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**University of Michigan**

Ann Arbor, Michigan 48109

[redacted]

# The Use of Outside Information in Econometric Forecasting

E. Philip Howrey
Saul H. Hymans
Mark N. Greene

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Abstract

This paper examines the potential value of using outside information such as monthly data to update or modify quarterly econometric forecasts. A best linear unbiased updating procedure is introduced and applied to a small experimental macro-econometric model patterned after the Michigan Quarterly Econometric Model. It is found that accurate outside information has the potential for substantial improvement in the accuracy of short-term forecasts. Based on a common definition of dynamic accuracy this gain appears to be concentrated in near-term forecasts and is much less pronounced as the forecast horizon is extended.

# The Use of Outside Information in Econometric Forecasting[1]

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## 1. Introduction

Outside information is invariably used to produce timely econometric forecasts. When a quarterly econometric model is used, an important source of outside information is the monthly data that become available during the quarter. As monthly data on such variables as retail sales, the unemployment rate, interest rates, industrial production, personal consumption expenditure, and personal income are released, the quarterly forecast is revised to be consistent with this new information.

Outside information is typically used in an informal, *ad hoc* way to modify econometric forecasts. The purpose of the research described in this paper is to investigate the properties of a scientifically sound approach to the use of outside information. In the following section, the forecasting procedure is described. The methods are then applied to a small macroeconometric model to determine the maximum potential gain in forecast accuracy that can be expected from the use of outside information.

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[1] Howrey and Hymans are Professors of Economics and Statistics and Faculty Associates at the Research Seminar in Quantitative Economics at the University of Michigan; Greene is a graduate student in Economics and Research Assistant at RSQE. The authors gratefully acknowledge the comments of Professor Jan Kmenta on an earlier version of this paper. Research on this topic has been supported by NSF Grant SOC78-09475.

## 2. The Use of Outside Information

In this section we describe a procedure for utilizing outside information to improve the forecast accuracy of an econometric model. This procedure provides a rationale for using monthly data, for example, to update or modify quarterly econometric model forecasts. An attractive feature of this proposed method of sequential revision of quarterly forecasts is that it does not require modification of the quarterly forecasting model. Thus it is possible, at least in principle, to obtain improved forecasts simply by augmenting a quarterly forecasting model with an auxiliary outside-information system.

To provide the motivation for the proposed forecasting procedure, we consider first a simple, suboptimal method of using outside information to revise or improve a forecast. Suppose an econometric model produces an unbiased forecast of the $g^{th}$ endogenous variable, $\hat{Y}_g$, with a forecast error variance $\hat{\sigma}_g^2$, and outside information such as current monthly data can be used to obtain another unbiased forecast $\tilde{Y}_g$ with forecast error variance $\tilde{\sigma}_g^2$. By combining the two forecasts according to

$$\bar{Y}_g = \hat{k}\hat{Y}_g + \tilde{k}\tilde{Y}_g, \tag{1}$$

an improved forecast may be obtained. Indeed, it is trivial to verify that if the two forecast errors are uncorrelated, the forecast weights

$$\tilde{k} = \hat{\sigma}_g^2/(\hat{\sigma}_g^2 + \tilde{\sigma}_g^2) \tag{2}$$

and

(3) $$\hat{k} = \tilde{\sigma}_g^2/(\hat{\sigma}_g^2 + \tilde{\sigma}_g^2) = 1 - \tilde{k}$$

will yield a minimum variance (linear) unbiased forecast of $Y_g$ with a prediction error variance equal to

(4) $$\bar{\sigma}_g^2 = \hat{\sigma}_g^2\tilde{\sigma}_g^2/(\hat{\sigma}_g^2 + \tilde{\sigma}_g^2).$$

The way in which the forecast based on outside information is used to revise the model forecast is best seen by rewriting the combined forecast (using the above weights) as

(5) $$\bar{Y}_g = \hat{Y}_g + \tilde{k}(\tilde{Y}_g - \hat{Y}_g).$$

This shows that the revised forecast $\bar{Y}_g$ is adjusted toward $\tilde{Y}_g$ by adding the fraction $\tilde{k}$ of the discrepancy between $\tilde{Y}_g$ and $\hat{Y}_g$ to the original model forecast. The proportional reduction in $\hat{\sigma}_g^2$ achieved by using the outside information is $\hat{\sigma}_g^2/(\hat{\sigma}_g^2 + \tilde{\sigma}_g^2)$. Thus the value of outside information clearly depends on the variances $\hat{\sigma}_g^2$ and $\tilde{\sigma}_g^2$.

The major shortcoming of this approach is that $\tilde{Y}_g$ affects only the forecast of $Y_g$. It would be preferable to recognize that $\tilde{Y}_g$ may contain information about other variables as well and thus to allow the outside information to modify the forecasts of all of the variables in the model. In order to develop this more general approach, we introduce explicitly a forecasting model and an auxiliary outside-information system. Predictions from these two systems are then combined to obtain an optimal forecast.

The structural form of a quarterly linear forecasting model is written as

(6) $$CY(t) = AY(t-1) + BX(t) + U(t), \quad t = 1, 2, \ldots$$

where

$Y(t)$ = a $G * 1$ vector of endogenous variables valued at time t

$X(t)$ = a $K * 1$ vector of exogenous variables valued at time t

A, B, C = conformable matrices of structural parameters

$U(t)$ = a $G * 1$ vector of disturbances with mean zero and covariance matrix $\Sigma_u$.

The reduced form used for forecasting is written as

(7) $$Y(t) = PY(t-1) + QX(t) + V(t)$$

where

$$D = C^{-1}$$

$$P = DA$$

$$Q = DB$$

and

(8) $$V(t) = DU(t).$$

The one-quarter ahead forecast of $Y(t)$ given $Y(t-1)$ and $X(t)$ is

(9) $$\hat{Y}(t) = PY(t-1) + QX(t)$$

so that

(10) $$Y(t) = \hat{Y}(t) + V(t)$$

where

$V(t)$ = a $G * 1$ vector of disturbances with mean zero and covariance matrix $\Sigma_v = D \Sigma_u D'$.

This last equation summarizes the stochastic relationship between the predicted and realized values of the endogenous variables implied by the forecasting model.

We assume that some outside information is also available and refer specifically to monthly observations on some of the endogenous variables in the quarterly model to illustrate the approach. Let $y_g(t, i)$ denote the $g^{th}$ variable in Y for month i of quarter t. Then, assuming $Y_g(t)$ is a flow variable,

$$Y_g(t) = \sum_{i=1}^{3} y_g(t, i). \tag{11}$$

If $\tilde{y}_g(t, i)$ denotes the predicted value of $y_g(t, i)$ based on monthly data,[2] the predicted quarterly aggregate is

$$\tilde{Y}_g(t) = \sum_{i=1}^{3} \tilde{y}_g(t, i). \tag{12}$$

Finally, let $\tilde{Y}(t)$ denote the $H * 1$ vector of forecasts obtained from the monthly data and let $\theta$ denote a "selection" matrix which picks out the $H \le G$ elements of $Y(t)$ for which forecasts based on the monthly data are made. Then the outside-information system is summarized by

$$\tilde{Y}(t) = \theta Y(t) + W(t) \tag{13}$$

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[2] If the value of $y_g(t, i)$ is known, then $\tilde{y}_g(t, i) = y_g(t, i)$.

where

$W(t)$ = a $H * 1$ vector of disturbances with mean zero and covariance matrix $\Sigma_w$.

Collecting the results for the quarterly forecasting model and the outside-information system, we have

$$(10) \qquad Y(t) = \hat{Y}(t) + V(t)$$

$$(13) \qquad \tilde{Y}(t) = \theta Y(t) + W(t).$$

By analogy with our earlier result, we write the combined predictor as

$$(14) \qquad \bar{Y}(t) = \hat{Y}(t) + K[\tilde{Y}(t) - \theta\hat{Y}(t)].$$

This combined forecast can be shown to have two desirable properties; namely, under certain conditions indicated below, it is the minimum variance linear unbiased predictor of $Y(t)$ and it also satisfies the identities of the econometric model.

The optimality of the combined predictor can be established using the following lemma.

<u>Lemma 1</u>. Let $x$ be a random vector with mean $\mu$ and covariance matrix $\Sigma$. Let $x_1$ denote the first $n_1$ elements of $x$ and let $x_2$ denote the remaining $n_2 > 0$ elements of $x$. Partition $\mu$ and $\Sigma$ conformably and suppose $\Sigma_{22}$ is nonsingular. The predictor

$$\hat{x}_1 = \mu_1 + K(x_2 - \mu_2)$$

where

$$K = \Sigma_{12}\Sigma_{22}^{-1}$$

is the best linear unbiased predictor of $x_1$ given $x_2$.

Proof. Consider the linear predictor $k_1 + k_2x_2$. This predictor is (unconditionally) unbiased for $x_1$ if

$$E(x_1 - k_1 - k_2x_2) = \mu_1 - k_1 - k_2\mu_2 = 0.$$

Thus $k_1 = \mu_1 - k_2\mu_2$ and unbiasedness requires that the forecast take the form $\tilde{x}_1 = \mu_1 + k_2(x_2 - \mu_2)$. If $k_2 = K + \delta$, the covariance matrix of $x_1 - \tilde{x}_1$ is

$$V \equiv E[(x_1 - \tilde{x}_1)(x_1 - \tilde{x}_1)']$$

$$= \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} + \delta\Sigma_{22}\delta'.$$

It follows that the covariance matrix of $x_1 - \hat{x}_1$ is $\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$ and the prediction error covariance matrix of any other unbiased linear predictor exceeds this by a positive semi-definite matrix. ■

We state the optimality of the predictor $\tilde{Y}(t)$ in the following theorem.

<u>Theorem 1</u>. Given the system

$$Y(t) = \hat{Y}(t) + V(t)$$

$$\tilde{Y}(t) = \theta Y(t) + W(t)$$

where $V(t) \sim (0, \Sigma_v)$, $W(t) \sim (0, \Sigma_w)$, and $V(t)$ and $W(t)$ are mutually as well as serially uncorrelated, the best linear unbiased predictor of $Y(t)$ given $\tilde{Y}(t)$ is

$$(14) \qquad \bar{Y}(t) = \hat{Y}(t) + K[\tilde{Y}(t) - \theta\hat{Y}(t)]$$

where

$$(15) \qquad K = \Sigma_v\theta'(\theta\Sigma_v\theta' + \Sigma_w)^{-1},$$

and the prediction error covariance matrix of $\bar{Y}(t)$ is

$$(16) \qquad \Omega = (I - K\theta)\Sigma_v.$$

Proof. Write the system as

$$x_1 \equiv \hat{Y}(t) + V(t)$$

$$x_2 \equiv \theta Y(t) + W(t) = \theta\hat{Y}(t) + \theta V(t) + W(t)$$

so that

$$\mu_1 = E(x_1) = \hat{Y}(t)$$
$$\mu_2 = E(x_2) = \theta\hat{Y}(t)$$
$$\Sigma_{11} = E[(x_1 - \mu_1)(x_1 - \mu_1)'] = \Sigma_v$$
$$\Sigma_{12} = E[(x_1 - \mu_1)(x_2 - \mu_2)'] = \Sigma_v\theta'$$

and

$$\Sigma_{22} = E[(x_2 - \mu_2)(x_2 - \mu_2)'] = \theta\Sigma_v\theta' + \Sigma_w.$$

The results of the theorem follow directly from Lemma 1 provided $\Sigma_{22}$ is nonsingular. ■

In order for the combined forecast to be reasonable, it should satisfy the identities of the structural model. The forecast $\hat{Y}(t)$ will automatically satisfy the identities but there is no obvious reason for the outside-information forecast $\tilde{Y}(t)$ to satisfy them. Thus the following result is reassuring.

Theorem 2. The optimal predictor $\bar{Y}(t)$ defined in Theorem 1 will satisfy any identities that appear in the structural form of the econometric model.

Proof. The structural model is rewritten as

$$C_1Y(t) = A_1Y(t-1) + B_1X(t) + U_1(t)$$

$$C_2Y(t) = A_2Y(t-1) + B_2X(t)$$

so that the first $G_1$ equations are stochastic and the remaining $G_2 = G - G_1$ equations are identities. Let

$$\eta(t) = A_2Y(t-1) + B_2X(t).$$

The forecast $\bar{Y}(t)$ will satisfy the identities if and only if

$$C_2\bar{Y}(t) = \eta(t).$$

Now

$$C_2\bar{Y}(t) = C_2\hat{Y}(t) + C_2K[\tilde{Y}(t) - \theta\hat{Y}(t)].$$

Since $\hat{Y}(t)$ is a forecast obtained from the structural model, it necessarily satisfies the identities, i.e.,

$$C_2\hat{Y}(t) = \eta(t).$$

Moreover, since $C_2K = C_2\Sigma_v\theta'(\theta\Sigma_v\theta' + \Sigma_w)^{-1}$, it follows that $C_2K = 0$. To see this, consider

$$C_2\Sigma_v = C_2D\Sigma_uD'$$

$$= C_2 \begin{bmatrix} C_1 \\ \hline C_2 \end{bmatrix}^{-1} \Sigma_uD'$$

$$= [0 : I] \left[\begin{array}{c|c} \Sigma_{11} & 0 \\ \hline 0 & 0 \end{array}\right] D'$$

$$= 0.$$ ■

We conclude this discussion of the properties of the combined forecast by noting that, provided V(t) and W(t) are uncorrelated, the potential gain in precision associated with using outside information along with a given econometric model is maximized as $\Sigma_w \to 0$, i.e., as the quality of the outside information approaches that of a perfect forecast. According to (16), the reduction in the prediction error variance that accompanies the use of outside information is

$$\Delta = K\theta\Sigma_v$$

$$= \Sigma_v\theta'(\theta\Sigma_v\theta' + \Sigma_w)^{-1}\theta\Sigma_v,$$

a positive semi-definite matrix. As $\Sigma_w \to 0$, this approaches the value

$$\Delta^* = \Sigma_v\theta'(\theta\Sigma_v\theta')^{-1}\theta\Sigma_v.$$

Since the difference between $\Delta^*$ and $\Delta$,

$$\Delta^* - \Delta = \Sigma_v\theta'[(\theta\Sigma_v\theta')^{-1} - (\theta\Sigma_v\theta' + \Sigma_w)^{-1}]\theta\Sigma_v,$$

is positive semidefinite,[3] we conclude that the maximum contribution of outside information occurs at $\Sigma_w = 0$.

The preceding results are based on a linear forecasting model. Since most econometric models are nonlinear, these techniques are not directly applicable. The usual procedure in the case of nonlinear models is to apply the results of the linear theory to a linear approximation to the nonlinear model.[4] Suppose, as is usually the case, that the structural model is a system of nonlinear equations of the form

$$F[Y(t), Y(t-1), X(t)] = U(t). \tag{17}$$

The one-step ahead forecast given Y(t-1) and X(t) is typically obtained by solving the system

$$F[\hat{Y}(t), Y(t-1), X(t)] = 0 \tag{18}$$

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[3] This follows from the proposition that if A and B are positive definite and B - A is positive semidefinite, then $A^{-1} - B^{-1}$ is positive semidefinite. For proof, see Dhrymes (1978), pp. 494-495.

[4] For a discussion of this approach to nonlinear systems, see Schweppe (1973), Chapter 13 and the references cited there.

for $\hat{Y}(t)$. If $F[Y(t), Y(t-1), X(t)]$ is expanded in a Taylor series about the point $[\hat{Y}(t), Y(t-1), X(t)]$, the result is

$$(19)\quad F[Y(t), Y(t-1), X(t)] = F[\hat{Y}(t), Y(t-1), X(t)] + [\partial F/\partial Y(t)]\,[Y(t) - \hat{Y}(t)] + R(t).$$

Defining $C_t = \partial F/\partial Y(t)$ and setting $F[\hat{Y}(t), Y(t-1), X(t)] = 0$, we have

$$(20)\quad U(t) = C_t[Y(t) - \hat{Y}(t)] + R(t)$$

or

$$(21)\quad Y(t) = \hat{Y}(t) + D_tU(t) - D_tR(t)$$

where

$$(22)\quad D_t = C_t^{-1}.$$

Assuming that $D_tR(t)$ is negligible, the forecasting model can be expressed as

$$Y(t) = \hat{Y}(t) + V(t)$$

where

$$(23)\quad V(t) = D_tU(t).$$

Thus with a nonlinear model the time-invariant covariance matrix $\Sigma_v$ of the linear model is replaced by $\Sigma_v(t) = D_t\Sigma_uD_t'$. The combined forecast is now given by

$$\bar{Y}(t) = \hat{Y}(t) + K_t[\tilde{Y}(t) - \theta\hat{Y}(t)]$$

where $K_t$ and $D_t$ now vary with t.[5]

## 3. Some Empirical Results

The procedures introduced in the preceding section for utilizing outside information in econometric forecasting are applied to an experimental model in this section. The model, a simplified version of the Michigan Quarterly Econometric Model, is summarized first. This is followed by an analysis of static one-quarter ahead forecasts. Finally, the accuracy of dynamic forecasts is investigated.

### 3.1 A Simple Macroeconometric Forecasting Model

In order to investigate the magnitude of the potential improvement in forecast accuracy resulting from the availability of outside information, we constructed a 14-equation econometric model based on the Michigan Quarterly Econometric Model of the U.S. Economy (MQEM). The test model consists of simplified versions of the consumption, employment, and income sectors of MQEM. The model is closed with identities so that the resulting dynamic simultaneous equation system determines GNP, personal income, corporate profits, consumption, and the unemployment rate. We thus obtained a model of manageable size, yet characteristic of the macroeconometric models currently in regular use.

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[5] Optimality of the combined forecast in this case is dependent upon the assumption that the Taylor Series remainder term, $D_t R(t)$, is indeed negligible. This seems reasonable for most applied work.

The definitions of the variables are shown in Table 1 and the equations are listed in Table 2. A simplified flow diagram of the model is shown in Figure 1 to provide an overview of the structure of the model. The figure indicates that the three consumption functions are driven primarily by real disposable income. Gross national product is obtained by adding exogenous expenditure $X_1$ to aggregate consumption expenditure. Nominal personal income is obtained by first converting real GNP to nominal terms and then subtracting corporate profits and other net withdrawals $X_2$ from the income stream. Finally, personal taxes are subtracted from personal income to yield disposable income which in turn is converted to real disposable income. The unemployment rate is determined by GNP but has no contemporaneous feedback effects on the real variables in the model.

The parameter values shown in Table 2 were obtained by the method of ordinary least squares for the period 1954.1 through 1966.4. Although the least squares estimator is not the preferred estimator, the parameter values so determined do not differ in any important ways from the two-stage least-squares estimates. In the experiments reported subsequently, the model was re-estimated (but not re-specified) each "year" before forecasts for the next year were generated. This successive re-estimation helps to keep the model correctly calibrated over the period of the experiments and corresponds to the re-estimation that operating models undergo periodically.

Table 1. Endogenous and Exogenous Variables in the Experimental Model

ENDOGENOUS:

| | | |
|---|---|---|
| CD72 | - | Consumption: Durables (billions of 1972 $s) |
| CN72 | - | Consumption: Nondurables (billions of 1972 $s) |
| CS72 | - | Consumption: Services (billions of 1972 $s) |
| GNP | - | Gross National Product (billions of current $s) |
| GNP72 | - | Gross National Product (billions of 1972 $s) |
| RUM | - | Unemployment Rate: Male, 20 and over (%) |
| TW | - | Taxes Withheld (billions of current $s) |
| YCP | - | Corporate Profits (billions of current $s) |
| CD | - | Durable Consumption (billions of current $s) |
| CN | - | Nondurable Consumption (billions of current $s) |
| YD | - | Disposable Income (billions of current $s) |
| YD72 | - | Disposable Income (billions of 1972 $s) |
| YP | - | Personal Income (billions of current $s) |

EXOGENOUS:

| | | |
|---|---|---|
| DAS | - | Auto Strike Dummy |
| PC | - | Consumption Deflator (1972 = 100) |
| PCD | - | Durable Consumption Deflator (1972 = 100) |
| PCN | - | Nondurable Consumption Deflator (1972 = 100) |
| RAAA | - | Corporate AAA Interest Rate (%) |
| TAXCHG | - | Tax Revenue Affects of Statutory Tax Changes (billions of current $s) |
| TIME | - | Time Trend |
| $X_1$ | - | Investment, Net Exports, and Government Purchases (billions of 1972 $s) |
| $X_2$ | - | Capital Consumption Allowance and Indirect Business Taxes (billions of current $s) |

Table 2. Equations in the Experimental Model[a]

1. Personal Income

$$YP \equiv GNP - YCP - X_2$$

2. Disposable Income

$$YD \equiv YP - TW$$

3. Real Disposable Income

$$YD72 \equiv \frac{YD}{PC} * 100$$

4. Real Nondurable Consumption

$$CN72 = \underset{(36)}{74} + \underset{(0.06)}{0.22}\ \Delta\, YD72 + \underset{(0.04)}{0.11}\ YD72_{-1}$$

$$- \underset{(29)}{47}\ \frac{PCN_{-1}}{PC_{-1}} - \underset{(73)}{66}\ \Delta\, \frac{PCN}{PC} + \underset{(0.13)}{0.59}\ CN72_{-1}$$

5. Real Services Consumption

$$CS72 = \underset{(6)}{15} + \underset{(0.18)}{0.12} \sum_{i=1}^{4} YD72_{-i} + \underset{(0.04)}{0.14}\ \Delta\, YD72$$

$$+ \underset{(0.11)}{0.27}\ TIME + \underset{(0.09)}{0.80}\ CS72_{-1}$$

6. Real Durable Consumption

$$CD72 = \underset{(1.7)}{-1.1} + \underset{(0.05)}{0.20}\ [YD72_{-1} - \underset{(0.09)}{0.83}\ YD72_{-2}]$$

$$- \underset{(1.8)}{2.4}\ [RAAA_{-1} - \underset{(0.09)}{0.83}\ RAAA_{-2}] - \underset{(0.7)}{1.7}\ RUM_{-1} - \underset{(0.09)}{0.83}\ RUM_{-2}$$

$$+ \underset{(0.6)}{1.6}\ DAS - \underset{(0.6)}{0.9}\ DAS_{-1} + \underset{(0.11)}{0.73}\ CD72_{-1}$$

7. Corporate Profits

$$YCP = \underset{(0.7)}{2.3} + \underset{(0.04)}{0.49}\ \Delta\, GNP - \underset{(1.02)}{3.66}\ \Delta\, PC + \underset{(0.02)}{0.92}\ YCP_{-1}$$

Table 2
(continued)

8. Tax Withholdings

$$\Delta TW = \underset{(0.15)}{-0.07} + \underset{(0.02)}{0.12}\ \Delta YP + TAXCHG$$

9. Unemployment Rate (Okun's Law)

$$\Delta RUM = \underset{(0.05)}{0.36} - \underset{(3.4)}{24.6}\ \frac{GNP72}{GNP72_{-1}} \underset{(4.8)}{-15.5}\ \frac{GNP72_{-1}}{GNP72_{-2}}$$

$$+ \underset{(0.10)}{0.23}\ \Delta RUM_{-1}$$

10. Real GNP

$$GNP72 \equiv C72 + X1$$

11. Nominal GNP

$$GNP \equiv GNP72 * PC/100$$

12. Nondurable Consumption

$$CN \equiv CN72 * PCN/100$$

13. Durable Consumption

$$CD \equiv CD72 * PCD/100$$

14. Total Consumption

$$C72 \equiv CN72 + CS72 + CD72$$

[a] The sample period for the parameter estimates is 1954.1-1966.4. Estimated standard errors are shown in parentheses.

Despite the fact that this is an enormously simplified version of an operating econometric model, it does contain the basic features of the parent model. For this reason we feel it is a useful vehicle for investigating the contribution of outside information to forecast accuracy.

### 3.2 The Potential Value of Outside Information: Static Forecasts

The covariance matrix $\Sigma_v$ of reduced-form disturbances provides a measure of the forecast error variance of <u>ex post</u> forecasts generated by the econometric model.[6] The availability of outside information offers the potential to reduce this prediction error variance. In this section we consider the limits on the potential improvement in forecast accuracy that could be achieved within the context of the experimental model described above.

Outside information is available on four of the thirteen endogenous variables in our model. Monthly observations on the unemployment rate and personal income are directly available. In addition, monthly retail sales data can be used to construct estimates of nominal purchases of consumer durables and non-durables.[7] As noted previously, the maximum gain in forecast accuracy is achieved when the outside information is exact. Thus

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[6] Given the exogenous variables in the model, the diagonal elements of $\Sigma_v$ are the forecast error variances of one-quarter ahead forecasts if the sampling variability of the estimated coefficients is negligible.

[7] Beginning in 1980, monthly estimates of nominal purchases of consumer durables, nondurables, and services are available in the Survey of Current Business.

we can investigate the maximum gain in forecast accuracy for our test model by assuming that the values of these four variables are known in advance.

The theoretical results are summarized in Table 3 for three different sample periods. The first column of entries in the table contains the theoretical standard errors of one-quarter ahead _ex post_ forecasts of selected endogenous variables implied by the model as estimated over the period 1954.1-1966.4. Thus the entry 2.36 for GNP72 indicates that the approximate 95% error bounds for the predicted value of real GNP in the first quarter of 1967 would be ± 4.72 (= 2 x 2.36) billion 1972 dollars. If personal income, consumption expenditure on durables and non-durables, and the unemployment rate were known in advance, the 95% error bounds would be only ± 1.38 (= 2 x 0.69). Thus a size-able gain in forecast accuracy is potentially available through the use of outside information. Similar gains in forecast precision are achieved for corporate profits (YCP) and real disposable income (YD72).

The entries in Table 3 for the two other sample periods are similar in magnitude. Thus the rather dramatic gain in forecast accuracy achievable through the use of outside information is not peculiar to the specific parameter estimates of the 1954.1-1966.4 sample period. Parenthetically, we observe that the forecast error variance is increasing over time. There are two reasons for this. The first is that the error variance of the structural equations of the model increases over time. The structural equations explain the data less well at the end of the sample period than at the beginning of the period. The second reason is that the model is nonlinear and many of the variables are trending upward over time. These variables enter the model in such a way as to increase the forecast error variance.

As a practical matter it is useful to know what specific outside information is the most useful for improving the accuracy of econometric forecasts. This can be determined by varying the selection matrix $\theta$ in the observational model. In the case of the experimental model used here, it was found that the spillover effects of the unemployment rate are virtually zero. This is perhaps not too surprising in view of the fact that there is no simultaneous feedback from the unemployment rate to the other variables in this model. The marginal contribution of each of the other three variables is clearly significant, however, as shown in Table 4. This suggests that as far as this model is concerned, a high priority should be given to the development of operational observation equations for consumer expenditures on durables and nondurables and for personal income.

### 3.3 The Realized Value of Outside Information: Static Forecasts

The parameter estimates of our experimental model were used in the previous section to determine the potential value of outside information as implied by the model. This potential may not be realized if the assumptions of the model are not satisfied. In this section we investigate the extent to which the potential improvements in forecast accuracy can be realized in practice.

Table 3. Potential value of Outside Information[a]
(CD, CN, YP, and RUM given)

| | Sample Period | | | | | |
|---|---|---|---|---|---|---|
| | 1954.1-1966.4 | | 1954.1-1971.4 | | 1954.1-1976.4 | |
| | $\hat{\sigma}$ | $\bar{\sigma}$ | $\hat{\sigma}$ | $\bar{\sigma}$ | $\hat{\sigma}$ | $\bar{\sigma}$ |
| CD72 | 1.29 | ------ | 1.49 | ------ | 2.12 | ------ |
| CN72 | 1.37 | ------ | 1.49 | ------ | 1.68 | ------ |
| CS72 | 0.82 | 0.69 | 0.96 | 0.71 | 1.11 | 0.91 |
| RUM | 0.19 | ------ | 0.18 | ------ | 0.20 | ------ |
| YCP | 1.29 | 0.56 | 1.47 | 0.70 | 2.77 | 1.23 |
| GNP72 | 2.36 | 0.69 | 2.57 | 0.71 | 3.43 | 0.91 |
| YD72 | 1.98 | 0.55 | 2.15 | 0.90 | 2.59 | 1.08 |

[a] $\hat{\sigma}$ is the standard error of the one-quarter ahead *ex post* forecast implied by the econometric model; $\bar{\sigma}$ is the same standard error with CD, CN, YP, and RUM known in advance. $\hat{\sigma}$ entries are calculated from the covariance matrix $\Sigma_v$ of reduced-form disturbances; $\bar{\sigma}$ entries are calculated from $\Omega$ defined in equation (16) with $\Sigma_w = 0$.

Table 4. Potential Value of Outside Information[a]
(CD, CN, and YP given)

| | Sample Period | | | | | |
|---|---|---|---|---|---|---|
| | 1954.1-1966.4 | | 1954.1-1971.4 | | 1954.1-1976.4 | |
| | $\hat{\sigma}$ | $\bar{\sigma}$ | $\hat{\sigma}$ | $\bar{\sigma}$ | $\hat{\sigma}$ | $\bar{\sigma}$ |
| CD72 | 1.29 | ------ | 1.49 | ------ | 2.12 | ------ |
| CN | 1.37 | ------ | 1.49 | ------ | 1.68 | ------ |
| CS72 | 0.82 | 0.70 | 0.96 | 0.72 | 1.11 | 0.91 |
| RUM | 0.19 | 0.18 | 0.18 | 0.18 | 0.20 | 0.20 |
| YCP | 1.29 | 0.56 | 1.47 | 0.70 | 2.77 | 1.23 |
| GNP72 | 2.36 | 0.69 | 2.57 | 0.72 | 3.43 | 0.91 |
| YD72 | 1.98 | 0.58 | 2.15 | 0.90 | 2.59 | 1.09 |

[a] $\hat{\sigma}$ is the standard error of the one-quarter ahead <u>ex post</u> forecast implied by the econometric model; $\bar{\sigma}$ is the same standard error with CD, CN, and YP known in advance. $\hat{\sigma}$ entries are calculated from the covariance matrix $\Sigma_y$ of reduced-form disturbances; $\bar{\sigma}$ entries are calculated from $\Omega$ defined in equation (16) with $\Sigma_w = 0$.

A sequence of one-quarter-ahead forecasts was calculated for the period 1967.1-1977.1, both with and without the benefit of prior knowledge of personal income, expenditures on durables and nondurables, and the unemployment rate. The parameters of the model were re-estimated at the end of each calendar year. Thus the forecasts are all out-of-sample predictions generated by a recently estimated model.

Table 5 summarizes the results of the experiment. Columns headed "Model" refer to forecasts generated by the simplified version of MQEM presented above. Columns headed "Modified" refer to model forecasts modified by outside information on YP, CN, CD, and RUM. The error statistics of the Modified forecasts are clearly superior to those of the original model. Thus, the use of outside information yields the reductions in forecast error variances that the "potential improvement" calculations of Table 3 led us to expect.

The forecast standard errors recorded in Table 5 exceed the corresponding values in Table 3. One possible explanation for this is that the contribution of the sampling variability of the estimated parameters to the forecast error variance is not negligible as assumed in Table 3. Suppose that $S_v$ represents the omitted positive (semi-) definite component of the prediction error covariance matrix, so that the covariance matrix $\Sigma_v^* = \Sigma_v + S_v$ should have been used to determine K, giving a value $K^*$ say. Using the suboptimal value for K determined in equation (15) gives a covariance matrix $\Omega$ of the forecast errors that exceeds the optimal covariance matrix $\Omega^*$ by a positive semi-

Table 5. Summary statistics of 1-Quarter Forecast Errors, Given YP, CN, CD, RUM[a]

| | Mean Error | | Standard Deviation[b] | | Minimum | | Maximum | |
|---|---|---|---|---|---|---|---|---|
| | Model | Modified | Model | Modified | Model | Modified | Model | Modified |
| CD72 | 1.07 | ------ | 3.05 | ------ | -0.35 | ------ | 6.60 | ------ |
| CN72 | 0.06 | ------ | 2.45 | ------ | -4.66 | ------ | 5.95 | ------ |
| CS72 | 0.71 | 0.56 | 1.55 | 1.54 | -3.69 | -3.67 | 3.99 | 4.04 |
| RUM | -0.04 | ------ | 0.22 | ------ | -0.72 | ------ | 0.41 | ------ |
| YCP | 1.05 | 0.72 | 4.11 | 1.74 | -5.66 | -3.69 | 14.40 | 4.93 |
| GNP72 | 1.83 | 0.58 | 5.28 | 1.58 | -12.37 | -3.71 | 12.53 | 4.30 |
| YD72 | 0.65 | -0.27 | 4.30 | 2.31 | -10.39 | -11.38 | 8.98 | 4.19 |

[a] These statistics are based on successive 1-quarter-ahead forecasts from 1967.1-1977.1 with the model re-estimated at the end of each calendar year.

[b] These entries are comparable to $\hat{\sigma}$ and $\bar{\sigma}$ shown in Table 3.

definite matrix. The improvement in forecast accuracy given in Table 3, $\Delta = \Sigma_v - \Omega$, will understate the potential gain, $\Delta^* = \Sigma_v^* - \Omega^*$, since

$$\Delta^* - \Delta = (\Sigma_v^* - \Omega^*) - (\Sigma_v - \Omega)$$

$$= (\Sigma_v^* - \Sigma_v) + (\Omega - \Omega^*)$$

is the sum of two positive semi-definite matrices. In addition, the gain in forecast accuracy shown in Table 5 also understates the potential because if $\Sigma_v^* \neq \Sigma_v$, a suboptimal value of K has been used to obtain the forecasts. We conclude that the dramatic improvements in forecast accuracy shown in both Tables 3 and 5 may even understate the potential value of the outside information.

### 3.4 The Potential Value of Outside Information: Dynamic Forecasts

The static forecast results indicate that outside information can make a valuable contribution to forecast accuracy. We now consider the impact of outside information on a dynamic forecast. The basic question is the extent to which the improvement due to the use of outside information in the first quarter will persist over the horizon of a four or eight-quarter forecast.

In the absence of outside information, a dynamic forecast is generated recursively using the reduced form of the model. The first period forecast of Y(t) given Y(t-1) and X(t) is given by

(24) $$\hat{Y}(t) = PY(t-1) + QX(t)$$

as noted previously, Forecasts for the following periods are then obtained from

$$\hat{Y}(t+h) = P\hat{Y}(t+h-1) + QX(t+h) \tag{25}$$

for h = 1, 2 ... . The prediction error of $\hat{Y}(t+h)$ is

$$\begin{aligned} \hat{V}(t+h) &= Y(t+h) - \hat{Y}(t+h) \\ &= P[Y(t+h-1) - \hat{Y}(t+h-1)] + V(t+h) \\ &= P\hat{V}(t+h-1) + V(t+h). \end{aligned} \tag{26}$$

It follows that the prediction error covariance matrix of $\hat{Y}(t+h)$ can be obtained recursively according to

$$\Sigma(h) = P\Sigma(h-1)P' + \Sigma_v \tag{27}$$

with the initial condition $\Sigma(0) = \Sigma_v$.

With the outside information available only in the first forecast period, the first quarter forecast is

$$\bar{Y}(t) = \hat{Y}(t) + K[\tilde{Y}(t) - \theta\hat{Y}(t)] \tag{28}$$

and the forecast error covariance matrix is $\Omega = (I - K\theta)\Sigma_v$. Forecasts for the following periods are again generated recursively using (25) but with $\bar{Y}(t)$ as the initial value. The covariance matrix of $\bar{Y}(t+h)$ is obtained by solving (27) with the initial condition $\Sigma(0) = \Omega$.

It follows that the difference between these two prediction error covariance matrices is

$$\Delta(h) = P\Delta(h-1)P' \tag{29}$$

with $\Delta(0) = K\theta\Sigma_v$. The solution to this matrix equation can be written explicitly as

(30) $$\Delta(h) = P^h \Delta(0) P^{h\prime}.$$

Provided the econometric model is stable, it follows that

(31) $$\lim_{h \to \infty} \Delta(h) = 0$$

since in this case the characteristic roots of P are less than one in absolute value, which implies that $P^h \to 0$ as $h \to \infty$.[8] This shows that as the forecast horizon is extended, the outside information forecast loses its relative advantage over the pure model forecast.

The forecast standard errors for the test model are shown in Table 6. The striking feature of this table is that the advantage of the outside information forecast diminishes very quickly with the forecast horizon. For real GNP, for example, the standard error of the two-quarter ahead model forecast is 2.91 while the outside information forecast has a standard error of 2.43. Recall that the respective one-quarter forecast standard errors are 2.36 and 0.69. Thus the rather decided advantage

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[8] Let R be the matrix of characteristic vectors of P and let $\Lambda$ be a diagonal matrix with the characteristic roots of P on the diagonal. Then

$$P = R\Lambda R'$$

and

$$P^h = R\Lambda^h R'.$$

Equation (31) follows from the fact that $\Lambda^h \to 0$ as $h \to \infty$.

of the one-quarter outside information forecast is much less pronounced in the second quarter of the forecast horizon and all but disappears in the 4-quarter forecast.

Summary statistics for 2-, 4-, and 8-quarter forecast errors are shown in Table 7. These statistics are based on a set of 8-quarter dynamic simulations beginning with the first quarter of 1967 and running through the first quarter of 1977, a total of 41 forecast intervals. The model was re-estimated each year and a new set of outside-sample forecasts was generated. These results are therefore indicative of the forecast performance that would have been observed if this model had been used continuously over this period to generate dynamic forecasts.

Two important conclusions emerge from this table. The first is that the simulation results conform to the theoretical finding that the value of current outside information diminishes as the forecast horizon lengthens. This can be seen by looking at the entries for the standard error or root mean squared error for the real GNP forecasts for two, four, and eight quarters in advance. The more dramatic finding, however, is that the standard deviations of the forecast errors are far in excess of the values given in Table 6. As remarked previously, we expect the simulation standard errors to exceed the theoretical values if the sampling errors of the coefficients are not negligible.[9] For the

[9] With the usual assumptions, the contribution of sampling variability of the coefficients is of the order 1/T where T is the sample size. with $T \geq 50$, one would ordinarily expect this source of forecast error to be small relative to the variance of the disturbances.

Table 6. Potential Value of Outside Information for Dynamic Forecasts [a]
(CD, CN, and YP given)

| | | 1954.1-1966.4 | | | 1954.1-1971.4 | | | 1954.1-1976.4 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 4 | 8 | 2 | 4 | 8 | 2 | 4 | 8 |
| CD72 | $\hat{\sigma}$ | 1.37 | 1.64 | 3.40 | 1.55 | 1.71 | 2.69 | 2.19 | 2.30 | 4.78 |
| | $\bar{\sigma}$ | 1.30 | 1.49 | 2.79 | 1.50 | 1.63 | 2.36 | 2.13 | 2.26 | 4.71 |
| CN72 | $\hat{\sigma}$ | 1.68 | 1.92 | 2.23 | 1.84 | 2.15 | 2.46 | 2.04 | 2.34 | 2.68 |
| | $\bar{\sigma}$ | 1.38 | 1.84 | 2.17 | 1.49 | 2.04 | 2.41 | 1.69 | 2.24 | 2.65 |
| CS72 | $\hat{\sigma}$ | 1.05 | 1.25 | 1.38 | 1.23 | 1.47 | 1.61 | 1.58 | 2.26 | 3.27 |
| | $\bar{\sigma}$ | 0.99 | 1.23 | 1.37 | 1.11 | 1.43 | 1.60 | 1.44 | 2.18 | 3.21 |
| RUM | $\hat{\sigma}$ | 0.31 | 1.63 | 2.07 | 0.30 | 0.64 | 2.07 | 0.35 | 0.78 | 2.88 |
| | $\bar{\sigma}$ | 0.19 | 0.46 | 1.59 | 0.18 | 0.46 | 1.60 | 0.21 | 0.55 | 2.21 |
| YCP | $\hat{\sigma}$ | 1.58 | 1.94 | 2.47 | 1.77 | 2.13 | 2.61 | 3.33 | 4.01 | 4.78 |
| | $\bar{\sigma}$ | 1.37 | 1.82 | 2.36 | 1.58 | 2.20 | 2.52 | 3.02 | 3.87 | 4.71 |
| GNP72 | $\hat{\sigma}$ | 2.91 | 3.62 | 5.61 | 3.18 | 3.92 | 5.30 | 4.16 | 5.16 | 6.62 |
| | $\bar{\sigma}$ | 2.43 | 3.34 | 4.99 | 2.65 | 3.65 | 4.95 | 3.61 | 4.90 | 6.50 |
| YD72 | $\hat{\sigma}$ | 2.75 | 3.75 | 5.35 | 2.98 | 4.09 | 5.76 | 3.58 | 4.92 | 6.77 |
| | $\bar{\sigma}$ | 2.09 | 3.38 | 5.01 | 2.38 | 3.76 | 5.51 | 2.90 | 4.58 | 6.63 |

[a] $\hat{\sigma}$ is the standard error of the 2-, 4-, 8-quarter ahead <u>ex post</u> forecast implied by the econometric model; $\bar{\sigma}$ is the same standard error with CD, CN, and YP known in advance.

Table 7. Summary Statistics of 2-, 4-, and 8-Quarter Forecast Errors Given YP, CD, CN, and RUM[a]

| | | 2 | | | 4 | | | 8 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | mean | se | rmse | mean | se | rmse | mean | se | rmse |
| CD72 | M | 2.23 | 4.73 | 5.23 | 4.22 | 6.56 | 7.80 | 7.67 | 8.52 | 11.46 |
| | F | 1.16 | 3.25 | 3.45 | 3.45 | 5.86 | 6.80 | 7.29 | 8.38 | 11.11 |
| CN72 | M | 0.10 | 3.42 | 3.42 | 0.36 | 4.34 | 4.35 | 1.17 | 4.94 | 5.08 |
| | F | -0.06 | 2.64 | 2.64 | 0.14 | 3.99 | 3.99 | 0.99 | 5.00 | 5.10 |
| CS72 | M | 1.38 | 2.42 | 2.79 | 2.74 | 3.57 | 4.50 | 6.09 | 5.84 | 8.43 |
| | F | 1.27 | 2.36 | 2.68 | 2.65 | 3.67 | 4.53 | 5.99 | 5.99 | 8.47 |
| RUM | M | -0.12 | 0.41 | 0.43 | -0.27 | 0.57 | 0.63 | -0.66 | 0.76 | 1.01 |
| | F | -0.04 | 0.22 | 0.22 | -0.20 | 0.51 | 0.55 | -0.61 | 0.75 | 0.97 |
| YCP | M | 2.19 | 6.79 | 7.13 | 4.24 | 11.06 | 11.84 | 7.40 | 14.95 | 16.68 |
| | F | 1.84 | 5.62 | 5.91 | 4.05 | 10.13 | 10.91 | 7.41 | 14.48 | 16.27 |
| GNP72 | M | 3.71 | 8.67 | 9.43 | 7.33 | 12.67 | 14.89 | 14.94 | 16.90 | 22.56 |
| | F | 2.36 | 6.66 | 7.07 | 6.24 | 11.73 | 13.29 | 14.27 | 17.05 | 22.23 |
| YD72 | M | 1.30 | 7.07 | 7.19 | 2.81 | 10.74 | 11.10 | 6.61 | 15.53 | 16.88 |
| | F | 0.33 | 5.55 | 5.56 | 1.95 | 10.02 | 10.21 | 5.97 | 15.53 | 16.64 |

[a] The entries in the M row correspond to unadjusted model forecast errors; the entries in the F row correspond to outside information forecast errors.

one period ahead forecasts, the simulation results did exhibit this tendency, but it was not nearly so pronounced as with the four and eight quarter forecasts shown here. On the basis of these results, it appears to be necessary to re-examine the usual (time-series) practice of ignoring the sampling variability of coefficient estimates when evaluating the standard error of forecasts.

### 3.5 The 1974-75 Recession and Recovery

The performance of the dynamic forecasts of real GNP during 1974-75 recession and recovery is summarized in Table 8. The cyclical peak experienced in the fourth quarter of 1973 is correctly forecast seven quarters in advance by both forecasting methods. Similarly, the lower turning point in 1975.1 is correctly anticipated seven quarters in advance by both forecasting procedures. Moreover, there were no false turning-point forecasts generated by either forecasting procedure over this period.

The dynamic forecast paths are quite similar for both forecast procedures. The use of outside information does get the model "on track" at the beginning of each forecast period, but this has relatively little effect on where the forecast winds up

Table 8. Dynamic Forecasts of Real GNP During the 1974-75 Recession and Recovery[a]

| Date | Actual | Predicted | | | | | |
|---|---|---|---|---|---|---|---|
| | | M | F | M | F | M | F |
| 1972.2 | 1163 | 1154 | 1163 | | | | |
| .3 | 1178 | 1165 | 1173 | | | | |
| .4 | 1202 | 1182 | 1189 | | | | |
| 1973.1 | 1230 | 1203 | 1209 | | | | |
| .2 | 1231 | 1212 | 1217 | | | | |
| .3 | 1236 | 1218 | 1223 | 1239 | 1236 | | |
| .4 | 1243 | 1232 | 1236 | 1254 | 1252 | | |
| 1974.1 | 1230 | 1222 | 1226 | 1245 | 1243 | | |
| .2 | 1224 | | | 1236 | 1234 | | |
| .3 | 1217 | | | 1222 | 1220 | | |
| .4 | 1200 | | | 1218 | 1217 | | |
| 1975.1 | 1172 | | | 1188 | 1187 | 1167 | 1168 |
| .2 | 1190 | | | 1200 | 1199 | 1179 | 1178 |
| .3 | 1220 | | | | | 1208 | 1207 |
| .4 | 1228 | | | | | 1214 | 1211 |
| 1976.1 | 1260 | | | | | 1238 | 1235 |
| .2 | 1267 | | | | | 1247 | 1244 |
| .3 | 1277 | | | | | 1252 | 1250 |
| .4 | 1288 | | | | | 1254 | 1251 |

[a] The entries in the columns headed M and F are 8-quarter dynamic forecasts generated with the econometric model (M) and with outside information (F) on YP, CD, CN, and RUM.

four or eight quarters later. This is, of course, precisely what would be expected from the summary measures of dynamic forecast accuracy reviewed earlier.[10]

A striking feature of Table 8 is that the econometric model is able to forecast the cyclical turns very well with no outside information. The major reason for this is that in the test model used here, the cyclically volatile components of GNP, namely, the various categories of investment expenditure, are exogenous and assumed to be known in advance. This may explain why the model forecasts the 1974-75 recession so well and why the value of outside information appears to deteriorate so rapidly in our simulations.

## 4. Conclusion

The results summarized in this paper indicate that the potential gain in forecast accuracy achievable through the use of outside information is not trivial. However, this gain appears to be concentrated in near-term forecasts and is of much less importance for longer-term forecasts. Several important issues remain to be considered, however.

(1) The gain in forecast accuracy using an operational outside information model needs to be investigated.

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[10] Note that dynamic accuracy is here being measured by the error in predicting GNP (or Profits, or ...) at some quarter in the future. If dynamic accuracy were somehow measured as a function of the errors in predicting quarterly changes in GNP along the time path of the forecast, relatively more benefit would be attached to the procedure which got on track more quickly for any given degree of end-point accuracy.

(2) The gain in forecast accuracy using an operating model such as MQEM needs to be explored, especially to see the effects of a greater degree of endogeneity.

(3) The approach described here could also be applied to anticipations data and possibly also leading indicators.

(4) The importance of sampling variability of coefficient estimates which is typically overlooked in studies of this kind needs to be investigated more fully.

Despite the limitations of this study, it clearly provides the motivation for more detailed study of these issues. These further developments promise to offer new evidence on the potential for using outside information to improve the forecasting performance of econometric models.

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