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AN ANALYTIC SOLUTION TO THE STEADY-STATE DOUBLE ADIABATIC EQUATIONS

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Abstract. 20 moment transport equations applicable to low β plasmas of thermal origin in rotating planetary magnetospheres are presented. An analytic solution is found for a set of simplified equations describing the steady-state transport of collisionless plasma neglecting heat flow effects, which is also known as the double adiabatic situation [*Chew et al.*, 1956]. The new element of this analytic solution is a closed form solution for the variation of the parallel flow velocity, $u_{//}$, along magnetic field lines. The solution is significantly different from the often used assumption that, in the collisionless regime, the divergence of the parallel flow velocity is negligible (cf. [*Burgers*, 1969]). It is shown that only $T_\perp/B$ remains constant along the field line (as predicted by earlier calculations), while the density, parallel temperature and parallel Mach number vary as $n/B \propto 1/u_{//}$, $T_{//} \propto 1/u_{//}^2$, and $M_{//} \propto u_{//}^2$, respectively (in contrast to earlier predictions of $n/B=const$, $T_{//}=const$, and $M_{//}=const.$).

1. Introduction

The evolution and transport of plasmas of thermal origin is usually described by generalized transport equations. These equations are derived by taking velocity moments of the plasma kinetic equation (cf. *Burgers*, [1969]) and truncating the resulting set of coupled, partial differential equations using closing assumptions of varying degrees of sophistication. *Chew et al.*, [1956] were the first to derive transport equations based on a bi-Maxwellian distribution (neglecting heat flow) and this work was extended by a series of authors for a collisionless anisotropic plasma (cf. *Gombosi and Rasmussen* [1991] and references therein).

The transport of plasmas of thermal origin in rotating planetary magnetospheres (such as Jupiter or Saturn) has been discused in a long series of papers (cf. [*Barbosa*, 1990; *Hill et al.*, 1983; *Richardson and Eviatar*, 1988; *Vasyliunas*, 1983; *Wilson and Waite*, 1989] and references therein). These studies used transport equations of varying sophistication to describe various aspects of magnetospheric plasmas, such as radial diffusion, pressure balance, field aligned flow, etc.

This paper presents a set of generalized transport equations applicable to low β plasmas of thermal origin in rotating planetary magnetospheres (β = random energy density over magnetic energy density). These equations are based on a recent paper of *Gombosi and Rasmussen* [1991], who published a set of 20 moment equations applicable to space plasma flows of thermal origin in the presence of externally imposed strong magnetic fields.

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The newly-derived equations are considered for a simplified steady-state situation neglecting the effects of collisions and heat flow. This scenario corresponds to the well-known double adiabatic approximation [*Chew et al.*, 1956] for rotating magnetospheres. By neglecting the effects of heat flow the double adiabatic approximation oversimplifies the physics and therefore it should be treated with great care. On the other hand, the double adiabatic solution offers simple scaling laws extensively used by theoretical models describing plasma flows in the magnetospheres of the giant planets, the terrestrial polar wind and the plasmasphere (see the reviews of *Schunk* [1975; 1977; 1988], *Vasyliunas* [1983]). The widely used double adiabatic scaling laws do not represent a full solution of the simplified equations, because the momentum equation is not solved. This paper presents a full analytic solution to the steady-state double adiabatic equations. It is shown that only $T_\perp/B$ remains constant along the field line (as predicted by earlier calculations), while the density, parallel temperature and parallel Mach number vary as $n/B \propto 1/u_{//}$, $T_{//} \propto 1/u_{//}^2$, and $M_{//} \propto u_{//}^2$, respectively.

2. 20-Moment Transport Equations for a Rotating Magnetosphere

In a gyration dominated plasma the bulk flow velocity vector, the pressure and heat flow tensors of species "s" ($u_{s,i}$, $P_{s,ij}$, and $Q_{s,ijk}$, respectively) can be expressed in the following form: (cf. *Gombosi and Rasmussen* [1991])

$$u_s = u_B + u_{s//} b \quad (1a)$$

$$P_{s,ij} = \delta_{ij} p_{s\perp} + (p_{s//} - p_{s\perp}) b_i b_j \quad (1b)$$

$$Q_{s,ijk} = h_{s\perp}(\delta_{ij} b_k + \delta_{ik} b_j + \delta_{jk} b_i) + (h_{s//} - 3h_{s\perp}) b_i b_j b_k \quad (1c)$$

where δ_{ij} is the Kronecker delta, $b=B/B$ is the unit vector along the magnetic field line, u_B is the convection velocity of magnetic field lines perpendicular to the macroscopic magnetic field, $u_{s//}$ is the plasma velocity component parallel to the macroscopic magnetic field, $p_{s//}$ and $p_{s\perp}$ represent the parallel and perpendicular pressure components, while $h_{s//}$ and $h_{s\perp}$ denote the field aligned flow of parallel and perpendicular random energy. The 20-moment set of transport equations for gyration dominated plasmas reduce to six governing equations describing the evolution of plasma density, n_s, as well as the field aligned velocity, and the pressure and heat flow components [*Gombosi and Rasmussen*, 1991]. These equations are not repeated here.

We evaluate the terms on the right-hand sides of equations (20) through (25) of *Gombosi and Rasmussen* [1991] using the following assumptions: (i) the magnetic field is quasi-steady-state ($\partial B/\partial t \approx 0$), so that $E = -\nabla\Psi$, where Ψ is the electric potential; (ii) the angular velocity vector, ω, is constant and time independent along any given magnetic field line (Ferraro's theorem), i.e., $\partial\omega/\partial z=0$ ($\partial/\partial z=\mathbf{b}\cdot\nabla$, where z denotes the distance along magnetic field lines). It is also assumed that the magnitude of ω might be different for different L values, but its direction is the same for all field lines, $\omega=\omega(L)\mathbf{e}_3$, where $\mathbf{e}_3$ is the unit vector along the fixed rotation axis. This means that the perpendicular flow velocity, $\mathbf{u}_B$, can be expressed as $\mathbf{u}_B=\omega\times\mathbf{r}=\omega r\sin\Theta\mathbf{\Phi}$, where $\mathbf{\Phi}$ is the unit vector in the azimuthal direction and Θ is the angle between ω and $\mathbf{r}$; and finally (iii) the magnetic field has a negligible azimuthal component, $|B_\Phi|<<B$.

It should be also noted that the gravitational acceleration can be expressed as the negative gradient of the gravitational potential, $\mathbf{g}=-\nabla(GM_0/r)$, where G is the gravitational constant and M_0 is the mass of the planet.

First, however, several expressions need to be evaluated. First of all, the divergence of $\mathbf{u}_B$ is zero, $\nabla\cdot\mathbf{u}_B=0$, because ω_Φ is zero and ω is independent of azimuth, Φ. Second, the convective derivative of B is also zero, $D_BB/Dt=0$ ($D_B/Dt=\partial/\partial t+\mathbf{u}_B\cdot\nabla$), because B is independent of both time and azimuth angle. Third, it can be shown that the variation of $\mathbf{u}_B$ along the field line is always perpendicular to the field line itself, $\mathbf{b}\cdot(\partial\mathbf{u}_B/\partial z)=0$, because we assumed that the magnetic field unit vector, $\mathbf{b}$, has no azimuthal component, and that $\partial\mathbf{\Phi}/\partial z=0$. Finally, it can be shown that

$$\mathbf{u}_B\cdot\frac{D_s\mathbf{b}}{Dt}=r\omega\sin\Theta\mathbf{\Phi}\cdot(\omega\times\mathbf{b})=\frac{\partial}{\partial z}\left[\frac{1}{2}(\omega\times\mathbf{r})^2\right] \tag{2}$$

(where $D_s/Dt=D_B/Dt+u_{s//}\,\partial/\partial z$) because $D_B\mathbf{b}/Dt=\omega\times\mathbf{b}$.

Using these relations the transport equations can be written as:

$$\frac{D_B}{Dt}\left(\frac{m_sn_s}{B}\right)+u_{s//}\frac{\partial}{\partial z}\left(\frac{m_sn_s}{B}\right)+\frac{m_sn_s}{B}\frac{\partial u_{s//}}{\partial z}=\frac{m_s\dot{n}_s}{B} \tag{3a}$$

$$\frac{D_Bu_{s//}}{Dt}+u_{s//}\frac{\partial u_{s//}}{\partial z}+\frac{B}{m_sn_s}\frac{\partial}{\partial z}\left(\frac{p_{s//}}{B}\right)=\dot{u}_{s//}$$
$$-\frac{p_{s\perp}}{m_sn_s}\frac{1}{B}\frac{\partial B}{\partial z}-\frac{\partial}{\partial z}\left(\frac{GM_0}{r}+\frac{e_s}{m_s}\Psi-\frac{1}{2}(\omega\times r)^2\right) \tag{3b}$$

$$\frac{D_B}{Dt}\left(\frac{p_{s//}}{B}\right)+u_{s//}\frac{\partial}{\partial z}\left(\frac{p_{s//}}{B}\right)+3\frac{p_{s//}}{B}\frac{\partial u_{s//}}{\partial z}+$$
$$\frac{\partial}{\partial z}\left(\frac{h_{s//}}{B}\right)=\frac{\dot{p}_{s//}}{B}-2\frac{h_{s\perp}}{B}\frac{1}{B}\frac{\partial B}{\partial z} \tag{3c}$$

$$\frac{D_B}{Dt}\left(\frac{p_{s\perp}}{B}\right)+u_{s//}\frac{\partial}{\partial z}\left(\frac{p_{s\perp}}{B}\right)+\frac{p_{s\perp}}{B}\frac{\partial u_{s//}}{\partial z}+\frac{\partial}{\partial z}\left(\frac{h_{s\perp}}{B}\right)=$$
$$=\frac{\dot{p}_{s\perp}}{B}+\frac{u_{s//}p_{s\perp}+h_{s\perp}}{B}\frac{1}{B}\frac{\partial B}{\partial z} \tag{3d}$$

$$\frac{D_B}{Dt}\left(\frac{h_{s//}}{B}\right)+u_{s//}\frac{\partial}{\partial z}\left(\frac{h_{s//}}{B}\right)+4\frac{h_{s//}}{B}\frac{\partial u_{s//}}{\partial z}+$$
$$3\frac{p_{s//}}{B}\frac{\partial}{\partial z}\left(\frac{p_{s//}}{m_sn_s}\right)=\frac{\dot{h}_{s//}}{B}-3\frac{p_{s//}}{B}\dot{u}_{s//} \tag{3e}$$

$$\frac{D_B}{Dt}\left(\frac{h_{s\perp}}{B}\right)+u_{s//}\frac{\partial}{\partial z}\left(\frac{h_{s\perp}}{B}\right)+\frac{2h_{s\perp}}{B}\frac{\partial u_{s//}}{\partial z}+\frac{p_{s//}}{B}\frac{\partial}{\partial z}\left(\frac{p_{s\perp}}{m_sn_s}\right)$$
$$=\frac{\dot{h}_{s\perp}}{B}-\frac{\dot{u}_{s//}p_{s\perp}}{B}+\left[\frac{u_{s//}h_{s\perp}}{B}+\frac{p_{s\perp}}{m_sn_s}\frac{p_{s//}-p_{s\perp}}{B}\right]\frac{1}{B}\frac{\partial B}{\partial z} \tag{3f}$$

where m_s is the particle mass, while the $\dot{n}_s$, $\dot{u}_{s//}$, $\dot{p}_{s//}$, $\dot{p}_{s\perp}$, $\dot{h}_{s//}$, and $\dot{h}_{s\perp}$ quantities represent the appropriate moment of the collision term (cf. *Gombosi and Rasmussen* [1991]).

Equations (3a) through (3f) describe gyration dominated plasma flows in a frame of reference moving with the $\omega\times r$ convection velocity. This reference frame is explicitly denoted by the use of the convective derivative, D_B/Dt.

Next we consider a simplified set of equations describing a steady-state collisionless situation.

3. Collisionless Steady-State Flow Without Heat Flow: An Analytic Solution

Let us consider a simple steady-state case and neglect collisions and all heat flow effects. By neglecting the heat flow we practically reduced the series expansion of the distribution function from third order to second order. In other words, the truncation expression is no longer Grad's fourth order condition ($mnR_{ijkl}=P_{ij}P_{kl}+P_{ik}P_{jl}+P_{il}P_{jk}$, where R is the fourth moment of the distribution function), but the third order $Q_{ijk}=0$ relation (cf. *Gombosi and Rasmussen* [1991]). The resulting transport equations are no longer part of the twenty moment expansion: they represent the gyration dominated limit of the ten moment approximation. This means that the two heat flow equations have to be dropped from our set of transport equations (otherwise one would have six independent equations for four unknown functions). After some manipulation the remaining equations (3a-3d) can be written in the following form:

$$\frac{\partial}{\partial z}\left(u_{s//}\frac{m_sn_s}{B}\right)=0 \tag{4a}$$

$$u_{s//}\frac{\partial u_{s//}}{\partial z}+\frac{B}{m_sn_s}\frac{\partial}{\partial z}\left(\frac{p_{s//}}{B}\right)=-\frac{p_{s\perp}}{m_sn_s}\frac{1}{B}\frac{\partial B}{\partial z}-$$
$$\frac{\partial}{\partial z}\left(\frac{GM_0}{r}+\frac{e_s}{m_s}\Psi-\frac{1}{2}(\omega\times r)^2\right) \tag{4b}$$

$$\frac{\partial}{\partial z}\left(u_{s//}^3\frac{p_{s//}}{B}\right)=0 \qquad \frac{\partial}{\partial z}\left(u_{s//}\frac{p_{s\perp}}{B^2}\right)=0 \tag{4c}$$

It should be noted that equations (4a), and (4c) can immediately be combined to yield the well-known double adiabatic relations [*Chew et al.*, 1956]:

$$\frac{\partial}{\partial z}\left(\frac{B^2p_{s//}}{n_s^3}\right)=0 \qquad \frac{\partial}{\partial z}\left(\frac{p_{s\perp}}{n_sB}\right)=0 \tag{5}$$

Equations (4a), (4b) and (4c) can be readily integrated:

$$\frac{m_s n_s u_{s//}}{B} = \frac{m_s n_{s0} u_{s//0}}{B_0}$$

$$u_{s//} \frac{p_{s\perp}}{B^2} = u_{s//0} \frac{p_{s\perp 0}}{B_0^2} \qquad u_{s//}^3 \frac{p_{s//}}{B} = u_{s//0}^3 \frac{p_{s//0}}{B_0} \tag{6}$$

where the subscript *0* refers to values at a reference point, B_0. With the help of equation (6) the momentum equation can be expressed as

$$\frac{\partial}{\partial z}\left[\frac{1}{2}u_{s//}^2 + \frac{3p_{s//0}}{2m_s n_{s0}}\frac{u_{s//0}^2}{u_{s//}^2} + \frac{p_{s\perp 0}}{m_s n_{s0}}\frac{B}{B_0} + \frac{GM_0}{r} + \frac{e_s}{m_s}\Psi - \frac{1}{2}(\omega \times r)^2\right] = 0 \tag{7}$$

Equation (7) means that the expression in the bracket is a conserved quantity along the magnetic field line. The electron momentum equation (4b) can be used to express the field aligned electric potential drop, Ψ-Ψ_0:

$$\Psi - \Psi_0 = \frac{3}{2}\frac{p_{e//0}}{e n_{e0}}\left(\frac{u_{e//0}^2}{u_{e//}^2} - 1\right) + \frac{p_{e\perp 0}}{e n_{e0}}\left(\frac{B}{B_0} - 1\right) \tag{8}$$

Most space plasmas are quasineutral and the electron number density can be expressed as the sum of all the ion number densities (assuming single ionized ions). If one treats the field aligned current density, *j*, as an externally specified quantity, one can also express the electron velocity in terms of the ion velocities.

Substituting equation (8) into (7) yields the following algebraic equation for the parallel flow velocity of an ion species, *s*:

$$u_{s//0}^2\left(\frac{u_{s//}^2}{u_{s//0}^2} - 1\right) + \frac{2kT_{s\perp 0}}{m_s}\left(\frac{B}{B_0} - 1\right) + \frac{2kT_{e\perp 0}}{m_s}\left(\frac{B}{B_0} - 1\right) + \frac{3kT_{s//0}}{m_s}\left(\frac{u_{s//0}^2}{u_{s//}^2} - 1\right) + \frac{3kT_{e//0}}{m_s}\left(\frac{u_{e//0}^2}{u_{e//}^2} - 1\right) + 2GM_0\left(\frac{1}{r} - \frac{1}{r_0}\right) - (\omega \times r)^2 + (\omega \times r_0)^2 = 0 \tag{9}$$

where *k* is the Boltzmann constant and *T* represents temperature. In the case of a multiion plasma relation (9) (together with the quasineutrality condition and the prescribed field aligned current density) represents a set of transcendent algebraic equation for the ion parallel velocities. In the simple case, when we have a single ion plasma and a negligible field aligned current, $n_e = n_i = n$ and $u_{e//} = u_{i//} = u_{//}$ and consequently equation (9) is simplified to

$$u_{//}^4 - u_{//}^2\left[u_{//0}^2 + a_{//0}^2 - a_{\perp 0}^2\left(\frac{B}{B_0} - 1\right) + (\omega \times r)^2 - (\omega \times r_0)^2 - 2GM_0\left(\frac{1}{r} - \frac{1}{r_0}\right)\right] + a_{//0}^2 u_{//0}^2 = 0 \tag{10}$$

where the parallel and perpendicular ion-acoustic speeds are defined as

$$a_{//0}^2 = \frac{3k(T_{i//0} + T_{e//0})}{m_i} \qquad a_{\perp 0}^2 = \frac{2k(T_{i\perp 0} + T_{e\perp 0})}{m_i} \tag{11}$$

Equation (10) can be solved for the magnitude of the parallel velocity:

$$u_{//}^2 = \begin{cases} \frac{1}{2}\left(C + \sqrt{C^2 - 4u_{//0}^2 a_{//0}^2}\right) & \text{if } u_{//0}^2 > a_{//0}^2 \\ \frac{1}{2}\left(C - \sqrt{C^2 - 4u_{//0}^2 a_{//0}^2}\right) & \text{if } u_{//0}^2 < a_{//0}^2 \end{cases} \tag{12}$$

where

$$C = u_{//0}^2 + a_{//0}^2 - a_{\perp 0}^2\left(\frac{B}{B_0} - 1\right) - 2GM_0\left(\frac{1}{r} - \frac{1}{r_0}\right) + (\omega \times r)^2 - (\omega \times r_0)^2 \tag{13}$$

It should be emphasized that this solution is valid along a given magnetic field line.

4. Discussion

Equations (3a) through (3f) represent the 20-moment set of generalized transport equations to be applied to the ionosphere - inner magnetosphere systems of rotating planets. These equations are capable to describe the transition from collision dominated to collisionless plasma flows. Typical examples are the polar wind or interhemispheric flows along plasmaspheric field lines. It is interesting to note that the $u_B \cdot D_s b/Dt$ term in the momentum equation eventually yields a term describing the change of the centrifugal potential along the magnetic field line. Such a term has been intuitively used in earlier work (cf. [*Vasyliunas*, 1983]), but here we derived it directly from the Boltzmann equation (using the results of *Gombosi and Rasmussen* [1991]). In general these equations are quite complicated and only numerical solutions are feasible.

Analytic solutions are not only aesthetic, but they also help us to gain physical insight to the problem. The analytic solution to the full steady-state double adiabatic problem in a rotating inner magnetosphere is particularly useful, because double adiabatic relations are frequently used in magnetospheric physics to provide a first approximation of various problems (such as temperature profiles in polar wind flows with anisotropic temperatures).

Let us first consider a dipole field line and assume that the plasma flow velocity is negligible at a reference latitude, λ_0, so that $u_{//}(\lambda_0) = u_{//0} = 0$. In this case the parallel flow velocity satisfies the following equation:

$$u_{//}^2\left[u_{//}^2 - a_{//0}^2 + a_{\perp 0}^2\left(\frac{B}{B_0} - 1\right) + 2GM_0\left(\frac{1}{r} - \frac{1}{r_0}\right) - (\omega \times r)^2 + (\omega \times r_0)^2\right] = 0 \tag{14}$$

It can be easily seen that only the trivial solution of this equation satisfies the $u_{\parallel}(\lambda_0)=u_{\parallel 0}=0$ condition. This means that if the field aligned flow velocity is zero at any point along a collisionless portion of a field line, it is zero everywhere in the collisionless region.

Next let us consider ionospheric outflow solutions, which characterizes polar wind or plasmaspheric flow conditions. In such a case the reference point is located at high latitude. It can be shown that the $C(\lambda)$ function increases toward the equator and consequently the magnitude of the parallel velocity of the outflowing plasma, $u_{\parallel}^2$, exhibits different behavior depending on its initial value. If $|u_{\parallel}(\lambda_0)|$ is larger than the local parallel ion-acoustic speed, $a_{\parallel 0}$, than $u_{\parallel}^2$ increases toward the equator, while in the case when the initial flow velocity is smaller than the local ion-acoustic speed, $u_{\parallel}^2$ decreases. These solutions are illustrated in Figure 1, which shows parallel flow velocity profiles for an L=4 Saturnian field line.

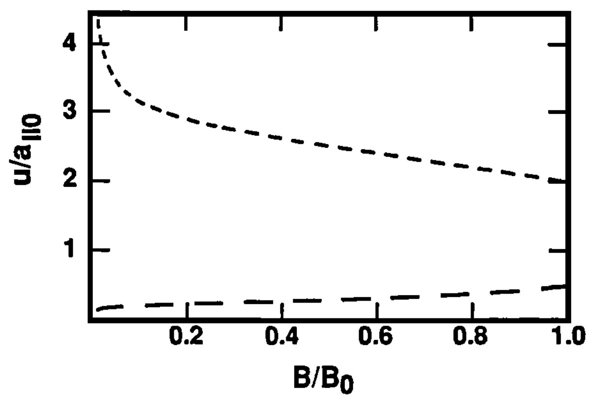

Fig. 1. The variation of the parallel flow velocity along an L=4 field line in the Saturnian magnetosphere from a reference point (located at $\lambda=58°$) toward the magnetic equator for initial velocities of $0.5a_{\parallel 0}$ (dashed line) and $2.0a_{\parallel 0}$ (dotted line) ($a_{\parallel 0}$ is the parallel ion acoustic speed at the reference point).

The actual solution is primarily determined by the choice of external boundary conditions. In our case this means flow conditions as $B\rightarrow 0$. It can be shown that only the subsonic solution is physical if the plasma pressure is finite at infinity, while in the case of zero external pressure the physical solution is supersonic flow. These boundary conditions are "disguised" in the plasma parameters at the reference point. A supersonic flow corresponds to a polar wind type solution along open magnetic field lines, while a subsonic flow describes plasma along closed plasmaspheric field lines.

5. Conclusion

The solution discussed in this paper provides not only the standard double adiabatic relations for steady-state conditions (given in equation (5)), but it also predicts that the field aligned plasma flow velocity varies along magnetic field lines. This prediction is significantly different from the often used assumption, that in the collisionless regime, the divergence of the parallel flow velocity is negligible. It is shown that only $T_{\perp}/B$ remains constant along the field line (as predicted by earlier calculations), while the density, parallel temperature and parallel Mach number vary as $n/B\propto 1/u_{\parallel}$, $T_{\parallel}\propto 1/u_{\parallel}^2$, and $M_{\parallel}\propto u_{\parallel}^2$, respectively (in contrast to earlier predictions of $n/B=const$, $T_{\parallel}=const$, and $M_{\parallel}=const$.).

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