Ross School of Business at the University of Michigan

Independent Study Project Report

TERM : **Fall 1998**

COURSE : **SMS 750**

PROFESSOR : **John Liechty**

STUDENT : **Won-Jae Lee**

TITLE : **Stability tests for identifying structural changes in the key performance measures**

Independent Study (1998)

Stability Tests for Identifying Structural Changes in the Key Performance Measures

November 6, 1998

Won-Jae Lee
University of Michigan Business School

This paper has been prepared under the supervision of Professor John Liechty

Evaluation of Won-Jae Lee's Research Paper

Completed for SMS 750; Fall 1998 Semester
Under the Direction of Dr. John Liechty

Mr. Lee's paper titled "Stability Tests for Identifying Structural Changes in the Key Performance Measures" is based on questions that arose during his summer internship at American Express.

In this paper Mr. Lee uses a collection of statistical models and tests to identify structural changes in time-series data.

The organization of the paper is appropriate. First he explains the project objectives. He then reviews several methods that can be used for testing if a structural change has occurred. Although his discussions are not technically detailed, the detail seems sufficient to demonstrate that he has a reasonable understanding of these methods and tests.

After reviewing these methods he shows how they can be used to detect changes in credit risk as measured by a credit reserve ratio.

In general Mr. Lee's work is complete and his writing style is understandable. It should be noted that English is not Mr. Lee's native language and that he had to do a considerable amount of work to make the text readable.

Executive Summary

When we use regression method to analyze the relationship between variables using time series data, it is important to check whether these data come from the same structure. If data from different time horizons have different underlying population, then the assumption of the regression will be violated, thus, resulting in unjustified estimations of parameters. In this report, I summarize a variety of methods checking structural change in the data and their relationships. Those include from simple tests such as t-test and F-test to more sophisticates methods of CUSUM test and other variable parameter models. Recently, new approaches using Bayesian inference, leading to dynamic model. I will cover all of these briefly.

In order to do empirical test, I use Credit Reserve ratio (CR ratio) of American Express Company, Corporate card division. In this data set, there can be no clear independent variables to explain the movement of CR-ratio, I depend more on time series analysis and simple test.

When I apply some applicable methods, t-test, f-test and Bayesian Time Series Analysis to check the structural changes in the CR ratio series, I can conclude that some significant change s has happened in the series.

I. Objectives

Generally, it is very important for policy makers to understand and verify that his policy changes have identifiable effect on his target. For this, they frequently use various types of statistical methods such as questionnaires, data analysis, etc. In the same way, it is also significant for risk managers to have a clear idea of the effectiveness of their policy change on the key risk related measures such as loss rate or CR-ratio, etc.

For this purpose, in Part II, I deal with some major theoretical advancement to address this issue, mainly by summarizing various statistical methods to assess the significance of the change of underlying regime, which I call structure. In this part, I can categorize those theories to static model and dynamic models. In the static world, the true coefficients are assumed to be constant within the same regime, while in dynamic models, the parameters are assumed to be variable all the times.

In Part III, I will focus on effectiveness of a series of policy changes made in American Express Risk Management Division ("Amex", hereinafter) since 1996, by testing their influences on CR-Rate. Its testing methods are based on the tests and models describe in the Part II, mainly by testing stability in parameters of the models, or figuring out some outliers in the time series models, which lead to conclusion.

II. Theoretical Background

1. Overview

From the viewpoint of standard linear regression models, identifying structural changes includes two meanings. Those include verifying the change itself, which means to check whether a change has happened or not, and checking the change points, at which point of time the change has happened, if happened.

In this case, if we have a prior information on the change point, then the task will be reduced to just to test the change itself. Pooled-variance t-test as explained in the sections below, Chow's F-test, and Tests using Dummy variables will be suitable for this purpose. Instead, if we do not have any prior information about the change point, we have to find out the change points firstly and then based on the point, we have to decide the significance of the structural change. For this purpose, more sophisticated models, such as CUSUM and variable parameters and Bayesian Analysis of Time Series ("BATS") are more appropriate. Because the assumption that we do not know change point is more realistic, the latter approach will be more desirable in real settings.

To pick up the change point in problem, We can assume two ways. One is that change has happened in a day, so the point will explain everything. The other assumption is that change is happened gradually and continuously over time. If we assume the former, that will be static models, whereas if we assume the latter, we have to depend on the dynamic models to figure out the structural change.

I want to refer to static models first, and then, in the latter part, dynamic models will be discussed Goodness-of-fit test, Quandt's Likelihood ratio test, CUSUM test can be said to be under static category. Three other models such as Random Coefficient Models, Adaptive Regression Models, and BATS are to be in dynamic world.

As a summary, I present the following table, which contains all above.

	Static Models	Dynamic Models
We Know *change point(s)*	-Pooled-Variance t-test(2)* -Chow's F-test(3) -Tests using Dummy(4) .Spline regression .Time Trending Regression	
We do not know it/them	-Goodness-of-fit test(ANOVA) -Likelihood ratio test(5) -CUSUM test(6)	- Random Coefficient Models(7) -Adaptive Regression Models(8) -BATS Model(9)

* Figures mean the section number it will be explained.

2. Pooled-variance t-test

This is the easiest and simplest way to test the mean difference between the two sets of data. Let $\mu_1 = \mu_2$ be the means of Group 1 and 2, respectively, under the assumption variance is equal, we can test whether $\mu_1 = \mu_2$. So,

$H_0: \mu_1 = \mu_2$.
$H_1: \mu_1 \neq \mu_2$.

For this test, we can use following statistic:

$$t = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where:

$$S_p^2 = \frac{(n_1-1)\, S_1^2 + (n_2-1)S_2^2}{(n_1-1) + (n_2-1)}$$

X_1, X_2 are sample means of group 1 and 2, S_1, S_2 are sample standard deviations of each group. And then, this t-value follows t-distribution with $n_1 + n_2 - 2$ degrees of freedom.

3. Chow's F-test

In the previous section, we did not use any form of regression equation or explanatory variables. However, in this section, we use regression equation as a base form. Chow, G.C., (1960) wrote a famous article on the stability test using F-distribution, which became the most important and classic testing method among several other tests using F-test, for the equality of two sets of coefficients in linear regression model. Another exposition of this procedure is Fisher (1970).

$$Y_1 = X_1 * \beta_1 + u_1 \qquad (1)$$
$$Y_2 = X_2 * \beta_2 + u_2 \qquad (2)$$

The null hypothesis(H_0) to be tested is that $\beta_1 = \beta_2 = \beta$. Under H_0, the above equations reduces to

$$Y = X * \beta + u \qquad (3)$$

where $Y = (Y_1, Y_2)'$, $X = (X_1, X_2)'$ and $u=(u_1,u_2)'$, then $E(uu') = \sigma_2 I$.

Let SSRi from each equation be the Sum of Squared Residuals of the regression i. The following F-test statistic

$$F = \frac{(SSR_3 - SSR_1 - SSR_2)/k}{(SSR_1 + SSR_2)/(n-2k)}$$

is known to be distributed as F(k,n-2k).

4. Tests using dummy variables

Tests using dummy variables are one of prevalent methods to detect structural changes. Compared with other methods, these dummy methods focus mainly on the stability of coefficients. Many kinds of variation may be possible, depending on how change patterns of coefficient are assumed. However, in this section, only spline regression and time trending regression will be dealt with.

<1> Spline Regression

When we know change point, t*, in advance, we can use spline function to regress. Let Period 1 be the period before t*, and Period 2 be that after t*; in each period,

Period 1:	$y_t = \alpha_1 + \beta_1 t + u_t$	$t < t^*$
Period 2	$y_t = \alpha_2 + \beta_2 t + u_t$	$t > t^*$

Then, we can integrate these two equations in one equation using dummy variable, D.

$$y_t = \alpha_1 + (\alpha_2 - \alpha_1)D + \beta_1 t + (\beta_2 - \beta_1)t^*D + u_t$$

In this case, test statistic is the t-value of coefficients, $(\alpha_2 - \alpha_1)$, $(\beta_2 - \beta_1)$ which include D and t*D, as dummy variables.

<2>Time trending Regression

This technique introduces time variation into the regression model explicitly by allowing the regression coefficients to become polynomials in time, i.e.

(0) $y_t = x_t'\beta_0 + \varepsilon_t$

(1) $y_t = x_t'(\beta_0 + \beta_1 t) + \varepsilon_t$

(2) $y_t = x_t'(\beta_0 + \beta_1 t + + \beta_2 t^2) + \varepsilon_t$

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(e) $y_t = x_t'(\beta_0 + \beta_1 t + \ldots + \beta_e t^e) + \varepsilon_t \quad t = 1,\ldots\ldots, T$

where x_t, β are vectors of 1,k, while y_t, ε_t are scalars, each.

Here, we can choose as a best solution t* which attains a maximum adjusted R^2.

5. Quandt's Log-likelihood Ratio test

In the previous tests, we assumed that we have a prior information on the change point. However, it is more common that we do not know anything about change point(s). From this section, I will discuss more general situation identifying the points, in which we do not assume a prior information on the point(s).

Chow's F-test is classical in testing hypothesis that change has happened, so much as is Quandt's Log-likelihood Ratio test in finding out change point(s).

The simplest case of switching regimes is based on the assumption of just two different regimes.

$$\begin{array}{ll} \text{Regime 1}: y_t = \alpha_1 + \beta_1 x_t + u_{1t} & \text{holds for } t < t^* \\ \text{Regime 2}: y_t = \alpha_2 + \beta_2 x_t + u_{2t} & \text{holds for } t > t^* \end{array}$$

where $t = 1, \ldots\ldots, n$

Then the log likelihood of these equations are

$$\ln L = - n/2^* \ln 2\pi - n/2 - t^*/2 \ln s_1^2 - (n-t^*)/2 \ln s_2^2$$

where s_1 and s_2 are the sample standard deviations of the regime 1 and 2, each. Choosing t* that maximize this likelihood is the first phase of the test. And then the null hypothesis (H_0) that no switch occurred may be examined by the likelihood ratio test. Let

$$\lambda = L(\omega)/L(\Omega)$$

where $L(\omega)$ is maximum likelihood value of the observation given H_0 (under restricted), and $L(\Omega)$ is maximum likelihood value of the observation given H_1 (under unrestricted). Then,

$$\lambda = \frac{\text{EXP}(- n/2^* \ln 2\pi - n/2 - n/2 * \ln s^2)}{\text{EXP}(- n/2^* \ln 2\pi - n/2\ t^*/2 \ln s_1^2 - (n - t^*)/2 * \ln s_2^2)}$$

$$\Rightarrow \quad \ln \lambda = t^*/2 \ln s_1^2 + (n - t^*)/2 * \ln s_2^2 - n/2 * \log s^2$$

where H_1 is the hypothesis that the observation in the time segments (1,..., r) and (r+1,..., T) comes from two different regressions. At minimum λr, we can identify the change point.

6. CUSUM test

Another important and influential paper on stability test is that of Brown, RL, Durbin, J. and Evans, JM(1975). They suggested using recursive residuals, Zr, instead of using OLS(ordinary least-squares) residuals, z_t, such that $Zr = \Sigma_1^r z_t/s$, $r = 1, \ldots, T$; where s is OLS estimate of variance σ.

The recursive residuals have a number of important applications, which include (1) the Chow's test of structural change, where the second sample contains fewer than k observations; (2) testing for heteroscadasticity, (3) auto-correlation, (4) a test of some possible forms of mis-specification. However, the most important application of this is in testing for structural change over time, i.e., BDE test, suggested by Brown, Durbin and Evans.

The authors suggest a pair of tests.
The first statistic, Wr, called CUSUM, is based on w^r.

Let H_0:

$$\beta_1 = \beta_2 = \ldots = \beta_T = \beta$$
$$\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_T^2 = \sigma^2$$

and let w_r be the standard prediction error of y_r when predicted from $y_1, \ldots y_{r-1}$, such that

$$w_r = \frac{y_r - x_r' b_{r-1}}{\sqrt{(1 + x_r'(X_{r-1}'X_{r-1})^{-1} x_r)}}$$

,where r= k+1 ,...., T,

Then, w_r follows $N(0, \sigma^2)$

For ease of calculation, we can use

$$w_r = \sqrt{(S_r - S_{r-1})}$$

, where $S_r = (y_r - X_r b_r)'(y_r - X_r b_r)$. And let

$$Wr = \frac{\Sigma_{k+1}^{r} w_j}{s}, \qquad \text{for } r = k+1, \ldots, T; \quad \text{where } s^2 = S_T/(T-k).$$

While the significance of the departure of Wr from the zero line may be assessed by two lines which pass through the points:

Upper limit: {k, $a\sqrt{(T-k)}$} and {T, $3a\sqrt{(T-k)}$}
Lower limit: {k, $-a\sqrt{(T-k)}$} and {T, $-3a\sqrt{(T-k)}$}

But, there is some evidence that the cusum test is less powerful than cusum of squares test. Therefore, the second test statistic is based on cumulative sums of the squared residuals, namely,

$$s_r = \frac{\Sigma_{k+1}^{r} w_j^2}{\Sigma_{k+1}^{T} w_j^2} = \frac{S_r}{S_T}, \qquad r = k+1, \ldots, T$$

This test uses the squared recursive residuals, w_r^2 and on H_0, s_r is known to have a beta distribution with mean (r-k)/(T-k). The criteria are given by the authors' article, however, I will use the table of "Significance values for c_0 in the cusum of squares test" in the Johnston's book for simplicity. Cusum test can also be applied to backward as well as forward recursive residuals.

7. Random Coefficient Models

In the above models from Section 2 to Section 6, we assumed that the parameters are fixed or constant in some time period, i.e., between change points. But in the models of the section 7, 8 and 9, we will assume that the parameters including coefficients are variable continuously in each time period, such that the latter models are called as 'dynamic models', while the former models are called as 'static models'. All the dynamic models use Generalized Least Squares (GLS) estimation.

We can think coefficient vector β is assumed to be stochastic, such that:

$$Y_j = x'_j\beta_j = x'_j(\beta + v_j) = x'_j\beta + x'_j v_j = x'_j\beta + u_j, \quad j = 1,\ldots\ldots,n \qquad (1)$$

where $u_j = x'_j v_j$ $x'_j = [1, x_{2j}, \ldots, x_{kj}]$ and $v'_j = [v_j\ v_j, \ldots, v_j]$.

With respect to v_j :

$E(v_j)=0$ $\qquad j = 1,\ldots, n$

$$E(v_j v_j') = \begin{bmatrix} \alpha_1,0,0,\ldots\ldots,0 \\ 0,\alpha_2,0,\ldots\ldots,0 \\ \ldots\ldots\ldots\ldots\ldots \\ 0,0,0,\ldots\ldots,\alpha_n \end{bmatrix} = A,$$

For j = 1, ..., n.

Since A is diagonal, var(u_j) simplifies to :

$$\sigma_j^2=E(u_j^2)=\Sigma_{i=1}^{k} x_{ij}^2\alpha_i$$

i.e.$\sigma^2 = X * \alpha$ where $\sigma^2 = [\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2]'$, $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K]$ and X* is obtained from X by squaring each element. Also, it can be shown that:

$$E(e^*) = (I-X(X'X)^{-1}X')^2 X * \alpha$$

, where e* is the square vector of residuals from OLS estimation of equation (1) Using the above logic, we can get α - estimate, σ_j^2-estimate,and thus, using the estimate of α and σ_j^2, we can get GLS estimate of β in equation (1).

8. Adaptive Regression Models

Cooley and Prescott, who mainly used Kalman filtering, developed this model.

For example, consider the relation

$$y_t = \alpha_t + \beta x_t + u_t$$

On this equation, they made the additional assumption that the coefficient (the intercept in this example) terms is subject to change according to $\alpha_t = \alpha_{t-1} + v_{t-1}$
Then $y_t = \alpha_{n+1} + \beta x_t + \omega$ where $\omega = u_t - \Sigma_{s=t}^{n} v_s$

Estimation of this equation is simplified by re-parameterizing the disturbance variances as

$$\sigma_u^2 = (1-\gamma)\sigma^2, \quad \sigma_v^2 = \gamma\sigma^2, \quad \text{where } 0 <= \gamma <= 1$$

Then, the variance matrix $E(\mathbf{ww'}) = \sigma^2\Omega$, where

$$\Omega(\gamma) = (1-\gamma)\begin{bmatrix} 1,0,0,........,0 \\ 0,1,0,........,0 \\ .... \\ 0,0,0,........,1 \end{bmatrix} + \gamma \begin{bmatrix} n-0, n-1, n-2, ...,3,2,1 \\ n-1, n-1, n-2, ...,3,2,1 \\ .... \\ 3,3,3, ...,3,2,1 \\ 2,2,2, ...,2,2,1 \\ 1,1,1, ...,1,1,1 \end{bmatrix}$$

We can get the optimal γ^* by maximizing following log likelihood function

$$\begin{aligned} \ln L &= -n/2 * \ln 2\pi - n/2 * \ln \sigma^2 - \ln|\Omega|/2 - (y-X\beta)'\Omega^{-1}(y-X\beta)/2\sigma^2 \\ &= \text{constant} - n/2 \ln s^2 - \ln|\Omega|/2 \end{aligned}$$

By maximizing ln L over γ, we can get γ^*, which then gives Ω^*.

The feasible GLS estimators are then

$$b_* = (X'\Omega^{*-1}X)^{-1} X'\Omega^{*-1}y \text{ and}$$
$$s^2 = (y - Xb_*)'\,\Omega^{*-1}(y - Xb_*)$$

This idea of adaptive coefficients can obviously be extended to slopes as well as intercepts as in the section 9.

9. Bayesian Dynamic Linear Model

Bayesian dynamic linear model is the most generalized approach in that the coefficient vector, θ_t follows the system equation, described as below. Let the original regression equation be observation equation and the movement of its coefficients matrix follows system equation:

$$\text{Observation Equation}: \quad y_t = F_t'\theta_t + \nu_t \qquad \nu_t \sim N[0, V_t]$$
$$\text{System Equation}: \quad \theta_t = G_t\theta_{t-1} + \omega_t \qquad \omega_t \sim N[0, W_t].$$

For simplicity, consider when k = 2, leading to straight line and simple regression.

$$y_t = \alpha_t + \beta_t X_t + \nu_t \qquad \nu_t \sim N[0, V]$$

$$\alpha_t = \alpha_{t-1} + \omega_{1t}$$
$$\beta_t = \beta_{t-1} + \omega_{2t} \qquad \omega_t = (\omega_{1t}, \omega_{2t})' \sim N[0,W].$$

In this model, we can decide structural changes by the frequency and size of outliers as well as the change of θ_t over time. Whereas t-test is very straightforward, Bayesian model requires some adjustments of the parameters in the program to best fit the model, in the sense that it should minimize the forecasting errors.

When I do test using this model, I use BATS software in the book [9] *Pole, Andy, West M. and Harrison Jeff, Applied Bayesian Forecasting and Time Series Analysis, 1994.* In using BATS, I assume that Observation covariance $\mathbf{V}_t$ is constant, while evolution covariance $\mathbf{W}_t$ are determined using the block discounting strategy.

In order to calculate them, I have to set values for Observation covariance. In that software, I can set the value by setting **component discount factors**. There are three discount factors in the model; trend, seasonal, variance discount factor. For example if the trend factor is 1, the trend is constant. Or if that is 0, the trend is completely a random variable. Thus, I set values of the three factors 0.98, 0.98, and 0.99 respectively.

III. Empirical tests

1. Test Methods:

As described in the Part I and Part II, there are several possibilities to check the structural changes of the dependent variables. In my report, I will use CR-ratio as dependent variable. However, I cannot find some meaningful independent variable from Amex data set, mainly from time limit. Thus, tests and models basically based on regression such as Chow-test, CUSUM test, are not tried. Even if there are no significant explanatory variables, time variable will an alternative, because data is time series data.

Thus, we have to depend only upon ①simple t-test and ②time series analysis. For the t-test, I used Excel/Tools/Data Analysis/ 't-Test: Two Samples Assuming Equal Variances'. I think that t-Test seems to be powerful and robust in some cases, even if it is restrictive in many cases. For the time series analysis, I used BATS (Bayesian Analysis of Time Series) software from the book by Pole, et al. [9]. In addition, I use ③regression method with time as independent variable, to try to use spline regression to determine time trend and structural change.

2. Data Sets

American Express has two layers of databases. One can be thought as primary data sources, maintained by Mainframe computer. This database includes every detail information about customer and their transactions. The other one is the secondary databases, mainly used for policy making and decision making for the upper level. One of these upper and secondary databases is DSNET. For DSNET, I selected four most important time series of CR(Credit Reserve) ratios. CR-rate is defined as follows:

> A ratio of the dollar amount going into a collection status (accounts typically enter collection sat 90 days past due) to the average of the past four months' billed charges. For example, if the CR rate for the current months is 1.15%, then for an average of $100 in billed charges in the last four months, $1.15 has enter collections. This is used as a leading indicator of losses or write-offs.

From products and categories available, Large Market Card, Middle Market Card are chosen; from categories available, Selective protection All Tenures, Selective protection Tenures 0-6 months, Expanded protection All Tenures, Expanded protection Tenures 0-6 months are selected.

Amex (Corporate card division) divides their market by the size of card members in their corporate clients, and protection type. If the card-members in a company is over 250, the business is classified as large market, while less than 250, its management belongs to Middle market. As well, the type of protection divides their markets. When Company and its holders share the default risk of specific cards, they call the protection as "Selective Protection". Otherwise if the default risks of the cards and their members are solely taken by the cardholders, they call it as Expanded Protection, The time period is monthly from Jan. 1996 to July 1998.

Using these categories, I selected 8 major and most frequent followed time series as follows:

Those are:

3.LEO	Large market card, expended protection, Tenure = 0-6 months
4.LEA	Large market card, expended protection, Tenure = All
5.LSO	Large market card, selective protection, Tenure = 0-6 months
5.LSA	Large market card, selective protection, Tenure = All.
6.MEO	Middle market card, expended protection, Tenure = 0-6 months
7.MEA	Middle market card, expended protection, Tenure = All
8.MSO	Middle market card, selective protection, Tenure = 0-6 months
8.MSA	Middle market card, selective protection, Tenure = All.

3. LEO

Before that, I would like to describe the content of policy change. From September 1996, The Company started using external information system on the card-members belonging to Expanded Protection for Paper Applications. Also, from October 1997, the use of external sources was extended to the cards of selective protections. For the unsigned applications, it is some legal problems to apply any external investigation with out the consensus of the card-members. But, from this August, they complemented the limitation of using external investigation on the unsigned applicants.

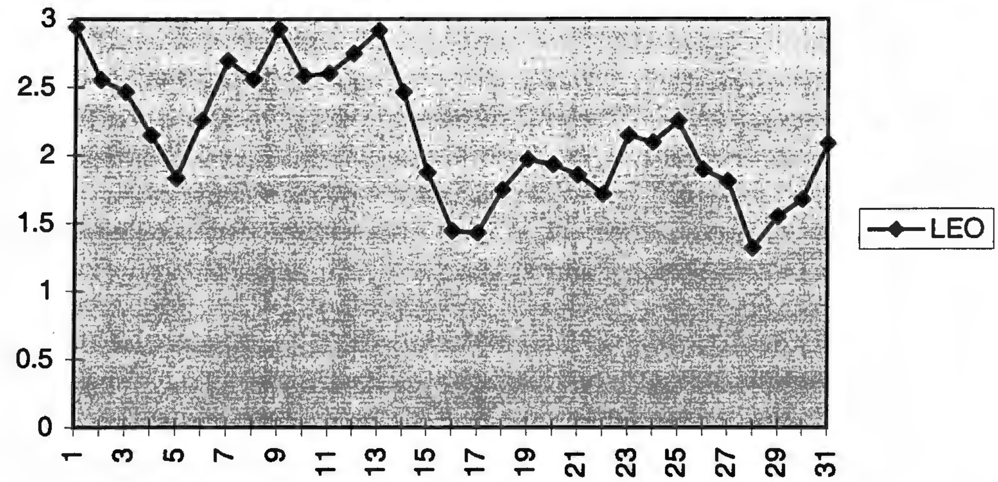
The content of policy changes:

	for Paper Applications	for Unsigned Applications
Expanded Protection	September, 1996	August, 1998
Selective Protection	October, 1997	August, 1998

So, we can expect the effect of the policy change is most effective on the low tenure and expanded protection, especially on LEO and MEO Series. I will apply ❶t-test, ❷Spline regreesion and ❸BATS to assess the structural change initiated by the policy change in Fall, 1996.

(Overall Description) Let us see the following graph. From March of 97, there seems to be downshift in trend. And there seems to be a consistent seasonal variation. As well the policy change in the 96's fall appears to be very effective in lowering the CR rate.

LEO



❶ t –test:

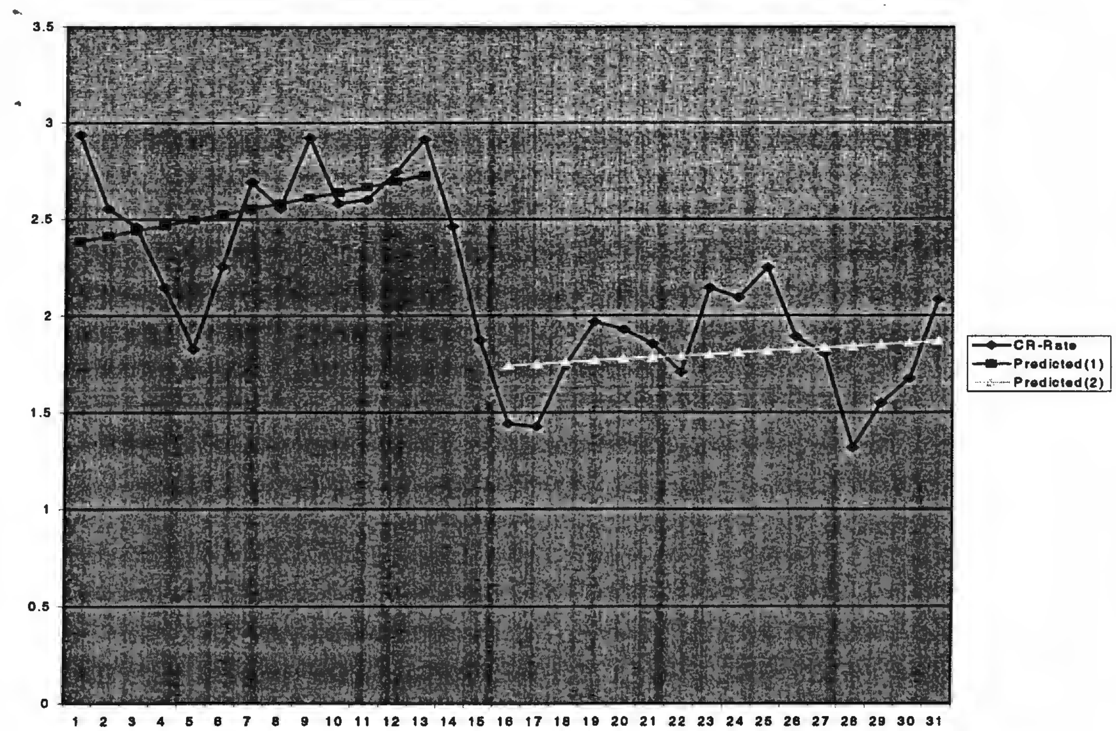
Using the Microsoft Excel™, the test T-statistic is 7.1 and p-value is below 0.01; thus there is a significant difference between the Period I and Period II.

t-Test: Two-Sample Assuming Equal Variances		
	Period 1	Period 2
Period	96.1-97.2.	97.3-98.7.
Mean	2.54705	1.809492
Variance	0.097341	0.071149
Observations	14	17
Pooled Variance	0.08289	
Hypothesized Mean Difference	0	
df	29	
t Stat	**7.098272**	
P(T<=t) one-tail	4.13E-08	
t Critical one-tail	1.699127	
P(T<=t) two-tail	8.25E-08	
t Critical two-tail	2.045231	

So, I can conclude that there is a significant difference between the Period I and II.

❷Spline regression

To spline the regime, I use a little different criteria on the splitting the regimes. Form Jan. 96 to Jan 97 belongs to Period I, while from Apr. 97 to July 98 is in Period II.(I deleted two months between Period I and II). The overall form are shown on the following graph.



And, the regression results of the Period I and Period II are as follows:

SUMMARY OUTPUT

Regression Statistic	*Period I*	Period II
Multiple R	0.34	0.15
R Square	0.12	0.02
Adjusted R Square	0.04	-0.05
Standard Error	0.32	0.28
Observations	13	16

ANOVA

	Period I	*Period II*
SSR/MSR	0.15/0.15	0.02/0.02
SSE/MSE	1.11/0.10	1.11/0.08
SST	1.26	1.13
F-Value	1.45	0.31
Significance of F	0.25	0.59

	Period I	*Period II*
Intercept Coefficient	**2.36**	**1.73**
T-Stat.	12.59	11.74
P-value	7.07E-08	1.23E-08
Time Coefficient	**0.028**	**0.008**
T-Stat.	1.2	0.55
P-value	0.25	0.59

From the above graph and Table, I can decide that a significant change has happened after policy change in Fall, 96.

❸BATS

I tried to check the structural change using BATS. But, since the software seems to have some problem in printing out the result, I cannot present graphs appropriate.

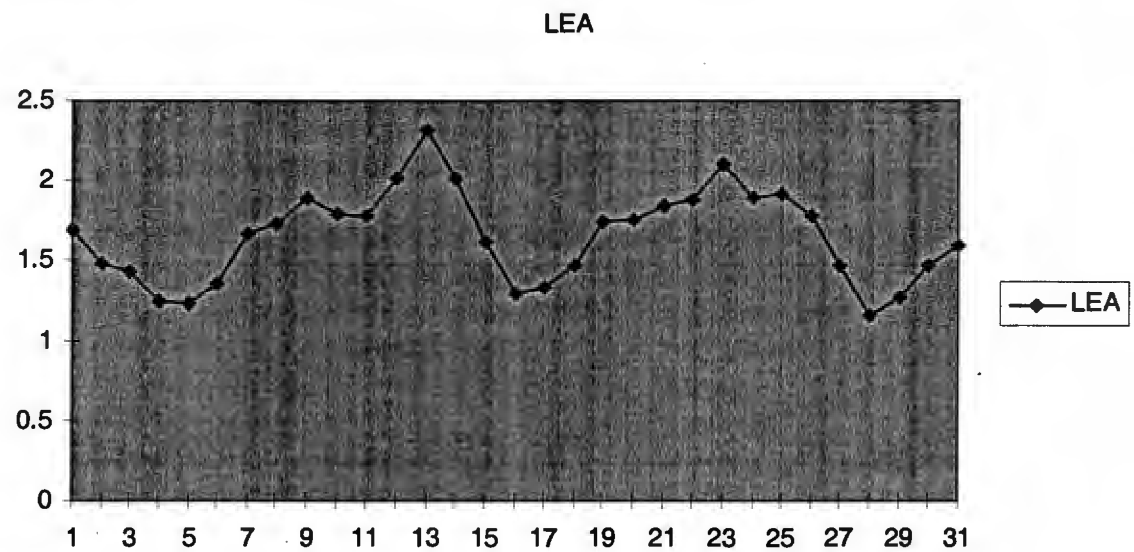
(Trend):

The series have no growth factor in the series. So, constant would be better.

(Seasonal):
There seems to have some consistent seasonal pattern. From Jan-to May it goes down. From June to September, it goes up again, In October, it goes down again, since then it goes up.

(Forecasting Power)
Because BATS has to learn the seasonal pattern, the predictive power of the first year is very poor. But, the retrospective forecasting power is reasonably good. In this case, more flexible discount (deep discount) has better MSE (Mean Squared Error).
From all of these method, test and models, I can conclude that in LEO series, some significant change has happened during the end of 1997m as a result of more tight screening of application.

4. <u>LEA</u>
(Overall description) From the below graph, we cannot see any increasing/decreasing trend nor any policy effect in the LEA.



❶ t –test:

From the below table, t- value is 0.62 and its p-value is 26.7% and 53%. Therefore, we cannot say that Period 1 and period 2 are different. In other words, I cannot reject the null hypothesis that there is no change in the series.

t-Test: Two-Sample Assuming Equal Variances		
	Period 1	*Period 2*
Period	*96.1-97.2*	*97.3-98.7*
Mean	1.68766	1.621641
Variance	0.099156	0.073215
Observations	14	17
Pooled Variance	0.084843	
Hypothesized Mean Difference	0	
df	29	
t Stat	**0.628011**	
P(T<=t) one-tail	0.267456	
t Critical one-tail	1.699127	
P(T<=t) two-tail	0.534911	
t Critical two-tail	2.045231	

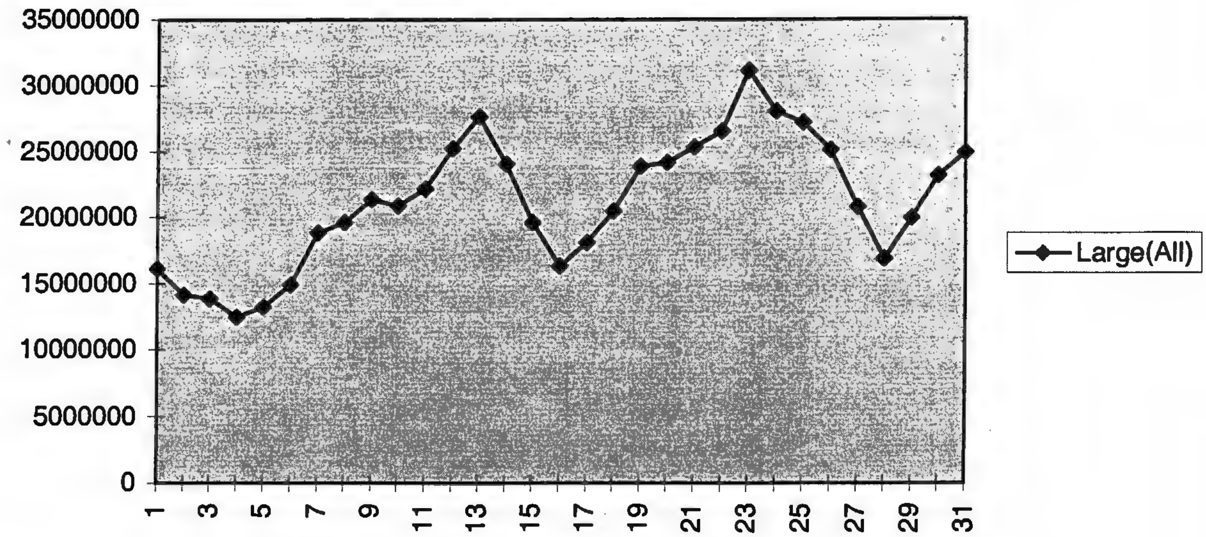
❸BATS
(Trend): There seems to be constant trend; no growth factor.

(Seasonal)
There may be strong seasonal variation in the series. It goes down from Jan to April/May. And then, it goes up again to the Jan. of the next year. There seems to have a consistent seasonal pattern. When I use 0.95,0.95,0.99 as trend, seasonal, variance discount factor, the value of Jan 1997 was an outlier.

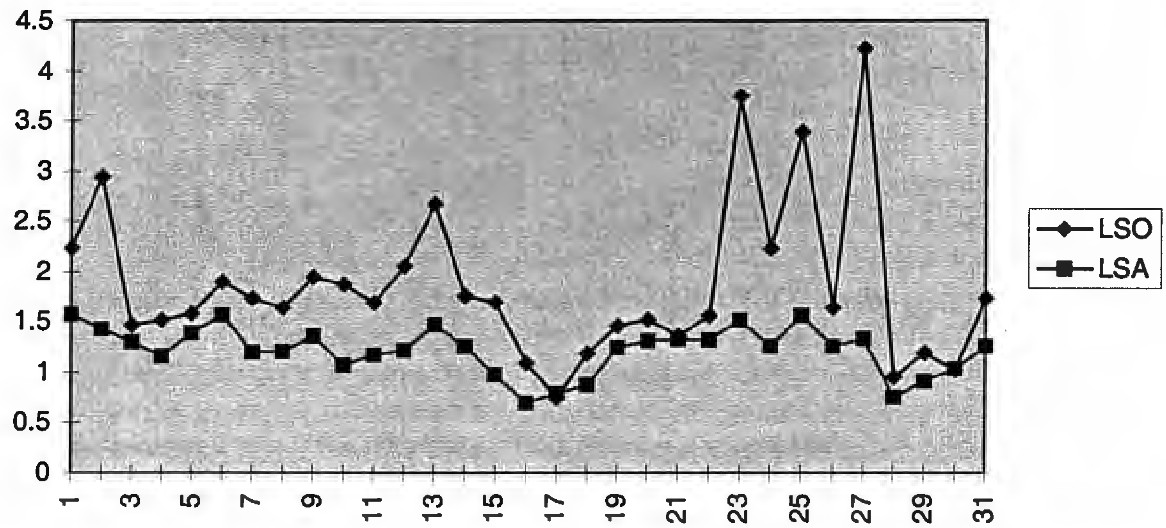
❹Comparisons between LEO and LEA:

When we compare LEO and LEA, the policy changes in 1996 had a strong effect on the low tenure of the expanded protection. But in the all tenure, there appears to ineffective. If the policy effect consistently lowers the CR rate in the young group, it will influence gradually as time goes on. However, we cannot find the relationship. There may be two possibilities. One is that the proportion of the young is so small to be influential to the whole group. And the other is that the policy may have some adverse effect on the older group.
The next graph represent the CR $ for LEA. For this graph we can know that there is very strong pattern in the seasonal variation the $ amount.

Large(All)



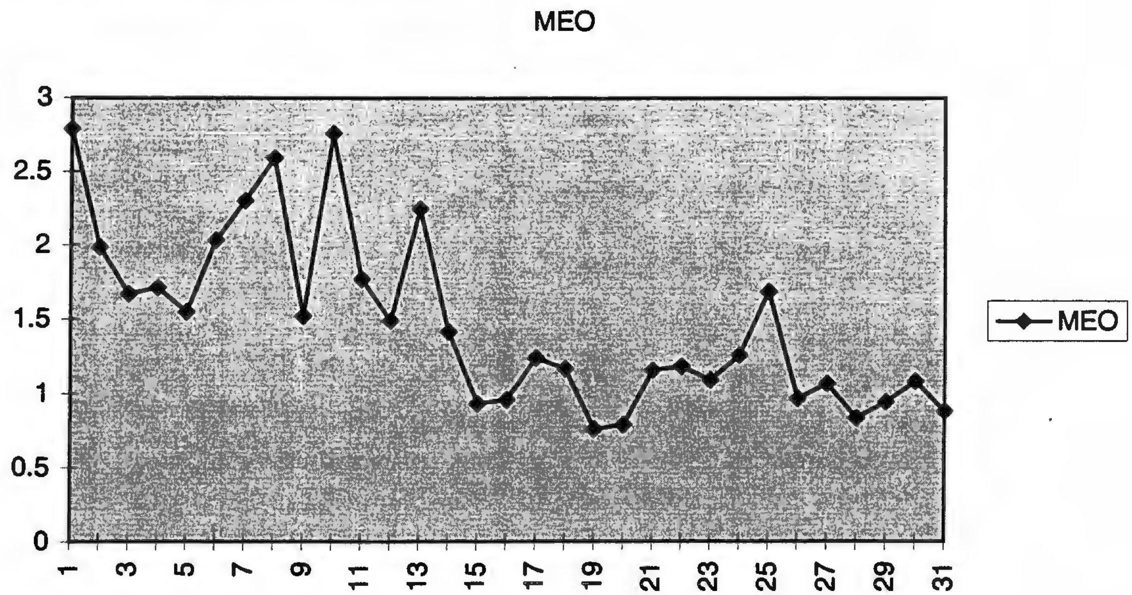
5. LSO & LSA



(Overall description)
In case of selective protection, there seem no structural changes in the series. All tenure has s stable and low rate, while 0-6 month shows very erratic behavior. I do not know why.
The five values from November of 97 to March of 98 are so high the average z-value is 2.86, thus we can say that some erratic variation has happened during the period.
To introduce some explanatory variable can be helpful to explain those variations.
(T-value) In both series, two t-values were not significant to assert the differences.

6. <u>MEO</u>

Now, let's turn to the middle market.



(Overall description)

Firstly, middle market & Expended protection and Low tenure: this series is assumed to be the most sensitive to the policy change. Agreeing with our prediction, among the eight time series, the most significant plummet has happened in this MEO from 1.99% to just 1.06%. The policy changes were the most effective in the category. Its seasonal pattern is also drastically changed, if the pattern in 1996 is typical one.

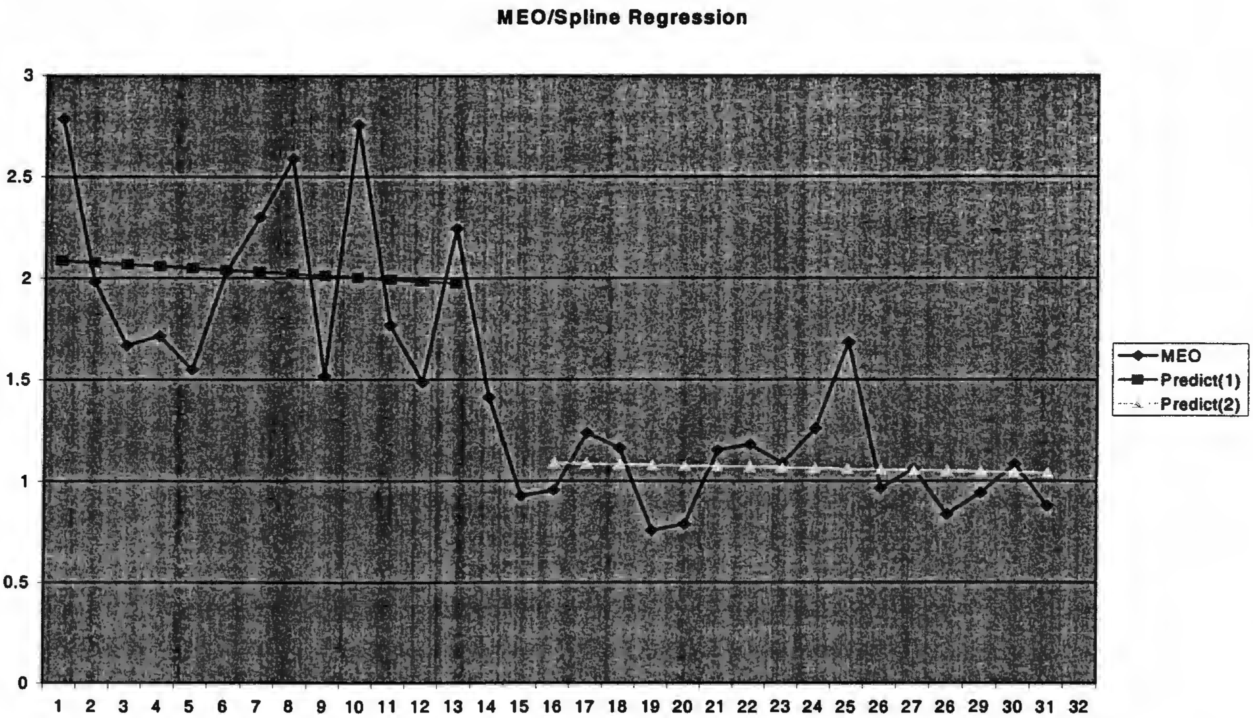
❶ t –test:

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Period	*96.1-97.2*	*97.3-98.7*
Mean	1.985924	1.056414
Variance	0.229008	0.049734
Observations	14	17
Pooled Variance	0.130098	
Hypothesized Mean Difference	0	
df	29	
t Stat	7.140445	
P(T<=t) one-tail	3.69E-08	
t Critical one-tail	1.699127	
P(T<=t) two-tail	7.38E-08	
t Critical two-tail	2.045231	

When we examine the t-value, we see that the difference is very outstanding, with t-value over 7.

❷Spline regression



Seen from the graph, the two regimes is significantly different from each other, leading to the effectiveness of policy change. Incidentally, the slope of left-hand regression is –0.009(t:-0.26), while that of right-hand is –0.003(t:0.25).

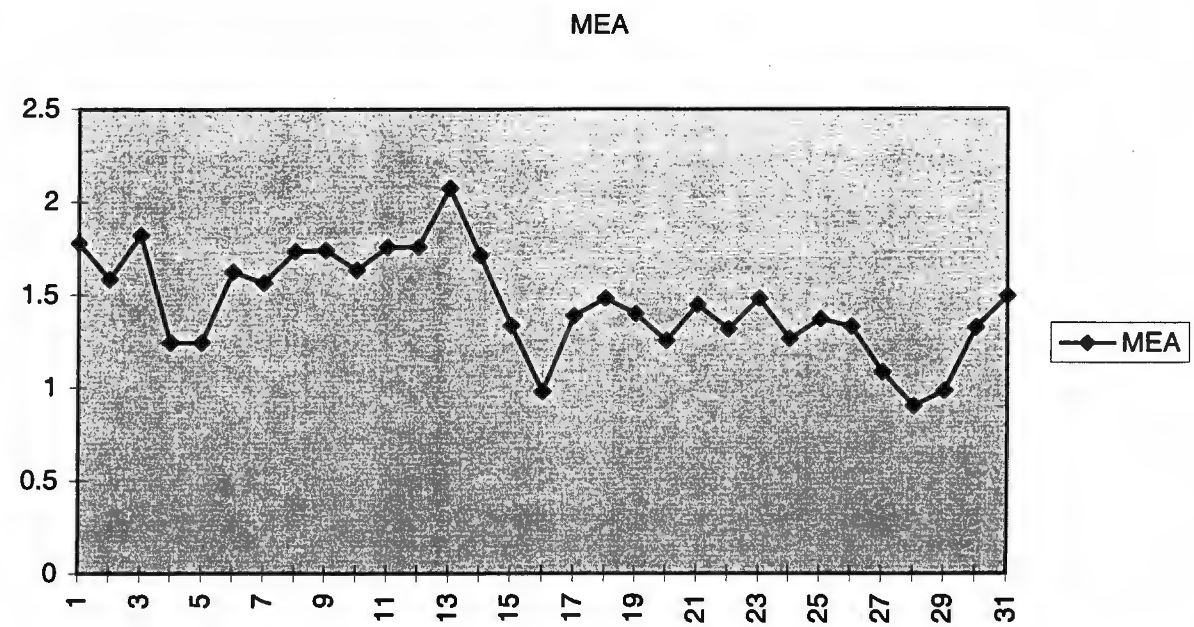
❸BATS

(Trend) : constant trend no growth

(Seasonal) : There maybe is no consistent seasonal pattern, except that January always shows high CR rate. Rather, the random portion of the variation excluding the trend seems to be most among the eight series.

7. MEA

(Overall description)
In case of all tenure, there has been a significant improvement on the CR-rate, even if the rate is not so much different as that of MEO. The more similarity of these two series, MEO and MEA may come from two factors: One is that the portion of young tenure is significantly higher than that of large market. The other possibility is that in middle market, the policy effectiveness can be observed from all tenures.



As we can know from the below table, tenure 0-6 months are below 20% and its weight are decreasing. But there is no much difference between the Large Market 0-6 tenure and Middle Market 0-6 tenure. So, the first possibility is not correct. The second hypothesis seems to be correct from below table.

Tenures	$ Weight(%)	Rate mean	Period 1 mean	Period 2 mean	Difference	t-value
0-6	14.7	1.48	1.99	1.06	0.93	7.14
7-12	25.4	2.26	2.73	1.86	0.87	5.37
13-18	17.9	1.96	2.19	1.78	0.41	2.04
19-24	12.6	1.73	1.78	1.68	0.1	0.78
25+	29.3	0.93	0.95	0.92	0.03	0.35
All	100	1.45	1.66	1.28	0.38	5.26

❶ t –test:

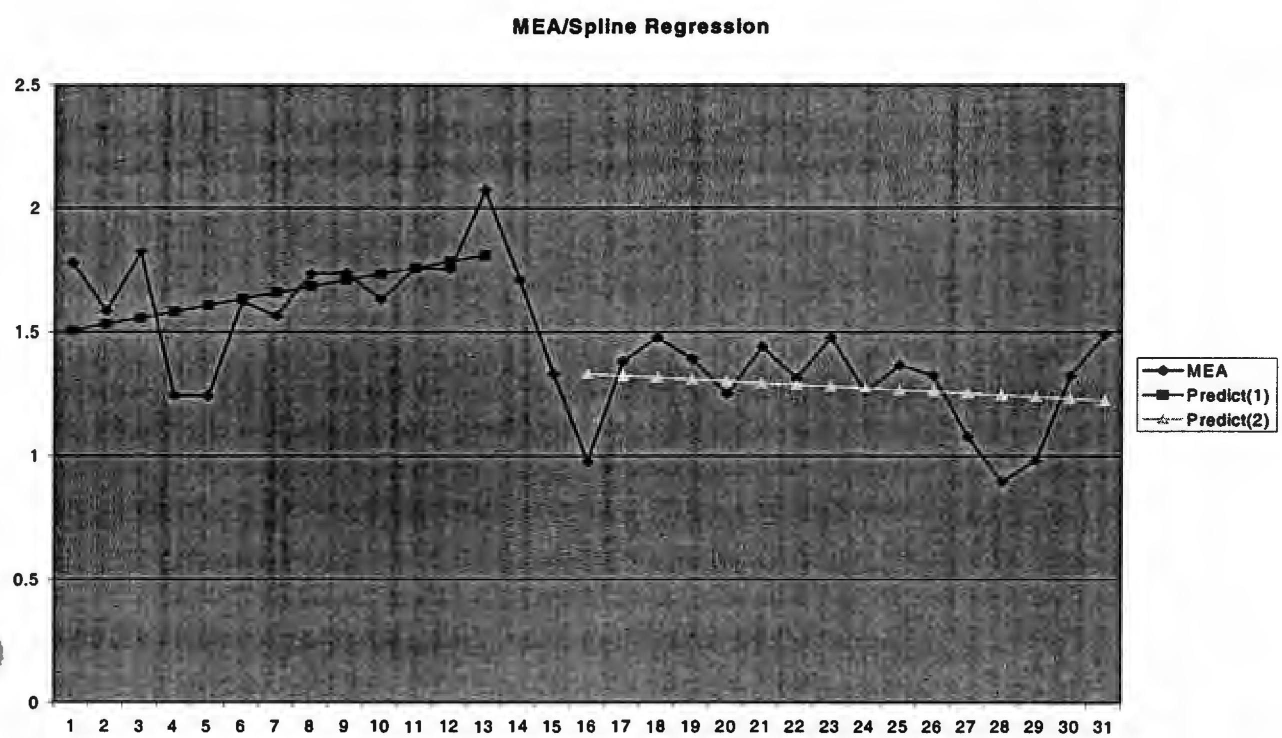
The t-value is also very high, implying the difference between two-time period.

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Period	*96.1-97.2*	*97.3-98.7*
Mean	1.6602	1.279382
Variance	0.046967	0.034826
Observations	14	17
Pooled Variance	0.040269	
Hypothesized Mean Difference	0	
df	29	
t Stat	5.258248	
P(T<=t) one-tail	6.19E-06	
t Critical one-tail	1.699127	
P(T<=t) two-tail	1.24E-05	
t Critical two-tail	2.045231	

❷Spline regression

Seen from the graph below, the two regimes is significantly different from each other, leading to the effectiveness of policy change. However, the interesting thing is that the slope of left-hand regression (Period I) goes up with the figure of 0.025(t: 1.62), whereas the slope after policy actions goes down with the number of –0.007(t:-0. 67).



❸BATS

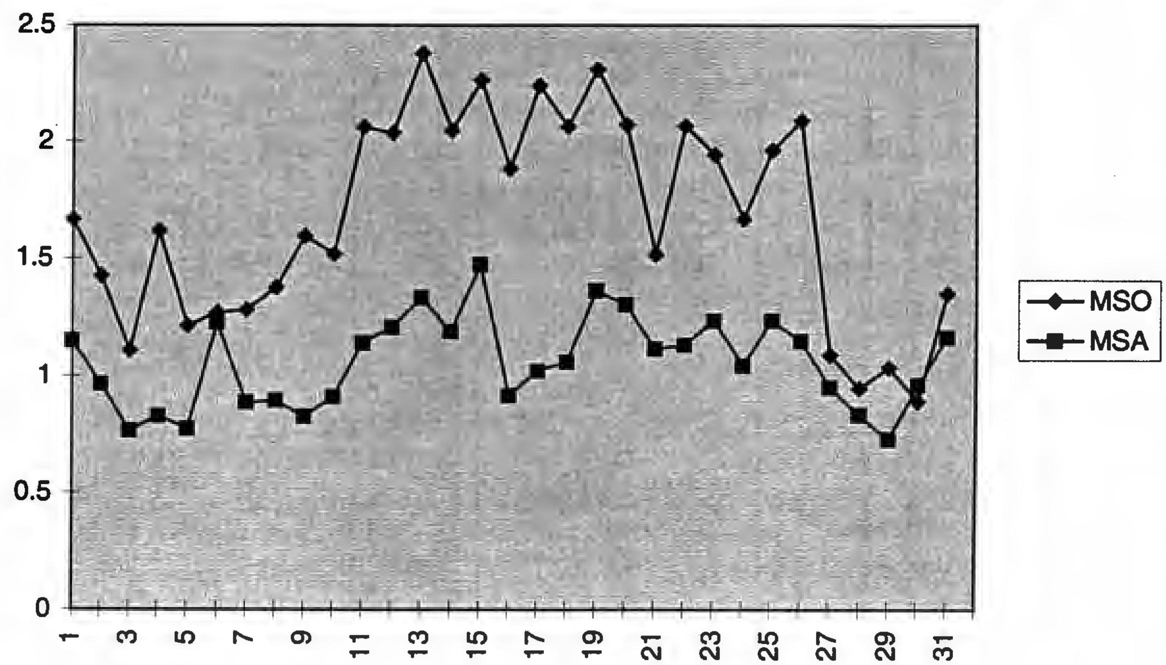
(Trend) I set 0.98, 0.98 and 0.99 as three discount values, respectively; while other parameter were set as default. In retrospective analysis, the trends slightly going down in the Period 1, however, it goes up again in the latter part.

(Seasonal) It shows consistent seasonal factors. Combined with the trend and seasonal variables, the MSE and Log-likelihood value was minimal, compared with other series.

8. MSO & MSA

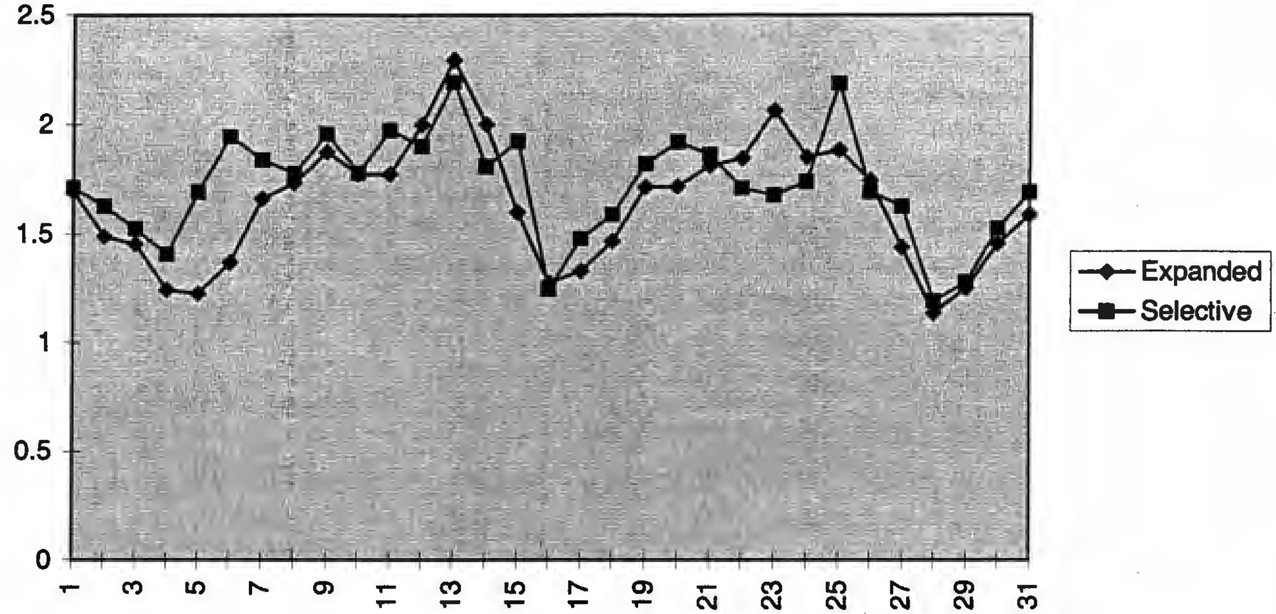
(Overall description)

As with large market, these series seems to have no structural change. And, the fluctuations are also erratic. In tenure 0-6, From January of 1997 to February of 1998, the CR rate is erratically high. In the middle market and in the selective protection category, new customer with tenure 0-6 months showed higher CR rate than before and after. To explain those change, we need more explanatory variables.



9. Empirical Results

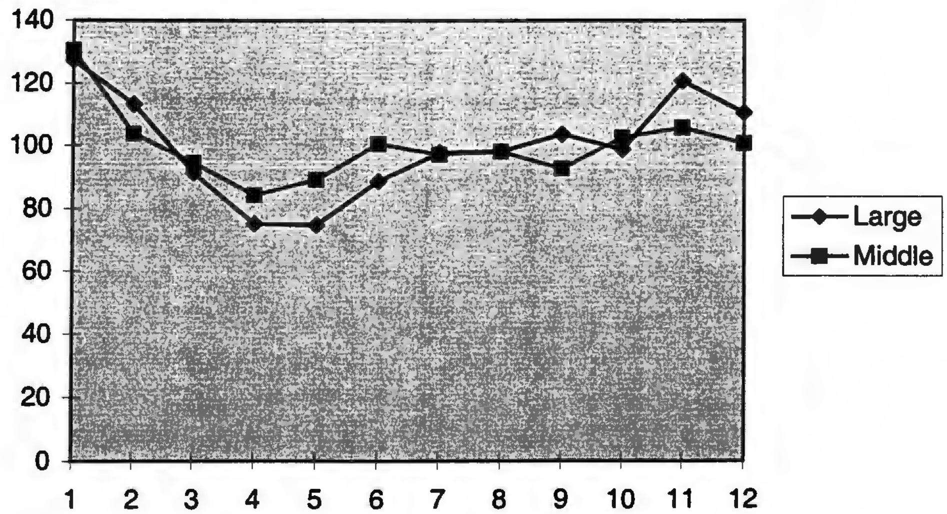
(1) The graph below comes from selective, all tenure for all corporate and expanded, all tenure for all corporate. From this graph, we can know that there seems to be no significant decrease in the CR rate during the whole period. However, there is a strong and systematic seasonal pattern in the series, with a peak in January and though point in April. There appears to be no difference between the selective and expanded. In overall sense, there seems no meaningful effect of the policy changes for these time period.



(2)Expanded vs. Selective Protection

I did t-test with the same time division, Jan.96-Feb.97 as Period I and Mar. 97 - Jul. 98 as Period 2. In expanded, the average CR rate decreased from 1.80 to 1.66 and its t-value is 1.64(p = 5%) while in selective protection, the average CR rate goes down from 1.96% to 1.60%, with its t-value is 0.85.
In other words, the policy changes in the fall of 96 have more deep impact on the expended protection than selective protection. The effectiveness of the 1998's policy is not clear from the data, because of data insufficiency.

(3)Seasonal Factor
This graph shows the average seasonal factors for the large market and middle market during 1996and 1997. Seasonal fluctuation of the middle market is milder than that of large market. Also, from this graph, in the first five months, it goes steeply down to the 75%(Large) and 85%(Middle) of the annual average. But after that point, it goes up. The reason why this kind of seasonal variation has happened is not so clear right now. However, two possibilities can be inferred. One is the effectiveness of policy changes initiated in the fall of 95 or 96. The other is reflecting pure cyclical characters such as credit behaviors or cash flow pattern of the company. It needs to be more studies.



IV. Conclusion

The goal of this work is to verify changes in a time series data using tests and models. However, there are no clear-cut methods for this purpose. Nevertheless, statistical approaches as explained in this report give us more convincing argument than just eye-view. The pros and cons of each method can, however, depend on the nature of the data. Thus, users have to decide which models or which test methods are the most suitable for their specific tasks.

In this study, even if I explained eight methods including static and dynamic approaches, I can apply only three methods in real data, t-test, spline regression and BATS. In this case, various methods I have used support that significant change ahs happened after policy change. The result can be summarized as follows. The younger the tenure, the more deep influence from the change. The expanded series are more influenced by the change than selective series. In this sense, MEO and LEO are the most sensitive series, which was my assumption before starting this study. However, the consistent seasonal fluctuation is the most remarkable features. To explain the seasonal factor, we need more deep understanding of the payment behavior of our clients.

Even though the effectiveness of checking structural change due to the policy change depends on the nature of data, situation. There are also something to be more refined. The first one is that I cannot use the functions of BATS with full sufficiency. If I can use printing facility more, the quality of this report could be enhanced. As the second one, It would be helpful to have more explanatory variables to explain the seasonal pattern. If I can detect some powerful explanatory variables, I could use more powerful methods such as CUSUM test and Quandt's test as well as BATS, which also supports regression function. Lastly, time period is a little shorter, to be fully check to change effectiveness, because in the All tenures, change effects will be shown only gradually. And to check the later change effectiveness, data has no enough information.

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