# Great Expectations, Greater Disappointment: Disappointment Aversion Preferences in General Equilibrium Asset Pricing Models

by

Stefanos Delikouras

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Business Administration)
in The University of Michigan
2013

Doctoral Committee:

Associate Professor Robert F. Dittmar, Chair
Professor Miles S. Kimball
Professor Reuven Lehavy
Associate Professor Paolo Pasquariello
Professor Tyler G. Shumway

*Σα βγεῐς στον πηγαιμό για την Ιθάκη,*
*να εύχεσαι να 'ναι μακρύς ο δρόμος,*
*γεμάτος περιπέτειες, γεμάτος γνώσεις.*
*Τους Λαιστρυγόνας και τους Κύκλωπας,*
*τον θυμωμένο* Ποσειδῶνα *μη φοβᾶσαι*

On your way to Ithaka, you should hope
that there lies a long journey ahead of you,
full of adventures, full of knowledge.
Of the Lestrygonians and the Cyclops,
of the angry Poseidon, have no fear

extract from the poem "Ithaca" by Constantine P. Cavafy (1863-1933)

Odysseus and the cyclops Polyphemus by Arnold Böcklin (1827-1902). 1896. Oil on panel.
75.5 x 148.5 cm. Private collection

To my beloved family, Nickolaos, Eleni and Eirini. I could have never come thus far without your unconditional encouragement, love, and support over the years.

To Lavrentia, my only reason for undertaking and completing this journey.

## ACKNOWLEDGEMENTS

This dissertation could not have been completed, had it not been for so many people. First and foremost, I am grateful to Robert Dittmar, my doctoral committee chair, whom I first met almost ten years ago when I was a student at the Financial Engineering program. His contribution to my Doctorate degree has been crucial long before this journey even began. Had it not been for his recommendation letter, it would have been difficult for me to get accepted into the Doctoral program at the Ross School of Business. Robert has been a catalyst in my academic development from a PhD student to a young scholar. Besides the numerous research opportunities he has given me through our co-authored papers, he has always taken a sincere interest on my research agenda, and has spent a lot of time providing me with valuable feedback on my research endeavors.

I am particularly grateful to Paolo Pasquariello, whom I also met while being a student at the Financial Engineering program, for his help and support. His advice has been critical in improving my presentation skills, and successfully navigating through the job market. Moreover, during my teaching semester, Paolo's suggestions were indispensable in assembling a novel curriculum for the undergraduate course in International Finance.

I would like to thank Tyler Shumway because his behavioral finance class inspired me to introduce elements of behavioral economics in my orthodox view of a rational world. I am also grateful to Francisco Palomino who taught me the basic tools for solving dynamic stochastic general equilibrium models with non-separable preferences. This dissertation is actually based on a term project for Francisco's special topics class. Finally, I owe Miles Kimball a huge part of my economics background, and I would like to thank him for teaching me how to communicate finance concepts to an economic audience.

I would also like to thank Sugato Battacharrya, Mattias Cattaneo, Joseph Conlon, Amy Dittmar, Lutz Kilian, Reuven Lehavy, NP Narayanan, Amiyatosh Purnandadam, Uday Rajan, and Lu Zhang for their time, attention, and research discussions during the last five years. My research profile would not be the same if it weren't for all these wonderful scholars who helped shape my academic personality. Finally, I would like to thank Achilleas Anastasopoulos, Samir Nurmohamed, Robert Smith, and Daniel Weagley for their support and advice. Special thanks to all the people who put together a nice LaTeX template for me to use for my dissertation, and to Alex Hsu for letting me know that such a template exists.

# PREFACE

Two years ago, during the weekly finance seminar at Ross, I was having lunch with a well-known scholar in the field of asset pricing. When I asked him whether it would be possible to introduce a stochastic discount factor based on a "habit model" with forward-looking reference levels, he immediately replied that such a model would be really hard to solve.

For the following year I was trying to wrap my head around the problem of introducing preferences with expectation-based reference levels into asset pricing models. Unfortunately, Gul had already done that back in 1991 when he introduced disappointment aversion preferences. Disappointment aversion relies on the simple and intuitive fact that people feel really sad whenever things turn out worse than expected. Although the theoretical framework for expectation-based utility functions was established more than twenty years ago, disappointment aversion preferences have been largely overlooked in favor of Kahneman's and Tversky's (1979) loss aversion model.

Disappointment aversion preferences combine well established behavioral patterns for decision making under uncertainty, such as reference-based utility and asymmetric marginal utility, with a number of economically tractable properties. For instance, unlike most behavioral models, disappointment aversion preferences do not violate first-order stochastic dominance, transitivity of preferences or aggregation of investors, and can therefore help us shed additional light on the link between financial markets and aggregate economic activity, while maintaining investor rationality.

# TABLE OF CONTENTS

# LIST OF FIGURES

**Figure**

# LIST OF TABLES

Table

# LIST OF APPENDICES

**Appendix**

# LIST OF ABBREVIATIONS

**AMEX** American Stock Exchange

**APT** Arbitrage Pricing Theory

**AR** Autoregressive

**BARC** Barclays

**BEA** Bureau of Economic Analysis

**BM** Book-to-Market

**BofA** Bank of America

**bps** basis points

**CAPM** Capital Asset Pricing Model

**c.d.f.** cumulative distribution function

**CRRA** Constant Relative Risk Aversion

**CRSP** Center for Research on Stock Prices

**DA** Disappointment Aversion

**EBIT** Earnings Before Interest and Taxes

**EBITDA** Earnings Before Interest, Taxes, Depreciation, and Amortization

**EIS** Elasticity of Intertemporal Substitution

**EP** Earnings-to-Price

**EZ** Epstein-Zin

**FF** Fama-French

**GDA** Generalized Disappointment Aversion

**GMM** Generalized Method of Moments

**GP** Gross Profits

**HML** High Minus Low

**i.i.d.** independent and identically distributed

**LDA** Linear Disappointment Aversion

**LTCM** Long-Term Capital Management

**MOM** Momentum

**NASDAQ** National Association of Securities Dealers Automated Quotations

**NBER** National Bureau of Economic Research

**NYSE** New York Stock Exchange

**OECD** Organisation for Economic Co-operation and Development

**OLS** Ordinary Least Squares

**PCE** Personal Consumption Expenditures

**p.d.f.** probability distribution function

**SMB** Small Minus Big

**SDF** Stochastic Discount Factor

**TR** Thomson-Reuters

**WRDS** Wharton Data Research Services

**yr** year

# ABSTRACT


Great Expectations, Greater Disappointment: Disappointment Aversion Preferences in General Equilibrium Asset Pricing Models

by

Stefanos Delikouras

Chair: Associate Professor Robert Dittmar

For a long time, most financial economists have largely ignored experimental evidence on decision making under risk, mainly because introducing behavioral elements into asset pricing models while preserving investor rationality is a very challenging task. This thesis focuses on a relatively novel set of preferences that exhibit attitudes toward risk termed disappointment aversion preferences. These preferences are able to capture well documented patterns for risky choices, such as asymmetric marginal utility over gains and losses, without violating first-order stochastic dominance, transitivity of preferences or aggregation of investors. In my dissertation, I employ disappointment aversion preferences in an attempt to resolve two of the most prominent puzzles in asset pricing: the equity premium puzzle in the cross-section of expected stock returns, and the credit spread puzzle in corporate bond markets.

The first chapter of my dissertation explains the cross-section of expected stock returns for the U.S. economy using an empirically tractable solution for the disappointment aversion discount factor. The consumption-based asset pricing framework introduced in the first chapter does not rely on additional risk processes, backwards-

looking state variables, or extremely persistent macroeconomic shocks to generate large equity risk premia. In contrast, estimation results highlight the importance of disappointment events, defined as periods during which consumption growth drops below its forward-looking certainty equivalent. Finally, the disappointment aversion model is able to generate smaller in- and out-of-sample pricing errors than popular factor-based models using aggregate consumption growth as the only independent variable.

Structural models of default are unable to generate measurable Baa-Aaa credit spreads, when these models are calibrated to realistic values for default rates and losses given default. Motivated by recent results in behavioral economics, the second chapter proposes a consumption-based asset pricing model with disappointment aversion preferences in an attempt to resolve the credit spread puzzle. Simulation results suggest that as long as losses given default and default boundaries are countercyclical, then the disappointment model can resolve the Baa-Aaa credit spread puzzle using preference parameters that are consistent with experimental findings. Further, the disappointment aversion discount factor can almost perfectly match key moments for stock market returns, the price-dividend ratio, and the risk-free rate.

## CHAPTER I

# Disappointment Events in Consumption Growth, and the Cross-Section of Expected Stock Returns

***"Blessed is he who expects nothing, for he shall never be disappointed."***
Alexander Pope (1688 – 1744)

## 1.1 Abstract

This paper explains the cross-section of expected stock returns for the U.S. economy using an empirically tractable solution for the disappointment aversion discount factor. The consumption-based asset pricing framework introduced in this paper does not rely on additional risk processes, backwards-looking state variables, or extremely persistent macroeconomic shocks to generate large equity risk premia. In contrast, estimation results highlight the importance of disappointment events, defined as periods during which consumption growth drops below its forward-looking certainty equivalent. Finally, the disappointment aversion model is able to generate smaller in- and out-of-sample pricing errors than popular factor-based models using aggregate consumption growth as the only independent variable.

## 1.2 Introduction

This paper examines whether recent experimental results on choices under uncertainty can help explain the cross-section of expected stock returns. Towards this goal, I focus on a relatively novel set of preferences that exhibit attitudes toward risk termed disappointment aversion preferences. Introducing behavioral models into a general equilibrium framework has always been a challenging task due to the fact that these models tend to violate fundamental preference axioms. Disappointment aversion preferences on the other hand are able to capture well documented patterns for risky choices, such as asymmetric marginal utility over gains and losses, without violating first-order stochastic dominance, transitivity of preferences or aggregation of investors. The disappointment framework can therefore help us shed additional light on the link between expected stock returns and aggregate economic activity, while maintaining investor rationality.

Although several consumption-based asset pricing models have proposed frameworks that generate risk premia consistent with empirical observations, these frameworks rely on unobserved or hard-to-measure quantities. In contrast, the empirical results obtained here depend only on the standard measure of growth in per capita consumption of nondurables and services. The disappointment aversion model defines bad states of the economy endogenously, and generates risk premia by amplifying contemporaneous covariances between equity returns and consumption growth through first-order risk aversion. Estimation results suggest that the disappointment aversion framework captures book-to-market, size, earnings-to-price, and market-wide risk premia, while maintaining low first and second moments for the risk-free rate. Further the disappointment aversion framework generates smaller in- and out-of-sample pricing errors than popular factor-based pricing models, such as the four-factor Fama-French-Carhart (Fama and French 1993 & 1996, Carhart 1997) model, using aggregate consumption growth as the only independent variable.

The disappointment model is centered around a single parameter, the disappointment aversion parameter, and a single explanatory variable, disappointment events in consumption growth, defined as periods during which consumption growth falls below its forward-looking certainty equivalent. If consumption growth is i.i.d., then disappointment events happen whenever annual consumption growth is less than 0.84%. In contrast, if consumption growth is an AR(1) process, then the disappointment threshold is time-varying. In the postwar sample, disappointment years happen with a 16% probability. These disappointment events tend to pre-date NBER recessions. For instance, if year $t$ is a disappointment year, then the probability that year $t+1$ will have more than three NBER recession months rises from 15% to 88%. Moreover, stock market crises, such as the one in 1987 or the 1998 LTCM bailout, which do not spill over to the real economy are not particularly important for the pricing of equity claims, because these periods are not associated with disappointment events in consumption growth.

Finally, this paper is one of the first to estimate disappointment aversion parameters using stock market data. Parameter estimates are higher than those estimated in clinical experiments. Nevertheless, the interaction between disappointment and second-order risk aversion results in lower estimates for the coefficient of relative risk aversion relative to preferences with second-order risk aversion alone. Under CRRA or Epstein-Zin (Epstein and Zin 1989) preferences with i.i.d. consumption growth, the annual point estimate for the relative risk aversion coefficient in my sample is 55 (140 for quarterly data). By incorporating disappointment aversion, point estimates for the coefficients of relative risk aversion fall between 10 and 16, depending on the persistence of consumption growth, the sample frequency, and the sample period. Moreover, even though risk aversion estimates for second-order risk aversion preferences are very sensitive to sample frequency, preference parameters for the disappointment aversion model remain constant across frequencies.

Disappointment aversion preferences were first introduced by Gul (1991) in order to resolve the Allais paradox (Allais 1953)[1]. These preferences belong to a broader class of preferences which are usually referred to as first-order risk aversion preferences. One way to describe first-order risk aversion preferences is by non-differentiable utility functions with asymmetric slopes around a reference point for gains and losses. On the other hand, preferences that are characterized by smooth, continuously differentiable utility functions are usually referred to as second-order risk aversion preferences.

Routledge and Zin (2010) and Bonomo et al. (2011) employ a generalized version of disappointment aversion preferences in order to explain the stock market premium for the U.S. economy. Furthermore, Bonomo et al. (2011) also find that the disappointment aversion framework can closely replicate predictability patterns found in stock market data and price-dividend ratios. However, neither paper addresses the cross-section of equity returns. In contrast, Ostrovnaya et al. (2006) use disappointment aversion preferences to explain the cross-section of stock returns, and focus on monthly returns for book-to-market, size and industry portfolios. Even though they emphasize the importance of consumption growth as a state variable, the authors also rely on aggregate stock market returns as a proxy for returns on aggregate wealth. Ostrovnaya et al. (2006) conclude that the addition of consumption growth to the disappointment aversion discount factor enhances the ability of the stock market index to explain the cross-section of stock returns.

This paper employs the same disappointment aversion framework as Routledge and Zin (2010) and Ostrovnaya et al. (2006), but further augments their contribution by solving for the value function solely in terms of consumption growth. A closed-form solution for the value function, and hence the pricing kernel, in terms of consumption growth is significant for three reasons. First, characterizing the pricing kernel in

[1]The Allais paradox is related to the empirical finding that people tend to violate the independence axiom for choices under uncertainty.

terms of consumption growth alone, rather than consumption growth and market returns, forces the model to confront asset pricing moments using macroeconomic data alone. Consequently, the model does not fit equity returns to reasonable preference parameters simply by increasing the volatility and correlation of the pricing kernel through the use of market returns. Second, contrary to the calibration approach undertaken by Routledge and Zin (2010) or Bonomo et al. (2011), I estimate rather than calibrate the disappointment model allowing consumption and stock return data to decide on the statistical and economic significance of disappointment aversion. Finally, due to the closed-form solution for the stochastic discount factor and the use of real data, I am able to actually identify disappointment events in the post-war sample.

Although, Ang et al. (2006) theoretically motivate their discussion on the downside risk CAPM based on disappointment aversion preferences, they do not provide a framework that directly links the disappointment aversion utility function to their asymmetric CAPM. Lettau et al. (2013) also employ the downside CAPM to explain the cross-sectional dispersion for an impressively broad set of assets: equities, currencies, commodities, corporate bonds. Despite the analytical tractability of the downside CAPM, by estimating the disappointment model via GMM on consumption-Euler equations, I do not have to explicitly transform the disappointment aversion pricing kernel into a linear factor model, thus preserving the economic content of preference parameters. Finally, even though I only consider a single class of assets (equities), I conduct a number of statistical tests (out-of-sample, different frequencies, different reference levels) which highlight the model's successes as well as its shortcomings.

The use of disappointment aversion preferences is motivated by strong experimental and field evidence from aspects of economic life that are not directly related to portfolio choices[2]. There are many asset pricing models that can efficiently explain stylized facts in equity markets, yet these models usually have questionable out-of-sample performance. The strategy of this paper is to impose more discipline on investor preferences, and provide solid micro-foundations for a universal discount factor by taking into account recent experimental results for choices under uncertainty. These results emphasize the importance of expectation-based reference-dependent utility. This paper also adds to the relatively limited strand of literature that incorporates elements of behavioral economics into a consumption-based asset pricing model without violating key assumptions of the traditional general equilibrium framework.

## 1.3 Recursive utility with disappointment aversion preferences

### 1.3.1 Disappointment aversion and the portfolio-consumption problem

Consider a discrete-time, single-good, closed, endowment economy. Disappointment aversion preferences are homothetic. Therefore, if all individuals have identical preferences, then a representative investor exists, and equilibrium prices are independent of the wealth distribution[3]. Implicit in the representative agent framework lies the assumption of complete markets. There is no productive activity, yet at each point in time the endowment of the economy is generated exogenously by $n$ "tree"-assets as in Lucas (1978). There is also a market where equity claims on these assets can be traded. In addition to rational expectations, I will also assume that there are no restrictions on individual asset holdings, no transaction costs, and that all agents can borrow and lend at the same risk-free rate.

At each point in time, the infinitely-lived, representative investor chooses con-

[2]See Section 1.5 for a complete set of references.
[3]Chapter 1 in Duffie (2000), and Chapter 5 in Huang and Litzenberger (1989).

sumption ($C_t$) and asset holdings ($\{w_{i,t}\}_{i=1}^n$) in order to maximize her lifetime utility $V_t$[4]:

$$V_t = \max_{C_t,\ \{w_{i,t}\}_{i=1}^n} \left[(1-\beta)C_t^\rho + \beta\mu_t(V_{t+1};\ V_{t+1} < \delta\mu_t)^\rho\right]^{\frac{1}{\rho}}, \tag{1.1}$$

with

$$\mu_t(V_{t+1};\ V_{t+1} < \delta\mu_t)^{-\alpha} = \mathbb{E}_t\left[\frac{V_{t+1}^{-\alpha}\ (1 + \theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\} + \theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\right], \tag{1.2}$$

subject to the usual budget and transversality constraints.

Lifetime utility $V_t$ is strictly increasing in wealth, globally concave[5], and homogeneous of degree one. Dolmas (1996) shows that homothetic preferences are a necessary condition for balanced growth of the economy[6]. This is an appealing characteristic of disappointment aversion relative to other types of first-order risk aversion preferences: disappointment preferences can successfully explain the cross-section of expected returns without violating key economic implications for the macroeconomy. Another important issue with reference-based utility in a dynamic framework is time-consistency. The disappointment aversion framework is time-consistent since $\frac{\partial V_t}{\partial V_{t+1}} > 0$[7].

$\mu_t$ in equation (1.2) is the disappointment aversion certainty equivalent which generalizes the concept of expected value. $\mathbb{E}_t$ is the conditional expectation operator. The denominator in (1.2) is a normalization constant such that $\mu_t(\mu_t) = \mu_t$. $\mathbf{1}\{\}$ is the disappointment indicator function that overweighs bad states of the world (disappointment events). In a dynamic setting, the reference point for disappointment is

[4]In Barberis et al. (2001) and Easley and Yang (2012), investors draw utility from consumption as well as from investing in risky assets. Here, investors draw utility from consumption alone.

[5]Contrary to Kahneman and Tversky's (1979) prospect theory, the objective function in (1.1) is globally concave, and the second-order conditions for maximization are satisfied.

[6]Along balanced growth paths for the economy, the consumption-wealth ratio $C_t/W_t$ is a stationary process.

[7]Andries (2011), p. 12 and pp. 50-55.

forward-looking and proportional to the certainty equivalent for next period's lifetime utility $\mu_t\big(V_{t+1}\big)$. According to (1.2), disappointment events happen whenever lifetime utility $V_{t+1}$ is less than some multiple $\delta$ of its certainty equivalent $\mu_t$[8].

$\delta > 0$ is the generalized disappointment aversion (GDA) multiplier introduced in Routlegde and Zin (2010). The parameter $\delta$ is associated with the threshold below which disappointment events occur. In Gul (1991) $\delta$ is 1, and disappointment events happen whenever utility falls below its certainty equivalent: $V_{t+1} < \mu_t(V_{t+1})$. On the other hand, according to the GDA framework, disappointment events may happen below or above the certainty equivalent, $V_{t+1} < \delta\mu_t(V_{t+1})$, depending on whether the GDA parameter $\delta$ is lower or greater than one respectively[9]. I set $\delta = 1$ as in Gul (1991) in order to solve $V_t$ analytically.

$\alpha \geq -1$ is the Pratt (1964) coefficient of second-order risk aversion which affects the smooth concavity of the objective function. $\theta \geq 0$ is the disappointment aversion parameter which characterizes the degree of asymmetry in marginal utility over above and below the reference level. If $\theta$ is positive[10], then a an additional one-dollar-loss in consumption below the reference point hurts approximately $1 + \theta$ times more than a an additional one-dollar-loss in consumption above the reference point. When $\theta = 0$ investors have symmetric preferences, and the effects of first-order risk aversion vanish.

$\beta \in (0, 1)$ is the rate of time preference. In the deterministic steady-state of the economy, an additional \$1 of consumption tomorrow is worth \$$\beta$ today. $\rho \leq 1$ characterizes the elasticity of intertemporal substitution (EIS) for consumption between two consecutive periods since EIS $= \frac{1}{1-\rho}$. The EIS also measures the responsiveness of consumption growth to the real interest rate. The sign of $\rho$ and the magnitude of the EIS have important implications for asset pricing models. In Bansal and Yaron

[8] I explicitly write $V_{t+1} < \delta\mu_t$ as a parameter in the certainty equivalent function to keep track of the disappointment threshold.

[9] For $\delta > 1$ in (1.2), $\theta(\delta^\alpha - 1) < 1$ is a sufficient condition for decreasing marginal utility.

[10] If $\theta$ is negative, then investor preferences are characterized by convex utility functions, losses hurt less than gains give joy, and investors are usually referred to as "elation seekers".

(2004), $\rho$ is positive, and the EIS is greater than 1. However, in a time-additive context, Hall (1988) finds that $\rho$ is negative, and that the EIS is a very small number. Here, I set $\rho = 0$ (EIS=1) in order to analytically solve the value function $V_t$ in terms of consumption growth.

Since the focus of this paper is the cross-sectional dimension of stock returns and not the time-series, setting $\rho$ equal to zero does not significantly affect empirical results while keeping the number of free parameters to a minimum. Fixing $\rho$ to zero essentially implies that current consumption expenditures and future lifetime utility are compliments (log-aggregator for consumption at different points of time), that consumption is always a fixed fraction of wealth, and that consumption growth moves one for one with the interest rate. Log-time preferences have been heavily exploited in the literature precisely because they lead to closed-form solutions for lifetime utility $V_t$. Piazzesi and Schneider (2006), Hansen et al. (2007), Hansen and Heaton (2008) are a few examples in which the elasticity of intertemporal substitution is equal to one. This paper is the first to show that an EIS equal to one allows for closed form solutions even in the case of disappointment aversion preferences.

The expression for the disappointment aversion intertemporal marginal rate of substitution[11] between two consecutive periods is given by

$$M_{t,t+1} = \underbrace{\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{\rho-1}}_{\text{time correction}} \underbrace{\Big[\frac{V_{t+1}}{\mu_t\big(V_{t+1};\ V_{t+1} < \delta\mu_t\big)}\Big]^{-\alpha-\rho}}_{\text{second-order risk correction}} \times \tag{1.3}$$
$$\underbrace{\Big[\frac{1+\theta\mathbf{1}\{V_{t+1} < \delta\mu_t\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta > 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1} < \delta\mu_t\}]}\Big]}_{\text{disappointment (first-order risk) correction}}.$$

$M_{t,t+1}$ essentially corrects expected values by taking into account investor preferences over the timing and riskiness of stochastic payoffs. The first term in (1.3) corrects for the timing of uncertain payoffs (resolution of uncertainty) which happen at a fu-

[11]See also Hansen et al. (2007), and Routledge and Zin (2010).

ture date. The second term adjusts future payoffs for investors' dislike towards risk (second-order risk aversion). When investors' preferences are time-additive, adjustments for time and risk are identical ($\alpha = \rho$)[12], and the second term vanishes. The third term in equation (1.3) corrects future payoffs for investors' aversion towards disappointment events, defined as periods during which lifetime utility $V_{t+1}$ drops below some multiple $\delta$ of its certainty equivalent $\mu_t$.

According to the expression in (1.3), if household preferences are not separable across time (Kreps and Porteus 1978), then the stochastic discount factor is a function of consumption growth as well as of lifetime utility (investor's value function). Epstein and Zin (1989) were the first to show that these lifetime utility terms can be replaced by returns on aggregate wealth. However, because aggregate wealth is hard to measure, various approaches have been suggested for measuring its returns. Campbell (1996) log-linearizes the budget constraint and expresses returns on wealth as a function of consumption growth. Lettau and Ludvigson (2001) infer returns on wealth by exploiting the co-integration of macroeconomic variables such as investment, consumption and production. Ostrovnaya et al. (2006) use stock market returns as a proxy for returns on wealth. Finally, Weil (1989) assumes a discrete state space for consumption growth, and solves a system of non-linear equations that yield wealth returns for each state of the world. Contrary to all the above, this paper analytically characterizes investors' lifetime utility in terms of consumption growth by building upon the methodology used in Hansen and Heaton (2008), and exploiting the fact that the EIS is set equal to one.

[12]When investors have time-additive preferences, the Bellman equation in (1.1) reads $V_t = -\frac{C_t^{-\alpha}}{\alpha} + \beta \mathbb{E}_t[V_{t+1}]$, $\beta \in (0,1)$, $\alpha \geq -1$.

### 1.3.2 Log-linear disappointment aversion preferences

For $\rho = 0$ and $\delta = 1$ in equations (1.1), (1.2) and (1.3), the disappointment aversion pricing kernel becomes

$$M_{t,t+1} = \underbrace{\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-1}}_{\text{time correction}} \underbrace{\Big(\frac{V_{t+1}}{\mu_t\big(V_{t+1}; V_{t+1} < \mu_t(V_{t+1})\big)}\Big)^{-\alpha}}_{\text{second-order risk correction}} \underbrace{\frac{1 + \theta\ \mathbf{1}\{V_{t+1} < \ \mu_t(V_{t+1})\}}{\mathbb{E}_t[1 + \theta\ \mathbf{1}\{V_{t+1} < \mu_t(V_{t+1})\}]}}_{\text{disappointment correction}}. \quad (1.4)$$

Suppose now that all the randomness in the economy can be summarized by consumption growth which follows an AR(1) process[13] with constant volatility

$$\Delta c_{t+1} = \mu_c(1 - \phi_c) + \phi_c \Delta c_t + \sqrt{1 - \phi_c^2}\sigma_c \epsilon_{t+1}. \quad (1.5)$$

$\mu_c = \mathbb{E}[\Delta c_{t+1}] \in \mathbb{R}$, $\sigma_c^2 = \mathbf{Var}(\Delta c_{t+1}) \in \mathbb{R}_{>0}$, $\phi_c = \rho(\Delta c_{t+1}, \Delta c_t) \in (-1, 1)$ are the unconditional mean, variance, and first-order autocorrelation coefficient for consumption growth[14]. Shocks to consumption growth $\epsilon_{t+1}$ are i.i.d. $N(0, 1)$ variables. The $R^2$ for the AR(1) model is 21.96% for annual data and 10.79% for quarterly data. Mehra and Prescott (1985) as well as Routledge and Zin (2010) also employ an AR(1) model for consumption growth.

The goal now is to obtain an empirically tractable version of the disappointment aversion stochastic discount factor in (1.4). This is done by expressing lifetime utility $V_t$ in terms of the observable consumption growth process $\Delta c_t$.

***Proposition 1:*** For $\rho = 0$, $\delta = 1$ and consumption growth dynamics in (1.5), the log utility-consumption ratio, $v_t - c_t$ is affine in consumption growth: $v_t - c_t = \mu_v + \phi_v \Delta c_t \ \forall t$, where

[13] Lowercase letters denote logs of variables: $c_t = logC_t,\ v_t = logV_t$.

[14] Following Hansen and Heaton (2008), the AR(1) framework in (1.5) can be extended to allow for consumption growth to be a function of multiple state variables which in turn can be described by VAR processes. Also for $\phi_c$=0, the AR(1) models nests the i.i.d. case. Appendix A.2 analyzes a linear version of the disappointment model in which I analytically express lifetime utility in terms of changes in consumption ($\Delta C_{t+1} = C_{t+1} - C_t$) rather than consumption growth ($\Delta c_{t+1} = log\frac{C_{t+1}}{C_t}$).

- $\mu_v = \frac{\beta}{1-\beta}\Big\{(\phi_v+1)\mu_c(1-\phi_c) + d_1(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\Big\},\ \mu_v \in \mathbb{R}$,

- $\phi_v = \frac{\beta\phi_c}{1-\beta\phi_c},\ \phi_v \in \mathbb{R}$,

- $d_1 \in \mathbb{R}$ is the solution to the fixed point problem

$$d_1 = \underbrace{-\frac{\alpha}{2}(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c}_{\text{risk}} \underbrace{- \frac{1}{\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c} log\Big[\frac{1+\theta N\big(d_1+\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\big)}{1+\theta N\big(d_1\big)}\Big]}_{\text{disappointment}}. \tag{1.6}$$

***Proof.*** *See Appendix A.4.1*

$\mu_v$ is the constant term in the log utility-consumption ratio which depends on the drift term for consumption growth $\mu_c(1-\phi_c)$ appropriately corrected for risk and disappointment, $d_1(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c$. $\phi_v$ is the sensitivity of the log utility-consumption ratio to consumption growth, and depends on consumption growth persistence ($\phi_c$). Finally, $d_1$ is the disappointment threshold for consumption growth shocks $\epsilon_{t+1}$. According to (1.6), the disappointment threshold $d_1$ consists of two terms: the first term depends only on the risk aversion coefficient $\alpha$, whereas the second term depends on both risk and disappointment aversion parameters, $\alpha$ and $\theta$. For positive $\theta$, if the coefficient of risk aversion is also positive ($\alpha > 0$), then the disappointment threshold is definitely negative $d_1 < 0$[15]. On the other hand, for $-1 \leq \alpha < 0$ we may have $d_1 \geq 0$.

An immediate consequence of *Proposition 1* is that disappointment events can now be expressed in terms of consumption growth $\Delta c_{t+1}$ rather than lifetime utility

[15]For this result to hold we also need $\beta \in (0,1)$ and $\phi_c \in (-1,1)$ so that $\phi_v + 1 > 0$. Empirical results suggest that these conditions hold.

$V_{t+1}$:

$$\Delta c_{t+1} < \underbrace{\mu_c(1-\phi_c) + \phi_c \Delta c_t + d_1\sqrt{1-\phi_c^2}\sigma_c}_{\text{certainty equivalent for } \Delta c_{t+1}} \tag{1.7}$$

The right-hand side in (1.7) is the certainty equivalent for next period's consumption growth which takes into account investors' aversion towards risk and disappointment. $(1-\phi_c)\mu_c + \phi_c\Delta c_t$ is the expected value for next period's consumption growth[16], whereas $d_1\sqrt{1-\phi_c^2}\sigma_c$ captures the disappointment and risk correction terms. Since consumption growth is assumed an AR(1) process, simple algebra shows that disappointment events happen whenever shocks to consumption $\epsilon_{t+1}$ are less than the disappointment threshold $d_1$[17]. Note that analytical solutions for the disappointment aversion stochastic discount factor are not limited to the AR(1) specification, but include any linear model for consumption growth with homoscedastic, normally distributed shocks.

Equation (1.7) implies that disappointment events occur whenever next period's consumption growth is lower than some quantity which depends on current consumption growth. At a first glance this result may be reminiscent of a habit model, like the one in Campbell and Cochrane (1999). However, the threshold value for disappointment events $\mu_t\big(\Delta c_{t+1}\big)$, which is also the certainty equivalent for consumption growth, is forward-looking. *Proposition 1* exploits the log-linear structure of the value function $V_{t+1}$ in order to express the forward-looking disappointment threshold $\mu_t\big(V_{t+1}\big)$ in terms of the autoregressive consumption growth process, and consequently, in terms of current consumption growth. Nevertheless, this dependence does not imply a habit mechanism. Note also that in the habit model of Campbell and Cochrane (1999) consumption never drops below its habit, otherwise marginal utility becomes infinity. On

[16] In this paper, expectations about future consumption growth are based on the AR(1) framework. It would be interesting to consider alternative expectation measures such as analyst forecasts.

[17] Estimation results suggest that $d_1 \approx -0.80$. Disappointment events happen whenever shocks to consumption growth are less that $-0.80$.

the other hand, for disappointment aversion preferences it is precisely periods during which consumption growth falls below its certainty equivalent that are important for asset prices.

Using the results in *Proposition 1*, the disappointment aversion discount factor becomes

$$
\begin{aligned}
M_{t,t+1} = exp\Big[ & \underbrace{log\beta - \Delta c_{t+1}}_{\text{time correction}} \\
& \underbrace{+\frac{\alpha\mu_c}{1-\beta\phi_c}(1-\phi_c) - \frac{\alpha^2\sigma_c^2}{2(1-\beta\phi_c)^2}(1-\phi_c^2) - \frac{\alpha}{1-\beta\phi_c}\Delta c_{t+1} + \frac{\alpha}{\beta}\phi_v\Delta c_t\Big]}_{\text{second-order risk correction}} \\
& \underbrace{\times\frac{1+\theta\mathbf{1}\{\Delta c_{t+1} \ < \ \mu_c(1-\phi_c)+\phi_c\Delta c_t + d_1\sqrt{1-\phi_c^2}\sigma_c\}}{1+\theta\mathbb{E}_t\big[\mathbf{1}\{\Delta c_{t+1} \ < \ \mu_c(1-\phi_c)+\phi_c\Delta c_t + d_1\sqrt{1-\phi_c^2}\sigma_c + \alpha(\phi_v+1)(1-\phi_c^2)\sigma_c^2\}\big]}}_{\text{disappointment (first-order risk) correction}},
\end{aligned}
\tag{1.8}
$$

$M_{t,t+1}$ in (1.8) corrects expected future payoffs for timing, risk and disappointment[18], much like the discount factor in (1.4). The crucial difference between the two expressions is that in equation (1.8) unobservable lifetime utility $V_{t+1}$ is expressed in terms of the observable consumption growth $\Delta c_{t+1}$. The empirically relevant terms in (1.8) which affect expected excess stock returns are future consumption growth terms ($\Delta c_{t+1}$), and the disappointment aversion indicator function.

The disappointment model yields an analytical solution for the risk-free rate as

[18]Excluding time-correction terms, $exp\big(log\beta - \Delta c_{t+1}\big)$, the expected value of the remaining terms in (1.8) should equal one, since the risk and disappointment correction terms induce a new probability measure on the space of asset returns and consumption growth.

well. According to (1.8), the one-period, log risk-free rate is equal to

$$r_{f,t+1} = \underbrace{-log\beta + 1 \cdot \mu_c(1-\phi_c) + 1 \cdot \phi_c \Delta c_t}_{\text{impatience and future prospects}} \tag{1.9}$$

$$\underbrace{\underbrace{-\frac{1}{2}[2\alpha(\phi_v+1)+1](1-\phi_c^2)\sigma_c^2}_{\text{second-order risk aversion}} + \underbrace{log\frac{1+\theta N\big(d_1+\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\big)}{1+\theta N\big(d_1+[\alpha(\phi_v+1)+1]\sqrt{1-\phi_c^2}\sigma_c\big)}}_{\text{disappointment aversion}}}_{\text{precautionary savings motive}}.$$

If agents are impatient with low $\beta$, then they would require a high interest rate as compensation for foregone consumption in the current period. Consumption growth terms $\big(\mu_c(1-\phi_c), \phi_c\Delta c_t\big)$ in (1.9) are multiplied by unity, because the EIS is assumed equal to one, and consumption growth moves one-for-one with interest rates. The last two terms in (1.9) reflect the precautionary motive for investors to save. This motive depends on both risk and disappointment aversion. Notice that second-order risk aversion terms depend on consumption growth variance ($\sigma_c^2$), while disappointment aversion terms depend on consumption growth volatility ($\sigma_c$) due to the first-order risk aversion mechanism[19].

## 1.4 Estimation

### 1.4.1 Historical data

For the empirical analysis I use annual and quarterly data. Personal consumption expenditures (PCE), and PCE index data are from the BEA. Per capita consumption expenditures are defined as services plus non-durables. Each component of aggregate

[19] The expression in (1.9) underestimates the unconditional volatility of the risk-free rate since $\widehat{\mathbf{Vol}}(r_{f,t+1}) = 2.428\% > \hat{\phi}_c\widehat{\mathbf{Vol}}(\Delta c_t) = 0.572\%$ (Table 1.7.1). In contrast, an important drawback for most consumption-based asset pricing models is an extremely volatile risk-free rate. For example, in the time-additive CRRA case with AR(1) consumption growth, the expression for the log risk-free rate reads $r_{f,t+1} = -log\beta + (\alpha+1)\mu_c(1-\phi_c) + (\alpha+1)\phi_c\Delta c_t - \frac{1}{2}(\alpha+1)^2(1-\phi_c^2)\sigma_c^2$. Given that the risk aversion parameter $\alpha$ in the CRRA model needs to be around 60 to match the stock market premium, CRRA models severely overestimate risk-free rate volatility since $60 \cdot 0.45 \cdot \widehat{\mathbf{Vol}}(\Delta c_t) = 34.320\% >> \widehat{\mathbf{Vol}}(r_{f,t+1}) = 2.428\%$.

consumption expenditures is deflated by its corresponding PCE price index (base year is 2004). Population data are from the U.S. Census Bureau. Recession dates are from the NBER. Asset returns, factor returns, and interest rates are from Kenneth French's (whom I kindly thank) website. Stock returns and interest rates have been adjusted for inflation by subtracting the growth rate of the PCE price index[20]. For quarterly data, I follow the "beginning-of-period" convention as in Campbell (2003) and Yogo (2006) because beginning-of-quarter consumption growth is better aligned with stock returns.

Annual consumption data are from 12/31/1948 to 12/31/2011, whereas quarterly consumption data are from 1948.Q1 to 2011.Q4. Annual asset returns are cum-dividend, equal-weighted returns from 12/31/1949 to 12/31/2011 with the exception of earnings-to-price portfolios which start on 12/31/1952. Quarterly returns are from 1948.Q2 up to 2011.Q4. Following Liu et al. (2009), I focus on equal-weighted portfolios which exhibit more pronounced cross-sectional dispersion, and do not overweigh large firms. Following Yogo (2006), I start the sample in the late 40's in order to allow sufficient time for Second World War shocks to die out. The use of post-war data is motivated by the possibility of a structural break in the U.S. economy after the Second World War, as well as by the fact that consumption and population measurements during the first half of the 20th century may not be accurate[21].

### 1.4.2 Estimation methodology

My analysis is focused on portfolios double sorted on size and book-to-market (BM). Ever since Fama and French (1993 & 1996) documented that these two variables capture most of the cross-sectional variation in equity returns, much of the

[20] $R_{\text{real},t+1} = exp(logR_{nom,t+1} - log\frac{PCE_{t+1}}{PCE_t})$, $R$ are gross returns.

[21] This study focuses on 25 portfolios double sorted on book-to-market and size. Estimation results for 10 BM portfolios, 10 size portfolios, 10 BM and 10 size portfolios combined, value-weighted portfolios, nominal consumption growth and nominal stock returns, as well as results for the 1930-2011 period are available upon request.

asset pricing literature in the past two decades has focused on explaining the size and value factors. Parameters to be estimated are the rate of time preference $\beta$, the second-order risk aversion parameter $\alpha$, and the disappointment aversion parameter $\theta$. The key insight for disappointment aversion preferences is that the reference point for disappointment $d_1$ is endogenous. According to equation (1.6), $d_1$ will be identified once preference parameters and consumption growth moments have been estimated. Consumption growth moments (mean $\mu_c$, autocorrelation $\phi_c$, volatility $\sigma_c$) are estimated in advance, and are considered inputs for the GMM estimation[22].

Estimation is conducted using the generalized method of moments (GMM, Hansen and Singleton 1982) in which the unconditional consumption-Euler equations serve as moment restrictions

$$g(\beta,\alpha,\theta) = \begin{bmatrix} \mathbb{E}\big[\tilde{M}_{t,t+1}\big(R_{i,t+1} - R_{f,t+1}\big)\big] \text{ for } i = 1, 2, ..., n-1 \\ \\ \mathbb{E}\Big[\frac{\tilde{M}_{t,t+1}}{1+\theta\mathbb{E}_t\Big[\mathbf{1}\{\Delta c_{t+1}<\phi_c\Delta c_t+\mu_c(1-\phi_c)+d_1\sqrt{1-\phi_c^2}\sigma_c+\alpha(\phi_v+1)(1-\phi_c^2)\sigma_c^2\}\Big]} R_{f,t+1}\Big] - 1 \end{bmatrix}, \quad (1.10)$$

with

$$\begin{aligned}\tilde{M}_{t,t+1} &= exp\Big[log\beta + \alpha(\phi_v+1)\mu_c(1-\phi_c) - \frac{\alpha^2}{2}(\phi_v+1)^2(1-\phi_c^2)\sigma_c^2 \qquad (1.11)\\ &-[\frac{\alpha}{1-\beta\phi_c}+1]\Delta c_{t+1} + \frac{\alpha}{\beta}\phi_v\Delta c_t\Big]\big(1+\theta\mathbf{1}\{\Delta c_t < \mu_c(1-\phi_c)+\phi_c\Delta c_{t+1}+d_1\sqrt{1-\phi_c^2}\sigma_c\}\big),\end{aligned}$$

$R_{i,t}$ are one-period, real, cum-dividend, gross returns for portfolio $i$, and $R_{f,t}$ is the one period risk-free rate. It is important to emphasize that, contrary to the majority of cross-sectional results in the literature, moment conditions include the Euler equation for the risk-free rate in order to examine whether the disappointment model can explain the cross-section of expected stock returns while generating realistic first and

[22] In untabulated results, I also consider the case where consumption moments are part of the GMM objective function, and results still go through.

second moments for the risk-free rate[23].

We can also use the unconditional consumption-Euler equations in (1.10), and the definition of covariances[24] to obtain an explicit formula for model-implied expected returns

$$\hat{\hat{\mathbb{E}}}[R_{i,t+1}] = \hat{\mathbb{E}}[R_{f,t+1}] - \frac{1}{\hat{\mathbb{E}}[\tilde{M}_{t,t+1}]}\widehat{\mathbf{Cov}}[R_{i,t+1} - R_{f,t+1}, \tilde{M}_{t,t+1}], \qquad (1.12)$$

$$m.a.p.e. = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{\hat{\mathbb{E}}}[R_{i,t+1}] - \hat{\mathbb{E}}[R_{i,t+1}]\right|.$$

$\tilde{M}_{t,t+1}$ is from (1.11), $\hat{\hat{\mathbb{E}}}[R_{i,t}]$ are model-implied expected returns, and $\hat{\mathbb{E}}[R_{i,t}]$ are sample expected returns. Mean absolute prediction error (m.a.p.e.) is a metric which shows how well the model fits expected returns.

Parameters are estimated by minimizing the sample analogue of the GMM objective function ($\hat{g}(\beta, \alpha, \theta)$) with respect to the unknown preference parameters

$$\min_{\{\beta,\ \alpha,\ \theta\}} \hat{g}(\beta, \alpha, \theta)'\ W\ \hat{g}(\beta, \alpha, \theta). \qquad (1.13)$$

Moment conditions are weighted by the identity matrix (first-stage GMM). According to Cochrane (2001) and Liu et al. (2009), first-stage GMM preserves the economic structure of the GMM objective function. Furthermore, according to Ferson and Foerster (1994), second-stage GMM estimates are distorted in finite samples. Hayashi (2000, p. 229) and references therein also provide a discussion on small sample GMM estimators, and suggest the use of first-stage GMM in finite samples. Although first-stage GMM estimates are consistent (Cochrane 2001, p. 203), standard errors need to be adjusted for the fact that first-stage GMM does not use the minimum variance weighting matrix (Cochrane 2001, p. 205).

[23]The risk-free rate is assumed conditionally risk-free. Unconditionally, the risk-free rate becomes a random variable.

[24]$\mathbf{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.

Estimation of the disappointment model is challenging because the discount factor in (1.8) is not continuous. However, Newey and McFadden (1994) and Andrews (1994) have shown that continuity and differentiability of the GMM objective function can be replaced by the less stringent conditions of continuity with probability one (Theorem 2.6 p. 2132 in Newey and McFadden 1994) and stochastic differentiability (Theorems 7.2 p. 2186, and 7.3 p. 2188, in Newey and McFadden 1994). As shown in Appendix A.3, both of these conditions are satisfied by the disappointment aversion stochastic discount factor provided that log-consumption growth and log-stock returns are continuous random variables (no mass points) with bounded first and second moments, and a well defined moment generating function. In this case, discontinuities are zero probability events.

For comparison purposes, I estimate five additional models: the market discount factor (Lintner 1965), the four factor Fama-French-Carhart (FF) model (Fama and French 1996, Carhart 1997) model, the time-additive CRRA discount factor defined over consumption (Mehra and Prescott 1985)

$$M_{t,t+1}^{(CRRA)} = \beta e^{-(\alpha+1)\Delta c_{t+1}}, \tag{1.14}$$

the Epstein-Zin (EZ) pricing kernel with AR(1) consumption growth and log-time aggregator (Epstein and Zin 1989, Hansen and Heaton 2008)[25]

$$M_{t,t+1}^{(EZ)} = \tag{1.15}$$

$$exp\Big[log(\beta) + \frac{\alpha\mu_c}{1-\beta\phi_c}(1-\phi_c) - \frac{\alpha^2\sigma_c^2}{2(1-\beta\phi_c)^2}(1-\phi_c^2) - \frac{\alpha}{1-\beta\phi_c} + 1]\Delta c_{t+1} + \frac{\alpha}{\beta}\phi_v\Delta c_t\Big],$$

[25]The EIS in Epstein-Zin preferences is not necessarily one as it is assumed here. However, throughout the paper I will refer to the non-separable model with second-order risk aversion and log-time preferences as the Epstein-Zin model. The discount factor in (1.15) is derived along the lines of *Proposition 1* with the additional assumption that the coefficient of disappointment aversion $\theta$ is zero (no first-order risk aversion effects).

and finally, a linear version of the disappointment aversion discount factor[26][27]:

$$M_{t,t+1}^{(LDA)} = \beta \frac{1 + \theta\ \mathbf{1}\{\Delta C_{t+1} < \mu_C + d_1 \Sigma_C\}}{1 + \theta\ \mathbb{E}_t[\mathbf{1}\{\Delta C_{t+1} < \mu_C + d_1 \Sigma_C\}]}. \tag{1.16}$$

The market and Fama-French-Carhart specifications are considered benchmark models among practitioners and academics. According to Cochrane (2001, p. 442), the Fama-French-Carhart (FF) model can be regarded as an arbitrage pricing theory model (APT) "rather than a macroeconomic factor model." However, due to its popularity, I include it in the set of asset pricing models. The time-additive CRRA discount factor in (1.14) requires extremely large values for the second-order risk aversion and rate of time preference parameters in order to match equity returns. The Epstein-Zin framework does not account for disappointment aversion, yet it relies on second-order risk aversion and consumption growth persistence in order to generate realistic equity premia. The linear disappointment model in (1.15) with i.i.d. changes in consumption highlights the explanatory power of disappointment aversion alone, without considering second-order risk aversion or persistence in consumption growth. Consumption models in (1.14) - (1.16) are essentially nested by the benchmark model in (1.8).

### 1.4.3 Estimation results for annual stock returns

Table 1.7.2 shows estimation results for the the 25 Fama-French portfolios and the disappointment aversion discount factor. According to the $J$-test and $p$-value statistics (20.087 and 0.636 respectively), the null hypothesis that all moment conditions are jointly zero cannot be rejected at conventional confidence levels. The rate

[26] The linear version of the disappointment aversion discount factor is discussed in Appendix A.2, and derived in Appendix A.4.2. $\mu_C$ and $\Sigma_C$ are the unconditional mean and standard deviation respectively for consumption in first differences ($\Delta C_{t+1}$) which, in turn, is assumed to be an i.i.d. process with normal shocks. $d_1$ is the disappointment threshold for the linear disappointment aversion discount factor, and is defined in Appendix A.4.2 (equation A.17).

[27] An undesirable aspect of the linear disappointment models is the non-zero, but infinitesimally small, probability of negative consumption.

of time preference $\beta$ is equal to 0.977 ($t$-statistic 2.868), whereas the disappointment aversion coefficient $\theta$ is 4.606 ($t$-statistic 3.883). The estimated value for $\theta$ implies that an extra dollar of consumption during disappointment years is approximately 5.5 times more valuable in terms of marginal utility than an extra dollar of consumption during normal times. The second-order risk aversion coefficient is 9.929, yet the low t-statistic ($t$-stat. 0.574) suggests that $\alpha$ cannot be accurately estimated by GMM.

Kahneman and Tversky (1992) estimate the loss aversion coefficient to be 1.25, and the second-order risk aversion parameter $\alpha$ to be -0.88. Barberis et al. (2001) also use a loss aversion parameter of 1.25, yet they set the second-order risk aversion parameter equal to zero (log-preferences over risk) and prescribe preferences over consumption as well as individual asset returns, whereas here investors have preferences over consumption alone. In order to explain the market-wide equity premium, Routledge and Zin (2010) set $\theta$ equal to 9 with $\alpha$ equal to -1 (second-order risk neutrality), whereas in Bonomo et al. (2011) $\theta$ is 2.33 and $\alpha$ is 1.5 because the authors assume a very persistent process for resumption growth variance, whereas here consumption growth variance is constant.

Choi et al. (2007) conduct clinical experiments on portfolio choice under uncertainty, and find disappointment aversion coefficients that range from 0 to 1.876, with a mean of 0.39. They also estimate second-order risk aversion parameters that range from -0.952 to 2.871, with a mean of 1.448. Using experimental data on real effort provision, Gill and Prowse (2012) estimate disappointment aversion coefficients ranging from 1.260 to 2.070. Ostrovnaya et al. (2006) estimate disappointment aversion parameters from stock market data using market wide stock market returns as the explanatory variable, instead of consumption growth. Their estimates for $\theta$ range from 1.825 to 2.783. However, the authors rely on aggregate stock market returns as an explanatory variable, which are much more volatile than consumption growth.

The main reason as to why parameter estimates may deviate from those obtained in clinical experiments is probably limited stock market participation. It has been well documented (Mankiw and Zeldes 1991, Jorgensen 2002) that only a fraction of households participate in the stock market. If aggregate consumption is less volatile than stock-market participants' consumption, then parameter estimates using aggregate consumption will be upwards biased.

According to Table 1.7.2, the disappointment threshold $d_1$ is -0.780, which means that disappointment events happen whenever annual consumption growth is less than $1.031\% + 0.463\Delta c_t - 0.780 \cdot 1.120\%$. These events happen with a 15.873% probability in the post-war sample[28]. This is in sharp contrast to the disaster literature (Barro 2006) which indicates that disasters happen with probability 1.7% per year, and to the results in Ostrovnaya et al. (2006) which identify only 4 disappointment months for a period from 1960 to 2005. Barro (2006) calibrates the disaster process, an additional risk process, to OECD log-output data, whereas here disappointment events arise endogenously from investor preferences over consumption. In Ostrovnaya et al. (2006), disappointment events happen rarely because reference levels for disappointment, in terms of the generalized disappointment aversion coefficient $\delta$, are low. In their model, the aggregate investor penalizes extreme events since $\delta < 1$, whereas here $\delta$ is 1.

Table 1.7.2 also shows GMM estimation results for the extended set of discount factors. The constant term in the market model is positive (4.377), whereas the coefficient on the market factor is negative (-3.132). Both parameters are statistically significant ($t$-statistics 3.661 and -2.991 respectively), yet the null hypothesis that all moment conditions are jointly satisfied can be rejected ($p$-value 0.009). Statistically significant estimates for the Fama-French-Carhart model include the constant term (3.659, $t$-stat. 2.627), the market parameter (-2.268, $t$-stat. 1.931), and the HML coefficient (-3.956, $t$-stat. -3.058). The null hypothesis for the Fama-French-Carhart

[28] Disappointment years for the log-linear disappointment aversion discount factor happened in 1953, 1956, 1959, 1973, 1980, 1990, 1999, 2007, 2008, 2010.

model is also rejected ($p$-value 0.023). According to Hayashi (2000, p. 229), the low $J$-statistics across all asset pricing models in Table 1.7.2 can be attributed to the fact that first-stage GMM tests of overidentifying restrictions tend to reject the null more often than they should.

Results for time-separable preferences (CRRA model) reaffirm the equity premium puzzle in Mehra and Prescott (1985) since the second-order risk aversion parameter is extremely high (55.171[29], $t$-stat. 2.561). With time-separable CRRA preferences, a large coefficient of risk aversion is the only way to map consumption growth risk into equity premia. Moreover, the rate of time preference $\beta$ is significantly larger than one (2.172[30], $t$-stat. 3.334) so that the unconditional mean for the risk-free rate remains low despite the large risk aversion coefficient. Nevertheless, a risk aversion parameter equal to 55 implies an extremely volatile risk-free rate. Finally, the null hypothesis for this model is rejected at conventional confidence levels ($p$-value 0.002).

Contrary to the CRRA case, the estimated rate of time preference for the Epstein-Zin model is lower than one (0.983, $t$-stat. 9.395). Also, the second-order risk aversion parameter (35.550[31], $t$-stat. 3.336) is smaller than for CRRA utility because, with Epstein-Zin preferences, consumption growth risk is amplified by consumption growth persistence. However, in untabulated results for i.i.d., instead of AR(1), consumption growth, the risk aversion estimate for Epstein-Zin preferences is 55.171 ($t$-stat. 2.537), exactly identical to the time-additive CRRA case.

The Epstein-Zin discount factor can explain the cross-section of returns with low values for the second-order risk aversion parameter $\alpha$ provided that consumption growth is extremely persistent. A number of recent asset pricing results rely on highly persistent shocks to expected consumption growth. In Bansal and Yaron (2004),

[29]Cochrane (2001) argues that time-additive CRRA preferences can explain the unconditional equity premium provided that the risk aversion parameter is larger than 50.

[30]Liu et al. (2009) and Yogo (2004) also estimate $\beta$ larger than one for time-additive CRRA preferences.

[31]In Routldege and Zin (2010), the risk aversion parameter $\alpha$ for the Epstein-Zin model is calibrated to 31.542.

shocks to expected consumption growth have a half-life of approximately 3 years[32], whereas, according to BEA data from Table 1.7.1, shocks to consumption growth have a half-life of less than a year. Of course, consumption growth persistence and expected consumption growth persistence are two different quantities. Nevertheless, the persistent shocks in expected consumption growth assumed by the Bansal-Yaron model are hard to detect empirically (Beeler and Campbell 2012). Furthermore, a number of authors (Campbell and Cochrane 1999, Cochrane 2001) suggest that consumption growth is most likely an i.i.d. process.

When preferences are time-separable, expected excess log-returns are a function of covariances between stock returns and consumption growth. According to the expression in (1.14), these covariances are amplified by the second-order risk aversion coefficient $\alpha$[33]:

$$\mathbb{E}[r_{i,t+1} - r_{f,t+1}]^{CRRA} \approx (\alpha + 1)\mathbf{Cov}\big(\Delta c_{t+1}, r_{i,t+1} - r_{f,t+1}\big). \tag{1.17}$$

When preferences are non-separable (Epstein-Zin model), then expected excess log-returns are still generated by covariances between stock returns and consumption growth. However, according to the expression in (1.15), the second-order risk aversion coefficient $\alpha$, which amplifies covariances, is divided by $1-\beta\phi_c$, the term that captures consumption growth persistence

$$\mathbb{E}[r_{i,t+1} - r_{f,t+1}]^{EZ} \approx (\frac{\alpha}{1-\beta\phi_c} + 1)\mathbf{Cov}\big(\Delta c_{t+1}, r_{i,t+1} - r_{f,t+1}\big). \tag{1.18}$$

If consumption growth persistence $\phi_c$ or the rate of time preferences $\beta$ are high enough so that $1 - \beta\phi_c \approx 0$, then covariances of consumption growth with stock returns can generate plausible equity risk premia, even if the coefficient of risk aversion

[32] The half-life of consumption growth shocks when consumption growth follows an AR(1) process is equal to $\log(0.5)/log(\phi_{AR(1)})$ in which $\phi_{AR(1)}$ is the first-order autocorrelation coefficient.

[33] $r_{i,t} = logR_{i,t}$

$\alpha$ is low. For $\phi_c = 0$ however, risk aversion estimates for the Epstein-Zin model are the same as in the time-separable case. If additionally we allow the elasticity of intertemporal substitution to be greater than one, instead of unitary EIS as is assumed here, then the effects of consumption growth persistence will be even more pronounced. Beeler and Campbell (2012) highlight the interaction between expected consumption growth persistence and an EIS higher than one as the main driving force behind equity risk premia in the long-run risk model of Bansal and Yaron (2004). In the long-run risk model, equity premia are almost zero if the EIS is lower than one or if consumption growth is i.i.d.[34], unless one assumes extremely high values for the coefficient of risk aversion $\alpha$.

Turning to the linear disappointment model in (1.16), the disappointment threshold $d_1$ is -0.913, higher than the threshold for the log-linear case (-0.780 in Table 1.7.2). Similarly, disappointment events for the linear model happen with probability 11.111%, and are less frequent relative to the log-linear case[35]. The rate of time preference for the linear disappointment aversion model is 0.987 ($t$-stat. 340.996)[36], and the disappointment aversion coefficient $\theta$ is 9.331[37] ($t$-stat. 1.070). The GMM cannot accurately estimate the disappointment aversion for the linear model probably because the GMM function remains constant for a range of $\theta$ values. Nevertheless, with a $p$-value of 0.074 the null hypothesis for the linear disappointment model cannot be rejected at a 5% confidence level.

Table 1.7.2 also shows mean absolute prediction errors (m.a.p.e.) across all models, and Figure 1.8.1 shows fitted and sample expected returns according to the expression in (1.12). Prediction errors for the disappointment aversion discount factors (log-

[34] Table 4, p. 23 in Bonomo et al. (2011).

[35] Disappointment years for the linear disappointment aversion discount factor happened in 1957, 1973, 1979, 1980, 1990, 2007, 2008.

[36] The high $t$-statistic is due to the fact that the linear disappointment model exactly pins down the rate of time preference $\beta$ from the moment condition $\mathbb{E}[R_{f,t+1}\beta] = 1$.

[37] For their version of the linear model, Routledge and Zin (2010) set the disappointment aversion parameter equal to 9.

linear m.a.p.e. 0.99%, linear m.a.p.e. 0.99%) are smaller than for the rest of the models. The market model is the least accurate model since average prediction error is 2.38% and fitted returns in Figure 1.8.1 (graph b) are almost parallel to the horizontal axis. The Fama-French-Carhart model does a better job than the market model (FF m.a.p.e. 1.12%), and its accuracy is superior to consumption models (CRRA m.a.p.e. 1.51%, EZ m.a.p.e. 1.35%). However, in-sample prediction errors for the Fama-French-Carhart specification are slightly lager than the errors for the disappointment aversion models. In accordance to m.a.p.e. results, fitted expected returns for the disappointment models (plots a & f in Figure 1.8.1) are aligned in an orderly fashion along the 45° line.

Relative to the time-additive CRRA and Epstein-Zin models in (1.14) and (1.15), the log-linear disappointment aversion discount factor in (1.8) has an additional free parameter, the disappointment aversion coefficient $\theta$. We would therefore expect the disappointment aversion discount factor to fit the data better than traditional consumption models. However, results in Table 1.7.2 and Figure 1.8.1 suggest that the linear disappointment discount factor performs better than the CRRA and Epstein-Zin discount factors while maintaining the same number of free parameters.

The empirical performance of disappointment aversion preferences can be explained by three important characteristics. The first one is common to all consumption models, and is related to consumption smoothing. During bad times, when consumption growth is low, the discount factor is high. According to equation (1.12), assets that covary positively with the stochastic discount factor $M_{t,t+1}$, that is assets that perform well in states of the world for which consumption growth is low, essentially provide insurance to investors. These assets command low, even negative, expected returns. On the other hand, assets which do well when consumption growth is high, but perform poorly when consumption growth is low (negative covariance with the stochastic discount factor), command high expected returns so as to entice the aggregate investor to include these assets in her portfolio.

Second, disappointment averse investors are reluctant to take small bets due to non differentiable preferences with asymmetric marginal utility over gains and losses. Aggregate consumption growth exhibits extremely low time-series variability, which in turn implies very low covariances between assets returns and consumption growth[38]. If investors' preferences are described by continuously differentiable functions, then these functions need to be extremely concave in order to generate the observed equity premia. In contrast, with disappointment aversion preferences, whenever disappointment events occur, there is an upwards jump in marginal utility. Even though these jumps in marginal utility are smoothed out by the expectation operator, first-order risk aversion terms amplify shocks to consumption growth, and generate realistic risk premia with preference parameters which are smaller in magnitude than those in second-order risk aversion models.

The third characteristic is related to the reference point for disappointment events. According to the expression in (1.7), reference levels for disappointment and gains are endogenously defined, and depend on preference parameters $\alpha$ and $\theta$. Furthermore, in a dynamic setting the expectation-based reference point for disappointment aversion preferences is forward-looking which matches perfectly the forward-looking nature of asset prices. On the other hand, most first-order risk aversion models assume reference points which are exogenously specified. Relative to other first-order risk aversion models, the disappointment framework seems to provide a more accurate description of what investors consider gains and losses.

The sceptical reader might argue that by introducing a non-differentiable utility function, one can reduce the required magnitude of the risk aversion coefficient because second-order risk aversion and disappointment aversion are perfect substitutes. While this might be partially true, the discussion in the introductory and literature

[38]Table 1.7.1.

review parts of this paper, and references therein, emphasize important theoretical differences between the two concepts. First-order risk aversion can resolve a number of stylized facts about decisions under uncertainty which cannot be explained by smooth utility functions. If second-order risk aversion and disappointment aversion were perfect substitutes, then prediction errors in Table 1.7.2 for the two types of consumption models should be identical. Moreover, expected returns for traditional consumption models (graphs d and e in Figure 1.8.1) should perfectly match those for disappointment aversion preferences (graphs a and f).

### 1.4.4 Disappointment events and NBER recessions

Figure 1.8.2 plots consumption growth, disappointment years, and NBER recession dates. Disappointment events are estimated from the Euler equations for the 25 Fama-French portfolios plus the risk-free rate, and are highlighted with ellipses. When consumption growth is i.i.d, the disappointment threshold is constant across time (the flat line in Figure 1.8.2) and equal to 0.84%[39]. When consumption growth is AR(1), the disappointment threshold is time-varying (the dashed line in Figure 1.8.2). Overall, disappointment events are connected to real economic activity. The stock market crisis of 1987 or the LTCM bailout in 1998 are not considered disappointment events since the financial meltdowns did not spill over to aggregate consumption. Disappointment events emphasize an important aspect of consumption asset pricing models: financial assets are priced according to the co-movement of these assets with aggregate consumption and the real economy. Financial crises are therefore priced into asset returns only to the extent that these crises spill over to the real sector. This is exactly what happened during the recent 2007-2009 recession.

[39]For i.i.d. consumption growth, disappointment events are characterized by the threshold $\mu_c + d_1\sigma_c \approx 0.84\%$. $\mu_c$ is the unconditional expected consumption growth (1.922% from Table 1.7.1), $\sigma_c$ is the unconditional standard deviation for consumption growth (1.264% from Table 1.7.1), and $d_1$ is the disappointment threshold (-0.854 in untabulated results for i.i.d. consumption growth and the set of the 25 Fama-French portfolios plus the risk-free rate).

According to Figure 1.8.2, disappointment events tend to pre-date NBER recession years. In order to test how often disappointment events are followed by recessions, I run logistic regressions in which the dependent variable is an indicator function depending on whether there are at least three NBER recession months in year $t$

$$Y = \mathbf{1}\{\text{at least three months in year } t \text{ are NBER recession months}\}.$$

The explanatory variable is also an indicator function depending on whether year $t-1$ was a disappointment year

$$X = \mathbf{1}\{\text{year } t-1 \text{ was a disappointment year}\}.$$

Disappointment years are estimated for the set of 25 BM-size portfolios and the disappointment discount factor in (1.8) with AR(1) consumption growth (the ellipses in Figure 1.8.2). Panel A in Table 1.7.3 presents results for the logistic regression. If year $t-1$ is a disappointment year, then the probability that there will be more than three NBER recession months during year $t$ increases from $(1+e^{1.727})^{-1} = 15.09\%$[40] to $(1+e^{1.727-3.806})^{-1} = 88.88\%$. Furthermore, since the $p$-value for the log-likelihood test is almost zero, we can reject the null hypothesis that the two logistic regression models, with and without disappointment events as an explanatory variable, have the same overall fit.

In order to emphasize the fact that disappointment events precede NBER recessions, I repeat the above exercise, but now the explanatory variable is an indicator function depending on whether year $t$ is also a disappointment year.

$$X = \mathbf{1}\{\text{year } t \text{ is also a disappointment year}\}.$$

[40] 15.09% is the probability that at least three months in year $t$ are NBER recession months given that year $t-1$ was not a disappointment year.

Results in Panel B suggest that disappointment events do not overlap with NBER recessions since regression coefficients are statistically insignificant (0.251, $t$-stat. 0.330). Moreover, the high $p$-value (0.743) indicates that including contemporaneous disappointment events to the logistic model does not improve the overall fit relative to the model with the constant term alone. The above results establish that the set of disappointment events is different than the set of NBER recessions, and that disappointment events tend to pre-date NBER recessions.

### 1.4.5 Out-of-sample performance

Consumption-based stochastic discount factors are usually structural models that rely on deep economic parameters such as the rate of time preference, first or second-order risk aversion parameters, the elasticity of intertemporal substitution, the elasticity of substitution across different consumption goods, the Frisch elasticity of labor supply and so on. Estimates for these parameters should remain roughly the same across time[41] and across assets. In this section, the set of asset pricing models is submitted to a series of out-of-sample performance tests. Besides providing additional information for the disappointment model, out-of-sample tests can also help address the critique in Lewellen et al. (2010) on the structural nature of book-to-market portfolios.

Using estimation results for the 25 Fama-French portfolios in Table 1.7.2, I calculate prediction errors according to the expression in (1.12) when the estimated asset pricing models are applied to 10 equal-weighted earnings-to-price (EP) portfolios. Earnings-to-price portfolios have also been used by Fama and French (1993) as testing assets. The stock market portfolio is also included as an out-of-sample testing asset for consumption models only, since the Fama-French and market models already include market returns as an asset pricing factor. For the market portfolio tests, I

[41] The possibility of exogenous time variation in preference parameters is generally unappealing to most economists.

also set preference parameters in the log-linear disappointment aversion model equal to the clinical estimates from Choi et al. (2007): the disappointment aversion parameter $\theta$ is 1.876, and the second-order risk aversion coefficient $\alpha$ is 2.871. Choi et al. (2007) perform their clinical experiments in an atemporal setting, and do not provide any guidance on the choice of $\beta$ which I set equal to 0.99. Finally, for the Choi et al. (2007) parametrization, I assume an extremely persistent process for consumption growth in which the autocorrelation coefficient $\phi_c$ is equal to 0.968.

Panel A in Table 1.7.4 shows out-of-sample results for the set of discount factors considered in this study and the 10 EP portfolios. Disappointment aversion models seem to outperform all other models in terms of prediction errors (linear m.a.p.e. 0.40%, log-linear m.a.p.e. 0.80%). According to graph a in Figure 1.8.3, predicted and sample returns for the disappointment aversion model are almost perfectly aligned across the diagonal. In terms of the market-wide equity premium (Panel B), disappointment models outperform standard consumption models, and can almost perfectly replicate stock market expected returns (linear m.a.p.e. 0.24%), even though preference parameters have been estimated from the set of 25 Fama-French portfolios. Prediction errors for the Choi et al. (2007) model are also very low (0.15%), but this is mainly due to consumption growth autocorrelation, which is set equal to 0.968. Fitted expected returns for the Choi et al. (2007) parametrization with extremely persistent consumption growth prove that, according to the expression in (1.18), if consumption growth persistence $\phi_c$ or the rate of time preference $\beta$ are large enough, clinical estimates for risk and disappointment aversion parameters can fully rationalize the equity premium.

In addition to cross-sectional out-of-sample tests, I also study the out-of-sample accuracy of the asset pricing models across the time-series dimension. First, I estimate model parameters for the extended set of discount factors using stock returns from 1949 to 1978. Then, I use the estimated parameters to generate model-implied expected returns according to (1.12) for the second half of the sample. For these tests, I set consumption growth moments (autocorrelation, mean, standard deviation) equal to the full sample estimates from Table 1.7.1.

Table 1.7.5 shows GMM results for the 1949-1978 sample. Parameter estimates for the market and Fama-French-Carhart specifications are statistically significant, and are comparable to the full-sample results from Table 1.7.2, with the exception of the momentum coefficient (-6.607 vs. 0.268 for the full sample in Table 1.7.2). The risk aversion estimate for the CRRA model during the 1949-1978 period is 61.229 ($t$-stat. 2.179), slightly larger than for the full sample. The rate of time preference for the Epstein-Zin model is higher than one (1.104, $t$-stat. 5.508), and the second-order risk aversion estimate is 30.014 ($t$-stat. 2.706), which is lower than the one obtained for the full sample in Table 1.7.2. Finally, estimates for the disappointment aversion parameter $\theta$ in the log-linear and linear disappointment models are 3.990 ($t$-stat. 2.768) and 6.810 ($t$-stat. 1.980) respectively. None of the models is rejected since all $p$-values are large. Nevertheless, standard errors are not reliable, and test statistics should be interpreted with caution since there are only 30 observations in the sample.

Table 1.7.5 also shows out-of-sample mean absolute prediction errors for the four models during the 1979-2011 period. The Fama-French-Carhart model cannot price expected returns out of sample since the mean absolute prediction error for 1979-2011 period is 13.59%. The market, CRRA, and linear disappointment aversion models do not do well either, since average out-of-sample errors are equal to 4.10%, 3.22%, and 2.67% respectively. In contrast, the log-linear disappointment aversion and Epstein-Zin models outperform all other specifications with average prediction errors of 2.17% and 1.99% respectively. Figure 1.8.4 and Figure 1.8.5 show expected stock returns for the first and second half of the sample. According to Figure 1.8.4, the Fama-French-Carhart specification clearly performs better than all other specifications in terms of in-sample accuracy. However, plot c in Figure 1.8.5 shows that the Fama-French-Carhart model cannot explain out-of-sample expected returns.

Figure 1.8.6 and Figure 1.8.7 show expected stock returns for 10 book-to-market portfolios during the first and second half of the sample respectively. Estimation results can be found in Table 1.7.6. In terms of point estimates, results in Table 1.7.6 are quite similar to the ones obtained for the 25 portfolios in Table 1.7.2. Figure 1.8.6 highlights the impressive in-sample performance of the Fama-Frech discount factor (FF in-sample m.a.p.e. 0.21%). However, out-of-sample prediction errors for the second half are extremely large (FF out-of-sample m.a.p.e. 10.70%). According to Figure 1.8.6 and Figure 1.8.7, consumption models exhibit more consistent performance across samples than the Fama-French model, and this is probably due to the structural nature of these models.

Large out-of-sample errors for the Fama-French-Carhart model do not imply that we should automatically dismiss this model, but rather that its unconditional version fails to capture time variation in risk premia. Following Ferson and Harvey (1991), and Jagannathan and Wang (1996), there is a large literature on time-varying betas which seem to improve the performance of factor-based asset pricing models. On the other hand, the disappointment model delivers out-of-sample performance with constant preferences parameters, since time-variation in risk aversion, and therefore in expected risk premia, is hardwired into disappointment aversion terms. The impressive out-of-sample performance for the disappointment model should also be attributed to better consumption measurements towards the end of the sample, and the realization of particularly important disappointment events in 1990 and 2007-2008.

### 1.4.6 Estimation results for first-order risk aversion preferences with alternative reference points for gains and losses

The empirical evidence in this paper emphasize the importance of endogenous reference points for gains and losses in explaining the cross-section of expected stock returns. In this section, I estimate three additional consumption models which are very similar to the disappointment aversion stochastic discount factor in (1.8). However, unlike the disappointment aversion framework, reference points for gains and losses are no longer equal to the certainty equivalent for consumption growth.

The first-order risk aversion discount factor specification to be tested is

$$\begin{aligned} M_{t,t+1} = exp\Big[ & \underbrace{log\beta - \Delta c_{t+1}}_{\text{time correction}} \\ & \underbrace{+\frac{\alpha\mu_c}{1-\beta\phi_c}(1-\phi_c) - \frac{1}{2}\Big(\frac{\alpha\sigma_c}{1-\beta\phi_c}\Big)^2(1-\phi_c^2) - \frac{\alpha}{1-\beta\phi_c}\Delta c_{t+1} + \frac{\alpha}{\beta}\phi_v\Delta c_t\Big]\times}_{\text{second-order risk correction}} \\ & \underbrace{\frac{1+\theta\mathbf{1}\{\Delta c_{t+1} \ < \ \bar{d}\}}{1+\theta\mathbb{E}_t\big[\mathbf{1}\{\Delta c_{t+1} < \bar{d} + \alpha(\phi_v+1)(1-\phi_c^2)\sigma_c^2\}\big]}}_{\text{first-order risk correction}}. \end{aligned} \tag{1.19}$$

in which $\bar{d}$ is the exogenous reference point for gains and losses. It is straightforward to show that $M_{t,t+1}$ in (1.19) is non-negative and that

$$\begin{aligned} &\mathbb{E}_t\Big\{exp\Big[\alpha(\phi_v+1)\mu_c(1-\phi_c) - \tfrac{\alpha^2}{2}\sigma_c^2(1-\phi_c^2)(\phi_v+1)^2 - \tfrac{\alpha}{1-\beta\phi_c}\Delta c_{t+1} + \tfrac{\alpha}{\beta}\phi_v\Delta c_t \\ &-log\big(1+\theta\mathbb{E}_t\big[\Delta c_{t+1} < \bar{d} + \alpha(\phi_v+1)(1-\phi_c^2)\sigma_c^2\big]\big) + log(1+\theta\mathbf{1}\{\Delta c_{t+1} \ < \ \bar{d}\})\Big]\Big\} = 1, \end{aligned}$$

provided that i) consumption growth is log-normal, ii) its dynamics are given by the expression in (1.5), and iii) $\phi_v = \frac{\beta\phi_c}{1-\beta\phi_c}$. Note that utility functions corresponding to the discount factor in (1.19) are hard, or even impossible, to aggregate because preferences are no longer homothetic.

Disappointment events are defined in equation (1.7) as years during which consumption growth drops below its certainty equivalent. Similarly, we can define loss events as periods during which consumption growth drops below the threshold $\bar{d}$. I consider four different values for $\bar{d}$: i) the log risk-free rate $r_{f,t+1}$, ii) current period's consumption growth $\Delta c_t$, iii) zero consumption growth, and iv) $\bar{d}$ is a free parameter to be estimated. The above parameter values are intuitively appealing, and have been previously used in the literature[42].

Table 1.7.7 shows results for the discount factor in (1.19). When $\bar{d}$ is equal to the log risk-free rate, the probability of a loss event is 33.333%, the rate of time preference is 0.917 ($t$-stat. 5.143), the second-order risk aversion estimates is quite high ($\alpha$ = 46.784, $t$-stat. 3.015), and the disappointment aversion parameter is negative ($\theta$ = -0.827, $t$-stat. -3.073). When $\bar{d}$ is equal to current consumption growth, the probability of a loss event is 53.568%, the rate of time preference is larger than one (1.215, $t$-stat. 6.590), the second-order risk aversion estimate (54.227, $t$-stat. 13.369) is almost equal to the time-separable CRRA case from Table 1.7.2, and the first-order risk aversion parameter is negative (-0.916, $t$-stat. -7.794). We can therefore conclude that whenever the reference point $\bar{d}$ is equal to either the log risk-free rate or current consumption growth, then loss events happen so often that: i) they become irrelevant for asset pricing, ii) the first-order risk aversion parameter is negative, and iii) the second-order risk aversion parameter is similar in magnitude to the time-additive CRRA estimates from Table 1.7.2.

Results are more economically sensible when the reference point for consumption growth is zero (the status quo). This reference point can also be interpreted as the outcome of a reference mechanism for consumption in levels: $C_{t+1} < C_t$. In this case, loss events happen rarely with probability 6.349% because the loss threshold is quite low. The rate of time preference is lower than one (0.919, $t$-stat. 11.963), the first-order risk aversion estimate is quite low (1.511, $t$-stat. 0.428), and the second-order

[42]Barberis et al. (2001) and Piccioni (2011) use the risk-free rate as a reference point, whereas in Bernatzi and Thaler (1995) the reference point is zero.

risk aversion parameter is equal to 19.297 ($t$-stat. 0.793).

Finally, estimates for the free threshold model are very similar to the benchmark disappointment aversion model from Table 1.7.2. The rate of time preference is lower than one (0.903, $t$-stat. 4.444), while the first and second-order risk aversion parameters are equal to 4.113 ($t$-stat. 2.018) and 13.043 ($t$-stat. 0.875) respectively. The estimated reference point for consumption growth, $\hat{\bar{d}} = 0.47\%$, is greater than zero but lower than the i.i.d. disappointment reference level of 0.84% in Figure 1.8.2. Empirical results for the disappointment aversion and free threshold models suggest that loss events in consumption-based asset pricing models are triggered by positive thresholds rather than zero or negative consumption growth.

None of the models in Table 1.7.7 is rejected. However, mean absolute prediction errors across different models indicate that the relatively high $p$-values for first-order risk aversion models are mainly driven by large covariance estimates, rather than zero means for the error terms. Similarly, mean absolute prediction errors for first-order risk aversion models are larger than those for the disappointment model in Table 1.7.2. For $\bar{d} = r_{f,t+1}$ m.a.p.e. is 2.02%, for $\bar{d} = \Delta c_t$ m.a.p.e. is 1.84%, for $\bar{d} = 0$ m.a.p.e. is 1.54%, and for the free threshold model with $\hat{\bar{d}} = 0.47\%$ m.a.p.e. is 1.23%.

Figure 1.8.8 shows fitted expected returns for first-order risk aversion models plus the disappointment aversion discount factor from (1.8). According to Figure 1.8.8, the free threshold and disappointment aversion discount factors outperform the rest of the models in terms of fitted expected returns. The above results highlight the fact that asymmetric marginal utility alone does not improve the performance of consumption-based asset pricing models. First-order risk aversion preferences must be combined with an accurate description of investors' perception of losses in order to achieve accurate asset pricing moments.

### 1.4.7 Estimation results for quarterly stock returns

Discrete-time models do not provide any guidelines as to how often investors should evaluate their wealth, and adjust their consumption. If an optimal consumption rebalancing frequency exists, then it will undoubtedly affect the empirical performance of consumption-based asset pricing models. This section studies the performance of asset pricing models at the quarterly frequency in order to shed more light on the relevant frequency of consumption adjustments by disappointment averse individuals.

Table 1.7.8 shows GMM results for the 25 Fama-French portfolios and the set of discount factors. The intercept for the market discount factor is economically and statistically significant (4.317, $t$-stat. 4.342), while the loading on market returns (-3.253, $t$-stat. -3.392) is similar to the one estimated from annual data. For the Fama-French-Carhart model, all terms are statistically significant. According to Table 1.7.8, the equity premium puzzle is more pronounced for quarterly data since second-order risk aversion parameters for the time-additive CRRA and Epstein-Zin discount factors are extremely large: 138.538 ($t$-stat. 3.368) and 147.910 ($t$-stat. 1.406) respectively[43]. Notice also that the rate of time preference $\beta$ for CRRA utility is higher than one (1.483, $t$-stat. 13.030).

Why are second-order risk aversion estimates for the CRRA and Epstein-Zin models so large? Gabaix and Laibson (2002) propose a continuous-time model in which at each point in time only a fraction of investors adjust consumption for a period of $D$ time-units. The authors show that adjustment delays cause covariances of aggregate consumption with asset returns to be very low. According to Gabaix and Laibson (2002), second-order risk aversion parameters should be divided by $6D$ ("$6D$ bias") with $D$ being the adjustment period[44]. If we believe the $6D$ bias, and investors adjust

[43] Aït-Sahalia et al. (2004) and Yogo (2006) obtain even larger estimates for the second-order risk aversion parameter using quarterly data.

[44] Breeden et al. (1989) suggest dividing the second-order risk aversion estimate by 2 in order to

their consumption every 4 quarters, then quarterly estimates for risk aversion parameters should be equal to $138/(6\cdot 4) \approx 5.75$ for CRRA preferences, and $148/(6\cdot 4) \approx 6.16$ for the Epstein-Zin model. However, Piazzesi (2002) shows that adjustment delays in consumption are not enough to generate plausible equity premia, and that the Gabaix-Laibson model has a number of undesirable implications.

Unlike second-order risk aversion models, preference parameters for the disappointment aversion discount factor remain roughly equal to their annual counterparts. The disappointment aversion coefficient $\theta$ in the linear model is 7.932 ($t$-stat. 1.412), whereas for the log-linear disappointment aversion model $\theta$ is 5.274 ($t$-stat. 2.861) and $\alpha$ is 14.376 ($t$-stat. 0.352). The disappointment threshold $d_1$ is -0.858 for the linear model, and -0.774 for the log-linear case, while the probabilities of disappointment events are 15.294% and 16.862% respectively. Disappointment thresholds and disappointment event probabilities for quarterly data are similar to those obtained for annual data in Table 1.7.2 because preference parameters for the two samples are almost identical. The fact that disappointment aversion parameters remain constant across frequencies, while risk aversion triples in magnitude, emphasizes that first and second-order risk aversion models are not perfect substitutes, and that the two specifications have both quantitative and qualitative differences.

According to Table 1.7.8, the CRRA model achieves the lowest mean absolute prediction error (0.40%) among consumption models, probably because the AR(1) specification in non-separable preferences does not fit quarterly consumption growth well (Table 1.7.1). The Fama-French-Carhart specification generates the lowest prediction error among all models (0.24%). Figure 1.8.9 shows predicted and sample expected returns at the quarterly frequency. Although there is a weak alignment pattern between predicted and sample expected returns for the disappointment discount factors (graphs a & f), the latter models tend to overestimate expected returns for

correct for the summation bias in consumption measures.

low book-to-market portfolios (portfolios 1, 6, 11, and 16).

An important issue that emerges from quarterly data is the disappointing performance of the disappointment models. Bernatzi and Thaler (1995) combine loss aversion with narrow framing[45] under the term "myopic loss aversion". They provide evidence that stock market equity premia can be explained by a model in which loss averse investors evaluate portfolio performance and rebalance consumption infrequently:

*The longer the investor intends to hold the asset, the more attractive the risky asset will appear, so long the investment is not evaluated frequently.* Bernatzi and Thaler (1995), p. 75.

My results also suggest that the disappointment aversion discount factor performs much better at low frequencies. Disappointment aversion preferences do not seem to work well for high frequencies simply because individuals do not adjust their consumption often enough. The fact that disappointment models fail at the quarterly frequency may also be related to the results in Dillenberger (2004) and Artstein-Avidan and Dillenberger (2011) where the authors show that disappointment averse individuals prefer one-shot over gradual resolution of uncertainty. According to these results, investors prefer to evaluate their portfolios once a year (one-shot resolution of uncertainty) rather than gradually accumulate information about portfolio performance every quarter, and adjust their consumption accordingly.

Aït-Sahalia et al. (2004) provide an alternative explanation for the failure of consumption models at higher frequencies which is related to consumption measurement. They claim that consumption pricing models should focus on consumption of luxury goods because these goods are more responsive to changes in wealth, and constitute a better measure for stock market participants' consumption. Yogo (2006) successfully explains quarterly expected returns for 25 Fama-French portfolios using durables

[45] The fact that investors tend to evaluate new risks in isolation instead of pooling new risks together with old ones is usually referred to as "narrow framing".

consumption, even though estimated coefficients for second-order risk aversion are extremely large (around 200). It might well be the case that consumption of nondurables and services, which is used here, is unresponsive to wealth performance on a quarterly basis, while other measures of consumption that include luxury or durable goods covary better with equity returns. Note also that this study uses seasonally adjusted consumption data from the BEA. Ferson and Harvey (1992) show that the implied smoothing in seasonally adjusted quarterly data will affect the empirical performance of consumption-based models.

Overall, results for quarterly data raise two very important questions which are left for future research: i) What determines optimal consumption rebalancing intervals when investors are disappointment averse? ii) Why are quarterly estimates for disappointment aversion parameters almost equal to annual estimates, whereas second-order risk aversion coefficients for time-additive and Epstein-Zin preferences triple in magnitude?

## 1.5 Related literature

Before concluding the discussion about disappointment aversion preferences, I will briefly relate the disappointment framework to previous results on first-order risk aversion, and to the current state of consumption-based asset pricing literature.

### 1.5.1 First-order risk aversion preferences

Starting with the seminal paper by Kahneman and Tversky (1979), there has been an abundance of experimental evidence in favor of first-order risk aversion preferences (Duncan 2010, Pope and Schweitzer 2011). Kahneman and Tversky (1979) were also among the first to introduce the concept of loss aversion which describes first-order risk aversion behavior by means of piece-wise utility functions with exogenous reference points for gains and losses. However, piece-wise utility functions are not the only way to obtain first-order risk aversion preferences. Epstein and Zin (1990) show that first-order risk aversion behavior also occurs when investors use concave functions to rescale cumulative distribution functions of random payoffs. These types of preferences are usually referred to as rank-dependent preferences (Epstein and Zin 1990).

Even though loss aversion is probably the most widely known approach for modeling first-order risk aversion preferences, there are a number of important issues which until recently have been overlooked by the literature. First, loss aversion preferences may lead to violations of the continuity and transitivity axioms for choices under uncertainty (Gul 1991). Second, the original loss aversion framework does not provide theoretical arguments as to what reference points for gains and losses should be or how these reference points should be dynamically updated. Towards the end of their paper, Kahneman and Tversky (1979) essentially discuss time-varying reference points. However, they do not provide further guidelines on how to construct endogenous reference points within the loss aversion framework. Third, contrary to the well behaved aggregation properties of the disappointment model, Ingersoll (2011) shows that loss aversion preferences cannot be aggregated under the standard assumptions of general equilibrium models.

Segal and Spivak (1990), who were among the first to introduce the term first-order risk aversion, discuss the full insurance problem[46] which can be rationalized by first-order risk aversion preferences, but cannot be explained by smooth utility functions. Rabin (2000) argues that smooth utility functions imply an approximately risk-neutral behavior "not just for negligible stakes, but for quite sizeable and economically important stakes"[47]. He also explains why second-order risk aversion preferences have unappealing implications for large scale risks, a result known as the

[46]The full insurance puzzle is related to the fact that it is never optimal to purchase full insurance when insurance policies are not actuarially fairly priced, but in practice people do so (Mossin 1968).

[47]Rabin (2000), p. 1281.

calibration theorem[48]. First-order risk aversion models are not immune to calibration theorems. Safra and Segal (2008) extend Rabin's (2000) critique on expected utility to non-expected utility models, like the disappointment aversion model, in which they assume the presence of background risk (Theorem 2, p. 1151 in Safra and Segal 2008 )[49].

Recent empirical results indicate that endogenous reference points are a very important aspect of first-order risk aversion preferences. Choi et al. (2007) identify disappointment aversion behavior during clinical experiments on portfolio decisions under uncertainty. Post et al. (2008) suggest that players' choices in the TV show "Deal or No Deal" can be explained by reference-based preferences in which reference points are affected by previous outcomes experienced during the game. Using a questionnaire experiment with stock prices, Arkes et al. (2008) identify an asymmetric adaptation process for reference points which is a function of past decision outcomes (gains vs. losses).

Doran (2010) and Crawford and Meng (2011) find evidence that taxi drivers set daily income goals (reference points) which are affected by expectations (slow day vs. a good day), and these goals change during the course of the day (dynamic updating). Choice-acclimating reference-dependent preferences have also been well documented in the context of effort provision by Abeler et al. (2011), while Gill and Prowse (2012) identify disappointment aversion preferences in real effort competition. They argue that

*Disappointment at doing worse than expected can be a powerful emotion. This emotion may be particularly intense when the disappointed agent exerted effort in competing for a prize [...] Furthermore, a rational agent who anticipates possible disappointment*

[48] Appendix A.1 also provides a brief discussion about key differences between first and second-order risk aversion preferences.

[49] Nevertheless, Chapman and Polkovnichenko (2011) show that if this background risk is a discrete random variable and investors have rank-dependent preferences, then Safra and Segal's (2008) critique cannot be applied.

*will optimize taking into account the expected disappointment arising from her choice.* Gill and Prowse (2012), p. 469.

Finally, Artstein-Avidan and Dillenberger (2011) show that their dynamic disappointment aversion framework can explain why individuals tend to pay overpriced fees in order to insure electric appliances.

First-order risk aversion preferences have already been used in prior attempts to resolve asset pricing puzzles. Epstein and Zin (1990), Bernatzi and Thaler (1995), Barberis et al. (2001), Andries (2011), Piccioni (2011), Easley and Yang (2012) are papers which use loss aversion models or some form of asymmetric marginal utility over gains and losses in order to explain the equity premium puzzle. However, none of these papers focuses on the importance of reference points for gains and losses. Epstein and Zin (2001) integrate models of first-order risk aversion into a recursive intertemporal asset-pricing framework and find that "*risk preferences that exhibit first-order risk aversion accounts for significantly more of the mean and autocorrelation properties of the data than models that exhibit only second-order risk aversion*" (Epstein and Zin 2001, p. 537). Campanale et al. (2010) introduce disappointment aversion preferences in a production economy to match the unconditional market-wide equity premium.

Ang et al. (2005) compare loss and disappointment aversion models, and emphasize the tractability of disappointment aversion preferences relative to loss aversion. The authors also argue that if expected excess returns are positive, then smooth utility functions will necessarily generate positive holdings of risky assets, while first-order risk aversion preferences can admit corner solutions: zero holdings of risky assets in spite of positive expected excess returns (non-participation effect). In a similar way, Khanapure (2012) uses disappointment aversion preferences to rationalize the fact that investors drastically cut their portfolio allocations on stocks after retirement, a puzzling behavior that cannot be explained by smooth (CRRA) preferences.

Finally, the theoretical framework in this paper assumes identical preferences across individuals which can then be aggregated due to linear homogeneity of disappointment aversion. Nevertheless, Chapman and Polkovnichenko (2009) show that in models with first-order risk aversion preferences the equity premium and the risk-free rate are sensitive to preference heterogeneity, an important implication which is ignored by the representative agent model.

### 1.5.2 Consumption-based asset pricing

Throughout this paper, I maintain that BEA consumption accurately depicts economic conditions. A number of papers have tried to improve on BEA measures of consumption by focusing on consumption of stock market participants in Mankiw and Zeldes (1991), luxury goods consumption like in Aït-Sahalia et al. (2004), consumption of durable goods in Yogo (2006), or even garbage output as in Savov (2011). An extremely important aspect of consumption measurement is limited stock market participation. According to Jorgensen (2002), stock market participants are a small sub-sample of the total population. Using aggregate consumption as a proxy for stock market participants' consumption may lead to inconsistent estimates for preference parameters. The above strand of literature is complimentary to ours. Combining more accurate measures of consumption with disappointment aversion preferences will probably resolve a number of stylized facts in financial markets. Furthermore, improving upon measures of consumption will also decrease the estimated magnitudes for risk and disappointment aversion parameters.

It has been well documented that consumption models with time-additive CRRA preferences require implausibly high values for the risk aversion parameter (Mehra and Prescott 1985) in order to explain expected stock returns. However, Bansal and Yaron (2004) show that with non-separable preferences and a persistent mean in consumption growth, consumption risk can explain stock return moments with plausible parameter values. Furthermore, Bansal et al. (2005) use the concept of long-run risk and are able to explain 60% of the cross-sectional variation in risk premia for BM, size and momentum portfolios. However, the persistent shocks in expected consumption growth implied by the long-run risk framework are difficult to detect empirically. According to the results for the linear disappointment aversion discount factor in which consumption changes are i.i.d. (Table 1.7.2 and Figure 1.8.1), disappointment events can explain stock returns even if there are no risks for the long-run, and changes in consumption are unpredictable. van Binsbergen et al. (2011) also find that short-term risks may be more important than long-term ones for the pricing of dividend strips.

Habit models, like the one proposed by Campbell and Cochrane (1999), are a promising answer to asset pricing puzzles, mainly because they allow for time-variation in expected returns. Nevertheless, according to Ljungqvist and Uhlig (2009), these models imply a weird behavior from the social planner's point of view: government interventions that destroy part of the endowment may lead to an increase in welfare. Disappointment events should not be confused with Barro's (2006) rare disaster framework either. First, contrary to rare disasters, which are not present in the post-war U.S. economy, disappointment events can be easily identified and happen relatively often. Second, disappointment events are endogenously characterized by investor preferences, and are not exogenously specified as an additional source of uncertainty.

Ju and Miao (2012) address the equity premium puzzle using the concept of smooth ambiguity aversion introduced by Klibanoff et al. (2005). Ambiguity essentially refers to uncertainty about the"true" probability distribution of stochastic variables. Klibanoff et al. (2005) propose a smooth concave "utility" function over the set of possible distributions for stochastic payoffs which implies that investors overweigh unfavorable prior distributions. Epstein (2010) highlights some unappealing characteristics of the smooth ambiguity aversion model, and proposes the multiple priors approach by Gilboa and Scmeidler (1989) instead. Although, uncertainty about "true" probability distributions for macroeconomic variables and asset returns is a realistic assumption, I abstain from such considerations, and assume a rational expectations framework with no uncertainty about probability distributions in order to focus on the performance of disappointment aversion preferences alone.

## 1.6 Conclusion

According to Kocherlacota (1996), in order to resolve the equity premium puzzle (at least) one of the following three assumptions needs to be relaxed: i) CRRA preferences, ii) market completeness, iii) transaction costs. Although, I maintain the last two assumptions, this paper focuses on the first one, and introduces disappointment aversion preferences in a general equilibrium framework. This paper is the first to obtain closed-form solutions for the stochastic discount factor in terms of consumption growth when investors are disappointment averse. Analytical solutions, in turn, allow for a wide range of empirical tests, including comparisons with more traditional asset pricing models. Unlike exogenous reference levels proposed by the majority of first-order risk aversion models, my results highlight that endogenous, expectation-based reference points for gains and losses, as suggested by disappointment aversion preferences, are important in explaining the cross-section of equity returns.

At the annual frequency, the disappointment aversion discount factor can explain expected returns for portfolios sorted on book-to-market, size, earnings-to-price, as well as the aggregate market portfolio. Comparative results also suggest that at the annual frequency disappointment aversion preferences outperform traditional asset pricing models in terms of prediction errors, and that disappointment events tend to predate NBER recessions. Nevertheless, at higher frequencies the performance of the disappointment model deteriorates, and this is probably related to the myopic loss aversion effect of Bernatzi and Thaler (1995) or due to consumption measurement issues. Directions for future research include the pricing of fixed income securities subject to default risk, introducing disappointment aversion preferences in a production economy in order to study investment, production and employment during disappointment years, or even combining disappointment aversion preferences with better measures for consumption. Finally, this study establishes that small and value firms covary more with macroeconomic conditions, and consequently, that these firms are riskier than big and growth firms respectively. However, a very important question that remains unanswered by the literature is what are the fundamental firm-level characteristics which expose small and value firms to aggregate risk.

## 1.7 Tables

### Table 1.7.1 Summary statistics

**Consumption and the risk-free rate**

| | annually | | | quarterly | | |
|---|---|---|---|---|---|---|
| | $\mathbf{\Delta c_{t+1}}$ | $\mathbf{\Delta C_{t+1}}$ | $\mathbf{r_{f,t+1}}$ | $\mathbf{\Delta c_{t+1}}$ | $\mathbf{\Delta C_{t+1}}$ | $\mathbf{r_{f,t+1}}$ |
| $\hat{\mathbb{E}}$ | 1.922% | \$291.432 | 1.174% | 0.484% | \$73.254 | 0.295% |
| $\hat{\sigma}$ | 1.264% | \$223.447 | 2.338% | 0.502% | \$77.705 | 0.644% |
| $\hat{\rho}\Big(\Delta c_{t+1},\ .\Big)$ | 1 | 0.841 | 0.087 | 1 | 0.876 | 0.057 |
| $\hat{\rho}_{t,t-1}$ | 0.463 | 0.503 | 0.701 | 0.328 | 0.509 | 0.736 |
| $R^2\ AR(1)$ | 21.968% | 24.751% | 49.261% | 10.796% | 25.909% | 54.172% |

**Panel B: 25 Fama-French portfolios**

| | | **Small/ Value** | **Medium/ Growth** | **Medium** | **Medium/ Value** | **Big/ Growth** |
|---|---|---|---|---|---|---|
| **25 BM-Size** | $\hat{\mathbb{E}}[\tilde{R}_{i,t+1}]$ | 20.766% | 8.419% | 11.971% | 15.770% | 8.204% |
| annual | $\widehat{\mathbf{Cov}}(\Delta c_{t+1}, \tilde{R}_{i,t+1})$ | 0.0017 | 0.0009 | 0.0011 | 0.0013 | 0.0010 |
| **25 BM-Size** | $\hat{\mathbb{E}}[\tilde{R}_{i,t+1}]$ | 4.667% | 2.120% | 2.880% | 3.678% | 2.006% |
| quarterly | $\widehat{\mathbf{Cov}}(\Delta c_{t+1}, \tilde{R}_{i,t+1})$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |

**Panel C: stock market, HML, SMB, and MOM factors**

| | | **market** | **HML** | **SMB** | **MOM** |
|---|---|---|---|---|---|
| annual | $\hat{\mathbb{E}}[\tilde{R}_{i,t+1}]$ | 8.930% | 5.126% | 2.669% | 9.374% |
| | $\widehat{\mathbf{Cov}}(\Delta c_{t+1}, \tilde{R}_{i,t+1})$ | 0.0010 | 0.0004 | 0.0002 | -0.0000 |
| quarterly | $\hat{\mathbb{E}}[\tilde{R}_{i,t+1}]$ | 1.778% | 1.110% | 0.504% | 2.319% |
| | $\widehat{\mathbf{Cov}}(\Delta c_{t+1}, \tilde{R}_{i,t+1})$ | 0.0001 | 0.0000 | 0.0000 | -0.0000 |

Table 1.7.1 presents summary statistics for the variables used in this study. $\Delta c_{t+1} = log\big(C_{t+1}/C_t\big)$ in Panel A is real consumption growth, $\Delta C_{t+1} = C_{t+1} - C_t$ is real consumption in first differences, and $r_{f,t+1} = log\big(R_{f,t+1}\big)$ is the real log risk-free rate. $\hat{\mathbb{E}}$ is the sample mean, $\hat{\sigma}$ is the sample standard deviation, and $\hat{\rho}\Big(\Delta c_{t+1},\ .\Big)$ is the sample correlation coefficient with consumption growth. $\hat{\rho}_{t,t-1}$ is the autocorrelation coefficient estimate, and $R^2\ AR(1)$ is the $R$-square for the $AR(1)$ model. Panel B shows summary statistics for real, cum-dividend, equity returns $\tilde{R}_{i,t}$ for the 25 Fama-French portfolios. $HML$, $SMB$, $MOM$ in Panel C are the value, size and momentum factors respectively. $\widehat{\mathbf{Cov}}$ are covariance estimates. More details on consumption data and stock returns can be found in subsection 1.4.1.

**Table 1.7.2 GMM results for the 25 Fama-French portfolios and the risk-free rate (annual data)**

| | market | Fama-French | CRRA | Epstein-Zin | linear DA | log-linear DA |
|---|---|---|---|---|---|---|
| $\hat{d}_1$ | | | | | -0.913 | -0.780 |
| $\hat{\mathbb{E}}[\mathbf{1}\{\text{disap.}\}]$ | | | | | 11.111% | 15.873% |
| $\hat{\beta}$ | | | 2.172 | 0.983 | 0.987 | 0.977 |
| | | | (3.334) | (9.395) | (340.966) | (2.868) |
| $\hat{\alpha}$ | | | 55.171 | 35.550 | | 9.929 |
| | | | (2.561) | (3.336) | | (0.574) |
| $\hat{\theta}$ | | | | | 9.331 | 4.606 |
| | | | | | (1.070) | (3.883) |
| $\hat{a}_0$ | 4.377 | 3.659 | | | | |
| | (3.661) | (2.627) | | | | |
| $\hat{b}_M$ | -3.132 | -2.268 | | | | |
| | (-2.991) | (-1.931) | | | | |
| $\hat{b}_{HML}$ | | -3.956 | | | | |
| | | (-3.058) | | | | |
| $\hat{b}_{SMB}$ | | -1.077 | | | | |
| | | (-0.978) | | | | |
| $\hat{b}_{MOM}$ | | 0.268 | | | | |
| | | (0.202) | | | | |
| $J$-test | 43.005 | 35.780 | 47.881 | 45.464 | 37.029 | 20.087 |
| $d.o.f.$ | 24 | 21 | 24 | 24 | 24 | 23 |
| *p-value* | 0.009 | 0.023 | 0.002 | 0.005 | 0.074 | 0.636 |
| m.a.p.e. | 2.38% | 1.12% | 1.51% | 1.35% | 0.99% | 0.99% |

Table 1.7.2 presents first-stage GMM results for the 25 Fama-French portfolios and the risk-free rate. $\hat{d}_1$ are disappointment thresholds for consumption growth and consumption in first differences, and are defined in (1.6) and (A.5) respectively. $\hat{\mathbb{E}}\Big[\mathbf{1}\{\text{disap.}\}\Big]$ is the (unconditional) probability for disappointment events. $\hat{\beta}$ is the rate of time preference, $\hat{\alpha}$ is the second-order risk aversion parameter, $\hat{\theta}$ is the disappointment aversion coefficient, and $\hat{b}_i$'s are factor coefficients. $t$-statistics are in parenthesis. $J$-test is a $\chi^2$ random variable that tests for over-identifying restrictions. $d.o.f.$ (degrees of freedom) is the number of over-identifying restrictions. *p-value* is the probability of obtaining a $J$-test statistic at least as large as the one estimated here, assuming the null hypothesis that all moment conditions are jointly zero is true. m.a.p.e. $(\frac{1}{n}\sum_{i=1}^{n}|\hat{\hat{E}}[R_{i,t+1}] - \hat{E}[R_{i,t+1}]|)$ are mean absolute prediction errors. $\hat{\hat{E}}[R_{i,t+1}]$ are fitted expected returns according to (1.12), and $\hat{E}[R_{i,t+1}]$ are sample expected returns from Table 1.7.1.

**Table 1.7.3 NBER recessions and disappointment years (annual data)**

| | $X^{(t-1)}$ | $X^{(t)}$ |
|---|---|---|
| *const.* | -1.727 | -1.098 |
| | (-4.501) | (-3.430) |
| $\hat{b}_X$ | 3.806 | 0.251 |
| | (3.374) | (0.330) |
| $LL$ | -25.629 | -35.350 |
| $LL_{null}$ | -35.403 | -35.403 |
| $LR$ | 19.547 | 0.106 |
| *p-value* | 0.000 | 0.743 |

Table 1.7.3 presents logistic regression results for NBER recession years and disappointment events. The dependent variable is an indicator function depending on whether at least three months in year $t$ have been characterized as recession months by the NBER. The explanatory variable is an indicator variable depending on whether year $t-1$ ($X^{(t-1)}$) or year $t$ ($X^{(t)}$) is a disappointment year. Disappointment years are estimated from 25 Fama-French portfolios and the risk-free rate in Table 1.7.2. *const.* is the constant term in the logistic regression, and $\hat{b}_X$ is the regression parameter for disappointment events. $t$-statistics are in parenthesis. $LL$ is the log-likelihood value, and $LL_{null}$ is the log-likelihood value for the logistic regression which includes the constant term only. $LR = -2LL_{null} - (-2LL)$ is the likelihood-ratio statistic, a $\chi^2$ random variable. *p-value* is the probability of obtaining a $LR$-test statistic at least as large as the one estimated here, assuming the null hypothesis that the two models (with and without disappointment events as an explanatory variable) have the same overall fit is true

.

## Table 1.7.4 Out-of-sample expected stock returns for 10 earnings-to-price portfolios and the stock market (annual data)

**Panel A: expected returns for earnings-to-price portfolios**

| | sample | market | FF | CRRA | EZ | linear DA | log-linear DA |
|---|---|---|---|---|---|---|---|
| **Low** | 9.010% | 13.500% | 10.394% | 11.649% | 11.637% | 8.173% | 9.987% |
| **3** | 10.720% | 11.514% | 10.617% | 11.393% | 11.656% | 11.072% | 11.117% |
| **Medium** | 12.925% | 11.001% | 11.650% | 11.939% | 12.052% | 12.226% | 12.225% |
| **7** | 13.996% | 11.072% | 12.587% | 12.652% | 13.105% | 13.835% | 13.800% |
| **High** | 18.974% | 13.867% | 16.406% | 15.886% | 15.806% | 17.774% | 16.068% |
| m.a.p.e. | | 2.595% | 1.049% | 1.343% | 1.356% | 0.398% | 0.806% |

**Panel B: expected returns for the value-weighted market portfolio**

| | sample | CRRA | EZ | linear DA | log-linear DA | Choi et al. (2007) |
|---|---|---|---|---|---|---|
| | 8.93% | 10.29% | 9.43% | 9.17% | 8.43% | 9.08% |
| m.a.p.e. | | 1.36% | 0.50% | 0.24% | 0.50% | 0.15% |

Table 1.7.4 shows out-of-sample expected returns for the market, Fama-French (FF), CRRA, Epstein-Zin (EZ), and disappointment aversion (DA) stochastic discount factors. Model parameters were estimated using the 25 Fama-French portfolios and the risk-free rate. Parameter estimates can be found in Table 1.7.2. Fitted expected returns are calculated according to the expression in (1.12). Out-of-sample testing assets in Panel A are 10 equal-weighted earnings-to-price portfolios. In Panel B, the market portfolio is included as an out-of-sample testing asset for consumption models only. Choi et al. (2007) corresponds to the log-linear disappointment aversion discount factor in (1.8) with parameter values from Choi et al. (2007): $\theta = 1.876$ and $\alpha = 2.871$. The rate of time preference $\beta$ for the Choi et al. (2007) model is set equal to 0.99, and the autocorrelation parameter $\phi_c$ for consumption growth is equal to 0.968. m.a.p.e. are mean absolute prediction errors.

**Table 1.7.5 GMM results for the 25 Fama-French portfolios and the risk-free rate during the 1949-1978 period (annual data)**

| | market | Fama-French | CRRA | Epstein-Zin | linear DA | log-linear DA |
|---|---|---|---|---|---|---|
| $d_1$ | | | | | -0.807 | -0.788 |
| $\hat{\mathbb{E}}[\mathbf{1}\{\text{disap.}\}]$ | | | | | 13.334% | 16.667% |
| $\hat{\beta}$ | | | 3.506 | 1.104 | 0.998 | 0.926 |
| | | | (1.678) | (5.508) | (338.014) | (2.964) |
| $\hat{\alpha}$ | | | 61.229 | 30.014 | | 15.893 |
| | | | (2.179) | (2.706) | | (0.923) |
| $\hat{\theta}$ | | | | | 6.810 | 3.990 |
| | | | | | (1.980) | (2.768) |
| $\hat{a}_0$ | 4.008 | 4.845 | | | | |
| | (2.836) | (2.641) | | | | |
| $\hat{b}_M$ | -2.789 | -2.632 | | | | |
| | (-2.265) | (-1.654) | | | | |
| $\hat{b}_{HML}$ | | -4.781 | | | | |
| | | (-2.222) | | | | |
| $\hat{b}_{SMB}$ | | -1.118 | | | | |
| | | (-0.873) | | | | |
| $\hat{b}_{MOM}$ | | -6.607 | | | | |
| | | (-1.708) | | | | |
| $J$-test | 28.769 | 28.808 | 28.737 | 29.281 | 25.822 | 4.494 |
| *d.o.f.* | 24 | 21 | 24 | 24 | 24 | 23 |
| *p-value* | 0.228 | 0.118 | 0.230 | 0.209 | 0.362 | 0.999 |
| m.a.p.e. | 2.06% | 1.10% | 2.44% | 2.48% | 1.83% | 2.12% |
| m.a.p.e. *1979-2011* | 4.10% | 13.59% | 3.22% | 1.99% | 2.67% | 2.17% |

Table 1.7.5 presents first-stage GMM results for the risk-free rate and 25 equal-weighted portfolios double sorted on BM and size. Portfolio returns are from 1949 to 1978 (30 years). $\hat{d}_1$ is the disappointment threshold. $\hat{\mathbb{E}}\Big[\mathbf{1}\{\text{disap.}\}\Big]$ is the (unconditional) probability for disappointment events. $\hat{\beta}$ is the rate of time preference, $\hat{\alpha}$ is the risk aversion parameter, $\hat{\theta}$ is the disappointment aversion coefficient, and $\hat{b}_i$'s are factor coefficients. $t$-statistics are in parenthesis. m.a.p.e. are in-sample mean absolute prediction errors, and m.a.p.e. *1979-2011* are the out-of-sample mean absolute prediction errors for the 1979 - 2011 period.

**Table 1.7.6 GMM results for 10 Book-to-Market portfolios and the risk-free rate during the 1949-1978 period (annual data)**

| | market | Fama-French | CRRA | Epstein-Zin | linear DA | log-linear DA |
|---|---|---|---|---|---|---|
| $d_1$ | | | | | -0.833 | -0.825 |
| $\hat{\mathbb{E}}[\mathbf{1}\{\text{disap.}\}]$ | | | | | 13.334% | 13.334% |
| $\hat{\beta}$ | | | 3.758 | 1.121 | 0.998 | 1.021 |
| | | | (1.636) | (5.203) | (338.017) | (2.521) |
| $\hat{\alpha}$ | | | 64.517 | 31.656 | | 14.450 |
| | | | (2.208) | (2.502) | | (4.118) |
| $\hat{\theta}$ | | | | | 7.350 | 4.521 |
| | | | | | (1.741) | (3.707) |
| $\hat{a}_0$ | 4.346 | 3.132 | | | | |
| | (2.790) | (1.747) | | | | |
| $\hat{b}_M$ | -3.093 | -1.427 | | | | |
| | (-2.272) | (-0.888) | | | | |
| $\hat{b}_{HML}$ | | -6.066 | | | | |
| | | (-2.937) | | | | |
| $\hat{b}_{SMB}$ | | -1.589 | | | | |
| | | (-0.773) | | | | |
| $\hat{b}_{MOM}$ | | -1.731 | | | | |
| | | (-0.499) | | | | |
| $J$-test | 15.135 | 2.454 | 16.213 | 18.833 | 15.414 | 17.288 |
| $d.o.f.$ | 9 | 7 | 9 | 9 | 9 | 8 |
| *p-value* | 0.087 | 0.873 | 0.062 | 0.026 | 0.080 | 0.027 |
| m.a.p.e. | 2.14% | 0.21% | 2.00% | 2.33% | 1.28% | 1.98% |
| m.a.p.e. *1979-2011* | 5.04% | 10.70% | 3.91% | 1.82% | 2.45% | 1.63% |

Table 1.7.6 presents first-stage GMM results for the risk-free rate and 10 equal-weighted portfolios sorted on BM. Portfolio returns are from 1949 to 1978 (30 years). $t$-statistics are in parenthesis. m.a.p.e. are in-sample mean absolute prediction errors, and m.a.p.e. *1979-2011* are the out-of-sample mean absolute prediction errors for the 1979 - 2011 period.

**Table 1.7.7 GMM results for first-order risk aversion preferences with alternative reference points for gains and losses (annual data)**

$$M_{t,t+1} = exp\Big[\underbrace{log\beta - \Delta c_{t+1}}_{\text{time correction}}$$

$$\underbrace{+\alpha(\phi_v+1)\mu_c(1-\phi_c) - \frac{\alpha^2}{2}(\phi_v+1)^2(1-\phi_c^2)\sigma_c^2 - \alpha(\phi_v+1)\Delta c_{t+1} + \frac{\alpha}{\beta}\phi_v\Delta c_t\Big]}_{\text{second-order risk correction}}\times$$

$$\underbrace{\frac{1+\theta\mathbf{1}\{\Delta c_{t+1} \ < \ \bar{d}\}}{1+\theta\mathbb{E}_t\big[\mathbf{1}\{\Delta c_{t+1} < \bar{d} + \alpha(\phi_v+1)(1-\phi_c^2)\sigma_c^2\}\big]}}_{\text{first-order risk correction}},$$

| | $\bar{d} = r_{f,t+1}$ | $\bar{d} = \Delta c_t$ | $\bar{d} = 0$ | $\hat{\bar{d}} = 0.47\%$ |
|---|---|---|---|---|
| $\hat{\mathbb{E}}\big[\mathbf{1}\{\text{loss}\}\big]$ | 33.333% | 53.968% | 6.349% | 11.111% |
| $\hat{\beta}$ | 0.917 | 1.215 | 0.919 | 0.903 |
| | (5.143) | (6.590) | (11.963) | (4.444) |
| $\hat{\alpha}$ | 46.784 | 54.277 | 19.2976 | 13.043 |
| | (3.015) | (13.369) | (0.793) | (0.875) |
| $\hat{\theta}$ | -0.827 | -0.916 | 1.511 | 4.113 |
| | (-3.073) | (-7.794) | (0.428) | (2.018) |
| $J$-test | 33.061 | 6.058 | 21.017 | 7.191 |
| *d.o.f.* | 23 | 23 | 23 | 22 |
| *p-value* | 0.080 | 0.998 | 0.580 | 0.998 |
| m.a.p.e. | 2.02% | 1.84% | 1.54% | 1.23% |

Table 1.7.7 presents first-stage GMM for the 25 Fama-French portfolios and the risk-free rate. Models in Table 1.7.7 are characterized by first-order risk aversion preferences with alternative reference points for gains and losses. Reference points are: i) the log risk-free rate, $\bar{d} = r_{f,t+1}$, ii) current period's consumption growth, $\bar{d} = \Delta c_t$, iii) zero consumption growth, $\bar{d} = 0$, and iv) $\bar{d}$ is a free parameter to be estimated. $\hat{\mathbb{E}}\Big[\mathbf{1}\{\text{loss}\}\Big]$ is the (unconditional) probability for loss events ($\mathbf{1}\{\Delta c_{t+1} < \bar{d}\}$). $\hat{\beta}$ is the rate of time preference, $\hat{\alpha}$ is the second-order risk aversion coefficient, and $\hat{\theta}$ is the first-order risk aversion parameter. $t$-statistics are in parenthesis. m.a.p.e. are mean absolute pricing errors.

**Table 1.7.8 GMM results for the 25 Fama-French portfolios and the risk-free rate (quarterly data)**

| | **market** | **Fama-French** | **CRRA** | **Epstein-Zin** | **linear DA** | **log-linear DA** |
|---|---|---|---|---|---|---|
| $d_1$ | | | | | -0.858 | -0.774 |
| $\hat{\mathbb{E}}[\mathbf{1}\{\text{disap.}\}]$ | | | | | 15.294% | 16.862% |
| $\hat{\beta}$ | | | 1.483 | 0.917 | 0.997 | 0.996 |
| | | | (13.030) | (5.110) | (2,486) | (4.173) |
| $\hat{\alpha}$ | | | 138.538 | 147.910 | | 14.376 |
| | | | (3.368) | (1.406) | | (0.352) |
| $\hat{\theta}$ | | | | | 7.932 | 5.274 |
| | | | | | (1.412) | (2.861) |
| $\hat{a}_0$ | 4.317 | 6.892 | | | | |
| | (4.342) | (4.296) | | | | |
| $\hat{b}_M$ | -3.253 | -5.406 | | | | |
| | (-3.392) | (-3.604) | | | | |
| $\hat{b}_{HML}$ | | -8.625 | | | | |
| | | (-4.527) | | | | |
| $\hat{b}_{SMB}$ | | -4.534 | | | | |
| | | (-1.865) | | | | |
| $\hat{b}_{MOM}$ | | -11.055 | | | | |
| | | (-2.227) | | | | |
| $J$-test | 78.741 | 43.351 | 71.640 | 70.984 | 35.234 | 8.921 |
| *d.o.f.* | 24 | 21 | 24 | 24 | 24 | 23 |
| *p-value* | 0.000 | 0.002 | 0.000 | 0.000 | 0.065 | 0.996 |
| m.a.p.e. | 0.60% | 0.24% | 0.40% | 0.48% | 0.43% | 0.42% |

Table 1.7.8 presents first-stage GMM results for the 25 Fama-French portfolios and the risk-free rate at the quarterly frequency.

## 1.8 Figures

**Figure 1.8.1 Expected returns for the 25 Fama-French portfolios and the risk-free rate (annual data)**

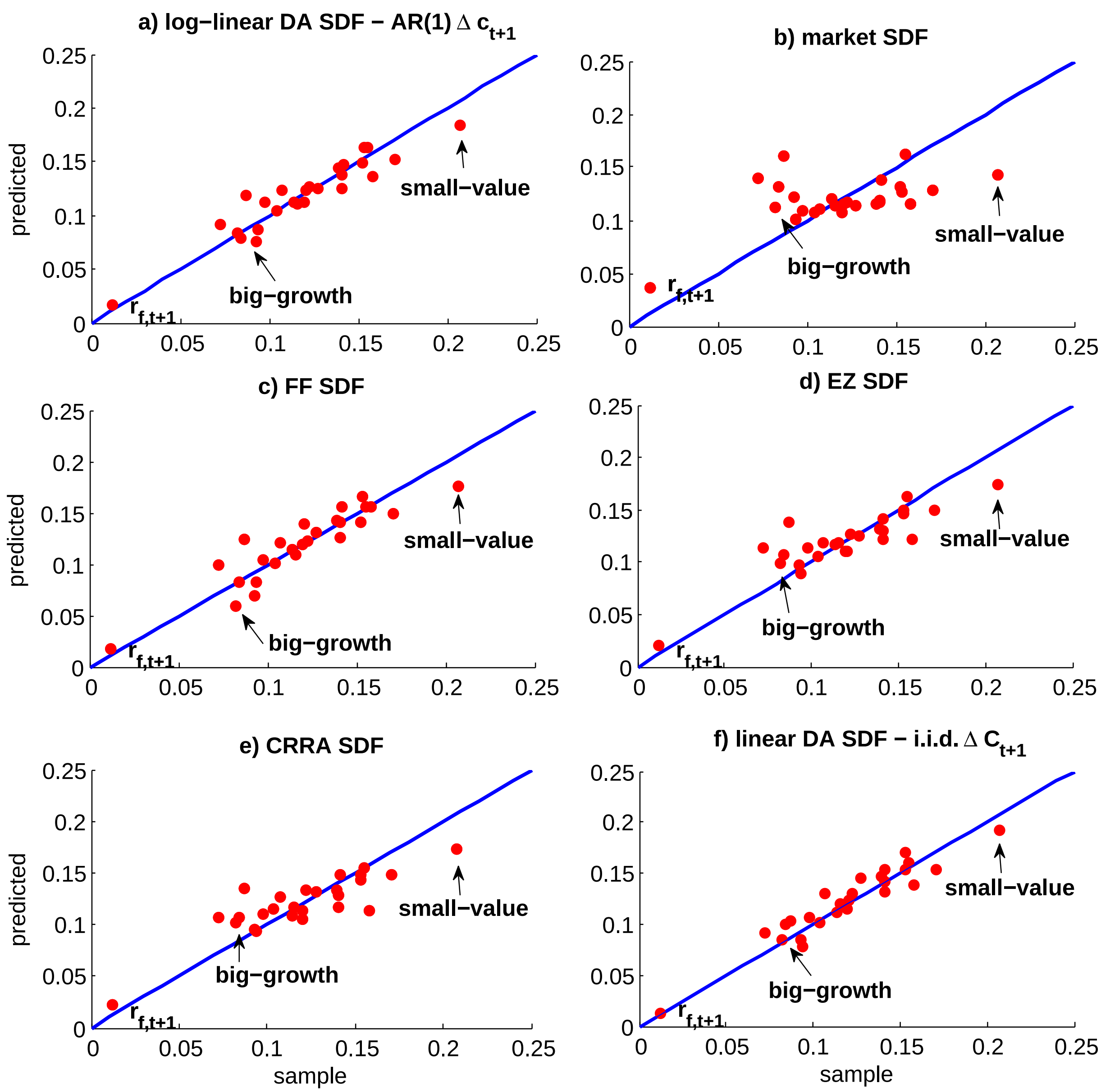


Figure 1.8.1 plots fitted (vertical axis) and sample (horizontal axis) expected equity returns for the 25 Fama-French portfolios and the risk-free rate. Estimation results can be found in Table 1.7.2. Sample expected stock returns are from Table 1.7.1, while fitted expected returns are calculated according to the expression in (1.12).

**Figure 1.8.2 Annual consumption growth, disappointment events, and NBER recession dates (annual data)**

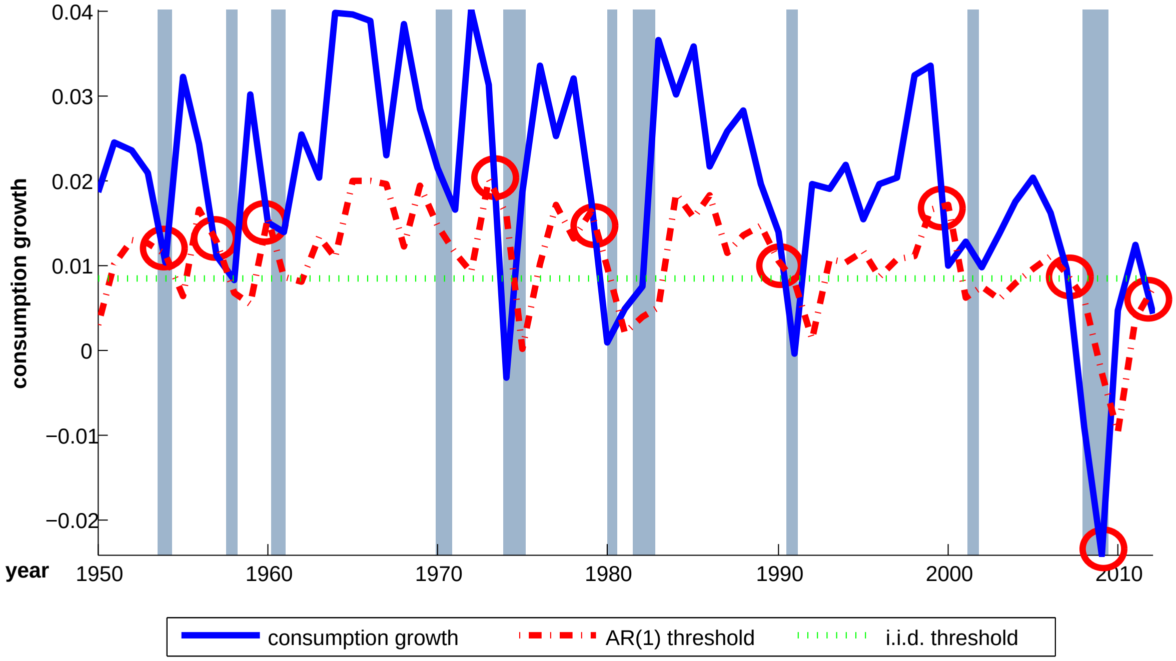


Figure 1.8.2 plots time-series for consumption growth and disappointment events. Shaded areas are NBER recession dates. Disappointment events are estimated from the 25 Fama-French portfolios plus the risk-free rate, and are highlighted by ellipses. The disappointment threshold for AR(1) consumption growth is given by the expression $\hat{\mu}_c(1-\hat{\phi}_c)+\hat{\phi}_c\Delta c_{t-1}+\hat{d}_{1,AR(1)}\sqrt{1-\hat{\phi}_c^2}\hat{\sigma}_c$ ($\Delta c_{t+1} < 1.031\% + 0.463\Delta c_t - 0.780 \cdot 1.120\%$). Moment estimates $(\hat{\mu}, \hat{\sigma}, \hat{\phi}_c)$ for consumption growth are from Table 1.7.1 and $d_{1,AR(1)}$ is from Table 1.7.2. The flat line shows the disappointment threshold when consumption growth is i.i.d.. In this case, the disappointment threshold is constant, and equal to $\hat{\mu}_c + \hat{d}_{1,i.i.d.}\hat{\sigma}_c$ ($\Delta c_{t+1} < 1.922\% - 0.854 \cdot 1.264\%$).

**Figure 1.8.3 Out-of-sample expected stock returns for 10 earnings-to-price portfolios (annual data)**

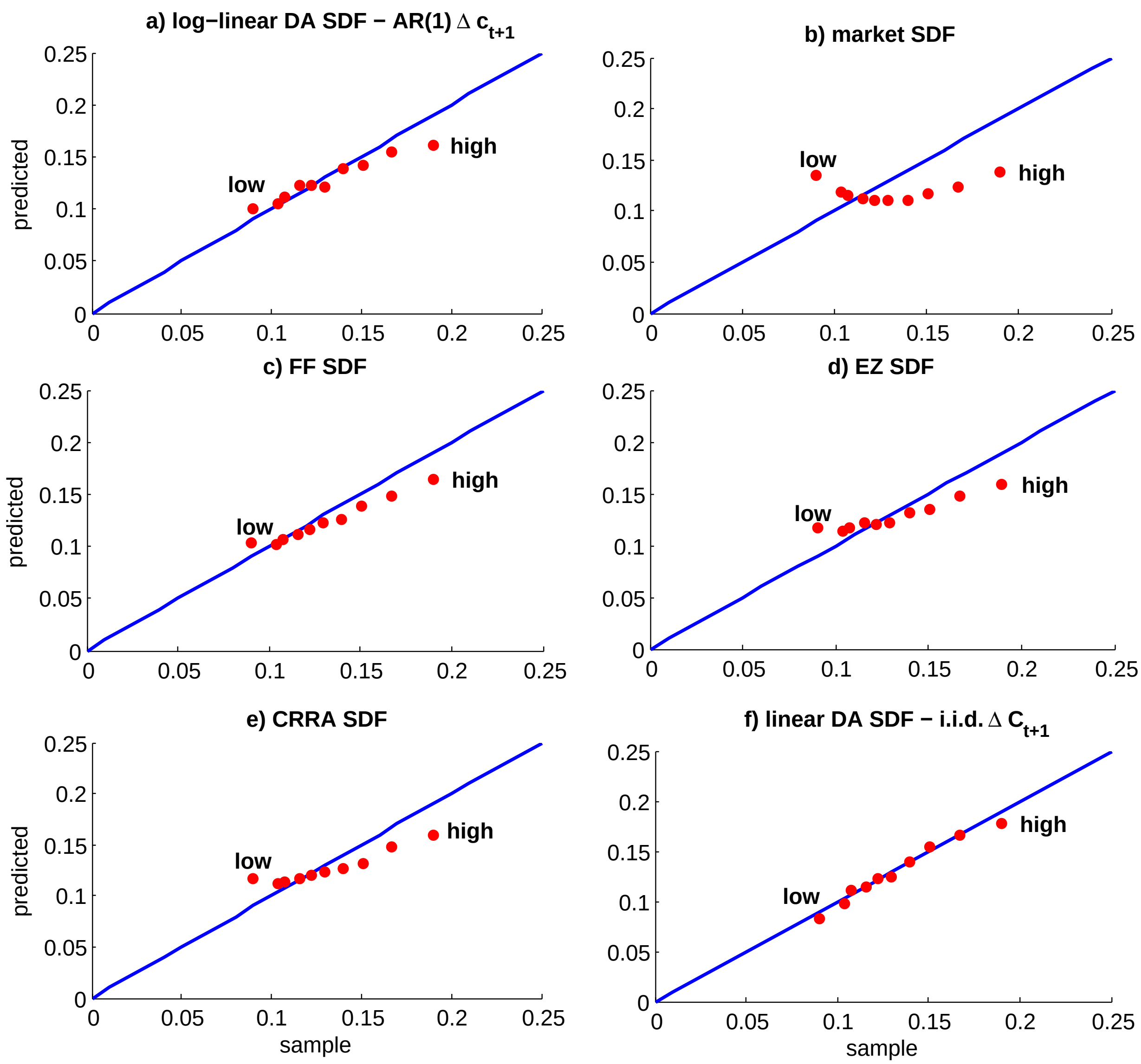


Figure 1.8.3 plots predicted and sample equity returns for 10 equal-weighted earnings-to-price portfolios. Model parameters have been estimated using the 25 Fama-French portfolios. Estimation results are shown in Table 1.7.2. Predicted expected returns for the earnings-to-price portfolios were calculated according to the expression in (1.12), and can be found in Table 1.7.4.

**Figure 1.8.4** In-sample expected returns for the 25 Fama-French portfolios and the risk-free rate during the 1949-1978 period (annual data)

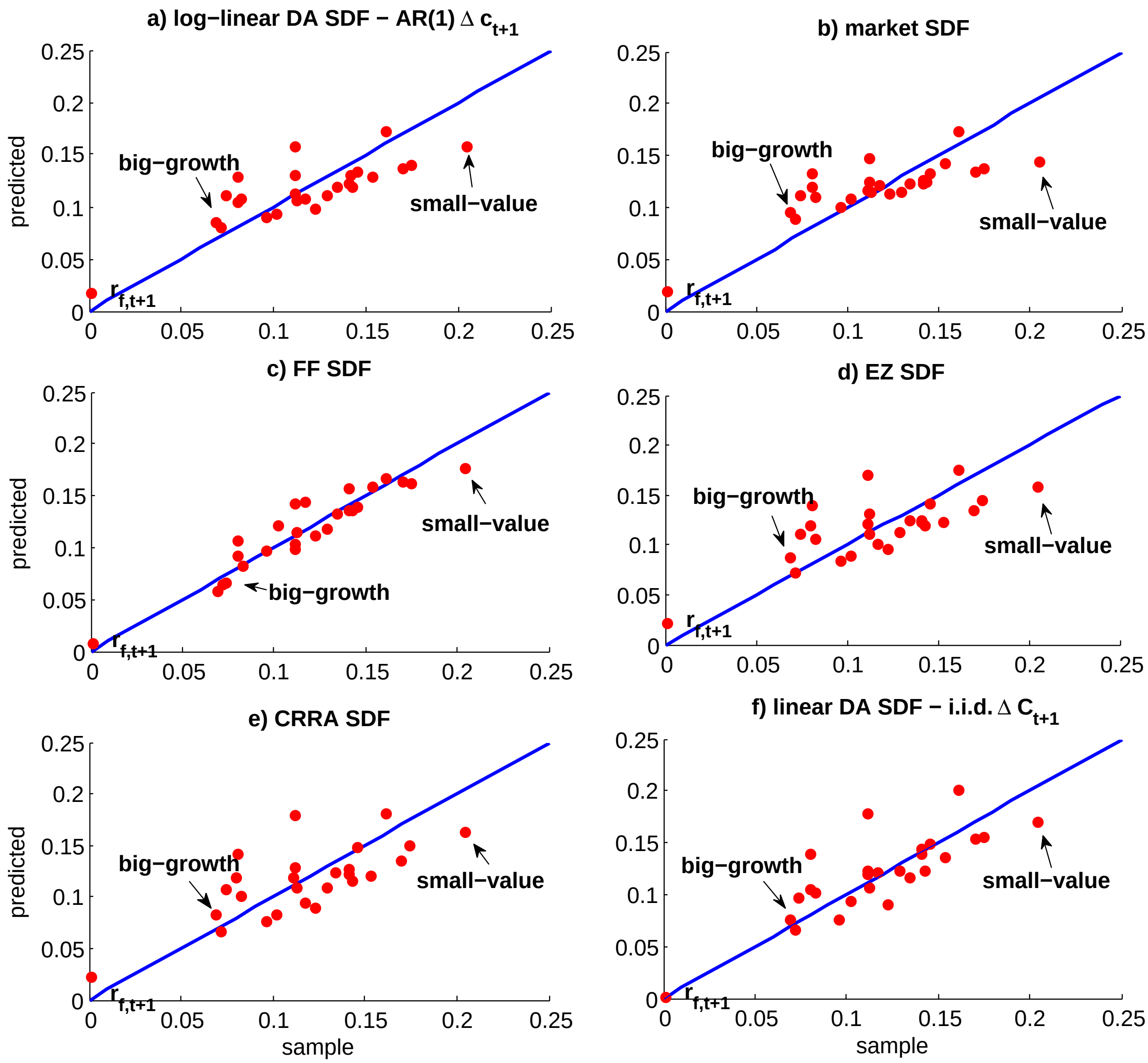


Figure 1.8.4 plots fitted and sample expected equity returns for the 25 Fama-French portfolios and the risk-free rate. I use the first thirty years of the sample to estimate model parameters, and calculate in-sample fitted expected returns according to equation (1.12). Estimation results for each model can be found in Table 1.7.5.

**Figure 1.8.5 Out-of-sample expected returns for the 25 Fama-French portfolios during the 1979-2011 period (annual data)**

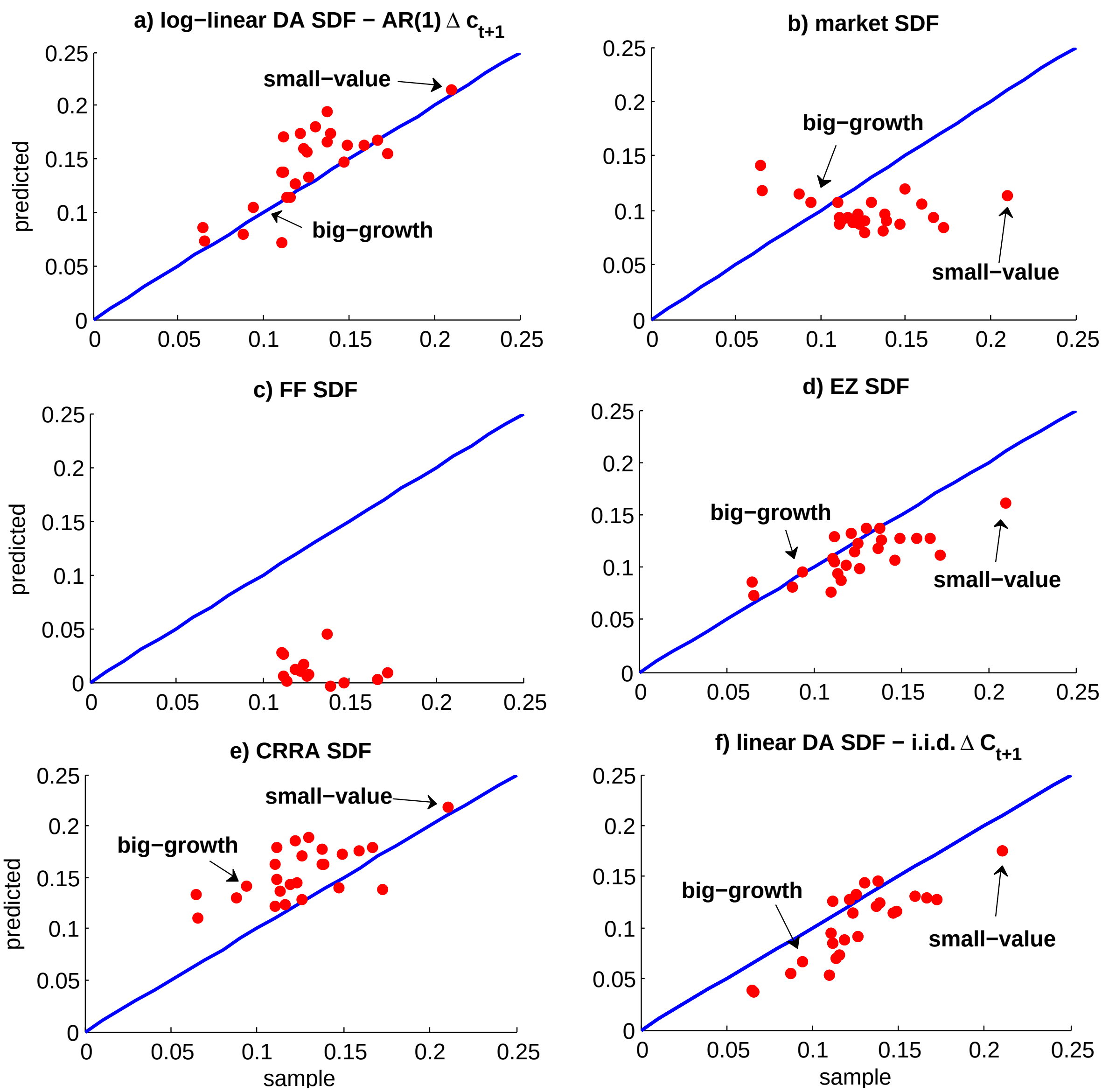


Figure 1.8.5 plots predicted and sample expected equity returns for the 25 Fama-French portfolios. I use the first thirty years of the sample to estimate model parameters, and the 1979-2011 period to test out-of-sample predictions. Predicted expected returns are derived according to the expression in (1.12), and can be found in Table 1.7.5.

**Figure 1.8.6 In-sample expected returns for 10 BM portfolios and the risk-free rate during the 1949-1978 period (annual data)**

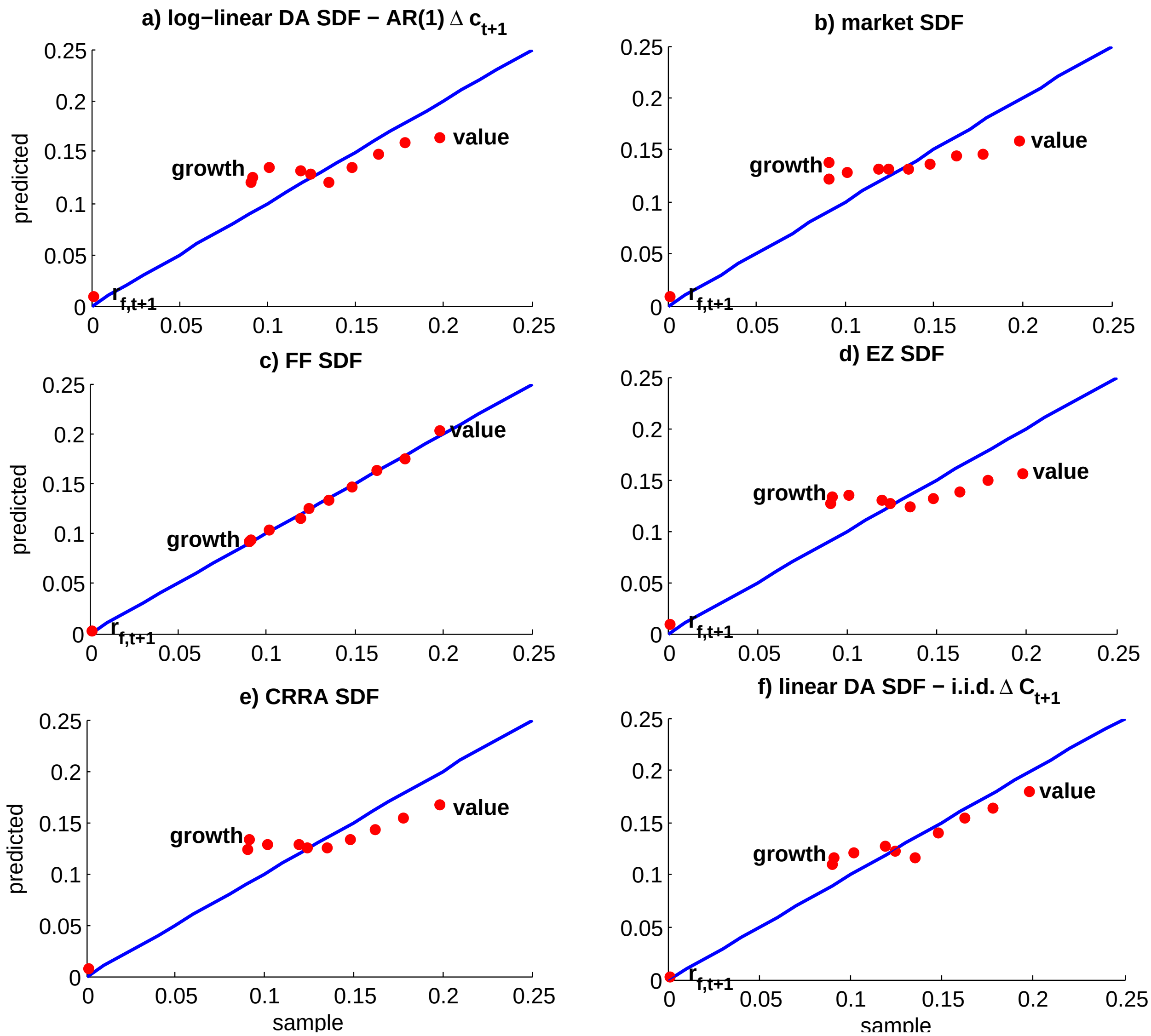


Figure 1.8.6 plots fitted and sample expected equity returns for 10 book-to-market portfolios and the risk-free rate. I use the first 30 years of the sample (1949-1978) to estimate model parameters, and calculate in-sample fitted expected returns according to equation (1.12). Estimation results for each model can be found in Table 1.7.6.

## Figure 1.8.7 Out-of-sample expected returns for 10 BM portfolios during the 1979-2011 period (annual data)

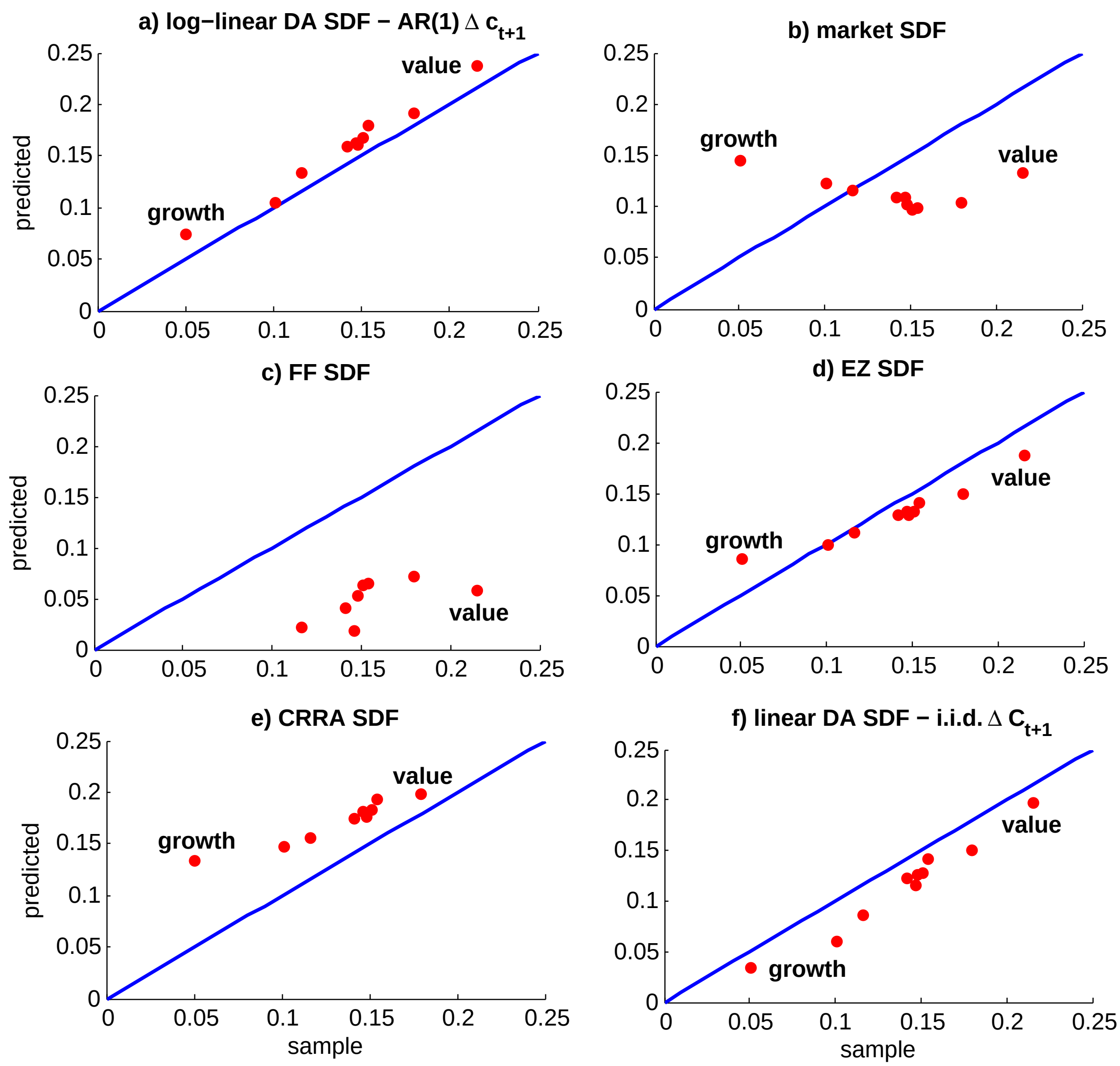


Figure 1.8.7 plots predicted and sample expected equity returns for 10 book-to-market portfolios. I use the first thirty years of the sample (1949-1978) to estimate model parameters, and the 1979-2011 period to test out-of-sample predictions. Predicted expected returns are derived according to the expression in (1.12), and can be found in Table 1.7.6.

**Figure 1.8.8 Expected returns for first-order risk aversion preferences with alternative reference points for gains and losses (annual data)**

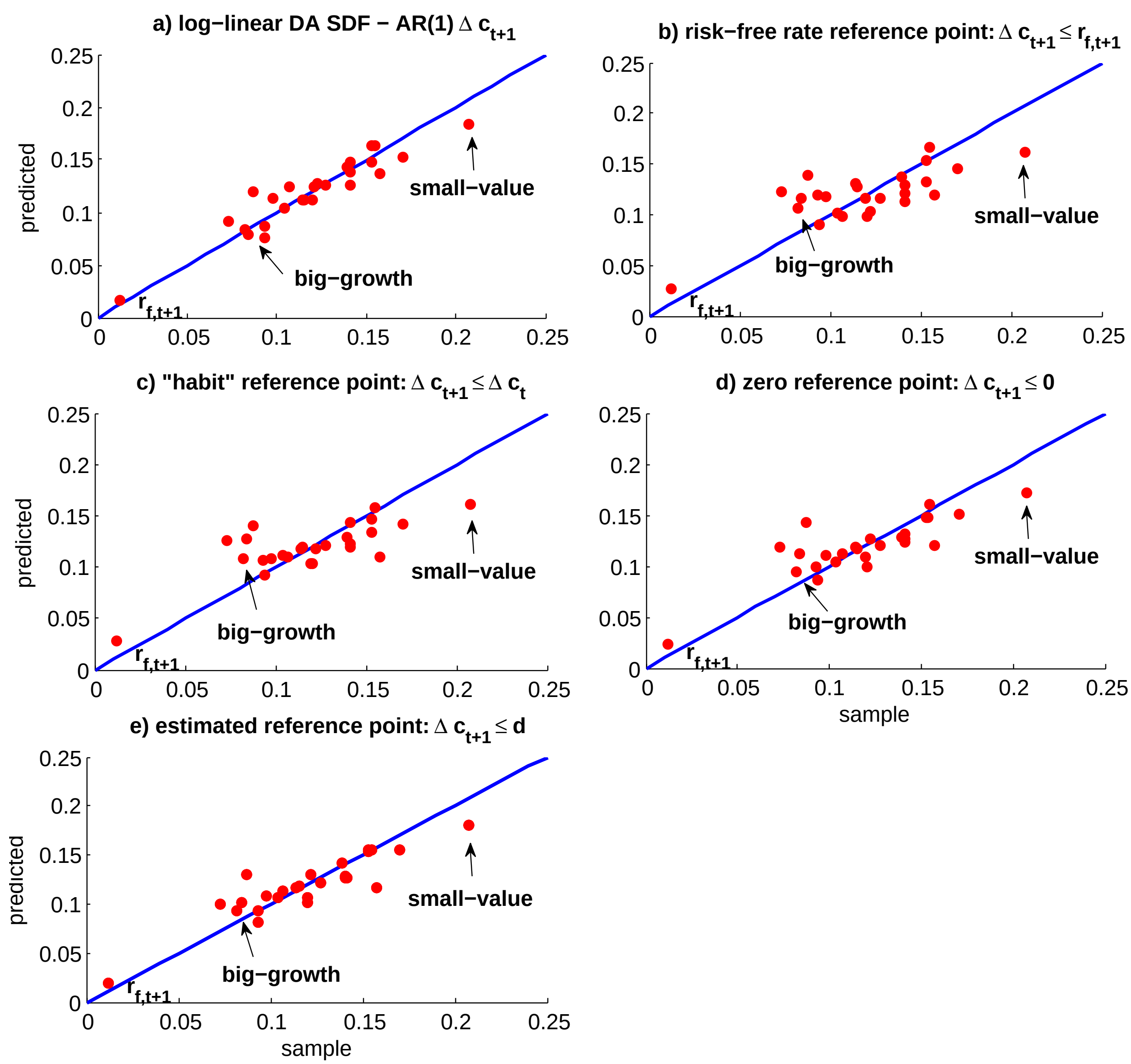


Figure 1.8.8 plots fitted and sample expected equity returns for the 25 Fama-French portfolios and the risk-free rate. According to the expression in (1.19), discount factors are characterized by first-order risk aversion preferences with alternative reference points for gains and losses. These reference points are: i) the log risk-free rate, $\bar{d} = r_{f,t+1}$, ii) previous period's consumption growth, $\bar{d} = \Delta c_t$, iii) zero consumption growth, $\bar{d} = 0$, and iv) $\bar{d}$ is a free parameter to be estimated. Estimation results for each model can be found in Table 1.7.7.

## Figure 1.8.9 Expected returns for the 25 Fama-French portfolios and the risk-free rate (quarterly data)

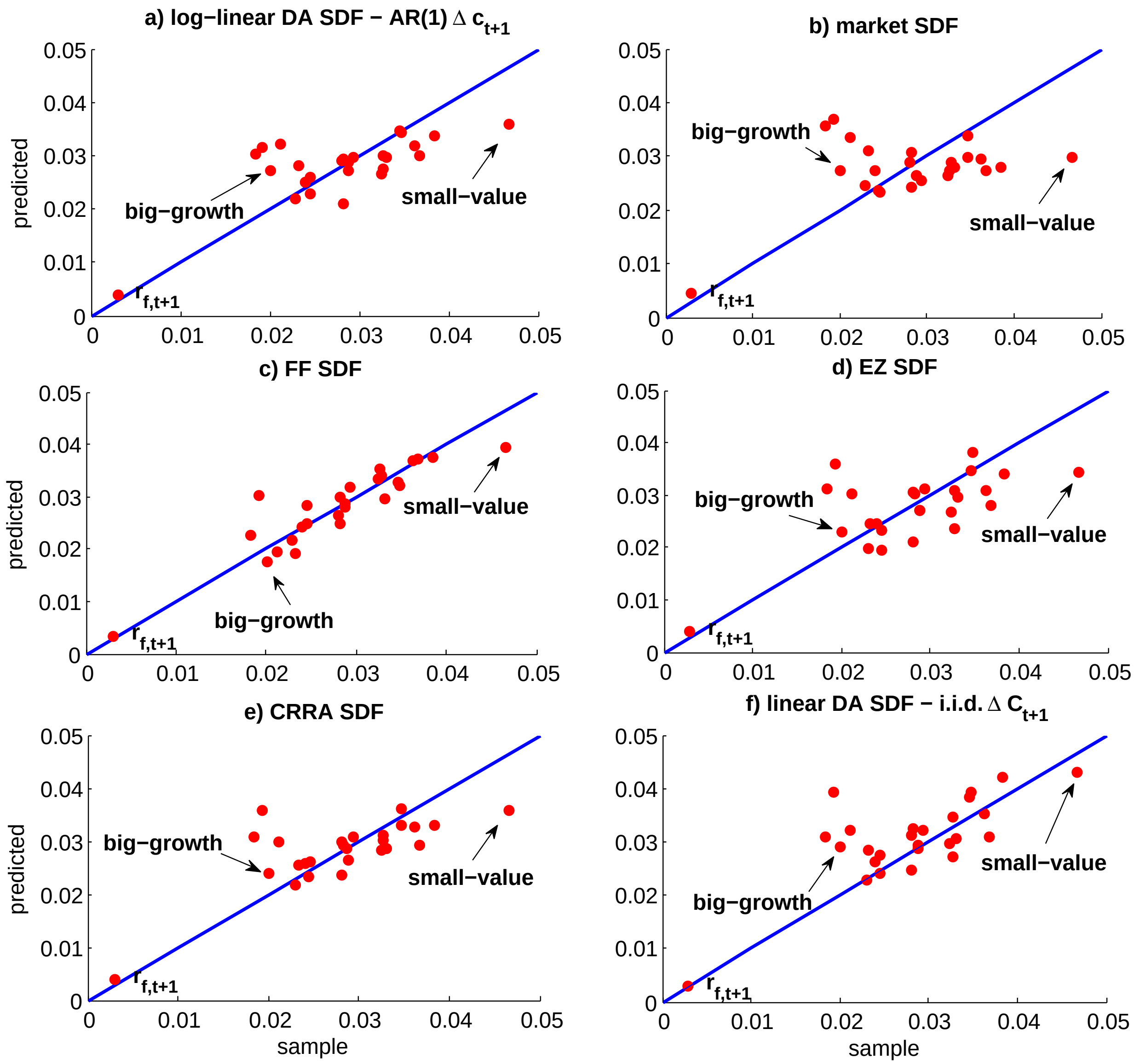


Figure 1.8.9 plots fitted and sample expected equity returns for the 25 Fama-French portfolios and the risk-free rate at the quarterly frequency. Estimation results for each model are shown in Table 1.7.8. Fitted expected returns are derived according to equation (1.12), while sample expected returns are from Table 1.7.1.

# CHAPTER II

# Disappointment Aversion Preferences, and the Credit Spread Puzzle

*ουκ αν λάβοις παρά του μη έχοντος*

***"You cannot receive anything by someone who has nothing"***

*"Dialogues of the Dead"*, Lucian ($125 - 175$ *A.D.*)

## 2.1 Abstract

Structural models of default are unable to generate measurable Baa-Aaa credit spreads, when these models are calibrated to realistic values for default rates and losses given default. Motivated by recent results in behavioral economics, this paper is the first to propose a consumption-based asset pricing model with disappointment aversion preferences in an attempt to resolve the credit spread puzzle. Simulation results suggest that as long as losses given default and default boundaries are countercyclical, then the disappointment model can explain Baa-Aaa credit spreads using preference parameters that are consistent with experimental findings. Further, the disappointment aversion discount factor can match key moments for stock market returns, the price-dividend ratio, and the risk-free rate.

## 2.2 Introduction

When traditional structural models of default[1] are calibrated to realistic values for default rates and losses given default, then these models are unable to generate measurable Baa-Aaa credit spreads, an empirical conundrum also known as the credit spread puzzle. Moreover, recent results[2] suggest that state-of-the-art consumption-based asset pricing models cannot rationalize corporate bond spreads, even if they are successful in explaining equity premia. Nevertheless, a universal stochastic discount that can resolve the equity premium puzzle should also be able to fit credit spreads in corporate bond markets.

Although behavioral theories have been extensively used to explain equity risk premia[3], this is the first paper to address the credit spread puzzle from a behavioral perspective. Towards this objective, I use a general equilibrium model of an endowment economy populated by disappointment averse investors in order to price zero-coupon corporate bonds subject to default. Disappointment aversion preferences were first introduced by Gul (1991), and are able to capture well documented patterns for risky choices, such as asymmetric marginal utility over gains and losses or reference-based evaluation of stochastic payoffs[4], without violating first-order stochastic dominance, transitivity of preferences or aggregation of investors. The disappointment aversion framework can therefore help us shed additional light on the link between credit-spreads and aggregate economic activity while maintaining investor rationality.

Disappointment averse investors are characterized by first-order risk aversion[5] preferences with endogenous expectation-based reference points for gains and losses. Due to the linear homogeneity of these preferences, I am able to obtain approximate

[1] e.g. Merton (1974)

[2] Chen et al. 2009.

[3] Epstein and Zin (1990), Bernatzi and Thaler (1995), Barberis et al. (2001), Andries (2011), Piccioni (2011), Easley and Yang (2012), Delikouras (2013).

[4] Kahneman and Tversky (1979), Duncan (2010), Pope and Schweitzer (2011).

[5] Segal and Spivak (1990).

analytical solutions for the price-payout ratios in the economy which are log-linear functions of three state variables: consumption growth, consumption growth volatility, and consumption growth variance. Explicit solutions for price-payout ratios, in turn, facilitate the simulation algorithm, and provide valuable intuition. The main mechanism in place for disappointment aversion preferences is related to asymmetric marginal utility, and the fact that disappointment averse investors penalize losses below the endogenous reference level three times more than they do for losses above the reference level.

The disappointment aversion model highlights the interaction between default rates and periods of worse-than-expected aggregate macroeconomic conditions when marginal utility is high. During these periods there is an upwards jump in marginal utility. Almeida and Philipon (2007) also document that distress costs are most likely to happen during times when marginal utility is high. Figure 2.9.1 shows Baa-Aaa credit spreads, Baa default rates, and NBER recessions for the 1946-2011 period. Two things become immediately clear from Figure 2.9.1. First, credit spreads are strongly countercyclical. Second, Baa default rates are zero during most of the time, and tend to spike up at or after a recession. Through first-order risk aversion, the disappointment model amplifies very small risks, such as the almost zero default risk for Baa firms, and is able to generate measurable Baa-Aaa credit spreads despite the very low default rates.

Although several consumption-based asset pricing models have proposed frameworks that generate credit spreads consistent with empirical observations, with the exception of the habit model in Chen et al. (2009), either preference parameters (eg. the risk aversion coefficient) in these models are much larger than those estimated in clinical experiments[6], or these models cannot perfectly match other asset pricing

[6]Chen (2010), p. 2190, assumes a risk aversion parameter equal to 6.5 and an EIS larger than 1. Bhamra et al. (2010) remain silent on preference parameters, and focus on risk-neutral pricing.

moments such as equity risk premia[7]. On the other hand, preference parameters for the disappointment model in this paper are calibrated to values which are consistent with recent experimental results[8]: the risk aversion parameter is equal to 1.8, and the disappointment aversion coefficient is equal to 2.03.

By providing evidence that the disappointment model can contribute to the resolution of the credit spread puzzle, this paper compliments a growing literature which argues that disappointment aversion preferences are able address a variety of stylized facts in financial markets such as the equity premium puzzle (Routledge and Zin 2010, Bonomo et al. 2011), the cross-section of expected returns (Ostrovnaya et al. 2006, Delikouras 2013), or limited stock market participation (Ang et al. 2005, Khanapure 2012). Simulation results suggest that as long as losses given default and default boundaries are countercyclical, then the disappointment model can explain the credit spread puzzle, and generate expected Baa-Aaa credit spreads equal to 100 bps for four-year maturities, contrary to 51 bps for the benchmark model which is based on Merton's framework (1974), and it is derived in discrete time. Nevertheless, the disappointment model seems to overpredict expected credit spreads for long maturities (15yr+).

Ever since Merton's model (Merton 1974), most results on corporate bond pricing (Leland 1994, Leland and Toff 1996, Goldstein et al. 2001, Bhamra et al. 2010) rely directly on the risk-neutral probability measure for asset returns, while being silent on investor preferences and the stochastic discount factor. In contrast, this paper adds to recent works by Chen et al. (2009), and Chen (2010) who approach the equity premium and credit spread puzzles in a unified manner, explicitly using a universal consumption-based stochastic discount factor across all financial markets. Taking a stance on the functional form of the stochastic discount factor is particu-

[7]The equity premium in Bhamra et al. (2010), p. 682, is 3.19%, whereas the sample equity premium for the 1946-2011 period is around 5.7%.

[8]Choi et al. (2007), Gill and Prowse (2012).

larly important for two reasons. First, we can identify whether a particular set of preferences is able to generate plausible asset pricing moments across different markets. For instance, besides explaining the credit spread puzzle, the disappointment aversion discount factor in this paper matches moments for aggregate state variables, stock market returns, and the risk-free rate. Second, estimates for preference parameters can be compared to recent experimental findings for choices under uncertainty in order to assess the empirical plausibility of the model.

There are many asset pricing models that can efficiently explain stylized facts in financial markets, yet these models usually explain asset prices one market at a time. The strategy of this paper is to impose more discipline on investor preferences, and provide solid micro-foundations for a universal discount factor across different markets by taking into account recent experimental results for choices under uncertainty. These results emphasize the importance of expectation-based reference-dependent utility. The use of disappointment aversion preferences is therefore motivated by strong experimental and field evidence from aspects of economic life that are not directly related to financial markets[9]. This paper also adds to the relatively limited strand of literature that incorporates elements of behavioral economics into a consumption-based asset pricing model without violating key assumptions of the traditional general equilibrium framework.

[9] Choi et al. (2007), Gill and Prowse (2012), Artstein-Avidan and Dillenberger (2011).

## 2.3 The credit spread puzzle

### 2.3.1 Historical data

Average default rates for the 1970-2011 period[10] and recovery rates are from the Moody's 2012 annual report. Data on recovery rates start in 1982. Corporate bond yields are obtained from Datastream and the St. Louis Fed website for four different sets of indices: two Moody's indices[11], four Barclays indices[12], six BofA indices[13], and eighteen Thomson-Reuters corporate bond indices[14].

In terms of aggregate variables, personal consumption expenditures (PCE), and PCE index data are from the BEA. Per capita consumption expenditures are defined as services plus non-durables. Each component of aggregate consumption expenditures is deflated by its corresponding PCE price index (base year is 2004). Population data are from the U.S. Census Bureau. Recession dates are from the NBER. Interest rates are from Kenneth French's (whom I kindly thank) website. Market returns, dividends, and price-dividend ratios are obtained through the CRSP-WRDS database for the value weighted AMEX/NYSE/NASDAQ index.

Earnings are gross profits (item GP) from the merged CRSP-Compustat database. I use gross profits as a measure of earnings because Compustat EBIT (or EBITDA) growth rates are very volatile[15]. Earnings have been exponentially detrended due to the increasing number of firms in the Compustat sample over time. Stock market

[10] Average default rates in the Moody's report are calculated for three different periods: 1920-2011, 1970-2011, and 1983-2011. Average default rates for the 1983-2011 sample are almost identical to the ones used in this study. However, average default rates for the 1920-2011 period are substantially higher than for the 1970-2011 or the 1983-2011 samples due to the inclusion of the Great Depression.

[11] Moody's Seasoned Aaa and Baa Corporate Bond Indices (1920-2011).

[12] US Agg. Corp. Intermediate Aaa and Baa Indices, US Agg. Corp. Long Aaa and Baa Indices (1974-2011).

[13] US Corp. 1-5y Aaa and Baa, US Corp. 7-10y Aaa and Baa, and US Corp. 15y+ Aaa and Baa Indices (2001-2011).

[14] US Corp. AAA and BBB Indices for maturities from 2yr up to 10yr (2003-2011). Even though BofA indices use S&P ratings (AAA, BBB), for the practical purposes of this study, BBB (AAA) and Baa (Aaa) ratings are considered equivalent. See also Cantor and Packer (1994).

[15] Compustat EBIT growth volatility is around 12%. Earnings growth volatility from Shiller's website is around 30%.

returns, dividend growth, earnings growth, and interest rates have been adjusted for inflation by subtracting the growth rate of the PCE price index[16]. Aggregate variables and market data are sampled for the 1946-2011 period, with the exception of earnings data that start in 1950 and end in 2010. Earnings growth for year $t$ have been aligned with consumption for year $t-1$ because in the 1950-2010 sample, the contemporaneous correlation coefficient between earnings growth and consumption growth is low. All variables have been sampled or simulated at the annual frequency.

### 2.3.2 A benchmark model for credit spreads

Consider a discrete-time, single-good, closed, endowment economy in which the aggregation problem has been solved. Implicit in the representative agent framework lies the assumption of complete markets. There is no productive activity, yet at each point in time the endowment of the economy is generated exogenously by $n$ "tree-"assets as in Lucas (1978). There are also markets where equity, debt, and claims on the total output of these "tree-"assets can be traded. In addition to rational expectations, I will also assume that there are no restrictions on individual asset holdings or transaction costs, that preferences over risky payoffs can be described by power utility, and that all agents have can borrow and lend at the same risk-free rate. This paper focuses on zero-coupon bonds because, according to Chen et al. (2009, p. 3384), the inclusion of coupon payments does not really affect credit spreads.

Consider also a $T$-period, zero-coupon bond written on a firm's assets. This bond pays \$1 if the firm remains solvent at time $t+T$, and $\$(1-L) < \$1$ otherwise. According to Appendix B.1, expected yields for zero-coupon, corporate bonds are

[16] $R_{\text{real},t+1} = exp(logR_{\text{nom},t+1} - log\frac{PCE_{t+1}}{PCE_t})$.

given by[17]

$$\mathbb{E}[y_{i,t,t+T}] = r_f - \frac{1}{T} log\Big[1 - LN\Big(N^{-1}(\pi^{\mathbb{P}}_{i,T}) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big]. \tag{2.1}$$

$y_{i,t,t+T}$ and $r_f$ are the continuously compounded yield-to-maturity and risk-free rate respectively, $L$ are losses given default, $N()$ is the standard normal c.d.f. and $N^{-1}()$ is the inverse of the standard normal c.d.f., $\pi^{\mathbb{P}}_{i,T}$ is the physical probability of default, while $\tilde{\mu}_i$ and $\sigma_i$ are the expected value and standard deviation for asset log-returns. Expected corporate bond yields in (2.1) depend on Sharpe ratios ($\frac{\tilde{\mu}_i - r_f}{\sigma_i}$), physical probabilities of default ($\pi^{\mathbb{P}}_{i,T}$), losses given default ($L$), and bond maturity ($T$). In calibrating the model, I set the Sharpe ratio equal to 0.22 which is the Sharpe ratio for the median Baa firm in Chen et al. (2009)[18]. Losses given default $L$ are set equal to 54.9% to match the average recovery rate of 45.1% for senior unsecured bonds in the Moody's report[19]. Finally, Panel A in Table 2.8.1 shows average default probabilities for Aaa and Baa bonds during the 1970-2011 period.

Panel B in Table 2.8.1 shows average Baa-Aaa credit spreads estimated in previous studies, as well as mean spreads for the four sets of bond indices (Moody's, Barclays, BofA, Thomson-Reuters)[20]. Following the credit spread puzzle literature, this paper focuses on Baa-Aaa spreads because Aaa yields seem to encompass parts of credit spreads such as liquidity, callability, or tax issues which are unrelated to default risk, and are ignored by the model in (2.1)[21]. According to Panel B, the average Baa-Aaa spread in the Huang and Huang sample (2012) is around 103 bps for short matu-

[17] This expression is identical to the continuous-time one in Chen et al. (2009) p. 3377. However, Appendix B.1 derives the expression in (2.1) for a discrete-time economy with CRRA investors.

[18] The Sharpe ratio in (2.1) is the Sharpe ratio for the firm's assets in place, not the equity Sharpe ratio. However, because returns for assets in place are hard to measure, I follow Chen et al. (2009, p. 3375) who proxy asset Sharpe ratios with equity Sharpe ratios.

[19] Chen et al. (2009) use an average recovery rate of 44.1%.

[20] See subsection 2.3.1.

[21] Longstaff, Mithal and Neis (2005) find evidence in favor of a liquidity component in the spreads of corporate bonds over treasuries, while Ericsson and Renault (2006) suggest part of the spread over treasuries can also be attributed to taxes.

rities, 131 bps for medium maturities, whereas expected credit spreads for the long maturity Barclays indices is 112 bps. However, due to different sample periods, there is significant variation in average credit spreads estimates across different studies. In Duffee (1998), average credit spreads are low because the sample is short (1985-1995), and is heavily influenced by the 1990-1995 period which, according to Figure 2.9.1, is characterized by very low spreads (around 50 bps). In contrast, average credit spreads for the BofA and Thomson-Reuters indices are high because average credit spreads for the these indices are also calculated over a short sample (2001-2011), and mean spreads are affected by high credit spreads during the 2009 recession (Figure 2.9.1).

For the rest of the paper, target expected credit spreads will be 103 bps for 4yr maturities and 131 bps for 10yr maturities from Huang and Huang (2012), because these spreads are frequently cited in the literature, and have been calculated over a relatively long period (1973-1993). Note that 4yr expected credit spreads from Huang and Huang are very similar to 4yr spreads in Chen et al. (2009) (107 bps for the 1970-2001 period), while 10yr expected credit spreads from Huang and Huang are very close to 10yr spreads in the Barclays sample (129 bps for the 1974-2011 period). Finally, the target spread for long maturities (15yr) is 112 bps from the long-term Barclays indices. Expected credit spreads for the long-term Barclays indices, in turn, are similar to the Moody's sample (118 bps for 1920-2011 data).

The second-to-last line in Panel B of Table 2.8.1 shows average Baa-Aaa credit spreads generated by the benchmark model in (2.1). Expected Baa bond yields were calculated using default probabilities for Baa firms from Panel A, a Sharpe ratio of 0.22, and losses given default equal to 54.9%. Expected Aaa bond yields were estimated with the same values for the Sharpe ratio and losses given default, but Aaa default probabilities were used instead. Expected Baa-Aaa spreads generated by the model in (2.1) are substantially smaller in magnitude than those observed in the data. For instance, model implied expected credit spreads for short maturites (4yr) are almost half the average spreads observed in practice (51 bps vs. 103 bps in Huang and Huang 2012)[22].

The credit spread puzzle is clearly illustrated in Figure 2.9.2. The dotted line shows expected credit spreads according to the expression in (2.1). The scattered dots in Figure 2.9.2 are mean Baa-Aaa spreads from Huang and Huang (2012), and the three sets of bond indices shown in Table 2.8.1 (Barclays, Thomson-Reuters, BofA). If the expression in (2.1) were able to fit expected credits spread reasonably well, then the credit spread curve should intersect with the scattered points. According, to Figure 2.9.2, the credit spread puzzle is particularly pronounced for short maturities up to 10 years. However, as maturity ($T$) increases, the term $\frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}$ in (2.1) becomes larger, and the benchmark model is able to fit credit spreads better.

Besides the implicit assumption of CRRA preferences, the model in (2.1) imposes three very important limitations that can explain its problematic empirical performance. First, even though time-variation in expected asset returns is considered a key mechanism for resolving a number of stylized facts in financial markets, asset returns in (2.1) are normally distributed with constant mean ($\tilde{\mu}_i$) and variance ($\sigma_i$). Ferson and Harvey (1991) emphasize the importance of time-varying expected returns, while Campbell and Cochrane (1999), Bansal and Yaron (2004), and Ostrovnaya et al. (2006) describe different mechanisms (habit, time-varying macroeconomic uncertainty, generalized disappointment aversion) which can generate time-variation in investors' risk attitudes, and, consequently, time-varying expected returns[23].

Second, recovery rates $(1 - L)$ in (2.1) are also constant. Table 2.8.2 shows OLS regression results for recovery rates and aggregate consumption growth during the

[22]Nevertheless, the benchmark model is doing quite well in matching the Duffee (1998) sample or longer maturities.

[23]Besides constant moments, the normal distribution also appears to be a restrictive assumption. Nevertheless, Huang and Huang (2012) and Chet et al. (2009) show that introducing jumps and relaxing the normality assumption cannot resolve the credit spread puzzle.

1982-2011 period. The regression coefficient is positive (4.461), and statistically significant ($t$-stat. 3.036, $R^2$ 24.767%), suggesting that recovery rates are most likely procyclical. Figure 2.9.3 also indicates that recovery rates decrease substantially during recessions[24]. Appendix B.2 shows that if recovery rates comove with aggregate economic conditions (consumption growth) in a linear way[25]

$$1 - L_{t+T} = a_{rec,0} + a_{rec,c}\Delta c_{t+T-1,t+T},$$

then the benchmark model becomes

$$\mathbb{E}[y_{i,t,t+T}] = r_f - \frac{1}{T}log\Big[1 - \Big(\mathbb{E}[L_{t+T}] + \underbrace{a_{rec,c}\frac{\tilde{\mu}_m - r_f}{\rho_{m,c}\sigma_m}\sigma_c}_{\text{risk premia for } L_{t+T}}\Big)N\Big(N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big]. \quad (2.2)$$

$\frac{\tilde{\mu}_m - r_f}{\sigma_m}$ in (2.2) is the stock market Sharpe ratio (0.378 from Table 2.8.6), $\rho_{m,c}$ is the correlation coefficient between stock market returns and consumption growth (0.463 in Table 2.8.6), and $\sigma_c$ is consumption growth volatility (1.914% in Table 2.8.4)[26].

According to the expression in (2.2), risk averse individuals adjust (decrease) expected values for recovery rates $1 - \mathbb{E}[L_{t+T}]$ because these rates are procyclical ($a_{rec,c} > 0$). This risk adjustment term depends on the risk aversion parameter of the CRRA power utility. However, Appendix B.2 shows that we can use the consumption-Euler equation for stock market returns (eqn. B.6 in Appendix B.6.1) in order to substitute the risk aversion parameter with the stock market Sharpe ratio $\frac{\tilde{\mu}_m - r_f}{\sigma_m}$ adjusted for the correlation ($\rho_{m,c}$) between stock market returns and consumption growth. Nevertheless, the last line in Panel B suggests that the addition of procyclical

[24]Evidence in favor of procyclical recovery rates can be found in Altman et al. (2005), and Acharya et al. (2007) among others. Shleifer and Vishny (1992) also provide theoretical arguments in favor of procyclical recovery rates.

[25]Throughout the paper, recovery rates do not change across all firms, even though they can vary through time.

[26]For comparison purposes with the disappointment model in subsection 1.4.3, values for $\frac{\tilde{\mu}_m - r_f}{\sigma_m}$, $\rho_{m,c}$, and $\sigma_c$ are from the simulated economy.

recovery rates leads to a small increase in credit spreads (10 bps across maturities) relative to the benchmark model in (2.1), either because recovery rates do not covary much with aggregate consumption (low $a_{rec,c}$ in 2.2), or because the standard power utility framework does not penalize enough recovery rate risk.

The third drawback of the benchmark model in (2.1) is related to the constant and exogenous default boundary. In the original Merton model, default boundaries are constant, and equal to the face value of debt. In (2.1), the default boundary is also assumed constant but not necessarily equal to the face value of debt, because a number of studies[27] suggest that default happens below the debt level. For instance, Chen et al. (2009) argue that since average recovery rates are around 45%, if default happened at the face value of debt, then default costs would amount to 55% of face value, an extremely large number. Contrary to the constant default case, Chen et al. (2009) set an exogenous default boundary which comoves negatively with surplus consumption[28]. Chen (2010) and Bhama et al. (2010), on the other hand, endogenize default boundaries exploiting the smooth pasting conditions in a continous-time framework. Although default boundaries are hard to measure, it seems that time-variation in these boundaries is an important ingredient for resolving the credit spread puzzle.

In a continuous-time setting, the derivation of the benchmark models in (2.1) and (2.2) hinges on continuous trading so that, under the risk-neutral probability measure, expected returns ($\tilde{\mu}_i$) are replaced by the risk-free rate [29]. However, for discrete-time models, in which continuous trading is not an option, replacing the mean with the risk-free rate while preserving log-normality of asset returns necessarily requires that investor preferences are characterized by power utility[30]. Hence, in a discrete-time

[27]Leland (2004), Davydenko (2012).
[28]The assumption of countercyclical default boundaries in Chen et al. (2009) is necessary for positive comovement between default rates and credit spreads.
[29]Black and Scholes (1973).
[30]Brennan (1979) and Appendix B.6.1.

world, the models in (2.1) and (2.2) are essentially a statement about investor preferences, despite the absence of the risk aversion parameter. The aim of this paper is to examine whether relaxing the CRRA assumption, and introducing disappointment aversion preferences can help us resolve the credit-spread puzzle. Unfortunately, by introducing more complicated preference structures, we are no longer able to derive simple pricing formulas for corporate bond yields like the ones in (2.1) and (2.2).

## 2.4 Recursive utility with disappointment aversion preferences

### 2.4.1 Disappointment aversion stochastic discount factor

For the main model of this paper, I maintain the same assumptions as in subsection 2.3.2, with the crucial difference that now the model economy is populated by disappointment averse, instead of CRRA, individuals. Disappointment aversion preferences are homothetic. Therefore, if all individuals have identical preferences, then a representative investor exists, and equilibrium prices are independent of the wealth distribution[31]. The expression for the disappointment aversion intertemporal marginal rate of substitution between periods $t$ and $t+1$[32] is given by

$$M_{t,t+1} = \underbrace{\underbrace{\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{(\rho-1)}}_{\text{time correction}} \quad \underbrace{\Big[\frac{V_{t+1}}{\mu_t\big(V_{t+1}\big)}\Big]^{-\alpha-\rho}}_{\text{second-order risk correction}}}_{\text{Epstein-Zin terms}} \quad \times \tag{2.3}$$

$$\underbrace{\Big[\frac{1+\theta\mathbf{1}\{V_{t+1} \;<\; \delta\mu_t\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta \;>\; 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1} \;<\; \delta\mu_t\}]}\Big]}_{\text{disappointment (first-order risk) correction}}.$$

[31] Chapter 1 in Duffie (2000), and Chapter 5 in Huang and Litzenberger (1989).
[32] See also Hansen et al. (2007), Routledge and Zin (2010), and Delikouras (2013).

with

$$\mu_t(V_{t+1}) = \mathbb{E}_t\Big[\frac{V_{t+1}^{-\alpha}\ (1+\theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\Big]^{-\frac{1}{\alpha}}. \qquad (2.4)$$

The derivation of the disappointment aversion discount factor is shown in Appendix B.3.

$V_t$ is lifetime utility from time $t$ onwards. $\mu_t$ in equation (2.4) is the disappointment aversion certainty equivalent which generalizes the concept of expected value. $\mathbb{E}_t$ is the conditional expectation operator. The denominator in (2.4) is a normalization constant such that $\mu_t(\mu_t) = \mu_t$. $\mathbf{1}\{\}$ is the disappointment indicator function that overweighs bad states of the world (disappointment events). According to (2.4), disappointment events happen whenever lifetime utility $V_{t+1}$ is less than some multiple $\delta$ of its certainty equivalent $\mu_t$. The parameter $\delta$ is associated with the threshold below which disappointment events occur. In Gul (1991) $\delta$ is 1, whereas in Routlegde and Zin (2010), disappointment events may happen below or above the certainty equivalent, $V_{t+1} < \delta\mu_t(V_{t+1})$, depending on whether the GDA parameter $\delta$ is lower or greater than one respectively. Here, I follow Gul (1991), and set $\delta$ equal to 1 for analytical tractability.

$\alpha \geq -1$ is the Pratt (1964) coefficient of second-order risk aversion which affects the smooth concavity of the objective function. $\theta \geq 0$ is the disappointment aversion parameter which characterizes the degree of asymmetry in marginal utility above and below the reference level. $\beta \in (0, 1)$ is the rate of time preference. $\rho \leq 1$ characterizes the elasticity of intertemporal substitution (EIS) for consumption between two consecutive periods since EIS $= \frac{1}{1-\rho}$. In order to facilitate the derivation of analytical solutions, I set the EIS equal to unity ($\rho = 0$). For $\rho = 0$ and $\delta = 1$ in (2.3), the disappointment aversion stochastic discount factor becomes[33]

$$M_{t,t+1} = \underbrace{\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-1}}_{\text{time correction}} \underbrace{\Big(\frac{V_{t+1}}{\mu_t(V_{t+1})}\Big)^{-\alpha}}_{\text{second-order risk correction}} \underbrace{\frac{1+\theta\ \mathbf{1}\{V_{t+1}< \ \mu_t(V_{t+1})\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{V_{t+1}<\mu_t(V_{t+1})\}]}}_{\text{disappointment correction}}. \tag{2.5}$$

$M_{t,t+1}$ in (2.3) and (2.5) essentially corrects expected values by taking into account investor preferences over the timing and riskiness of stochastic payoffs. The first term in (2.3) and (2.5) corrects for the timing of uncertain payoffs (resolution of uncertainty) which happen at a future date. The second term adjusts future payoffs for investors' dislike towards risk (second-order risk aversion). When investors' preferences are time-additive, adjustments for time and risk are identical ($\alpha = \rho$), and the second term vanishes. The third term in equations (2.3) and (2.5) corrects future payoffs for investors' aversion towards disappointment events, defined as periods during which lifetime utility $V_{t+1}$ drops below its certainty equivalent $\mu_t$.

### 2.4.2 Approximate analytical solutions for the disappointment aversion discount factor

Since lifetime utility $V_t$ in (2.5) is unobservable, it is hard to test the empirical performance of the disappointment model. The analysis will become much easier if we are able to express lifetime utility as a function of observable state variables.

Suppose that at each point in time, expected consumption growth is a function of a state variable $x_t$. For simplicity, I will assume that $x_t$ is equal to current consumption growth $\Delta c_{t-1,t}$. Suppose also that there is a second state variable $\sigma_t$ which drives aggregate economic uncertainty. Based on those two assumptions, our model economy

[33]The reader is referred to Delikouras (2013) for a more thorough analysis of the disappointment model.

is described by the following system of equations

$$\Delta c_{t,t+1} = \mu_c + \phi_c \Delta c_{t-1,t} + \sigma_t \epsilon_{c,t+1}, \tag{2.6}$$

$$\sigma_{t+1} = \mu_\sigma + \phi_\sigma \sigma_t + \nu_\sigma \epsilon_{\sigma,t+1}, \tag{2.7}$$

$$\Delta o_{m,t,t+1} = \mu_o + \phi_o \Delta c_{t-1,t} + \sigma_o \sigma_t \epsilon_{o,t+1}. \tag{2.8}$$

According to (2.6), consumption growth is an AR(1) process with time-varying volatility. $\phi_c \in (-1, 1)$ is the first-order autocorrelation coefficient, $\mu_c$ is the constant term, and $\epsilon_{c,t+1}$ are i.i.d. $N(0, 1)$ shocks. Although, the AR(1) model for consumption growth is quite common in the asset pricing literature (Mehra and Prescott 1985, Routledge and Zin 2010), a number of authors (Campbell and Cochrane 1999, Cochrane 2001) suggest that consumption growth is i.i.d., and $\phi_c$ in (2.6) is zero.

Time-varying macroeconomic uncertainty[34] is captured by consumption growth volatility $\sigma_t$ which is stochastic. Following Chen et al. (2009), $\sigma_t$ is an AR(1) process in which $\epsilon_{\sigma,t+1}$ are i.i.d. $N(0, 1)$ shocks, $\phi_\sigma \in (-1, 1)$ is the first-order autocorrelation coefficient, $\mu_\sigma \in \mathbb{R}_{>0}$ is the constant term, and $\nu_\sigma \in \mathbb{R}_{>0}$ captures the conditional volatility in macroeconomic uncertainty. Bansal and Yaron (2004), Bansal et al. (2007), Lettau et al. (2007), and Bonomo et al. (2011) all use similar autoregressive models for macroeconomic uncertainty, although they consider consumption growth variance instead of consumption growth volatility. Because shocks in (2.7) are normally distributed, the probability of negative volatility is non-zero. However, consumption growth variance $\sigma_t^2$ is always positive[35].

The last equation describes the evolution of aggregate payout growth. Depending on the asset we want to price, $o_{m,t}$ represents different kinds of cashflows. For

[34]In addition to the asset pricing implications of stochastic volatility, Bloom (2009) and Bloom et al. (2012) propose a model in which stochastic second moments in TFP shocks are the single cause for business cycle fluctuations.

[35]Hsu and Palomino (2011) resolve the issue of negative variance by assuming an autoregressive gamma process as in Gourieroux and Gasiak (2006).

aggregate equity claims, the relevant payout is dividends ($o = d$). For the valuation of aggregate assets in place, the relevant payout is earnings ($o = e$). According to (2.8), expected payout growth depends on aggregate consumption $\Delta c_{t-1,t}$ through $\phi_o \in \mathbb{R}$. For $\phi_o > 1$ aggregate payout is a levered claim to consumption, whereas for $\phi_o = 0$, payout growth is i.i.d.. $\sigma_o \in \mathbb{R}_{>0}$ is the volatility parameter for payout growth. This specification for aggregate payout growth is very similar to the one in Bansal and Yaron (2004) where aggregate dividend growth depends on the long-run risk variable. Finally, for algebraic convenience, I will assume that shocks to consumption growth, consumption growth volatility, and payout growth $(\epsilon_{c,t}, \epsilon_{\sigma,t}, \epsilon_{o,t})$, are mutually uncorrelated.

Using the system of equations in (2.6) and (2.7), and the log-linear structure of investor's lifetime utility, I can derive an analytical expression for the log utility-consumption ratio $v_t - c_t$ in terms of consumption growth $\Delta c_{t-1,t}$ and aggregate uncertainty $\sigma_t$.

***Proposition 1:*** For $\rho = 0$, $\delta = 1$ in (2.3), and macroeconomic dynamics in (2.6) and (2.7), the log utility-consumption ratio, $v_t - c_t = log(V_t/C_t)$, is approximately affine in consumption growth, consumption growth volatility, and consumption growth variance: $v_t - c_t \approx A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2 \ \forall t$, where

- $A_1 = \frac{\beta \phi_c}{1-\beta\phi_c}$,
- $A_2 = \frac{\overbrace{-\theta\beta n(\bar{x})(A_1+1)(1+2\alpha A_3 \nu_\sigma^2)}^{\text{disappointment aversion}} + 2\beta A_3 \mu_\sigma \phi_\sigma}{1+2\alpha A_3 \nu_\sigma^2 - \beta\phi_\sigma}$,
- $A_3 = \frac{-[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2] + \sqrt{[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2 - 4\beta\alpha^2(A_1+1)^2\nu_\sigma^2}}{4\alpha\nu_\sigma^2}$,
- $A_0 = \frac{\beta}{1-\beta}[(A_1+1)\mu_c + \frac{1}{1+2\alpha A_3\nu_\sigma^2}(A_2\mu_\sigma + A_3\mu_\sigma^2 - 0.5\alpha A_2^2\nu_\sigma^2) + \frac{1}{2\alpha} log(1+2\alpha A_3 \nu_\sigma^2)]$,

and $n(.)$ is the standard normal p.d.f..

***Proof.*** *See Appendix B.6.4*

$A_1$ is the consumption growth multiplier. The sign and magnitude of $A_1$ depend on consumption growth autocorrelation $\phi_c$. If consumption growth is i.i.d. then $A_1$ is zero. $A_3$ is the multiplier for consumption growth variance $\sigma_t^2$. If the risk aversion coefficient $\alpha$ is positive, then $A_3$ is negative[36]. For $A_3$ to be real, we require that the terms inside the square root are positive, and that $\alpha$ is different than zero[37]. $A_2$ is the multiplier for consumption growth volatility $\sigma_t$, and captures first-order risk aversion through the $\theta n(\bar{x})$ term. For $A_3$ negative and positive $\theta$, then $A_2$ is also negative. $A_0$ is the constant term in the log utility-consumption ratio. For $A_0$ to be well defined, we require $1 + 2\alpha A_3 \nu_\sigma^2$ to be positive, and that $\alpha$ is non-zero. Finally, if consumption growth is positively autocorrelated ($\phi_c > 0$), and preference parameters ($\alpha, \theta > 0$) are also positive, then the log utility-consumption ratio is procyclical since $A_1$ is positive, and $A_2, A_3$ are negative.

An immediate consequence of *Proposition 1* is that we can express the disappointment aversion stochastic discount factor in (2.5) as a function of consumption growth $\Delta c_{t-1,t}$, and consumption growth volatility $\sigma_t$

$$M_{t,t+1} \approx \underbrace{e^{log\beta - \Delta c_{t,t+1}}}_{\text{time correction}} \underbrace{e^{-\alpha\left\{A_0(1-\frac{1}{\beta})+[(A_1+1)\Delta c_{t,t+1}-\frac{1}{\beta}A_1\Delta c_{t-1,t}]+A_2(\sigma_{t+1}-\frac{1}{\beta}\sigma_t)+A_3(\sigma_{t+1}^2-\frac{1}{\beta}\sigma_t^2)\right\}}}_{\text{second-order risk correction}} \times \tag{2.9}$$

$$\underbrace{\frac{1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2 < \ \frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2 < \ \frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}]}}_{\text{disappointment correction}}.$$

$M_{t,t+1}$ in (2.9) corrects expected future payoffs for timing, risk and disappointment, much like the one in (2.5). The crucial difference between the two expressions is that in equation (2.9) unobservable lifetime utility $V_{t+1}$ is expressed in terms of state variables.

Armed with the expression for the stochastic discount factor, we can also solve

[36] Appendix B.6.4.

[37] A detailed discussion on parameter restrictions can be found in Appendix B.6.4. The requirement $\alpha \neq 0$, implies that preferences need to be non-separable across time.

for the one-period log risk-free rate (see Appendix B.6.5)

$$r_{f,t,t+1} \approx \underbrace{-log\beta + 1 \cdot \mu_c + 1 \cdot \phi_c \Delta c_{t-1,t}}_{\text{impatience and future prospects}} \underbrace{\underbrace{-0.5[2\alpha(A_1+1)+1]\sigma_t^2}_{\text{second-order risk aversion}} - \underbrace{\theta n(\bar{x})\sigma_t.}_{\text{disappointment aversion}}}_{\text{precautionary savings motive}} \quad (2.10)$$

Consumption growth terms $\big(\mu_c(1-\phi_c),\, \phi_c\Delta c_{t-1,t}\big)$ in (2.10) are multiplied by unity, since the EIS is assumed equal to one, and thus consumption growth moves one-for-one with interest rates. The last two terms in (2.10) reflect the precautionary motive for investors to save. This motive depends on both risk and disappointment aversion. Notice that second-order risk aversion terms depend on consumption growth variance ($\sigma_t^2$), while disappointment aversion terms depend on consumption growth volatility ($\sigma_t$) due to the first-order risk aversion effect. For $\alpha, \theta$ positive, higher uncertainty (high values for $\sigma_t$ and $\sigma_t^2$) will force investors to save more in the risk-free technology, and therefore decrease interest rates.

Turning now to risky financial assets, let $R_{m,t}$ be the cum-payout, one-period, gross return for a claim on a stream of aggregate payments (dividends or earnings). If claims are traded in complete, frictionless markets, then the consumption-Euler equation implies that

$$\mathbb{E}_t\Big[M_{t,t+1}R_{m,t,t+1}\Big] = 1.$$

Using the results in Appendix B.4, aggregate log-returns $r_{m,t,t+1}$ can be written as a linear function of log price-payout ratios, and the Euler equation becomes

$$\mathbb{E}_t\Big[M_{t,t+1}e^{\kappa_{m,0}+\kappa_{m,0}z_{m,t+1}-z_{m,t}+\Delta o_{m,t,t+1}}\Big] = 1,$$

where $\kappa_{m,0}$ and $\kappa_{m,1}$ are linearization constants, and $z_{m,t} = log\frac{P_{m,t}}{O_{m,t}}$ is the log price-payout ratio.

In order to provide valuable intuition, we can further express the log price-payout ratio $z_{m,t}$ as a linear function of the state variables $\Delta c_{t,t+1}$ and $\sigma_t$ using *Proposition 2*.

***Proposition 2:*** For $\rho = 0$, $\delta = 1$ in (2.3), and the dynamics in (2.6) - (2.8), the log price-payout ratio $z_{m,t} = log(P_{m,t}/O_{m,t})$ for a claim on a stream of aggregate payments is approximately affine in consumption growth, consumption growth volatility, and consumption growth variance: $z_{m,t} \approx A_{m,0} + A_{m,1}\Delta c_{t-1,t} + A_{m,2}\sigma_t + A_{m,3}\sigma_t^2 \ \forall t$, where

- $A_{m,1} = \frac{\phi_o - \phi_c}{1-\kappa_{m,1}\phi_c}$,
- $A_{m,2} \approx \frac{\overbrace{\theta n(\bar{x})(1-\kappa_{m,1}A_{m,1})}^{\text{disappointment aversion}} + 2\kappa_{m,1}A_{m,3}\mu_\sigma\phi_\sigma}{1-\kappa_{m,1}\phi_\sigma}$,
- $A_{m,3} \approx \frac{1}{2}\frac{(1-\kappa_{m,1}A_{m,1})^2 + 2\alpha(A_1+1)(1-\kappa_{m,1}A_{m,1}) + \sigma_o^2}{1-\kappa_{m,1}\phi_\sigma^2}$,
- $A_{m,0} \approx \frac{1}{1-\kappa_{m,1}}\left[log\beta + \kappa_{m,0} + \mu_o + (\kappa_{m,1}A_{m,1}-1)\mu_c + \kappa_{m,1}A_{m,2}\mu_\sigma + \kappa_{m,1}A_{m,3}\mu_\sigma^2\right]$,

***Proof.*** *See Appendix B.6.6*

Note that the values for $A_{m,2}$, $A_{m,3}$ and $A_{m,0}$ above are approximations assuming that the variance for consumption growth volatility ($\nu_\sigma^2$) is a number close to zero. Exact solutions can be found in Appendix B.6.6. The above approximations preserve the intuition without the notational burden. However, for the simulation part of this study, I use the exact solutions. Moreover, the multipliers $A_{m,1}, A_{m,2}$, $A_{m,3}$ and $A_{m,0}$ for the price-dividend ratio are different than the multipliers for the price-earnings ratio because aggregate dividend growth dynamics are different than aggregate earnings growth dynamics ($\phi_d \neq \phi_e$ or $\mu_d \neq \mu_e$ or $\sigma_d \neq \sigma_e$).

As long as $\phi_o \neq \phi_c$, the multiplier for consumption growth $A_{m,1}$ will be non-zero, and the price-payout ratio $z_{m,t}$ will depend on consumption growth, even if $\phi_c = 0$ and consumption growth is i.i.d.. The sign of $A_{m,1}$ essentially depends on $\phi_o - \phi_c$

because $1-\kappa_{m,1}\phi_c$ is positive[38]. $A_{m,3}$ is the multiplier for $\sigma_t^2$, and depends on the risk aversion coefficient $\alpha$, as well as on the persistence of aggregate shocks through the terms $\phi_\sigma^2$ and $A_{m,1}$. Unlike $A_3$ in *Proposition 1*, which is always negative for positive values of the risk aversion parameter $\alpha$, $A_{m,3}$ can turn positive even if $\alpha$ is positive, provided that $\sigma_o$ is a large number. For $A_{m,3}$ positive, an increase in consumption growth variance will increase the price-payout ratio.

$A_{m,2}$ is the stochastic volatility multiplier. If investors are not disappointment averse ($\theta = 0$) and $A_{m,3}$ is positive, then $A_{m,2}$ is also positive, and an increase in aggregate uncertainty will lead to an increase in the price-payout ratio. However, for positive $\theta$, $A_{2,m}$ can be negative, even if $A_{m,3}$ is positive. In this case, the effects of aggregate uncertainty on the price-payout ratio operate in two different directions thought the first and second-order risk aversion mechanisms. This is a subtle, but important, difference between disappointment aversion and the traditional Epstein-Zin (Epstein and Zin 1989) framework with no first-order risk aversion effects. Finally, $A_{0,m}$ is the constant term in the price-divided ratio, and is essentially equal to the sum of the constant terms ($\mu_m,\ \mu_c$) from (2.6) and (2.8) adjusted for disappointment ($A_{m,2}\mu_\sigma$) and uncertainty ($A_{m,3}\mu_\sigma^2$).

The results in *Proposition 2* are particularly important, since we can use the price-payout approximation in Appendix B.4 to express asset log-returns as a function of the state variables

$$r_{m,t,t+1} \approx \kappa_{m,0} + \kappa_{m,1} z_{m,t+1} - \underbrace{z_{m,t}}_{A_{m,0}+A_{m,1}\Delta c_{t-1,t}+A_{m,2}\sigma_t+A_{m,3}\sigma_t^2} + \Delta o_{m,t,t+1},\ \forall t. \quad (2.11)$$

Asset returns in (2.11) correspond to aggregate claims. In order to describe firm-level

[38] For consumption growth to be stationary $\phi_c \in (-1,1)$. Additionally, $\kappa_{m,1} < 1$ from (B.4), and thus $1-\kappa_{m,1}\phi_c > 0$.

asset returns, we need to introduce idiosyncratic shocks as follows

$$r_{i,t,t+1} \approx \underbrace{\kappa_{m,0} + \kappa_{m,1} z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1}}_{\text{systematic component}} + \underbrace{\sigma_i \epsilon_{i,t+1}}_{\text{idiosyncratic part}}, \tag{2.12}$$

for cum-payout returns, and

$$r^x_{i,t,t+1} = z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1} + \sigma_i \epsilon_{i,t+1}, \tag{2.13}$$

for ex-payout returns. $\epsilon_{i,t+1}$ are i.i.d. N(0,1) idiosyncratic shocks, orthogonal to the rest of the aggregate shocks in (2.6)-(2.8). The above specification for firm-level returns matches perfectly a long-standing concept in finance according to which asset returns can be decomposed into a systematic part, and an idiosyncratic one. Note that for equity returns the relevant payout in (2.11) - (2.13) is dividends ($o = d$), whereas for assets in place returns the relevant payout is earnings ($o = e$).

## 2.5 Simulation results for the disappointment aversion discount factor

### 2.5.1 Preference parameters, and state variable moments for the simulated economy

The EIS and the GDA parameters for the disappointment aversion discount factor in (1.3) are assumed equal to one for analytical tractability. For the remaining parameters, I set the risk aversion coefficient $\alpha$ equal to 1.8 and the disappointment aversion parameter $\theta$ equal to 2.030. These values are within the range of clinical estimates[39], and are very similar to those used in Bonomo et al. (2011). The value for $\theta$ implies that whenever lifetime utility is below its certainty equivalent (disappointment events), investors penalize losses 3 times more than during normal times.

[39] Gill and Prowse (2012).

Finally, the rate of time preference $\beta$ is equal to 0.9955. In the deterministic steady-state of the economy, an additional \$1 of consumption tomorrow is worth \$0.9955 today.

In order to explain the market-wide equity premium, Routledge and Zin (2010) employ a constant consumption growth variance framework, and set $\theta$ equal to 9 with $\alpha$ equal to -1 (second-order risk neutrality). In Bonomo et al. (2011) consumption growth variance is stochastic, $\theta$ is 2.33, and $\alpha$ is 1.5. Choi et al. (2007) conduct clinical experiments on portfolio choice under uncertainty, and find disappointment aversion coefficients that range from 0 to 1.876, with a mean of 0.39. They also estimate second-order risk aversion parameters that range from -0.952 to 2.871, with a mean of 1.448. Using experimental data on real effort provision, Gill and Prowse (2012) estimate disappointment aversion coefficients ranging from 1.260 to 2.070. Ostrovnaya et al. (2006) estimate disappointment aversion parameters from stock market data using market wide stock market returns as the explanatory variable, instead of consumption growth. Their estimates for $\theta$ range from 1.825 to 2.783. Finally, Delikouras (2013) assumes constant consumption growth volatility, and provides $\theta$ estimates around 4.6, and risk aversion estimates that range from 10 up to 16.

Table 2.8.3 summarizes parameter values for state variable dynamics in (2.6)-(2.8). These values are carefully chosen so that simulated moments match those observed in real data. Many of these parameters have been used in previous studies. For instance, the consumption growth multipliers $\phi_d$ and $\phi_e$ in (2.8) are equal to 3 as in Bansal and Yaron (2004). Earnings are considered a levered claim to consumption ($\phi_e > 1$) because the endowment model ignores other claims to earnings such as salaries, depreciation, and taxes that need to be paid out before interest and dividends[40]. Volatility parameters for dividends and earnings growth ($\sigma_d = 7.1664$ and

[40]Also, for uncorrelated macroeconomic shocks in (2.6)-(2.8), letting $\phi_e$ ($\phi_d$) be larger than one is the only way to obtain plausible correlations between earnings (dividend) growth and consumption growth. Chen et al. (2009), p. 3404, set $\phi_d$ equal to 3.5 and $\phi_e$ equal to 2.7.

$\sigma_e = 2.2011$) are larger than one, because dividend and earnings growth are much more volatile than consumption growth. The autocorrelation parameter for aggregate consumption growth volatility is 0.971 because, according to previous results[41], aggregate uncertainty is a very persistent process. Finally, idiosyncratic volatility $\sigma_i$ is set equal to 0.210 so as to match the Sharpe ratio for the median Baa firm which is 0.220 (Chen et al. 2009, p. 3377).

Despite the similarities with previous studies, there are a few notable exceptions. First, in Bansal and Yaron (2004) and Bansal et al. (2007), expected consumption growth is a very persistent process, whereas in Chen et al. (2009) and Bonomo et al. (2011) consumption growth is i.i.d. ($\phi_c$=0). Here, I set the autocorelation parameter $\phi_c$ equal to 0.5 to match the persistence in BEA consumption data. Second, the volatility parameter $\mu_\sigma$ in Bansal and Yaron (2004) and Bonomo et al. (2011) is quite high. Their values for $\mu_\sigma$ imply that annual consumption growth volatility is approximately 3%, which is more than two times the volatility observed in the BEA sample (1.3% from Table 2.8.4). In this study, $\mu_\sigma$ is equal to a very small value (0.0004) so that consumption growth volatility remains low. Finally, the linearization constant $\bar{z}_m$ for log price-payout ratios in (B.4) is equal to 3, which is very close to the unconditional mean for the stock market log price-dividend ratio (Table 2.8.6).

Table 2.8.4 shows simulated and sample moments for all macroeconomic variables. Simulated values for the state variables are according to the system in (2.6)-(2.8), using parameter values from Table 2.8.3[42]. Simulated moments for aggregate consumption growth are very close to actual ones (mean 1.834% vs. 1.838% in the data, autocorrelation 0.504 vs. 0.502), with the exception of consumption growth volatility which is higher for the simulated economy (1.914% vs. 1.346% in the data)[43].

[41] Bansal and Yaron (2004), Bansal et al. (2007), Lettau et al. (2007), Chen et al. (2009), and Bonomo et al. (2011).

[42] Because (2.7) admits negative volatility, if at some point volatility becomes negative, then the negative observation is replaced with the previous observation.

[43] In Chen et al. (2009) consumption growth volatility is around 1.5%. In Bansal and Yaron (2004) and Bonomo et al. (2011) consumption growth volatility is 3%, whereas consumption growth

Simulated moments for aggregate dividend growth are very realistic as well (mean 1.796% vs. 2.107%, volatility 13.232% vs. 13.079% in the data). However, the autocorrelation for the simulated aggregate dividend growth process is positive (0.093), whereas dividend growth in the data is a mean reverting process (-0.278). Finally, the simulated dividend growth and consumption growth processes are positively autocorrelated much like in the 1946-2011 sample (0.218 vs. 0.286 in real data[44]).

Expected earnings growth for the simulated economy is positive (1.819%), and similar to the to expected value for consumption and dividend growth. Even though, in the long-run, expected growth rates should be almost identical because dividends and earnings are cointegrated, Belo et al. (2012) explain how endogenous capital decisions can make dividends riskier than earnings in the short-run. Expected earnings growth in the sample is negative (-3.831%), and approximately equal to expected inflation, because CRSP-Compustat nominal earnings have been exponentially detrended due to the increasing number of firms in the Compustat sample over time. Earnings growth volatility is lower than in the 1946-2011 sample (6.784% vs. 7.057%). Similarly, the simulated correlation coefficient between earnings growth and consumption growth is lower than the sample one (0.425 vs. 0.487)[45]. Macroeconomic uncertainty is hard to measure, and, therefore, there aren't any readily available data to benchmark simulation results for $\sigma_t$. Nevertheless, parameter values for uncertainty dynamics in (2.8) are based on previous results commonly used in the asset pricing literature[46].

---

volatility in Shiller's data is 1.8%.

[44]For their habit model, Chen et al. (2009), p. 3377, assume that the correlation coefficient between aggregate dividends and aggregate consumption growth is equal to 0.60, more than twice the estimated value 0.286 in Table 2.8.4.

[45]The correlation coefficient between consumption growth and earnings growth in Chen et al. (2009) is 0.48 (p. 3377).

[46]Bansal and Yaron (2004), Bansal et al. (2007), Chen et al. (2009), and Bonomo et al. (2011).

### 2.5.2 Simulation results for Baa-Aaa credit spreads

The main pricing equation used in this study is the consumption-Euler equation for zero-coupon, corporate bonds that are subject to default at the expiration date

$$\mathbb{E}[y_{i,t,t+T}] = \mathbb{E}\Big[-\frac{1}{T}log\mathbb{E}_t\big[\big(\prod_{j=1}^{T} M_{t+j-1,t+j}\big)\big(1 - L_{t+T}\mathbf{1}\{r^x_{i,t,t+T} < D_{i,t+T}\}\big)\big]\Big], \quad (2.14)$$

in which $M_{t,t+j}$ is the disappointment aversion stochastic discount factor from (2.9), $L_{t+T}$ are losses given default, $r^x_{i,t,t+T}$ are ex-payout, log-returns for assets in place (not equity returns) according to (2.13), and $D_{i,t+T}$ is the default boundary. Although bond yields in Table 2.8.1 are measured in nominal terms, the model economy has been simulated in real terms, and thus, model implied spreads are inflation-free. To the extend that inflation risk premia are approximately equal for Baa and Aaa bonds, then nominal Baa-Aaa credit spreads should be very similar to real Baa-Aaa credit spreads. Unlike the model in (2.1), losses given default $L_{t+T}$ and default boundaries $D_{i,t+T}$ are allowed to vary over time, and also be functions of the state variables

$$1 - L_{t+T} = a_{rec,0} + a_{rec,c}\Delta c_{t+T-1,t+T}, \quad (2.15)$$

$$D_{i,t+T} = a_{i,def,0} + a_{def,c}\big(\Delta c_{t+T-1,t+T} - \frac{\mu_c}{1-\phi_c}\big) + a_{def,\sigma}\big(\sigma_{t+T} - \frac{\mu_\sigma}{1-\phi_\sigma}\big). \quad (2.16)$$

Table 2.8.5 shows the main empirical results in this study which have been obtained through the simulation process discussed in Appendix B.5. Panel A in Table 2.8.5 specifies values for the Baa and Aaa default boundaries which are expressed in terms of asset log-returns. For example, the 4yr constant Baa default boundary is equal to -0.998 which means that the value of assets in place needs to decrease to $e^{-0.998} = 36.861\%$ of initial value before a Baa firm defaults[47].

Simulated default probabilities in Panel B are practically indistinguishable from

[47] Chen et al. 2009 also assume a similar constant default boundary for 4 year Baa bonds (p. 3384).

default rates in the Moody's report due to appropriately selecting default boundaries. The default rates in Panel B guarantee that the stochastic discount factor in (2.9) generates plausible credit spreads because investors severely penalize default states through the disappointment aversion mechanism, and not because default probabilities are abnormally high. Finally, Panel C in Table 2.8.5 shows average credit spreads implied by the disappointment aversion discount factor in (2.9) with preference parameters from Table 2.8.3, and aggregate state variable dynamics according to the system in (2.6)-(2.8). In order to address the shortcomings of the benchmark model in (2.1), I consider four different cases: I) constant recovery rates and default boundaries, II) procyclical recovery rates according to (2.15) and constant default boundaries, III) constant recovery rates and countercyclical default boundaries according to (2.16), and IV) procyclical recovery rates and countercyclical default boundaries.

Expected credit spreads for the disappointment aversion discount factor in case I are larger than those for the benchmark model (average increase across maturities 15 bps) because disappointment averse investors heavily penalize periods during which lifetime utility is less than its certainty equivalent (disappointment events). During these periods, Baa defaults happen more often than defaults for Aaa firms, which are fairly acyclical. In other words, Baa corporate bonds expose the aggregate investor to more disappointment risk than Aaa bonds. Therefore, in order for Baa bonds to be part of the aggregate portfolio, these claims should be discounted at higher rates than Aaa bonds.

Relative to the benchmark model in (2.1), case I in Panel C is different in two very important ways. First, as explained by *Lemma 1* in Appendix B.6.1, the benchmark model implicitly assumes CRRA preferences. Although concave CRRA utility functions overweigh unfavorable outcomes, they do not capture asymmetries in marginal utility, because CRRA preferences are isoelasic. On the other hand, the disappointment model relies heavily on investors penalizing losses that happen during periods when lifetime utility is below the certainty equivalent $1+\theta$ times more than loses during normal times. Second, disappointment aversion preference induce time-variation in risk attitudes. This time-variation is further amplified by stochastic consumption growth volatility in (2.7) to generate substantially time-variation in expected returns and Sharpe ratios. Recent results in asset pricing suggest that time-variation in Sharpe ratios is almost a necessary condition for resolving a number of prominent asset pricing puzzles[48], including the credit spread puzzle.

Nevertheless, disappointment aversion alone cannot fully rationalize expected Baa-Aaa credit spreads, especially for very short maturities, since, according to Table 2.8.5, 41 bps in expected credit spreads for 4yr bonds remain unexplained by the disappointment model. These results should not cast any doubt on the explanatory power of disappointment aversion. According to Chen et al. (2009), neither the habit, nor the long-run risk models can explain credit spreads[49], unless we assume time-varying recovery rates or stochastic default boundaries.

Table 2.8.2 provides evidence that recovery rates are procyclical. The assumption of constant recovery rates therefore ignores an important risk source for credit spreads. Case II in Table 2.8.5 relaxes this assumption, and, based on the results of Table 2.8.2, assumes that losses given default $L_{t+T}$ are a linear function of aggregate consumption growth as in (2.15) in which $a_{rec,c}$ is set equal to 4.464 from Table 2.8.2. The addition of procyclical recovery rates increases Baa-Aaa spreads implied by the disappointment model by 34 bps on average across maturities relatively to the benchmark model in (2.1), and by 23 bps relatively to the benchmark model with procyclical recovery rates in (2.2).

In the case of countercyclical losses given default, corporate bonds need to compensate the disappointment averse investor for two sources of systematic risk. The

[48]Campbell and Cochrane (1999), Bansal and Yaron (2004), Verdelhan (2010), Routledge and Zin (2010), Bansal and Shaliastovich (2013).

[49]Chen et al. (2009) p. 3384 and p. 3405.

first one is related to the fact that during economic downturns default frequencies for Baa firms increase more than default frequencies for Aaa bonds. The second source of systematic risk captures the fact that during disappointment periods recovery rates decrease. Moreover, disappointment aversion preferences punish the procyclicality of recovery rates more severely than power utility which is implicitly assumed by the model in (2.2). The first-order risk aversion mechanism amplifies recovery rate risk, despite the relatively low covariance between recovery rates and consumption growth. However, despite the improvement relatively to the benchmark case in (2.2), even with countercyclical recovery rates, 26 bps in 4yr expected credit spreads (17 bps for 10yr maturities) cannot be explained by case II of the disappointment model.

Cases III and IV in Table 2.8.5 assume stochastic default boundaries. Since these boundaries are hard to measure, parameters for the stochastic default boundary have been calibrated so that default rates for the simulated economy match actual ones. Unlike Chen (2010) or Bhamra et al. (2010), but similar to Chen et al. (2009), default boundaries in this study are exogenous, even though they are functions of state variables. The calibrated values for the default boundary functions in (2.16) imply that these boundaries are strongly countercyclical, since they co-move negatively with consumption growth, and positively with macroeconomic uncertainty. In bad times, when consumption growth (volatility) is lower (higher) than its unconditional mean, default boundaries are low in absolute value, and thus managers find it easier to declare bankruptcy. In good times, when consumption growth (volatility) is higher (lower) than its mean, default boundaries are high in absolute value, and firms do not default as easily as in bad times.

Countercyclical default boundaries lead to a larger number of defaults during economic downturns, and fewer number of defaults during good times, yet, unconditionally, average default rates are equal to the ones observed in actual data. Countercyclical default boundaries essentially imply that default events covary more with aggregate macroeconomic conditions relative to cases I and II. The combination of disappointment aversion preferences with countercyclical default boundaries (case III) improves the fit of the baseline disappointment model (case I), and also increases model implied expected credit spreads by 25 bps across maturities relative to the benchmark model in (2.1). Nevertheless, the increase in credit spreads induced by stochastic default boundaries in case III is less than the increase due to procyclical recovery rates in case II, and leaves 29 bps in 4yr expected credit spreads (25 bps in 10yr bonds) unexplained.

Finally, the disappointment model with procyclical recovery rates and countercyclical boundaries (case IV) can fit average credit spreads for short (100 bps vs. 103 bps for 4yr spreads) and medium maturities (129 bps vs. 131 bps for 10yr spreads), but severely overestimates credit spreads at the long end of the term structure (148 bps vs. 112 bps for 15yr bonds). Countercyclical default boundaries increase the frequency of defaults during bad times, while procyclical recovery rates increase losses given default during periods of low economic growth. Because periods of high Baa default rates and high losses given default are also associated with disappointment events (lifetime utility below its certainty equivalent), disappointment averse investors require larger compensation for holding Baa bonds than Aaa bonds.

Overall, results in Table 2.8.5 suggest that as long as we allow for procyclical recovery rates and countercyclical default boundaries, disappointment aversion preferences are able to resolve the credit spread puzzle using risk and disappointment aversion parameters that are consistent with recent experimental results. However, as shown in Figure 2.9.4, by fitting mean credit spreads for short and medium maturities, the disappointment model overestimates mean credit spreads for maturities longer than 15 years. The credit spread literature mostly considers 4yr or 10yr bonds, and does not provide any results on long maturities. Therefore, we cannot assess the relative performance of the disappointment aversion model for long maturities. Moreover, matching average credit spreads for very short maturities (1-3yr) still remains an open question[50].

Although, the goal of this paper is not a horse race between prominent asset pricing models, we need to highlight that the disappointment aversion mechanism is unique. First, disappointment aversion preferences fully encompass recent clinical and field evidence for behavior under uncertainty which emphasize the importance of expectation-based reference-dependent utility[51]. The key mechanism in disappointment aversion is asymmetric marginal utility over gains and losses. Gains and losses are, in turn, endogenously characterized by the forward-looking certainty equivalent for lifetime utility.

Asymmetric marginal utility is not present in the habit model, which assumes a backwards-looking unobservable habit process, and, according to Ljungqvist and Uhlig (2009), leads to policy inconsistencies for the central planner. Furthermore, in the habit model of Campbell and Cochrane (1999) consumption never drops below its habit, otherwise marginal utility becomes infinity. On the other hand, for disappointment aversion preferences it is precisely periods during which consumption growth falls below its certainty equivalent that are important for credit spreads. Asymmetric marginal utility is not captured by the long-run risk model either which assumes a highly persistent mean in expected consumption growth[52].

### 2.5.3 Equity premium, and the risk-free rate

By assuming extremely high risk premia, one could possibly improve the performance of consumption-based models in fitting credit spreads. However, high risk premia would also imply abnormally high expected returns for the stock market.

[50]According to Table 2.8.1, default rates for for 1 up to 3 years are almost zero. Because no asset pricing model can map zero default rates for short term bonds into measurable yields, the credit spread literature focuses on medium to long term maturities (4-10yr).

[51]Choi et al. (2007), Post et al. (2008), Doran (2010), Crawford and Meng (2011), Abeler et al. (2011), Gill and Prowse (2012).

[52]Beeler and Campbell (2012), Bonomo et al. (2011).

In this section, I show that the disappointment aversion model in (2.9) can match moments for the equity premium, the price-dividend ratio, and the risk-free rate reasonably well, with the same preference parameters and state variable dynamics from Table 2.8.3. Equity returns, the risk-free rate, and the price-dividend ratio, have been simulated according to the expressions in (2.11), (2.10), and *Proposition 2* respectively, while sample moments are calculated using the data described in subsection 2.3.1.

According to Table 2.8.6, simulated stock market returns for the disappointment aversion model have a high mean (6.653% vs. 6.581% in the data), are quite volatile (15.049% vs. 17.216% in the data) and i.i.d. $\big(\rho(r_{m,t,t+1}, r_{m,t-1,t}) = 0.035$ vs. -0.030 in the data $\big)$, and are positively correlated with consumption growth $\big(\rho(r_{m,t,t+1}, \Delta c_{t-1,t}) = 0.463$ vs. 0.503 in the data $\big)$. The disappointment model also predicts a highly autocorrelated (0.650 vs. 0.696 in the data) and low mean (0.962% vs. 0.928% in the data) risk-free rate, yet the variance for the simulated risk-free rate is substantially smaller than the sample estimate (1.163% vs. 2.727%). Finally, even though results for the price-dividend ratio are fairly accurate, especially in terms of persistence (0.891 vs. 0.950 in the data), the simulated price-dividend in the disappointment averse economy has lower mean (3.000 vs. 3.433), and is less volatile (0.227 vs. 0.467) than the one obtained from the CRSP database.

Traditional consumption-based asset pricing models with time-separable power utility need exorbitant values for the risk aversion coefficient, around 50 for annual data[53], and around 150 for quarterly data[54], in order to match expected stock market returns. Further, extremely large risk aversion parameters lead to very volatile risk-free rates[55]. Non-separable Epstein-Zin preferences without first-order risk aversion

[53] Mehra and Prescott (1985), Cochrane (2001), Yogo (2004), Liu et al. (2009), Routldege and Zin (2010), Delikouras (2013).

[54] Aït-Sahalia et al. (2004), Yogo (2004), Delikouras (2013).

[55] Weil (1989), Delikouras (2013).

effects, also require large coefficients of risk aversion, around 30[56] to match expected stock market returns, unless we assume a very persistent process for expected consumption growth[57]. These empirical discrepancies are ingeniously concealed by the benchmark models in (2.1) and (2.2) or any other model that directly uses risk-neutral pricing because these models do not explicitly model investor preferences. In contrast, the disappointment aversion discount factor in (2.9) can generate realistic asset pricing moments using parameter values that are consistent with clinical results for behavior under uncertainty.

### 2.5.4 Comparative results for alternative preference parameters

The main goal of the paper is to examine whether disappointment aversion preferences can explain asset prices across different financial markets with risk and disappointment aversion parameters calibrated to experimental findings. This section performs a sensitivity analysis on preference parameters for the disappointment aversion discount factor in (2.9). Comparative results focus on the two parameters that affect risky choices, the risk and disappointment aversion parameters $\alpha$ and $\theta$, while the rest of the parameters in Table 2.8.3 as well as model dynamics from (2.6)-(2.8) are kept constant.

The choice of alternative parameter values for the disappointment aversion model serves three purposes. First, alternative parameters need to be close to clinical estimates. Second, alternative parameter values should be able to identify the marginal importance of the first and second-order risk aversion channels. Finally, the choice of these alternative values ought to guarantee that the multipliers $A_0 - A_3$, and $A_{m,0} - A_{m,3}$ are well defined and real. The systems of equations in *Proposition 1* and *Proposition 2* impose constraints on the magnitude of the risk aversion parameter. For instance, if $\alpha$ is greater than 8.7, then the solutions to the quadratic equations for

[56] Routldege and Zin (2010), Delikouras (2013).

[57] Bonomo et al. (2011), Beeler and Campbell (2012), Delikouras (2013)

$A_3$ and $A_{m,3}$ are imaginary numbers, unless we specify different parameters for the state variable dynamics in (2.6)-(2.8). In contrast, there are no constraints imposed on $\theta$, because $A_2$ and $A_{m,2}$ are solutions to linear equations.

For the first alternative scenario, the risk aversion parameter is set equal to -1 (second-order risk neutrality), and the disappointment aversion parameter is equal to 3. By setting $\alpha$ equal to -1, we are essentially downgrading the importance of consumption growth variance $\sigma_t^2$ as a state variable. This is done through the parameters $A_3$ and $A_{m,3}$ which significantly decrease in magnitude, and even turn positive due to second order risk neutrality. For the baseline disappointment model in Table 2.8.5 and Table 2.8.6 where $\alpha$ is positive, $A_3$ and $A_{m,3}$ are large in absolute value and negative.

According to Table 2.8.7, if we turn off the risk aversion channel, and increase the magnitude for the disappointment aversion parameter, then the expected risk-free rate decreases relative to the baseline scenario (0.519% vs. 0.962%) because the first-order precautionary savings motive intensifies. In contrast, expected equity premia remain essentially the same relative to the baseline disappointment model (5.676% vs. 5.691% for the baseline model). Even though the reduction in expected excess stock returns is almost zero, the decrease in expected credit spreads relative to the baseline scenario in Table 2.8.7 is quite impressive, approximately -29 bps for 4yr maturities across all four cases. Results in Table 2.8.7 suggest that although equity premia are insensitive to the second-order risk aversion channel, credit-spreads are hugely affected by setting $\alpha$ equal to -1. Because Baa defaults are very rare events, even the slightest change in systematic risk can lead to substantial changes in credit spreads. On the other hand, equity premia are not sensitive to second-order risk-neutrality because stock market returns are not related to rare events.

For the second alternative scenario, the disappointment aversion channel is turned off ($\theta = 0$), and the risk aversion parameter is set equal to 5. Although 5 is a reasonable value in the asset pricing literature, experimental results imply that $\alpha$ cannot be greater than 2.8[58]. In the absence of the first-order risk aversion mechanism, there is an important decrease in average credit spreads relative to the baseline calibration for the disappointment model, approximately -38 bps for 4yr maturities across all four cases. Furthermore, expected excess stock returns are almost zero, while the expected risk-free rate doubles in magnitude (2.000%), because, without disappointment aversion, the precautionary savings motive attenuates. Table 2.8.7 highlights the importance of both first- and second-order risk aversion terms in generating measurable credit spreads. Asset pricing models that do not include disappointment aversion preferences, usually substitute first-order risk aversion effects with highly persistent shocks to the stochastic discount factor through the habit or the long-run risk mechanisms[59].

## 2.6 Related literature

Before concluding the discussion on the credit spread puzzle, I will briefly relate the disappointment framework to some key results in the corporate bond literature. Merton (1974) was one of the first authors to propose a unified framework for the valuation of corporate securities, bonds and equities, which are priced as contingent claims written on a firm's assets in place. Previous results on the inability of the Merton model to match credit spreads date back to Jones et al. (1984), while Huang and Huang (2012) show that the credit puzzle is robust to a variety of specifications for the risk-neutral dynamics of asset returns.

In Merton's early framework, there were no taxes, no bankruptcy costs, and capital structure choices were irrelevant. Leland (1994) and Leland and Toft (1996), extend Merton's framework to account for tax benefits of debt, bankruptcy costs, and optimal

[58] Choi et al. (2007).

[59] Campbell and Conchrane (1999), Bansal and Yaron (2004).

leverage decisions. Goldstein et al. (2001) also propose an asset pricing model for corporate bonds in which the government, bondholders, and equityholders all have stakes in the firm's EBIT-generating process. In the Goldstein et al. model, bond coupons, default, and leverage are all endogenous decisions. However, all these papers rely directly on risk-neutral dynamics, remain silent on investor preferences, and do not really focus on the empirical performance of these models across financial markets.

Bhamra et al. (2010) also propose a unified framework to explain the equity premium and the credit-spread puzzle. Even though they use risk-neutral dynamics, and do not focus on investor preferences either[60], they provide a comprehensive model with endogenous capital structure and default decisions in order to resolve the equity premium and credit spread puzzles. Nevertheless, their model generates a credit spread of only 45 bps for 5yr maturities (75 bps for 10yr maturities, p. 670), and an equity risk premium of 3.19%.

Chen et al. (2009) compare the habit model of Campbell and Cochrane (1999), and the long-run risk model of Bansal and Yaron (2004) for their ability to explain the credit spread puzzle while generating possible moments for the stock market. Although, both models can resolve the equity premium puzzle, the long-run risk model has difficulties in generating measurable credit-spreads, while the habit-model needs to be combined with countercyclical default boundaries or procyclical recovery rates in order to fit Baa-Aaa credit spreads. Finally, Chen (2010) provides a parsimonious general equilibrium model in order to resolve the credit spread and underleverage puzzles, while matching moments for equity risk premia. However, he focuses only on 10yr maturities, while he sets the risk aversion coefficient equal to 6.5, and the EIS equal to 1.5., even though a number of empirical results suggest that[61] the EIS cannot be larger than one.

Table 2.8.8 shows model implied credit spreads and expected equity premia calcu-

[60] Although they assume Epstein-Zin utility for the aggregate investor.

[61] Hall (1988), Bonomo et al. (2011), and Beeler and Campbell (2012).

lated in previous works. Notice that almost all results focus on 4yr or 10yr maturities and remain silent on longer maturities. Furthermore, this paper is the first to impose a stochastic discount factor which is microfounded on experimental evidence for behavior under risk.

## 2.7 Conclusion

The aim of this paper is to examine whether disappointment aversion preferences can help us resolve the credit spread puzzle within a consumption-based asset pricing framework of an endowment economy. Given the relative success of first-order risk aversion preferences in explaining other stylized facts in financial markets, the disappointment aversion discount factor seems a natural candidate for correctly pricing corporate bonds. However, the first-order risk aversion mechanism implied by disappointment aversion is not powerful enough to map low probabilities of default into measurable Baa-Aaa credit spreads. Only when the disappointment model is combined with countercyclical losses given default and default boundaries, can disappointment aversion preferences resolve the credit spread puzzle. This is in line with the conclusions in Chen et al. (2009), according to which neither the habit nor the long-run risk models can price Baa corporate bonds, unless we assume additional sources of risk such as procyclical recovery rates, countercyclical default boundaries or stochastic idiosyncratic volatility.

Furthermore, by fitting credit spreads for the short and medium term, the disappointment model tends to overestimate credit spreads for long maturities (15yr+). Traditional consumption-based asset pricing models (habit, long-run risk) have only been tested against 4yr or 10yr bond maturities. It would be interesting to examine the predictions of these models for longer maturities, as well as for the ultra short run. Another direction for future research is to introduce disappointment aversion preferences in a world where capital structure choices matter so as to endogenize default decisions. In spite of all these issues, the disappointment model is quite successful in explaining not just corporate bond prices, but also key moments for stock market returns, the risk-free rate, and the price-dividend ratio using preference parameters that are consistent with experimental data for choices under uncertainty.

## 2.8 Tables

**Table 2.8.1 Average default rates, and expected credit spreads for Baa and Aaa bonds**

**Panel A: average default rates for Aaa and Baa bonds (1970-2011)**

| | 1 year | 4 year | 10 year | 15 year | 20 year |
|---|---|---|---|---|---|
| **Aaa** | 0.000% | 0.035% | 0.476% | 0.884% | 1.045% |
| **Baa** | 0.181% | 1.379% | 4.649% | 8.632% | 12.315% |

**Panel B: average Baa-Aaa credit spreads (bps)**

| | **sample period** | **maturity** | | |
|---|---|---|---|---|
| | | **short** | **medium** | **long** |
| Moody's Baa-Aaa Corp. Bond Yield | 1920-2011 | | | 118 |
| Barclays US Agg. Corp. Baa-Aaa | 1974-2011 | | 128 | 112 |
| BofA US Corp. BBB-AAA | 2001-2011 | 155 | 128 | 102 |
| Thomson-Reuters US Corp. Baa-Aaa | 2003-2011 | 157 | 180 | |
| Duffee (1998) | 1985-1995 | 75 | 70 | 105 |
| Chen et al. (2009) | 1970-2001 | 109 | | |
| Huang and Huang (2012) | 1973-1993 | 103 | 131 | |
| benchmark model in (2.1) | | 51 | 77 | 97 |
| stochastic recovery rates in (2.2) | | 58 | 87 | 112 |

Table 2.8.1 Average default rates for Baa and Aaa-rated bonds in Panel A are from the Moody's 2012 annual report. Panel B summarizes sample average credit spreads used in previous studies, as well as expected credit spreads implied by the models in (2.1) and (2.2). In Duffee (1998), short maturity is 2yr-7yr, medium is 7yr-15yr, and long maturity is 15yr-30yr. Chen et al. (2009) consider 4yr maturities, while Huang and Huang (2012) consider 4yr and 10yr maturities. For the Moody's indices, long maturity is between 20yr and 30yr. For the Barclays indices, medium maturity is 1yr-10yr, and long maturity is 10yr+. For the BofA indices, short maturity is 1yr-5yr, medium is 7yr-10yr, and long maturity is 15yr+. For the Thomson-Reuters indices, short maturity is 4yr and medium maturity is 10yr. Finally, for the benchmark and stochastic recovery rates models in (2.1) and (2.2), short maturity is 4yr, medium maturity is 10yr, and long maturity is 15yr.

**Table 2.8.2 OLS regression of recovery rates on aggregate consumption growth (1982-2011)**

| | |
|---|---|
| $\hat{a}_{rec,c}$ | 4.461 |
| | (3.036) |
| $R^2$ | 24.767% |

Table 2.8.2 shows results for the OLS regression of recovery rates on contemporaneous consumption growth. Recovery rates for senior subordinate debt are from the Moody's 2012 report. $\hat{a}_{rec,c}$ is the OLS estimate with the $t$-statistic in parenthesis.

**Table 2.8.3 Preference parameters, and state variable dynamics for the baseline disappointment model**

| variable | variable description | variable value |
|---|---|---|
| $EIS$ | elasticity of intetemporal substitution | 1 |
| $\delta$ | generalized disappointment aversion | 1 |
| $\beta$ | rate of time preference | 0.9955 |
| $\alpha$ | risk aversion | 1.8000 |
| $\theta$ | disappointment aversion | 2.0303 |
| $\mu_c$ | consumption growth constant | 0.0091 |
| $\phi_c$ | consumption growth autocorrelation | 0.5026 |
| $\mu_\sigma$ | volatility constant | 0.0004 |
| $\phi_\sigma$ | volatility autocorrelation | 0.9715 |
| $\nu_\sigma$ | volatility of volatility | 0.0017 |
| $\mu_d$ | dividend growth constant | -0.0367 |
| $\phi_d$ | leverage parameter for dividend growth | 3 |
| $\sigma_d$ | volatility parameter for dividend growth | 7.1664 |
| $\mu_e$ | earnings growth constant | -0.0367 |
| $\phi_e$ | leverage parameter for earnings growth | 3 |
| $\sigma_e$ | volatility parameter for earnings growth | 2.2011 |
| $\sigma_i$ | idiosyncratic return volatility | 0.2100 |
| $\bar{z}_m$ | linearization constant for the price-payout ratio in (B.4) | 3 |
| $\bar{x}$ | linearization constant for the normal c.d.f. in (B.9) | 0 |

Table 2.8.3 summarizes preference parameters for the disappointment aversion stochastic discount factor in (2.9), as well as model parameters for aggregate state dynamics in equations (2.6)-(2.8).

**Table 2.8.4 Simulation results for aggregate state variables**

| | | |
|---|---|---|
| $\mathbb{E}[\Delta c_{t,t+1}]$ | 1.838% | 1.834% |
| $\textbf{Vol}(\Delta c_{t,t+1})$ | 1.346% | 1.914% |
| $\rho(\Delta c_{t-1,t}, \Delta c_{t,t+1})$ | 0.502 | 0.504 |
| $\mathbb{E}[\Delta d_{m,t,t+1}]$ | 2.107% | 1.796% |
| $\textbf{Vol}(\Delta d_{m,t,t+1})$ | 13.079% | 13.232% |
| $\rho(\Delta d_{m,t-1,t}, \Delta d_{m,t,t+1})$ | -0.278 | 0.093 |
| $\rho(\Delta d_{m,t,t+1}, \Delta c_{m,t,t+1})$ | 0.286 | 0.218 |
| $\mathbb{E}[\Delta e_{m,t,t+1}]$ | -3.831% | 1.819% |
| $\textbf{Vol}(\Delta e_{m,t,t+1})$ | 7.057% | 6.784% |
| $\rho(\Delta e_{m,t-1,t}, \Delta e_{m,t,t+1})$ | 0.114 | 0.360 |
| $\rho(\Delta e_{m,t,t+1}, \Delta c_{m,t,t+1})$ | 0.487 | 0.425 |
| $\mathbb{E}[\sigma_t]$ | | 1.498% |
| $\textbf{Vol}(\sigma_t)$ | | 0.691% |
| $\rho(\sigma_t)$ | | 0.967 |

Table 2.8.4 shows sample and simulated moments for aggregate state variables. $\mathbb{E}$ is expected value, **Vol** is volatility, and $\rho$ is the correlation coefficient. $\Delta c_{t-1,t}$, $\Delta d_{m,t-1,t}$, and $\Delta e_{m,t-1,t}$ are real consumption, real dividend, and real earnings growth respectively. $\sigma_t$ is consumption growth volatility. Variables have been simulated for 100,000 years.

## Table 2.8.5 Default boundaries, average default rates, and expected Baa-Aaa credit spreads for the disappointment model

**Panel A: default boundaries for the simulated economy**

**cases I & II: constant default boundary**

| | **4 yr** | | **10 yr** | | **15 yr** | |
|---|---|---|---|---|---|---|
| | Baa | Aaa | Baa | Aaa | Baa | Aaa |
| $a_{def,0}$ | -0.998 | -1.600 | -1.108 | -1.832 | -1.007 | -1.970 |

**cases III & IV: time-varying default boundary**

| | **4 yr** | | **10 yr** | | **15 yr** | |
|---|---|---|---|---|---|---|
| | Baa | Aaa | Baa | Aaa | Baa | Aaa |
| $a_{i,def,0}$ | -1.085 | -1.790 | -1.150 | -1.920 | -1.032 | -2.040 |
| $a_{def,c}$ | -4 | -4 | -4 | -4 | -4 | -4 |
| $a_{def,\sigma}$ | 4 | 4 | 4 | 4 | 4 | 4 |

**Panel B: average default rates for the simulated data**

| | **4 year** | | **10 year** | | **15 year** | |
|---|---|---|---|---|---|---|
| | | Aaa | Baa | Aaa | Baa | Aaa |
| **case I** | 1.379% | 0.036% | 4.655% | 0.469% | 8.626% | 0.888% |
| **case II** | 1.378% | 0.035% | 4.665% | 0.471% | 8.627% | 0.891% |
| **case III** | 1.374% | 0.035% | 4.666% | 0.469% | 8.640% | 0.881% |
| **case IV** | 1.385% | 0.036% | 4.651% | 0.472% | 8.663% | 0.869% |
| **1970-2011 sample** | 1.375% | 0.035% | 4.649% | 0.476% | 8.632% | 0.884% |

**Panel C: expected Baa-Aaa credit spreads according to the disappointment model**

| | **4 year** | | | **10 year** | | | **15 year** | | |
|---|---|---|---|---|---|---|---|---|---|
| | Baa-$r_f$ | Aaa-$r_f$ | Baa-Aaa | Baa-$r_f$ | Aaa-$r_f$ | Baa-Aaa | Baa-$r_f$ | Aaa-$r_f$ | Baa-Aaa |
| **case I** | 65 | 3 | 62 | 122 | 27 | 95 | 153 | 40 | 113 |
| **case II** | 81 | 4 | 77 | 151 | 37 | 114 | 191 | 55 | 136 |
| **case III** | 78 | 4 | 74 | 139 | 33 | 106 | 167 | 47 | 120 |
| **case IV** | 106 | 6 | 100 | 176 | 47 | 129 | 215 | 67 | 148 |
| **eq.** (2.1) | | | 51 | | | 77 | | | 97 |
| **eq.** (2.2) | | | 58 | | | 87 | | | 112 |
| **sample** | | | 103 | | | 131 | | | 112 |

Table 2.8.5 Default boundaries for the simulated economy (Panel A) are expressed in terms of asset log-returns. I consider four different cases for the disappointment aversion discount factor: i) constant recovery rates and default boundaries, ii) procyclical recovery rates according to (2.15) and constant default boundaries, iii) constant recovery rates and countercyclical default boundaries according to (2.16), and iv) procyclical recovery rates and countercyclical default boundaries. $a_{i,def,0}$ is a constant, and $a_{def,c}$, $a_{def,\sigma}$ are the multipliers for consumption growth and consumption growth volatility respectively in the expression for default boundaries (2.16). Panel B shows average default rates for the simulated data as well as for the Moody's sample. Finally, Panel C shows expected Baa-Aaa credit spreads for the simulated disappointment model. Benchmark expected credit spreads are from the models in (2.1) and (2.2). Sample average credit spreads are from Huang and Huang (2012) for 4yr and 10yr bonds, and from the Barclays corporate indices for long maturity bonds.

**Table 2.8.6 Simulation results for the stock market and the risk-free rate according to the disappointment model**

| | **1946-2011** | **simulated economy** |
|---|---|---|
| $\mathbb{E}[r_{m,t,t+1}]$ | 6.581% | 6.653% |
| $\mathbf{Vol}(r_{m,t,t+1})$ | 17.216% | 15.049% |
| $\rho(r_{m,t-1,t}, r_{m,t,t+1})$ | -0.030 | 0.035 |
| $\rho(r_{m,t,t+1}, c_{t,t+1})$ | 0.503 | 0.463 |
| Sharpe | 0.328 | 0.378 |
| $\mathbb{E}[r_{f,t,t+1}]$ | 0.928% | 0.962% |
| $\mathbf{Vol}(r_{f,t,t+1})$ | 2.727% | 1.163% |
| $\rho(r_{f,t-1,t}, r_{f,t,t+1})$ | 0.696 | 0.650 |
| $\mathbb{E}[z_{m,t}]$ | 3.433 | 3.000 |
| $\mathbf{Vol}(z_{m,t})$ | 0.427 | 0.227 |
| $\rho(z_{m,t}, z_{m,t-1})$ | 0.950 | 0.891 |
| Baa Sharpe ratio | 0.220 | 0.218 |

Table 2.8.6 shows sample and simulated moments for the stock market, and the risk-free rate. $r_{m,t,t+1}$ are real stock market returns, $r_{f,t,t+1}$ is the one-year real risk-free rate, $z_{m,t}$ is the aggregate price-dividend ratio, and Baa Sharpe ratio is the equity Sharpe ratio for the median Baa firm according to Chen et al. (2009).

**Table 2.8.7 Simulation results for alternative preference parameters in the disappointment model**

| | **baseline** $\theta = 2.03, \alpha = 1.8$ | **scenario I** $\theta = 3, \alpha = -1$ | **scenario II** $\theta = 0, \alpha = 5$ |
|---|---|---|---|
| **case I** Baa-Aaa 4yr | 62 | 43 | 33 |
| **case II** Baa-Aaa 4yr | 77 | 48 | 39 |
| **case III** Baa-Aaa 4yr | 74 | 43 | 38 |
| **case IV** Baa-Aaa 4yr | 100 | 61 | 51 |
| $\mathbb{E}[r_{m,t,t+1} - r_{f,t,t+1}]$ | 5.691% | 5.676% | 0.000% |
| **Vol**$(r_{m,t,t+1})$ | 15.049% | 16.275% | 14.367% |
| $\mathbb{E}[r_{f,t,t+1}]$ | 0.962% | 0.519% | 2.000% |
| **Vol**$(r_{f,t,t+1})$ | 1.163% | 1.247% | 0.987% |

Table 2.8.7 shows simulation results for expected Baa-Aaa credits spreads and the stock market when the disappointment aversion discount factor is calibrated to alternative preference parameters. In the baseline case, $\theta = 2.03$ and $\alpha = 1.8$. For the first alternative scenario, $\theta$ is 3 and $\alpha$ is -1 (second-order risk neutrality). In the second alternative scenario, $\theta$ is zero (no disappointment aversion effect) and $\alpha$ is 5.

## Table 2.8.8 Model implied expected credit spreads and expected equity risk premia in the literature

| | **model characteristics** | **maturity** | | | |
|---|---|---|---|---|---|
| | | **4yr** | **10yr** | **15yr** | $\mathbb{E}[r_{m,t,t+1} - r_{f,t,t+1}]$ |
| Chen et al. (2009) | habit, $\alpha = 2.45$ countercyclical boundaries | 107 | 123 | | 7.30% |
| Chen et al. (2009) | long-run risk, EIS=2, $\alpha = 7.5$ | 52 | | | 7.40% |
| Chen (2010) | endogenous default, EIS=1.5, $\alpha = 6.5$ | | 105 | | 6.71% |
| Bhamra et al. (2010) | endogenous default, no preferences | 45(5yr) | 75 | | 3.19% |
| Huang & Huang (2012) | Goldstein et al. (2001) model | 31 | 40 | | |
| case IV in Table 1.7.2 | EIS=1, $\alpha = 1.8$, $\theta = 2.03$, countercyclical boundaries & losses given default | 100 | 129 | 148 | 6.65% |

Table 2.8.8 shows model implied expected credit spreads (bps) and equity risk premia calculated in prior works. “no preferences” implies that expected credit spreads have been calculated using risk-neutral measures, without modeling investor preferences. $\alpha$ is the risk aversion parameter, EIS is the elasticity of intertemporal substitution, and $\theta$ is the disappointment aversion parameter. $\mathbb{E}[r_{m,t,t+1} - r_{f,t,t+1}]$ is the expected equity risk premium.

## 2.9 Figures

### Figure 2.9.1 Baa-Aaa credit spreads, and Baa default rates for the 1946-2011 period

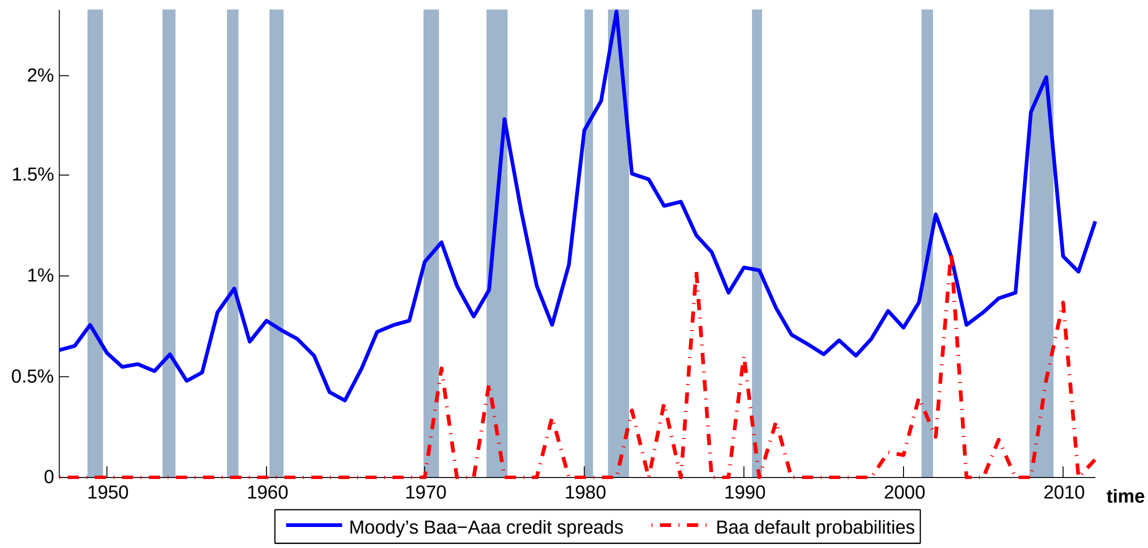


Figure 2.9.1 The solid line in Figure 1.8.2 shows Baa-Aaa credit credit spreads for the Moody's Seasoned Aaa and Baa Corporate Bond Indices. The dashed line shows annual Baa default rates from the Moody's 2012 report. Shaded areas are NBER recessions.

**Figure 2.9.2 Sample and fitted expected Baa-Aaa credit spreads according to the benchmark model in (2.1)**

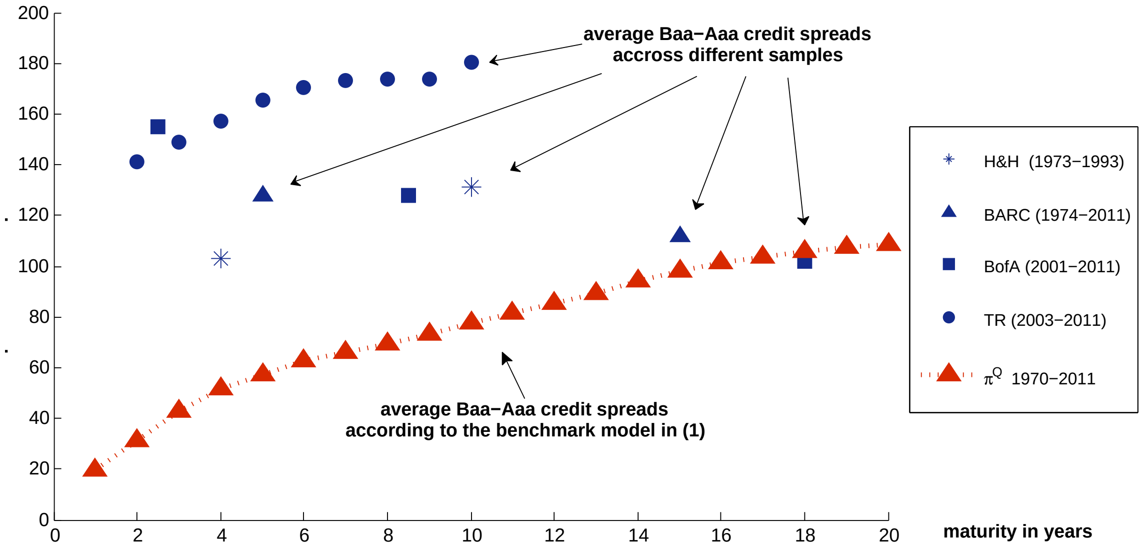


Figure 2.9.2 The dotted line in Figure 2.9.2 shows expected credit spreads (bps) according to the benchmark model in (2.1) for maturities from 1 up to 20 years. The scattered points are mean Baa-Aaa credit spreads for three sets of corporate bond indices (Barclays, BofA, and Thomson-Reuters) and the Huang and Huang (2012) sample.

**Figure 2.9.3 Recovery rates for senior subordinate bonds during the 1982-2011 period**

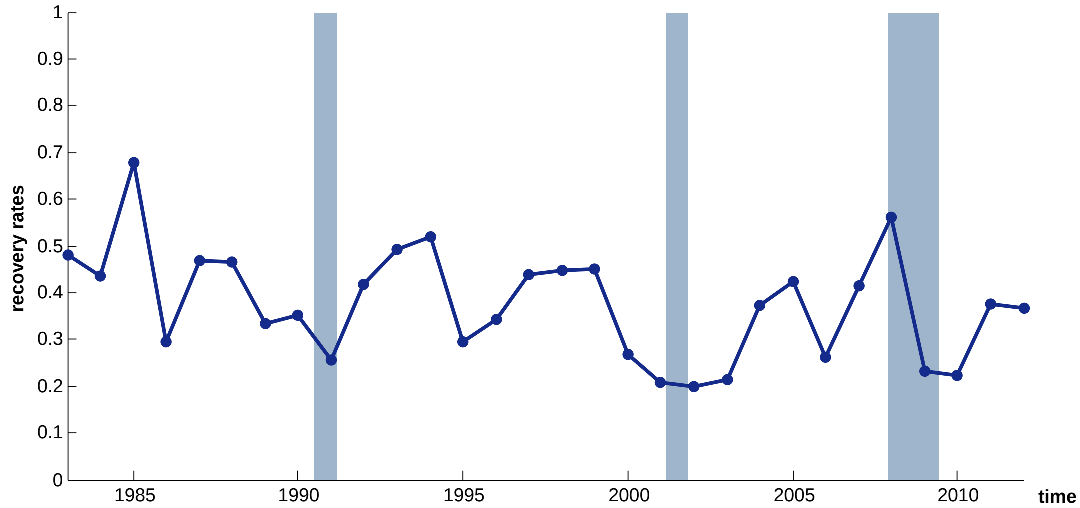


Figure 2.9.3 shows recovery rates for senior subordinate bonds from the Moody's 2012 report. Shaded areas are NBER recessions.

**Figure 2.9.4 Sample and fitted expected Baa-Aaa credit spreads according to the disappointment model in** (2.14)

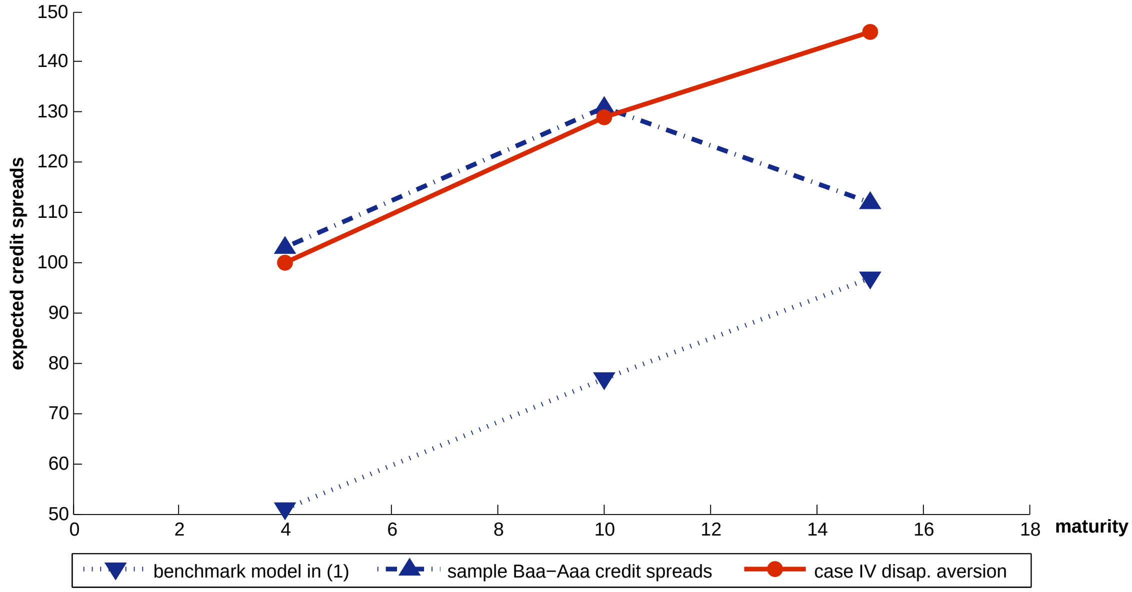


Figure 2.9.4 shows model implied expected Baa-Aaa credit spreads according to the benchmark model in (2.1) and case IV of the disappointment model in (2.14). Sample expected credit spreads are from Huang and Huang (2012) for 4yr and 10yr maturities. Sample credit spreads for 15yr maturities are from the Barclays corporate indices.

# APPENDICES

# APPENDIX A

# Disappointment Events in Consumption Growth, and the Cross-Section of Expected Stock Returns

## Appendix A.1 Preferences over stochastic payoffs

Assume that conditions for expected utility hold[1]. Suppose also that an investor is endowed with \$1, and is presented with the following dilemma: consume \$1 for sure or spend \$1 to buy a ticket to a lottery that pays either $\$1+\sigma$ or $\$1-\sigma$ ($\sigma > 0$) with equal probability. Consider first a risk-neutral agent whose preferences over risky payoffs $x$ can be expressed by a linear utility function $U_1(x) = x$. Since $\mathbb{E}[U_1(x)] = U_1\big(\mathbb{E}[x]\big) = 1$, the risk neutral agent is indifferent between taking the actuarial fair bet or not.

Now suppose that preferences over risky payoffs $x$ can be represented by a strictly increasing, strictly concave, twice differentiable utility function $U_2(x)$ such that $U_2(1) = 1$. The second-order Taylor approximation for expected utility $\frac{1}{2}U_2(1+\sigma)+\frac{1}{2}U_2(1-\sigma)$ around the point $\sigma_0 = 0$ is given by

$$\frac{1}{2}U_2(1+\sigma) + \frac{1}{2}U_2(1-\sigma) = U_2(1) + 0.5U_2''(1)\sigma^2 + O(\sigma^3). \tag{A.1}$$

[1] See von Neumann and Morgenstern (1944). For disappointment averse agents, the independence axiom can be relaxed with a betweenness axiom (Gul 1991).

Notice that when preferences are represented by smooth utility functions there are no linear $\sigma$-terms in equation (A.1) because $U_2(x)$ is twice differentiable, and the probability mass function for the random payoff is symmetric. Ignoring $O(\sigma^3)$ terms, as long as $U_2(x)$ is strictly concave everywhere, then $U_2''(x) < 0\ \forall x$, and $\frac{1}{2}U_2(1+\sigma) + \frac{1}{2}U_2(1-\sigma) < U_2(1) = 1$. The risk averse individual would reject the lottery, unless the lottery ticket were cheaper than \$1 (risk premium[2]).

Taking the limit of the Taylor expansion in (A.1) as the variance becomes zero, we conclude that

$$\lim_{\sigma^2 \downarrow 0} \left( U_2(1) + \frac{1}{2}U_2''(1)\sigma^2 + O(\sigma^3) \right) = U_2(1) = 1$$

When the dispersion of possible outcomes is very small, risk averse investors become indifferent between participating in an actuarial fair lottery or not, much like a risk neutral agent[3].

The utility function for a loss averse individual is given by

$$U_3(x) = \begin{cases} x + \tilde{\theta}(x-1), & x < 1,\ \tilde{\theta} > 0, \\ x, & x \geq 1, \end{cases}$$

in which $\tilde{\theta}$ is the coefficient of loss aversion. Since loss aversion theory does not provide any guidelines on the selection of the reference point, we set it equal to one, the value of investor's current wealth. Expected utility over lottery payoffs for the loss averse individual is equal to

$$\mathbb{E}[U_3(x)] = 1 - \frac{1}{2}\tilde{\theta}\sigma.$$

For $\tilde{\theta} > 0$, loss averse individuals would reject the fair bet, unless the ticket to enter

[2] The risk premium depends on the magnitude of $U_2''(1)$ which is associated with the Arrow-Pratt coefficient of second-order risk aversion (Pratt 1964).

[3] See also the discussion in Backus et al. (2005) p. 334.

the lottery were cheaper than \$1 (loss premium).

Consider finally an individual whose preferences over payoffs are described by a utility function of the form

$$U_4(x;\mu) = \begin{cases} x + \theta(x-\mu), & x < \mu,\ \theta > 0,\ \mu = \mathbb{E}\big[x\big] + \mathbb{E}\big[\theta(x-\mu)\mathbf{1}\{x<\mu\}\big], \\ x, & x \geq \mu, \end{cases}$$

in which $\theta > 0$ is the coefficient of disappointment aversion, and $\mu$ is the certainty equivalent of the random payoff[4]. Notice that $\mu$ is also the threshold for disappointment events. For $\theta > 0$, $\mu \in (1-\sigma, 1+\sigma)$ and in particular

$$\mu = 1 - \frac{\frac{1}{2}\theta}{1+\frac{1}{2}\theta}\sigma < 1. \tag{A.2}$$

The disappointment averse agent would also reject the actuarially fair bet, unless the price to enter the lottery were cheaper than \$1 (disappointment premium).

Let $\epsilon$ be an extremely small positive number close to zero, and consider the limit in (A.2) as $\sigma^2$ approaches $\epsilon$

$$\lim_{\sigma^2 \downarrow \epsilon} \mu = 1 - \frac{\frac{1}{2}\theta}{1+\frac{1}{2}\theta}\sqrt{\epsilon} < 1.$$

A similar expression holds for loss averse individuals

$$\lim_{\sigma^2 \downarrow \epsilon} \big(1 - \frac{1}{2}\tilde{\theta}\sqrt{\sigma^2}\big) = 1 - \frac{1}{2}\tilde{\theta}\sqrt{\epsilon} < 1.$$

[4]According the the generalized disappointment aversion model of Routledge and Zin (2010), the reference level for gains and losses is a multiple $\delta$ of the certainty equivalent $\mu$

$$U_4(x;\mu) = \begin{cases} x + \theta(x-\delta\mu), & x < \delta\mu,\ \delta \in (0,1],\ \theta > 0,\ \mu = \mathbb{E}\big[x\big] + \mathbb{E}\big[\theta(x-\delta\mu)\mathbf{1}\{x<\delta\mu\}\big], \\ x, & x \geq \delta\mu. \end{cases}$$

For the case $\delta > 1$, the reader is referred to Routledge and Zin (2010).

Finally, for the continuously differentiable concave utility function, it follows that

$$\lim_{\sigma^2 \downarrow \epsilon} \left( U_2(1) + \frac{1}{2} U_2''(1)\sigma^2 + O(\sigma^3) \right) = U_2(1) + \frac{1}{2} U_2''(1)\epsilon \approx 1.$$

As $\sigma^2$ approaches zero, $\sigma$ also approaches zero but at a far slower rate than $\sigma^2$ [5]. First-order risk aversion effects in disappointment or loss aversion preferences do not vanish immediately as $\sigma^2 \downarrow 0$. In contrast, as $\sigma^2 \downarrow 0$, a second-order risk averse individual would be indifferent between accepting actuarial fair lotteries or not. When the dispersion of lottery payoffs is small, first-order risk aversion induces a more conservative risk taking behavior than second-order risk aversion.

In the context of consumption-based asset pricing, smooth utility functions need to be extremely concave in order to generate realistic equity risk premia because aggregate consumption growth exhibits very low variability ($\sigma_c^2 \downarrow 0$). On the other hand, even if consumption growth variance is almost zero, there might still be measurable consumption growth volatility terms ($\sigma_c > 0$). These volatility terms can be combined with disappointment aversion preferences to generate measurable equity risk premia.

The figure below shows utility plots for all four types of preferences considered here. When uncertainty is low, linear utility becomes tangential to the CRRA utility function, and CRRA investors behave as if they were risk-neutral.

### Appendix A.2 Linear disappointment aversion preferences

One of the key insights of non-separable utility functions is that preferences over the timing and uncertainty of payoffs need not be characterized by the same parameter. The disappointment aversion framework can separate preferences over risk and time, while preserving the additive form for lifetime utility $V_t$. Suppose that in equa-

[5] $\sigma$ approaches zero at a square root rate, yet as $\sigma^2$ eventually becomes zero, $\sigma$ will also become zero.

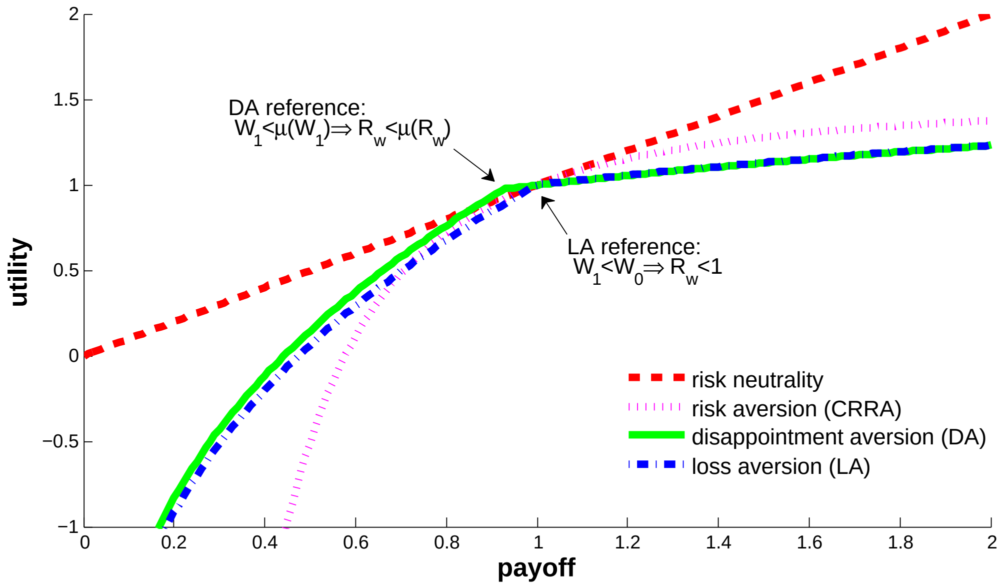


Figure A.1 The utility functions shown in are: i) linear utility (risk neutrality), ii) power utility (second-order risk aversion), iii) piece-wise utility over gains and losses (loss aversion, LA), iv) piece-wise utility with endogenous reference for gains and losses (disappointment aversion, DA). $W_0$ and $W_1$ are wealth before and after the gamble respectively. $R_w = W_1/W_0$ are returns on wealth after the gamble, and $\mu()$ is the disappointment aversion certainty equivalent.

tions (1.1), (1.2), and (1.3) $-\alpha = \delta = \rho = 1$. The restriction $-\alpha = \rho$ implies that preferences are essentially time-additive. However, the expected value operator $\mathbb{E}_t$ in the recursive equation (1.1) is replaced by the disappointment aversion certainty equivalent $\mu_t$, and the discount factor in (1.3) becomes

$$M_{t,t+1} = \beta \frac{1 + \theta\ \mathbf{1}\{V_{t+1} < \ \mu_t(V_{t+1})\}}{1 + \theta\ \mathbb{E}_t[\mathbf{1}\{V_{t+1} < \mu_t(V_{t+1})\}]}. \tag{A.3}$$

Suppose now that consumption in first differences follows an AR(1) process

$$\Delta C_{t+1} = \mu_C(1 - \Phi_C) + \Phi_C \Delta C_t + \sqrt{(1 - \Phi_C^2)} \Sigma_C \epsilon_{t+1}. \tag{A.4}$$

$\mu_C = \mathbb{E}[\Delta C_{t+1}] \in \mathbb{R}$, $\Sigma_C^2 = \mathbf{Var}(\Delta C_{t+1}) \in \mathbb{R}_{>0}$, $\Phi_C = \rho(\Delta C_{t+1}, \Delta C_t) \in (-1, 1)$ are the unconditional mean, variance, and first-order autocorrelation for consumption in first differences respectively. $\epsilon_{t+1}$ are i.i.d. $N(0, 1)$ variables. The $R^2$ for the above

AR(1) model is 24.571% at the annual frequency, and 25.909% for quarterly data. Since $V_{t+1}$ is unobservable, the use of the disappointment aversion discount factor in (A.3) for empirical purposes becomes problematic. However, we can express lifetime utility $V_t$ in terms of consumption changes $\Delta C_t$,

*Proposition 2:* Given the consumption dynamics in (A.4), then for $-\alpha = \rho = \delta = 1$ in equations (1.1), (1.2) and (1.3), investors' lifetime utility $V_t$ can be expressed as $V_t = C_t + \mu_V + \Phi_V \Delta C_t \; \forall t$, where

- $\Phi_V = \frac{\beta \Phi_C}{1-\beta\Phi_C}, \; \Phi_V \in \mathbb{R}$,
- $\mu_V = \frac{\beta}{1-\beta}\left[\mu_C(1-\Phi_C)(\Phi_V+1) - \frac{\theta n(d_1)}{1+\theta N(d_1)}(\Phi_V+1)\sqrt{1-\Phi_C^2}\Sigma_C\right], \; \mu_V \in \mathbb{R}$,
- $d_1 \in \mathbb{R}_{<0}$ is the solution to the fixed point problem

$$d_1 = -\frac{\theta n(d_1)}{1+\theta N(d_1)}, \tag{A.5}$$

and $N(.)$ and $n(.)$ are the standard normal cdf and pdf respectively.

***Proof.*** *See Appendix A.4.2.*

*Proposition 2* tells us that lifetime utility minus current consumption $(V_t - C_t)$ is an affine function of consumption in first differences $(\Delta C_t)$. $\mu_V$ is the constant term, and $\Phi_V$ is the slope coefficient for consumption changes. Notice the linear $(d_1\sqrt{1-\Phi_C^2}\Sigma_C)$, instead of quadratic, structure of the disappointment aversion correction in $\mu_v$. Comparative statics for $\mu_V$ imply that high uncertainty about the economy (large $\Sigma_C$) or a large disappointment aversion coefficient $\theta$ would lead to large (in absolute magnitude) corrections to $\mu_V$[6].

[6] $\partial\mu_V/\partial\theta = -\frac{n(d_1)}{1+\theta N(d_1)}\left(1-\frac{\theta N(d_1)}{1+\theta N(d_1)}\right) < 0$ since the inequality $1 > \frac{x}{1+x}$ holds trivially $\forall \; x \in \mathbb{R}_{>0}$.

From ***Proposition 2*** disappointment events happen whenever

$$\Delta C_{t+1} < \underbrace{\underbrace{\mu_C(1-\Phi_C)+\Phi_C\Delta C_t}_{\text{expected value}} + \underbrace{d_1\sqrt{1-\Phi_C^2}\Sigma_C}_{\text{disappointment aversion adjustment}}}_{\text{certainty equivalent}}. \tag{A.6}$$

For $\theta = 12$ in (A.5), then $d_1 \approx -1$, and disappointment events happen whenever shocks to consumption in first differences $\epsilon$ are less than -1, or consumption changes drop one standard deviation below the expected value. If shocks to consumption changes were normally distributed, then for $\theta = 12$ disappointment events would happen around 16% of the time.

Using *Proposition 2*, the discount factor in (A.3) now becomes

$$M_{t,t+1} = \beta\frac{1+\theta\ \mathbf{1}\{\Delta C_{t+1} < \mu_C(1-\Phi_C)+\Phi_C\Delta C_t + d_1\sqrt{1-\Phi_C^2}\Sigma_C\}}{1+\theta\ \mathbb{E}_t[\mathbf{1}\{\Delta C_{t+1} < \mu_C(1-\Phi_C)+\Phi_C\Delta C_t + d_1\sqrt{1-\Phi_C^2}\Sigma_C\}]},$$

and its conditional expectation is equal to

$$\mathbb{E}_t[M_{t,t+1}] = \beta.$$

The risk-free rate for the linear disappointment model is a constant, and the real yield-curve is always flat.

Finally, expected asset returns for the linear disappointment model are equal to

$$\mathbb{E}[R_{i,t+1}] = \underbrace{\mathbb{E}[R_{f,t+1}]}_{\frac{1}{\beta}} - \underbrace{\frac{\theta\mathbf{Cov}[R_{i,t+1},\mathbf{1}\{\Delta C_{t+1} < \mu_C(1-\Phi_C)+\Phi_C\Delta C_t + d_1\sqrt{1-\Phi_C^2}\Sigma_C\}]}{1+\mathbb{E}[\mathbf{1}\{\Delta C_{t+1} < \mu_C(1-\Phi_C)+\Phi_C\Delta C_t + d_1\sqrt{1-\Phi_C^2}\Sigma_C\}]}}_{\text{risk premium: a function of } \theta \text{ alone}},$$

The risk-free rate depends only on the rate of time preference $\beta$. On the other hand, the disappointment aversion coefficient $\theta$ affects risk premia, but not the risk-free rate. An individual characterized by linear disappointment aversion preferences is only worried about consumption in first differences dropping below the certainty equivalent. On the other hand, an investor with log-linear disappointment aversion preferences (section 1.3.2) cares about consumption growth falling below the certainty equivalent, as well as about the actual level of consumption growth.

## Appendix A.3 Consistency and asymptotic normality of GMM estimators when the GMM objective function is not continuous

In order to prove consistency and asymptotic normality for GMM estimators, standard applications require differentiability of the GMM objective function. However, continuity and differentiability are violated when moment restrictions are associated with indicator functions.

Let $z_t$ be a vector of random variables and $x$ a vector of parameters. Consider the GMM objective function

$$Q_0 = \mathbb{E}\big[q(z_t, x)\big]' W \ \mathbb{E}\big[q(z_t, x)\big], \tag{A.7}$$

and its sample analogue

$$\hat{Q}_T = \big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\big]' \hat{W} \big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\big]. \tag{A.8}$$

For the disappointment aversion model $x = \{\beta, \alpha, \theta\}$, $z_t = \big\{\Delta c_{t+1}, \{r_{i,t+1}\}_{i=1}^{n-1}, r_{f,t+1}\big\}$, and

$$q(z_t, \theta) = \tilde{M}_{t,t+1}\big(e^{r_{i,t+1}} - e^{r_{f,t+1}}\big) \text{ for } i = 1, 2, ..., n, \tag{A.9}$$

with

$$\tilde{M}_{t,t+1} = exp\Big[log(\beta) + \alpha(\phi_v+1)\mu_c(1-\phi_c) - \frac{\alpha^2}{2}(\phi_v+1)^2(1-\phi_c^2)\sigma_c^2$$
$$-[\alpha(\phi_v+1)+1]\Delta c_{t+1} + \frac{\alpha}{\beta}\phi_v\Delta c_t\Big]\big(1+\theta\mathbf{1}\{\Delta c_{t+1} < \mu_c(1-\phi_c)+\phi_c\Delta c_t + d_1\sqrt{1-\phi_c^2}\sigma_c\}\big),$$

and

$$d_1 = -\frac{\alpha}{2}(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c - \frac{1}{\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c}log\Big[\frac{1+\theta N\big(d_1+\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\big)}{1+\theta N\big(d_1\big)}\Big].$$

We can assume that $x = \{\beta, \alpha, \theta\}$ takes values in a compact space $\mathbf{X} \in \mathbb{R}^3$. Economic theory suggests that for disappointment averse investors $\beta \in (0,1)$, $\alpha \in (-1, B_\alpha)$, and $\theta \in (0, B_\theta)$. $B_\alpha < +\infty$ and $B_\theta < +\infty$ are upper bounds for the coefficients of risk and disappointment aversion respectively. In general, risk preference parameters $\alpha$ and $\theta$ cannot assume infinite values, and are bounded from above by some positive real numbers ($B_\alpha$ and $B_\theta$) which may be arbitrarily large but finite. We will also assume that $z_t = \big\{\Delta c_{t+1}, \{r_{i,t+1}\}_{i=1}^{n-1}, r_{f,t+1}\big\}$ is characterized by a continuous probability distribution function, and a well-defined moment generating function $\forall x \in \mathbf{X}$[7]. Finally, let $x_0$ be the minimizer in (A.7), and $\hat{x}_T$ the minimizer in (A.8).

**Identification.** We will assume that the GMM objective function in (A.7) satisfies the conditions in Lemma 2.3, p. 2126 in Newey and McFadden (1994), so that $x_0$ is globally identified. Because it is quite hard to verify identification, for the practical purposes of our estimation we will simply assume it[8].

**Consistency.** For consistency of GMM estimators when the GMM objective function is not continuous, we refer to Theorem 2.6, p. 2132 in Newey and McFadden (1994). We essentially require that:

1. $z_t$ is stationary and ergodic

[7] For $\beta \in (0,1)$ and $\Delta c_{t+1}$ stationary, it also follows that $1 - \phi_c\beta \neq 0$.

[8] See also the discussion in Newey and McFadden (1994), p. 2127 on the Hansen and Singleton (1982) model.

2. $\hat{W} \xrightarrow{p} W$, $W$ is positive definite, and $W\mathbb{E}[g(z, x_0)] = 0$ only if $x = x_0$

3. $\mathbf{X}$ is compact

4. $q(z_t, x)$ is continuous with probability one.

5. $\mathbb{E}\big[sup_{x\in\mathbf{X}}||q(z_t, x)||\big] < +\infty$

Stationarity and ergodicity are reasonable properties for the random variables $\big\{\Delta c_{t+1}, \{r_{i,t+1}\}_{i=1}^{n-1}, r_{f,t+1}\big\}$ at the quarterly and annual frequencies. The second condition is satisfied because the GMM weighting matrix is constant, and equal to the identity matrix. Moreover, according to the identification assumption above, the GMM objective function has a unique minimizer $x_0$ which can be identified. Economic theory suggests that the parameter space $\mathbf{X}$ is compact. The fourth condition is also satisfied since the only point of discontinuity in expression (A.9) is

$$\Delta c_{t+1} = \phi_c \Delta c_t + \mu_c(1 - \phi_c) + d_1\sqrt{1 - \phi_c^2}\sigma_c,$$

which is a zero-probability event as long as consumption growth is a continuous random variable. Finally, condition five is satisfied because $\mathbf{X}$ is compact, and the distribution of $z_t$ has a well-defined moment generating function $\forall x \in \mathbf{X}$.

**Asymptotic normality.** Theorems 7.2, p. 2186, and 7.3, p. 2188 in Newey and McFadden (1994) provide conditions for asymptotic normality of GMM estimates when the GMM objective function is not continuous. These conditions are

1. $\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\big]'\hat{W}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\big] \leq \inf_{x\in\mathbf{X}}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\big]'\hat{W}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\big]$

2. $\hat{W} \xrightarrow{p} W$, $W$ is positive definite

3. $\hat{x} \xrightarrow{p} x_0$

4. $x_0$ is in the interior of $\mathbf{X}$

5. $\mathbb{E}[g(z, x_0)] = 0$

6. $\left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x_0)\right] \xrightarrow{d} N(0, \Sigma)$

7. $\mathbb{E}[g(z, x)]$ is differentiable at $x_0$ with derivative $G$, and $G'WG$ is non-singular

8. for $\delta_N \to 0$, then

$$\sup_{||x-x_0||\leq\delta_n} \frac{\sqrt{n}\left|\left|\left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\right] - \left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x_0)\right] - \mathbb{E}\big[g(z, x_0)\big]\right|\right|}{1 + \sqrt{n}||x - x_0||} \xrightarrow{p} 0. \quad \text{(A.10)}$$

The first condition is related to identification. The second condition is satisfied since $\hat{W} = I$. The third condition is satisfied by the consistency theorem above. Conditions 4, 5, and 6 are standard GMM assumptions. The seventh condition is satisfied provided that the joint probability density function of asset returns and consumption growth is continuous, and that the moment generating function is well-defined. The critical condition for asymptotic normality is condition 8, the stochastic equicontinuity condition.

Andrews (1994) provides primitive conditions in order to verify stochastic equicontinuity. These conditions are related to Pollard's entropy condition (Pollard 1984). Fortunately, the GMM objective function in (A.9) is a mixture of functions that satisfy the entropy condition. According to Theorem 2, p. 2272 in Andrews (1994), indicator functions (which are "type I" functions, p. 2270 in Andrews 1994) satisfy Pollard's conditions. A second class of functions ("type II" functions, p. 2271 in Andrews 1994) that satisfy Pollard's conditions are functions which depend on a finite number of parameters, and are Lipschitz-continuous[9] with respect to these parameters.

[9] Lipschitz continuity is also exploited in Theorem 7.3, p. 2188, in Newey and McFadden (1994) as a primitive condition to show stochastic equicontinuity.

The GMM $q(z_t, x)$ vector-valued function in equation (A.9) consists of exponential terms which, in turn, are functions of a finite number of preference parameters. Exponentially functions are only locally Lipschitz-continuous. However, the exponential terms in the GMM objective function are Lipschitz-continuous on the compact parameter space $\mathbf{X}$, since the rate of change of the exponential functions remains bounded as long as variables take values in compact spaces. Therefore, exponential functions defined on the compact set $\mathbf{X}$ belong to the "type II" class of functions. We conclude that the disappointment aversion GMM objective function in equation (A.9) contains terms which individually satisfy Pollard's entropy condition.

According to Theorem 3, p. 2273 in Andrews (1994), elementary operations among "type I" and "type II" functions result in functions which also satisfy Pollard's entropy condition. Consequently, the disappointment aversion GMM objective function in (A.9), which is a product of "type I" and "type II" functions, satisfies the stochastic equicontinuity condition, and GMM estimates for the disappointment model are therefore asymptotically normally distributed.

The above discussion confirms that even though $q(z_t, x)$ in (A.9) is not continuous with respect to $x = \{\beta, \alpha, \theta\}$, standard results from GMM asymptotic theory can still be applied provided that certain regularity conditions are satisfied. These conditions are in general associated with "continuity" and "differentiability" of the function $\mathbb{E}\big[q(z_t, x)\big]$ rather than the function $q(z_t, x)$ itself.

Finally, even if $q(z_t, x)$ is not continuous or continuously differentiable, we can still proceed with hypothesis testing as usual by replacing derivatives with finite differences approximations. Theorem 7.4, p. 2190 in Newey and McFadden (1994) suggests that numerical derivatives for $\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)$ will asymptotically converge in probability to the derivative of $\mathbb{E}[q(z_t, x)]$. We can, therefore, obtain consistent asymptotic variance estimators using finite differences. However, a practical problem with numerical derivatives is the choice of perturbation parameters used in the denominator.

Unfortunately, econometric theory does not provide a clear answer to this problem.

## Appendix A.4 Proofs

### Appendix A.4.1 Proof of *Proposition 1*

For $\rho = 0$ and $\delta = 1$, equation (1.1) implies that along an optimal consumption path[10]

$$\Big(\frac{V_t}{C_t}\Big)^{\frac{1}{\beta}} = \mu_t\big(\frac{V_{t+1}}{C_t};\ \frac{V_{t+1}}{C_t} < \mu_t(\frac{V_{t+1}}{C_t})\big).$$

Taking logs in both sides of the equation, and using the definition of the disappointment aversion certainty equivalent $\mu_t$ in (1.2), we obtain

$$\frac{1}{\beta}(v_t - c_t) = -\frac{1}{\alpha} log \mathbb{E}_t \Big\{ exp\big[-\alpha(v_{t+1} - c_t)\big] \frac{1 + \theta \mathbf{1}\{v_{t+1} - c_t < \frac{1}{\beta}(v_t - c_t)\}}{1 + \theta \mathbb{E}_t[\mathbf{1}\{v_{t+1} - c_t < \frac{1}{\beta}(v_t - c_t)\}]} \Big\}.$$

Letting $v_t - c_t = \mu_v + \phi_v \Delta c_t \ \forall t$, then

$$\frac{1}{\beta}(\mu_v + \phi_v \Delta c_t) = -\frac{1}{\alpha} log \mathbb{E}_t \Big\{ e^{-\alpha\big[\mu_v + (\phi_v+1)\Delta c_{t+1}\big]} \frac{1 + \theta \mathbf{1}\{\mu_v + (\phi_v + 1)\Delta c_{t+1} < \frac{1}{\beta}(\mu_v + \phi_v \Delta c_t)\}}{1 + \theta \mathbb{E}_t[\mathbf{1}\{\mu_v + (\phi_v + 1)\Delta c_{t+1} < \frac{1}{\beta}(\mu_v + \phi_v \Delta c_t)\}]} \Big\}.$$

We can use (1.5) to express $\Delta c_{t+1}$ in terms of $\Delta c_t$

$$\frac{1}{\beta}(\mu_v + \phi_v \Delta c_t) = -\frac{1}{\alpha} log \mathbb{E}_t \Big\{ exp\Big[-\alpha\big[\mu_v + (\phi_v + 1)\big(\mu_c(1-\phi_c) + \phi_c \Delta c_t + \sqrt{1-\phi_c^2}\sigma_c \epsilon_t\big)\big]\Big] \times \frac{1+\theta\mathbf{1}\{\mu_v + (\phi_v+1)(\mu_c(1-\phi_c)+\phi_c\Delta c_t + \sqrt{1-\phi_c^2}\sigma_c\epsilon_{t+1}) < \frac{1}{\beta}(\mu_v+\phi_v\Delta c_t)\}}{1+\theta\mathbb{E}_t[\mathbf{1}\{\mu_v + (\phi_v+1)(\mu_c(1-\phi_c)+\phi_c\Delta c_t + \sqrt{1-\phi_c^2}\sigma_c\epsilon_{t+1}) < \frac{1}{\beta}(\mu_v+\phi_v\Delta c_t)\}]} \Big\}.$$

Partial moments for log-normal random variables imply that

$$\begin{aligned} \frac{1}{\beta}(\mu_v + \phi_v \Delta c_t) &= \mu_v + (\phi_v + 1)\big(\mu_c(1-\phi_c) + \phi_c \Delta c_t - \frac{\alpha}{2}(\phi_v + 1)^2(1-\phi_c^2)\sigma_c^2\big) \quad \text{(A.11)} \\ &-\frac{1}{\alpha} log\Big[1 + \theta N\Big(\frac{\frac{1}{\beta}(\mu_v+\phi_v\Delta c_t) - \mu_v - (\phi_v+1)(\mu_c(1-\phi_c)+\phi_c\Delta c_t)}{(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c} + \alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\Big)\Big] \\ &+\frac{1}{\alpha} log\Big[1 + \theta N\Big(\frac{\frac{1}{\beta}(\mu_v+\phi_v\Delta c_t) - \mu_v - (\phi_v+1)(\mu_c(1-\phi_c)+\phi_c\Delta c_t)}{(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c}\Big)\Big]. \end{aligned}$$

[10] Lower case letters denote logs of variables.

Ignoring for the moment the last two log-terms, $\phi_v$ must satisfy

$$\phi_v = \frac{\beta\phi_c}{1-\beta\phi_c}.$$

For $\phi_v = \frac{\beta\phi_c}{1-\beta\phi_c}$, the two log-terms in (A.11) do not depend on $\Delta c_t$. Hence, the constant term $\mu_v$ in (A.11) must be equal to

$$\mu_v = \frac{\beta}{1-\beta}\Big\{\frac{\mu_c(1-\phi_c)}{1-\beta\phi_c} - \frac{\alpha(1-\phi_c^2)\sigma_c^2}{2(1-\beta\phi_c)^2}$$
$$-\frac{1}{\alpha}log\Big[\frac{1+\theta N\Big(\frac{\frac{1-\beta}{\beta}\mu_v-(\phi_v+1)\mu_c(1-\phi_c)}{(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c}+\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\Big)}{1+\theta N\Big(\frac{\frac{1-\beta}{\beta}\mu_v-(\phi_v+1)\mu_c(1-\phi_c)}{(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c}\Big)}\Big]\Big\}.$$

We can define the disappointment event threshold as[11]

$$d_1 = \frac{\frac{1-\beta}{\beta}\mu_v - (\phi_v+1)\mu_c(1-\phi_c)}{(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c}. \quad \text{(A.12)}$$

Then $\mu_v$ becomes

$$\mu_v = \frac{\beta}{1-\beta}\Big\{(\phi_v+1)\mu_c(1-\phi_c) - \frac{\alpha}{2}(\phi_v+1)^2(1-\phi_c^2)\sigma_c^2$$
$$-\frac{1}{\alpha}log\Big[\frac{1+\theta N\big(d_1+\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\big)}{1+\theta N\big(d_1\big)}\Big]\Big\}.$$

Plugging back the above expression for $\mu_v$ into the definition of $d_1$ in (A.12), $d_1$ is the solution to the fixed-point problem

$$d_1 = -\frac{\alpha}{2}(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c - \frac{1}{\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c}log\Big[\frac{1+\theta N\big(d_1+\alpha(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\big)}{1+\theta N\big(d_1\big)}\Big].$$

[11] If $\delta \neq 1$ in (1.2), then $d_1$ would be a function of $\Delta c_t$, and the linearity of the log-value function in terms of consumption growth would brake down.

Finally,

$$\mu_v = \frac{\beta}{1-\beta}\Big\{(\phi_v+1)\mu_c(1-\phi_c) + d_1(\phi_v+1)\sqrt{1-\phi_c^2}\sigma_c\Big\}.$$

### Appendix A.4.2 Proof of *Proposition 2*

For $-\alpha = \rho = \delta = 1$, equation (1.1) implies that along an optimal consumption path

$$\frac{1}{\beta}\big(V_t - C_t\big) = \mu_t\Big(V_{t+1} - C_t;\ V_{t+1} - C_t\ <\ \frac{1}{\beta}\big(V_t - C_t\big)\Big).$$

Assume that $V_t - C_t = \mu_V + \Phi_V \Delta C_t\ \forall t$, then

$$\frac{1}{\beta}\big(\mu_V + \Phi_V\Delta C_t\big) = \mu_t\big(\mu_V + (\Phi_V+1)\Delta C_{t+1};\ \mu_V + (\Phi_V+1)\Delta C_{t+1}\ <\ \frac{1}{\beta}[\mu_V + \Phi_V\Delta C_t]\big).$$

Plugging the dynamics for $\Delta C_{t+1}$ from equation (A.4), it follows that

$$\frac{1}{\beta}\big(\mu_V + \Phi_V\Delta C_t\big) = \mathbb{E}_t\Big[\big(\mu_V + (\Phi_V+1)\tilde{\mu}_C + (\Phi_V+1)\Phi_C\Delta C_t + (\Phi_V+1)\tilde{\Sigma}_C\epsilon_{t+1}\big)$$
$$\big(1+\theta\mathbf{1}\{\mu_V + (\Phi_V+1)\tilde{\mu}_C + (\Phi_V+1)\Phi_C\Delta C_t + (\Phi_V+1)\tilde{\Sigma}_C\epsilon_{t+1} < \frac{1}{\beta}(\mu_V + \Phi_V\Delta C_t)\}\big)\Big] \times$$
$$\Big\{1+\theta\mathbb{E}_t\Big[\mathbf{1}\{\mu_V + (\Phi_V+1)\tilde{\mu}_C + (\Phi_V+1)\Phi_C\Delta C_t + (\Phi_V+1)\tilde{\Sigma}_C\epsilon_{t+1} < \frac{1}{\beta}(\mu_V + \Phi_V\Delta C_t)\}\Big]\Big\}^{-1},$$

where $\tilde{\mu}_C = \mu_c(1-\Phi_C)$ and $\tilde{\Sigma}_C = \Sigma_c\sqrt{1-\Phi_C^2}$. Since error terms $\epsilon_{t+1}$ are normally distributed, we conclude that[12]

$$\frac{1}{\beta}\big(\mu_V + \Phi_V\Delta C_t\big) = \frac{1}{1+\theta\ N(d_1)} \times \Big\{\mu_V + (\Phi_V+1)\tilde{\mu}_C \qquad \text{(A.13)}$$
$$+(\Phi_V+1)\Phi_C\Delta C_t + \theta\Big[\big(\mu_V + (\Phi_V+1)\tilde{\mu}_C + (\Phi_V+1)\Phi_C\Delta C_t\big)N(d_1)\Big] - \theta\ n(d_1)\big(\Phi_V+1\big)\tilde{\Sigma}_C\Big\},$$

[12] Winkler et al. (1972) derive simple expressions for partial moments of normally and log-normally distributed random variables.

in which $N(.)$ and $n()$ are the standard normal c.d.f. and p.d.f. respectively, and

$$d_1 = \big[\frac{1}{\beta}(\mu_V + \Phi_V \Delta C_t) - \mu_V - (\Phi_V + 1)\tilde{\mu}_C - (\Phi_V + 1)\Phi_C \Delta C_t\big] \cdot \big((\Phi_V + 1)\tilde{\Sigma}_C\big)^{-1}, \quad \text{(A.14)}$$

is the disappointment threshold.

Equation (A.13) simplifies to

$$\frac{1}{\beta}\big(\mu_V + \Phi_V \Delta C_t\big) = \big[\mu_V + \tilde{\mu}_C(\Phi_V + 1) + (\Phi_V + 1)\Phi_C \Delta C_t\big] - \frac{\theta n(d_1)\big(\Phi_V + 1\big)\tilde{\Sigma}_C}{1 + \theta\ N(d_1)}. \quad \text{(A.15)}$$

Ignoring for the moment the last term, $\Phi_V$ must satisfy

$$\Phi_V = \frac{\beta \Phi_C}{1 - \beta \Phi_C}.$$

For $\Phi_V = \frac{\beta \Phi_C}{1 - \beta \Phi_C}$[13], then $d_1$ in (A.14) is equal to

$$d_1 = \big[\frac{1 - \beta}{\beta}\mu_V - (\Phi_V + 1)\tilde{\mu}_C\big] \cdot \big((\Phi_V + 1)\tilde{\Sigma}_C\big)^{-1}. \quad \text{(A.16)}$$

Thus, for $\Phi_V = \frac{\beta \Phi_C}{1 - \beta \Phi_C}$, there are no $\Delta C_t$ terms in the expression for the disappointment threshold $d_1$. Collecting constant terms from (A.15), $\mu_v$ is equal to

$$\mu_V = \frac{\beta}{1 - \beta}\big[(\Phi_V + 1)\mu_C(1 - \Phi_C) - \frac{\theta n(d_1)}{1 + \theta N(d_1)}\sqrt{1 - \Phi_C^2}(\Phi_V + 1)\Sigma_C\big].$$

Plugging the equation for $\mu_V$ back into the equation for $d_1$ in (A.16), $d_1$ now becomes the solution to the fixed point problem[14]

$$d_1 = -\frac{\theta n(d_1)}{1 + \theta N(d_1)} < 0. \quad \text{(A.17)}$$

[13] The scalar $1 - \beta \Phi_C$ is non-zero since $\Phi_C$ lies within the $(-1, 1)$ interval, and $\beta \in (0, 1)$.

[14] Given the continuity and monotonicity of the function $h(x) = x + \frac{\theta n(x)}{1 + \theta N(x)}$ for $\theta > 0$, the fixed point problem is well defined and has a negative solution.

# APPENDIX B

# Disappointment Aversion Preferences, and the Credit Spread Puzzle

## Appendix B.1 Bond yields according to the benchmark model in (2.1)

Suppose that single-period, cum-payout, asset log-returns for firm $i$ $r_{i,t,t+1}$ are i.i.d. normal random variables with constant mean $\tilde{\mu}_i - \frac{1}{2}\sigma_i^2 \in \mathbb{R}$, and volatility $\sigma_i \in \mathbb{R}_{>0}$. Let $\Delta_i$ be the constant log-payout yield $\big(\Delta_i = log(1 + \frac{O_{i,t+1}}{P_{i,t+1}})\big)$[1]. Ex-payout, log-returns $r^x_{i,t,t+1}$ are equal to cum-payout log-returns minus the log-payout yield ($r^x_{i,t,t+1} = r_{i,t,t+1} - \Delta_i$). Hence, $r^x_{i,t,t+1}$ are also normal random variables, and, in a discrete-time setting, can be expressed as

$$r^x_{i,t,t+1} = \tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2 + \sigma_i \epsilon_{i,t+1},$$

with $\epsilon_{i,t+1}$ i.i.d. $N(0,1)$ shocks. Moreover, $T$-period, ex-payout returns are also i.i.d. normal random variables with mean $(\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T$ and volatility $\sigma_i\sqrt{T}$.

Suppose that the single-period, log risk-free rate is constant and equal to $r_f$. Assume also that there are no taxes, and that default boundaries $D_{i,T}$ as well as

[1] $O_{i,t+1}$ is the payout, and $P_{i,t+1}$ is the price of assets in place.

losses given default $L$ are constant. Let $\pi^{\mathbb{P}}_{i,t,t+T}$ be the physical probability of default for a $T$-period, zero-coupon bond

$$\pi^{\mathbb{P}}_{i,t,t+T} = \mathbb{P}_t\Big(P_{i,t+T} < D_{i,T}\Big).$$

$P_{i,t}$ is the value of assets in place for firm $i$. Similarly to the original Merton model, default can only happen at the expiration date $t+T$, but unlike the Merton model, the default boundary is not necessarily equal to the face value of debt. Normalizing current period firm value $P_{i,t}$ to one, the physical probability of default $\pi^{\mathbb{P}}_{i,t,t+T}$ can be expressed in terms of asset log-returns $r^x_{i,t,t+1}$

$$\pi^{\mathbb{P}}_{i,t,t+T} = N\Big(\frac{logD_{i,T} - (\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\Big),$$

in which $N()$ is the standard normal c.d.f.. Because asset log-returns are i.i.d. with constant mean and standard deviation, $\pi^{\mathbb{P}}_{i,t,t+T}$ depends only on maturity $T$, hence $\pi^{\mathbb{P}}_{i,t,t+T} = \pi^{\mathbb{P}}_{i,T}$. Finally, using the inverse of the normal c.d.f. $N^{-1}()$, we can express the log-default boundary $logD_{i,T}$ in terms of the physical probability of default $\pi^{\mathbb{P}}_{i,T}$, expected returns for assets in place $\tilde{\mu}_i$, and asset return volatility $\sigma_i$

$$logD_{i,T} = (\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T + N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big)\sigma_i\sqrt{T}. \tag{B.1}$$

The continuous-time framework in Black and Scholes (1973) allows for frictionless trading and hedging between underlying and derivative securities. An immediate consequence of continuous trading is that if asset returns under the physical measure are normally distributed with constant mean and volatility, then asset returns under the risk-neutral measure are also normally distributed with the same variance, and mean equal to the risk-free rate.

In a discrete-time setting, continuous trading is not possible. However, according to *Lemma 1* in Appendix B.6.1, the risk-neutral density for asset returns is normal, provided that aggregate preferences over consumption are described by a CRRA utility function, and that aggregate consumption growth is a log-normal random variable. Hence, assuming that all conditions for *Lemma 1* hold, $T$-period, ex-payout asset log-returns under the risk-neutral measure are normally distributed with mean $(r_f - \Delta_i - \frac{1}{2}\sigma_i^2)T$, and volatility $\sigma_i\sqrt{T}$.

Let $y_{i,t,t+T}$ be the continuously compounded yield to maturity for a $T$-period, zero-coupon bond written on firm $i$ at time $t$. Then, under the risk-neutral measure

$$e^{-Ty_{i,t,t+T}} = e^{-Tr_f}\Big(1 - LN\big(\frac{logD_{i,T} - (r_f - \Delta_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\big)\Big). \tag{B.2}$$

Taking logs in (B.2), and substituting $logD_{i,T}$ with the expression from (B.1), we get that

$$y_{i,t,t+T} - r_f = -\frac{1}{T}log\Big[1 - LN\Big(N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

Since the right-hand side above and the risk-free rate are constants, we conclude that

$$\mathbb{E}[y_{i,t,t+T}] - r_f = -\frac{1}{T}log\Big[1 - LN\Big(N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

## Appendix B.2 Bond yields according to the model in (2.2) with time-varying recovery rates

Suppose that recovery rates are the same across all bonds, and depend only on consumption growth

$$1 - L_{t+T} = a_{rec,0} + a_{rec,c}\Delta c_{t+T-1,t+T}.$$

Suppose also that all the assumptions in Appendix B.1 hold. Then, the yield-to-maturity for a zero-coupon, T-period bond is given by[2]

$$e^{-Ty_{i,t,t+T}} = e^{-Tr_f}\mathbb{E}_t^{\mathbb{Q}}\Big[\mathbb{E}_t^{\mathbb{Q}}\big[1-(1-a_{rec,0}-a_{rec,c}\Delta c_{t+T-1,t+T})\mathbf{1}\big\{r_{i,t,t+T}<logD_{i,T}\big\}|\Delta c_{t+T-1,t+T}\big]\Big],$$

in which $\mathbb{E}_t^{\mathbb{Q}}$ is the expectation under the risk-neutral measure. Further algebra implies that

$$e^{-Ty_{i,t,t+T}} = e^{-Tr_f}\mathbb{E}_t^{\mathbb{Q}}\Big[1-(1-a_{rec,0}-a_{rec,c}\Delta c_{t+T-1,t+T})N\big(\frac{logD_{i,T}-(r_f-\Delta_i-\frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\big)\Big]$$

According to Appendix B.6.3, under the risk neutral measure, log-consumption growth is a normal random variable with volatility $\sigma_c$, and mean $\tilde{\mu}_c-\frac{\tilde{\mu}_m-r_f}{\rho_{m,c}\sigma_m}\sigma_c$. $\frac{\tilde{\mu}_m-r_f}{\sigma_m}$ is the stock market Sharpe ratio, and $\rho_{m,c}$ is the correlation between stock market returns and consumption growth. Using the expression for the default boundary $logD_{i,T}$ from (B.1), we obtain

$$e^{-T(y_{i,t,t+T}+r_f)} = \Big[1-\Big(\underbrace{1-a_{rec,0}-a_{rec,c}\tilde{\mu}_c}_{\mathbb{E}[L_{t+T}]}+a_{rec,c}\frac{\tilde{\mu}_m-r_f}{\rho_{m,c}\sigma_m}\sigma_c\Big)N\Big(N^{-1}\big(\pi_{i,T}^{\mathbb{P}}\big)+\frac{\tilde{\mu}_i-r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

Since the right-hand side and the risk-free rate are constants, we conclude that

$$\mathbb{E}[y_{i,t,t+T}]-r_f = -\frac{1}{T}log\Big[1-\Big(\mathbb{E}[L_{t+T}]+a_{rec,c}\frac{\tilde{\mu}_m-r_f}{\rho_{m,c}\sigma_m}\sigma_c\Big)N\Big(N^{-1}\big(\pi_{i,T}^{\mathbb{P}}\big)+\frac{\tilde{\mu}_i-r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

[2] Under the risk neutral measure $\mathbb{Q}$, asset returns $r_{i,t,t+1}$ and consumption growth are independent.

## Appendix B.3 Intertemporal marginal rate of substitution for disappointment aversion preferences

Along an optimal consumption path, the Bellman equation for the representative investor's consumption-investment problem implies that

$$V_t = \left[(1-\beta)C_t^{\rho} + \beta\mu_t\big(V_{t+1}\big)^{\rho}\right]^{\frac{1}{\rho}},$$

where $\mu_t$ is the disappointment aversion certainty equivalent from (2.4). The expression for the stochastic discount factor is given by

$$M_{t,t+1} = \frac{\partial V_t/\partial C_{t+1}}{\partial V_t/\partial C_t},$$

in which

$$\partial V_t/\partial C_t = \frac{1}{\rho}V_t^{1-\rho}(1-\beta)\rho C_t^{\rho-1},$$

and

$$\begin{aligned}
&\partial V_t/\partial C_{t+1} = \frac{1}{\rho}V_t^{1-\rho}\beta\rho\mu_t\big(V_{t+1}\big)^{\rho-1} \times \\
&(-\frac{1}{\alpha})\mathbb{E}_t\Big[\frac{V_{t+1}^{-\alpha}\ (1+\theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\Big]^{-\frac{1}{\alpha}-1} \times \\
&(-\alpha)V_{t+}^{-\alpha-1}\frac{1+\theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\frac{1}{\rho}V_{t+1}^{1-\rho}(1-\beta)\rho C_{t+1}^{\rho-1},
\end{aligned}$$

to conclude that

$$M_{t,t+1} = \beta\Big(\frac{C_{t+1}}{C_t}\Big)^{(\rho-1)}\Big[\frac{V_{t+1}}{\mu_t\big(V_{t+1}\big)}\Big]^{-\alpha-\rho}\Big[\frac{1+\theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\Big].$$

### Appendix B.4 Asset returns and the price-payout ratio

Let $P_{m,t}$, $O_{m,t}$, $Z_{m,t} = (P/O)_{m,t}$ be the price, payout, and price-payout ratio of a generic financial claim $m$ written on a stream of aggregate payments. Depending on the asset we want to price, payouts can be aggregate dividends (equity), aggregate earnings (assets in place), or even aggregate consumption (claim on aggregate consumption). Let $R_{m,t+1}$ be the cum-payout, gross return for claim $m$, then

$$R_{m,t+1} = \frac{P_{m,t+1} + O_{m,t+1}}{P_{m,t}}.$$

Dividing and multiplying the numerator with $O_{m,t+1}$, the denominator with $O_{m,t}$, and taking logs, we can express log-returns $r_{m,t,t+1}$ in terms of log price-payout ratios $z_{m,t}$

$$r_{m,t,t+1} = log[e^{z_{m,t+1}} + 1] - z_{m,t} + \Delta o_{m,t,t+1}.$$

Using a first-order Taylor series approximation for $log[e^{z_{m,t+1}} + 1]$ around the point $z_{m,t+1} = \bar{z}_m$, asset returns can be expressed as

$$r_{m,t,t+1} \approx \kappa_{m,0} + \kappa_{m,1} z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1}, \tag{B.3}$$

where

$$\kappa_{m,1} = \frac{e^{\bar{z}_m}}{e^{\bar{z}_m} + 1} \in (0,1), \tag{B.4}$$

and

$$\kappa_{m,0} = log[e^{\bar{z}_m} + 1] - \frac{e^{\bar{z}_m}}{e^{\bar{z}_m} + 1}\bar{z}_m. \tag{B.5}$$

Following a similar line of arguments[3], ex-payout, asset log-returns are given by

$$r^x_{m,t,t+1} = z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1}.$$

## Appendix B.5 Simulation

### Appendix B.5.1 Simulation methodology

The consumption-Euler equation for a $T$-period, zero-coupon bond written on firm's $i$ assets reads

$$e^{-Ty_{i,t,t+T}} = \mathbb{E}_t\Big[\Big(\prod_{j=1}^{T} M_{t+j-1,t+j}\Big)\Big(\mathbf{1}\{r^x_{i,t,t+T} \geq D_{i,t+T}\} + (1-L_{t+T})\mathbf{1}\{r^x_{i,t,t+T} < D_{i,t+T}\}\Big)\Big].$$

Unlike the model in (2.1), the default barrier $D_{i,t+T}$, which is expressed in terms of ex-payout asset returns, and losses given default $L_{t+T}$ are allowed to vary over time, and be functions of the state variables

$$D_{i,t+T} = a_{i,def,0} + a_{def,c}\big(\Delta c_{t+T-1,t+T} - \frac{\mu_c}{1-\phi_c}\big) + a_{def,\sigma}\big(\sigma_{t+T} - \frac{\mu_\sigma}{1-\phi_\sigma}\big),$$

and

$$1 - L_{t+T} = a_{rec,0} + a_{rec,c}\Delta c_{t+T-1,t+T}.$$

The first step in the simulation exercise is to discretize the consumption growth and consumption growth volatility space into $N_{\Delta c} = 20$ and $N_\sigma = 20$ equidistant points with a pace of $d_{\Delta c}$ and $d_\sigma$ respectively. The consumption growth space is truncated from above and below by $\hat{\mathbb{E}}[\Delta c_{t-1,t}] \pm 3\widehat{\mathbf{Vol}}(\Delta c_{t-1,t})$, whereas the volatility space is truncated from above and below by $\hat{\mathbb{E}}[\sigma_t] \pm 1.9\widehat{\mathbf{Vol}}(\sigma_t)$. The lower bound for the volatility space guarantees that initial values for volatility are always positive. $\hat{\mathbb{E}}[]$

[3] For ex-dividend returns, no linearization is needed, since $r^x_{m,t,t+1} = log\frac{P_{m,t+1}/O_{m,t+1}}{P_{m,t}/O_{m,t}}\frac{O_{m,t+1}}{O_{m,t}}$.

and $\widehat{\mathbf{Vol}}()$ are the simulated unconditional mean and standard deviation from Table 1.7.3.

The second step is to choose starting values for consumption growth and consumption growth volatility. To do so, I iterate though all possible pairs of $\{\Delta c_l, \sigma_k\}$, $l = 1, 2, ..., N_{\Delta c}$, $k = 1, 2, ..., N_\sigma$. For each pair of starting values, I simulate $N = 10,000$ [4] paths for consumption growth, consumption growth volatility, and aggregate payout growth according to the system in (2.6)-(2.8), as well as idiosyncratic volatility shocks. Each path contains $T$ nodes, as many nodes as the life of of the zero-coupon security. Negative volatility observations are replaced with the lowest positive observation $\big(\hat{\mathbb{E}}[\sigma_t] - 1.9\widehat{\mathbf{Vol}}(\sigma_t)\big)$ from the initial grid.

At each node of the simulated paths for $\Delta c_{t-1,t}$ and $\sigma_t$, I can obtain values for the stochastic discount factor $M_{t+j-1,t+j}$ from (2.9), price-payout ratios according to *Proposition 2*, one-period, ex-payout asset log-returns for the median firm from (2.13), as well as losses given default and default boundaries according to (2.15) and (2.16). $T$-period, ex-payout, asset log-returns are simply given by the sum of single-period returns $r^x_{i,t,t+T} = \sum_{j=1}^T r^x_{i,t,t+j}$. Finally, for each simulated path, the discounted cashflow of a zero-coupon corporate bond is $\Big(\prod_{j=1}^T M_{t+j-1,t+j}\Big)\Big(\mathbf{1}\{r^x_{i,t,t+T} \geq D_{i,t+T}\} + (1 - L_{t+T})\mathbf{1}\{r^x_{i,t,t+T} < D_{i,t+T}\}\Big)$. Averaging across all $N$ simulated paths, we obtain a value for the yield to maturity given the initial values for $\Delta c_{t,t-1}$ and $\sigma_t$

$$\hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k) \approx -\frac{1}{T} log\Big[\frac{1}{n}\sum_{n=1}^{N}\Big(\prod_{j=1}^{T} M^{(n)}_{t+j-1,t+j}\Big) \times \\ \Big(\mathbf{1}\{r^{x\ (n)}_{i,t,t+T} \geq D^{(n)}_{i,t+T}\} + (1 - L^{(n)}_{t+T})\mathbf{1}\{r^{x\ (n)}_{i,t,t+T} < D^{(n)}_{i,t+T}\}\Big)\Big].$$

The objective is to match unconditional first moments for credit spreads. We therefore need to calculate unconditional expected values over the grid of starting values for consumption growth and consumption growth volatility using the p.d.f's.

[4] Simulation results are not affected by the number of simulation paths $N$ or the number of grid points ($N_{\delta c}$, $N_\sigma$), provided of course that these numbers are relatively large.

for $\Delta c_{t-1,t}$, $\sigma_t$, and $\sigma_{t-1}$

$$\mathbb{E}[\hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k)] \approx$$
$$\sum_{j=1}^{N_\sigma} \Big\{ \sum_{k=1}^{N_\sigma} \Big[ \sum_{l=1}^{N_{\Delta c}} y_{i,t,t+T}(\Delta c_l, \sigma_k) f(\Delta c_l|\sigma_k, \sigma_j) d'_{\Delta c} \Big] f(\sigma_k|\sigma_j) d'_\sigma \Big\} f(\sigma_j) d''_\sigma,$$

where $f(\Delta c_l|\sigma_k, \sigma_j)$, $f(\sigma_k|\sigma_j)$, and $f(\sigma_j)$ are the p.d.f.'s for $\Delta c_{t-1,t}$, $\sigma_t$, and $\sigma_{t-1}$, while $d'_{\Delta c}$, $d'_\sigma$ and $d''_\sigma$ are constants such that $\sum_{l=1}^{N_{\Delta c}} f(\Delta c_k|\sigma_l, \sigma_j) d'_{\Delta c} = 1$, $\sum_{k=1}^{N_\sigma} f(\sigma_k|\sigma_j) d'_\sigma = 1$, and $\sum_{j=1}^{N_\sigma} f(\sigma_j) d''_\sigma = 1$. The p.d.f.'s for $\Delta c_{t-1,t}$, $\sigma_t$, and $\sigma_{t-1}$ are derived in Appendix B.5.2.

### Appendix B.5.2 Unconditional p.d.f. for consumption growth, and consumption growth volatility

According to (2.7), consumption growth volatility $\sigma_{t-1}$ is unconditionally normally distributed with mean $\mu_\sigma/(1-\phi_\sigma)$ and variance $\nu_\sigma^2/(1-\phi_\sigma^2)$. According to (2.6), conditional on $\sigma_{t-1}$, $\Delta c_t$ is normally distributed with long-run mean

$$\mathbb{E}[\Delta c_{t-1,t}|\sigma_{t-1}] = \frac{\mu_c}{1-\phi_c},$$

and long-run variance

$$\mathbf{Var}(\Delta c_{t-1,t}|\sigma_{t-1}) = \frac{\sigma_{t-1}^2}{1-\phi_c^2}.$$

Using the above results and equations (2.6)-(2.7), we conclude that the long-run p.d.f. for $\sigma_{t-1}$ is equal to

$$f(\sigma_{t-1}) = \frac{1}{\sqrt{2\pi}(\nu_\sigma/\sqrt{1-\phi_\sigma^2})} e^{-\frac{(\sigma_{t-1}-\frac{\mu_\sigma}{1-\phi_\sigma})^2}{2\nu_\sigma^2/(1-\phi_\sigma^2)}}.$$

The p.d.f. for $\sigma_t|\sigma_{t-1}$ is equal to

$$f(\sigma_t|\sigma_{t-1}) = \frac{1}{\sqrt{2\pi}\nu_\sigma} e^{-\frac{(\sigma_t - \mu_\sigma - \phi_\sigma \sigma_{t-1})^2}{2\nu_\sigma^2}}.$$

The long-run p.d.f for $\Delta c_{t-1,t}$ conditional on $\sigma_t$ and $\sigma_{t-1}$ is equal to

$$f(\Delta c_{t-1,t}|\sigma_t, \sigma_{t-1}) = \frac{1}{\sqrt{2\pi}(\sigma_{t-1}/\sqrt{1-\phi_c^2})} e^{-\frac{(\Delta c_t - \mu_c/(1-\phi_c))^2}{2\sigma_{t-1}^2/(1-\phi_c^2)}}.$$

The joint p.d.f. for $\Delta c_{t-1,t}$, $\sigma_t$ and $\sigma_{t-1}$ is therefore equal to

$$f(\Delta c_{t-1,t}, \sigma_t, \sigma_{t-1}) = f(\Delta c_{t-1,t}|\sigma_t, \sigma_{t-1}) f(\sigma_t|\sigma_{t-1}) f(\sigma_{t-1}) \Leftrightarrow$$
$$f(\Delta c_{t-1,t}, \sigma_t, \sigma_{t-1}) = \frac{1}{\sqrt{2\pi}(\nu_\sigma/\sqrt{1-\phi_\sigma^2})} \frac{1}{\sqrt{2\pi}\nu_\sigma} \frac{1}{\sqrt{2\pi}(\sigma_{t-1}/\sqrt{1-\phi_c^2})} \times$$
$$e^{-\frac{(\Delta c_{t-1,t} - \mu_c/(1-\phi_c))^2}{2\sigma_{t-1}^2/(1-\phi_c^2)}} e^{-\frac{(\sigma_t - \mu_\sigma - \phi_\sigma \sigma_{t-1})^2}{2\nu_\sigma^2}} e^{-\frac{(\sigma_{t-1} - \mu_\sigma/(1-\phi_\sigma))^2}{2\nu_\sigma^2/(1-\phi_\sigma^2)}}$$

## Appendix B.6 Proofs

### Appendix B.6.1 Lemma 1

*Lemma 1:* Suppose that one-period, cum-dividend, asset log-returns $r_{i,t,t+1}$ are i.i.d. normal random variables with constant mean $\tilde{\mu}_i - \frac{1}{2}\sigma_i^2$ and volatility $\sigma_i$. Suppose also that financial markets are complete, that there exists a representative investor with CRRA (power utility) defined over consumption[5], that log-consumption growth $\Delta c_{t,t+1}$ is a normal random variable with constant mean $\tilde{\mu}_c$ and constant volatility $\sigma_c$, and that the correlation coefficient between $r_{i,t,t+1}$ and $\Delta c_{t,t+1}$ is $\rho_{i,c}$. Then, the log risk-free rate $r_f$ is constant, and also cum-payout, asset log-returns under the risk-neutral measure $\mathbb{Q}$ are i.i.d. normal random variables with constant mean $r_f - \frac{1}{2}\sigma_i^2$ and volatility $\sigma_i$.

[5]More on the aggregation properties of the CRRA utility function can be found in Chapter 1 of Duffie (2000), and Chapter 5 in Huang and Litzenberger (1989).

*Proof:*

In equilibrium, the consumption-Euler equation for asset log-returns implies that

$$\mathbb{E}_t\left[\beta e^{-\alpha \Delta c_{t,t+1}} e^{r_{i,t,t+1}}\right] = 1 \Leftrightarrow \tilde{\mu}_i + log\beta - \alpha\tilde{\mu}_c + \frac{1}{2}\alpha^2\sigma_c^2 - \alpha\rho_{i,c}\sigma_c\sigma_i = 0. \tag{B.6}$$

in which $\beta \in (0,1)$ is the rate of time-preference, and $\alpha \geq -1$ is the risk aversion parameter in the CRRA power utility function. Similarly, for the log risk-free rate

$$r_f + log\beta - \alpha\tilde{\mu}_c + \frac{1}{2}\alpha^2\sigma_c^2 = 0. \tag{B.7}$$

which is constant since $\mu_c$ and $\sigma_c$ are also constant.

We can rewrite the consumption-Euler equation in (B.6) using the p.d.f. for $\Delta c_{t+1}$ conditional on $r_{i,t,t+1}$

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{log\beta} e^{r_{i,t,t+1}} e^{-\frac{(r_{i,t,t+1}-\tilde{\mu}_i+0.5\sigma_i^2)^2}{2\sigma_i^2}} e^{-\alpha[\tilde{\mu}_c+\rho_{i,c}\frac{\sigma_c}{\sigma_i}(r_{i,t,t+1}-\tilde{\mu}_i+0.5\sigma_i^2)]+\frac{1}{2}\alpha^2(1-\rho_{i,c}^2)\sigma_c^2} dr_{i,t,t+1} = 1.$$

Exploiting the consumption-Euler conditions in (B.6) and (B.7), we obtain

$$e^{-r_f}\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{r_{i,t,t+1}} e^{-\frac{(r_{i,t,t+1}-r_f+0.5\sigma_i^2)^2+(\alpha\rho_{i,c}\sigma_i\sigma_c)^2-2(r_{i,t,t+1}-r_f+0.5\sigma_i^2)\alpha\rho_{i,c}\sigma_i\sigma_c}{2\sigma_i^2}} \times$$
$$e^{-\alpha\rho_{i,c}\frac{\sigma_c}{\sigma_i}(r_{i,t,t+1}-\tilde{\mu}_i+0.5\sigma_i^2)} e^{-\frac{1}{2}\alpha^2\rho_{i,c}^2\sigma_c^2} dr_{i,t,t+1} = 1.$$

Further algebra yields

$$e^{-r_f}\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{r_{i,t,t+1}} e^{-\frac{(r_{i,t,t+1}-r_f+0.5\sigma_i^2)^2}{\sigma_i}} e^{-\frac{1}{2}\alpha^2\rho_{i,c}^2\sigma_c^2+(r_{i,t,t+1}-r_f+0.5\sigma_i^2)\alpha\rho_{i,c}\frac{\sigma_c}{\sigma_i}} \times$$
$$e^{-\alpha\rho_{i,c}\frac{\sigma_c}{\sigma_i}(r_{i,t,t+1}-r_f-\alpha\rho_{i,c}\sigma_i\sigma_c+0.5\sigma_i^2)} e^{-\frac{1}{2}\alpha^2\rho_{i,c}^2\sigma_c^2} dr_{i,t,t+1} = 1.$$

Cancelling out terms, we conclude that

$$e^{-r_f}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_i}e^{r_{i,t,t+1}}e^{-\frac{(r_{i,t,t+1}-r_f+0.5\sigma_i^2)^2}{\sigma_i}}dr_{i,t,t+1}=1.$$

### Appendix B.6.2 Lemma 2

*Lemma 2:* Let $x$ be a normal random variable with mean $\mu\in\mathbb{R}$ and standard deviation $\sigma\in\mathbb{R}_{>0}$. Let $A$ and $B$ two real numbers with $B>-\frac{1}{2\sigma^2}$, then

$$\mathbb{E}\left[e^{-Ax-Bx^2}\right]=e^{\frac{0.5A^2\sigma^2-A\mu-B\mu^2}{1+2B\sigma^2}}\frac{1}{\sqrt{1+2B\sigma^2}}. \tag{B.8}$$

*Proof:*

$$\mathbb{E}\left[e^{-Ax-Bx^2}\right]=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty}e^{\frac{-2A\sigma^2x-2\sigma^2Bx^2-x^2-\mu^2+2\mu x}{2\sigma^2}}dx.$$

Completing the square in the right-hand side

$$\mathbb{E}\left[e^{-Ax-Bx^2}\right]=e^{\frac{\left(\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\right)^2-\mu^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty}e^{\frac{-(1+2B\sigma^2)x^2+2\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\sqrt{1+2B\sigma^2}x-\left(\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\right)^2}{2\sigma^2}}dx.$$

After a change of variables $\tilde{x}=\sqrt{1+2B\sigma^2}x$, we conclude that

$$\mathbb{E}\left[e^{-Ax-Bx^2}\right]=e^{\frac{0.5A^2\sigma^2-A\mu-B\mu^2}{1+2B\sigma^2}}\frac{1}{\sqrt{1+2B\sigma^2}}.$$

### Appendix B.6.3 Risk-neutral density for consumption growth under CRRA preferences

Following *Lemma 1* in Appendix B.6.1, assume that consumption growth is log-normally distributed with constant mean $\tilde{\mu}_c$ and volatility $\sigma_c$, and that aggregate investor preferences can be described by a CRRA power utility function. Let $f_t(\Delta c_{t,t+1})$

be the normal p.d.f. for log-consumption growth, then the risk-neutral density $f_t^{\mathbb{Q}}(\Delta c_{t,t+1})$ is given by

$$f_t^{\mathbb{Q}}(\Delta c_{t,t+1}) = \frac{M_{t,t+1}^{CRRA}}{\mathbb{E}_t[M_{t,t+1}^{CRRA}]} f_t(\Delta c_{t,t+1}).$$

Following a similar line of arguments as in *Lemma 1*, we obtain

$$f_t^{\mathbb{Q}}(\Delta c_{t,t+1}) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{\left(\Delta c_{t,t+1} - (\tilde{\mu}_c - \alpha\sigma_c^2)\right)^2}{2\sigma_c^2}}.$$

Exploiting the consumption-Euler equations for stock market returns and the risk-free rate in (B.6) and (B.7), we can substitute out the term $\alpha\sigma_c^2$ with the stock market Sharpe ratio adjusted for the correlation between the stock market and consumption growth

$$\alpha\sigma_c^2 = \frac{\tilde{\mu}_m - r_f}{\sigma_m \rho_{m,c}} \sigma_c,$$

to conclude that

$$f_t^{\mathbb{Q}}(\Delta c_{t,t+1}) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{\left(\Delta c_{t,t+1} - (\tilde{\mu}_c - \frac{\tilde{\mu}_m - r_f}{\sigma_m \rho_{m,c}}\sigma_c)\right)^2}{2\sigma_c^2}}.$$

### Appendix B.6.4 Proof of Proposition 1

For $\rho = 0$, the Bellman recursion for the aggregate investor's consumption problem becomes

$$V_t = C_t^{1-\beta} \mu_t \Big(V_{t+1}\Big)^{\beta}.$$

$\mu_t$ is the disappointment aversion certainty equivalent from (2.4) with $\delta = 1$. Suppose that $log\frac{V_t}{C_t} = v_t - c_t = A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2$. Then, the Bellman equation reads

$$exp\Big[\frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2)\Big] =$$
$$\mathbb{E}_t\Big\{exp\Big[-\alpha[A_0 + (A_1+1)\Delta c_{t,t+1} + A_2\sigma_{t+1} + A_3\sigma_{t+1}^2]\Big] \times$$
$$\frac{1+\theta\mathbf{1}\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2\sigma_{t+1} + A_3\sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2)\}}{1+\theta\mathbb{P}_t\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2\sigma_{t+1} + A_3\sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2)\}}\Big\}^{-\frac{1}{\alpha}}.$$

Dividing both parts by the left-hand side,

$$1 = \mathbb{E}_t\Big\{exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)\Delta c_{t,t+1} - \frac{1}{\beta}A_1\Delta c_{t-1,t}]$$
$$-\alpha(A_2\sigma_{t+1} - \frac{1}{\beta}A_2\sigma_t) - \alpha(A_3\sigma_{t+1}^2 - \frac{1}{\beta}A_3\sigma_t^2)\Big] \times$$
$$\frac{1+\theta\mathbf{1}\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2\sigma_{t+1} + A_3\sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2)\}}{1+\theta\mathbb{P}_t\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2\sigma_{t+1} + A_3\sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1\Delta c_{t-1t} + A_2\sigma_t + A_3\sigma_t^2)\}}\Big\}^{-\frac{1}{\alpha}}.$$

Recall that $\epsilon_{c,t+1}$ and $\epsilon_{\sigma,t+1}$ from (2.6) and (2.7) are independent. We can use the law of total expectation to rewrite the above expression as

$$1 = \mathbb{E}_t\Big\{\mathbb{E}_t\Big\{exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)\Delta c_{t,t+1} - \frac{1}{\beta}A_1\Delta c_{t-1,t}]$$
$$-\alpha(A_2\sigma_{t+1} - \frac{1}{\beta}A_2\sigma_t) - \alpha(A_3\sigma_{t+1}^2 - \frac{1}{\beta}A_3\sigma_t^2)\Big] \times$$
$$\frac{1+\theta\mathbf{1}\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2\sigma_{t+1} + A_3\sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2)\}}{1+\theta\mathbb{P}_t\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2\sigma_{t+1} + A_3\sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2)\}}|\epsilon_{\sigma,t+1}\Big\}\Big\}.$$

Using the dynamics of consumption growth $\Delta c_{t,t+1}$ in (2.6), and partial moments for the normal distribution, the above expression becomes

$$1 = \mathbb{E}_t\Big\{exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c\Delta c_{t-1,t}) - \frac{1}{\beta}A_1\Delta c_{t-1,t}] + \frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2$$
$$-\alpha(A_2\sigma_{t+1} - \frac{1}{\beta}A_2\sigma_t) - \alpha(A_3\sigma_{t+1}^2 - \frac{1}{\beta}A_3\sigma_t^2)\Big] \times$$
$$\frac{1+\theta N\Big(\frac{\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)-A_0-(A_1+1)\mu_c-(A_1+1)\phi_c\Delta c_{t-1,t}-A_2\sigma_{t+1}-A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t} + \alpha(A_1+1)\sigma_t\Big)}{1+\theta N\Big(\frac{\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)-A_0-(A_1+1)\mu_c-(A_1+1)\phi_c\Delta c_{t-1,t}-A_2\sigma_{t+1}-A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t}\Big)}\Big\}.$$

For $\theta = 2$ and $N()$ a small number[6], we can use the following approximation $1 + \theta N(y) \approx e^{\theta N(y)}$ to get

$$\mathbb{E}_t\Big\{exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta}A_1 \Delta c_{t-1,t}] + \frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2$$
$$-\alpha(A_2\sigma_{t+1} - \frac{1}{\beta}A_2\sigma_t) - \alpha(A_3\sigma_{t+1}^2 - \frac{1}{\beta}A_3\sigma_t^2)\Big] \times$$
$$e^{\theta N\left(\frac{\frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2) - A_0 - (A_1+1)\mu_c - (A_1+1)\phi_c\Delta c_{t-1,t} - A_2\sigma_{t+1} - A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t} + \alpha(A_1+1)\sigma_t\right)} \times$$
$$e^{-\theta N\left(\frac{\frac{1}{\beta}(A_0 + A_1\Delta c_{t-1,t} + A_2\sigma_t + A_3\sigma_t^2) - A_0 - (A_1+1)\mu_c - (A_1+1)\phi_c\Delta c_{t-1,t} - A_2\sigma_{t+1} - A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t}\right)}\Big\} = 1,$$

Further, we can use a first-order linear approximation for the difference of the two standard normal c.d.f.'s in the above equation, provided that this difference is small[7],

$$N(x) - N(y) \approx n(\bar{x})(x - y),$$

to obtain

$$exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c\Delta c_{t-1,t}) - \frac{1}{\beta}A_1\Delta c_{t-1,t}] + \frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2 + \qquad \text{(B.9)}$$
$$\alpha\theta n(\bar{x})(A_1+1)\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big]\mathbb{E}_t\Big\{exp\Big[-\alpha A_2\sigma_{t+1} - \alpha A_3\sigma_{t+1}^2\Big]\Big\} = 1,$$

in which $n()$ is the standard normal p.d.f..

Combining the dynamics for aggregate uncertainty $\sigma_{t+1}$ in (2.7) with *Lemma 2* from Appendix B.6.2, the Bellman equation becomes

$$e^0 = exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c\Delta c_{t-1,t}) - \frac{1}{\beta}A_1\Delta c_{t-1,t}] \qquad \text{(B.10)}$$
$$+\frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2 + \alpha\theta n(\bar{x})(A_1+1)\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big] \times$$
$$exp\Big[\frac{0.5\alpha^2A_2^2\nu_\sigma^2 - \alpha A_2\mu_\sigma - \alpha A_2\phi_\sigma\sigma_t - \alpha A_3\mu_\sigma^2 - \alpha A_3\phi_\sigma^2\sigma_t^2 - 2\alpha A_3\mu_\sigma\phi_\sigma\sigma_t}{1 + 2\alpha A_3\nu_\sigma^2}\Big]\frac{1}{\sqrt{1 + 2\alpha A_3\nu_\sigma^2}}.$$

[6]In simulations, the probability of disappointment events is less than 0.5

[7]Essentially we require that $\frac{\alpha}{1-\beta\phi_c}\sigma_t$ to be small.

We can now solve for $A_0$, $A_1$, $A_2$, and $A_3$ using the method of undetermined coefficients. We first collect $\Delta c_{t-1,t}$ terms to get

$$A_1 = \frac{\beta\phi_c}{1-\beta\phi_c}. \tag{B.11}$$

Note that for $\beta \in (0,1)$ and $\phi_c \in (-1,1)$, then $A_1+1$ is positive. Also, for $\beta \in (0,1)$, the sign of $A_1$ depends only on the sign of $\phi_c$.

Similarly, collecting $\sigma_t^2$ terms yields

$$2\alpha\nu_\sigma^2 A_3^2 + [1-\beta\phi_\sigma^2 + \beta\alpha^2(A_1+1)^2\nu_\sigma^2]A_3 + \frac{1}{2}\beta\alpha(A_1+1)^2 = 0. \tag{B.12}$$

For $\alpha \neq 0$, the solution for to the quadratic equation is

$$A_3 = \frac{-[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2] \pm \sqrt{[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2 - 4\beta\alpha^2(A_1+1)^2\nu_\sigma^2}}{4\alpha\nu_\sigma^2}. \tag{B.13}$$

The ratio of the constant term over the quadratic coefficient in the above quadratic equation is a positive number $\left(\beta(A_1+1)^2/4\nu_\sigma^2\right)$. Hence, the roots of the quadratic equation will be of the same sign. Furthermore, since $\beta \in (0,1)$ and $\phi_\sigma \in (-1,1)$, then $1-\beta\phi_\sigma^2$ is positive, $-[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]$ is negative, and the solutions to the quadratic equation are therefore negative. We will pick the largest negative root so that the quadratic solution in (B.13) is very close to the linear approximation in (B.14) below.

For $A_3$ to be a real number, we require that

$$[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2 - 4\beta\alpha^2(A_1+1)^2\nu_\sigma^2 > 0.$$

We cannot really examine whether the above inequality holds without having calibrated model parameters. However, $\nu_\sigma^2$ is a very small number close to zero ($0.00177^2$), and for $\nu_\sigma^2 \approx 0$ the determinant in (B.13) is approximately equal to

$$\lim_{\nu_\sigma^2 \downarrow 0}[1 - \beta\phi_\sigma^2 + \beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2 - 4\beta\alpha^2(A_1+1)^2\nu_\sigma^2 \approx [1-\beta\phi_\sigma^2]^2 > 0.$$

The restriction that $\nu_\sigma$ is a very small number is associated with higher consumption growth moments being well defined. Parameter values for the simulated economy ensure that the determinant in (B.13) is well defined, and that $1 + 2\alpha A_3 \nu_\sigma^2 > 0$ as required by *Lemma 2* in Appendix B.6.2. Finally, for $\nu_\sigma^2 \approx 0$, equation (B.12) becomes linear yielding an approximate solution for $A_3$

$$A_3 \approx -\frac{1}{2}\frac{\beta\alpha(A_1+1)^2}{1-\beta\phi_\sigma^2}. \tag{B.14}$$

Collecting $\sigma_t$ terms in (B.10), we obtain the solution for $A_2$

$$A_2 = \frac{-\theta\beta n(\bar{x})(A_1+1)(1+2\alpha A_3\nu_\sigma^2) + 2\beta A_3\mu_\sigma\phi_\sigma}{1+2\alpha A_3\nu_\sigma^2 - \beta\phi_\sigma}. \tag{B.15}$$

It is easy to verify that for negative $A_3$, then $A_2$ is also negative. As $\nu_\sigma^2 \downarrow 0$, an approximate solution for $A_2$ reads

$$A_2 \approx \frac{-\theta\beta n(\bar{x})(A_1+1) + 2\beta A_3\mu_\sigma\phi_\sigma}{1-\beta\phi_\sigma}. \tag{B.16}$$

Finally, the remaining constant terms in (B.10) are grouped under $A_0$

$$A_0 = \frac{\beta}{1-\beta}[(A_1+1)\mu_c + \frac{1}{1+2\alpha A_3\nu_\sigma^2}(A_2\mu_\sigma + A_3\mu_\sigma^2 - 0.5\alpha A_2^2\nu_\sigma^2) + \frac{log(1+2\alpha A_3\nu_\sigma^2)]}{2\alpha}, \tag{B.17}$$

with the approximation for $\nu_\sigma^2 \downarrow 0$

$$A_0 \approx \frac{\beta}{1-\beta}[(A_1+1)\mu_c + A_2\mu_\sigma + A_3\mu_\sigma^2]. \tag{B.18}$$

### Appendix B.6.5 The log risk-free rate

The Euler condition for the log risk-free rate reads

$$e^{-r_{f,t,t+1}} = \mathbb{E}_t\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-1}\Big(\frac{V_{t+1}}{\mu_t\big(V_{t+1}\big)}\Big)^{-\alpha}\frac{1+\theta\ \mathbf{1}\{V_{t+1} < \ \mu_t(V_{t+1})\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{V_{t+1} < \mu_t(V_{t+1})\}]}\Big].$$

Repeating all the steps that lead to equation (B.9) in Appendix B.6.4, we obtain

$$e^{-r_{f,t,t+1}} = exp\Big[log\beta - \alpha(A_0 - \frac{1}{\beta}A_0) - \big[[\alpha(A_1+1)+1](\mu_c + \phi_c\Delta c_{t-1,t}) - \frac{1}{\beta}A_1\Delta c_t\big] +$$
$$\frac{1}{2}[\alpha(A_1+1)+1]^2\sigma_t^2 + \theta n(\bar{x})[\alpha(A_1+1)+1]\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big]$$
$$\mathbb{E}_t\Big\{exp\Big[-\alpha A_2\sigma_{t+1} - \alpha A_3\sigma_{t+1}^2\Big]\Big\}.$$

But from (B.9) we know that

$$exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c\Delta c_{t-1,t}) - \frac{1}{\beta}A_1\Delta c_{t-1,t}] + \frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2 +$$
$$\alpha\theta n(\bar{x})(A_1+1)\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big]\mathbb{E}_t\Big\{exp\Big[-\alpha A_2\sigma_{t+1} - \alpha A_3\sigma_{t+1}^2\Big]\Big\} = 1.$$

Therefore, the log risk-free rate must be approximately equal to

$$r_{f,t,t+1} \approx -log\beta + \mu_c + \phi_c\Delta c_{t-1,t} - \frac{1}{2}[2\alpha(A_1+1)+1]\sigma_t^2 - \theta n(\bar{x})\sigma_t$$

### Appendix B.6.6 Proof of Proposition 2

We conjecture that the log price-payout ratio $z_{m,t}$ for a financial claim on a stream of aggregate payments (dividends or earnings) is an affine function of the state variables $\Delta c_{t-1,t}$, $\sigma_t$, $\sigma_t^2$

$$z_{m,t} = A_{m,0} + A_{m,1}\Delta c_{t-1,t} + A_{m,1}\sigma_t + A_{m,2}\sigma_t^2.$$

Combining equation (B.3) with our conjecture about $z_{m,t}$, the Euler equation for asset returns becomes

$$\mathbb{E}_t\Big[M_{t,t+1}e^{\kappa_{m,0}+\kappa_{m,1}(A_{m,0}+A_{m,1}\Delta c_{t,t+1}+A_{m,2}\sigma_{t+1}+A_{m,3}\sigma_{t+1}^2)-(A_{m,0}+A_{m,1}\Delta c_{t-1,t}+A_{m,2}\sigma_t+A_{m,3}\sigma_t^2)+\Delta o_{m,t,t+1}}\Big]=1.$$

Substituting the result for the disappointment aversion discount factor $M_{t,t+1}$ from (2.9), we can re-write the Euler equation as

$$\begin{aligned}&\mathbb{E}_t\Big[e^{\log\beta-\Delta c_{t,t+1}}e^{-\alpha\left\{A_0(1-\frac{1}{\beta})+[(A_1+1)\Delta c_{t,t+1}-\frac{1}{\beta}A_1\Delta c_{t-1,t}]+A_2(\sigma_{t+1}-\frac{1}{\beta}\sigma_t)+A_3(\sigma_{t+1}^2-\frac{1}{\beta}\sigma_t^2)\right\}}\times \\ &\frac{1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2<\ \frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2<\ \frac{1}{\beta}(A_0+A_1\Delta c_t+A_2\sigma_t+A_3\sigma_t^2)\}]}\times \\ &e^{\kappa_{m,0}+\kappa_{m,1}(A_{m,0}+A_{m,1}\Delta c_{t,t+1}+A_{m,2}\sigma_{t+1}+A_{m,3}\sigma_{t+1}^2)-(A_{m,0}+A_{m,1}\Delta c_{t-1,t}+A_{m,2}\sigma_t+A_{m,3}\sigma_t^2)+\Delta o_{m,t,t+1}}\Big]=1.\end{aligned} \tag{B.19}$$

Following the same line of arguments as in Appendix B.6.4, the Euler equation becomes

$$\begin{aligned}&exp\Big[log(\beta)-\alpha(A_0-\frac{1}{\beta}A_0)-\big[[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}](\mu_c+\phi_c\Delta c_{t-1,t})+\alpha\frac{1}{\beta}A_1\Delta c_{t-1,t}\big] \\ &+\frac{1}{2}[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]^2\sigma_t^2+\theta n(\bar{x})[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]\sigma_t+\alpha\frac{1}{\beta}A_2\sigma_t+\alpha\frac{1}{\beta}A_3\sigma_t^2 \\ &+\kappa_{m,0}+A_{m,0}(\kappa_{m,1}-1)-A_{m,1}\Delta c_{t-1,t}-A_{m,2}\sigma_t-A_{m,3}\sigma_t^2+\mu_m+\phi_m\Delta c_{t-1,t}+\frac{1}{2}\sigma_m^2\sigma_t^2\Big]\times \\ &exp\Big[\frac{0.5(\alpha A_2-\kappa_{m,1}A_{m,2})\nu_\sigma^2-(\alpha A_2-\kappa_{m,1}A_{m,2})\mu_\sigma-(\alpha A_2-\kappa_{m,1}A_{m,2})\phi_\sigma\sigma_t}{1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2}\Big]\times \\ &exp\Big[\frac{-(\alpha A_3-\kappa_{m,1}A_{m,3})\mu_\sigma^2-(\alpha A_3-\kappa_{m,1}A_{m,3})\phi_\sigma^2\sigma_t^2-2(\alpha A_3-\kappa_{m,1}A_{m,3})\mu_\sigma\phi_\sigma\sigma_t}{1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2}\Big]\times \\ &\frac{1}{\sqrt{1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2}}=e^0.\end{aligned} \tag{B.20}$$

We are now able to solve for $A_{m,0}$, $A_{m,1}$, $A_{m,2}$, and $A_{m,3}$ using the method of undetermined coefficients. Specifically, for $A_{m,1}$ we get

$$-\phi_c-\alpha(A_1+1)\phi_c+\frac{1}{\beta}\alpha A_1+\kappa_{m,1}A_{m,1}\phi_c-A_{m,1}+\phi_m=0.$$

Using the expression for $A_1$ from (B.11), we conclude that

$$A_{m,1} = \frac{\phi_m - \phi_c}{1 - \kappa_{m,1}\phi_c}. \tag{B.21}$$

Collecting $\sigma_t^2$ terms from (B.19), $A_{m,3}$ must satisfy the quadratic equation

$$\frac{1}{2}\beta\big[[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]^2+\sigma_m^2\big][1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2]+\alpha A_3(1+2\alpha A_3\nu_\sigma^2-\beta\phi_\sigma^2)$$
$$-2\alpha A_3\kappa_{m,1}A_{m,3}\nu_\sigma^2-\beta A_{m,3}-2\alpha A_3\nu_\sigma^2\beta A_{m,3}+2\beta\kappa_{m,1}\nu_\sigma^2A_{m,3}^2+\beta\kappa_{m,1}\phi_\sigma^2A_{m,3}=0.$$

After tedious algebra, the solution for $A_{m,3}$ is equal to

$$A_{m,3} = \frac{-\tilde{b} \pm \sqrt{\tilde{b}^2 - 4\tilde{a}\tilde{c}}}{2\tilde{a}}, \tag{B.22}$$

with

$$\tilde{a} = 2\beta\kappa_{m,1}\nu_\sigma^2,$$
$$\tilde{b} = -\beta + \beta\kappa_{m,1}\phi_\sigma^2 - 2\alpha A_3\kappa_{m,1}\nu_\sigma^2 - 2\alpha\beta A_3\nu_\sigma^2,$$
$$\tilde{c} = \frac{1}{2}\beta\big[[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]^2+\sigma_m^2\big](1+2\alpha A_3\nu_\sigma^2)+\alpha A_3(1+2\alpha A_3\nu_\sigma^2-\beta\phi_\sigma^2).$$

We will pick the largest negative root so that the quadratic solution in (B.22) is very close to the linear approximation in (B.23) below. As in Appendix B.6.4, we need to make sure that $1 + 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2$ is positive, and that the determinant in (B.22) is well defined. Both conditions are satisfied for very small $\nu_\sigma^2$, and reasonable values for the risk aversion coefficient $\alpha$. Finally, since $\nu_\sigma^2$ is a small number close to zero, we can obtain an approximate solution for $A_{m,3}$ using equation (B.14) for $A_3$

$$A_{m,3} \approx \frac{1}{2}\frac{[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]^2+\sigma_m^2-\alpha^2(A_1+1)^2}{1-\kappa_{m,1}\phi_\sigma^2}. \tag{B.23}$$

Collecting $\sigma_t$ terms from (B.19), the solution for $A_{2,m}$ is given by

$$A_{m,2} = \frac{\theta\beta n(\bar{x})[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}][1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2]}{\beta+2\beta(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2-\beta\kappa_{m,1}\phi_\sigma} \tag{B.24}$$
$$\frac{+\alpha A_2[1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2-\beta\phi_\sigma]-2\beta(\alpha A_3-\kappa_{m,1}A_{m,3})\mu_\sigma\phi_\sigma}{\beta+2\beta(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2-\beta\kappa_{m,1}\phi_\sigma}.$$

For $\nu_\sigma^2 \approx 0$, and the approximate expressions for $A_3$ and $A_2$ in (B.14) and (B.16) respectively, we conclude that

$$A_{m,2} \approx \frac{\theta n(\bar{x})(1-\kappa_{m,1}A_{m,1})+2\kappa_{m,1}A_{m,3}\mu_\sigma\phi_\sigma}{1-\kappa_{m,1}\phi_\sigma}. \tag{B.25}$$

Finally, collecting all the constant terms in (B.20), we get

$$A_{m,0} = \frac{1}{1-\kappa_{m,1}}\Big[log\beta+\kappa_{m,0}+\mu_m-\alpha A_0\frac{\beta-1}{\beta}-[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]\mu_c \tag{B.26}$$
$$-\frac{(\alpha A_2-\kappa_{m,1}A_{m,2})\mu_\sigma+(\alpha A_3-\kappa_{m,1}A_{m,3})\mu_\sigma^2-0.5(\alpha A_2-\kappa_{m,1}A_{m,2})^2\nu_\sigma^2}{1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2}$$
$$-0.5log\big(1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2\big)\Big].$$

Exploiting the fact that $\nu_\sigma^2 \approx 0$, and the expression for $A_0$ in (B.18), an approximation for $A_{m,0}$ is

$$A_{m,0} \approx \frac{1}{1-\kappa_{m,1}}\big[log\beta+\kappa_{m,0}+\mu_m+(\kappa_{m,1}A_{m,1}-1)\mu_c+\kappa_{m,1}A_{m,2}\mu_\sigma+\kappa_{m,1}A_{m,3}\mu_\sigma^2\big]. \tag{B.27}$$

# BIBLIOGRAPHY

# BIBLIOGRAPHY


Abeler, J., A. Falk, L. Goette, and D. Huffman (2011), Reference points and effort provision, *American Economic Review*, *101*(2), 470–492.

Acharya, V., S. Bharath, and A. Srinivasan (2007), Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries, *Journal of Financial Economics*, *85*, 787–821.

Aït-Sahalia, Y., J. A. Parker, and M. Yogo (2004), Luxury goods and the equity premium, *Journal of Finance*, *59*(6), 2959–3004.

Allais, M. (1953), Le comportement de l' homme rationnel devant le risque: Critique des postulats et axiomes de l' École Américaine, *Econometrica*, *21*(4), 503–546.

Almeida, H., and T. Philippon (2007), The risk-adjusted cost of financial distress, *Journal of Finance*, *62*, 2557–2586.

Altman, E. I., B. Brady, A. Resti, and A. Sironi (2005), The link between default and recovery rates: Theory, empirical evidence, and implications, *Journal of Business*, *78*, 2203–2228.

Andrews, D. (1994), Empirical process methods in econometrics, in *Handbook of Econometrics*, vol. 4, chap. 37, pp. 2248–2292, Elsevier B.V., Amsterdam.

Andries, M. (2011), Consumption-based asset pricing with loss aversion, *Working paper.*

Ang, A., G. Bekaert, and J. Liu (2005), Why stocks may disappoint, *Journal of Financial Economics*, *76*(3), 471–508.

Ang, A., J. Chen, and Y. Xing (2006), Downside risk, *Review of Financial Studies*, *19*(4), 1191–1239.

Arkes, H. R., D. Hirshleifer, D. Jiang, and S. Lim (2008), Reference point adaptation: Tests in the domain of security trading, *Organizational Behavior and Human Decision Processes*, *105*, 67–81.

Artstein-Avidan, S., and D. Dillenberger (2011), Dynamic disappointment aversion, *Working paper.*

Backus, D. K., B. R. Routledge, and S. E. Zin (2005), Exotic preferences for macroeconomists, in *NBER Macroeconomics Annual*, vol. 19, pp. 319–405, MIT Press, Cambridge, MA.

Bansal, R., and I. Shaliastovich (2013), A long-run risks explanation of predictability puzzles in bond and currency markets, *Review of Financial Studies*, *26*(1), 1–33.

Bansal, R., and A. Yaron (2004), Risks for the long-run: a potential resolution of asset pricing puzzles, *Journal of Finance*, *59*(4), 1481–1509.

Bansal, R., R. F. Dittmar, and C. T. Lundblad (2005), Consumption, dividends, and the cross-section of equity returns, *Journal of Finance*, *60*(4), 1639–1671.

Bansal, R., D. Kiku, and A. Yaron (2007), Risks for the long-run: Estimation and inference, *Working Paper*.

Barberis, N., M. Huang, and T. Santos (2001), Prospect theory and asset prices, *Quarterly Journal in Economics*, *116*(1), 1–53.

Barro, R. (2006), Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics*, *121*(3), 823–866.

Beeler, J., and J. Y. Campbell (2012), The long-run risks model and aggregate asset prices: an empirical assessment, *Critical Finance Review*, *1*, 141–182.

Belo, F., P. Collin-Dufresne, and R. S. Goldstein (2012), Endogenous dividend dynamics and the term structure of dividend strips, *Working Paper*.

Bernatzi, S., and R. Thaler (1995), Myopic risk aversion and the equity premium puzzle, *Quarterly Journal of Economics*, *110*(1), 73–92.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2010), The levered equity risk premium and credit spreads: a unified framework, *Review of Financial Studies*, *23*(2), 645–703.

Black, F., and M. Scholes (1973), The pricing of options and corporate liabilities, *Journal of Political Economy*, *81*(3), 637–654.

Bloom, N. (2009), The impact of uncertainty shocks, *Econometrica*, *77*, 623–685.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2012), Really uncertain business cycles, *Working paper*.

Bonomo, M., R. Garcia, N. Meddahi, and R. Tedongap (2011), Generalized disappointment aversion, long-run volatility risk, and asset prices, *Review of Financial Studies*, *24*(1), 82–122.

Breeden, D., M. Gibbons, and R. Litzenberger (1989), Empirical tests of the consumption-oriented CAPM, *Journal of Finance*, *44*(2), 231–262.

Brennan, M. J. (1979), The pricing of contingent claims in discrete time models, *Journal of Finance*, *34*(1), 53–68.

Campanale, C., R. Castro, and G. L. Clementi (2010), Asset pricing in a production economy with Chew-Dekel preferences, *Review of Economic Dynamics*, *13*, 379–402.

Campbell, J. (2003), Consumption-based asset pricing, in *Handbook of the Economics of Finance*, vol. 1b, chap. 13, pp. 801–885, Elsevier B.V., Amsterdam.

Campbell, J. Y. (1996), Understanding risk and returns, *Journal of Political Economy*, *104*(2), 298–345.

Campbell, J. Y., and J. H. Cochrane (1999), By force of habit: a consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy*, *102*(2), 205–251.

Cantor, R., and F. Packer (1994), The credit rating industry, *Federal Reserve Bank of New York Quarterly Journal*, *2*, 1–26.

Carhart, M. (1997), On persistence in mutual fund performance, *Journal of Finance*, *52*(1), 57–82.

Chapman, D. A., and V. Polkovnichenko (2009), First-order risk aversion, heterogeneity, and asset market outcomes, *Journal of Finance*, *64*(4), 141 – 153.

Chapman, D. A., and V. Polkovnichenko (2011), Risk attitudes toward small and large bets in the presence of background risk, *Review of Finance*, *15*, 909 – 927.

Chen, H. (2010), Macroeconomic conditions and the puzzles of credit spreads and capital structure, *Journal of Finance*, *65*(6), 2171–2219.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein (2009), On the relation between the credit spread puzzle and the equity premium puzzle, *Review of Financial Studies*, *22*(9), 3367–3409.

Choi, S., R. Fisman, D. Gale, and S. Kariv (2007), Consistency and heterogeneity of individual behavior under uncertainty, *American Economic Review*, *97*(5), 1921 – 1938.

Cochrane, J. (2001), *Asset pricing*, Princeton University Press, Princeton, NJ.

Crawford, V. P., and J. Meng (2011), New York City cab drivers' labor supply revisited: Reference-dependent preferences with rational-expectations targets for hours and income, *American Economic Review*, *101*(5), 1912–1932.

Davydenko, S. (2012), When do firms default? A study of the default boundary, *Working paper.*

Delikouras, S. (2013), Disappointment events in consumption growth and the cross-section of expected stock returns, *Working Paper*.

Dillenberger, D. (2004), Preferences for one shot resolution of uncertainty and Allais-type behavior, *Econometrica*, *78*(6), 1973–2004.

Dolmas, J. (1996), Balanced-growth-consistent recursive utility, *Journal of Economic Dynamics and Control*, *20*(4), 657–680.

Doran, K. B. (2010), Wages, daily income goals and daily labor supply, *Working paper*.

Duffee, G. R. (1998), The relation between treasury yields and corporate bond yield spreads, *Journal of Finance*, *53*, 2225–2241.

Duffie, D. (2000), *Dynamic Asset Pricing Theory*, Princeton University Press, Princeton, NJ.

Duncan, L. R. (2010), *Utility of Gains and Losses: Measurement-Theoretical, and Experimental Approaches*, Lawerence Erlbaum Associates Inc., Mahwah, NJ.

Easley, D., and L. Yang (2012), Loss aversion, survival and asset prices, *Working paper*.

Epstein, L. (2010), A paradox for the "smooth" ambiguity model of preference, *Econometrica*, *78*(6), 2085–2099.

Epstein, L. G., and S. Zin (1990), First-order risk aversion and the equity premium puzzle, *Journal of Monetary Economics*, *26*, 387–407.

Epstein, L. G., and S. E. Zin (1989), Substitution, risk aversion, and the temporal behavior of consumption and asset returns, *Econometrica*, *57*(4), 937–969.

Epstein, L. G., and S. E. Zin (2001), The independence axiom and asset returns, *Journal of Empirical Finance*, *8*, 537–572.

Ericsson, J., and O. M. Renault (2006), Liquidity and credit risk, *Journal of Finance*, *61*, 2219–2250.

Fama, E. F., and K. R. French (1993), Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, *33*, 3–56.

Fama, E. F., and K. R. French (1996), Multifactor explanations of asset pricing anomalies, *Journal of Finance*, *50*(1), 131–156.

Ferson, W., and C. R. Harvey (1991), The variations of economic risk premia, *Journal of Political Economy*, *99*, 385–415.

Ferson, W. E., and S. R. Foerster (1994), Finite sample properties of the generalized method of moments in tests of conditional asset pricing, *Journal of Financial Economics*, *36*, 29–55.

Ferson, W. E., and C. R. Harvey (1992), Seasonality and consumption based asset pricing, *Journal of Finance*, *47*(2), 511–551.

Gabaix, X., and D. Laibson (2002), The 6-D bias and the equity premium puzzle, in *NBER Macroeconomics Annual*, vol. 16, pp. 257–312, MIT Press, Cambridge, MA.

Gilboa, I., and D. Schmeidler (1989), Maxmin expected utility with non-unique priors, *Journal of Mathematical Economics*, *18*, 141–153.

Gill, D., and V. Prowse (2012), A structural analysis of disappointment aversion in a real effort competition, *American Economic Review*, *102*(1), 469–503.

Goldstein, R., N. Ju, and H. E. Leland (2001), An EBIT-based model of dynamic capital structure, *Journal of Business*, *74*, 483–512.

Gul, F. (1991), A theory of disappointment aversion, *Econometrica*, *59*(3), 667–686.

Hall, R. (1988), Intertemporal elasticity of consumption, *Journal of Political Economy*, *96*(2), 339–357.

Hansen, L. P., and J. C. Heaton (2008), Consumption strikes back? Measuring long-run risk, *Journal of Political Economy*, *116*(2), 260–302.

Hansen, L. P., and K. J. Singleton (1982), Generalized instrumental variables estimation of non-linear rational expectations models, *Econometrica*, *50*(5), 1269–1286.

Hansen, L. P., J. C. Heaton, J. Lee, and N. Russanov (2007), Intertemporal substitution and risk aversion, in *Handbook of Econometrics*, vol. 6A, chap. 61, pp. 3968–4056, Elsevier B.V., Amsterdam.

Hayashi, F. (2000), *Econometrics*, Princeton University Press, Princeton, NJ.

Hsu, A., and F. Palomino (2011), A simple solution method for models with time-varying volatility, *Working Paper*.

Huang, C.-F., and R. H. Litzenberger (1989), *Foundations for Financial Economics*, North-Holland, New York, NY.

Huang, J.-Z., and M. Huang (2012), How much of the corporate-treasury yield spread is due to credit risk?, *Review of Asset Pricing Studies*, *2*(2), 153–202.

Ingersoll, J. E., Jr. (2011), Cumulative prospect theory and the representative investor, *Working paper*.

Jagannathan, R., and Z. Wang (1996), The conditional CAPM and the cross-section of expected returns, *Journal of Finance*, *51*(1), 3–51.

Jasiak, J., and C. Gourieroux (2006), Autoregressive gamma processes, *Journal of Forecasting*, *25*(2), 129–152.

Jones, E. P., S. P. Mason, and E. Rosenfeld (1984), Contigent claims analysis of corporate capital structures: an empirical investigation, *Journal of Finance*, *39*(2), 611–625.

Ju, N., and J. Miao (2012), Ambiguity, learning, and asset returns, *Econometrica*, *2*, 559 – 591.

Kahneman, D., and A. Tversky (1979), Prospect theory: an analysis of decisions under risk, *Econometrica*, *47*(2), 263–292.

Kahneman, D., and A. Tversky (1992), Advances in prospect theory: Cumulative representation of uncertainty, *Journal of Risk and Uncertainty*, *5*, 297–323.

Khanapure, R. (2012), Life-cycle portfolio allocation for disappointment averse agents, *Working paper*.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005), A smooth decision model of decision making under uncertainty, *Econometrica*, *73*(6), 1849–1892.

Kocheracota, N. (1996), The equity premium: It's still a puzzle, *Journal of Economic Literature*, *34*, 42–71.

Kreps, D. M., and E. L. Porteus (1978), Temporal resolution of uncertainty and dynamic choice, *Econometrica*, *46*(1), 185–200.

Leland, H. E. (1994), Corporate debt value, bond covenants, and capital structure, *Journal of Finance*, *49*, 1213–1252.

Leland, H. E. (2004), Predictions of default probabilities in structural models of default, *Journal of Investment Management*, *2*, 5–20.

Leland, H. E., and K. B. Toft (1996), Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *Journal of Finance*, *51*, 987–1019.

Lettau, M., S. Ludvigson, and J. Wachter (2007), The declining equity premium: What role does macroeconomic risk play?, *Review of Financial Studies*, *21*, 1653–1687.

Lettau, M. L., and S. C. Ludvigson (2001), Consumption, aggregate wealth, and expected stock returns, *Journal of Finance*, *56*(3), 815–849.

Lettau, M. L., M. Maggiori, and M. Weber (2013), Conditional risk premia in currency markets and other asset classes, *Working paper*.

Lewellen, J., S. Nagel, and J. Shankenc (2010), A skeptical appraisal of asset pricing tests, *Journal of Financial Economics*, *96*(2), 175–194.

Lintner, J. (1965), The valuation of assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, *47*(1), 13–37.

Liu, L. X., T. M. Whited, and L. Zhang (2009), Investment-based expected stock returns, *Journal of Political Economy*, *117*, 1105–1139.

Ljungqvist, L., and H. Uhlig (2009), Optimal endowment construction under Campbell-Cochrane habit formation, *Working paper*.

Longstaff, F. A., S. Mithal, and E. Neis (2005), Corporate yield spreads: Default risk or liquidity? New evidence form the credit default swap market, *Journal of Finance*, *60*, 2213–2253.

Lucas, R. (1978), Asset prices in an exchange economy, *Econometrica*, *46*(6), 1429–1445.

Mankiw, G. N., and S. P. Zeldes (1991), The consumption of stockholders and non-stockholders, *Journal of Financial Economics*, *29*(1), 97–112.

Mehra, R., and E. C. Prescott (1985), The equity premium: a puzzle, *Journal of Monetary Economics*, *15*, 145–161.

Merton, R. C. (1974), On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance*, *29*, 449–470.

Mossin, J. (1968), Aspects of rational insurance purchasing, *Journal of Political Economy*, *76*, 533–568.

Newey, W., and D. McFadden (1994), Large sample estimation and hypothesis testing, in *Handbook of Econometrics*, vol. 4, chap. 36, pp. 2112–2241, Elsevier B.V., Amsterdam.

Ostrovnaya, A., B. R. Routledge, and S. E. Zin (2006), Endogenous countercyclical risk aversion and the cross-section, *Working Paper*.

Piazzesi, M. (2002), The 6-D bias and the equity premium puzzle: Comment, in *NBER Macroeconomics Annual*, vol. 16, pp. 317–329, MIT Press, Cambridge, MA.

Piazzesi, M., and M. Schneider (2006), Equilibrium yield curves, in *Macroeconomics Annual 2006*, pp. 389–442, MIT Press, Cambridge, MA.

Piccioni, I. (2011), Rationalizing size, value, and momentum effects with an asymmetric CAPM, *Working paper*.

Pollard, D. (1984), *Convergence of Stochastic Processes*, Springer, New York.

Pope, D. G., and M. E. Schweitzer (2011), Is Tiger Woods loss averse? Persistent bias in the face of experience, competition, and high stakes, *American Economic Review*, *101*(1), 129–157.

Post, T., M. J. van den Assem, G. Baltussen, and R. H. Thaler (2008), Deal or No Deal? Decision making under risk in a large-payoff game show, *American Economic Review*, *98*(1), 38–71.

Pratt, J. W. (1964), Risk aversion in the small and in the large, *Econometrica*, *32*(1/2), 122–136.

Rabin, M. (2000), Risk aversion and expected-utility theory: a calibration theorem, *Econometrica*, *68*(5), 1281–1292.

Routledge, B., and S. Zin (2010), Generalized disappointment aversion and asset prices, *Journal of Finance*, *65*(4), 1303–1332.

Safra, Z., and U. Segal (2008), Calibration results for non-expected utility theories, *Econometrica*, *76*(5), 1143–1166.

Savov, A. (2011), Asset pricing with garbage, *Journal of Finance*, *76*(1), 177–201.

Segal, U., and A. Spivak (1990), First order versus second order risk aversion, *Journal of Economic Theory*, *51*(1), 111–125.

Shleifer, A., and R. W. Vishny (1992), Liquidation values and debt capacity, *Journal of Finance*, *47*, 1343–1366.

van Binsbergen, J. H., W. H. Hueskes, R. S. Koijen, and E. B. Vrugt (2011), Equity yields, *Working paper*.

Verdelhan, A. (2010), A habit-based explanation of the exchange rate risk premium, *Journal of Investment Management*, *65*, 123?145.

Vissing-Jorgensen, A. (2002), Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy*, *110*(4), 825–853.

von Neumann, J., and O. Morgenstern (1944), *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ.

Weil, P. (1989), The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics*, *24*, 401–421.

Winkler, R. L., G. M. Roodman, and R. R. Britney (1972), The determination of partial moments, *Management Science*, *19*(3), 290–296.

Yogo, M. (2004), Essays on consumption and expected returns, Ph.D. thesis, Harvard University.

Yogo, M. (2006), A consumption-based explanation of expected stock returns, *Journal of Finance*, *61*, 540–579.