Center for Research on Economic and Social Theory

and

Department of Economics

Working Paper Series

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David D. Li

November, 1992
Number 93-09

DEPARTMENT OF ECONOMICS
University of Michigan
Ann Arbor, Michigan 48109-1220

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David D. Li
University of Michigan

October 1991

This Version: November 1992

[1]I am grateful to Eric Maskin and Andrei Shleifer, who provided me with invaluable advice. Thanks are also due to Carliss Baldwin, Jerry Green, Bart Lipman and Andreu Mas-Colell for helpful comments. Comments from seminar participants at Harvard, University of Michigan, New York University Business School and the Queen's University have proved helpful.

Abstract

The purpose of this paper is to show that efficient corporate control can be maintained in the absence of large shareholders, provided that there exist active arbitragers in a properly functioning financial market. To illustrate the point, the focus is on the contest for corporate control, in which arbitragers, who buy large amounts of shares after a takeover announcement, become temporary large shareholders of the firm. It is shown that the presence of these temporary large shareholders makes otherwise impossible takeovers successful. The model also explains a range of widely observed phenomena in the contest for corporate control, including the relationship between jumps in share price and those in trading volume. The optimal takeover premium is shown to be a fixed portion of the value improvement which the takeover can realize. This division rule implies that the market for corporate control can be efficient, thanks to the participation of arbitragers. The importance of financial markets and arbitragers for corporate control is emphasized in the context of large scale privatization.

JEL Classification Code: G34

Keywords: Mergers, Corporate Control, Arbitrage

1. Introduction

A tenet of the modern theory of ownership and property rights states that a firm owned by millions of small shareholders is bound to be inefficient, since each owner would rely on other owners to monitor the operation of the firm and in the end no one would exercise his/her control right. A well-known study is Grossman and Hart (1980). They argue that it is difficult to replace a widely recognized bad management of a publicly held firm, because all small shareholders would not sell their shares to the corporate raider in order to benefit from the potential share value increase[2]. Therefore, large shareholders are necessary to maintain effective corporate control, as was argued by Shleifer and Vishny(1986).

In this paper, I will argue that the aforementioned conventional wisdom is not always correct. Specifically, I will show that effective corporate control can be maintained in spite of diffused ownership, provided that there exists a properly functioning financial market with active arbitragers. To illustrate this point, the focus is on the contest for corporate control. After a takeover announcement, arbitragers take positions in the stock of the firm by buying large amounts of shares in the hope of making profits. Once in position, they become temporary large shareholders. Unlike small shareholders, they tend to sell their shares to the raider, since their decision to sell to the raider is crucial to the success of the takeover. Through this mechanism, profitable takeovers become possible and incompetent management can be replaced. Therefore, to a great extent, an efficient financial market with active arbitragers fills the vacuum of corporate control left by diffused owners.

Another aim of this paper is to explain certain empirical patterns in the highly dramatic contest for corporate control. A widely observed phenomenon is that, after

[2]Unless the price of the tender offer is as high as the improved value of the firm, but then no value-improving takeovers will ever be attempted, since the takeover process is not costless for the raider. This is because each shareholder is too small to be individually influential in the outcome of the takeover.

the announcement of the takeover, both the stock price and the transaction volume rise tremendously relative to their pre-announcement levels. Jensen and Ruback (1983) found in a comprehensive survey that the average jump in share price ranges from 17% to 35%[3]. Moreover, numerous case studies revealed that the increased trading volume is largely due to arbitraging activities[4]. Why is there such a great jump in the price of the shares? Why do the arbitragers participate to such a great extent during a takeover? What is the effect of their participation on the outcome of the takeover fight? Given the existence of these arbitragers, how does the raider choose his optimal takeover strategy?

In a two period model, a firm is initially owned by a large number of atomistic small shareholders. At the beginning of period one, a takeover bid is announced by a raider. The bidding price is chosen to maximize the raider's expected takeover surplus. Then the trading of the stock of the firm takes place. Arbitragers place orders for shares among small investors whose demand is given exogenously. The market clearing price incorporates the (incumbent) small shareholders' expectation

[3]They reported that the "abnormal return to target firms of successful tender offers in the month or two surrounding the offer ... ranges from 16.9% to 34.1% and the weighted average ... is 29.1% ". They also found that, for unsuccessful tender offers, the weighted average is 35.2%. It should be pointed out that these estimates came from samples that covered periods before the merger wave of the 1980's. It is suspected that if the same estimation procedures were repeated with the samples from the 1980's, significantly larger statistics would be reported.

[4]See Harvard Business School case 9-282-065: *Note on Hostile Takeover Bid Defense Strategies*, which was originally prepared by Merrill Lynch White Weld Capital Market Group. For specific examples see Harvard Business School (HBS) case 9-285-053: *Gulf Oil Corp—Takeover* , HBS case 9-285-018: *The Diamond Shamrock Tender for Natomas (A)* and D. Commons: *Tender Offer*, which is a description by the ex-CEO of Natomas. On the other hand, it is common knowledge among financial arbitragers that a takeover bid represents one of the best opportunities for them to operate. Not of the least importance is the book by Ivan Boesky, the king of arbitragers in the U.S. in the 1980's, *Merger Mania — Arbitrage: Wall Street's Best Kept Money-making Secret*. In this book, it is clear that trading in takeovers is a major activity of the arbitrager.

of the success rate of the takeover. This expectation is based on their forecast of the outcome of the game in the next period. In the second period when the trading session is closed, a tendering game is played. In this game, each arbitrager decides how many shares to tender to the raider, while all the other small shareholders keep their shares. An equilibrium of this tendering game generates the probability of success of the takeover. An equilibrium of the whole model determines the post-announcement trading price, the trading volume and the success rate of the takeover.

A major implication of this paper is for the privatization movement in the post-socialist transition. The privatization process is likely to generate a very diffused ownership structure for the firm, since the income distribution is very even in these economies. The conclusions in the paper suggests that financial markets and active arbitragers will be perhaps more important in these countries than in established market economies. One can imagine that in these countries, once the few wealthy investors become arbitragers, by moving from one firm to another rather than by staying in one firm, these investors can help monitor more firms and enhance their efficiency.

The analysis based on the model gives answers to the opening questions. The jump in share price is explained by the rational expectation that the takeover can be successful, even though the bidding price is below the improved value of the firm. The very participation of the arbitrager makes such otherwise impossible takeovers profitable for the raider.

The analysis in this paper also generates a sequence of practically observable and/or empirically testable conclusions. It is shown that the higher the bidding price, the higher the success rate of the takeover and consequently the higher the trading price. Also, in most cases, the higher the transaction volume, the higher the success rate and the trading price. Furthermore, it is found that a raider who can greatly improve the value of the target firm would offer a high bid. In fact, the tender offer premium is always a fixed portion of the value improvement. Thus, the market

of corporate control seems to be efficient from the shareholder's point of view.

Intuitively, the reason why financial arbitragers can be helpful to the raider in the takeover is simple. After taking positions in the stock of the firm, the arbitragers are much more willing than the small shareholders to tender their shares to the raider. This is because the arbitragers are now large shareholders. For them, the decision of whether or not to tender significantly influences the success of the takeover. In other words, the arbitragers are very likely to be pivotal to the success of the takeover[5].

But if it is common knowledge that the participation of the arbitragers may make the takeover bid successful, why would small shareholders trade with the arbitragers? In other words, why doesn't a small shareholder just hold on to his shares and wait for the success of the takeover? Or, to ask the same question from a different angle, how can the arbitrager afford to pay a price which is high enough to persuade small shareholders to give up their shares? It seems that the free rider problem changes from no tendering to the raider to no trade with the arbitragers. Here is an example to clarify this point. Assume that there are 100 thousand (100k) shares outstanding and that the share price is \$10 if the takeover fails and \$20 if the takeover is a success. \$18 is the tender offer. Suppose that after the takeover announcement, there is a total demand from the small investors and arbitragers for 60k shares of the firm. Also assume that from their *a priori* knowledge, small shareholders infer that out of this demand of 60k, the probability that at least 50k comes from the arbitragers is 0.5. Supposing that all arbitragers later tender their acquired shares, the probability of the success of the takeover will be 0.5. Therefore, in order for the small shareholders to give up their shares, the trading price must be as high as $0.5 \times 10 + 0.5 \times 20 = 15$.

[5] In addition, these arbitragers usually have high discount rates (because they are highly leveraged); free riding can be costly since, in general, it takes more time for an arbitrager to benefit from the value improvement which the raider brings. Of course, the hidden assumption is that even if the market evaluation of the captured firm goes up instantly after the consummation of the takeover, the arbitrager may not be able to benefit from this price jump quickly since an instant sale of shares by the arbitrager may be interpreted as information-related and drives down the price.

Suppose that $15 is the actual trading price, what is the profit of an arbitrager? If the takeover is indeed successful, he earns $18 - 15 = 3$ (per share)[6] ; if the takeover fails, he gets $10 - 15 = -5$. Clearly, the arbitrager's expected profit is -$1. Therefore, at the outset there should be no trade.

The reason why there exists heavy trade between the arbitragers and small shareholders is that there is a natural information asymmetry between the arbitragers and small shareholders. As a result, the arbitragers' evaluation of the stock is always higher than that of the small shareholders[7]. An arbitrager who places an order for the shares at least knows that there is a minimum of one arbitrager in play — himself[8]. On the other hand, small shareholders cannot be sure of this. Thus, the arbitrager's estimate of the total number of arbitragers in play should always be larger than that of a small shareholder. In other words, that the arbitrager is more optimistic about the success of the takeover.

The model of this paper can be adapted to explain the pre-announcement trade of the stock. Very often, before the actual announcement of a takeover, the share price and trading volume of the target firm already start to increase, because some speculators who possess private information about the chance of the announcement start to take positions. Being potentially large shareholders and trading among small

[6]If the arbitragers' demand actually constitutes more than 50k of the demand, then each of them can retain some shares to later benefit from the $20 price. However, since the tender price is only $18 < 20$, the average profit per share can never be over $20 - 15 = 5$. This guarantees that arbitragers will lose money for sure.

[7]There is another possible basis for trade. If the arbitragers are less risk averse than small shareholders, then the arbitragers' evaluation of the stock is again higher than that of the small shareholders. In other words, there is a risk-bearing mechanism in such trading. This may be a plausible scenario, since the arbitragers, unlike other investors, are known for their willingness to take risks. In fact, in the context of takeovers, they are called risk arbitragers. Although this consideration will strengthen the logic in the model, it is not necessary.

[8]It is very plausible that the arbitrager knows much more than this. In other words, the information asymmetry is much greater than it is argued here. Thus, the explanation can be reinforced.

investors, the speculators play a very similar role to that of arbitragers in the takeover.

Kyle and Vila (1991) created a model in which there is a single speculator who happens to be the potential raider. This speculator/raider manipulates the stock price by his pre-announcement trading. After the trading, he has an option to announce the takeover bid. The crucial difference between their model and a natural extension of the current model is that in the former the raider is the sole speculator who dominates the market. This set-up may not be realistic because the speculators and the raiders are usually different kinds of players in the financial market. In addition, even though a raider would like to trade before the announcement, he can only do so to a limited extent due to financial disclosure requirements[9]. Thus, the raider cannot easily become a dominant player in the market.

Finally, both Holmstrom and Nalebuff (1990) and Bagnoli and Lipman (1988) are concerned with simple tender offer games. Both papers assumed that the total number of shareholders is finite or that not all shareholders are atomistic. Their models and conclusions are similar to certain cases of the tendering game of the current model; i.e., when the total number of arbitragers is perfectly known[10].

Following the introduction, the model is described in detail. Next, section 3 studies the decisions of the arbitragers and the determination of the success rate of the takeover. Section 4 is a discussion about the rational behavior of the raider — such a raider should take the market reaction to his bid into account. Finally, the conclusion summarizes the results of the paper and discusses the implications of these

[9]It is felt that these financial disclosure requirements are necessary in order for the stock market to function properly. In other words, they can be justified in a context greater than that of takeovers.

[10]Of course, this case will not arise in this paper. In the current model, if the total number of arbitragers is known *ex ante* , there would be no trade after the announcement of the takeover.

results for the issue of privatization in the post-socialist transition.

2. The Model

Let S refer to the small shareholder, where the word "small" has the following connotation: a *small* shareholder controls so few shares that he believes that his decision to sell, to retain or to tender his shares will have no effect on either the trading price or the outcome of the takeover. Let R denote the raider and A the arbitrager. To begin with, S controls 100% of the outstanding shares. P_0 is the initial price per share (before the takeover announcement).

The above set-up is exactly the same as that of the model in Grossman and Hart (1980), except for the existence of the arbitrager. In order to accentuate the role of the arbitrager, the current model does not consider situations where there are large shareholders and/or the raider is initially a large shareholder. In such cases, the takeover is much easier and profitable for the raider than in the current set-up.

The timing of the model is the following. At time 0, R announces a cash tender offer of P_T for all shares, conditional on the outcome that more than 50% shares are tendered. If more than 50% shares are tendered, R purchases them all at the price P_T. At time 1, stock trading takes place — A's take positions, hidden among small investors. Finally, at time 2, a tendering game is played. In this game, all shareholders decide how many shares are to be tendered to the raider and consequently, the outcome of the takeover bid is determined.

Assume that both P_0 and P_T are readily observable to all. So is the value improvement per share that R can bring to the firm, ΔP. Naturally, assume

$$P_0 + \Delta P > P_T > P_0$$

That is, the bidding price is between the *status quo* share price and the potential improved value of the share. This implies that the takeover bid, if successful, is profitable to the raider.

In addition, assume that if the takeover bid proves to be a failure, the stock price of the firm goes back to P_0[11]. The implied assumption is that the occurrence of this takeover bid does not change the probability of new takeover bids and their success. The price P_0 should already accommodate such information.

All of R, S and A are assumed to be risk neutral. For the raider, this is purely a technical convenience. For S and A, this is a technical simplification of the assumption that they share the same attitude towards risk[12]. All of them maximize expected-utility.

The arbitragers are assumed to be homogeneous in their positions[13]. That is, should an arbitrager decide to arbitrage in the stock of the target firm, he buys a portion, δ, of the total outstanding shares of the firm. δ is exogenously fixed at a constant value[14]. The justifications are two-fold. On the one hand, there is an upper limit on the number of shares an arbitrager can purchase. This is due to financial disclosure requirements, such as the 13-D schedule filing in the U.S.. Obtaining more shares than δ results in significantly higher transaction costs (such as the loss of speed and flexibility in the disposition of shares). On the other hand, most of the costs associated with arbitraging in the takeover are likely to be fixed cost (like research and seeking bank finance); so that once an arbitrager decides to participate, it pays to place an order as large as is legally convenient.

Let n be the total number of arbitragers who have acquired shares of the firm. n is assumed to be a random variable, which is exogenously given. The rationale for this key assumption is that although the total number of arbitragers in the stock market is known to all, when the announcement of the takeover bid is made, some

[11] If the stock price falls to a different value than P_0, a similar analysis can still be performed.

[12] As the previous footnote indicates, this assumption can be relaxed so that A is no more risk averse than S.

[13] Alternatively, the arbitragers are of different sizes whose distributions are common knowledge. A model with this assumption should not change the major conclusions derived later.

[14] In the U.S., 5% would be a good estimate of δ.

of the arbitragers are engaged in other arbitraging operations. As a result, their financial resources, including their bounded debt capacity, is tied up. Due to the highly secretive nature of the arbitraging business, no one knows exactly how many arbitragers are financially liquid.

It is common knowledge that n follows a distribution $g(.)$. For technical tractability, n will be treated as a real (continuous) number and, likewise, $g(.)$ as a continuous function. There are conceptual justifications for doing so. Appendix I is devoted to a discussion of this issue.

Together with the arbitragers in placing their orders are the small traders, who exist due to external reasons (such as diversification of their investment portfolio)[15]. These small investors do not have access to accurate and timely information about the firm and the stock market. However, knowing that the market is efficient, they are sure that buying at the market price will not incur losses. As a consequence, aggregate demand from the small traders, ω, is random and independent of both the share price and the demand of arbitragers. ω is measured in terms of percentage of the total outstanding shares of the firm. It is common knowledge that ω follows a distribution with the density function of $f(.)$, which is independent of $g(.)$.

Let y be the total demand for the shares of the firm, then

$$y = \omega + n\delta$$

As technical assumptions, both $f(.)$ and $g(.)$ are concave. Since both n and ω are bounded within certain ranges, the above concavity assumption essentially implies the requirement that the distributions are single-peaked and thick-tailed. Also, as will be justified, $h(t) = g(t)f(z - \delta t)$ is assumed to be concave for all z. Of course, $h(.)$ is the probability density of the event: "$n = t$ and $y = z$"; $h(.)$ will appear in numerous calculations in the model. Formally, here are the assumptions.

[15]Also these small investors can be program or package traders whose decision to buy is based on information uncorrelated to the takeover process.

Assumption 1 *A)* $g : [0, \frac{1}{\delta}] \to R_+$, $g''(n) \leq 0$ *for all* n;
B) $f : [0, 1] \to R_+$, $f''(\omega) \leq 0$ *for all* ω.

Assumption 2 *Define* $h(t) = g(t)f(z - \delta t) : [0, \frac{z}{\delta}] \to R_+$, *then* $h''(t) \leq 0 \ \ \forall t$.

Notice that since $h''(t) = g''(t)f(z - \delta t) + \delta^2 g(t)f''(z - \delta t) - 2\delta g'(t)f'(z - \delta t)$, assumption 2 is almost guaranteed by its predecessors, when δ is small and $g'f'$ is bounded from below. In reality, δ is indeed small.

Following Kyle(1985), the trading process of the stock is as follows. First, arbitragers and small traders simultaneously place their orders to a market maker. The market maker, or auctioneer, sets the market price which equilibrates the supply and demand[16].

The market of the stock must be a competitive one, since the sellers are *small* shareholders as described above. That is, the equilibrium price P_1 must be unique and:

$$P_1 = \tau(P_0 + \Delta P) + (1 - \tau)P_0$$

where τ is the probability of success of the takeover bid, *perceived by small shareholders*. In other words, the share price at time 1 must be set at a level such that each small shareholder is indifferent between selling the share right now or wait for the possibility of free riding. Notice that in this pricing mechanism the small investor (buyer) does not lose. His evaluation of the stock should be the same as P_1.

The probability τ is endogenous to the model. It actually depends on the observed transaction volume, i.e. $\tau = \tau(y)$. The reason is that y conveys new information about the number of arbitragers in position and therefore how many shares will be tendered

[16]One may ask the question why the total demand of the arbitrager is exogenous, i.e., not responsive to the trading price. The implicit assumption is that the total number of the arbitrager is limited so that there is no over-entry of arbitragers and no concern of negative *ex ante* arbitrage profit. Therefore, as long as an arbitrager is not tied up in his current project, he is willing to place an order.

in the coming tendering game. Note that there is another probability of success of the takeover, τ^a, which is *calculated by an arbitrager* when he places his order. As argued before, $\tau^a > \tau$. This will be shown later.

After the trading session is closed, the arbitrager gets his shares and observes the transaction volume y. Now that he becomes a large shareholder, he has to make a decision regarding the portion of his shares to be tendered. Let the ratio of the portion be $\gamma \in [0, 1]$. For him, an optimal γ should strike a balance between two opposing forces. When γ is close to 0, it is highly likely that he may become the person who tenders too few shares and the whole takeover fails. In this case, a loss will be incurred (which is $P_0 - P_1$). When, to the contrary, γ is close to 1, it is likely that some of his tendered shares will not actually be needed for the raider to take control. Therefore, he should have tendered less so that he could get $P_0 + \Delta P$ per share for these retained shares. Of course the optimal γ depends on the arbitrager's belief on n and his belief on other arbitragers' γ. In a symmetric equilibrium, all arbitragers tender the same portion γ.

3. Arbitrage and the Success of the Takeover

The tendering game, which follows the trading session at time 1, is played among the arbitragers. Small shareholders stay out of the picture, since in order to free ride, the remaining small shareholders always decide not to tender their shares (by the Grossman-Hart argument). After obtaining the shares, the arbitrager also observes the transaction volume y of the trading session. From y, he updates his belief about n, the total number of arbitragers. Based on this and the belief on the ratios by which other arbitragers decide to tender their shares, the arbitrager decides how many of his total shares should be tendered. At a symmetric equilibrium, all arbitragers decide to tender the same portion of acquired shares. Consequently, the probability that the takeover is successful is determined.

In the following subsection, the *posterior* probabilities that the takeover is suc-

cessful are derived. As was argued before, there are two types of these probabilities: one for the small shareholder and one for the arbitrager. Then the decision of the arbitrager is analyzed. Subsequently, the existence and properties of the equilibrium are established. The equilibrium concept used throughout is that of the Symmetric Bayesian Equilibrium (SBE). In the last subsection, the focus is on the effects of changes of various factors on the success rate of the takeover bid.

Let A_i be the i'th arbitrager who considers tendering γ_i portion of his total shares. A_i believes that all other A's tender a portion γ of their shares[17].

3.1 The Posterior Probabilities of Success of the Takeover

After observing the transaction volume of the trading session, each arbitrager updates his belief as to how many arbitragers are actually in the takeover game. An arbitrager's calculated probability is different from that of a small shareholder.

Let $\tau^a{}_i$ be the *posterior* probability of success of the takeover bid calculated by the arbitrager A_i conditional on the observation of y. Since no small shareholders are expected to tender, when $y < 0.5$, $\tau^a{}_i = 0$. The following derivation assumes $y \geq 0.5$. In this case,

$$\tau^a{}_i = \tau^a{}_i(y, \gamma, \gamma_i, \delta) = Prob\ [\ \gamma(n-1)\delta + \gamma_i\delta \geq 0.5\ \mid y - \delta\ ]$$
$$= Prob\ [\ n - 1 \geq \frac{0.5}{\delta\gamma} - \frac{\gamma_i}{\gamma}\ \mid y - \delta\ ]$$

where $y - \delta$ is his observation of the total transaction volume excluding his own. Clearly, A_i needs to work out the conditional probability distribution of the number of arbitragers other than himself. Let d (.) denote probability density, then

$$d\ (n - 1 = s\ \mid y - \delta\ ) = \frac{d\ (n - 1 = s, \omega + (n - 1)\delta = y - \delta\ )}{d\ (\omega + (n - 1)\delta = y - \delta\ )}$$

[17] Equivalently, one can assume that A_i believes that A_j tenders γ_j. This seemingly more general assumption does not change the following derivation at all, since all that matters is the sum of the shares tendered by other arbitragers. In other words, γ in the formal assumption can be regarded as the average portion of shares tendered.

$$= \frac{d\ (n-1 = s, \omega = y-(s+1)\delta\ )}{d\ [\omega + (n-1)\delta = y - \delta\ ]}$$

where, $d(.)$ indicates probability density.

Under the assumption that, *ex ante*, n and ω are independent, it follows[18]

$$d\ (n-1 = s, \omega = y-(s+1)\delta) = g(s+1)f(y-(s+1)\delta)$$

and

$$d\ [\omega + (n-1)\delta = y-\delta\ ] = \int_0^{\frac{y-\delta}{\delta}} g(t+1)f[y-(t+1)\delta]dt$$

$$= \int_1^{\frac{y}{\delta}} g(t)f(y-t\delta)dt$$

Therefore, we have

$$= Prob\ [\ n-1 \geq \frac{0.5}{\delta\gamma} - \frac{\gamma_i}{\gamma}\ \mid y-\delta\ ] = \int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i}{\gamma}}^{\frac{y-\delta}{\delta}} g(s+1)f[y-\delta-s\delta]ds$$

and consequently,

$$\tau^a{}_i = \frac{\int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i}{\gamma}}^{\frac{y-\delta}{\delta}} g(s+1)f[y-\delta-s\delta]ds}{\int_1^{\frac{y}{\delta}} g(t)f(y-t\delta)dt}$$

$$= \frac{\int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i-\gamma}{\gamma}}^{\frac{y}{\delta}} g(s)f(y-s\delta)ds}{\int_1^{\frac{y}{\delta}} g(t)f(y-t\delta)dt} \tag{3.1}$$

As for a small shareholder and the raider, there is no basis for the belief that there is at least one arbitrager in the game. This is because they only observe the transaction volume y, which may solely come from small traders. If one repeats the above calculation, then the *posterior* probability of success of the takeover (conditional on y) for both the small shareholder and the raider, τ, is

$$\tau = \frac{\int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i-\gamma}{\gamma}}^{\frac{y}{\delta}} g(s)f(y-s\delta)ds}{\int_0^{\frac{y}{\delta}} g(t)f(y-t\delta)dt} \tag{3.2}$$

[18]To be rigorous, the following derivation should start with the event "{ $s-\epsilon_1 < n-1 < s+\epsilon_1 \mid y-\delta-\epsilon_2 < \omega+(n-1)\delta < y-\delta+\epsilon_2$ } " before taking limits with both ϵ_1 and ϵ_2 going to zero. It can be verified that this rigorous approach will yield exactly the same results, since all the density functions are smooth. Therefore, for ease of exposition, the intuitive approach is presented here.

It is clear by comparing (3.1) and (3.2), that

$$\tau^a{}_i \geq \tau.$$

This verifies the afore-mentioned intuition behind the trade between the small shareholder and the arbitrager.

3.2 An arbitrager's Tendering Decision

For arbitrager A_i, if the takeover proves to be successful, the averaged price per share is $P_T\gamma_i + (P_0 + \Delta P)(1-\gamma_i)$; if the takeover fails, the price is P_0. Clearly, his problem is

$$\delta P_i^*(y,\delta) = \delta\ MAX_{\gamma_i}\ [P_T\gamma_i + (P_0+\Delta P)(1-\gamma_i)]\ \tau^a{}_i + P_0(1-\tau^a{}_i)$$

or

$$\delta P_i^*(y,\delta) =\ MAX_{\gamma_i}\ \delta\ [\Delta P - \gamma_i(P_0 + \Delta P - P_T)]\ \tau^a{}_i + P_0$$

where $\gamma_i \in [0,1]$ and $\tau^a{}_i$ is the probability of success of the takeover.

Define

$$P_{\tau_i} = [\Delta P - \gamma_i(P_0 + \Delta P - P_T)]\tau^a{}_i \tag{3.3}$$

Then A_i is actually maximizing P_{τ_i}. In P_{τ_i}, $\tau^a{}_i$ is A_i's inferred probability that the takeover is going to be successful.

From (3.3) the first order derivative of P_{τ_i} is

$$\frac{\partial P_{\tau_i}}{\partial \gamma_i} = -(P_0 + \Delta P - P_T)\tau^a{}_i + [\Delta P - \gamma_i(P_0 + \Delta P - P_T)]\frac{\partial \tau^a{}_i}{\partial \gamma_i}$$

As it stands, the above expression makes intuitive sense. For an increase in γ_i, the first term represents the loss incurred by missed opportunity of free riding. For the same increase in γ_i, the second term stands for the marginal gain in payoff due to a marginal increase in the probability that the takeover is successful. Rearranging terms, one has

$$\frac{\partial P_{\tau_i}}{\partial \gamma_i} = (P_0 + \Delta P - P_T)\ [\frac{\Delta P}{P_0 + \Delta P - P_T} - \gamma_i - \frac{\tau^a{}_i}{\frac{\partial \tau^a{}_i}{\partial \gamma_i}}\ ]\ \frac{\partial \tau^a{}_i}{\partial \gamma_i}.$$

Define

$$c \equiv \frac{\Delta P}{P_0 + \Delta P - P_T}$$

$$q_i = q_i(y, \gamma, \gamma_i, \delta) \equiv \frac{\tau^a{}_i}{\frac{\partial \tau^a{}_i}{\partial \gamma_i}}$$

and

$$\Psi_i = c - \gamma_i - q_i(y, \gamma, \gamma_i, \delta) \tag{3.4}$$

Equation (3.4) is a crucial one in characterizing the behavior of the arbitragers. Later, it will be utilized periodically. Also note that $c > 1$, since by assumption:

$$0 \leq P_0 + \Delta P - P_T < \Delta P$$

From (3.1), the partial derivative of $\tau^a{}_i$ with respect to γ_i is:

$$\frac{\partial \tau^a{}_i}{\partial \gamma_i} = \frac{1}{\gamma} \frac{g(\frac{0.5}{\delta\gamma} - \frac{\gamma_i - \gamma}{\gamma}) f(y - \frac{0.5}{\gamma} + \frac{\gamma_i - \gamma}{\gamma}\delta)}{\int_1^{\frac{y}{\delta}} g(t) f(y - t\delta) dt}$$

and so q_i is actually

$$q_i(y, \gamma, \gamma_i, \delta) = \gamma \frac{\int_{\frac{0.5}{\delta\gamma} - \frac{\gamma_i - \gamma}{\gamma}}^{\frac{y}{\delta}} g(s) f(y - s\delta) ds}{g(\frac{0.5}{\delta\gamma} - \frac{\gamma_i - \gamma}{\gamma}) f(y - \frac{0.5}{\gamma} + \frac{\gamma_i - \gamma}{\gamma}\delta)} \tag{3.5}$$

The optimal γ_i of A_i is a reaction to other arbitragers' γ. The reaction function $\gamma_i = \gamma_i(\gamma)$ is such that:

$$\Psi_i(y, \gamma, 0, \delta) \leq 0 \quad if \ \gamma_i(\gamma) = 0$$

$$\Psi_i(y, \gamma, \gamma_i, \delta) = 0 \quad if \ 0 < \gamma_i(\gamma) < 1$$

$$\Psi_i(y, \gamma, 1, \delta) \geq 0 \quad if \ \gamma_i(\gamma) = 1$$

Apparently, the function q_i plays a crucial role in shaping the reaction function and, later, the equilibrium. The intuition behind q_i is that it is inversely related to the possibility that A_i's tendering is pivotal to the success of the takeover. Notice that q_i and $\tau^a{}_i$ are transformations of each other. Actually, in the case that $q_i(.)$ is continuous, it is not difficult to see that

$$\tau^a{}_i(y, \gamma, \gamma_i, \delta) = e^{-\int_{\gamma_i}^1 \frac{1}{q_i(y,\gamma,t,\delta)} dt}$$

The following lemma gives some properties of the function q_i. They are needed in order to characterize the properties of the reaction function $\gamma_i = \gamma_i(y, \gamma, \delta)$.

Lemma 1 *Under assumption 1,*

$$\frac{\partial q_i}{\partial y} > 0 \quad \frac{\partial q_i}{\partial \gamma_i} > 0$$

Proof: See Appendix II.

The proof of the above lemma is purely technical and thus omitted here. The explanation of this lemma is that when the transaction volume increases, an arbitrager's tendering is less likely to be pivotal; also when an arbitrager tenders more and more shares, it is less and less likely that his tendering is pivotal. With this lemma, the properties of the reaction function are simple and intuitive.

Proposition 1 *The reaction function $\gamma_i = \gamma_i(y, \gamma, \delta)$ is unique and non-increasing in y. Furthermore, define $\underline{y_i}$: max $\{ 0.5, \ y_1 : \Psi_i(y_1, \gamma, 1, \delta) = 0 \}$ and $\overline{y_i}$: min $\{1, y_2 : \Psi_i(y_2, \gamma, 0, \delta) = 0 \}$. Then,*

$$\gamma_i = 1, \quad \forall \ y \leq \underline{y_i}$$

$$0 < \gamma_i < 1, \quad when \ \underline{y_i} < y < \overline{y_i}$$

$$\gamma_i = 0, \quad \forall \ y \geq \overline{y_i}$$

Proof: Given the lemma, $\frac{\partial \Psi_i(\gamma_i, \gamma, y)}{\partial y} = -\frac{\partial q_i}{\partial y} < 0$; also $\frac{\partial \Psi_i(y, \gamma, \gamma_i, \delta)}{\partial \gamma_i} = -1 - \frac{\partial q_i}{\partial \gamma_i} < 0$.

Referring to the definition of $\underline{y_i}$, if $y \leq \underline{y_i}$, $\forall \gamma_i < 1$,

$$\Psi_i(y, \gamma, \gamma_i, \delta) > \Psi_i(\underline{y}, \gamma, 1, \delta) = 0$$

Recall that Ψ_i shares its sign with $\frac{\partial P_{\tau_i}}{\partial \gamma_i}$. Therefore, the best reaction to any γ is $\gamma_i = 1$ when $y \leq \underline{y}$.

The proof for the other cases are similar, and hence omitted here.

As for the uniqueness and non-increasingness of γ_i in y, it suffices to consider the case $\underline{y} < y < \overline{y}$. In this case, $\underline{y} < y < \overline{y}$, $\Psi_i(y, \gamma, 1, \delta) < 0 < \Psi_i(y, \gamma, 0, \delta)$. Coupled with the fact that Ψ_i is strictly decreasing in γ_i, the existence and uniqueness of γ_i is guaranteed, and

$$\Psi_i(y,\gamma,\gamma_i(y,\gamma,\delta),\delta) = 0 \tag{3.6}$$

Taking partial derivatives with respect to y for all terms in (3.6):

$$-\frac{\partial \gamma_i}{\partial y} - \frac{\partial q_i}{\partial y} - \frac{\partial q_i}{\partial \gamma_i}\frac{\partial \gamma_i}{\partial y} = 0$$

gives

$$\frac{\partial \gamma_i}{\partial y} = \frac{-q_{i_y}'}{1+q_{i_{\gamma_i}}'} < 0 \tag{3.7}$$

¶Q.E.D.

The above characterization of the reaction function is very intuitive. All it says is that given the portion of tendered shares for other arbitragers, the higher the level of transaction volume, the more arbitragers will be inferred by A_i, and therefore fewer shares will be tendered by him in order for him to take the opportunity to free ride.

3.3 The Existence and Properties of Symmetric Equilibria of the Tendering Game

At a symmetric Bayesian equilibrium (SBE), it must be true that $\gamma_i(\gamma) = \gamma$. In search for such equilibria, define $\Psi(y,\gamma,\delta) = \Psi_i(y,\gamma,\gamma,\delta)$. As will be seen, Ψ is crucial in determining the existence of the symmetric equilibrium of the tendering game. To be specific,

$$\Psi(y,\gamma,\delta) = c-\gamma-\gamma\frac{\int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i-\gamma}{\gamma}}^{\frac{y}{\delta}} g(s)f(y-s\delta)ds}{g(\frac{0.5}{\delta\gamma}-\frac{\gamma_i-\gamma}{\gamma})f(y-\frac{0.5}{\gamma}+\frac{\gamma_i-\gamma}{\gamma}\delta)}\Big|_{\gamma_i=\gamma}$$

$$= c-\gamma-\gamma\frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s)f(y-s\delta)ds}{g(\frac{0.5}{\delta\gamma})f(y-\frac{0.5}{\gamma})} = c-\gamma-q(y,\gamma,\delta) \tag{3.8}$$

where $q(y,\gamma,\delta)$ is defined as

$$q(y,\gamma,\delta) = \gamma\frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s)f(y-s\delta)ds}{g(\frac{0.5}{\delta\gamma})f(y-\frac{0.5}{\gamma})} \tag{3.9}$$

And finally, define $\tau^a = \tau^a{}_i(y,\gamma,\gamma,\delta)$, i.e.

$$\tau^a = \frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s)f(y-s\delta)ds}{\int_1^{\frac{y}{\delta}} g(t)f(y-t\delta)dt} \tag{3.10}$$

The following lemma is needed in the proof of Proposition 2 which characterizes the SBE of the game. Again, the proof of this lemma is too technical to warrant presentation in the main text.

Lemma 2 *Under assumption 1,*

$$\frac{\partial q}{\partial y} > 0 \quad \frac{\partial q}{\partial \gamma} > 0$$

$$\frac{\partial \tau}{\partial y} > 0 \quad \frac{\partial \tau}{\partial \gamma} > 0$$

and

$$\frac{\partial \tau^a}{\partial y} > 0 \quad \frac{\partial \tau^a}{\partial \gamma} > 0$$

Proof: See Appendix III.

In the above lemma, $\frac{\partial q}{\partial \gamma} > 0$ states that as the tendering portion goes up, the chance of each of them being pivotal goes down. This is perfectly intuitive — the same is true with $\frac{\partial q}{\partial y} > 0$: when all arbitragers tender more shares, the probabilities of success of the takeover should go up. Thus, $\frac{\partial \tau}{\partial \gamma} > 0$ and $\frac{\partial \tau^a}{\partial \gamma} > 0$. However, caution should be exercised when interpreting the inequalities $\frac{\partial \tau}{\partial y} > 0$ and $\frac{\partial \tau^a}{\partial y} > 0$, because the implicit condition behind them is that γ is held constant. In fact, γ may decrease when y increases, as is shown in the following proposition.

Proposition 2 *Given any y, there exists a symmetric Bayesian equilibrium $\gamma = \gamma(y)$, which is non-increasing in y* [19]. *In other words, when the transaction volume goes up, the arbitrager tends to free ride. Furthermore, define $\underline{y}$: $Max\{\ y_1$:*

[19]There exist other symmetric Bayesian equilibria, in which $\gamma_i = \gamma < \frac{0.5-\delta}{y-\delta}$. This is easy to see, since if all other arbitrager tender $\gamma < \frac{0.5-\delta}{y-\delta}$, then even if arbitrager i tenders all of his shares, the number of shares tendered will be $\delta + (n-1)\delta\gamma < \delta + (\frac{y}{\delta}-1)\delta\frac{0.5-\delta}{y-\delta} < \delta + 0.5 - \delta = 0.5$. Thus, the

$\Psi(y_1, 1, \delta) \geq 0$ }, *then*

$$\gamma = 1, \quad \forall\, y \leq \underline{y}$$

$$0 < \gamma < 1, \; \Psi(y, \gamma, \delta) = 0, \quad \textit{when } y > \underline{y}.$$

Proof: Notice that from the equation:

$$c - 1 - \frac{\int_{\frac{0.5}{\delta}}^{\frac{y_1}{\delta}} g(t) f(y_1 - t\delta) dt}{g(\frac{0.5}{\delta}) f(y_1 - 0.5)} \geq 0$$

it is clear that $y_1 > 0.5$ (since $c > 1$) and so $\underline{y} > 0.5$. By the definition given by (3.8)

$$\Psi(y, \gamma, \delta) = c - \gamma - q(y, \gamma, \delta)$$

and recall that Ψ shares sign with $\frac{\partial P_{r_i}}{\partial \gamma_i}$ and

$$\frac{\partial \Psi}{\partial y} = -q'_y < 0$$

$$\frac{\partial \Psi}{\partial \gamma} = -1 - q'_\gamma < 0$$

Also notice that $\Psi(y, 0, \delta) = c > 0$. When γ goes from 0 to 1, Ψ should travel from c to $\Psi(y, 1, \delta)$ in a strictly decreasing fashion. If $\Psi(y, 1, \delta) < 0$, then there is a unique equilibrating γ to make $\Psi(y, \gamma, \delta) = 0$; no other γ qualifies as an equilibrium. If $\Psi(y, 1, \delta) \geq 0$, then $\gamma = 1$ is the equilibrium, since in this case all arbitragers want to tender more, but $\gamma = 1$ is the upper limit. In this case, no $\gamma < 1$ can be another equilibrium. Thus, we have the existence and uniqueness of the equilibrium.

As for the non-increasingness of the equilibrium, take partial derivatives with respect to y at both sides of the equation $\Psi(\gamma(y, \delta), y) = 0$:

$$-\frac{\partial \gamma}{\partial y} - q'_y + q'_\gamma \frac{\partial \gamma}{\partial y} = 0$$

outcome of the game is always the same, that is, the takeover fails. Therefore, arbitrager i might as well tender the same proportion γ. Of course, such equilibria are highly unrealistic and can be easily ruled out. A rational arbitrager would never tender as low as the above γ, since such γ can only lead to the failure of the takeover which is not in the interest of the arbitragers at all. I owe this point to Bart Lipman of the Queen's University.

and therefore:

$$\frac{\partial \gamma}{\partial y} = \frac{-q'_y}{1 + q'_\gamma} < 0$$

¶Q.E.D.

3.4 The Determination of Success of the Takeover Bid

As illustrated in the last subsection, the arbitrager's decision of how many shares to tender is a result of balancing two opposite forces: the tendency to tender more in order to increase the probability of success of the takeover *versus* the tendency to tender less in order to get more private return by free riding. Likewise, any changes in the observed transaction volume y will have two opposite gross effects on the predicted probability of success of the takeover. On the one hand, when y increases, more arbitragers are likely to be in position and this promises a greater chance of success of the takeover. On the other hand, when y increases, each arbitrager's tendering is less pivotal to the success of the takeover; the arbitrager reacts by tendering less. In general, the net effect of an increase in the observed transaction amount on the probability of success of the takeover can be either positive or negative.

To express the above idea clearly, notice that $\tau = \tau(y, \gamma)$ and therefore

$$\frac{d\tau}{dy} = \frac{\partial \tau}{\partial y} + \frac{\partial \tau}{\partial \gamma}\frac{d\gamma}{dy}$$

As was shown in lemma 2, the direct effect of an increase in y on τ, $\frac{\partial \tau}{\partial y}$, is positive, and so is that of an increase in γ. From proposition 2, an increase in y will decrease γ. Therefore the net effect of an increase in y on τ cannot be easily determined.

One obvious and very important case is when $y < \underline{y}$. In this case, by proposition 2, there is no indirect effect on τ, since γ is always equal to 1. This is important because it seems to be the most likely situation. In reality, it is not common to see arbitragers trying to free ride.

The following proposition answers the question of whether a higher level of transaction volume in the trading session bodes for a high probability of success of the takeover.

Proposition 3 *Within the range $y < \underline{y}$, an increase in y will increase the predicted probabilities of success of the takeover bid; it will also increase the post-announcement share price. The probabilities are calculated by the arbitrager and by the raider or small shareholder. When $y \geq \underline{y}$, the effect of an increase in y on the probabilities can be either increasing or decreasing. In other words,*

$$\frac{d\tau}{dy} > 0, \quad \frac{d\tau^a}{dy} > 0, \quad and \quad \frac{dP_1}{dy} > 0, \quad for\ y < \underline{y}$$

where P_1 is the trading price of trading session.

Proof: (trivial, omitted)

It is interesting to note that $\underline{y}$ in the last two propositions is actually a function of P_T, since c in (4-8) is a function of P_T. To be specific, recall that the definition of $\underline{y}$, when it is less than 1, is actually:

$$c - 1 - \frac{\int_{\frac{0.5}{\delta}}^{\underline{y}/\delta} g f\, dt}{g(\frac{0.5}{\delta}) f(y - 0.5)} = 0$$

By lemma 2, it is easy to see that $\frac{\int_{\frac{0.5}{\delta}}^{\underline{y}/\delta} g f\, dt}{g(\frac{0.5}{\delta}) f(y-0.5)}$ is increasing in $\underline{y}$. Therefore, it is clear that an increase in c caused by an increase in P_T (see the proof of the next proposition for this point) will not decrease $\underline{y}$. Hence,

Corollary 1 *Any increase in the tender offer price P_T makes it more likely that an increase in the observed transaction volume y bodes an increased probability of success of the takeover bid. In other words, if $P_T^1 > P_T^2$, then $\frac{d\tau^2}{dy} > 0$ implies $\frac{d\tau^1}{dy} > 0$, and $\frac{d\tau^{a2}}{dy} > 0$ implies $\frac{d\tau^{a1}}{dy} > 0$.* ¶

Unlike the change in the observed transaction volume, an increase in the bidding price has an unambiguous positive effect on the probability of success of the takeover. This is very intuitive, since an increase in the tender offer will definitely lure arbitragers away from free riding.

Proposition 4 *At any level of the observed transaction volume y, the higher the bidding price, the higher the predicted probabilities of success of the takeover. In other words,*

$$\frac{d\tau}{dP_T} \geq 0 \quad and \quad \frac{d\tau^a}{dP_T} \geq 0$$

Proof: First of all, by the definition of c,

$$c' = \frac{dc}{dP_T} = \frac{\Delta P}{(\ P_0 + \Delta P - P_T\ )^2} > 0$$

In other words, an increase in P_T will have the same effect as an increase in c.

To the raider and small shareholders, the probability of takeover conditional on y is

$$\tau = \frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s) f(y - s\delta) ds}{\int_0^{\frac{y}{\delta}} g(t) f(y - t\delta) dt} \tag{3.10}$$

Therefore

$$\frac{d\tau}{dP_T} = \frac{\frac{0.5}{\delta\gamma^2} g(\frac{0.5}{\delta\gamma}) f(y - \frac{0.5}{\gamma})}{\int_0^{\frac{y}{\delta}} g(t) f(y - \delta t)} \frac{d\gamma}{dP_T} \tag{3.12}$$

As for $\frac{d\gamma}{dP_T}$, it is either equal to 0, when $y < \underline{y}$, or from the equilibrium condition

$$c - \gamma - q(y, \gamma, \delta) = 0$$

we have

$$\frac{d\gamma}{dP_T} = \frac{c'}{1 + q'_\gamma(y, \gamma, \delta)} > 0$$

Therefore it is true that $\frac{d\tau}{dP_T} > 0$.

The same procedure carries over to the proof of $\frac{d\tau^a}{dP_T} > 0$, and is omitted here. ¶ Q.E.D.

Proposition 4 implies that a higher bidding price will trigger a bigger price jump in the post-announcement trading session, since the price jump is positively dependent on the probability of success of the takeover. Therefore the following corollary must be true.

Corollary 2 *At any level of the observed transaction volume, the higher the bidding price, the higher the trading price after the takeover announcement. That is*

$$\frac{dP_1}{dP_T} > 0 \qquad \P$$

As a matter of fact, the proposition 4 also implies that a higher takeover bid will make the *ex ante* probability of success of the takeover higher. That is, without observing the transaction volume, the raider can confidently increase the success rate of his takeover bid by offering a higher bid. For the raider, this is really a very favorable situation which is caused by the participation of the arbitragers. The reason is that without these arbitragers, the situation becomes the one described by Grossman and Hart (1980). In their model, any increase in the bidding price is useless in improving the chance of takeover, unless the bidding price is already high enough so that the success rate of the takeover is 100%. But in such a bid, there is no surplus left for the raider. In short, without arbitragers, the success rate is either 0 or 1. In either case, the raider gets zero gross surplus and cannot recover his takeover cost.

Corollary 3 *Let T be the* <u>*ex ante*</u> *probability of success of the takeover bid, calculated by the raider before the beginning of the trading session (i.e. unconditional on y), then* $\frac{dT}{dP_T} \geq 0$.

Proof: Let $f_y(.)$ be the distribution of the transaction volume, and τ, as before, is the probability of takeover conditional on y. Clearly,

$$T = \int_0^1 \tau(t, P_T) f_y(t) dt$$

and therefore by proposition 4,

$$\frac{dT}{dP_T} = \int_0^1 \frac{\partial \tau(t, P_T)}{\partial P_T} f_y(t) dt \geq 0$$

¶Q.E.D.

4. The Raider's Optimal Bidding Strategy

In this section, the decision of a rational corporate raider is studied. Specifically,

the focus is on the the optimal bidding price of the raider. Obviously, in making this decision, the raider should take the existence of the arbitrager into account. The basic model above can be regarded as a game which is subsequent to the raider's decision.

In his search for the optimal bidding price, the raider faces a fundamental trade-off between the success rate of the takeover and the surplus from the raid. That is, when the raider offers a high tender offer price, the success rate of the takeover will be relatively high; however, at such a bidding price, the surplus per share is also relatively small. A rational raider would strike a balance between these two forces.

The raider maximizes his expected net profit by choosing a best bidding price. With the assumption that the raider is risk-neutral and that there is no cost to the bidding process (alternatively, all costs are sunk), his problem is

$$MAX_{P_T} \quad (P_0 + \Delta P - P_T) E[N(P_T)] \tag{4.1}$$

where $P_0 + \Delta P - P_T$ is the surplus per share to the raider from a successful raid. $E[N(P_T)]$ is the expected number of tendered shares, conditional on the event that it is greater than 50%. For a given transaction volume y, the number of shares tendered by each arbitrager is $\delta\gamma(y)$. Let t be the number of arbitragers; then the total shares tendered is $t\delta\gamma(y)$. Recall that for the raider, the distribution density of t conditional on y is

$$\frac{g(t) f(y - \delta t)}{\int_0^{\frac{y}{\delta}} g(s) f(y - \delta s) ds}$$

Thus, the expected number of shares tendered given y is

$$E[n_T(y)] = \frac{\int_{\frac{0.5}{\delta\gamma(y)}}^{\frac{y}{\delta}} s\delta\gamma(y) g(s) f(y - \delta s) ds}{\int_0^{\frac{y}{\delta}} g(s) f(y - \delta s) ds}$$

and the *ex ante* expectation of the total number of shares tendered (unconditional on y) is

$$E[N(P_T)] = \int_0^1 E[n_T(y)] f_y(y) dy = \int_0^1 \frac{\int_{\frac{0.5}{\delta\gamma(y)}}^{\frac{y}{\delta}} s\delta\gamma(y) g(s) f(y - \delta s) ds}{\int_0^{\frac{y}{\delta}} g(s) f(y - \delta s) ds} f_y(y) dy$$

But, as was derived above

$$f_y(y) = \int_0^{\frac{1}{\delta}} g(t)f(y-\delta t)dt$$

therefore

$$E[N(P_T)] = \int_0^1 dy \int_{\frac{0.5}{\delta\gamma(y)}}^{\frac{1}{\delta}} s\delta\gamma(y)g(s)f(y-\delta s)ds \tag{4.2}$$

and notice that in (4.2) $\gamma(y)$ is also a function of P_T.

In summary, in a conditional tender offer, the raider chooses the optimal bidding price by considering:

$$MAX_{P_T}\ (P_0 + \Delta P - P_T) \int_0^1 dy \int_{\frac{0.5}{\delta\gamma(y)}}^{\frac{1}{\delta}} s\delta\gamma(y)g(s)f(y-\delta s)ds \tag{4.3}$$

One of the important questions of concern to the shareholder is: would an increase in the value improvement ΔP make the raider bid a higher price for the control of the firm? Or, is the bidding price P_T positively related to the value improvement that a raider can bring to the firm? The answer to this question is important, because if a high value-improver bids lower than a low value-improver, then it is possible that under certain circumstances, the firm may end up in the hands of the low value improver.

The above issue is also of great interest to students of the contest for corporate control. Typically, empirical researchers in corporate finance use the bidding price to measure the potential improvement that the raider can bring to the firm. The reason seems to be that there are no better and easier proxies for this incremental in share price[20]. Therefore, if the tender offer price by an optimizing raider is not positively correlated with the value improvement, then the common practice in empirical research is problematic.

Fortunately, the undesirable situation where an increase in ΔP reduces P_T does not happen. As a matter of fact, in the current set-up where the value-improvement is commonly known, the raider's optimal pricing strategy is very simple.

[20]The difficulty associated with using the post-takeover share price appears to be that it has to rely heavily on the theory of efficient capital markets.

Proposition 5 *In a conditional tender offer, the higher the value improvement that the raider is able to bring to the firm, the higher the raider's bidding price. Specifically, the takeover premium offered by the raider, $P_T - P_0$, is proportional to the value improvement ΔP, i.e.,*

$$P_T - P_0 = \alpha\Delta P$$

where, $0 < \alpha < 1$ and it is independent of ΔP. Moreover, the higher the value improvement, the higher the raider's expected profit from the takeover attempt.

Proof: From (4.1), the expected profit to the raider when he offers a bidding price of P_T is

$$(P_0 + \Delta P - P_T\ )E[N(P_T)]$$

Define

$$\alpha = \frac{P_T - P_0}{\Delta P}$$

In general, given ΔP and P_0, α is an one-to-one function of P_T. Therefore, the raider's choice of P_T can be regarded as that of α. This way, the raider's profit is

$$\Delta P(1-\alpha)E[N(\alpha)] \tag{4.4}$$

Obviously, in order to prove the assertions about the raider's optimal choice of P_T, one needs only to prove that in the above expression, the term $E[N(\alpha)]$ is independent of ΔP. Indeed, this is true. To see why, by (4.2)

$$E[N] = \int_0^1 dy \int_{\frac{0.5}{\delta\gamma(y)}}^{\frac{1}{\delta}} s\delta\gamma(y)g(s)f(y-\delta s)ds \tag{4.2}$$

in which the only place ΔY can possibly play a role is $\gamma(y)$, which is the equilibrium tendering portion by the arbitrager. But from the arbitrager's problem described in the last section, $\gamma(y)$ is the arbitrager's rational choice by comparing the benefit and cost of tendering. The benefit of tendering comes from the increased probability of success of the takeover:

$$B_T = \delta\frac{\partial\tau^a}{\partial\gamma}\ [\ \gamma(P_T - P_0) + (1-\gamma)\Delta P\ ] \tag{4.5}$$

which says that for a unit of increase in the probability of takeover, the benefit comes from the realization of the tender offer and the takeover. γ is the tendered portion which gets a boost in value from P_0 to P_T; $1-\gamma$ is the retained part whose value increases from P_0 to $P_0 + \Delta P$. The cost of tendering is the lost opportunity of free riding:

$$B_F = \delta\tau^*(P_0 + \Delta P - P_T) \tag{4.6}$$

which says that when the takeover proves to be a success the untendered share gets $P_0 + \Delta P$, instead of P_T had it been tendered.

$\gamma(y)$ is chosen so that B_T is equal to B_F ($\gamma(y)$ is 1 when (4.5) is always larger than (4.6)). By Comparing (4.5) with (4.6), it is clear that this decision is solely dependent on α.

Alternatively, from (3.8), $\gamma(y)$ is determined by

$$c - \gamma - q(\gamma) \geq 0$$

(if for all $\gamma \leq 1$ it is > 0, then $\gamma(y) = 1$) where $c = \frac{\Delta}{P_0 + \Delta P - P_T} = \frac{1}{1-\alpha}$. Therefore, one can also see that $\gamma(y)$ is independent of ΔP.

Thus, the first parts of the proposition are proved. As for the last part, it is obvious from (4.2). ¶ Q.E.D.

This proposition actually provides a rule of dividing the takeover surplus between the raider and other parties (the incumbent small shareholder and the arbitrager). It says that the profit of the takeover to the raider is a fixed proportion of the total value improvement that he brings to the firm, while small shareholders and the arbitrager take the rest of the profit.

This division of the surplus of takeover has a certain implication for the efficiency of the market of corporate control. One can regard the raider as a producer of the value improvement of the firm. A variable cost of this production is the bidding price he has to pay in order to obtain the control of the firm. Assuming that the cost of the search for the value improvement is either fixed or sunk, then the raider would always

want to implement as much value improvement as possible. From the shareholder's point of view, this scenario is ideal. Therefore, it seems that with the participation of the arbitrager, the market of corporate control is efficient.

5. Conclusions and Implications

In summary, there are three major conclusions in this paper. First, it is demonstrated that arbitragers can enhance the efficiency of corporate control. Their participation in the contest for corporate control makes otherwise impossible takeover attempts successful. Second, the involvement of the arbitrager explains very well the widely observed stock market movements during the process of the takeover, including the dramatic rise in share price and in transaction volume. It predicts that in most cases, a high transaction volume is associated with a high success rate, as well as with a high post-announcement trading price. Also, the higher the bidding price, the higher the success rate and the bigger the jump in share price. Finally, the existence of the arbitrager implies that in the tender offer, the shareholder can always expect to get a fixed portion of the total value improvement by the raider. This surprisingly simple division rule of the takeover surplus has a lot of implications for related empirical research and indicates that the market for corporate control may be efficient.

A direct implication of these conclusions concerns public opinion about the arbitrager. Criticisms are occasionally launched by the general public against arbitragers on the basis that they are extremely greedy and getting unfairly high profit. However, what most people do not realize is that the arbitrager can contribute to the increase in the value of publicly traded firms.

A much less obvious implication of this paper applies to the current privatization movement in the post-socialist economies. In these economies, due to decades of socialism, wealth distribution is very even. Therefore, the privatization process will almost for sure generate a very diffused ownership structure for the privatized firm.

This is regarded as very undesirable in terms of corporate control and efficiency (see Kornai (1990)). One popular solution is the creation of "holding companies" (for example, Lipton and Sachs (1990) and Blanchard, *et cetera* (1990)). However, the scheme of "holding companies" has potential problems. Economically, the issue of how to discipline and control the management of these "holding companies" is left open. Politically, these "holding companies" may easily become (or be regarded as) the same entities as the old economic ministries under the old regimes. such proposals to win wide-spread support from the general public.

The conclusions in this paper suggest that financial markets with active arbitragers will be very important in mitigating this problem of privatization, since a few arbitragers in the stock market, who are very mobile in transferring from one firm to another, are sufficient to discipline the management of a lot of firms[21]. In other words, one can imagine that only a very few genuinely profit seeking large investors as arbitragers are enough to enhance the efficiency of almost all diffusely owned firms.

Appendix I. Justifications of Treating n as a Real Number

In the text of the paper, the random number of arbitragers n is treated as a real number and its distribution $g(.)$ is regarded as a continuous and smooth function. Although the obvious reason for doing so is technical convenience, there do exist conceptual justifications for this.

The following is an alternative framework for the model. It will be argued that this model will yield the same equilibria, under certain conditions.

Let n only take integer values. Assume that when an arbitrager A_i puts an demand of δ shares of the target firm, his broker may not be able to come up with exactly the

[21] After the privatization, the most likely and, perhaps, appropriate corporate raiders are the former management and bureaucrats since they know the best about the firm.

amount of shares δ. Instead, there is a slight discrepancy between what is demanded and what is obtained. Let the obtained amount of shares be δ_i. *Ex ante*, δ_i follows distribution $f_\delta(.)$, which is supported by an interval $[\delta - \epsilon, \delta + \epsilon]$. These distributions of the size of the arbitragers are independent and identical. Finally, each arbitrager only observes his own realization of δ_i, and cannot update his belief of other arbitragers' δ_j.

Let x be the *total* shares controlled in the hands of arbitragers. In the current set-up, x becomes a real number. Let the *ex ante* distribution of x be $g_x(.)$. Assume that all other arbitragers tender γ portion of their shares, then, using the same notation as before,

$$\tau^a{}_i = Prob\{\gamma_i\delta_i + (x - \delta_i)\gamma > 0.5 \mid y - \delta_i\} = \frac{\int_{\frac{0.5}{\gamma} - \frac{\gamma_i - \gamma}{\gamma}\delta_i}^{y-\delta_i} g_x(s + \delta_i) f(y - \delta_i - s) ds}{\int_0^{y-\delta_i} g_x(t + \delta_i) f(y - \delta_i - t) dt}$$

$$= \frac{\int_{\frac{0.5}{\gamma} - \frac{\gamma_i}{\gamma}\delta_i}^{y} g_x(s) f(y - s) ds}{\int_{\delta_i}^{y} g_x(t) f(y - t) dt}$$

The reaction function of A_i is:

$$\gamma_i = \gamma_i(y, \gamma, \delta_i)$$

In general, this reaction function depends on the realization of δ_i. However, if ϵ is small enough, the following is a good approximation:

$$\gamma_i = \gamma_i(y, \gamma, \delta_i) \approx \gamma_i(y, \gamma, \delta)$$

And the whole game can be approximated by the game where $\delta_i = \delta$ for all i. But when $\delta_i = \delta$, $n = \frac{x}{\delta}$ becomes a real number and we are back to the original game analyzed before.

Alternatively, if the distribution function $g_\delta(.)$ is *highly* concentrated around $\delta_i = \delta$, and that it is costly to calculate the full schedule $\gamma_i = \gamma_i(.,.,\delta_i)$ and if A_i has limited calculation capacity to adjust his reaction function between when δ_i is realized and the tender offer is due, then it is worthwhile for an arbitrager to play the game as if his δ_i is always equal to δ. Again, the game is degenerated to the original one where n is a real number.

The bottom line is that treating n as a real number can be either regarded as an approximation to the normal case where n is an integer or, alternatively, can be justified when all players of the game experience a certain level of bounded rationality.

Appendix II. Proof of Lemma 1:

A) Three Useful Inequalities

Before proving the lemma, let us establish the following facts: under the current assumptions of the model, for any $a < b$,

$$g^2(a) + g'(a)\int_a^b g(t)dt \ > \ 0 \qquad (a-1)$$

$$g(a)\int_a^b g'f \ - \ g'(a)\int_a^b gf < 0 \qquad (a-2)$$

and

$$f(a)\int_a^b gf' \ - \ f'(a)\int_a^b gf > 0 \qquad (a-3)$$

(where $f(t)$ refers to $f(y - \delta t)$)

<u>*Proof*</u>: When $g(.)$ is concave, $g'(a) \geq g'(t)$ for all $t > a$. Therefore

$$g'(a)\int g \ \geq \ \int_a^b g'(t)g(t)dt \qquad (a-4)$$

Note that

$$\int_a^b g'(t)g(t)dt = \int_a^b g(t)dg(t) = g^2(t)\mid_a^b - \int_a^b g(t)dg(t)$$

which gives

$$\int_a^b g'(t)g(t)dt = \frac{g^2(b) - g^2(a)}{2} \qquad (a-5)$$

Putting (a-4) and (a-5) together, one gets (a-1), since

$$g^2(a) + g'(a)\int_a^b g(t)dt \ \geq g^2(a) + \frac{g^2(b) - g^2(a)}{2}$$

$$= \frac{g^2(b) + g^2(a)}{2} \ > \ 0$$

As for (a-2), if $g'(a) < 0$, then $g'(t) < 0$ for all $t > a$ which implies that $g(a) > g(t)$ for all $t > a$. So

$$g(a)\int_a^b f - \int_a^b gf \ > \ 0 \qquad (a-6)$$

By the concavity of $g(.)$ and (a-6),

$$g(a)\int_a^b g'f \ - \ g'(a)\int_a^b gf \ \leq \ g'(a)\ (\ g(a)\int_a^b f - \int_a^b gf\ ) \ < \ 0$$

If $g'(a) \geq 0$, then without losing generality, let t_0 be such that $g'(t_0) = 0$, when $g'(b) < 0$ and $t_0 = b$ when $g'(b) \geq 0$. Clearly

$$g(a)\int_a^b g'f \ - \ g'(a)\int_a^b gf \ \leq \ g(a)\int_a^{t_0} g'f \ - \ g'(a)\int_a^{t_0} gf \qquad (a-7)$$

since both terms on the left-hand-side made concessions to their counterparts in the right-hand-side.

By the concavity of $g(.)$ and (a-7),

$$g(a)\int_a^{t_0} g'f - g'(a)\int_a^{t_0} gf \ \leq \ g'(a)\ (\ g(a)\int_a^{t_0} f - \int_a^{t_0} gf\ ) \ < \ 0$$

since, now that $g'(t) > 0$, for all $a \leq t \leq t_0$, which implies $g(a) > g(t)$, one has

$$g(a)\int_a^{t_0} f - \int_a^{t_0} gf \ < \ 0$$

Therefore in all cases, (a-2) should hold. The proof of (a-3) is almost the same as (a-2), except that $f(.)$ actually is $f(y - \delta t)$ which makes the signs reversed. The proof of (a-3) is omitted to save space.

B) Proof of $\frac{\partial q_i}{\partial y} \geq 0$:

Recall that

$$q_i(y, \gamma, \gamma_i, \delta) = \gamma \frac{\int_{\frac{0.5}{\delta\gamma} - \frac{\gamma_i - \gamma}{\gamma}}^{\frac{y}{\delta}} g(s)f(y - s\delta)ds}{g(\frac{0.5}{\delta\gamma} - \frac{\gamma_i - \gamma}{\gamma})f(y - \frac{0.5}{\gamma} + \frac{\gamma_i - \gamma}{\gamma}\delta)}$$

For notational ease, define

$$a = \frac{0.5}{\delta\gamma} - \frac{\gamma_i - \gamma}{\gamma}$$

and $b = \frac{y}{\delta}$. In addition, write $g(s)$ as g, $f(y - \delta s)$ as f, $g(s)f(y - \delta s)$ as h and $\int_a^b g(t)f(y - \delta t)dt$ as $\int h$.

Thus

$$\frac{\partial q_i}{\partial y} = \gamma \frac{A}{B} \qquad (a-8)$$

where $B = h^2(a)$ and

$$A = (\ \frac{1}{\delta}g(b)f(b) + \int gf'\ )\ g(a)f(a)\ -\ (\int gf)\ g(a)f'(a)$$

$$= \frac{1}{\delta}h(a)h(b) + g(a)\ (\ f(a)\int gf' - f'(a)\int gf\ ) \geq \frac{1}{\delta}h(a)h(b) > 0$$

The last but one step in the above derivations is due to (a-3).

Therefore $\frac{\partial q_i}{\partial y} > 0$

C) Proof of $\frac{\partial q_i}{\partial \gamma_i} > 0$:

$$\frac{\partial q_i}{\partial \gamma_i} = \gamma\frac{C}{B}$$

where B is the same as the one in (a-8), while

$$C = (\ -h^2(a) - h'(a)\int h\ )\ \frac{\partial a}{\partial \gamma_i} = \frac{h^2(a) + h'(a)\int h}{\gamma}$$

By assumption 1, h is concave. Therefore applying (a-1), $C > 0$, which gives

$$\frac{\partial q_i}{\partial \gamma_i} > 0$$

Appendix III. Proof of Lemma 2:

A) Proof of $\frac{\partial q}{\partial y} > 0$ and $\frac{\partial q}{\partial \gamma} > 0$

The validity of $\frac{\partial q}{\partial y} > 0$ is actually already established by the results of lemma 1 — $\frac{\partial q_i}{\partial y} > 0$; since the difference between q_i and q lies between the evaluation of γ_i, which is immaterial to the last inequality.

As for the second inequality, from the definition of (4-9),

$$\frac{\partial q}{\partial \gamma} = \frac{q}{\gamma} + \gamma\frac{-h^2(a) - h'(a)\int h}{h^2(a)}\frac{\partial a}{\partial \gamma} = \frac{q}{\gamma} + \gamma\frac{h^2(a) + h'(a)\int h}{h^2(a)}\frac{0.5}{\delta\gamma^2}$$

Therefore by (a-1) $\frac{\partial q}{\partial \gamma} > 0$.

B) Proof of $\frac{\partial \tau}{\partial y} > 0$ and $\frac{\partial \tau}{\partial \gamma} > 0$

The second inequality is obvious, since

$$\frac{\partial \tau}{\partial \gamma} = \frac{-h(a)}{(\int_0 h)}\frac{\partial a}{\partial \gamma} = \frac{h(a)}{(\int_0 h)^2}\frac{0.5}{\delta\gamma^2} > 0$$

As for the first one, from (4-2),

$$\frac{\partial \tau}{\partial y} = \frac{(\ \frac{1}{\delta}h(\frac{y}{\delta}) + \int_a gf'\ )\ \int_0 h\ -\ (\ \frac{1}{\delta}h(\frac{y}{\delta}) + \int_0 gf'\ )\int_a h}{(\ \int_0 h\ )^2}$$

So, in order to prove $\frac{\partial \tau}{\partial y} > 0$, one wants to establish

$$(\ \frac{1}{\delta}h(\frac{y}{\delta}) + \int_a gf'\ )\ \int_0 h\ -\ (\ \frac{1}{\delta}h(\frac{y}{\delta}) + \int_0 gf'\ )\int_a h > 0$$

But, since

$$\frac{1}{\delta}h(\frac{y}{\delta})\int_0 h\ -\ \frac{1}{\delta}h(\frac{y}{\delta})\int_a h > 0$$

It will suffice, if

$$\int_0 h\int_a gf'\ -\ \int_a h\int_0 gf'\ > 0$$

which will be true if

$$\phi(t) = \frac{\int_t gf'}{\int_t gf}$$

is an increasing function. Indeed, for $\phi'(t)$,

$$\phi'(t) = \frac{-gf'(t)\int_t gf + gf(t)\int_t gf'}{(\int_t h)^2} > 0$$

because of (a-3).

Therefore $\frac{\partial \tau}{\partial y} > 0$ is true.

C) Proof of $\frac{\partial \tau^a}{\partial y} > 0$ and $\frac{\partial \tau^a}{\partial \gamma} > 0$

Since the only difference between τ and τ^a lies in the lower bound of the integration of denominators, i.e. $\int_0 h$ v.s. $\int_1 h$, it is very easy to see that the proof in part B) can be repeated here without any difficulty. Therefore the actual proof is omitted to save space.

Appendix IV. Proof of Lemma 3

A) Proof that $\frac{\partial q}{\partial v} > 0$

Now that

$$q = \gamma\frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s - v)f(y - s\delta)ds}{g(\frac{0.5}{\delta\gamma} - v)f(y - \frac{0.5}{\gamma})}$$

then

$$\frac{\partial q}{\partial v} = \gamma \frac{-\int_a g'f \; gf \,|_a \; + \; \int_a gf \; g'f \,|_a}{(\; gf \,|_a\,)^2}$$

it suffice to prove that

$$-\int_a g'f \; gf \,|_a \; + \; \int_a gf \; g'f \,|_a > 0$$

which is true, indeed, by (a-2).

B) Proof that $\frac{d\gamma}{dv} \leq 0$

This is true when $y < \underline{y}$. When the opposite is true, from

$$c - \gamma - q(y, \gamma, \delta) = 0$$

one gets

$$\frac{d\gamma}{dv} = \frac{-q'_v}{1 + q'_\gamma} < 0$$

from the result of the last part of this appendix.

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