THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
DEPARTMENT OF NAVAL ARCHITECTURE AND MARINE ENGINEERING
SHIP HYDRODYNAMICS LABORATORY

Experimental Wave Component Analysis as Applied to Ship Wave Systems

Part I

Analysis of Available Methods and Evaluation of Some Experimental Data

HUN CHOL KIM
FINN C. MICHELSEN

Administered through:

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OFFICE OF RESEARCH ADMINISTRATION • ANN ARBOR

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ABSTRACT

Although the wave analysis methods do not seem to give correct wave resistance, the emphasis is placed on obtaining information on wave components with the objective of wave cancellation thereby ultimately leading to a better hull form. The existing theories are reviewed and applicability is discussed.

Sharma's longitudinal cut method is chosen as a initial try and a partial result of wave analysis on a 12-foot C-201 model is presented. As such, final conclusion is not drawn. A description of instrumentation is attached.

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INTRODUCTION

We take the view that the main objective of the wave analysis is to study the wave components and that the aggregate information, that is, wave resistance itself, can only be used in a qualitative, comparative sense.

The purpose of the present study is to investigate various theories, methods, techniques and their applications for the somewhat fundamental objective in the above. This report presents partial result of that effort: the first part containing the survey of theories and methods, and the error analysis; the second part showing the partial result on a 12-foot C-201 model by Sharma's longitudinal cut method using capacitance type wave measuring device; the third part containing the description of instrumentation.

SURVEY OF EXISTING THEORIES AND METHODS

Wave analysis theories assumes waves to be fait accompli. Resistance related to these waves can only be described by potential theory, and within that theory, depending on the mathematical expressions, fairly distinct differences exist. This results in what we shall refer to as different theories.

Although these theories differ in details, they are all developed under the assumption that:

1. the fluid is inviscid and the flow irrotational,
2. non-linearities can be neglected,
3. measurements are taken where the effect of the local wave system can be neglected,
4. the problem is steady,
5. the wave resistance, by definition, is given by the waves behind a body moving in the free surface of a real fluid.

In order to compare these different theories, we will categorize according to author, cut method, analysis method, etc.

A. Analysis based on the Amplitude Functions with respect to the Direction of Wave Propagation (Taniguchi's Method).

From Havelock's formulae, the wave resistance for a ship moving in the free surface is given by

$$R_w = \rho \pi U^2 L^2 \int_0^{\frac{\pi}{2}} \left[\bar{f}(\theta)^2 + \bar{g}(\theta)^2 \right] \cos^3\theta \, d\theta \qquad (1.1)$$

where U = forward speed of the ship

L = length of the ship

$\bar{f}(\theta)$ & $\bar{g}(\theta)$ = amplitude functions (non-dimensionalized).

Thus, if the amplitude functions are known, it is a simple matter to calculate the wave resistance. In a strictly analytical treatment, these functions are calculated from the singularity distribution representing the ship. In the present case, however, these functions are obtained experimentally from wave profiles, and in particular in Taniguchi's method, from one or several longitudinal cuts.

In general, wave profile in non-dimensional form is

$$\bar{\zeta}(\xi,\eta,0)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\Big\{\bar{f}(\theta)\sin\left[K_0 L\sec^2\theta(\xi\cos\theta+\eta\sin\theta)\right] + \bar{g}(\theta)\cos\left[K_0 L\sec^2\theta(\xi\cos\theta+\eta\sin\theta)\right]\Big\}\,d\theta \qquad (1.2)$$

where (x,y,z) coordinate system is fixed at the midship and x is directed to the stern, y port and z down; $\xi = x/L$, $\eta = y/L$; $k_0 = g/U^2$.

By assuming expansions of $\bar{f}(\theta)$ and $\bar{g}(\theta)$ by cosine and sine series of the form

$$\bar{f}(\theta) = \sum_{j=1}^{n} A_j \cos j\theta \qquad (1.3)$$

$$\bar{g}(\theta) = \sum_{j=1}^{n} B_j \cos j\theta$$

and defining functions

$$F_{A_j} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos j\theta \, \sin\left[K_0 L \sec^2\theta (\xi \cos\theta + \eta \sin\theta)\right] d\theta$$

$$F_{B_j} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos j\theta \, \cos\left[K_0 L \sec^2\theta (\xi \cos\theta + \eta \sin\theta)\right] d\theta \qquad (1.4)$$

the wave profile can be written as

$$\bar{\zeta}(\xi,\eta,0) = \sum_{j=1}^{n} (A_j F_{A_j} - B_j F_{B_j}). \qquad (1.5)$$

Here A_j and B_j are unknown arbitrary constants to be determined from the L.H.S. of Equation (1.5). A finite number of terms in A_j and B_j are chosen, and the least square technique may be applied. Once A_j and B_j are determined, $\bar{f}(\theta)$ and $\bar{g}(\theta)$ are known from Eq. (1.3) and the resistance is found from Eq. (1.1).

Note that his method can be used for any type of cuts, transverse, longitudinal or even descrete point data. Since it can be shown that the dummy variable θ in (1.1) actually corresponds to physical angles of wave propagation with respect to the body, (1.2) represents a summing up of all wave amplitudes with the direction of wave propagation properly considered.

Amplitude functions of this type, decomposed into the angles of propagation, have been used by Inui for selection of an optimum bulb for a given hull form. Crux of his method is to obtain the amplitude functions of the main hull and, for cancellation purposes, to find a bulb which has an amplitude function of approximately the same magnitude and distribution as the main hull but of opposite sign. In obtaining these amplitude functions for both the main hull and the bulb, he has resorted to an analytical method from the given singularity distributions. This technique is well understood in principle, but difficulties lie in the fact that the usual hull forms are necessarily too complex to render analytical computation of amplitude functions.

We suggest, therefore, that these amplitude functions be obtained experimentally and that the cancellation process be analized on the basis of obtained data all based on those following the same method as used by Inui. This experimental method has several advantages over a strictly analytical method in that the method, once developed, is much simpler and it does take into account to a degree, the viscosity effect.

To explain the latter, consider two submerged cylinders, spaced suitably apart, towed laterally at a constant speed (two-dimensional problem). Analytically, two-dimensional horizontal doublets may be used to find the waves, resulting in sine waves in the far rear. The distance between the two cylinders for cancellation condition can be found easily. This was tried out in the experiment with a rather poor result. In order to improve this, when the distance between the cylinders for cancellation condition was obtained from the measured wave of the first cylinder, the experiment showed an excellent cancellation at that speed. Further, slight changes in speed either below or above resulted in marked increase in wave height at the rear as it should have been. Of course, in three-dimensional cases, things are not so easy as this, but the fact that the experimental method offers possible means of taking account of viscosity should be explored.

The experimental method of finding amplitude functions, or more generally speaking, the experimental wave component analysis method requires some sort of systematic compilation of observed waves on conventional hull forms and bulbs. Such data must be presented in the best available manner, and not necessarily so with respect to the amplitude functions decomposed into the angles of propagation. We should try out the available methods, evaluate the functions, verify experimentally and so on.

Taniguchi has presented amplitude functions from measurements on a model for three different longitudinal cuts and result is quoted here (Figure 1). Figure 2 presents the wave resistance calculated from the result of Figure 1 by the present writers. The amplitude functions from different cuts appear to vary considerably and the wave resistance also. It is, however premature, to make a judgement on this basis alone. Results from investigations of other methods as well as experiments should be studied first.

Pien's method follows Taniguchi's except that the functions of the type given in (1.4) are considered with respect to bow and stern separately, contrary to Taniguchi's in which the functions are considered around the midship. Thus, in place of (1.5), we now have

$$\bar{\zeta}(\xi,\eta,0)=\sum_{j=1}^{n}\left(A_j^B F_{Aj}^B + B_j^B F_{Bj}^B + A_j^S F_{Aj}^S + B_j^S F_{Bj}^S\right) \tag{1.6}$$

where superscripts B and S indicate bow and stern, respectively. Notations are here as used by present writers.

B. Analysis based on the Amplitude Functions with respect to the Wave Numbers. (Eggers and Sharma's Method)

Referring to (1.1) and (1.2), if one makes the transformations

$$\begin{aligned} k &= K_0 \sec^2\theta \\ w &= K_0 \sec^2\theta \cos\theta \\ u &= K_0 \sec^2\theta \sin\theta \end{aligned} \tag{2.1}$$

and define the following Fourier transforms

$$\begin{matrix} S \\ C \end{matrix} = \int_{-\infty}^{\infty} \zeta \begin{matrix} \sin \\ \cos \end{matrix} (uy)\, dy \tag{2.2}$$

$$\begin{matrix} S_x \\ C_x \end{matrix} = \int_{-\infty}^{\infty} \frac{\partial \zeta}{\partial x} \begin{matrix} \sin \\ \cos \end{matrix} (uy)\, dy \tag{2.3}$$

$$\begin{matrix} LS \\ LC \end{matrix} = \int_{-\infty}^{\infty} \zeta \begin{matrix} \sin \\ \cos \end{matrix} (wx)\, dx, \tag{2.4}$$

the wave resistance coresponding to (1.1) is shown to be

(1) For a transverse cut

$$R_w = \frac{\rho g}{4\pi} \int_0^{\infty} \left\{ S^2 + C^2 + \frac{1}{k K_0} \left(S_x^2 + C_x^2 \right) \right\} \left(2 - \frac{K_0}{k} \right) du. \tag{2.5}$$

(2) For two transverse cuts close to each other

$$R_w = \frac{\rho g}{16\pi} \int_0^{\infty} \left\{ \frac{(C_1+C_2)^2 + (S_1+S_2)^2}{\cos^2 \frac{w}{2}(x_1 - x_2)} + \frac{(C_1-C_2)^2 + (S_1-S_2)^2}{\sin^2 \frac{w}{2}(x_1 - x_2)} \right\} \left(2 - \frac{K_0}{k} \right) du \tag{2.6}$$

(3) For a longitudinal cut

$$R_w = \frac{\rho g}{2\pi} \int_{k_0}^{\infty} (LS^2 + LC^2) \frac{k_0}{w} \sqrt{1 - \frac{k_0^2}{w^2}} \; dw. \qquad (2.7)$$

In the above the variable k is interpreted, according to Sharma, as the circular wave number of a component plane wave and u and w are the induced transverse and longitudinal wave numbers. Thus from (2.5), (2.6) and (2.7), one may say that this method is based on the breakdown of observed waves by amplitudes with respect to wave numbers.

This method does present an alternative to the Taniguchi method and should be tried out for the sake of comparison on the same experiment data.

C. Gadd and Hogben's Method.

From the momentum or energy balance, the wave resistance of a ship moving in an ideal incompressible fluid is given by

$$R_w = \frac{\rho g}{2} \int_{-\frac{b}{2}}^{\frac{b}{2}} \zeta^2(x_0, y)\, dy + \frac{1}{2} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{0}^{-h} (v^2 + w^2 - u^2)_{x = x_0}\, dz\, dy \qquad (3.1)$$

where b and h are the breadth and the depth of the rectangular tank, respectively, and x_0 is the location of the transverse cut.

This equation is exact, but the velocity $\bar{q}$ (u,v,w) cannot be measured in a real flow and even if one had a potential flow, the experiment is very much involved.

The authors suggest that the potential velocity $\bar{q}$ be calculated from the observed waves by following two methods.

(1) Weighted Mean Square Method

At the centerline, v = o and the mean-square spatial average of $(w^2 - u^2)$ is equal to zero at every depth. Hence

$$R_w = \frac{1}{2}\rho g \int_{-\frac{b}{2}}^{\frac{b}{2}} \bar{\zeta}^2 \, dy \qquad (3.2)$$

where $\bar{\zeta}^2$ is an average value, taken along the x-direction in a length greater than $\frac{2\pi U^2}{g}$.

Similarly for an oblique wave of angle θ (θ aligned with how).

$$R_w = (1 + \sin^2\theta)\frac{\rho g}{2}\int_{-\frac{b}{2}}^{\frac{b}{2}} \bar{\zeta}^2 \, dy \qquad (3.3)$$

and for a general wave pattern, by introducing a factor η,

$$R_w = \eta \frac{\rho g}{2} \int_{-\frac{b}{2}}^{\frac{b}{2}} \bar{\zeta} \, dy \qquad (3.4)$$

η is a measure of the relative importance of the divergent and transverse waves and is estimated from Havelock's theory (See appendix of authors' paper in Reference (1)).

Equation (3.4) requires data in a band of region at least $\frac{2\pi U^2}{g}$ wide in the longitudinal direction and enclosing the trailing waves athwartships completely. Thus, either transverse cuts, longitudinal cuts or photographic method of taking data may be used. This method, however, does not offer component analysis and hence will not be considered further.

(2) Fourier Analysis Method

The essentials of this method is to assume a Fourier series to a longitudinal wave profile of

$$z = f(x) \qquad (3.5)$$

as

$$f(x) = \frac{r_0}{2} + \sum_{n=1}^{\infty} r_n \cos\left(\frac{g}{U^2}R + \epsilon_n\right) \qquad (3.6)$$

where r_i = amplitude

$R = \sqrt{x^2+y^2}$, $x = R\cos\theta$ & $y = R\sin\theta$

ϵ_n = phase angle.

The energy is considered for each Fourier component, from which one can show the wave resistance to be

$$R_w = \frac{\rho g}{4}\int_{-\frac{b}{2}}^{\frac{b}{2}}\left[\sum_{n=1}^{\infty} r_n^2\,(2-\cos^2\theta)\right]dy \qquad (3.7)$$

This method, then, requires complete areawise data on waves behind. Hoghen and Gadd used a series of wave height gages arranged in a manner of a large transverse comb each taking a longitudinal cut. Instrumentation is rather involved, but this method appears very attractive for wave component analysis, particularly because it takes into account all waves behind the ships or model.

D. Photogrammetrical Analysis Method (Inui and Kajitani's Method).

From the engery consideration, the resistance for a plane wave is

$$R_w = \frac{1}{4}\rho g\, a^2 \qquad (4.1)$$

where a = amplitude.

Owing to the asymptotic character, when such character is advantageously used, the wave elevation such as (1.2) can be considered only for a very narrow band of angle, i.e.,

$$\zeta(x,y) = \sum_{\theta^*} h\,(x,y,\theta^*) \cdot \text{(phase factor)} \tag{4.2}$$

and the wave resistance for three dimensional waves can be written as

$$R_w = \frac{1}{4}\rho g \int_S h^2(x,y,\theta^*) \cos\theta^* \, ds \tag{4.3}$$

where $h\,(x,y,\theta^*)$ is visual amplitude progressing in θ^* direction.

The path of integration S is arbitrary, provided that the total path of integration include only once the whole band of directional elementary waves from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

Inui and Kajitani divide waves into transverse and diverging waves and select the path S along wave crest lines. Since the path is complicated, Inui and Kajitani use "photogrammetrical" method, by which they mean the waves are analyzed for contours of elevations from stereo photographs, and the paths and elevation taken afterwards.

Due to complicated instrumentation and volumous data required, and because other simpler methods are available, this approach does not appear attractive, although there remains one great benefit in that a person can, to quote Inui and Kajitani, "see and feel" the waves.

E. Newman's Method.

An alternate form of (1.1) is

$$R_w = \frac{16\rho K_0^2}{\pi}\int_0^{\frac{\pi}{2}} \left[P^2(\theta) + Q^2(\theta)\right] \sec^3\theta \, d\theta \tag{5.1}$$

Newman shows that by taking a Fourier transform of wave height along a longitudinal cut in the form

$$\int_{-\infty}^{\infty} \zeta(x, y_0)\, e^{iK_0 x}\, dx,$$

it can be shown that

$$P(\theta) + iQ(\theta) \simeq -U \sin\theta\cos\theta\, e^{iK_0 y_0 \sec^2\theta \sin\theta} \int_{-\infty}^{\infty} \zeta(x, y_0)\, e^{iK_0 x \sec\theta}\, dx \tag{5.2}$$

and

$$R_w = \frac{\rho g K_0}{\pi}\int_0^{\frac{\pi}{2}} \sin^2\theta \sec\theta \left| \int_{-\infty}^{\infty} \zeta(x, y_0)\, e^{iK_0 x \sec\theta}\, dx \right|^2 d\theta \tag{5.3}$$

In principle, this method up to this point is not different from that of Taniguchi's. The amplitude functions appearing between the absolute signs, and the wave components are considered with respect to wave propagation angle as before. Eq. (5.3), however, may be rewritten in a different form as

$$R_w = \frac{\rho g K_0}{\pi}\int_{-\infty}^{\infty} \zeta(x, y_0) \left[\int_{-\infty}^{\infty} \zeta(\xi, y_0)\, K\left[K_0(x-\xi)\right] d\xi \right] dx \tag{5.4}$$

where

$$K(z) = \int_0^{\frac{\pi}{2}} \sin^2\theta \sec\theta \cos(z \sec\theta)\, d\theta$$

In this form, the interpretation changes. The function K $[K_0(x-\xi)]$ is a kind of influence function for a unit wave height, which depends only upon a product of the wave number and the relative distance between the two points on the wave, that is, it depends only on the phase relationship. The integration of wave height with respect to ξ variable combined with Newman's K-function appearing under [ ] may thus be interpreted as a collective influcence of phase relationship of wave height ζ at a point by every other point on the cut. The wave profile as well as the K-function oscillate along the cut, and therefore a suitably selected wave profile would give a small contribution to the wave resistance.

It appears that this formulation still offers another method of wave analysis and can offer some beneficial result for the present objectives if phase relations are properly exploited.

F. Ward's Method.

Ward in his 1963 work considers a combination of a longitudinal and a transverse cut aft of the body. This would in a sense be a longitudinal cut method, since the contributions along the transverse cut is actually obtained from the longitudinal data and not measured directly.

From an engergy consideration, the wave resistance is given by

$$R_w = \rho g \int_{\substack{\text{transv}\\\text{cut}}} \zeta^2 dy - \rho \int_{-h}^{0} \int_{\substack{\text{transv}\\\text{cut}}} u^2 dy\, dz + 2\rho \int_{-h}^{0} \int_{\substack{\text{long't}\\\text{cut}}} uv\, dx\, dz \qquad (6.1)$$

The first term is due to potential and kinetic energy increase. The second and the third term are due to the work done by the waves in the region outside of control volume to the region within the control volume.

By writing the wave along the longitudinal cut as

$$\zeta = \sum_{i=1}^{n} \zeta_i = \sum_i A_i \cos\left[K_0 \sec^2\theta_i (x\cos\theta_i + y\sin\theta_i)\right] \qquad (6.2)$$

and expressing velocities in a similar manner Eq. (6.3) can be written as

$$R_w = \frac{1}{2}\rho g \int_{\text{trans cut}} \sum_{i=1}^{n} (2 - \cos^2\theta_1)\, \zeta_i^2\, dy + \int_{\text{long'l cut}} \sum_{i=1}^{n} \cos\theta_i \sin\theta_i\, \zeta_i^2\, dx. \qquad (6.3)$$

In measuring what the author calls "directed wave elevations", a suspended circular cylinder is placed stationary along the path of the model and X-Y components of force are measured, from which the elevation can be found via a suitable calibration.

This ingenious method has given some promising result in calculating total resistance, but for the purposes of wave component analysis, it has to the knowledge of the writers not been tried out. It appears that some technique towards this end can be developed from it but not until modifications in Eq. (6.3) have been made.

In summary, we can categorize as follows

1. According to the method of observations:

    a. Longitudinal cut method

b. Transverse cut method

c. Hybrid of above two

d. Streophotogrammetrical method.

Notably a part of Sharma's, Taniguchi's, Hogben and Gadd's, and in a certain sense also the contributions of Newman and Pien belong to the first category. Eggers is credited with the transverse cut method. Ward's X-Y method belongs to the third category and Inui and Kajitani have advocated the last method.

2. According to the Breakdown of Wave Components:

a. Amplitude functions in terms of angles of propagation

b. Amplitude functions in terms of wave numbers

c. Phase relationships.

Inui, Pien, Taniguchi and Ward's method belong to the first, Eggers and Sharm's to the second and Newman's to the third.

3. According to the Instrumentation:

a. Simple mechanical type

b. Resistance wire type

c. Capacitance type

d. Sonic probe type

e. Stereophoto type

f. A force measuring-cylinder type.

These categorizations, and the fact that such is necessary, emphasize the diversity in carrying out a wave component analysis. Often, however, a particular method may not be suitable to a ship designer for one reason or another.

We feel that, at least once, some of these be tried out systematically and consistently with an eye towards hull form improvement and in order that the applicability be established. This is particularly true of category 2.

DISCUSSIONS ON THE ERRORS, A COMPARISON

While it is difficult to give a direct comparison of theories, methods and instrumentations without first trying them out extensively, it was felt, nevertheless, that some aspects of such a comparison can be made now, namely,

(1) the simplicity in computation and measurement,

(2) the source of errors inherrent in a method, and

(3) evaluation of available instrumentation.

A. On the Measurement.

The longitudinal cut requires a single cut whereas other methods of wave measurement require a minimum of two cuts. Within the longitudinal cut method, there are several theories that can be applied and hence it offers a good chance to compare these theories. It has shortcomings in that, in a narrow tank, the data has to be trunkated rather prematurely. But one important advantage is that the wave gage is stationary with respect to the shore and therefore the error encountered in a moving wave wire is not present. Overall instrumentation is simplified, although in our case we already had a transversely travelling carriage mechanism and it would have been easy to obtain a transverse cut.

As an initial try, the longitudinal cut method was chosen but transverse and other cut should be tried out eventually.

B. On the Instrumentation.

A short inquiry indicated that wave wires provided for simple instrumentation. We have tried resistance type and capacitance type, and have finally chosen the latter as being the best suited for the present purpose. A commercially available dynamic capacitance measuring equipment has been procured resulting in a very economical instrumentation as well. Details are given below.

When more than one gage is used simultaneously, a cross coupling effect is noted due to the common water ground, a probelm which is yet to be resolved. Unless this cross coupling can be eliminated, our existing wave wire instrumentation can not be used to obtain the wave elevation along two cuts in one test run.

C. On the Errors.

1. "Universal" Error.

Sources of errors are many. Some are inherent in the assumption of the basic theory of potential flow and some are closely related to the difficulties arising in measurement and computation. As already pointed out all theories neglect the existence of viscosity. If we define the wave resistance of a body moving on the free surface of a viscous fluid to be based on the observed waves, this would include part of viscous effect since these waves have been influenced by the viscosity. On the other hand, if we define the wave resistance to be that predicted by the potential flow theory, we will never be able to measure this resistance exactly, as no flow in the nature is ever a potential flow.

In solving potential problem, the free surface boundary condition is linearized assuming that the waves are small. As a recent study for a two-dimensional foil indicates [5], the flow may be highly non-linear even at a relatively low speed when the disturbance is near the free surface.

Contribution from near field wave system is assumed to have died out. Landweber [1] points out that the main component of local waves in a tank may remain for sometime and casts doubt on the possibility of eliminating such effect in a conventional tank. In this respect, intuitively, the transverse cut method appears to be better than the longitudinal cut method.

While the above three sources of errors should be looked into, there appears no possibility of eliminating such sources as long as the wave analysis method is to be adhered to. It must be remembered that these errors are "universal".

2. Trunkation Error.

The errors introduced by a trunkation of wave record aft should be equivalent in all longitudinal cut methods. Newman has given the error estimate and based on this and others, it was felt that a usable data can be obtained in The University of Michigan towing tank. Pien [1] says on this that the wave record should have the length of about ten times the distance from the probe to the model centerline. This gives about 30 to 60 feet of record in our tank. Our records, as it turned out, had about 35 or 50 feet of usable data.

Perhaps the transverse cut and hybrid cut method should be tried out mainly due to this source of error.

3. Fitting Error.

When a record taken in a tank is to be analyzed for amplitude functions, wave numbers or phase difference, a series representation of function is required of the wave cut profile.

In general this profile has the form of random noise, and of necessity the infinite series must furthermore be trunketed at a finite number of terms. This introduces errors. Newman's method and Eggers and Sharma's method are equivalent in this respect and considered to be simpler than Taniguchi's and Inui's. Taniguchi has taken twenty terms for his data originally and the curve fitting was at best acceptable. Either taking many more terms or less terms would not in general guarantee a better fit, and further, one will have to look at every case to make a judgement.

Sharma was much more successful in fitting curves. This process is, however, a very time consuming task.

4. Transient Error.

In a towing tank, the model must necessarily start from rest and usual assumption in dynamometer towing test is that the steady state has reached after a short run. When the record that has to be taken is long, this problem becomes more serious. Here Newman's results can be used for estimating the required length of the run. We have located the probe at about 230 ft. down the tank from the start of the run and have assumed this is satisfactory. Nothing can be done to improve the conditions here unless the tank is lengthened.

5. Computational Difficulty.

Numerical integrations require approximate quadratures of one kind or another, and solving a set of simultaneious equations introduces an error. Although the errors involved in approximate computations here can readily be assessed, it is evidently true that the greater the number of numerical integration the more critical accuracy will be. Eggers and Sharma's formulations call for double numerical integration, Newman's three-fold integration and Taniguchi's two-fold plus a set of simultaneious equations. One of Newman's integration can be tabulated for a single variable. Taniguchi's functions require two parameters and are rather lengthy.

In conclusion, it is apparent that some of the errors are inherent and cannot be eliminated or reduced. Longitudinal cut is certainly simpler in terms of instrumentation, and considering the computation difficulties and the fitting errors, Newman's and Sharma's methods on the longitudinal cut appear to be about comparable. In what follows, then, we shall describe the present investigation on these two methods.

SHARMA'S LONGITUDINAL CUT METHOD ON C-201

C-201 hull is a mathematical full form originally developed by Takahei [6] and a 12-foot model is readily available in our towing tank. The model has approximately 17.4 inch beam and is double ended with sine curve waterlines. All tests were conducted on the main hull at full displacement.

At this time only longitudinal cut method has been attempted. The wave wire probe is stationed at a location in the tank and the model passes by in a straight course. A time dependent record is taken and with suitable calibration a steady record can be obtained.

The main reason for using Sharma's theory is, as explained earlier, to search for an alternate means of breaking down wave component since we are reasonably well aware of Taniguchi's method as to its merits and faults.

Four sets of systematic data were taken, namely:

(1) For various speeds at a set lateral distance of the probe on one side,

(2) For the same set but on the other side,

(3) For the same set but when the model and the probe has been moved close to the wall,

(4) For various lateral distances at a constant speed.

Of these, only the last set has been analyzed, since the data had to be read manually and punched on cards. (An analog digital device will be of great help here).

A photograph showing wave forms in set (4) is given in Figure 3. 22' - 5 5/8" marks indicate the longitudinal markers, and cuts are

3" apart starting from 13 5/8" lateral distance. All data are for a constant model speed of 6.5 ft./sec.

These have been analyzed by Sharma's formula (2.4) and (2.7) appearing on pages 6 and 7 of this report. The result is given in Figure 4 in terms of wave resistance coefficients. The Japanese data is taken from [4] and is shown in Figure 4. It is seen that the observed wave resistance falls short by about 50 percent. This amount of discrepancy is consistent with findings by other researchers. What bothers one here, however, is the fact that there is considerable scatter due to the location of the probe. The same type of scatter was noted earlier in Taniguchi's theory and was shown in Figure 2. This is being investigated further. Other researchers seem in the past to have simply averaged the result.

Figure 5 shows the computed amplitude functions analyzed in terms of wave numbers by (2.4). This method also appears to give considerable variation in amplitude functions depending on the cut. Although it is too early to tell, the result shows a predominant contribution from wave components with wave numbers between 10 and 12 for cosine component for cuts 1-13. This result shows some encouragement, perhaps more so than the wave propagation angle method, i.e. Taniguchi's method.

The study has not been carried out beyond this point at present, but some results will be available from other sets of data shortly.

NEWMAN'S LONGITUDINAL CUT METHOD ON C-201

The data as in Sharma's method are used for this case, but using (5.4). The function K (0) has a peak and the form of (5.4) makes it extremely difficult to carry through the first indicated numerical integration. In order to avoid this, we put

$$R_w = \frac{\rho g K_0}{\pi}\int_{-\infty}^{\infty} \zeta(x,y_0)\int_{-\infty}^{\infty} \zeta(\xi,y_0)\, K\left[K_0(x-\xi)\right] d\xi\, dx \qquad (1)$$

$$= \frac{\rho g K_0}{\pi}\int_{-\infty}^{\infty} \zeta(x,y_0)\left\{\int_{-\infty}^{\infty}\left[\zeta(\xi,y_0)-\zeta(x,y_0)\right] K\left[K_0(x-\xi)\right] d\xi\right.$$

$$\left. + \int_{-\infty}^{\infty} \zeta(x,y_0)\, K\left[K_0(x-\xi)\right] d\xi\right\} dx \qquad (2)$$

$$= \frac{\rho g K_0}{\pi}\int_{-\infty}^{\infty} \zeta(x,y_0)\left\{\int_{-\infty}^{\infty}\left[\zeta(\xi,y_0)-\zeta(x,y_0)\right] K\left[K_0(x-\xi)\right] d\xi\right.$$

$$\left. + \zeta(x,y_0)\left[\int_{-\infty}^{\infty} K\left[K_0(x-\xi)\right] d\xi\right]\right\} dx \qquad (3)$$

The second part in the above has a peak, but can be shown to be independent of x and ξ by a substitution, i.e.

$$\int_{-\infty}^{\infty} K\left[K_0(x-\xi)\right] d\xi = \frac{2.0}{K_0}\int_{0}^{\infty} K(z)\, dz \qquad (4)$$

and the numerical integration in (4) can be done once and for all. The first part of (3) is now free from a singularity of the integrand.

Unfortunately the integral

$$\int_{-\infty}^{\xi\to\infty}\left[\zeta(\xi,y_0)-\zeta(x,y_0)\right] K\left[K_0(x-\xi)\right] d\xi$$

was found to converge very slowly. We are currently working on the possibility of finding a remedy for this.

CAPACITANCE WAVE HEIGHT GAGE

The principle of wave height gage using the capacitance change in an insulating wire piercing the water surface is well explained in a paper such as

W. S. Campbell, An Electronic Wave-Height Measuring Apparatus, DTMB Report 859, 1953.

Instead of manufacturing a bridge carrier system, we were able to find a commercially available unit:

Wayne Kerr Autobalance Capacitance Bridge Model B-541.

This unit gave an excellent resolution. By a careful test, it was found that the dynamic response was somewhat low due primarily to the wetting of wire and the slow response property of the recording pen. The response was judged to be acceptable for five cycles or less per second.

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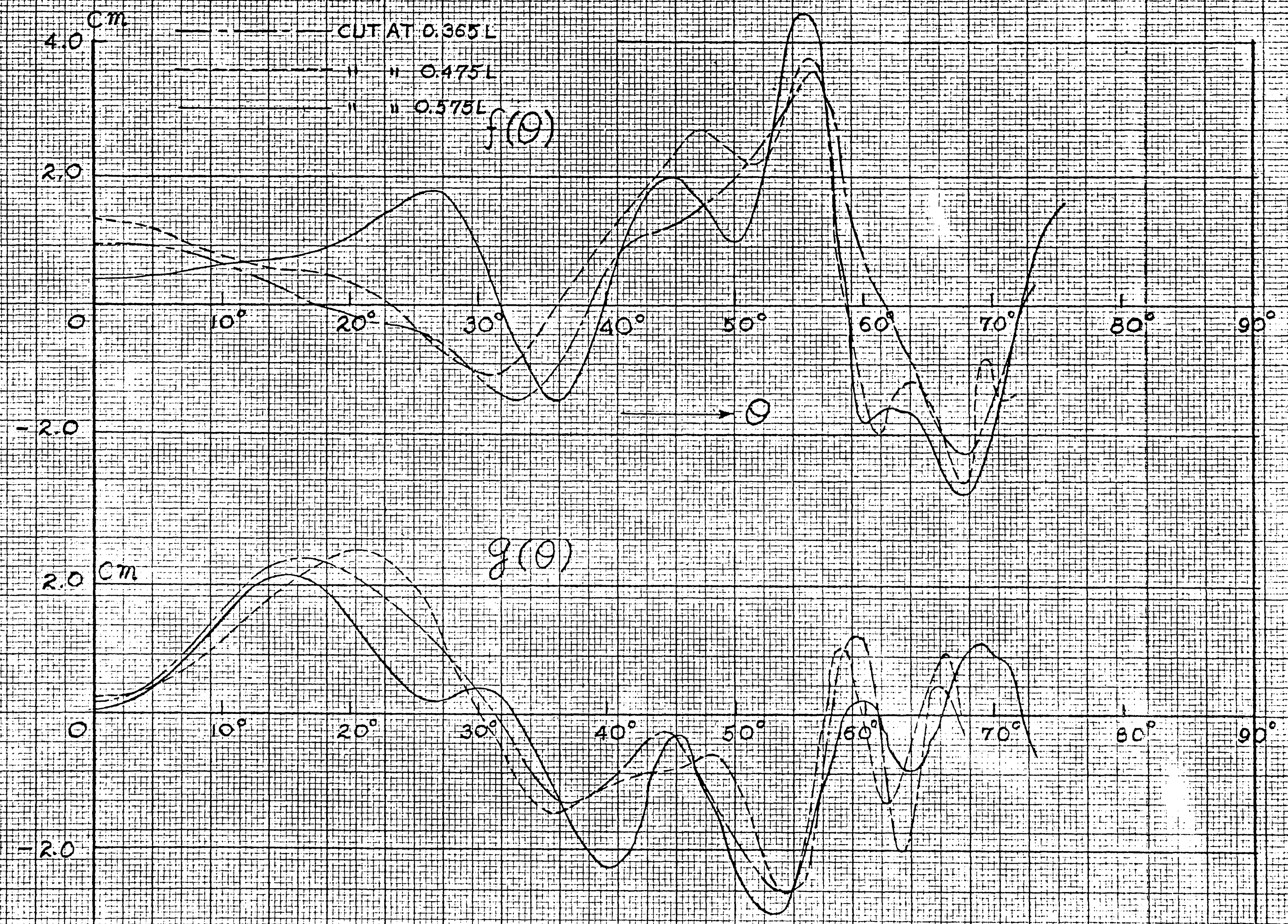

FIGURE 1 AMPITUDE FUNCTIONS ANALYZED FROM OBSERVED WAVES FOR THREE LONGITUDINAL CUTS FOR A CONVENSIONAL SHIP. (REFERENCE 2)

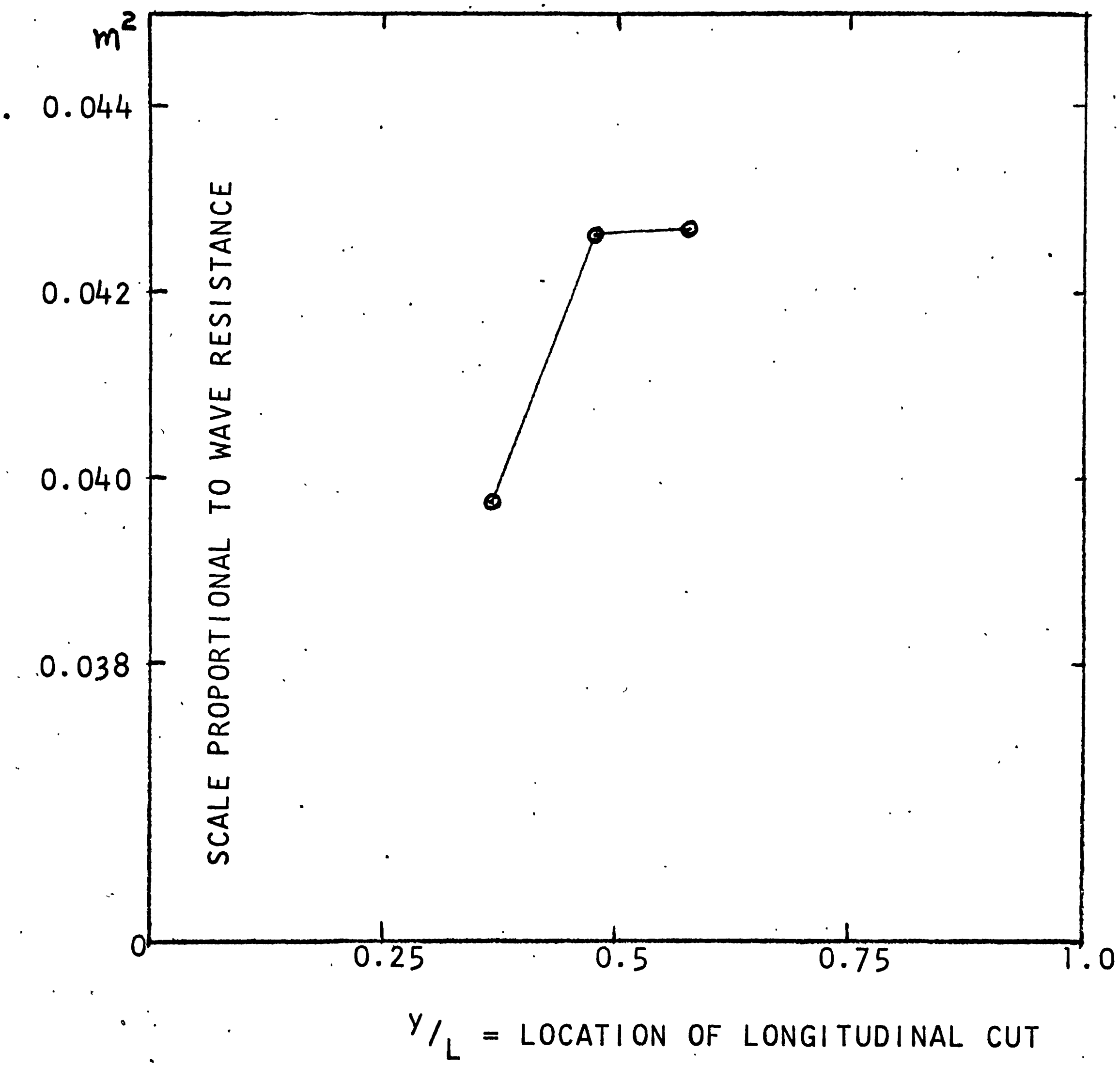

Figure 2 Variation of Wave Resistance Due to the Location of a Cut. (See Figure 1)

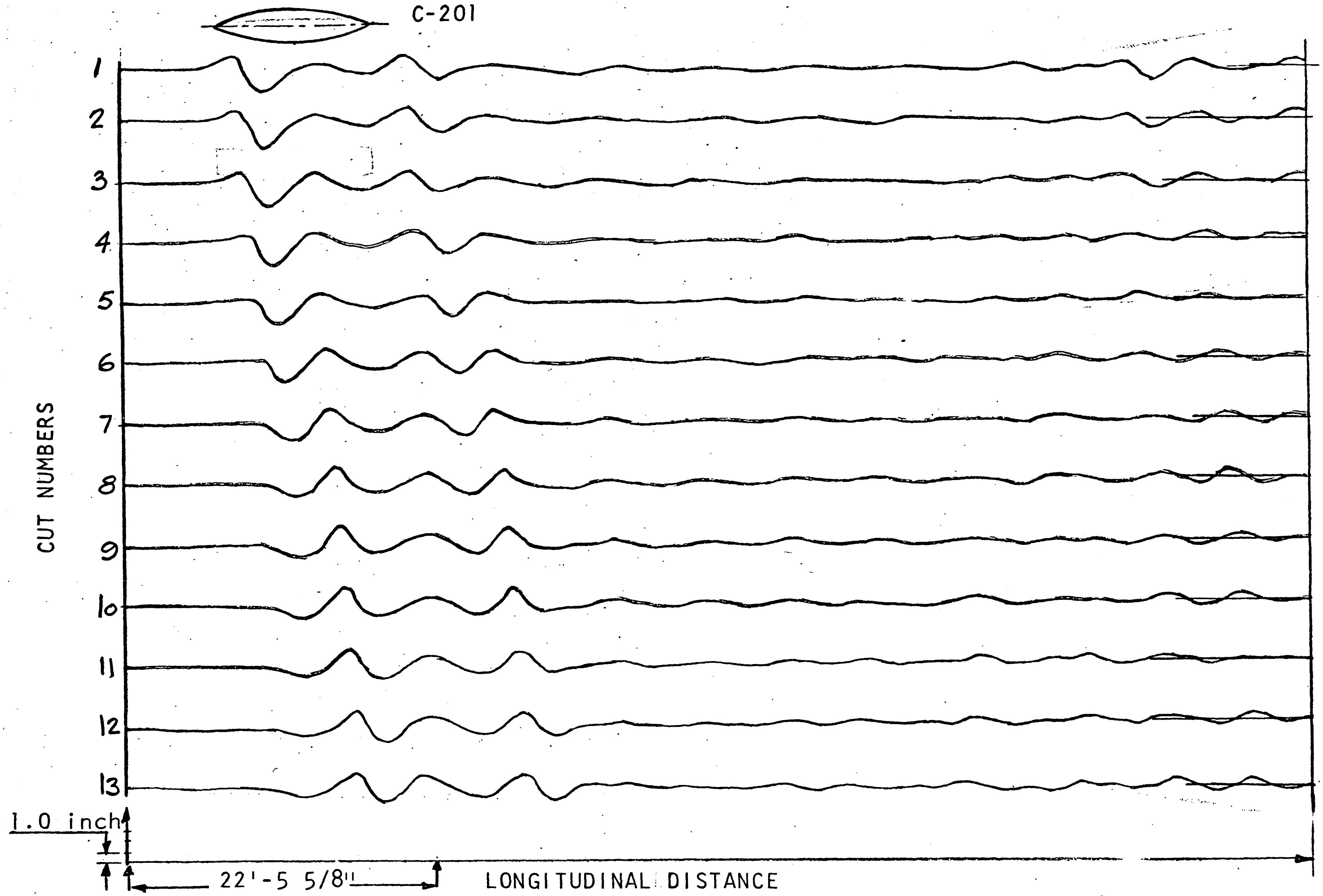

Figure 3 Trace from Phogograph Showing Wave Profiles for Various Lateral Positions of Longitudinal Cuts for C-201 Model at a Constant Speed of 6.5 ft/sec. (Top record is near the Hull. Approximate size and location of the Model is shown on upper left corner).

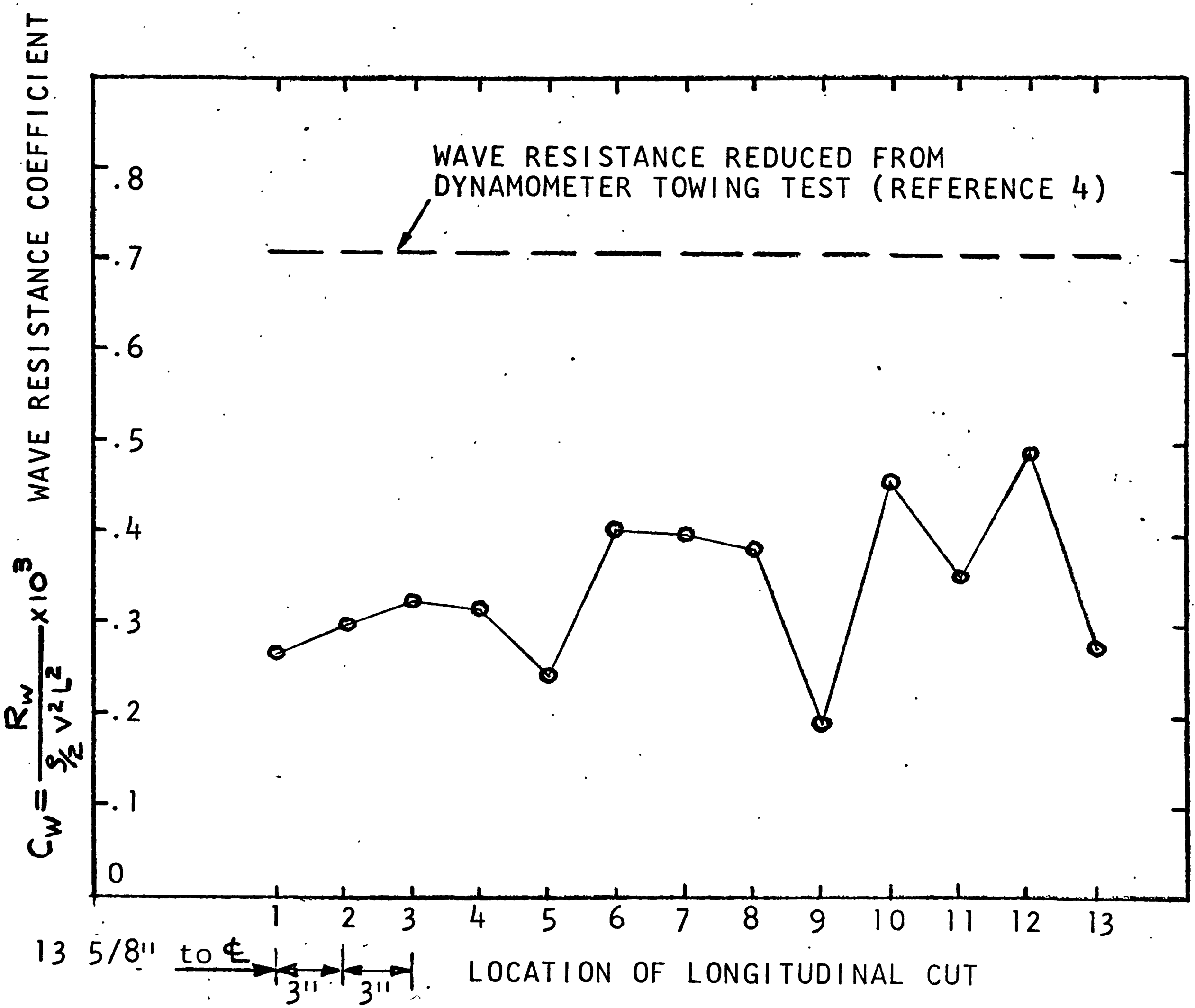

Figure 4 Wave Resistance from Observed Waves by Sharma's Method, C-201, Full Displacement v = 6.5 ft/sec.

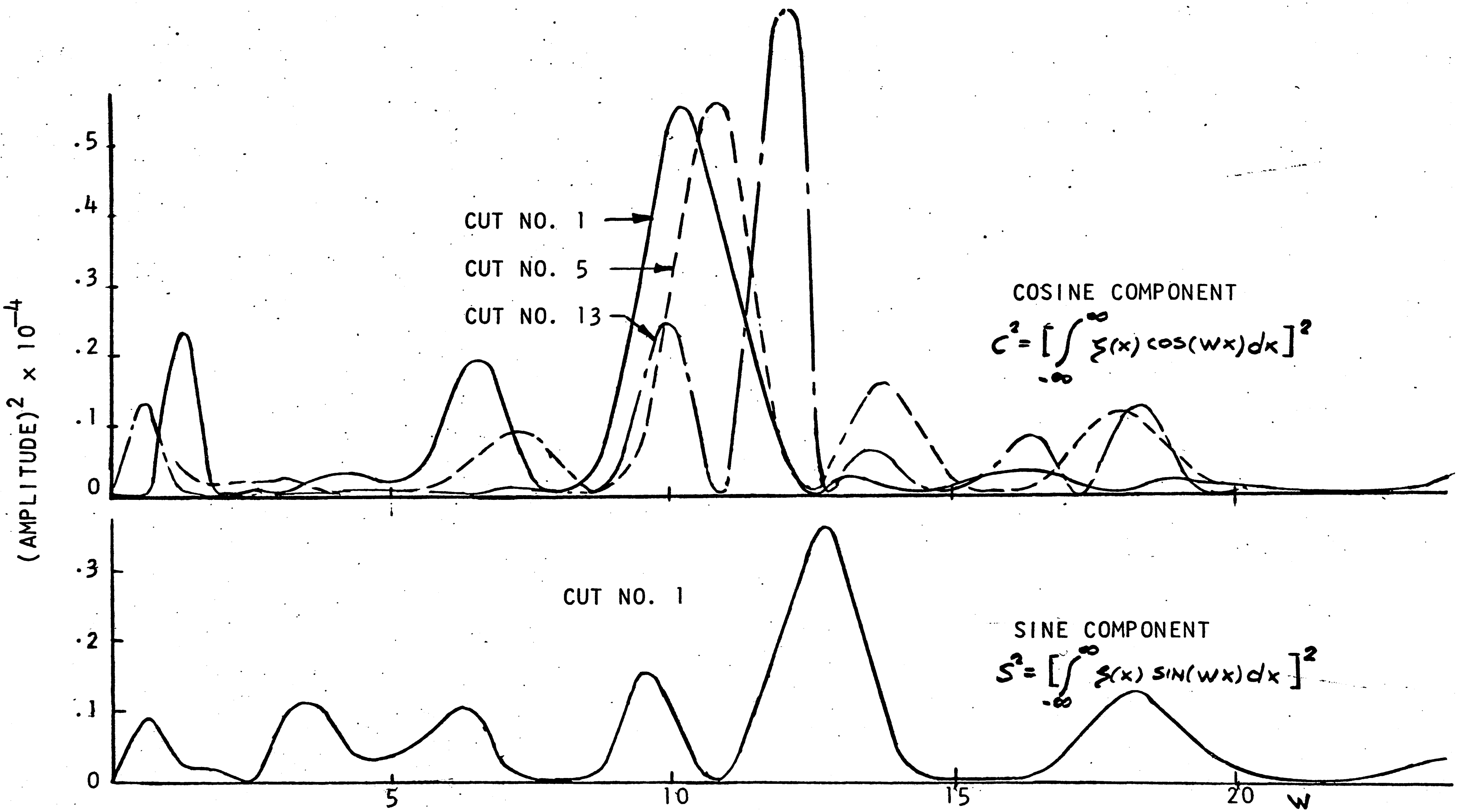

Figure 5 Amplitude Functions Analyzed from Observed Waves with Respect to Wave Numbers by Sharma's Theory on Model C-201 at a Speed v = 6.5 ft/sec.