

## Space-time analysis of TIMED Doppler Interferometer (TIDI) measurements

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**Abstract.** A technique is developed for reducing the amount of aliasing in the spectral analysis of TIDI observations, by ingestion of ground-based data into the satellite data set. A multi-dimensional (space-time) least squares fitting approach is applied to the satellite and ground-based data to determine the aliasing spectra. The addition of ground-based data to the TIDI data set reduces the aliased components in the aliasing spectrum. For example, at 20° latitude, the combined ground-based and TIDI data set of a sampled input semidiurnal (frequency of 2  $days^{-1}$ ) signal with zonal wavenumber 2 results in a factor of 2 reduction in the amount of power aliasing into a signal with zonal wavenumber 0 and frequency 0  $days^{-1}$ .

### Introduction

The National Aeronautics and Space Administration (NASA) Thermosphere-Ionosphere-Mesosphere, Energetics and Dynamics (TIMED) mission is scheduled for launch in 2000, with the goal to study Earth's Mesosphere and Lower Thermosphere (60-180 km), the least explored and understood region of our atmosphere. Observations from the TIMED-Doppler Interferometer (TIDI) will provide information on the global mesosphere-lower thermosphere-ionosphere (MLTI) dynamics, allowing an improved characterization of global tidal fields.

In this paper we have studied the application of space-time analysis of the TIDI observations in retrieving uncontaminated estimates of tidal harmonics. This paper presents a technique that combines space-borne and ground-based observations to optimize the retrieval of tidal harmonics from the observations, whilst effectively reducing the power leakage (aliasing) into the signal from aliased frequencies and wavenumbers.

### The TIMED-Doppler Interferometer

The TIMED-Doppler Interferometer [Killeen *et al.*, 1999] is the principal instrument aboard the TIMED satellite for investigating the dynamics of the MLTI region. The satellite will be placed into a 74.1° inclined orbit at an altitude of 625 km. The primary data products of TIDI will be the horizontal vector winds and neutral temperatures from 60-300

km. The instrument's line-of-sight (LOS) Doppler measurements will be combined to provide vector wind profiles in two swaths (sunward and anti-sunward) to either side of the TIMED satellite track. The details of the operation of TIDI can be found elsewhere [Killeen *et al.*, 1999].

### TIDI Sampling

Figure 1 shows the TIDI measurement tracks on the warm (sun-ward) and cold (anti sun-ward) sides of the spacecraft for one day in May 2000. The slow precession rate of the orbit (3° in longitude per day) allows 24 hours of local time coverage by TIDI in about 60 days using both the ascending and descending portions of the orbit.

Traditionally, tidal analyses of space-based measurements have involved the assimilation of data over multiple days/weeks to increase the temporal coverage [McLandress *et al.*, 1996; Burrage *et al.*, 1995]. However, the local time structure of the atmosphere is inherently non-stationary, and given the precession period of the TIMED satellite, this characteristic of the dynamics can lead to sampling and aliasing problems when attempting to deconvolve TIDI measurements into the constituent tidal harmonics.

### The Aliasing Spectrum

Salby [1982] showed that asymptotic (observations made at a single location at a given time) space-time satellite data can be spectrally analyzed in the wavenumber-frequency domain via a rotation from synoptic (simultaneous observations at different locations) to asymptotic coordinates and performing a Fourier transform. A significant advantage of this approach is that it directly maps the asymptotic data set to the equivalent space-time spectrum. However, since this approach relies on the discrete Fourier transform (DFT) operation, any irregularities in the sampling (non evenly spaced sampling), will destroy the orthogonality of discrete projections of the observations onto the Fourier expansion functions. To avoid the constraint of sampling uniformity, Wu *et al.* [1995] presented a least squares fitting technique to spectrally analyze a two-dimensional space-time data set. The least squares fitting technique allows the determination of the space-time spectrum from the asymptotic sampling patterns within the allowed region of wavenumber-frequency space defined by the Asymptotic Sampling Theorem [Salby, 1982].

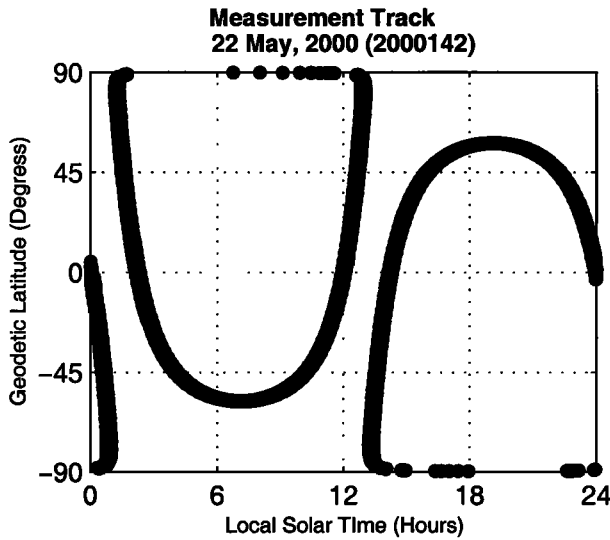
An observed wave field ( $y_i$ ),  $i = 1, \dots, N$ , where  $N$  is a positive integer, can be defined by a mathematical model given by:

$$y_i = A \cos[2\pi(\sigma t_i + s\lambda_i)] + B \sin[2\pi(\sigma t_i + s\lambda_i)] \quad (1)$$

where  $s$  is the wavenumber,  $\sigma$  is the frequency in  $days^{-1}$ ,  $t$  is the universal time in days, and  $\lambda$  is the longitude

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**Figure 1.** Coverage plot of TIDI instruments as a function of local solar time.

(normalized by  $360^\circ$ ). The 2D periodogram ( $P(\sigma, s) = A^2(\sigma, s) + B^2(\sigma, s)$ ) can be obtained by fitting for the coefficients  $A$  and  $B$ . The 2D periodogram of the sampling pattern constitutes the “aliasing spectrum” that provides an indication of the wavenumbers and frequencies that can contribute power to (or alias) the signal.

For each latitude and altitude grid point, satellite and ground-based measurements are ordered asynchronously in longitude and universal time (UT). The aliasing spectrum is computed for each asynchronously measured space-time series using 2D least squares fit for a given wavenumber and frequency. To construct a complete 2D periodogram (aliasing spectrum) that includes all the possible wavenumbers and frequencies and their corresponding aliases, the spectrum has to be computed for different values of  $s$  and  $\sigma$ .

### Aliasing Spectrum of an Input (2,2) Signal

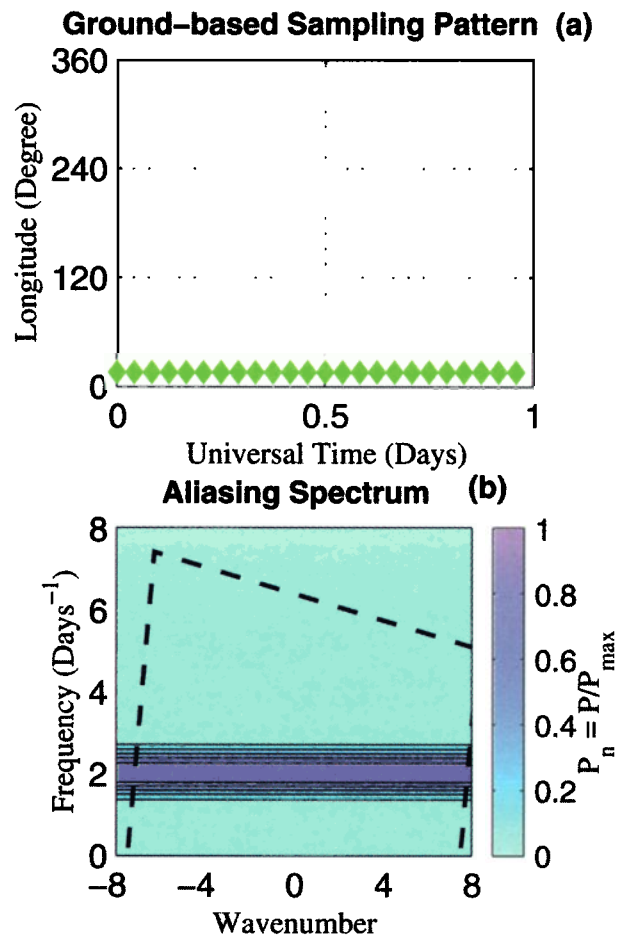
In their study of High Resolution Doppler Imager (HRDI) on board the Upper Atmospheric Research Satellite (UARS), *Wu et al.* [1995] identified random sampling as the best scheme for sampling stationary or slow moving waves. However, they also indicated that such a sampling scheme was not conceivable due to practical considerations. This paper presents an approach which introduces a more realistically achievable non-uniformity in the sampling pattern to exploit the idea of random sampling being the best scheme to reduce aliasing.

An artificial low white noise wave field was generated with the  $s = 2$  and  $\sigma = 2$  [ $days^{-1}$ ] to test this approach. A mock data set was created by sampling this  $(\sigma, s) = (2, 2)$  wave along the TIDI measurement tracks (on both the day and night side of the orbit) at about 100 km tangent height over one day. The aliasing spectrum for the (2,2) wave field was calculated for the TIDI sampling patterns at various latitude by scanning the wavenumber and frequency at an interval of 0.25 and 0.1  $day^{-1}$ , respectively. To introduce some degree of non-uniformity in the overall sampling pattern, hourly sampled data from various ground-

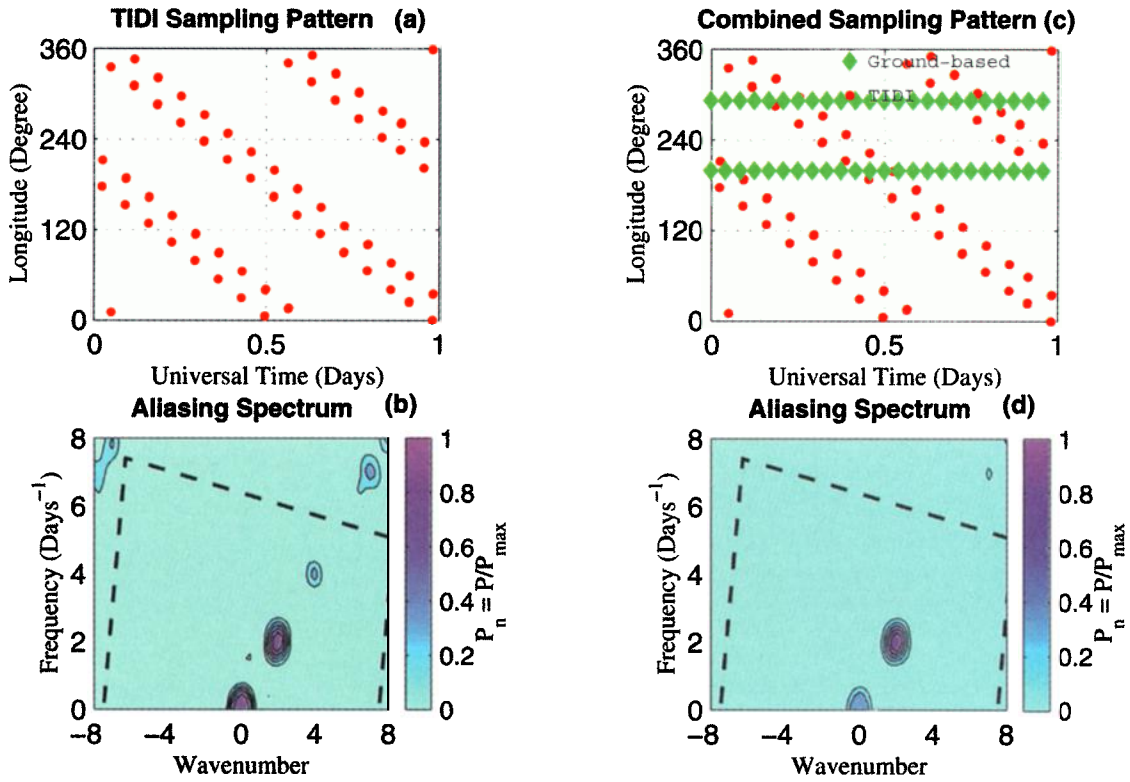
based medium frequency (MF) radar sites were added and the aliasing spectrum was re-calculated. In doing so, we have assumed that the observations made by TIDI agree with those of MF radar sites. To illustrate our approach, results from  $20^\circ$  latitude circle are presented here. It should be noted that this assimilative analysis technique is not limited to TIDI and MF radar data sets but can include any number and combinations of ground-based and space-borne data sets of perturbations in wind, temperature or density fields.

### Results

Figures 2 and 3 illustrate the effects on the aliasing spectrum resulting from combining the ground-based and space-borne measurements of an input (2,2) signal. For the ground-based observations, which are made from a geographically fixed location, the aliasing spectrum indicates that all the wavenumbers are aliased into the input (2,2) signal. Consequently, from a single ground-based site, it is impossible to resolve different wavenumbers in the spectral plane. The aliasing spectrum of the sampling pattern obtained from the TIDI measurements at  $20^\circ$  latitude is shown in Figure 3b. The dashed lines (also shown in Fig-



**Figure 2.** Sampling pattern and aliasing spectrum of ground-based samples only. The dashed lines describe the allowed region of resolvable wavenumber-frequency space defined by the asymptotic sampling theorem [Salby, 1982]



**Figure 3.** Sampling patterns for (a) TIDI at 20° latitude circle and (c) combined TIDI and ground based MF radar sites at 20° latitude circle and the corresponding aliasing spectrum of the (2,2) wave for (b) TIDI observations alone and (d) TIDI and MF radar sites at Hawaii, USA [22N, 160W] and Puerto Rico [19N, 67W]. The dashed lines describe the allowed region of resolvable wavenumber-frequency space defined by the asymptotic sampling theorem [Salby, 1982]. The periodogram is normalized to the power of the (2,2) signal.

ure 2b) define the boundary of asymptotic Nyquist region [Salby, 1982]. Any signal with wavenumber and frequency lying outside this region can not be unambiguously resolved given the TIDI sampling pattern. The space-time spectrum corresponding to the input signal (2,2) and a sample interval of 1 day, shows a spectral peak at wavenumber 2 and frequency 2 along with the aliased components. Examination of the aliasing spectrum indicates that the respective magnitudes of the aliases at (0,0) and (4,4) are about 95% and 46% of the magnitude of the spectral peak at (2,2). In other words, the signal at (0,0), i.e. the mean component, in the power spectrum will appear spuriously as a result of 95% of the (2,2) signal aliasing into the mean component. Similarly, 46% of the input (2,2) signal will alias into the (4,4)

signal. The combined sampling pattern, shown in Figure 3c, includes the observations from TIDI and two ground-based MF radar sites that are fixed at specific longitudes. The resulting aliasing spectrum for the combined sampling pattern and the same input wave field is shown in the lower right panel of Figure 3. The power spectrum at (0,0) indicates a reduction of power leakage to about 45% of the power at (2,2). The power spectrum indicates that the signal component at (4,4) has about 15% of the input signal power (a factor of 3 improvement in the amount of power aliased due the input signal). If, instead of two ground-based sites only one of them were combined with the TIDI observations, the resulting aliasing spectrum (not shown here) indicates spectral peaks at (0,0) and (4,4) to be 63% and 24% of the (2,2) signal, respectively. If, in addition to the two MF radar sites mentioned earlier, a third MF radar sites was chosen at, say 20°N and 239°E, the aliasing spectrum (not shown here) of the combined TIDI and three MF radar stations shows an improvement with (0,0) and (4,4) aliased component to be 34% and 10% of the input signal at (2,2).

In addition to the number of ground-based sites, the separation between the sites also impacts the aliasing spectrum. In general, the optimum separation between the sites is dependent upon what signal and aliased components are present in the spectrum. For an input (2,2) signal and aliased components at (0,0) and (4,4), best results were obtained when the longitudinal separation between the stations was greater than 45°. Tests, where the separation was arbitrarily reduced to less than 45°, indicated an increase of power at the aliased wavenumbers and frequencies.

**Table 1.** Percentage of aliased components in the spectrum.

Mode	Exp. 1	Exp. 2	Exp. 3	Exp. 4
(0,0)	95%	63%	45%	34%
(4,4)	46%	24%	15%	10%

This table shows the percentage of power at (2,2) aliased into the different modes shown. Exp. 1 corresponds to the case where only TIDI observations were used to calculate the aliasing spectrum. Exp. 2, Exp. 3, and Exp. 4 correspond to aliasing spectrum calculations using 1, 2, and 3 ground-based stations in addition to the TIDI observations, respectively.

While a single ground-based station is not capable of distinguishing different zonal wavenumbers due to its fixed geographical location, the combined ground-based radar and TIDI sampling is capable of separating the contributions from various zonal wavenumbers. The 2D least squares fitting algorithm for calculating the aliasing spectrum, presented here, contains no inherent assumptions regarding the composition of the perturbation fields. As a result, the aliasing spectrum for any input signal with any frequency and wavenumber (migrating or non-migrating tide) can be computed and optimized with the addition of ground-based sampling to reduce the aliased components in the spectrum.

## Conclusions

We have applied a least squares fitting approach to the space-time spectral analysis of TIMED-TIDI measurements. The results of spectral analyses of TIDI samples at 20° latitude indicate that contamination introduced by aliasing can be reduced by incorporating ground-based data into the TIDI data set. The salient results of this study are summarized in Table 1.

This study presents a simplified approach to address the issue of aliasing in the analysis of satellite observations of perturbation fields in the MLTI region. In order to better illustrate the results, the aliasing problem has been simplified by assuming the model atmosphere to be composed of a single perturbation mode i.e. the (2,2) component. However, this work can be easily expanded to include other wave perturbations that are present in the MLTI region. Also, observations from ground-based sites other than the MF radar locations reported here can be used to construct the combined data set. Further work is needed to determine the optimum configuration for the locations of ground-based instruments to support the unambiguous determination of

tidal harmonics at multiple locations using the combination of satellite (TIDI) and ground-based observations.

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