied housing stock was affected more than rental units.

#### Regression Model

To analyze the impact of the hurricane on migration patterns, price, investment and vacancy rates, I estimate difference-in-differences regressions for each series. Equations [8] and [9] are cross-sectional representations of the estimating equations for a single MSA *i*.

$$y_{it} = \sum_{j=2000}^{2008} \beta_j T_{j_t} + \sum_{j=2000}^{2008} \beta_j^{FEMA} FEMA_i * T_{j_t} + u_{it}$$
 [8]

$$y_{it} = \sum_{j=2000}^{2008} \beta_j T_{j_t} + \sum_{j=2000}^{2008} \beta_j^n Near_i * T_{j_t} + \sum_{j=2000}^{2008} \beta_j^m Medium_i * T_{j_t} + u_{it}$$
 [9]

In these equations,  $T_j$  are time dummies, j=2000, 2001, ..., 2008. In equation [8], FEMA interacted with time is a binary variable that takes value 1 if MSA i lies in the FEMA sub-sample of MSAs and the date is j; it takes value 0 otherwise. In equation [9] an MSA lies at 'Near' distance to the epicenter of the Hurricane if it lies within 100 miles of New Orleans. An MSA lies at 'Medium' distance if it is between 100 and 200 miles away from New Orleans. The interaction variables Near\*Time and Medium\*Time take value 1 if an MSA is 'Near' or 'Medium' respectively and the date is j.

In both specifications, the time trend captures the average time path for the comparison group of MSAs that are either non-FEMA or further than 200 miles from New Orleans city, respectively. The FEMA interaction terms isolate the impact of the Hurricane in excess of the trend, for all affected MSAs identified by FEMA as eligible for receiving public and individual assistance. 'Near' and 'medium' dummies similarly measure impact in excess of the trend on areas close to the epicenter of the storm.

I expect to see that the lower the proximity to New Orleans MSA, the less of an estimated deviation there will be for a particular group from the average trend, i.e.  $\beta_j^{NEAR} > \beta_j^{MEDIUM}$  for all j. Since the FEMA group comprises MSAs at varying distances from New Orleans, I also expect that  $\beta_j^{NEAR} > \beta_j^{FEMA}$  for all j.

Regression equations [8] and [9] are estimated by pooled OLS, or Prais-Winsten regression if there is serial correlation in the residuals. Variance estimates are corrected for panel-heteroskedasticity and serial correlation in the residuals where appropriate. I do not allow contemporaneous correlation in the errors across MSAs.

## Migration

Approximately 40 percent the population in occupied housing units in New Orleans moved from or within the metro area during September to December 2005. While 28 percent of movers relocated within the New Orleans area, 4.5 percent moved to Houston, TX and 11.6 percent to the remainder of Texas. 8.1 percent of movers from New Orleans moved to Baton Rouge, LA and 20.6 percent moved to the remainder of the U.S. excluding Gulf Coast states (Koerber, 2006). Less detailed data is available on migration patterns in other cities.

To analyze migration dynamics in hurricane-affected cities I use annual estimates from the Census Bureau from 2000 onwards. For the full panel of 57 MSAs, I estimate regression models [8] and [9] with net migration as a percentage of total population as the dependent variable. Coefficient estimates and panel-corrected standard errors are presented in Table 3.2.

Figure 3.1 plots the estimated FEMA and distance coefficients for net migration. Migration estimates are made from mid-year to mid-year, hence the vertical line indicating the hurricane date in both panels is drawn at mid-2006. This corresponds to estimated migration from mid-2005 to mid-2006.

Plots of the estimated FEMA and NEAR effects tell the same story and are equally stark. MSAs that bore the brunt of Hurricane Katrina can be seen to follow the trend prior to the hurricane. Between 2005 and 2006 in-FEMA MSAs clearly experience a large and statistically significant outflux of population, which is estimated at 9.7 percent in NEAR MSAs and 3.8 percent across FEMA metro areas. After 2006, net migration recovers only slightly in the treated group, indicating a persistent effect on the population in affected cities.

Notice that point estimates plotted for MEDIUM cities are statistically significant and smaller than the corresponding NEAR estimates. Hence, the greater the distance at which a city lies from New Orleans, the lower is the effect on its population size and hence, the market for housing.

#### House Price

I use the MSA-level quarterly Housing Price Index (HPI) from the Federal Housing Finance Authority (FHFA) website, formerly maintained by OFHEO. The HPI is a repeated-sales index and records the sales prices of only those houses that have been sold at least twice over. This price index provides data on the largest number of MSAs, which makes it preferable to other publicly available price index series.

Figure 3.2 presents estimated FEMA and distance coefficients for the housing price index from regressions [8] and [9]. NEAR and MEDIUM plotted estimates are normalized to zero in 2000Q1, so the plot in the lower panel in Figure 3.2 can be interpreted as the change in price in percentage points.

Just as with the migration data, FEMA coefficient estimates pre-2005Q3 illustrate the similarity of MSAs included in the sample. House prices in FEMA cities pre-Katrina were consistently within a 2 to 4 index point range above the trend for the comparison group. The subsequent increase above trend for in-FEMA cities can thus be attributed to the only completely unpredictable event that occurred after 2005Q2 in the treatment group: Hurricane Katrina. Had the shock been foreseeable or had the underlying change been region-wide, the pre-2005Q2 pattern suggests that prices across the whole sample would have moved in concord.

There is a similar clear and statistically significant positive effect on prices for NEAR MSAs after the hurricane date, starting in 2005Q4. Prices in FEMA MSAs increase by 3 percent from 2005Q3, while the jump is 7 percent for NEAR MSAs. As expected, there is a much smaller increase in MEDIUM cities of 1 percent right after the hurricane. In NEAR MSAs prices continue to rise, peaking at 24 percentage points above their 2005Q2 level at the end of 2007. Comparing this trajectory to that of the MEDIUM cities, Figure 3.2 clearly bears out the hypothesis that the closer the MSA lies to the epicenter of the hurricane, the higher the post-shock house price increase it will experience. Selected point estimates for these regressions are presented in Table 3.3.

#### Investment

I use annual data from the Census Bureau on building permits to proxy for residential investment in new single-family units. Due to sample size restrictions, the Census does not produce estimates of starts for geographic units below the Census Regions. However, since permits are the fundamental series on which Census estimates of housing starts are formed, they provide a second-best estimate of residential building activity even though they cannot be interpreted directly as housing starts.

Permits data provide a complete count of intended new residential building activity in the U.S., as opposed to a survey estimate. The Census notes that only about 2.5 percent of housing starts in the United States are built in non-permit areas. Building permits are issued only for new construction and do not include remodeling or repair to existing houses. The average lag between issuance of permit and start of construction for new residential buildings is 0.8 months, for 1976-2008. Averaging between 1994 and

2008, only 9.5 percent of projects for which residential permits are issued are not observed to have started construction by end of year (regardless of month of issue).

Using log permits by MSA as the dependent variable, I estimate regressions [8] and [9]. Selected point estimates and standard errors from these regressions are presented in Table 3.4. Figure 3.3 plots selected coefficient estimates for both specifications. In these plots, distance coefficient estimates for 2000 are normalized to zero so the plots can be interpreted as percent change in log permits over the previous year. The bottom panel of Figure 3.3 shows that NEAR MSAs saw an increase of 35 percent from 2005 to 2006, while permits issued in-FEMA MSAs increased by 19 percent after the hurricane which is marked by the red vertical line in each plot. These results establish that in the aftermath of Hurricane Katrina residential investment increased above average in affected cities.

#### Elasticity of Supply of Housing

The estimated change in prices and permits after the hurricane effectively provide an instrumental variables estimate of the flow elasticity of supply for areas near Katrina's epicenter. The destruction of housing stock by the hurricane provides an instrument for a shift in the demand for housing along a stable upward sloping flow supply curve. If Hurricane Katrina resulted in a shift of the flow supply curve as well as a shift in the demand curve, then the estimate would be unreliable due to simultaneity bias. Looking at the data, however, my estimations show that after Katrina permits recovered from the initial drop and continued to rise, while prices also increased. This would suggest that construction firms were travelling up the supply curve in response to a shift in demand alone. Had the hurricane shifted the supply curve inward, permits and prices would vary non-positively.

For conformity with the frequency of permits, I re-estimate [9] for log prices at annual frequency. Taking the ratio of the change in estimated coefficients on NEAR, between the date immediately prior to Katrina (annual observation for 2004) and immediately after Katrina (the observation for 2006), I have the estimated change in log permits over the change in log prices. This yields an estimated elasticity of 2 for flow supply of new housing units.

## Ownership Vacancy Rate

Time series data for ownership vacancy rates are available from 1986 to 2008 from the Housing Vacancy Survey for roughly the highest-populated 75 MSAs in the U.S. Only 2 hurricane-affected MSAs are covered, namely New Orleans and Houston. Of the remaining sample, there are 6 comparison MSAs for which vacancy rates are available. Although the number of cities for which vacancy data are available is smaller, the advantage of using panel data is that there are still a sufficiently large number of observations to make regression analysis viable for the subset of my sample. Appendix Table 3.8 lists the cities for which vacancy data are available.

The survey has varying sample size at quarterly frequencies, so I use annual observations. I also correct for shifting geographical boundaries of MSA definitions over the sample period by calculating heteroskedasticity-robust standard errors in the vacancy rate regressions.

Figure 3.4 illustrates clearly that there is a statistically significant rise in vacancy rates in New Orleans MSA in 2007 and 2008. Even though the estimated coefficient for NEAR2005 is not statistically significant, the difference between NEAR2004 and NEAR2005 is significantly different from zero, with a  $\chi^2(1)$  value of 26.57 (p-value of 0).

In-FEMA point estimates show a decline in the ownership vacancy rate after 2005, but coefficient estimates are never significantly different from zero. The plot is an average of the ownership vacancy rate between Houston (where the vacancy rate fell as Houston experienced immigration from New Orleans) and New Orleans (where the vacancy rate rose after the hurricane).

Table 3.5 reports regression results for data on ownership vacancy rates. The NEAR coefficient isolates the effect on the ownership vacancy rate for New Orleans, since that is the only city in the sub-sample used for these regressions to lie within 100 miles of the hurricane's epicenter. Vacancy rates are the average of quarterly rates, recorded at the end of the calendar year, so the vacancy rate in both 2005 and 2006 will show the effects of the hurricane on the ownership housing market.

Vacancy rates in general are a very small fraction of the housing stock, on the order of 1 to 2 percent nationally, for instance. A sizeable treatment effect in the city where the hurricane caused the greatest destruction to housing stock itself is not apparent.

Even if it were, it would be unclear whether vacancy rates have risen because the overall stock of housing has shrunk or because more houses lie vacant due to lower demand for existing housing units.

To emphasize the effect of large fractions of the stock of housing being destroyed by the hurricane on vacancies, I re-estimate the regressions after adjusting all in-FEMA observations for the percentage damage incurred. Since non-FEMA MSAs experience no damage to housing stock, each observation is multiplied by 1 (i.e. it is included without adjustment). I place a greater weight on New Orleans and Houston proportional to the percentage of occupied housing units destroyed there. In this way, the MSA that experienced more damage receive greater weight in the regression across all years. Results for actual weighted data in Table 3.5 show a positive and statistically significant FEMA coefficient in 2005. All estimated coefficients for specification [9] are statistically significant. Thus we can conclude that a rise in vacancy rates is observed in New Orleans after the hurricane, but in absolute magnitude the increase is not a sizeable one.

### Summary of Results

To summarize these findings, metro areas that lie in the path of Hurricane Katrina have experienced significantly higher prices and higher residential investment activity since 2005. Disaster-hit metro areas experienced a large out-migration of their resident population which has only been partially reversed thus far. The further a city lay from the epicenter of the hurricane, the lower the out-migration and resulting impact on housing demand is observed there. On average, vacancy rates show non-negative movement after the hurricane.

# IV. Hurricane Katrina in the Search and Matching Model of Housing

As in Chapter 2, I use the Andersen Moore algorithm to solve for a saddle point equilibrium for the log-linearized model. The model is calibrated at the quarterly frequency and all parameters values are taken from Chapter 2, Table 2.1. In steady state equilibrium, price and relative supply are normalized to equal 1 and searchers and vacancies are 5.8 percent of the total population and housing stock respectively.

The simulation exercise is as follows. For each of the 57 cities in my regression sample, I calibrate the model's baseline parameters to the same general specification as in Table 2.1 of Chapter 2. All cities are assumed to have a flow supply elasticity of 2, which is the estimated post-disaster elasticity discussed in Section III above. For each city, the model is in steady state when Hurricane Katrina arrives unanticipated at date t. When the hurricane occurs, the indicator variable  $\kappa$  takes value 1 and returns to zero immediately thereafter. In the quarter when  $\kappa$  takes the value 1, depreciation of the housing stock in each city rises above the steady state level to  $\delta + \eta \kappa$ , which is set equal to actual data on the percentage of housing stock destroyed by the hurricane for each city. Hence for all comparison cities,  $\eta = 0$ , when  $\kappa = 1$ , so depreciation is equal to its steady state level even when the hurricane occurs. When  $\kappa$  returns to 0, depreciation returns to its steady state level of  $\delta = 0.014$  for all cities.

The second aspect of the shock from the hurricane is the dramatic increase in out-migration from affected cities, as the empirical results in Section III show. To incorporate this into the model simulation, I use net migration data for my entire sample and input the actual observed in- and out-migration as a percentage of steady state population for the  $IM_t$  and  $EM_t$  shocks for each city in each period. Hence,  $EM_t$  jumps up from zero for in-FEMA MSAs when  $\kappa = 1$ , and  $IM_t$  rises in non-FEMA MSAs, per actual observed movements in each city. Once  $\kappa = 0$ ,  $IM_t$  and  $EM_t$  do not return to zero, but follow the actual pattern of migration observed in each MSA in my sample.

In total, 32 percent of the total population of New Orleans emigrated after the hurricane. The number of owner-occupiers who left amounted to 7.6 percent of the pre-hurricane population. Since the matching model has no place for permanent renters, it is important to account for the large percentage of renters that comprise the total emigration

from the city who should have no impact on the demand for ownership vacancies. I therefore mimic the number of owner-occupiers leaving New Orleans in the model to actual data and force all searchers (who are renters) to leave when the shock hits. While migration shocks for all other cities are exactly equal to observed data, migration into and out of New Orleans in the model is net of permanent renters and set to 42 percent of actual migration.

Simulating the model also requires an assumption about the fraction of emigrants whose houses are destroyed in Hurricane-hit MSAs. This requires choosing a value for the parameter  $\rho$  in the equation of motion of vacancies [3]. If  $\rho$  is zero, the correlation between destruction of housing and emigration is zero. For  $\rho = 1$ , all owner-occupier emigrants must have their houses destroyed. For the benchmark calibration, I set  $\rho$  to 0.5 and present robustness checks later in this section.

Under these assumptions, I run 57 replications of the model, one per MSA, and generate artificial data for from the model for the 13 quarters between 2005Q3 and 2008Q4. I discuss the implications of the shock for a single MSA and then proceed to summarize findings for all MSAs through regressions on simulated data that mimic those in Section III.

#### Impulse Response Function for New Orleans

Figure 3.5 displays impulse responses for New Orleans for the benchmark calibration. When the shock hits the city, the housing stock and population in the model fall by the amount they actually did. While emigration implies a decline in the number of searchers next period, the increase in demand for housing due to the magnitude of destruction of housing stock far outweighs this negative effect. Consequently searchers increase by 600 percent relative to steady state. Vacancies initially rise a mere 16 percent above steady state when the shock occurs, but plummet after that as  $\theta$  falls, raising the hazard rate for vacant units.

The matching friction induces inertia in the evolution of housing demand as captured by the number of searchers. Excess demand bids up price and the likelihood of selling a vacant house increases almost fourfold. Flow investment only increases by enough in this forward-looking model to produce the number of housing units the market

will match each period, rather than the number of total searchers looking for a house. This eliminates the stock of vacant housing upfront and raises the probability of selling a house over threefold. Since the matching technology does not allow immediate reallocation of housing units across searchers, the effects on price and investment are perpetuated as long as housing demand persists away from the steady state level.

The behavior of price in relation to the likelihood of selling is important to illustrate a more general point about the model's mechanisms. All else equal, the lower is the hazard rate for a vacant house, the lower the bargained price it will receive. In the impulse responses here, the ratio of vacancies to searchers falls on impact and slowly rises over time. Thus, the duration of vacancy for available housing units falls and the price a house is able to fetch rises dramatically. The model is therefore able to capture the effect of time-on-market on sales prices in a dynamic aggregate equilibrium framework.

#### Regressions on Simulated Data

To summarize the results of the Monte Carlo simulations and illustrate the predictive capability of the model, I create a combination panel dataset for prices, investment and the vacancy rate. Per MSA, I combine artificial data generated by the model for 13 quarters following the shock (2005Q4 to 2008Q4) with actual data for that MSA for the sample period before Katrina. Using these combination series for the whole panel of MSAs, I re-estimate the regressions for prices, vacancy rates and log investment.

I plot the simulated-data estimation results against actual results for prices, permits and vacancy rates in Figures 3.6 to 3.8, maintaining the convention that estimates based on actual data alone are represented by solid lines while simulated data estimates are represented by dotted lines. For the FEMA specification in each figure, I also present results from robustness checks in the simulations. These plots are stacked and are read as follows. The line labeled "Simulated "FEMA" BM" lies directly above the line labeled "Actual "FEMA". For example, the point estimate for "Simulated "FEMA" BM" line and data point directly below it on the "Actual "FEMA" line at 2005Q4. For ease of comparison, relevant point estimates for 2005 and 2006 for each calibration are also labeled in figures to show the jump predicted by the model on impact of the Hurricane.

Since point estimates prior to the Hurricane are identical for all specifications and simulations, by construction, the figures only plot coefficient estimates from simulated and actual data after 2005(Q3). Bottom panels in Figures 3.6 to 3.8 plot NEAR and MEDIUM point estimates for simulated and actual data sets.

Figure 3.6 presents point estimates for prices. In actual data, in-FEMA MSAs experience a 3.4 percent increase in 2005Q4 over the previous quarter. In contrast, prices in the model overshoot, increasing by 39 percent for in-FEMA MSAs in the benchmark calibration. The overshooting is even more pronounced in NEAR MSAs, where prices rise by 68 percent in the model, compared to about 7 percent in the data. The model seems to perform better for MEDIUM MSAs. This overshooting behavior is due to the size of the shift in demand for housing after the Hurricane. Even though a fraction of former homeowners leave affected cities after the shock, the remaining former owner-occupiers push up demand up to sevenfold because the destruction of housing stock is so large. The top panel in Figure 3.6 shows that the overshooting behavior of price in FEMA MSAs is robust to any assumption about the correlation of emigration and destruction of housing (captured by  $\rho$ ). By construction, the model returns to steady state after a shock, so simulated prices decline over time in Figure 3.6. In actual data, by contrast, prices in the affected metro areas continue to rise after the Hurricane.

Figure 3.7 similarly compares simulation and actual results for log permits. Here too, the change in permits predicted by the model between 2005 and 2006 is higher than that seen in the data. For in-FEMA MSAs permits rise by 33 percent in simulations while they rise by 19 percent in the data. In NEAR MSAs, the overshooting is more pronounced, but the model performs relatively well for MEDIUM cities. These effects are robust to any value chosen for  $\rho$ . Once more, however, permits in the model start to decline towards steady state, while they are seen to continue to rise above trend in actual data.

Finally, Figure 3.8 shows that the response of vacancy rates is predicted to be more extreme by the model compared to the data: predicted in-FEMA vacancy rates fall by more than in actual data. For in-FEMA areas, the vacancy rate falls by 24 basis points after the Hurricane, while in the model the vacancy rate declines by 2 percent. For New

Orleans alone, as summarized by the NEAR point estimates, the prediction of the model is counterfactual altogether.

#### Discussion

The model's predicted path for prices is incongruous with the data, both in magnitude of change and in the pattern of adjustment. In the model, perfect foresight dictates that price jumps up when the shock hits and then adjust slowly back to equilibrium. By contrast, in the data, price is still rising at the end of the sample period. In magnitude, the price response observed empirically is likely tempered by employment changes and financial constraints not included in the model. As for the observed timepath of prices in the data, it could be interpreted as the effect of learning about the extent of demand changes over time. Equally, actual price in affected cities could rise over time in response to repaired units coming on the market over time, improving prices of those units for sale post-hurricane.

Other concerns must be acknowledged about the price data itself. There might be a compositional effect in the data, biasing the observed response of prices. For instance, if the HPI excludes destroyed units located in a low-income residential area, like the badly-impacted lower Ninth Ward in New Orleans, this will result in a jump up in prices merely due to the exclusion. However, the HPI is constructed using a repeated-sales methodology which could potentially control for this effect. Similarly, though, while rents may have increased in affected cities, the intervention of FEMA would have dampened the response of rents by providing subsidized housing. Depending on whether emigrants from affected cities are the richest or the poorest households, rents could also behave unpredictably and drive the price response away from the model's prediction.

#### Robustness Checks

The top panels in Figures 3.6 to 3.8 also show the effect of assuming lower magnitude of migratory flows in all MSAs than in the benchmark model. Specifically, I restrict migration shocks to be 40 percent of actual size, which reflects the split between renters and owner-occupiers in emigrants from New Orleans. In this case, denoted  $\tau = 0.42$  in the figures, prices and permits overshoot by a couple of percentage points more in FEMA

areas, while the vacancy rate overshoots by less than the benchmark model, compared to actual data. This illustrates the incremental effect of migration on the housing market, in addition to that of destruction of housing units. Specifically, the lower is reallocation of households across MSAs, the more upward pressure there is on prices and permits to respond to higher demand for housing. More houses are produced for sale and sales from the stock of vacancies are replaced at a higher rate in affected MSAs because investment jumps to meet higher demand.

#### Summary of Simulation Results

The model propagates the effects of an unforeseen shock over time through slow moving searchers and vacancies, which are restricted in their speed of adjustment by the matching friction. The model's predicted response for prices and investment overshoots relative to the data. The model's predictions are driven by the assumption that hurricane impacted households immediately search for another house to purchase, causing demand to increase by several orders of magnitude in affected MSAs. In addition, the model makes the simplifying assumptions of exogenous rents and no policy interventions. Both of these assumptions are not true in the data. Rents in New Orleans increased in the aftermath of the hurricane. FEMA's response to the crisis would also impact rents and prices, by absorbing a fraction of the demand for housing that would otherwise have impacted private investment and price.

## V. Conclusion

This paper contributes a new model of the housing market to macroeconomic literature and provides evidence of its predictive capability. The model is a dynamic quantitative matching model relating new housing supply to existing vacancies, and forecasts the path of prices, vacancies and flow investment after an unanticipated shock to the housing stock from Hurricane Katrina. It propagates the effects of the shock over time through slow moving searchers and vacancies which are restricted in their speed of adjustment by the matching technology. The burden of adjustment therefore lies on jump variables, namely prices and investment. While investment tracks observed data quite closely, price

responses tend to be higher in magnitude relative to actual movement in the data for MSAs affected by Hurricane Katrina.

Considering simulation results for New Orleans as a case study of the worst-affected city in the sample, a number of important observations can be made about the model's dynamics. New Orleans experienced both the highest fraction of loss of housing stock and population due to Hurricane Katrina. While emigration implies a decline in the number of searchers in the model, the increase in demand for housing due to the magnitude of destruction of housing stock far outweighs this negative effect. Excess demand bids up price and the likelihood of selling a vacant house increases almost fourfold. The behavior of price in relation to the likelihood of selling illustrates a more general point about the model's mechanisms. All else equal, the higher is the hazard rate for a vacant house, the lower its duration of vacancy, and the higher the bargained price it will receive. When demand rises after the shock, relative supply of vacancies and hence the duration of vacancy for available units both fall and the price a house is able to fetch rises dramatically. The model is therefore able to capture the effect of time-on-market on sales prices in a dynamic aggregate equilibrium framework.

After the shock, the matching friction induces inertia in the evolution of housing demand as captured by the number of searchers. Flow investment only increases by enough in this forward-looking model to produce the number of housing units the market will match each period, rather than the number of total searchers looking for a house. Since the matching technology does not allow immediate reallocation of housing units across searchers, the effects on price and investment are perpetuated as long as housing demand persists away from the steady state level.

The model in this paper does not analyze the effects of movement in the rental price of houses. Furthermore, there is a possibility that the relatively small gap between the predicted response of new investment in the model and observed private residential investment in the data is being filled by public investment in housing after the Hurricane. Future work will address both of these issues.

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Table 3.1 – Estimated Number of Housing Units Damaged in MSAs Affected by Hurricane Katrina

	Owner- Occupied Housing Units	Renter- Occupied Housing Units	Number of Housing Units with Major Damage	Number of Housing Units with Severe Damage	Total Number of Housing Units Damaged	Major & Severely Damaged Houses as a Percentage of Occupied Housing Units (%)	Total Damaged Houses as a Percentage of Occupied Housing Units (%)
Baton Rouge, LA	190,591	83,389	855	117	36,004	0.35	13.14
Beaumont-Port Arthur, TX	97,507	47,055	32,752	5,318	122,494	26.33	84.73
Gulfport-Biloxi, MS	67,109	34,077	24,446	12,330	67,067	36.34	66.28
Hattiesburg, MS	31,822	16,806	1,805	237	21,167	4.20	43.53
Houma-Bayou Cane-Thibodaux, LA	52,458	17,515	2,803	162	20,728	4.24	29.62
Houston-Sugar Land-Baytown, TX	1,146,283	674,668	1,038	172	37,166	0.07	2.04
Jackson, MS	131,145	62,059	479	73	18,554	0.29	9.60
Lafayette, LA	65,859	29,745	193	25	7,213	0.23	7.54
Lake Charles, LA	52,597	22,271	6,678	2,285	47,411	11.97	63.33
Mobile, AL	101,298	49,293	2,814	363	44,869	2.11	29.80
New Orleans-Metairie-Kenner, LA	305,339	174,513	79,711	103,110	319,458	38.10	66.57
Pascagoula, MS	42,954	14,042	14,624	2,119	34,388	29.38	60.33
Tuscaloosa, AL	49,173	30,222	17	9	1,551	0.03	1.95

Notes: Counties are assigned to MSAs per November 2007 definitions by U.S. Office of Management & Budget (OMB). Major damage refers to flooding of up to 2 feet or where less than 50 percent of a house is damaged and requires extensive repair work for future occupancy. Severely damaged housing units are completely destroyed or flooded 2 feet or more.

Source: FEMA count of damaged units from HUD Report (2006); Estimated total housing units American Community Survey 2005, U.S. Census Bureau

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Table 3.2 - Pooled OLS Estimates for Net Migration as a Percentage of Population

	FEMA Specification	Distance S <sub>1</sub>	pecification
	FEMA	Near	Medium
2003	-0.28	-0.4	-0.23
	(0.9)	(1.97)	(0.28)
2004	-0.28	-0.21	-0.34
	(0.9)	(1.97)	(0.28)
2005	-0.32	-0.56	-0.31
	(0.9)	(1.97)	(0.28)
2006	-3.8***	-9.69***	0.09
	(0.9)	(1.97)	(0.28)
2007	0.17	0.56	-0.33
	(0.9)	(1.97)	(0.28)
2008	0.01	0.04	-0.28
	(0.9)	(1.97)	(0.28)

Notes: Coefficient estimates are starred to indicate statistical significance: \*\*\* indicates significance at 1% level; \*\* indicates significance at 5%; and \* indicates significance at 10% level. All estimates have panel-corrected standard errors in parentheses adjusted for panel-heteroskedasticity.

This table displays selected coefficient estimates from the peopled cross sectional regression for not migration in my semple of 57 MSAs, which includes 13 in

This table displays selected coefficient estimates from the pooled cross sectional regression for net migration in my sample of 57 MSAs, which includes 13 in-FEMA metro areas. The estimated regression is of the form  $y_{it} = \sum_{j=2000}^{2008} \beta_j T_j + \beta_j^D D_i T_j + u_{it}$ , where *i* subscripts MSA.  $T_j$  are time dummies, j=2000, 2001, ..., 2008 and  $D_i$  are dummies capturing the effect of distance from New Orleans city in three different ways. The alternative specifications are as follows: 1)  $D_i$  is a dummy identifying in-FEMA MSAs and is interacted with time. 2)  $D_i$  is further divided into dummies indicating 'near' distance (for an MSA that lies within 100 miles of New Orleans) or 'medium' distance (100-200 miles from New Orleans city) and interacted with time. See also Figure 3.1.

Table 3.3 – Pooled OLS Estimation Results for the Housing Price Index

	FEMA Specification		Distance Specification		
Date	FEMA	Near	Medium		
2005Q1	2.84	10.11*	-1.04		
	(2.86)	(5.28)	(1.82)		
2005Q2	3.52	11.53**	-0.48		
	(2.86)	(5.28)	(1.82)		
2005Q3	3.39	11.57**	0.58		
	(2.86)	(5.28)	(1.82)		
2005Q4	6.83**	18.55***	1.55		
	(2.86)	(5.28)	(1.82)		
2006Q1	8.8***	20.88***	3.13*		
	(2.86)	(5.28)	(1.82)		
2006Q2	12.45***	26.19***	6.33***		
	(2.86)	(5.28)	(1.82)		
2006Q3	14.08***	29.25***	8.48***		
	(2.86)	(5.28)	(1.82)		
2006Q4	14.86***	30.62***	8.51***		
	(2.86)	(5.28)	(1.82)		
2007Q1	15.8***	33***	8***		
	(2.86)	(5.28)	(1.82)		
2007Q2	15.14***	32.41***	8.63***		
	(2.86)	(5.28)	(1.82)		
2007Q3	16.53***	32.92***	9.85***		
	(2.86)	(5.28)	(1.82)		
2007Q4	17.17***	35.11***	9.76***		
	(2.86)	(5.28)	(1.82)		
2008Q1	17.29***	35.07***	10.42***		
	(2.86)	(5.28)	(1.82)		
2008Q2	17.82***	34.3***	10.4***		
	(2.86)	(5.28)	(1.82)		
2008Q3	16.49***	30.54***	9.81***		
	(2.86)	(5.28)	(1.82)		
2008Q4	17.68***	31.99***	9.5***		
-	(2.86)	(5.28)	(1.82)		

Notes: Statistical significance: \*\*\* indicates significance at 1% level; \*\* indicates significance at 5%; and \* indicates significance at 10% level. Panel-corrected standard errors are in parentheses, adjusted for panel-heteroskedasticity and serial correlation in residuals. This table displays selected coefficient estimates from the pooled cross sectional regression for house price index in 57 MSAs, which includes 13 in-FEMA metro areas. The estimated regression is of the form  $y_{it} = \sum_{j=2000}^{2008} \beta_j T_j + \beta_j^D D_i T_j + u_{it}$ , where i subscripts MSA.  $T_j$  are time dummies, j=2000, 2001, ..., 2008 and  $D_i$  are dummies capturing the effect of distance from New Orleans city in three different ways. The alternative specifications are as follows: 1)  $D_i$  is a dummy identifying in-FEMA MSAs and is interacted with time. 2)  $D_i$  is further divided into dummies indicating 'near' distance (for an MSA that lies within 100 miles of New Orleans) or 'medium' distance (100-200 miles from New Orleans city) and interacted with time. See also Figure 3.2.

**Table 3.4 – Pooled Cross Section OLS Estimates for Log Residential Building Permits** 

Table 3.4 – I toled Cross Section OLS Estimates for Log Residential Building Lethnits								
	Actual				Damage-Weighted Actual			
	Log Residential Building Permits					Log Residential Building Permits		
	FEMA Specification Distance Specification			_	FEMA Specification	Distance Specification		
	FEMA	Near	Medium	<u></u>		FEMA	Near	Medium
2005	0.33*	0.33	-0.34		2005	0.46**	0.53**	-0.17
	(0.2)	(0.22)	(0.24)			(0.2)	(0.23)	(0.24)
2006	0.52***	0.68***	-0.2		2006	0.66***	0.89***	-0.03
	(0.2)	(0.22)	(0.24)			(0.2)	(0.23)	(0.24)
2007	0.58***	0.55***	-0.1		2007	0.72***	0.76***	0.08
	(0.2)	(0.22)	(0.24)			(0.2)	(0.23)	(0.24)
2008	0.76***	0.68***	0.12		2008	0.89***	0.89***	0.29
	(0.2)	(0.22)	(0.24)			(0.2)	(0.23)	(0.24)

Notes: Coefficient estimates are starred to indicate statistical significance: \*\*\* indicates significance at 1% level; \*\* indicates significance at 5%; and \* indicates significance at 10% level. All estimates have panel-corrected standard errors in parentheses adjusted for panel-heteroskedasticity.

This table displays selected coefficient estimates from the pooled cross sectional regression for log permits in my sample of 57 MSAs, which includes 13 in-FEMA metro areas. The estimated regression is of the form  $y_{it} = \sum_{j=2000}^{2008} \beta_j T_j + \sum_{k=2000}^{2008} \beta_k D_i T_k + u_{it}$ , where *i* subscripts MSA.  $T_j$  are time dummies, j=2000, 2001, ..., 2008 and  $D_i$  are dummies capturing the effect of distance from New Orleans city in three different ways. The alternative specifications are as follows: 1)  $D_i$  is a dummy identifying in-FEMA MSAs and is interacted with time. 2)  $D_i$  is further divided into dummies indicating 'near' distance (for an MSA that lies within 100 miles of New Orleans) or 'medium' distance (100-200 miles from New Orleans city) and interacted with time. See also Figure 3.3.

Table 3.5 – Pooled Cross Section OLS estimates for Ownership Vacancy Rates

	Actual Vac	ancy Rate	Dama	nge-Weighted Vacancy Rate	
	FEMA	Distance		FEMA	Distance
	<b>Speciation</b>	<u>Specification</u>		<b>Specification</b>	<b>Specification</b>
	FEMA	Near		FEMA	Near
2000	0.72*	1.81***	2000	1.21***	2.78***
	(0.42)	(0.23)		(0.49)	(0.24)
2001	-0.5	-0.14	2001	-0.25	0.36
	(0.42)	(0.23)		(0.49)	(0.24)
2002	-0.97**	-1***	2002	-0.86*	-0.79***
	(0.42)	(0.23)		(0.49)	(0.24)
2003	-0.73*	-1.31***	2003	-0.63	-1.13***
	(0.42)	(0.23)		(0.49)	(0.24)
2004	-0.35	-1.56***	2004	-0.21	-1.32***
	(0.42)	(0.23)		(0.49)	(0.24)
2005	0.62	-0.21	2005	0.95**	0.39*
	(0.42)	(-0.23)		(0.49)	(0.24)
2006	0.38	0.21	2006	0.76	0.94***
	(0.42)	(0.23)		(0.49)	(0.24)
2007	0.82**	1.21***	2007	1.4***	2.34***
	(0.42)	(0.23)		(0.49)	(0.24)
2008	0.08	0.64***	2008	0.61	1.68***
	(0.42)	(0.23)		(0.49)	(0.24)

Notes: \*\*\* indicates significance at 1% level; \*\* indicates significance at 5%; and \* indicates significance at 10% level. Panel-corrected standard errors in parentheses adjusted for panel-heteroskedasticity. Table displays selected estimates from pooled cross sectional regression for ownership vacancy rate for 8 MSAs, including 2 in-FEMA areas. Regression equation:  $y_{it} = \sum_{j=2000}^{2008} \beta_j T_j + \beta_j^D D_i T_j + u_{it}$ , where *i* subscripts MSA.  $T_j$  are time dummies, j=2000:2008;  $D_i$  are dummies capturing distance from New Orleans city in two alternative specifications: 1)  $D_i$  is 1 for in-FEMA MSAs 2)  $D_i$  dummies indicating 'near' distance to New Orleans(shown) or 'medium' distance (not shown). The table displays estimation results for the regression run on actual vacancy rates and for an alternative specification that ranks vacancy rates in each MSA by the damange incurred in each MSA. See also Figure 3.4.

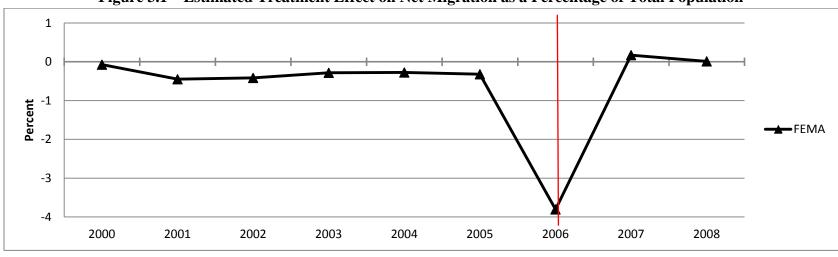
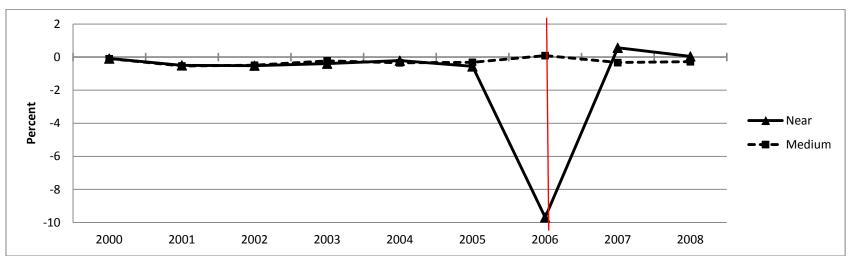


Figure 3.1 – Estimated Treatment Effect on Net Migration as a Percentage of Total Population



Notes: This figure plots point estimates for FEMA, NEAR and MEDIUM dummies interacted with time from regressions for actual migration data. See Table 3.2 for regression results and details on estimation.

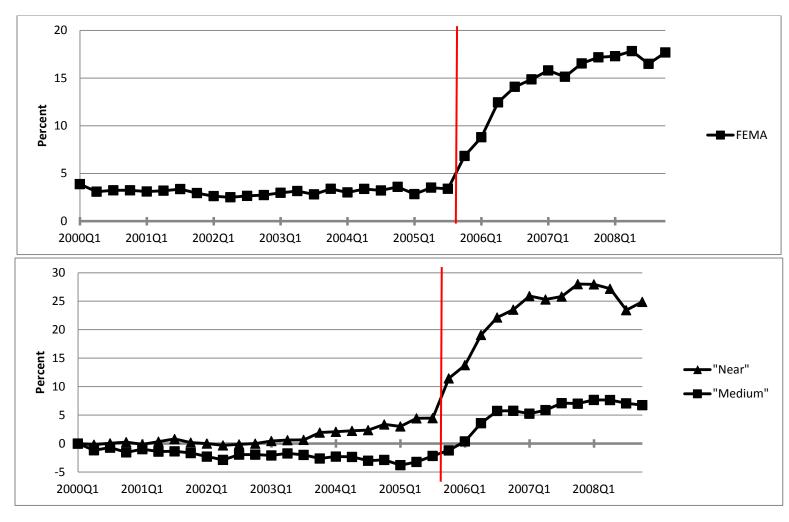
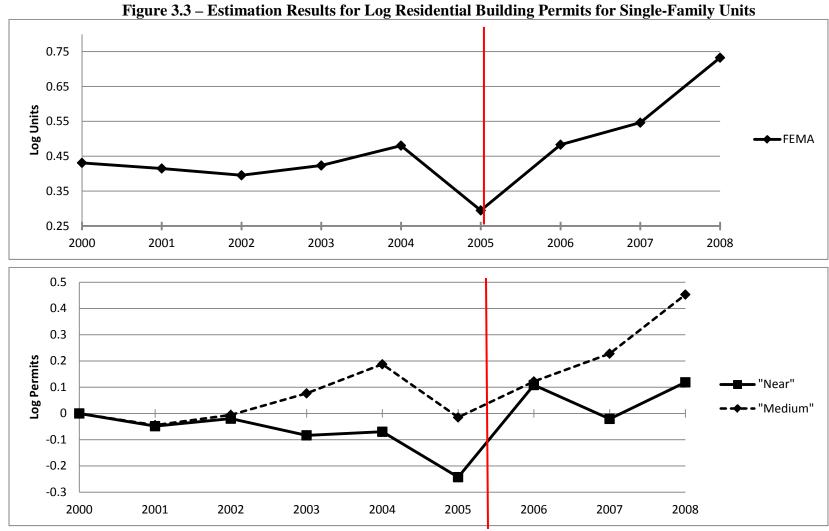


Figure 3.2 – Estimation Results for Quarterly House Price Index (1995Q1=100)

Notes: This figure plots point estimates for FEMA, NEAR and MEDIUM dummies interacted with time from regressions for actual price data. See Table 3.4 for regression results and details on estimation.



Notes: This figure plots point estimates for FEMA, NEAR and MEDIUM dummies interacted with time from regressions for actual permits data. See Table 3.5 for regression results and details on estimation.

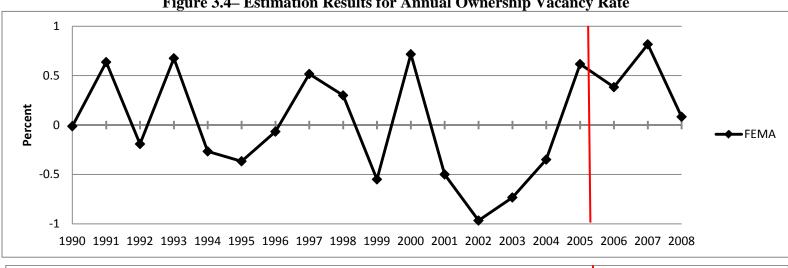
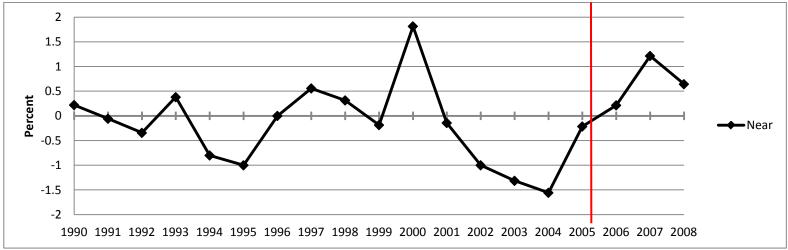
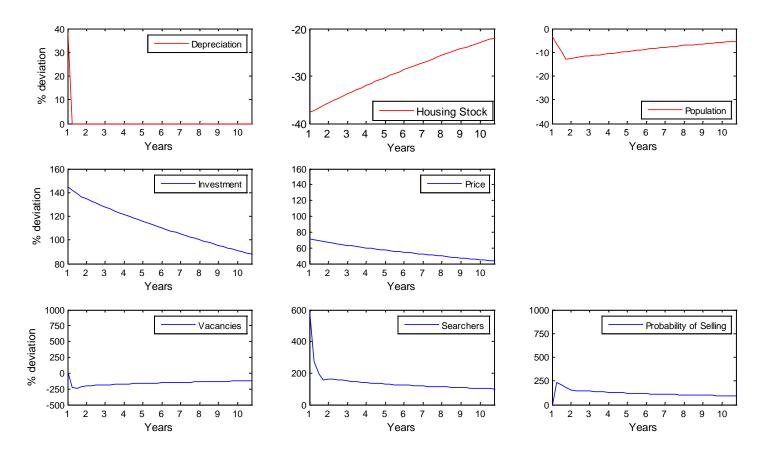


Figure 3.4– Estimation Results for Annual Ownership Vacancy Rate



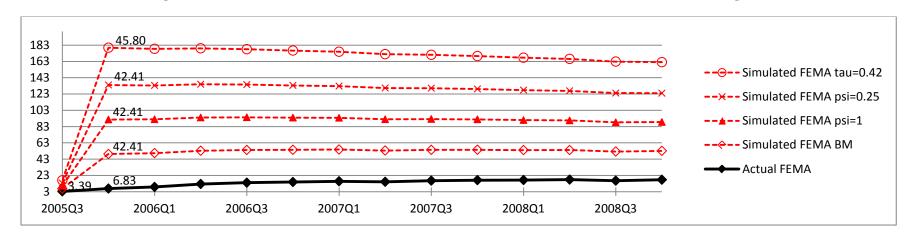
Notes: This figure plots point estimates for FEMA and NEAR dummies interacted with time from regressions for actual ownership vacancy rates. See Table 3.5 for regression results and details on estimation.

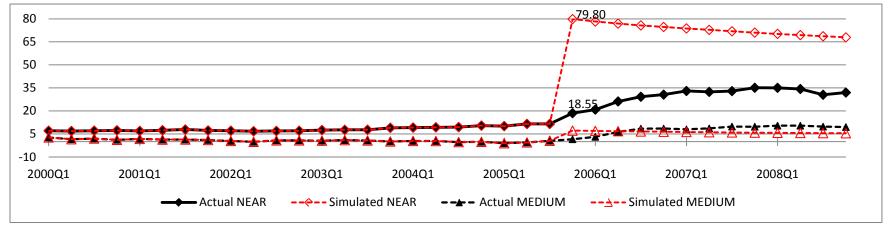
Figure 3.5 – FEMA MSAs Simulated Impulse Responses to Katrina Shock



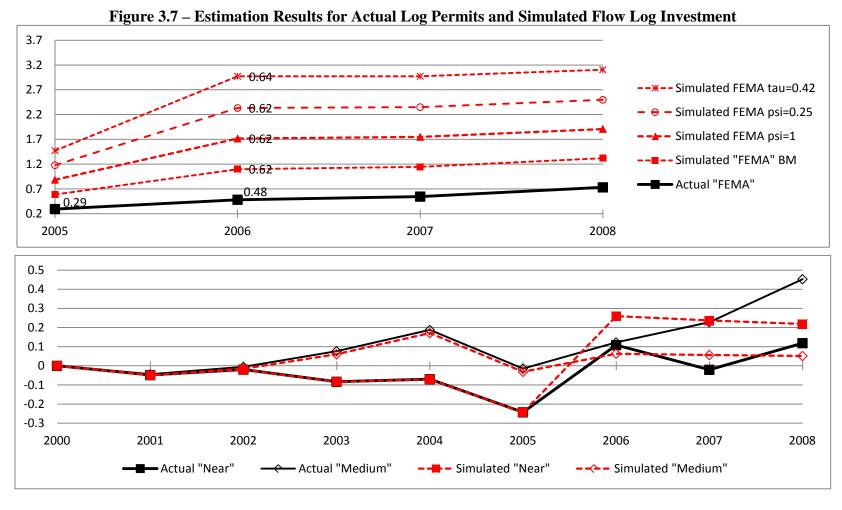
Notes: This figure plots impulse responses from the model simulation for New Orleans MSA. The top panel shows the depreciation shock, which is equal to 38 percent of the housing stock by construction from the data. Also in the top panel is the shock to population, which falls 13.4 percent in New Orleans on impact of the Hurricane. The second and third panels plot the impulse response for 10 years following the Hurricane, for selected variables from the model. Note that actual data are only available for 3 y ears after the Hurricane, which occurred on 29 August 2005, for comparison with these simulations.

Figure 3.6 – Estimation Results for Actual HPI and Simulated House Prices (Percentage)

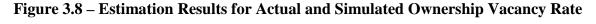


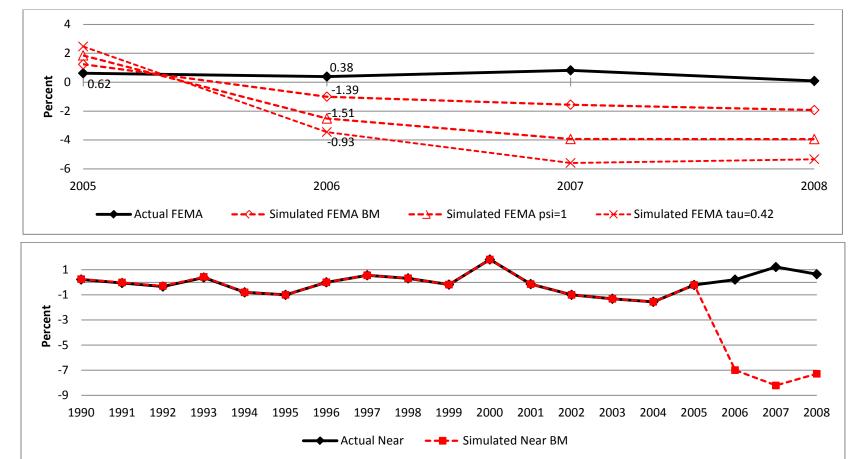


Notes: This figure plots regression results of actual versus artificial data on house prices. The dotted lines represent point estimates from artificial data. The top panel compares simulation results across different calibrations of the model and stacks each set of regression estimates one on top of the other. Hence the vertical distance between points at 2006Q1 for Actual FEMA and Simulated FEMA BM is the point estimate for Simulated FEMA BM. For the sake of graphical clarity, estimates prior to 2005Q3 are omitted in the top panel, because these are equal across actual and simulated data and across calibrations, by construction.



Notes: This figure plots regression results of actual versus artificial data on house prices. The dotted lines represent point estimates from artificial data. The top panel compares simulation results across different calibrations of the model and stacks each set of regression estimates one on top of the other. Hence the vertical distance between points at 2006Q1 for Actual FEMA and Simulated FEMA BM is the point estimate for Simulated FEMA BM. For the sake of graphical clarity, estimates prior to 2005Q3 are omitted in the top panel, because these are equal across actual and simulated data and across calibrations, by construction.





Notes: This figure plots regression results of actual versus artificial data on ownership vacancy rates. The dotted lines represent point estimates from artificial data. The top panel compares simulation results across different calibrations of the model and stacks each set of regression estimates one on top of the other. Hence the vertical distance between points at 2006Q1 for Actual FEMA and Simulated FEMA BM is equal to the numerical value of the point estimate for Simulated FEMA BM. For the sake of graphical clarity, estimates prior to 2005Q3 are omitted in the top panel, because these are equal across actual and simulated data and across calibrations, by construction.



Source: Census Atlas of the United States, http://www.census.gov/population/www/cen2000/censusatlas

# **Appendix to Chapter 3**

## **FEMA Damage Estimates by County**

Table 3.6 presents FEMA housing damage estimates by county for hurricane-affected MSAs. Out of a total of 117 counties affected by the hurricane, destruction of housing units was concentrated in 11 counties along the Louisiana and Mississippi coastlines. These counties mostly comprise New Orleans-Metairie-Kenner, LA and Gulfport-Biloxi, MS metropolitan statistical areas. Worst hit in New Orleans were Orleans, St. Bernard and Plaquemines Parishes, which lost between 35 and 50 percent of their housing stocks due to flooding. In Gulfport MSA, Hancock County was worst affected and lost almost 20 percent of its housing stock.

#### **FEMA Damage Estimates by Tenure**

Appendix Table 3.7 shows the breakdown of damage estimates by tenure, across the states affected by the hurricane. Damage estimates are calculated as a percentage of total *occupied* housing units estimated in the American Community Survey 2005. This breakdown by tenure gives an indication of how much of the displaced population from these states could be classified as 'permanent renters', outside the scope of our analysis.

FEMA inspections lasted until 12 February 2006, so these estimates include damage from Hurricane Rita. Hurricane Rita occurred on 24 September 2005 and caused repeated flooding of areas affected by Katrina. For the use of FEMA estimates in this paper, this does not pose an issue since other data series used are annual (or, in case of price data, quarterly). Having damage estimates for the third quarter of 2005 makes them more consistent with the rest of the analysis.

Table 3.6 – Appendix: Estimated Number of Housing Units Damaged in Core Counties\* Affected by Hurricane Katrina

	Major Damage	Severe Damage	Total Housing Units as of July 1, 2005	Percentage of Total Housing Stock Destroyed (%)	Percentage of Total Housing Stock with Major or Severe Damage (%)
Louisiana					
LAKE CHARLES MSA					
Cameron Parish	914	1,665	5,724	29.09	45.06
NEW ORLEANS-METAIRIE-KENNER M	MSA				
Jefferson Parish	29,643	4,677	192,373	2.43	17.84
Orleans Parish	26,405	78,918	213,137	37.03	49.42
Plaquemines Parish	1,190	3,994	11,290	35.38	45.92
St. Bernard Parish	5,938	13,748	27,292	50.37	72.13
St. Tammany Parish	15,948	1,682	88,791	1.89	19.86
ABBEVILLE MICROPOLITAN STATIST	ΓICAL AREA				
Vermilion Parish	2,372	207	23,562	0.88	10.95
Mississippi					
GULFPORT-BILOXI MSA					
Hancock County	7,185	4,611	23,530	19.60	50.13
Harrison County	16,829	7,618	88,138	8.64	27.74
Stone County	432	101	5,602	1.80	9.51
PASCAGOULA MSA					
Jackson County	14,259	2,043	56,732	3.60	28.74

Notes:\*Louisiana is divided into parishes, analogous to counties elsewhere. Counties are assigned to MSAs per November 2007 definitions by U.S. Office of Management & Budget (OMB). These estimates were finalized by February 12, 2006 & include storm surge damage from Rita. Hurricane Katrina hit New Orleans & Gulfport-Biloxi metro areas with unequaled force & most of this damage was initiated in September 2005. Lake Charles MSA lay directly in Rita's path & Rita also caused repeated flooding in areas previously hit by Katrina. Major damage refers to flooding of up to 2 feet or where less than 50 percent of a house is damaged and requires extensive repair work for future occupancy. Severely damaged housing units are completely destroyed or flooded ≥ 2 feet. Source: FEMA count of damaged units from HUD Report (2006); Estimated total housing units from Population Division, U.S. Census Bureau.

Table 3.7 Appendix: State-Level Estimated Damage to Occupied Housing Units in Hurricanes Katrina and Rita by Tenure

	Alabama		Louis	Louisiana Miss		sissippi	Texas	
	Owner- Occupied Units	Renter- Occupied Units	Owner- Occupied Units	Renter- Occupied Units	Owner- Occupied Units	Renter- Occupied Units	Owner- Occupied Units	Renter- Occupied Units
Flood Damage								
Major Damage	1,829	470	40,434	25,881	18,752	10,285	391	140
Severe Damage/Destroyed	156	56	61,137	41,361	7,366	4,707	37	17
Wind Damage								
Major Damage	765	172	18,589	13,182	12,137	4,602	7,700	2,292
Severe Damage/Destroyed	162	74	2,432	1,721	2,252	1,285	1,146	380
Total Occupied Units	1,261,475	527,217	1,136,873	539,726	757,446	326,588	5,162,604	2,815,491
Severe Damage								
as Percentage of	0.02	0.02	5.50	7.00	1 27	1.04	0.02	0.01
Occupied Units (%)	0.03	0.03	5.59	7.98	1.27	1.84	0.02	0.01
Severe & Major Damage as Percentage of								
Occupied Units (%)	0.23	0.15	10.78	15.22	5.35	6.39	0.18	0.11

Source: HUD Report (2006); American Community Survey 2005

Table 3.8 – Appendix: List of MSAs in Panel Data used for Empirical Analysis

CBSA		Miles to	HVS Sub-
Code	MSA Name	New Orleans	Sample
35380	New Orleans-Metairie-Kenner, LA	0.00	Yes
26380	Houma-Bayou Cane-Thibodaux, LA	45.75	
25060	Gulfport-Biloxi, MS	64.96	
12940	Baton Rouge, LA	72.68	
37700	Pascagoula, MS	95.33	
25620	Hattiesburg, MS	103.37	
29180	Lafayette, LA	118.22	
33660	Mobile, AL	130.85	
27140	Jackson, MS	162.64	
10780	Alexandria, LA	170.04	
29340	Lake Charles, LA	188.67	
33740	Monroe, LA	213.29	
13140	Beaumont-Port Arthur, TX	241.86	
46220	Tuscaloosa, AL	269.62	
33860	Montgomery, AL	278.73	
43340	Shreveport-Bossier City, LA	280.24	
20020	Dothan, AL	292.04	
13820	Birmingham-Hoover, AL	311.47	Yes
38220	Pine Bluff, AR	314.89	
26420	Houston-Sugar Land-Baytown, TX	318.07	Yes
30980	Longview, TX	326.10	
12220	Auburn-Opelika, AL	326.80	
45500	Texarkana, TX-Texarkana, AR	336.49	
17980	Columbus, GA-AL	346.96	
46340	Tyler, TX	351.04	
30780	Little Rock-North Little Rock-Conway, AR	353.23	
11500	Anniston-Oxford, AL	353.85	
32820	Memphis, TN-MS-AR	357.61	Yes
26300	Hot Springs, AR	358.35	
22520	Florence-Muscle Shoals, AL	364.11	
19460	Decatur, AL	365.13	
23460	Gadsden, AL	367.03	
10500	Albany, GA	367.91	
17780	College Station-Bryan, TX	378.02	
26620	Huntsville, AL	387.20	
27180	Jackson, TN	398.45	
27860	Jonesboro, AR	405.94	
46660	Valdosta, GA	409.95	

Table 3.8 (Contd.)
Appendix: List of MSAs in Panel Data used for Empirical Analysis

CBSA Code	MSA Name	Miles to New Orleans	HVS Sub- Sample
40660	Rome, GA	412.26	Sumpre
47580	Warner Robins, GA	422.70	
12060	Atlanta-Sandy Springs-Marietta, GA	424.30	Yes
47020	Victoria, TX	425.58	
31420	Macon, GA	428.28	
47380	Waco, TX	433.58	
19124	Dallas-Plano-Irving, TX (MSAD)	442.75	Yes
19140	Dalton, GA	445.91	
16860	Chattanooga, TN-GA	450.43	
12420	Austin-Round Rock, TX	460.13	Yes
18580	Corpus Christi, TX	467.82	
34980	Nashville-DavidsonMurfreesboroFranklin, TN	468.18	Yes
17420	Cleveland, TN	470.76	
23580	Gainesville, GA	472.36	
12020	Athens-Clarke County, GA	480.08	
15260	Brunswick, GA	518.69	
15180	Brownsville-Harlingen, TX	530.82	
12260	Augusta-Richmond County, GA-SC	534.74	
42340	Savannah, GA	552.00	

Notes: This table displays the list of metropolitan statistical areas (MSAs) that comprise the sample for empirical analysis in this paper, sorted by "Miles to New Orleans". The first column shows the CBSA code – the standard geographic identifier for each MSA per November 2007 MSA definitions. The last column indicates with entry "Yes" if the MSA is in the sub-sample for which ownership vacancy rates are available from the Housing Vacancy Survey.

## **Chapter 4**

# The Impact of the U.S. Federal Housing Tax Credit Scheme in a Search and Matching Model with Heterogeneous Matches

#### I. Introduction

In the aftermath of the subprime mortgage and financial crises, most cities across the U.S. economy have experienced falling prices and rising inventories of unsold vacant housing units. Between 2008 and 2010 the U.S. Federal government initiated the Home Buyer's Tax Credit policy to lower the cost of purchase for prospective home buyers with a lump sum tax credit. According to the IRS, in 2010 the program cost an estimated \$16 billion to the U.S. Treasury Department, with more than 2.2 million people filing for the credit, and \$3.6 billion in 2009 with just 479,622 claimants. This paper investigates the impact of such policy intervention on housing prices, time to sale and housing unit sales.

In this paper, I present a search and matching model of housing that incorporates searcher-buyer heterogeneity and durable housing units that are held vacant for sale in equilibrium. The crux of the model is that a searcher matched with a vacancy finds out whether or not it is a good fit for him. For a poor fit, the match is discarded and the searcher continues the search. For a good fit, the match proceeds to sale. Every time the market meets, a distribution of fits is realized and a cutoff fit can be identified below which there are no sales. When there is a positive incentive to purchase that lowers the effective price paid by the searcher, the market accepts sales of a lower quality that it otherwise would have. In the absence of such an incentive, the market prefers sales at a higher price, for a higher (average) fit.

In steady state, the equilibrium market average price and the marginal fit value are positively correlated. There is no determinate steady state correlation, however, between time to sale and the marginal fit value. A higher cutoff fit value does not necessarily imply that the market will have a higher equilibrium time to sale. The relationship

between time to sale and marginal fit value depends on the matching efficiency of the market, which is exogenous. In the absence of a matching friction with heterogeneous demand, equilibrium time to sale predicted by the model is 4 weeks, which is half the length of search typically observed for U.S. households.

In dynamic analysis, I observe that there is a composition effect in response to an adverse demand shock that mitigates the response of observed price to less than what it would be in the absence of searcher heterogeneity. I show that a failure to adjust prices down to reach equilibrium quickly is a reflection of the fact that when faced with a demand shock sellers can adjust in other respects rather than price alone. In the presence of heterogeneity and matching frictions, sellers endure a longer time to sale given the likelihood of finding a buyer with a higher fit in future. Reinforcing this point, model simulations illustrate that as the heterogeneity of fit increases, market average price becomes less variable.

The model shows that a tax credit scheme can successfully raise sales and the average price in the market while it is in place. Moreover, it lowers the selectivity that searchers and sellers display, by causing a lower cutoff fit value in the market and a commensurately shorter time to sale, given a fixed degree of matching efficiency.

In the model, all vacant units are assumed to be built to a homogeneous standard and heterogeneity stems from the searcher side of the market. For each seller-searcher paired by the market mechanism, the match surplus varies by the quality of fit between the housing unit and searcher. Fit value captures those elements of product differentiation which are not standardized by a seller or producer of housing ex ante and are idiosyncratic to the searcher-buyer matched with a particular housing unit.

As in the basic search and matching model, housing demand is exogenously determined by Poisson processes whereby a given fraction of the population enters the state of search each period. Hence, searchers leave their existing housing units when they receive an exogenous move or depreciation shock, automatically creating new demand for housing each period. Moreover, these shocks also partly drive the supply of vacant housing units. Since all searchers who enter the market thus are previously homeowners, their housing units also become vacant for sale if they have not depreciated. The stock supply of vacancies for sale also receives a flow of new vacant units from new

investment in housing. The total supply of vacancies is therefore generated by the fraction of homeowners exogenously separated from their housing units and new investment undertaken by profit maximizing construction firms.

Using the technology of the matching function, the market rations vacancies across searchers coincidentally yielding a random draw of fit values for all matched pairs. There is a positive probability of no match for any vacancy which increases in the ratio of vacancies to searchers. This externality effect generates persistence in the stock of vacant housing in the dynamic environment of this model.

In this paper, the quality of fit per searcher-vacancy pair is an independent identically distributed variable. Since fit value is i.i.d., neither the market nor the searcher can predict future fit values based on the current draw, which forces the searcher to consider a current match independently of all other future draws. So the model here is still a random search model wherein sellers cannot price discriminate ex ante to ensure a transaction through self-selection by searchers, unlike directed search models (see Shi, 2002, for an example of the latter).

Match surplus is increasing in quality-of-fit: the higher the fit value a searcher realizes with a matched vacancy, the higher is the searcher's payoff from purchasing the unit, and the higher is the total surplus associated with the match. Both parties to a transaction know the value of the searcher-buyer's realized fit once a match is made. For any matched searcher-vacancy pair with a non-negative surplus, the match culminates in a sale. The match which yields zero total surplus defines the cutoff fit value below which matches do not result in a sale. This marginal fit value thus becomes a sufficient statistic to determine the number of sales that will occur each period.

In this paper I present dynamic simulation results as well as comparative steady state analysis using the model. I also provide a policy application by analyzing the impact of the recent federal home buyers' tax credit scheme on market price, time to sale and sales.

In 2008, federal housing tax credits for new homebuyers were introduced as part of stimulus measures to revitalize the U.S. housing market in the aftermath of the subprime mortgage crisis. They were enacted as part of the 2008 American Housing

Rescue and Foreclosure Prevention Act; 2009 Worker, Homeownership and Business Assistance Act; and 2009 American Recovery and Reinvestment Act.

The so called first time home buyer tax credits were extended from April 9, 2008 through the end of May 1 2010, being in effect for just over two years in total. They were extended to qualifying first-time home buyers for a credit worth 10 percent of the housing unit's purchase price, capped at \$7500 in 2008 and \$8000 in 2009 and 2010. The credit was also extended to qualifying non-first-time home buyers, who could receive a maximum tax credit of \$6500. Additionally, the credit was available only to buyers whose gross income is less than \$95,000. The limit was revised in November 2009 to \$145,000. Finally, the maximum purchase price of a housing unit allowable under the tax credit scheme was \$800,000. Ignoring the finer details of the tax code, the terms of the scheme require that owners retain their housing units for 3 years after the purchase date, or repay the credit if they sell before then.

According to the IRS, in 2010 the program cost an estimated \$16 billion to the U.S. Treasury Department, with more than 2.2 million people filing for the credit, and \$3.6 billion in 2009 with just 479,622 claimants. An official audit of the program has revealed widespread misuse of the program, with people claiming the benefit on housing not yet purchased or without a valid purchased address; claiming the benefit in the name of people below the age of 18; claiming the benefit on housing units previously filed as being owned by a spouse, etc. This paper does not comment on the fraudulent application for funds from the scheme.

I apply the model to conduct the policy experiment qualitatively by introducing a housing tax credit for 2 years, at 10 percent of the average steady state price of a housing unit. Since the model is dynamic I can trace out the qualitative impact on time to sale, the marginal fit value, prices, and supply in response to a tax credit shock.

By distorting the effective surplus of a match upwards, the credit allows searcherbuyers with lower fit values to bid high enough that sellers agree to a sale. Hence, the threshold or marginal quality of matches in the market is lowered by the policy shock and it is effective in raising the number of sales. Since the model is in steady state prior to the shock, investment rises with the credit, keeping the number of vacancies per searcher roughly level. However, since people become more willing to accept lower-fit matches to take advantage of the credit, time on market for a vacancy falls. All else equal, the model predicts that the tax credit effectively greases the wheels of the housing market by raising sales and new flow investment in the market and decreasing the time to sale.

This paper brings together many elements that have featured in other search and matching models. Novy-Marx (2009) points out that a typical search and matching model equilibrium is a dual of a price and a vacancy rate. In response to a shock, the market adjusts both, rather than achieving clearing through price movements alone. In this paper I show that there are three endogenous variables that determine equilibrium if one explicitly takes heterogeneity into account: namely, price, vacancy rate and the minimum fit between searcher-buyer and vacant units.

Toshihiko and Sahin (2006) consider a heterogeneous labor-market model where transactions are rejected for any match with a negative surplus, leading to a natural cutoff match type in the market. This is in agreement with the way I analyze searcher-buyer heterogeneity in the housing market.

Shimer and Smith (2001) agree that introducing heterogeneity is an important part of analyzing whether matching is successful but ex ante heterogeneity makes the market solution inefficient (and renders the Hosios condition irrelevant). Since my paper models ex post heterogeneity, it skirts both the issue of ex ante directed search and the problem of inefficiency in a decentralized equilibrium with heterogeneity.

Lazear (2010) points out that downward price rigidity is observed and quite idiosyncratic to the housing market, in contrast with other major asset markets. I show with my model that the measured price in the housing market moves slowly in response to an adverse demand shock because of a composition effect. In particular, an unexpected decrease in rental rates does not elicit the full reduction in prices that a Walrasian market with identical agents might dictate, just as Lazear contends. However, I show that this failure to adjust prices down to reach equilibrium quickly is a reflection of the fact that sellers adjust in other respects rather than price alone: they withstand a longer time to sale, because of the possibility that they might sell to a searcher-buyer with a higher fit value in a downturn, as compared to the steady state. In other words, the heterogeneous matching model shows that the search and matching friction, as well as the additional friction imparted on the market from heterogeneous matches, widen the set of variables

that adjust in response to a negative demand shock and dampen the response of price alone.

The paper proceeds as follows. Section II lays out the search and matching model with heterogeneous matches. In Section III, I discuss numerical calibration of the model. Section IV analyzes the steady state solution of the model. In Section V, I use the model for a dynamic analysis of the impact of exogenous rental shocks in the housing market, discussing summary statistics from Monte Carlo simulations of the model and illustrating the composition effect in the market. In Section VI, I analyze the home buyers' tax credit scheme using the heterogeneous matching model. Section VII concludes.

# II. A Search and Matching Model with Heterogeneous Matches in the Housing Market

This section presents the search model with variable 'fit' between a matched searcher-vacancy pair. Matches are heterogeneous because searchers have idiosyncratic preferences in housing realized after a match has been made. When a searcher is matched with a vacancy, he realizes the degree to which the vacancy meets his preferences, which is reflected in an i.i.d quality-of-fit random variable. After the market pairs searchers and vacancies together matches can be ranked by their fit, even though both are ex ante homogenous.

A match only becomes a sale if it generates non-negative total surplus. This implies that the value of fit for the searcher is at least as high as a threshold value at which total match surplus would be zero. Should a searcher pass up a match he is currently offered, the market cannot predict his future fit based on his current fit with a vacancy because fit is a random independent and identically distributed variable. Hence, the continuation value of the search state is a probability weighted average of future fit outcomes, above the expected minimum fit next period. Since all searchers are identical before they are matched, the payoff from being in the search state is identical across searchers.

Analogously, the payoff from having a vacancy to sell is also based on the expected *average* sales price in the future compared to the current price if the vacancy is sold immediately.

The fit of a match is modeled as a stationary stochastic continuous variable,  $\zeta$ , with a cumulative distribution function denoted  $G(\zeta)$  and a probability distribution function  $g(\zeta)$ . A higher realized value corresponds to a higher quality match for the searcher. The threshold fit value at which total surplus from a match is zero is denoted by  $\bar{\zeta}$ .

#### Market Technology

The market matching technology is identical to the Cobb-Douglas constant returns to scale specification assumed in the basic model, where total matches, M, depend on the total number of searchers, S, and the total number of vacancies, V. The parameter  $\overline{m} \in (0,1)$  captures the efficiency of the market technology in pairing each searcher with a vacancy. The smaller the value of  $\overline{m}$ , the greater the degree of matching friction in the market.

$$M(S,V) = \overline{m}S^{\gamma}V^{1-\gamma}$$

The number of vacancies per searcher, which I will interchangeably refer to as relative supply, is denoted by  $\theta = \frac{V}{S}$ . The rate at which a vacancy is matched with a searcher can be written as the ratio of total matches to total vacancies available, denoted by  $q(\theta) = \frac{M}{V} = \bar{m}\theta^{-\gamma}$ .

Relative supply provides a measure of market saturation and allows a characterization of time-to-sale, which is the inverse of the hazard rate for vacancies. Unlike the basic search and matching model, in a heterogeneous matches environment  $q(\theta)$  is not the hazard rate for a vacancy. The hazard rate for a vacancy must account for the fit value of each matched vacancy-searcher pair and how it compares to some marginal fit value,  $\bar{\zeta}$ , which will be explained and derived shortly. Thus, the hazard rate for a vacancy is its matching rate multiplied by the probability that the match fit value equals or exceeds a cutoff fit value.

$$\Pr(vacancy\ sells) = (1 - G(\bar{\zeta})) * q(\theta).$$

Analogously, the rate at which a searcher is matched with a vacancy is given by the ratio of total matches to total searchers in the market. It is denoted by  $\mu(\theta) = \overline{m}\theta^{1-\gamma}$ . The hazard rate for a searcher is the matching rate for a searcher multiplied by the probability that the match fit value equals or exceeds the cutoff fit value.

$$Pr(search\ ends) = (1 - G(\bar{\zeta})) * \mu(\theta).$$

Finally, if every match need not materialize in a sale, the total number of sales is also conditional on the marginal fit value in the market. Hence, total sales per period are given by the number of matches per period times the fraction of matches that lie above the cutoff value,

$$Total\ Sales = (1 - G(\bar{\zeta})) * Total\ Matches.$$

#### Demand

As in the basic model, I assume that each owner-occupier has an equal chance of receiving a shock that separates him from his current housing unit, which is denoted by the Poisson rate  $\lambda$ . Each owner-occupier also faces an exogenous likelihood of complete depreciation, denoted by the Poisson rate  $\delta$ . Complete depreciation is essentially the exit of a house from the resale and ownership market and changes the state of an owner-occupier into that of a searcher. Since this is an aggregate model, this assumption allows for a non-zero level of new construction in steady state equilibrium investment. Since both the separation and depreciation shocks are Poisson rates, they can be interpreted as the fractions of the aggregate population that experiences separation and depreciation each period and enters the search state.

Given this predictable inflow into housing demand each period, we can express the number of searchers in the market next period,  $S_{t+1}$ , as the sum of searchers unable to transact in the previous period, which is the first and second terms in [1], and the number of new searchers due to separation and depreciation of the remaining population, which is the third term in the equation.

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<sup>&</sup>lt;sup>1</sup> One can also interpret this as a fraction  $\delta$  of each housing unit suffering wear and tear each period, but the maintenance costs being accumulated over time and realized by the owner occupier only at the point of sale as a cumulative loss in value. Hence, individual owners do not undertake maintenance expenditures in this model, thereby just 'eating through' their assets.

$$S_{t+1} = (1 - \mu_t)S_t + \mu_t G(\overline{\zeta_t})S_t + (\lambda + \delta(1 - \lambda))(N_t - (1 - \mu_t)S_t - \mu_t G(\overline{\zeta_t})S_t)$$
[1]

In the first two terms, out of current searchers,  $S_t$ , the fraction  $(1 - \mu_t)$  are those not matched with a vacant unit by the market and the fraction  $\mu_t G(\overline{\zeta_t})$  are matched but do not finalize a sale. In the third term,  $N_t$  is the total population. From  $N_t$ , we can subtract the total number of searchers still searching in time t. The remaining population comprises owner-occupiers, who are then subject to the exogenous separation and depreciation shocks.

# Supply

Housing supply in this model comes from new construction, referred to as investment, and the resale of existing housing units. Given the durable nature of housing, there is a distinction between the inflow of units into the available vacant stock, the outstanding number of vacancies that comprise the stock supply for sale, and the stock of total housing available in the aggregate.

The flow supply curve for new housing units in equation [2] is an upward sloping schedule that equates the expected sales value of a vacancy to the marginal cost of new construction. In the market for housing units in this model, vintage does not make a difference to the sales price. Hence  $A_t$  is the expected sales value of any vacancy for sale today, be it newly constructed or an existing unit coming onto the market.  $X_t$  refers to the number of new units constructed in period t, also referred to as investment. Finally,  $1/\xi$  is the elasticity of flow supply, which is less than 1. The shift parameter  $\alpha$  can be used to capture exogenous cost shocks for new construction and can be normalized to 1 without loss of generality.

$$A_t = \alpha X_t^{\xi} \tag{2}$$

The stock supply of vacancies for sale evolves according to equation [3]. The number of vacancies for sale next period,  $V_{t+1}$ , is the sum of unsold vacancies this period and new construction and existing housing units coming on to the market for sale.

$$V_{t+1} = (1 - q_t)(1 - \delta)V_t + q_t(1 - \delta)G(\overline{\zeta_t})V_t + \lambda(1 - \delta)[H_t - (1 - q_t)V_t - q_tG(\overline{\zeta_t})V_t] + X_t$$
[3]

In the first term on the right hand side of [3] are included all vacancies that remained unmatched in period t and escape depreciation. In the second term are vacancies that were matched by the market but did not proceed to sale and do not depreciate. In the third term, inside the square brackets, is the total housing stock net of unsold vacancies, all of which is owner-occupied in period t. A fraction  $\lambda(1-\delta)$  of these owner-occupied units enters the state of vacancy each period as their owners receive the exogenous move shock but escape the depreciation shock. Finally, new housing construction in period t,  $X_t$ , enters the stock of vacancies for sale in t+1.

Given new construction  $X_t$ , the total stock of housing evolves according to equation [4].

$$H_{t+1} = H_t(1 - \delta) + X_t$$
 [4]

Continuation Values for Owner, Sellers and Searchers

This model divides agents in the housing market in to four categories: searchers, owner-occupiers, sellers and firms. Firms are engaged in new construction and their optimal condition is described by the upward sloping supply curve in equation [2]. The optimal decision of the remaining three categories of agents depends on value functions that describe their expected real payoffs. These value functions are forward looking and recursive in structure.

Existing owner-occupiers are assumed to earn a continuation value,  $W_t$ , that is identical for all owner-occupiers, in equation [5].

$$W_t = R_t + b + \lambda (L_t + (1 - \delta)A_t) + \delta (1 - \lambda)L_t + \frac{(1 - \lambda)(1 - \delta)}{1 + r} E_t W_{t+1}$$
 [5]

The owner-occupier in period t earns implicit rent  $R_t$  and an additional benefit from owning his place of residence b that captures the ability to customize fixtures and fittings to his taste and make minor structural changes. In the third term, if the owner-occupier receives an exogenous move shock this period, he becomes a searcher looking for a replacement housing unit and earning a payoff  $L_t$  and a seller of his existing housing unit, also earning a seller's payoff,  $A_t$ . Similarly, the fourth term describes the agent's payoff should he lose his current housing unit to depreciation, only earning the payoff of a searcher in that case. The last term is the owner-occupier's expected payoff next period, which the agent receives if he does not experience the move shock or the depreciation

shock. The last term is discounted to present value at the discount rate r, which is common across all risk-neutral owner-occupiers.

Next, consider the value function of a seller. Whether by new construction or a separation shock, once a vacancy is created, the seller receives the opportunity to sell it only when the market meets at the start of next period. Given that the opportunity to sell does not arise until the following period, the value function for a seller is a forward looking equation that anticipates the payoffs from a successful sale, an unsuccessful match and no match at all next period.

When the market meets and searcher-vacancy pairs are matched, the idiosyncratic realization of fit for the searcher in the matched pair becomes known to both parties. The searcher earns the value of the fit at the time of sale if the match is successful and hence the fit value constitutes part of the match surplus. The sales price of a vacancy therefore depends on the fit value of the match and determines the seller's payoff in a successful match.

For any current match with fit value,  $\zeta_t$ , I will denote the idiosyncratic price as  $P(\zeta)_t$ . The expected value of a vacancy for sellers is denoted by  $A_t$ . The following equation describes  $A_t$  as the present discounted sum of the contingent payoffs received by the seller next period.

$$A_{t} = \frac{1}{1+r} E_{t} \left[ q_{t+1} \left( \int_{\bar{\zeta}_{t+1}}^{\infty} P(\zeta)_{t+1} g(\zeta) d\zeta \right) + q_{t+1} G(\bar{\zeta}_{t+1}) (1-\delta) A_{t+1} \right]$$

$$+ (1-q_{t+1})(1-\delta) A_{t+1}$$

[6]

Recall that a vacancy is matched by the market technology at a Poisson rate  $q_{t+1}$  in time period (t+1). The first term inside the square bracket in [6] captures the event that a vacancy is matched with fit value above the market cutoff. Then, the seller's expected payoff is the price he receives for any fit value greater than or equal to the expected market cutoff fit in (t+1). Since the fit of the match he will make in (t+1) is not known ex ante, his payoff is the expected price over all possible fit values above the cutoff. A match may not always become a successful sale. The searcher's realized fit value will lie below the market cutoff value at date (t+1) with probability mass

 $G(\bar{\zeta}_{t+1})$ . The second term thus writes the payoff to the searcher with an unsuccessful match, which is simply the expected value of a vacancy to sell next period if there is no depreciation shock. In the final term, with probability  $(1 - q_{t+1})$  a vacancy is not matched by the market at all. In this event, too, the seller simply receives the expected value of a vacancy to sell next period if there is no depreciation shock.

We can simplify the expression for the seller's payoff by defining the conditional average price in the market as follows in equation [7].

$$\bar{P}_t = \left( \int_{\bar{\zeta}_t}^{\infty} P(\zeta)_t g(\zeta) d\zeta \right) / \left( 1 - G(\bar{\zeta}_t) \right)$$
 [7]

Thus,  $\bar{P}_{t+1}$  is just the average of the observed prices in the market at time t. Since the market does not allow the sale of matches with fit values below the marginal fit value, the observed distribution of prices is truncated at the bottom at  $P(\bar{\zeta})_t$ . Hence the average observed price is simply the conditional mean of the distribution of prices, corresponding to all  $\zeta_t > \bar{\zeta}_t$ . Substituting [7] into the seller's payoff in equation [6] and collecting terms, I simplify  $A_t$  as follows.

$$A_{t} = \frac{1}{1+r} E_{t} \left[ (1 - G(\overline{\zeta}_{t+1})) q_{t+1} \, \overline{P}_{t+1} + \left( 1 - q_{t+1} (1 - G(\overline{\zeta}_{t+1})) \right) (1 - \delta) A_{t+1} \right]$$

Finally, the value function for ex ante homogeneous searchers in the model is written as the sum of payoffs across different contingencies in the next period. Flow rental payments are excluded from the searcher's payoff and included instead as imputed rent in the owner-occupier's value function. Treating rental payments like this focuses the model on the market for housing sales alone, rather than explicitly including a rental market as well.

The timing of the model dictates that an agent who is in the search state in period t will next have the opportunity to match with and purchase a vacancy in t+1. At time t+1, with probability  $\mu_{t+1}$ , a searcher is matched with a vacancy and realizes a random fit value  $\zeta_{t+1}$ . Because  $\zeta$  is independent and identically distributed, the searcher's previous realizations are no indication of what fit value he will realize in period t+1. The match is successful and results in a sale if  $\zeta_{t+1} \geq \bar{\zeta}_{t+1}$ .

Hence, the searcher's continuation value,  $L_t$ , is the present discounted value of expected contingent payoffs from purchasing or searching next period, as in equation [8].

Because all searchers are ex ante homogeneous and  $\zeta$  is independent and identically distributed, the continuation value of the search state is identical across searchers.

$$L_{t} = \frac{1}{1+r} E_{t} \left[ \mu_{t+1} \left( \int_{\bar{\zeta}_{t+1}}^{\infty} (W_{t+1} - P(\zeta)_{t+1} + C_{t+1} + \zeta_{t+1}) g(\zeta) d\zeta \right) + \mu_{t+1} G(\bar{\zeta}_{t+1}) L_{t+1} + (1-\mu_{t+1}) L_{t+1} \right]$$
[8]

The market technology matches a searcher with a vacancy at Poisson rate  $\mu_{t+1}$ . If the searcher has a fit above the marginal fit value for the market at t+1, he will purchase the housing unit and earn the payoff of an owner-occupier,  $W_{t+1}$ , plus the housing tax credit,  $C_{t+1}$ , and the fit value of the match,  $\zeta_{t+1}$ , less the sales price paid,  $P(\zeta)_{t+1}$ . This is captured by the first term of equation [8]. Since the searcher does not know ex ante what the realized fit will be, his payoff from purchasing is an expectation over all potentially successful fits. The second term in [8] captures the payoff from finding a match with fit below the market cutoff in period t+1. In that event, the searcher earns the continuation value of searching into the next period, with the composite probability weight  $\mu_{t+1}G(\bar{\zeta}_{t+1})$ . The last term in equation [8] shows that a searcher will also earn the continuation value of searching into the next period if he is not matched at all in t+1.

Just as I simplified equation [6], I can simplify the integral in [8]. Notice that  $W_{t+1}$  and  $C_{t+1}$  are independent of  $\zeta$ . Also, equation [7] provides us with a substitution for the conditional mean price in t+1. Hence, using that substitution and collecting terms, I can rewrite equation [8] as follows.

$$L_{t} = \frac{1}{1+r} E_{t} \left[ \mu_{t+1} \left( 1 - G(\bar{\zeta}_{t+1}) \right) \left( W_{t+1} - \bar{P}_{t+1} + C_{t+1} + \int_{\bar{\zeta}_{t+1}}^{\infty} \zeta_{t+1} g(\zeta) d\zeta / \left( 1 - G(\bar{\zeta}_{t+1}) \right) \right) + \left( 1 - \mu_{t+1} \left( 1 - G(\bar{\zeta}_{t+1}) \right) \right) L_{t+1} \right]$$

These closed form expressions for the continuation values of each state in the owner-occupier, seller and searcher categories enable us to specify the surplus created by a match and solve for the equilibrium sales price for any successful match in the market.

#### Prices and Threshold Fit Value

The total surplus created by any match is the sum of the net payoffs to searchers and sellers. For the searcher-buyer, the net payoff is the incremental earning from becoming an owner-occupier over the continuation value of the search state. For the seller, the net payoff is the difference between the sales price earned from a successful match this period and the continuation value of remaining a seller until next period. At time t, the total surplus created by a match with fit value  $\zeta$  is the sum of the searcher-buyer's and seller's surplus, as follows.

$$TS_t = (W_t + C_t + \zeta_t - P(\zeta)_t - L_t) + (P(\zeta)_t - (1 - \delta)A_t).$$

The searcher's surplus from a match linearly increases in the quality-of-fit, so matches can be ranked by the realized values of  $\zeta$  across the market. In each period, a distribution of  $\zeta$  values is realized and, therefore, there is a corresponding distribution of equilibrium sales prices,  $P(\zeta)$ , which we must determine. Since only those prices are observed which correspond to fit values above the market cutoff fit, it is necessary to identify this marginal fit value each period.

The market cutoff fit, or marginal fit value, is one where the total surplus from a match equals zero as in [9].

$$\bar{\zeta}_t = L_t + (1 - \delta)A_t - (W_t + C_t).$$
 [9]

For any fit value above the threshold,  $\bar{\zeta}_t$ , the searcher and seller proceed to Nash bargaining over the surplus. The Nash bargaining problem maximizes the weighted product of the surplus earned by searcher-buyer and seller with respect to the fit-specific price.

$$P(\zeta)_t = arg \max_{P(\zeta)_t} (W_t - P(\zeta)_t + C_t + \zeta_t - L_t)^{\phi} (P(\zeta)_t - (1 - \delta)A_t)^{1 - \phi}$$

The first term of the Nash product is the searcher-buyer's surplus from the match, where the searcher's bargaining weight is  $\phi$ . The second term is the seller's surplus. For  $\zeta_t > \overline{\zeta}_t$ , an equilibrium sales price is

$$P(\zeta)_t = \phi(1 - \delta)A_t + (1 - \phi)(W_t + C_t - L_t + \zeta_t).$$
 [10]

In each period a distribution of sales prices is observed that satisfies [10]. In this forward-looking model, however, both sellers' and searchers' continuation values rely on

the future market average price conditional on the future marginal fit value. Hence, the market average price is a sufficient statistic and one of the three key endogenous variables in the solution of the model.

To specify the market average price explicitly, I combine equations [7] and [10] as follows.

$$\bar{P}_t = \left( \int_{\bar{\zeta}_t}^{\infty} (\phi(1-\delta)A_t + (1-\phi)(W_t + C_t - L_t + \zeta_t)) g(\zeta) d\zeta \right) / (1 - G(\bar{\zeta}_t))$$

Once more,  $W_t$  and  $C_t$  are independent of  $\zeta$ . Moreover,  $A_t$  and  $L_t$  are independent of  $\zeta_t$  and  $\bar{\zeta}_t$ . Hence, the market average price at time t simplifies to the weighted sum of the seller, owner and searcher's continuation values plus the housing tax credit and conditional mean fit of all successful matches in the market, as expressed in equation [11].

$$\overline{P}_t = \phi(1-\delta)A_t + (1-\phi)(W_t + C_t - L_t) + (1-\phi)\int_{\overline{\zeta}_t}^{\infty} \frac{\zeta_t g(\zeta)d\zeta}{(1 - G(\overline{\zeta}_t))}$$
[11]

#### III. Model Calibration

In this section, I will briefly relate the sources for parameter values to which the model is calibrated, as laid out in Table 4.1, which I refer to as the benchmark calibration of the model. This calibration assists in numerically solving the for the steady state solution of the model, which is discussed in Section IV.

Numerical calibration of the model allows quantitative analysis of the tax credit shock using the Anderson-Moore algorithm. I use this algorithm to derive the reduced form from the log-linearized version of the structural equations. In the reduced form representation, each variable is expressed in terms of its own lags and the lags of other variables and shocks, in terms of deviation from the steady state solution. These are the impulse response functions of the model to any shock that moves it away from its steady state equilibrium.

The baseline model calibration outlined in Chapter 2, Table 2.1, is replicated in this chapter. Choosing values for the population N and joy-of-ownership b that will fix steady state average market price and  $\theta$  each at 1, I solve for the steady state threshold

value,  $\bar{\zeta}$ , by assuming that it is normally distributed with a standard deviation of 10 percent of the average sales price, and mean 0.

Given  $\theta = 1$ ,  $\overline{m}$  is calibrated to yield a search duration for the representative household in the model that matches the median observed length of search for home buyers in the U.S. according to the National Association of Realtors of 8 weeks.

I choose  $\gamma$ , the elasticity of the matching function with respect to searchers, to be 0.5. This implies that searchers and vacancies are equally substitutable in the matching function. Hence, an increase in the number of searchers or vacancies of the same magnitude will affect the total number of matches made to an equal extent.

In a centralized market, the substitution of vacancies for searchers depends only on the parameters of the matching function, mainly  $\gamma$ . Hosios (1990) shows that for a decentralized framework, both the Nash bargaining weight of searchers and their weight in the matching function will dictate the flexibility of the market to substitute between searchers and vacancies in equilibrium. He establishes that to achieve the social planner's solution in a decentralized market, a necessary and sufficient condition is that the searchers' Nash bargaining weight should equal the Cobb Douglas searcher elasticity of the matching function (Hosios, 1990). I therefore choose  $\phi$  to equal  $\gamma = 0.5$ . The results of the model are invariant to changing  $\phi$  and  $\gamma$ .

The frequency of separation for owner-occupiers from their existing housing unit is set to match the 6-year median length of stay in one house in the U.S. reported by the National Association of Realtors.

The depreciation rate is fixed to 1.14% per year, which implies a service life of 80 years for the average house, as reported by the Bureau of Economic Analysis (February 2008).

The subjective discount rate is fixed to the time-average of the real federal funds rate since agents are risk-neutral in the model. In the calibration of the model with heterogeneous matches, the joy of ownership parameter is tied to the equilibrium value chosen for the average price. Moreover, the sum of the rental rate and *b* is inversely related to the marginal fit value, so I choose an equilibrium annual rental rate at the lower range of observed real rents of 3 percent of average price so as to ensure a non-zero benefit of ownership.

Finally, a neutral steady state value  $\alpha = 1$  is fixed for the shifter in the firm's marginal cost. The firm's elasticity of supply for new units is fixed based on my own estimates in the basic search and matching model (see Chapters 1 and 2).

### IV. Steady State Equilibrium & Comparative Statics

The steady state counterparts for equations [1] through [11] can be written in terms of three key variables in the model  $(\bar{P}, \theta, \bar{\zeta})$  and exogenous parameters. There are 10 endogenous variables and 10 exogenous parameters and variables including the rental rate. Hence, to solve for the steady state I have sufficient degrees of freedom to choose  $\bar{P} = \theta = 1$  and solve for the corresponding marginal fit value, given parameter values listed in Table 4.1.

I normalize the steady state value of the home buyer's tax credit to zero, essentially treating it as an exogenous policy shock variable that is zero in expectation.

### Graphical Analysis of Steady State

To analyze the steady state of the model it is helpful to present the three key equations expressing steady state values of the market average price, relative supply and marginal fit value in terms of exogenous variables and parameters in equations [12], [13] and [14], respectively. The steady state equilibrium is completely characterized by these three equations in three unknowns.<sup>2</sup>

First, I can write the steady state counterpart of equation for the market average price in equation [11] by substituting in the steady state payoff values of selling, search and owner-occupation and steady state expected fit value. The steady state equilibrium market average bargained price can be expressed as a function of  $\theta$  and  $\bar{\zeta}$  as in equation [12].

<sup>3</sup> Note that I use the following substitution to simplify the algebra and presentation in the text for the remainder of this chapter:  $\psi = \lambda + \delta(1 - \lambda)$ . Hence,  $\psi$  is just the composite probability of separation and depreciation in one period.

<sup>2</sup> For all steady state values that I will discuss I substitute out  $\mu(\theta)$  and  $q(\theta)$  in terms of  $\overline{m}$ ,  $\gamma$  and  $\theta$  to keep the number of variables under discussion to a minimum.

$$\bar{P}(\theta,\bar{\zeta})$$

$$= \frac{(1-\phi)\left(r+\delta+\bar{m}\theta^{-\gamma}\left(1-G(\bar{\zeta})\right)(1-\delta)\right)\left\{(1+r)(R+b)+\frac{(r+\psi)}{(1-G(\bar{\zeta}))}\int_{\bar{\zeta}}^{\infty}sg(s)ds\right\}}{(r+\delta)\left(r+\psi+\phi\bar{m}\theta^{1-\gamma}\left(1-G(\bar{\zeta})\right)(1-\psi)+(1-\phi)\bar{m}\theta^{-\gamma}\left(1-G(\bar{\zeta})\right)(1-\psi)\right)}$$
[12]

The average bargained price as a function of relative supply is a downward sloping schedule,  $\partial \bar{P}(\theta, \bar{\zeta})/\partial \theta < 0$ . This curve specifies the average equilibrium price a housing unit will fetch for each equilibrium ratio of vacancies to searchers. As expected, the average bargained price schedule illustrates that the higher the number of vacancies per searcher, the greater the supply relative to demand and the lower the equilibrium price paid on average for a vacancy. It is illustrated in  $(\bar{P}, \theta)$  space as the solid downward sloping line in Figure 4.1.

Second, the steady state counterpart of equation [2] is the steady state equilibrium flow supply curve for new housing units, written as the market average price as a function of  $\theta$  and  $\bar{\zeta}$ , in equation [13]. This is derived by substituting in the steady state level of housing stock and investment

$$\bar{P}(\theta,\bar{\zeta}) = \frac{\alpha \delta^{\xi} N^{\xi}}{\bar{m}\theta^{-\gamma} (1 - G(\bar{\zeta}))} \Big[ r + \delta + \bar{m}\theta^{-\gamma} (1 - G(\bar{\zeta})) (1 -$$

[13]

The steady state supply curve expresses the relationship between equilibrium average price and supply of vacancies per searcher. The slope  $\partial \bar{P}(\theta, \bar{\zeta})/\partial \theta$  is positive in  $(\bar{P}, \theta)$  space, verifying that vacancies per searcher follow the law of supply. This curve is also represented as the dashed upward sloping line in Figure 4.1.

Finally, I write the steady state marginal fit value as a function of  $\bar{P}$ ,  $\theta$  and exogenous parameters by substituting out the steady state continuation values of selling, searching and owner-occupation in equation [9] which defines the marginal fit value. This yields equation [14], the steady state marginal fit value.

$$\bar{\zeta}(\theta,\bar{P}) = \frac{1}{\left(r + \psi + \bar{m}\theta^{1-\gamma}(1-\psi)\left(1-G(\zeta)\right)\right)\left(r + \delta + \bar{m}\theta^{-\gamma}(1-\delta)\left(1-G(\zeta)\right)\right)} \cdot \left\{ (r+\delta)(1-\psi)\left(1-G(\bar{\zeta})\right)(\bar{m}\theta^{-\gamma} - \bar{m}\theta^{1-\gamma})\bar{P} - (1+r)\left(r + \delta + \bar{m}\theta^{-\gamma}(1-\delta)\left(1-G(\bar{\zeta})\right)\right)(R+b) + (1-\psi)\left(r + \delta + \bar{m}\theta^{-\gamma}(1-\delta)\left(1-G(\bar{\zeta})\right)\right)\bar{m}\theta^{1-\gamma}\int_{\bar{\zeta}}^{\infty} sg(s)ds \right\}$$
[14]

Figure 4.1 shows the steady state equilibrium in  $(\bar{P}, \theta)$  space for the benchmark calibration of the model, as laid out in Table 4.1. In Figure 4.1, market average price is varied independently in exchange for endogenizing the joy-of-ownership parameter b; relative supply is varied along the x-axis in exchange for endogenizing the size of the total population N. Given calibrated values of the match efficiency, depreciation and separation, the risk-free interest rate and the equilibrium rental rate as in Table 4.1, the corresponding equilibrium value of the cutoff fit is 5.29 percent of the market average price. Hence, in Figure 4.1 both the average bargained price schedule and the flow supply curve are drawn holding the value of the cutoff fit constant  $\bar{\zeta} = 0.0529$ , given an exogenously determined matching efficiency rate,  $\bar{m}$ .

In this paper, searcher-buyer heterogeneity augments the basic matching friction whereby not all matched vacancy-searcher pairs proceed to a successful sale transaction. Hence, demand heterogeneity introduces a market cutoff for the match fit quality and associated price, below which vacancies are not traded in equilibrium. An analysis of the steady state equilibrium therefore must include an analysis of the static relationship between market average price and cutoff fit value, and between relative supply and cutoff fit value.

Since there are three endogenous variables, one way to do this might be to present a three-dimensional graph with  $\bar{\zeta}$  represented along the z-axis. An alternative way to show the same impact is to present the steady state equilibrium in  $(\bar{P}, \theta)$  space, for different equilibrium values of  $\bar{\zeta}$ . In other words, I will demonstrate how the average bargained price schedule and the supply curve are affected by different values of the marginal fit value in equilibrium.

In order to vary the cutoff value thus, I vary the matching efficiency parameter to satisfy the steady state equation for  $\bar{\zeta}$  in equation [14]. The underlying distribution and variance of the fit variable is unchanged and lies at its benchmark value as in Figure 4.1,  $\sigma_{\zeta}^2 = 0.01$ . In Figure 4.2A, I show the impact of a higher cutoff value on the two curves and in Figure 4.2B, the impact of a lower cutoff value. Notice that when  $\bar{m}$  is thus endogenized, it affects the slopes of both the average bargained price and the flow supply curves, noticeably reducing the slope of the latter in Figure 4.2 compared to Figure 4.1.

In varying  $\bar{\zeta}$ , I am interested in answering two questions. First, does the equilibrium relationship between the market cutoff fit and the market average price reflect the relationship between price and fit at the individual level? In other words, one expects that the higher the cutoff fit value, the higher is the cutoff price and hence the higher the resulting market average price. Second, is there an analogous static relationship between searcher heterogeneity and relative supply,  $\theta$ ?

Figure 4.2 answers both of these questions. In each of the two panels, the benchmark equilibrium value of  $\bar{\zeta}=0.0529$  is associated with the dashed curves. The solid curves in Figure 4.2A represent the average bargained price schedule and supply curve corresponding to a higher equilibrium value of  $\bar{\zeta}=0.0729$ . Similarly, the solid lines in Figure 4.2B represent the average bargained price schedule and supply curve corresponding to a lower equilibrium value of  $\bar{\zeta}=0.0329$ .

Figure 4.2A illustrates that for a higher equilibrium marginal fit value, the market average price curve shifts to the right and the equilibrium price at the intersection of the 'demand' and supply curves is higher than before. Figure 4.2B shows that for a lower equilibrium marginal fit value the market average price curve shifts to the left and the equilibrium price at the intersection of the 'demand' and supply curves is lower than before. Thus, the relationship between market price and marginal fit value is positive, as expected.

Figure 4.2A shows that a higher marginal fit value causes the supply curve to shift inwards. Hence, for a higher marginal fit value, new construction of housing is offered at a higher price because a higher cutoff fit implies fewer successful sales each period. Figure 4.2B shows the opposite is also true. While this does demonstrate that the supply schedule shifts to the left, it does not imply that the equilibrium level of  $\theta$  will deviate

from 1. The market average price curve and the supply curve still intersect at  $\theta = 1$  in both Figures 2A and 2B. Indeed, the endogenous change in matching efficiency for alternative values of  $\bar{\zeta}$  ensures that relative supply remains largely unchanged in equilibrium and it is equilibrium price rather than relative supply (and hence vacancy rates) that adjust in response to different values of the cutoff.

One might infer that for a higher marginal fit value, equilibrium duration will be higher. That would be true assuming a constant matching efficiency parameter. Choosing alternate values of  $\bar{\zeta}$  in the comparative statics of Figure 4.2 requires the use of one degree of freedom. I choose to vary matching efficiency endogenously in place of  $\bar{\zeta}$ . Figure 4.3 illustrates the changes in  $\bar{m}$  and hence vacancy duration that accompany the shifts of the market price and supply schedules (shown in Figure 4.2) for alternative values of  $\bar{\zeta}$ .

The underpinnings of Figure 4.3 are identical to those of Figure 4.2: in order to vary the cutoff value thus, I vary the matching efficiency parameter to satisfy the steady state equation for  $\bar{\zeta}$  in equation [14]. The underlying distribution and variance of the fit variable is unchanged and lies at its benchmark value as in Figure 4.1,  $\sigma_{\zeta}^2 = 0.01$ . The top panel of Figure 4.3 plots the equilibrium value of  $\bar{m}$  for the corresponding value of  $\theta$ , given  $\bar{\zeta} = 0.039$  (dotted curve) or  $\bar{\zeta} = 0.0529$  (benchmark case, solid line) or  $\bar{\zeta} = 0.0729$  (dashed line).

The bottom panel of Figure 4.3 plots vacancy duration, given in weeks by

$$Vacancy\ Duration = \frac{1}{\overline{m}\theta^{-\gamma}(1 - G(\overline{\zeta}))}$$

for each of the three values assumed for  $\bar{\zeta}$ . Again, the line markers correspond to the three cases:  $\bar{\zeta} = 0.039$  (dotted curve) or  $\bar{\zeta} = 0.0529$  (benchmark case, solid line) or  $\bar{\zeta} = 0.0729$  (dashed line).

Since the steady state equilibrium is where  $\theta = 1$ , consider the values for  $\overline{m}$  and vacancy duration at this point, for the different values of  $\overline{\zeta}$ . Figure 4.3 illustrates that for a higher cutoff fit value, the equilibrium matching efficiency rate is also higher. Hence, as searchers become more selective in terms of fit value in a match, the market compensates for it by increasing the rate at which it pairs searchers with vacancies.

The bottom panel of Figure 4.3 shows that there is a static tradeoff between time to sale and the marginal fit value in equilibrium. For an unchanged spread of possible fit values, a higher cutoff fit value is more than compensated for by the matching efficiency rate, so that the corresponding time to sale is actually lower. Moreover, notice that the three curves plotting vacancy duration against relative supply in the bottom panel of Figure 4.3 are not parallel. The margin for change of vacancy duration rises as the equilibrium number of vacant units per searcher increase. Market efficiency increases by more the higher the equilibrium vacant units per searcher, and vacancy duration falls by more, as the equilibrium cutoff fit increases.

Figure 4.3 illustrates two things. First, all else equal, as searchers become less choosy (there is a low marginal fit value) the market rations vacancies more actively by having a low matching efficiency in equilibrium. When searchers themselves are extremely picky, the market performs its matching function more efficiently to such an extent that the average vacant unit has a shorter time to sale in equilibrium. Second, however, the figure underlines that this static tradeoff between time to sale, the market cutoff fit and matching efficiency is under the assumption of relatively low demand heterogeneity. The remaining question is whether this equilibrium tradeoff will persist at higher degrees of heterogeneity among searchers?

Mean-Preserving Spread, Equilibrium Marginal Fit Value and Equilibrium Time to Sale To understand the effect of distributional assumptions, it is also important to see how a mean-preserving increase in the spread of the fit variable will impact the equilibrium cutoff fit value and hence the fraction of matches converting into sales. Figure 4.4 shows three related aspects of changing the variance of fit in three subplots. Each subplot is drawn for the static equilibrium where all parameter values are as laid out in Table 4.1 and  $\bar{P} = \theta = 1$ . All that changes in each subplot is the exogenously determined variance of  $\zeta$ , which is reported along the x-axis in each panel, in units of standard deviation.

The line plotted in the top panel of Figure 4.4 shows combinations of standard deviation of  $\zeta$  and equilibrium  $\bar{\zeta}$ . To make this plot, I calculate the equilibrium value of  $\bar{\zeta}$  from equation [14] as the variance of  $\zeta$  changes, given all other exogenous variable and

parameter values per Table 4.1 and  $\bar{P} = \theta = 1$ . The top panel clearly illustrates that as the spread of the fit variable increases, the market cutoff fit value also rises.

In the middle panel of Figure 4.4, I plot the cumulative probability of the equilibrium cutoff fit,  $G(\bar{\zeta})$ , against the standard deviation of  $\zeta$ . Again, the parameter values are unchanged from Table 4.1 and  $\bar{P}=\theta=1$ . This plot shows clearly that as the spread of  $\zeta$ , there is greater probability mass under the cutoff fit value. All else equal, particularly the degree of matching efficiency in the market, this would suggest that higher heterogeneity in the market lowers the hazard rate for vacancies and searchers and raise time to sale in equilibrium.

In the bottom panel in Figure 4.4, I plot the conditional expected fit value for matches that proceed to sale in equilibrium,  $(\int_{\bar{\zeta}}^{\infty} sg(s)ds)/(1-G(\bar{\zeta}))$ , against the standard deviation of  $\zeta$ . This plot illustrates that the greater the variation in fit values and the higher the equilibrium cutoff fit value, the higher the expected quality of fit will be for each match that proceeds to sale.

In summary, Figure 4.4 shows that the greater is searcher heterogeneity, as captured by a higher spread in distribution of  $\zeta$ , the higher is the cutoff fit value required for a successful match, the lower is equilibrium likelihood of sales. Moreover, given constant matching efficiency, greater searcher heterogeneity causes longer duration of vacancy in equilibrium.

## Robustness Check: Varying Matching Efficiency as Spread Increases

Figure 4.3 illustrated that matching efficiency could offset rationing in the market stemming from heterogeneity. I conclude there that market efficiency can offset increases in the marginal fit value to maintain or reduce vacancy duration. In this sub-section, I recalibrate the model for each alternative value of  $\bar{\zeta}$ , by matching moment of the model to the data. I therefore allow  $\bar{\zeta}$  and  $\bar{m}$  to vary as long as duration equals observed duration.

In the homogeneous search and matching model of housing developed in Chapter 2 of this dissertation, steady state search and vacancy duration are given by  $(\mu(\theta))^{-1} = 1/(\overline{m}\theta^{1-\gamma})$  and  $(q(\theta))^{-1} = 1/(\overline{m}\theta^{-\gamma})$ . Also,  $\theta=1$ , which implies there is only one vacancy per searcher in existence in the market.

In the heterogeneous matching model, by contrast, steady state duration of search is given by  $\left(\mu(\theta)\big(1-G(\bar\zeta)\big)\right)^{-1}=1/\big[\overline m\big(1-G(\bar\zeta)\big)\big]$ , given  $\theta=1$ . Analogously, steady state vacancy duration under heterogeneous matching is given by  $\left(\mu(\theta)\big(1-G(\bar\zeta)\big)\right)^{-1}=1/\big[\overline m\big(1-G(\bar\zeta)\big)\big]$ . Thus the only difference between the steady states of the two types of matching models is the absence of the cutoff fit value in the former. The steady state solution of each model is  $\bar P=\theta=1$ , given  $\bar m=0.8238$  at a quarterly frequency. Both versions of the matching model are calibrated on the empirical observation that it takes the average U.S. family 8 weeks to purchase a housing unit (see Table 4.1). In the heterogeneous matching model this implies an equilibrium value of  $\bar\zeta=0.0529$ .

I use two steady state equations that jointly express the steady state relationship between time to sale, matching efficiency and the marginal fit value. Because I am analyzing the steady state and only changing the exogenous time to sale, I maintain steady state  $\bar{P}=\theta=1$  and remaining parameter values per Table 4.1. The first equilibrium equation I employ is the equilibrium vacancy duration as a function of  $\bar{m}$  and  $\bar{\zeta}$ .

Steady State Vacancy Duration = 
$$\frac{1}{\overline{m}(1 - G(\overline{\zeta}))}$$

Second, I use the steady state equation for the marginal fit value in equation [13] with the substitution of  $\bar{P} = \theta = 1$ .

$$\left[r+\psi+\overline{m}\big(1-G(\bar{\zeta})\big)(1-\psi)\right]\bar{\zeta}+(1+r)(R+b)-(1-\psi)\overline{m}\int_{\bar{\zeta}}^{\infty}sg(s)ds=0$$

As before,  $\psi = \lambda + \delta(1 - \lambda)$  and s is the dummy of integration. I now need only to make an assumption about the variance of  $\zeta$  and solve these two equations simultaneously for any exogenously given vacancy duration.

Figure 4.5 plots these solutions for three possible values of  $var(\zeta)$ . The solid blue line is the benchmark case, where  $s.d.(\zeta) = 0.1$ , that is, 10 percent of the market average price. The green lines with circle markers in each subplot correspond to  $s.d.(\zeta) = 0.17$ . Finally, the black dotted lines correspond to  $s.d.(\zeta) = 0.22$ .

The first observation about Figure 4.5 is that as spread increases, both loci of equilibrium values of  $\overline{m}$  and  $\overline{\zeta}$  shift outward. For higher demand heterogeneity, the marginal fit value and the matching efficiency of the market are higher at every duration. We are interested in determining how a higher spread for the fit variable will impact equilibrium vacancy duration when matching efficiency is allowed to adjust endogenously to market conditions.

The benchmark calibration of the model uses the empirical observation that the average U.S. household searches for 8 weeks before purchasing a housing unit. When  $\theta = 1$ , as it is in the steady state of the model, search and vacancy duration are identical. The top panel of Figure 4.5 shows that equilibrium vacancy duration of 8 weeks corresponds to a negative marginal fit value if  $s.d.(\zeta) = 0.1$ .

A negative marginal fit value, while theoretically possible, does not seem entirely reasonable. It implies that a searcher will purchase a housing unit even when it yields disutility for him, simply because the market offers them the opportunity to do so at a low price. Imposing the requirement that the market cutoff fit should at least be non-negative implies that the equilibrium duration of vacancy will actually be shorter at 6.5 weeks, when at least 33 percent of the market values a vacancy differently from the sales price by 10 percent (i.e.  $s.d.(\zeta) = 0.1$ ). In other words, the market compensates for differences in valuation of searchers by raising the matching efficiency and lowering time to sale in equilibrium.

As the spread of the fit variable increases, however, equilibrium  $\bar{\zeta}$  and  $\bar{m}$  are both higher for any length of vacancy duration. Consider the second alternative, where the spread of the fit variable increases to  $s.d.(\zeta) = 0.17$ . If the market cutoff fit value is zero, equilibrium vacancy duration is 11 weeks. For a higher market cutoff, matching efficiency is higher and vacancy duration is lower in equilibrium.

There is also a limit to which the matching efficiency parameter can rise. A matching efficiency of 1 eliminates the role of the search and matching friction in the model, by ensuring that all searchers and vacancies will meet in the market. Hence, the lowest possible equilibrium vacancy duration must coincide with  $\overline{m} = 1$ , given the negative relationship between  $\overline{\zeta}$  and duration. Figure 4.5 shows that for any assumed

spread of the fit variable and no matching inefficiency, the lowest possible time to sale in the model is roughly 4 weeks.

To summarize the findings of this robustness check, the greater the degree of heterogeneity in the market, the more of a role idiosyncratic difference in valuation will play in determining the equilibrium time to sale. Taking demand heterogeneity and market matching frictions into account, time to sale will depend inversely on how selective searchers are. If searchers are more selective, the cutoff fit value is high. To compensate for this, the market has high matching efficiency thereby yielding a shorter duration of vacancy in equilibrium. If the matching friction is removed altogether, searcher heterogeneity accounts for at least half of the empirically observed length of search by U.S. households, i.e. duration of just over 4 weeks in equilibrium.

# V. Dynamic Behavior of the Model: Rental Shocks and Summary Statistics

I now turn to dynamic analysis of the model, using the Anderson Moore algorithm to solve for the reduced form of the model. I use this reduced form to provide summary statistics of the model in response to shocks to the rental rate, which is an exogenous variable of the model. This allows us to gain a sense of whether the model behaves reasonably in response to unforeseen macroeconomic shocks, and can credibly provide quantitative analysis of the home buyers' tax credit policy shock.

Table 4.2 displays summary statistics of the heterogeneous matching model. The statistics reported here are population averages over 1000 Monte Carlo simulations of the model. To do the simulations, I start by drawing 200 random, normally distributed shocks to the rental rate. The rental rate is assumed to follow an AR(1) process with persistence of 0.97 and a standard deviation of 4 percent (of sales price) at annual frequency, based on the seasonally adjusted BLS index of owner-occupied rents across all U.S. cities from 1990-2010.

For every Monte Carlo iteration, the model starts from steady state equilibrium and then is hit with a new shock to rent each quarter, for 15 years. In other words, each iteration is a realization of one "lifetime" of the housing market, which is 15 years long. As exogenous rental rates change by random amounts each quarter in a lifetime,

endogenous market variables are impacted. I record the resulting values of the variables of the model in terms of deviations away from steady state, which I refer to as "simulated data". From these data, I obtain the mean deviation from steady state (here-onward referred to as the "mean gap"), standard deviation and autocorrelations for variables of interest. I repeat this process for a total of 1000 Monte Carlo iterations, in effect imagining 1000 different lifetimes for the housing market, and record the mean gap, standard deviation and correlations in each lifetime. For the results presented in Table 4.2, I finally average over the 1000 lifetimes to get a population mean gap, standard deviations and correlations.

Table 4.2 reports these summary statistics for the fundamental variables of the model, namely market average price, relative supply (i.e. vacancies per searcher) and the market cutoff fit value. There are three specifications of the housing search and matching model presented here, by column. Each column show results for the heterogeneous matches model developed in this paper, with a different assumption for  $s.d.(\zeta)$ . In rows, the table reports mean deviation from steady state, average standard deviation and average first order autocorrelation from the Monte Carlo simulations.

First, the table verifies the stationarity of the model: each of the reported variables has a mean zero deviation from steady state, for all model specifications. This verifies that the calibrated model is functioning correctly and returns to the steady state after it has completely adjusted to a macroeconomic shock.

Next, Table 4.2 reports standard deviations for each variable of interest. Comparing the three specifications of the heterogeneous model, it is clear that as the spread of the underlying distribution of fit values  $G(\zeta)$  increases, the variability of average price decreases. When the marginal fit value in a high variance distribution changes in response to a rental shock, there will be less of a change in the proportion of matches converted into sales because there will be a smaller change in the weight under the cutoff value compared to a low-variance distribution. Hence there is a lower variability of average price when the spread of  $G(\zeta)$  is higher.

Finally, Table 4.2 presents population estimates of the persistence of average price, relative supply and the marginal fit value. We would expect that all three variables are highly persistent: price is known to follow a random walk; the persistence of

vacancies due to search and matching frictions as well as market heterogeneity is also to be expected. Since the marginal fit value is essentially reflective of the sales price and market saturation, it will also be persistent, if both average price and relative supply are highly autocorrelated. This is borne out in the quantitative estimates.

# Dynamic Response to Rental Rate Shocks

In order to provide a clearer picture of how an exogenous shock is propagated through the model, Figure 4.6 displays plots of the impulse responses of key variables from an unexpected 10 percent decline in the rental rate. The market is in steady state equilibrium prior to the shock and the graphs start at the impact responses to the shock, leading on to subsequent periods in time.

The negative and persistent rental shock lowers the value of ownership immediately, by almost 0.5 percent below the steady state value. In comparison, the values of searching and having a vacancy to sell decrease by far less, between 0.1 and 0.2 percent, respectively, because the impact of lower rental payments is enters into these payoffs with small probabilities. By lowering the payoff to owner-occupation, the rental rate shock decreases the surplus of all matches, for any realized fit. While this would have caused a decline of about 0.8 percent away from steady state in the homogeneous matches model, it causes the price to move by roughly half that in the heterogeneous matches model. The price subplot in Figure 4.6 also shows the adjustment in discounted stream of rental payments from a housing unit (red line with asterisk markers). This is the reaction of the neoclassical price to the shock, which is unequivocally larger than the matching bargained price response. Furthermore, the negative rental shock causes a jump up in the marginal fit value and an increase in time to sale.

The combined responses of price, marginal fit value and vacancy duration in Figure 4.6 tell a significant story. The muted response of the matching price compared to its neoclassical and homogeneous matching counterparts, is due to a compositional effect as follows. With lower rental rates earned from any housing unit, the price offered by a searcher of any fit will be lower. Were all searcher-buyers identical, sellers would have to accept a lower price for vacancies in order to sell (namely, the homogenous matching model price). With heterogeneous matches, however, sellers are not obliged to sell to a

searcher with the same fit value as they did prior to the shock. Instead, as rental rates fall, sellers adjust the marginal fit value upwards and sustain a longer time to sale of vacant units rather than adjusting to the shock through lower prices. Thus, the whole market becomes more selective, which is reflected in a smaller decline in the average price than would be observed with homogeneous demand.

Indeed, while the marginal fit value and hence the duration of vacancies show a smooth path of adjustment back down towards steady state after the impact response, the stock of searchers and vacancies rises for several quarters, reflecting the results of fewer sales each period after the shock and a decrease in new investment from its steady state level. As the average price rises back while investment is still below steady state, vacancies and searchers slowly recover towards the steady state too.

This analysis offers a simple explanation of why prices are observed to fall slowly while vacancies high and rising after an adverse demand shock. Failure to lower prices and reach equilibrium quickly is a reflection of the fact that sellers adjust in other respects rather than price alone: in a downturn, they will withstand a longer time to sale, if it implies that they can sell to a searcher-buyer with a higher fit value.

Table 4.3 presents population level correlations between average price, relative supply and marginal fit value based on the Monte Carlo simulations described above. These correlations represent the dynamic correlations between these variables as they adjust to rental shocks, much as in Figure 4.6, replicated 1000 times and then averaged. As Figure 4.6 illustrates, there is a negative correlation between average price and relative supply. Similarly, as we expect, the correlation between average price and marginal fit value in Table 4.3 is negative, while there is a positive correlation between marginal fit value and relative supply.

# VI. A Policy Experiment: Simulated Impact of the Home Buyer Tax Credit Shock

The heterogeneous matching model of the housing market introduces the important element of fit in a search and matching context, which cannot be affected by the supply side of the market. Because fit is idiosyncratic and random, construction firms cannot ex ante target a particular consumer base by customizing housing units to individual tastes and pursuing price discrimination. This means that once the market has performed the matching function, only those matches with non-negative total surplus actually proceed to sale. It is natural, then, to analyze government interventions in the market that change searcher-buyer payoffs and raise total match surplus, to see if they are effective in raising sales and possibly lowering time to sale. One such intervention is the Home Buyers' Tax Credit offered by the U.S. federal government to assist the housing market in the aftermath of the subprime mortgage and financial crises.

The so called "first time" home buyer tax credits were extended from April 9, 2008 through the end of May 1 2010, being in effect for just over two years in total. They were extended to qualifying first-time home buyers for a credit worth 10 percent of the housing unit's purchase price, capped at \$7500 in 2008 and \$8000 in 2009 and 2010. The credit was also extended to qualifying non-first-time home buyers, who could receive a maximum tax credit of \$6500. Additionally, the credit was available only to buyers whose gross income is less than \$95,000. The limit was revised in November 2009 to \$145,000. Finally, the maximum purchase price of a housing unit allowable under the tax credit scheme was \$800,000. Ignoring the finer details of the tax code, the terms of the scheme require that owners retain their housing units for 3 years after the purchase date, or repay the credit if they sell before then.

According to the IRS, in 2010 the program cost an estimated \$16 billion to the U.S. Treasury Department, with more than 2.2 million people filing for the credit. The number of claimants in 2009 was far smaller at 479,622, which amounted to a total of \$3.6 billion in payments by the IRS.

Figure 4.7 shows the impact of a tax credit shock equal to 10 percent of the steady state purchase price of a housing unit. I choose a 10 percent shock rather than 4 percent since the model scales up the effects linearly. In reality the credit shock equaled approximately 4 percent of the median U.S. house price. The dynamic response of the model discussed here is therefore suggestive of the directions of change in the U.S. housing market following the shock, and not the actual magnitude of the impact of the tax credit shock.

The model starts in steady state equilibrium where the tax credit is 0. The figure plots the model's response to an unexpected increase in the tax credit in percentage deviations from steady state for each variable, starting from the quarter in which the credit is first introduced.

When the credit shock occurs it exogenously raises the total surplus of any match, thereby allowing searcher with fit values below the pre-shock marginal fit value to pay a price acceptable to sellers. Hence, the marginal fit value in the market jumps down by about 8 percent compared to steady state, and the remainder of the credit shock being reflected in a 3.8 percent increase in average price.

The higher expected sales price over the life of the credit raises the expected value of a vacancy by 2.8 percent above steady state. Since the connection between new investment and expected sales value is mechanical in this model, with no credit constraints or other supply side restrictions, investment automatically jumps up in response to higher expected sales value.

Overall, the increase in match surplus and decrease in the market cutoff fit results in more sales being made, which lowers both the number of vacancies and searchers below steady state. These move in tandem sufficiently to keep the change in relative supply minimal at 0.5 percent above steady state.

Even though the tax credit is discontinued abruptly at the end of 2 years, the market's response to the shock is smoothed, given the assumptions of perfect foresight and perfect information in this model. Since sellers know the length of time for which the credit is in place, they only allow the average price of a vacancy to fall gradually over time. Thus progressively lower fit values, hence lower match surplus, is accepted only as the time for the credit to end draws near. Thus, in Figure 4.7 marginal fit value is gradually declining over the life of the tax credit shock, allowing more sales to occur for lower fits at lower prices as the credit shock draws to an end.

Nearly all variables return abruptly back to steady state when the tax credit expires. This is reasonable for each variable. All payoffs and prices are forward looking. The forward-looking behavior of sellers in the market drives average price back towards its steady state gradually, reflecting a forward looking expectation that the surplus from each match will be lower when the credit is eliminated completely. The payoffs to

owner-occupation, search and having a vacancy to sell all reflect this forward-looking pricing.

As soon as contribution of the tax credit to total surplus is removed, the marginal fit value returns to its steady state level because each matched searcher-seller pair receives the true surplus from the match, unaugmented by the lump sum credit payment. As soon as the market cutoff returns to steady state, time to sale also returns to its steady state level, reflecting the change in probability of a successful sale due to the greater probability mass that lies below the steady state cutoff. New investment is tied to price and once price returns to steady state, new investment does as well.

Indeed, the only two variables that do not immediately reflect the discontinuation of the credit are vacancies and searchers. Both of these are stock variables, which are very persistent and experience large declines while the credit is in effect. With investment back at steady state level when the credit ends and the stock of vacancies far below equilibrium, it takes several quarters for vacant units to make up for the depletion of vacant units during the shock period. Similarly, the stock of searchers is lower than equilibrium and begins to recover only slowly since new demand depends solely on separation and depreciation per quarter.

Hence, this model shows that a tax credit scheme can successfully raise sales and the average price in the market while it is in place. Moreover, it lowers the selectivity that searchers and sellers display, as summarized by a lower cutoff fit value in the market and a commensurately shorter time to sale. Since the market starts in steady state, the model also predicts an increase in new housing investment while the stimulus is in place. Once the tax stimulus is removed, vacancies per searcher, new investment and average price all return to their steady state equilibrium levels.

#### VII. Conclusion

In this final chapter I have presented a modification of the search and match model of housing that allows for variable quality of fit in matches, to reflect underlying heterogeneous preferences of searchers.

With idiosyncratic and random valuation by buyers, construction firms cannot ex ante target a particular consumer base or customize housing units to individual tastes. Searcher-buyers in a match with too low a fit value remain in the search state because their offer price is too low to guarantee non-negative total surplus. Hence, only those matches with a non-negative total surplus can proceed to sale, which identifies a marginal fit value below which matches by the market mechanism fail.

Comparative static analysis shows that there is a positive static relationship between the equilibrium market average price and the marginal fit value. However, equilibrium relative supply remains unchanged for alternative values of the cutoff fit.

The relationship between time to sale and the marginal fit value is mediated by the matching efficiency of the market. As the market becomes more selective and has a high equilibrium cutoff fit value, the market compensates by lowering the matching friction and hence lowering vacancy duration. In the absence of a matching friction, equilibrium time to sale predicted by the model is 4 weeks.

Monte Carlo simulations of the model illustrate that as the spread of the fit variable increases, market average price becomes less variable. This is the dynamic counterpart of the composition effect on price observed in response to demand shocks. In the impulse responses from a negative shock to the rental rate, the observed price decreases by less than a homogeneous model would predict because sellers increase the cutoff fit value and sell to higher fit-value searchers than before in exchange for a longer time to sale.

The model also allows one to discern the qualitative impact of the 2008-2010 home-buyers tax credit on the housing market. I find that because the credit distorts upwards the potential surplus of a match, it actually lowers the threshold quality of matches in the market and raises the number of sales. As market participants become more willing to accept lower valued matches to take advantage of the credit, the duration of vacancy and search commensurately falls. Hence, all else equal, the model predicts that the tax credit effectively greases the wheels of the housing market and raises sales in the market.

All of these results are presented in a scenario where demand and supply are essentially deterministic and agents are not financially constrained. To make the model

amenable to further policy application, one can incorporate endogenous move decisions and financial constraints for buyers and sellers. This would make future market conditions stochastic for sellers, which might provide quantitatively informative results. Introducing financial constraints on sellers would also temper the deterministic link between new construction and higher prices to bring issues of timing and financing to the fore.

On the searcher-buyer side, introducing financial constraints will allow analysis of the rent-versus-buy decision. Endogenizing the decision to move for searcher-buyers will provide greater policy insight into the housing market cycle. It might also allow a more direct analysis of housing slumps which feature overbuilding and credit-constrained bargain-shoppers who bid low in distressed markets.

Finally, the heterogeneous matching model also opens the door to a search model of housing that explicitly incorporates location, which commands a premium in the match surplus and is reflected in the distribution of prices. This chapter presents, therefore, an important precursor to a more complex model of heterogeneous matches in housing that focuses on location-based differentiation in the housing market.

The modified search and matching model presented in this paper is therefore an effective framework in which to think about macroeconomic shocks that impact the housing market business cycle. With the incorporation of the features mentioned above, we might begin to gain a better grasp of what drives housing market inventories, sales and construction.

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Table 4.1

Benchmark Calibration Values of the Matching Model with Heterogeneity

Symbol	Description	Value
$\overline{m}$	Matching efficiency	0.125 per quarter
γ	Searchers' elasticity in match function	0.5
α	Supply shifter	1
$1/\xi$	Elasticity of flow supply	2
λ	Separation rate	0.2 per annum
δ	Depreciation rate	0.0114 per annum
$\delta^{rent}$	Depreciation rate of rental unit	0.0228 per annum
$\phi$	Nash bargaining weight of households	0.5
r	Real discount rate	0.02 per annum
R	Steady state exogenous real rental rate	0.03 per annum
Z	Cost of converting vacancy to rental unit	10 percent of steady state
$\sigma_{\zeta}$	Standard deviation of fit value	sales price 0.1

**Notes**: This table shows the baseline values chosen for parameters and exogenous variables in the quantitative solution of the matching model.

Table 4.2
Summary Statistics from Monte Carlo Simulations

Specification			
Standard Deviation of Fit	0.1	0.173	0.224
Equilibrium Marginal Fit Value	0.0529	0.148	0.221
Mean Deviation from Steady State Level			
Market Average Price	0.001	0.00	0.00
Relative Supply (Vacancies per Searcher)	-0.026	0.000	0.007
Marginal Fit Value	-0.002	0.000	0.001
Standard Deviation			
Market Average Price	0.019	0.008	0.006
Relative Supply (Vacancies per Searcher)	0.375	0.283	0.245
Marginal Fit Value	0.029	0.022	0.019
First Order Autocorrelation			
Market Average Price	0.946	0.956	0.960
Relative Supply (Vacancies per Searcher)	0.995	0.996	0.996
Marginal Fit Value	0.936	0.944	0.946

**Notes**: This table shows summary statistics of the dynamic search and matching model with heterogeneity, using Monte Carlo simulations of the model with random shocks to the rental rate. The first three columns of data (from left to right) represent alternative assumptions about the distribution of fit values in the searcher-buyer population. The final column shows summary statistics for the search and matching model with homogeneous demand, using the same random rental shocks for Monte Carlo simulations.

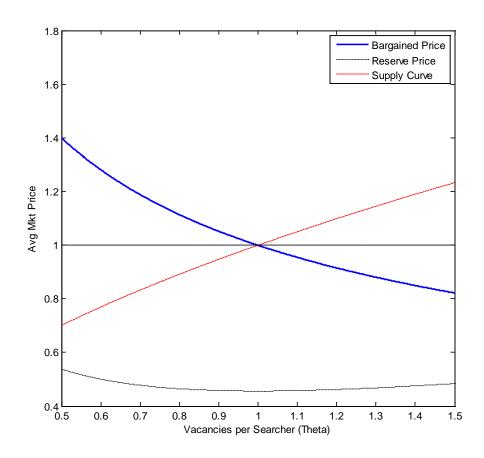
Table 4.3

Dynamic Cross-Correlations for Monte Carlo Simulations

0.0529 0	0.173 0.224 0.148 0.221 -0.818 -0.836	
0.0529 0	0.148 0.221	
	01 <b>-1</b> -1	
-0.776 -0	0.919 0.926	
-0.776 -0	0.010 0.026	
-0.776 -0	0.010 0.026	
-0.770 -0	-0.630	
0.70	0 = 40	
0.726	0.769	
-0.996 -0	-0.994 -0.992	
	0.720	0.70

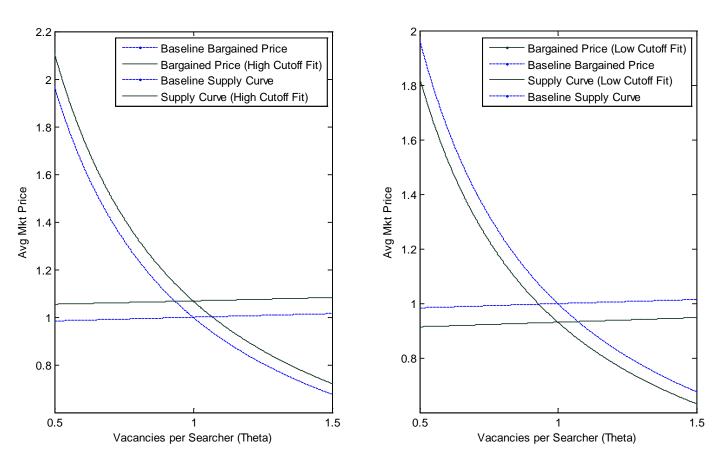
**Notes**: This table shows summary statistics of the dynamic search and matching model with heterogeneity, using Monte Carlo simulations of the model with random shocks to the rental rate. The first three columns of data (from left to right) represent alternative assumptions about the distribution of fit values in the searcher-buyer population. The final column shows summary statistics for the search and matching model with homogeneous demand, using the same random rental shocks for Monte Carlo simulations.

Figure 4.1
Steady State Equilibrium in the Search and Matching Model
with Hetereogeneous Matches



**Notes**: This figure plots the market average price schedule for different values of vacancies per searcher, given a steady state marginal fit value of 5 percent of the average price of a housing unit and  $s.d.(\zeta) = 0.1$ . It also plots the flow supply curve and the reserve price curve, which is the minimum price a housing unit must obtain in order for the seller to stay in the market and not convert the unit permanently to a rental unit. This minimum price is defined as  $\bar{P}_{Reserve} = \mathcal{R}\left(\frac{1+r}{(1-\delta)(r+\delta^{rent})}\right) - \frac{z}{1-\delta}$ . This price is derived in Chapter 2 and parameter values used to plot the price in this figure are provided in Table 4.1 of this paper.

Figure 4.2 – Equilibrium Relationship Between Average Price, Relative Supply and Fit Value
Figure 4.2A
Figure 4.2B



**Notes**: This figure shows comparative static results where alternative values of  $\bar{\zeta}$  are assumed and matching efficiency is adjusted in response. The figure plots the steady state equilibrium market average price schedule and flow supply curve for alternative equilibrium values of  $\bar{\zeta}$ . Fig.2A shows that a higher equilibrium  $\bar{\zeta}$  shifts the market average price curve shifts right and the supply curve inwards. Fig. 2B shows a lower  $\bar{\zeta}$  shifts the market price curve left and supply curve to the right. Comparing the equilibrium points for alternative assumed cutoff values, there is a positive relationship between equilibrium  $\bar{\zeta}$  and  $\bar{P}$  but equilibrium  $\theta$  (at the intersection point) is unchanged given alternative values of  $\bar{\zeta}$ .

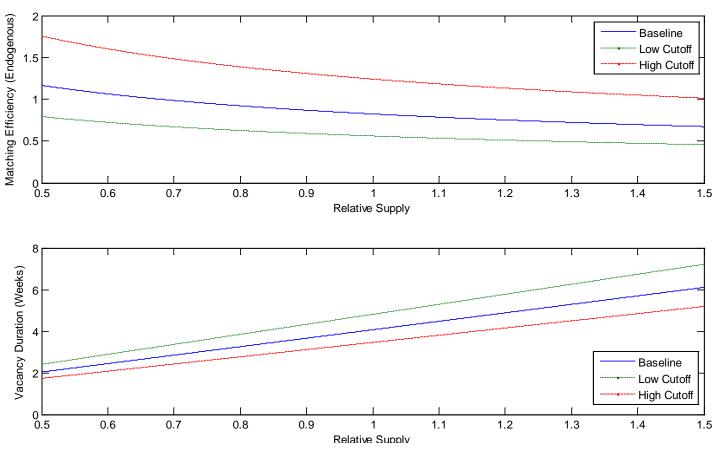


Figure 4.3 – Equilibrium Relationship Between Match Efficiency, Vacancy Duration and Fit Value

**Notes:** This figure is the companion to Figure 4.2 and shows the endogenous changes in equilibrium matching efficiency and equilibrium vacancy duration when alternative values of  $\bar{\zeta}$  are assumed. The top panel shows combinations of the matching efficiency parameter and relative supply for different given values of  $\bar{\zeta} = [0.0329; 0.0529; 0.0729]$ . The benchmark cutoff value is 0.0529, and the other two are labeled 'high' and 'low' correspondingly. For a high cutoff value, matching efficiency is higher in equilibrium for every possible value of  $\theta$ . The opposite is also true. The bottom panel similarly shows combinations of vacancy duration and  $\theta$  that occur in equilibrium, for the three alternative values of  $\bar{\zeta}$ . Because matching efficiency has a positive static relationship with  $\bar{\zeta}$ , vacancy duration has a negative static relationship with  $\bar{\zeta}$ .

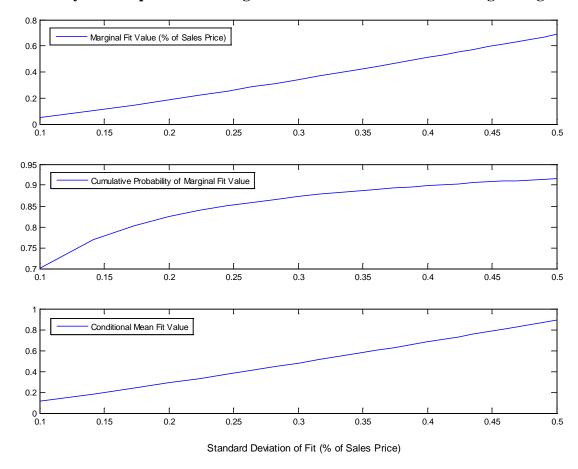


Figure 4.4 – Steady State Equilibrium Marginal Fit Value For Mean-Preserving Changes in Spread

**Notes**: This figure shows equilibrium  $\bar{\zeta}$  and its associated probability mass and conditional expectation of fit for different spreads assumed for the distribution of  $\zeta$ . The top panel in this figure plots equilibrium  $\bar{\zeta}$  against different spreads of  $\zeta$  (in units of s.d.), given  $\bar{P} = \theta = 1$ . The middle panel plots  $G(\bar{\zeta})$  against  $\mathrm{sd}(\zeta)$ . The bottom panel plots  $\int_{\bar{\zeta}}^{\infty} s \ g(s) ds / (1 - G(\bar{\zeta}))$ , where s is the dummy of integration. The figure shows that as the spread of the fit variable increases in the model, the cutoff fit value is higher (top panel) and hence the average expected fit that proceeds to sale, conditional on meeting the cutoff fit, is also higher (bottom panel).

Marginal Fit Value -0.2 <u></u> 2 3 5 9 10 11 12 13 Steady State Duration of Vacancy (Weeks) 1.8 Match Efficiency 1.2 0.8 0.6 0.4 0.2 2 3 5 10 11 12 13 Steady State Duration of Vacancy (Weeks)

Figure 4.5 Robustness Check: Varying Equilibrium Match Efficiency and Marginal Fit Value Given Vacancy Duration

Notes: This figure plots the steady state relationship of matching efficiency, marginal fit value & vacancy duration, given by  $Duration = 1/\overline{m}(1 - G(\overline{\zeta}))$  and the steady state solution of  $\overline{\zeta}$  in equation [13] of the text. Vacancy duration is exogenously determined and the two specified equations and parameter values are used to determine equilibrium matching efficiency and the market cutoff. The three curves in each subplot refer to different variances of the random fit variable. The solid blue line is the benchmark case, where s.d.( $\zeta$ )=0.1, i.e. 10% of average price. The green lines with circle markers in each subplot correspond to s.d.( $\zeta$ )=0.17; the black dotted lines correspond to sd( $\zeta$ )=0.22. In the benchmark case, each curve plots the corresponding equilibrium values of marginal fit value and match efficiency on the y-axis, for a given length of vacancy duration, marked along the x-axis. As the spread of the fit variable increases, each curve shifts outwards. If  $\overline{\zeta} \ge 0$  and  $\overline{m} \le 1$ , the duration of vacancy may be as low as 3.8 weeks or longer than a quarter, depending on the spread of  $\zeta$ . Time to sale depends inversely on searcher selectivity. If searchers are more selective,  $\overline{\zeta}$  is high. To compensate for this,  $\overline{m}$  is high and equilibrium duration is shorter. If the matching friction is removed altogether, searcher heterogeneity accounts for at least half of the empirically observed length of search by U.S. households, i.e. a duration of just over 4 weeks in equilibrium.

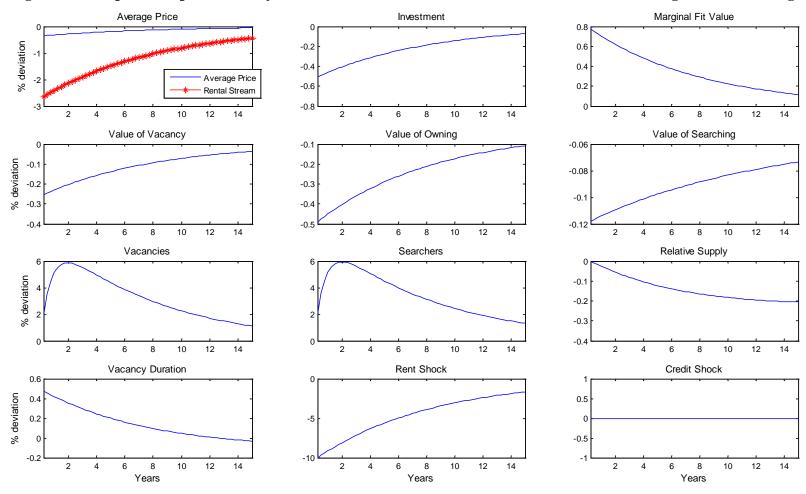
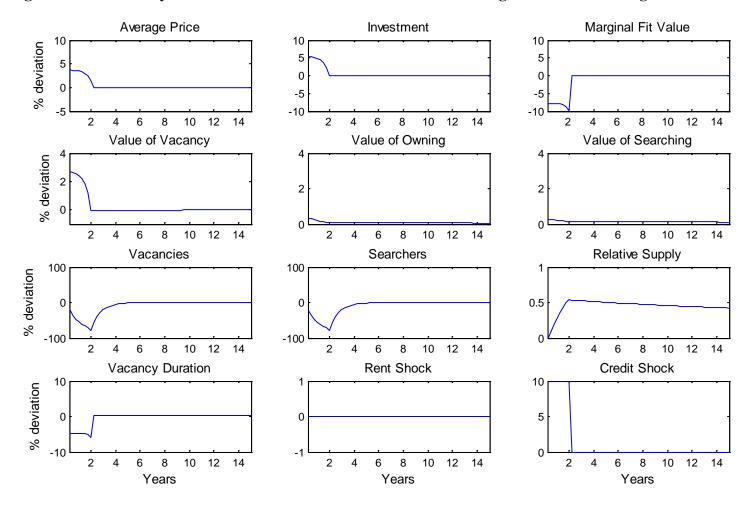


Figure 4.6 – Impulse Responses of Key Variables to 10 Percent Shock to Rental Rate with Heterogeneous Matching

**Notes:** This graph shows impulse responses of key model variables to a 10 percent decrease in the rental rate above steady state. The model is calibrated according to Table 4.1with  $s.d.(\zeta) = 0.1$ . The top leftmost window plots the neoclassical price (i.e. present discounted rental stream (red asterisked line)), as well as the bargained price of the matching model. The plot clearly illustrates that when sellers can choose over multiple equilibrating variables, the matching bargained price will not fall to the extent we would expect in a neoclassical setting; marginal fit value and vacancy duration both jump upwards.

Figure 4.7 – Home Buyers Tax Credit Shock in the Search and Matching Model with Heterogeneous Matching



**Notes:** This graph shows impulse responses of key model variables to a 10 percent housing tax credit in place for 2 years. The model is calibrated according to Table 4.1with  $s.d.(\zeta) = 0.1$ 

# **Chapter 5**

## **Conclusion**

This dissertation contributes two original dynamic structural models of decentralized search and matching in the housing market that incorporate demand and supply of existing vacancies and new residential investment and relate them to observed price and time on market. I have illustrated in Chapters 3 and 4 that these quantitative models are suitable for policy analysis, with ready empirical applications.

The success of the models developed in this dissertation lies in capturing the quality of price movements and investment responses correctly. It is also lies in the insight the search and matching framework provides about the basic of the dynamic relationship established between price, time to sale and vacancy rates, particularly under heterogeneity.

The models developed herein take a partial equilibrium approach in a perfect foresight setting and with no constraints on new construction. To develop the themes I have initiated in this dissertation further, I intend to model the locational choice of searcher-buyers, also perhaps incorporating income constraints. Also, the models can be strengthened by modifying new investment to account for time to plan, adjustment costs and financing constraints. By incorporating the relationship with employment and credit availability, the search and matching will be better suited to an analysis of the role of the housing sector in the business cycle.