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Goodness-of-Fit in Demand Analysis

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Goodness-of-Fit in Demand Analysis

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Abstract. Revealed preference analysis provides a definitive method to test for optimizing behavior. However, it has been criticized because it fails to allow for approximate satisfaction of optimizing behavior. In this paper, I outline some possible solutions to this problem. These solutions suggest some novel measurements of goodness-of-fit in parametric demand estimation.

Keywords. Nonparametric, demand analysis, revealed preference

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Goodness-of-Fit in Demand Analysis

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There are two approaches to the empirical analysis of consumer choice behavior. Parametric analysis proceeds by postulating a functional form for a utility function, deriving the associated demand equations, and estimating the parameters of the resulting system of equations. The estimates of the parameters can be used to test the maximization hypothesis, forecast demand, or do welfare analysis. Nonparametric analysis uses revealed preference techniques to achieve the same ends.

Samuelson (1938) and Houthakker (1950) were the first to develop the implications of the revealed preference idea for economic theory, but Afriat (1967) was the first to pursue its implications for empirical demand analysis. Subsequently Diewert (1973), Diewert and Parkan (1985), and Varian (1982a), (1982b) extended Afriat's analysis in a number of directions. More recently, several authors such as Browning (1984), Bronars (1987), Deaton (1985) Green and Srivastava (1985), (1986), Houtman and Maks (1987), Landsburg (1981), Manser and McDonald (1988), and Swofford and Whitney (1986) have contributed to nonparametric analysis.

The aspect of nonparametric analysis that I wish to examine in this paper has to do with the goodness-of-fit of the utility maximization model—what does it mean to say that some consumer behavior is "almost" consistent with maximization? To motivate this discussion let us consider the violation of the Generalized Axiom of Revealed Preference depicted in Figure 1. Here we have x^t revealed preferred to x^s and x^s revealed preferred to x^t. However, the size of the violation is not large: a small perturbation of the budget line through either observation would eliminate the problem. Hence we might want to consider this an insignificant violation of the maximization model.

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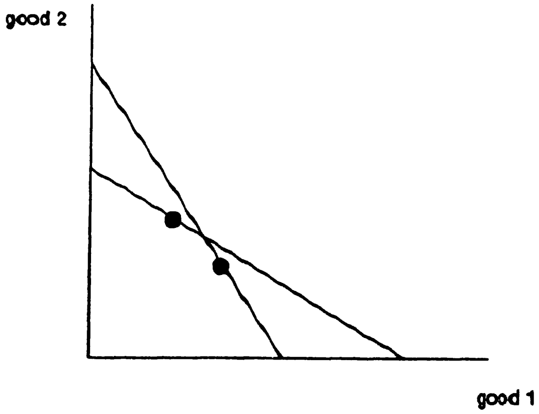

Figure 1. A "small" violation of GARP.

The notion of what is or is not significant implicitly relies on a statistical model of how the data were generated and what are the possible sources of error. In this paper I will investigate several proposed answers to this question. Some of these proposed answers have interesting implications for parametric demand analysis as well which we will consider at the end of the paper.

1. Finding the minimal perturbation

In Varian (1985) I examined how one might formalize the concept of significant violations in the context of measurement error. I will describe this method here in reasonably general terms, since it applies to a wide variety of problems involving testing inequality restrictions.

Let x be a vector of observations of consumer choices. The maintained hypothesis is that

$$x = X + \epsilon,$$

where X is the vector of true values and ϵ is a vector of error terms. For simplicity we take the components of ϵ to be IID Normal, with mean of zero and variance σ^2. The null hypothesis that we wish to test is that X lies in some region H. In the case of revealed preference analysis, H is simply the subset of R^n for which the data satisfy the revealed preference inequalities.

It is important to recognize that in our applications H will typically be a subset with the same dimension as the ambient space. This is in contrast to the standard theory of hypothesis testing in which the null hypothesis usually imposes restrictions of smaller dimensionality.

In Varian (1985) I proposed the following test statistic

$$T = \min_{z_i} \sum_{i=1}^{n} (x_i - z_i)^2/\sigma^2$$
$$\text{such that } z \in H.$$

This statistic is proportional to the (minimum) distance from x to the set H in the Euclidean metric. Intuitively, this number should be "small" if the data were in fact generated by maximization.

Standard hypothesis testing methodology would suggest that we calculate the distribution of this statistic under the null hypothesis and identify the critical region. The problem is how to calculate the sampling distribution of T. It is well-known that the sampling distribution for the optimized value function can be a very complicated function of error terms entering in the constraints.

However, the following simple observation makes at least some calculations possible: under the null hypothesis the distance from the observed data, x, to the true data, X, is *at least* as great as the value T, since under the null hypothesis X is an element of H and T is the minimal distance from x to H. It follows that for any critical value C,

$$\text{Prob}\{T > C\} \leq \text{Prob}\left\{\sum_{i=1}^{n} \frac{\epsilon_i^2}{\sigma^2} > C\right\}.$$

Note that the probability on the right-hand side of this inequality can be calculated; under the null hypothesis, it is simply a χ^2 variable with n degrees of freedom.

Suppose that we choose a test of size α and find the critical value C_α such that $\text{Prob}\left\{\sum_{i=1}^{n} \epsilon_i^2/\sigma^2 > C_\alpha\right\} = \alpha$. Then we know from the above inequality that

$$\text{Prob}\{T > C_\alpha\} \leq \alpha.$$

Hence if we reject the null hypothesis when $T > C_\alpha$ we are certain that the probability of rejecting the null when it is true is less than α. Hence our proposed test has size of at most α.

This proposed test has some advantages and disadvantages. Among its advantages are the following:

- It is a very general approach that can be applied in a large variety of cases.
- It actually identifies a perturbation of the data that satisfies the appropriate revealed preference conditions, and which can be interpreted as a maximum likelihood estimator of the true choices. (On this last point, see Varian (1985).)

Among its *disadvantages* are:

- One need to specify *a priori* the error variances.
- The test may be difficult to compute, even for relatively small data sets.

I find the last objection the most serious. If there are n observations, then finding the minimal perturbation of the data will involve solving a quadratic programming problem with n^2 constraints. This means that problems with more than 50 or so observations will demand significant computer resources for their solution.

Of course the problems under examination typically have a very special structure, and a deeper analysis could provide much improved algorithms. However, this computational problem seems significant enough that it warrants thinking about alternative approaches.

2. The Afriat index

The last section emphasized the idea that the "error term" in the optimization was due to measurement error or other sorts of "observational" problems. In this section I want to examine a different approach to the problem of how to account for violations of maximizing behavior. The approach is based on a notion of "almost maximizing" behavior that was first described by Afriat (1967). Initially I will describe the goodness-of-fit measure without referring to a statistical model of error generation, and then turn to a statistical model.

Afriat's measure is calculated in the following manner. For a given set of numbers (e^t), $t = 1, \ldots, T$, with $0 \leq e^t \leq 1$, define an extension of the standard direct revealed preference relation by

$$x^t \; R_e^0 \; x^s \text{ if and only if } e^t p^t x^t \geq p^t x^s.$$

If $e^t = 1$ this is the standard revealed preference relation; if $e^t = 0$ the relation is vacuous in the sense that observation t cannot be revealed preferred to any other observation. As e^t varies from 1 to 0 the number of observations revealed preferred to other observations monotonically decreases.

The number e^t is known as the *efficiency index*, and it can be thought of as how much less the potential expenditure on a bundle x^s has to be before we will consider it worse than the observed choice x^t. If e^t is .90, for example, we will only count bundles whose cost is less than 90% of an observed choice x^t as being revealed worse than x^t. Said another way: if e^t is .90 and x^s would cost only 5% less than x^t, we would not consider this a significant enough different to conclude that x^t was preferred by the consumer to x^s. We are allowing the consumer a "margin of error" of $(1 - e^t)$.

Given an arbitrary set of data (p^t, x^t), let us choose a set of efficiency indices (e^t) that are as close as possible to 1 in some norm. If the data satisfy the revealed preference conditions exactly, then we can choose $e^t = 1$ for all $t = 1, \ldots, T$. If we choose $e^t = 0$ for all $t = 1, \ldots, T$, then the data vacuously satisfy the revealed preference conditions, since no observation is revealed preferred to any other. Thus for any reasonable norm, there will be some set of (e^t) that are as close as possible to 1 in some norm that will summarize "how close" the observed choices are to maximizing choices.

In Afriat's (1967) original treatment of this idea, he considered choosing a single e that applied to all observations, rather than a different e^t for each observation. We refer to this as a *single index* model as opposed to the *multiple index* model described above. The advantage of Afriat's original proposal is that it is much easier to compute a single index e than the multiple indices (e^t).

Houtman and Maks (1987) suggest the following binary search. Start with $e = 1$ and test for violations of revealed preference using Warshall's algorithm as described in Varian (1982a). If the data fail to satisfy the strong axiom, try $e = 1/2$. If $e = 1/2$ doesn't work, try $e = 1/4$. If $e = 1/2$ *does* work, try $e = 3/4$, and so on. After n revealed preference tests, you are within $1/2^n$ of the actual efficiency index.

Computing the set of efficiency indices that are as close as possible to 1 in some norm is substantially more difficult. If we choose a quadratic norm, for example, we would have so solve a problem such as:

$$E = \min_{(e^t)} \sum_{t=1}^{T} (e^t - 1)^2 \tag{1}$$

subject to the constraint that the revealed preference relation R_e satisfies the Strong Axiom. This approach is significantly more demanding from a computational perspective.

3. The sampling distribution of the Afriat index

Afriat's original definition of the efficiency index was motivated by considerations of the goodness-of-fit of the optimization model. It provides a reasonable measure of how well a given set of data satisfy the optimization hypothesis. But without some specification of the reasons why the data fail to satisfy the optimization hypothesis in the first place, it is hard to know whether we have a significant or an insignificant violation of the model. However, if we formulate a stochastic model of describing how the data were generated, then we can view the Afriat index as a *statistic* and ask the usual sort of questions about the distribution of this statistic.

My preliminary investigations indicated that it is very difficult to say anything of much use analytically, except in very simple cases, so I have proceed to simulate the sampling distribution of the Afriat index under a standard stochastic model. Here I will present some of my initial findings.

I begin by constructing a random set of prices. I then calculate the set of demands implied using a parametric system of demand equations. In the particular set of simulations I describe below I used a CES utility function with parameter ρ. I then added an Normally distributed error term to the demand for each good. Finally, I took these data and calculated the implied Afriat index. I repeated this process a hundred times, and examined the resulting frequency distribution of the Afriat index.

My major concern was how the index varied with respect to the two major unknowns of the problem—the tastes, as measured by the CES parameter ρ, and the variance of the error term. Charts 1 and 2 present typical examples of the sampling distributions for different values of the error terms and CES parameter.

As you can see from the tables, it is quite unusual to observe a value of the Afriat efficiency index less than about .8($\approx$ 13/16) under the null hypotheses used to simulate

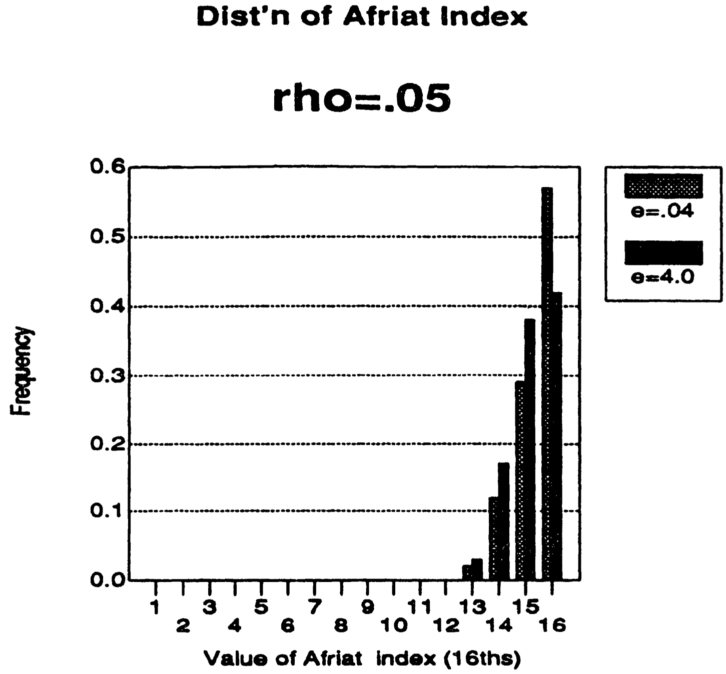

Chart 1. Sampling distribution of the Afriat index ($\rho = .05$).

the sampling distribution. Put somewhat differently: if the stochastic model is correct, then a value of the Afriat index less than .8 is strong evidence against the null hypothesis of utility maximization.

The method of simulating the sampling distribution under the null hypothesis can be extended in several directions. In any *particular* problem it would make sense to condition on the observed prices rather than generate the prices randomly, as I did in my simulations. Furthermore, if a specific alternative hypothesis is available, it makes sense to simulate the sampling distribution under that alternative. The resulting sampling distributions can be used for power calculations, as in Bronars (1987), or for calculating the posterior odds in favor of the null, as in the Bayesian approach to hypothesis testing.

The major difficulty that I have with this approach is that the null hypothesis is not really a nonparametric hypothesis. It is unfortunately a rather sharp parametric hypothesis: the observed choices are a perturbation of a particular CES utility function. I'm not sure exactly how to solve this problem. One could postulate a prior distribution over the parameter ρ and then (numerically) integrate the sampling distribution over this nuisance

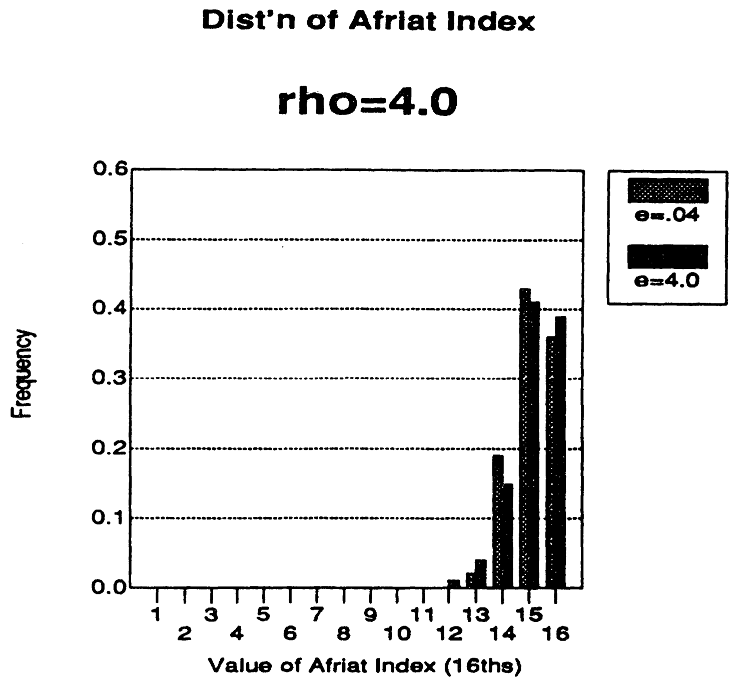

Chart 2. Sampling distribution of the Afriat index ($\rho = 4.0$).

parameter. Although this procedure seems theoretically sound, it appears somewhat *ad hoc* and computational demanding to carry out in practice.

My preferred approach is to estimate the CES parameter ρ and the variance of the error term using standard parametric methods and use these numbers to construct the simulations of the sampling distributions described above. This also suffers from the defect of being somewhat *ad hoc* but at least it seems like a sensible thing to do.

4. A characterization of the efficiency indices

There is a characterization of the set of (e^t) that minimize some norm that will be useful in what follows. In order to describe it, we need some formal definitions.

As above, define the relation R_e^0 by $x^t \; R_e^0 \; x$ iff $e^t p^t x^t \geq p^t x$, and let R_e be the transitive closure of this relation. Then define $GARP_e$ to mean

$$x^s \; R_e \; x^t \text{ implies } e^t p^t x^t \leq p^t x^s.$$

If $e^t = 1$ for all t then this reduces to the standard definition of GARP.

Here is another way to state this definition: if some data (p^t, x^t, e^t) satisfy $GARP_e$, then

$$\text{for all } x^s \; R_e \; x^t \text{ we have } e^t p^t x^t \leq p^t x^s.$$

This statement can be written as

$$e^t \leq \frac{p^t x^s}{p^t x^t} \text{ for all } x^s \; R_e \; x^t.$$

If we attempt to choose a set of (e^t) that are on the average as close as possible to 1, then this inequality will typically be binding for *some* observation s so we have:

$$e^t = \min_{x^s R_e x^t} \frac{p^t x^s}{p^t x^t}. \tag{2}$$

Note that this is not really an "operational" way to determine e^t, since e^t is implicitly involved in the relation R_e. Nevertheless, the characterization is still useful, as we shall see in the next section.

5. Parametric methods

The characterization of (e^t) described in the last section is useful because it can be extended to a novel way to estimate parametric demand systems. Suppose that one is willing to postulate that some observed demand behavior was generated by the maximization of a particular parametric utility function $u(x, \beta)$, where β is a vector of parameters.

Let $\succeq_\beta$ be the preferences generated by the utility function $u(x, \beta)$. Then it is natural to define a parametric generalization of Afriat's efficiency index by

$$i^t = \min_{x \succeq_\beta x^t} \frac{p^t x}{p^t x^t}.$$

All we have done is to replace the partial order R_e by the total order $\succeq_\beta$.

Using some constructs from duality theory allows for an easier statement of this definition. Given a preference relation $\succeq$, the money metric utility function $m(p, x)$ is defined to be

$$m(p, x) = \min_y \; py$$
$$\text{s.t. } y \succeq x.$$

In words, the money metric utility function measures the minimum expenditure at prices p the consumer would need to be as well off as he would be consuming the bundle x. For more on the money metric utility function see Samuelson (1974), King (1982), and Varian (1984). If utility is parameterized by β, then the money metric utility function depends on the same parameters and we write $m(p, x, \beta)$.

In terms of the money metric utility function, we can restate the definition of the efficiency index as

$$i^t = \frac{m(p^t, x^t, \beta)}{p^t x^t}.$$

In words, $m(p^t, x^t, \beta)$ gives the minimum expenditure necessary to achieve utility $u(x^t, \beta)$ while $p^t x^t$ gives the expenditure actually observed. Roughly speaking, the consumer is "wasting" a fraction $1 - i^t$ of his money.

An index of the degree of violation of maximization in the data set could be given by

$$I = \sum_{t=1}^{T} \left(\frac{m(p^t, x^t, \beta)}{p^t x^t} - 1 \right)^2.$$

This definition is directly analogous to equation (1).

The discussion to this point has proceeded under the assumption that β was known. But what if β is unknown? Then we would like to have an estimate of β—an estimate that provides the best fit to the maximizing model. A natural estimate is to find that value of β that minimizes the degree of violation of maximizing behavior as measured by the index I. This makes the average value of e^t as close as possible to 1, using the sum-of-squared-error norm. I believe that this estimator has several desirable characteristics.

First, it uses a sensible *economic* norm for goodness-of-fit. Conventional estimators of demand parameters use the sum-of-squared errors of the observed and predicted quantities demand, or some variant on this. But this has little *economic* content; a large difference between predicted and observed demand can easily be consistent with a small difference in utility. This is depicted in Figure 2. Here the observed choice is far from the predicted choice in Euclidean distance, but quite close in terms of money metric utility. The model is a bad fit in terms of Euclidean distance, but a good fit in the sense that the consumer really isn't that far from maximizing behavior in terms of money metric utility.

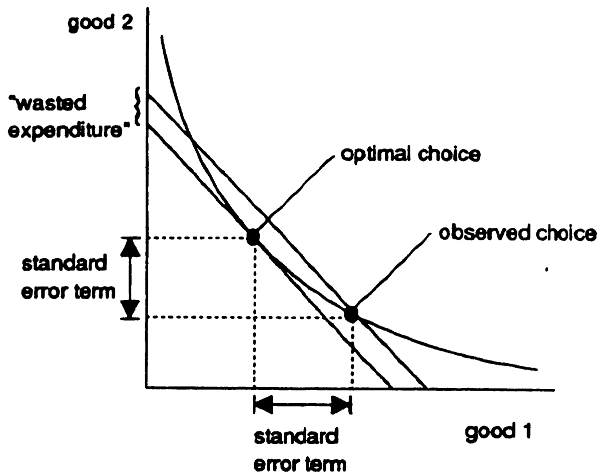

Figure 2. This is a good fit in terms of money metric utility although it is a bad fit in terms of the usual error terms.

Second, the minimized value of the objective function gives a meaningful economic measure of how close the observed choices are to maximizing choice for the particular parametric form involved. If the average value of e^t is .95 for example, then it is meaningful to say that the observed choice behavior was 95 percent as efficient as maximizing behavior.

Third, the mechanics of the estimation problem may be much simpler than they are using the conventional approach. Economic theory imposes the restriction that a money metric utility function must be an increasing, linearly homogeneous, and concave function of prices. These constraints are not terribly difficult to impose on the maximization problem. By contrast theory implies that a system of demand equations must have a symmetric negative semidefinite Slutsky substitution matrix. Imposing this restriction involves imposing nonlinear cross equation restrictions on a system of equations. In general this is a difficult thing to do.

Fourth, this same method can easily be applied to estimation of production relationships. If one starts with a null hypothesis of cost minimization, say, then it makes sense to measure the goodness-of-fit of estimation procedure by comparing the actual costs to the minimum costs implied by the estimated parameters. If it is thought that errors in optimization are a significant component of the error term, then it can make sense to estimate the parameters by choosing parameter estimates that minimize the difference between the observed costs and the minimum costs.

6. An example

In order to examine the money metric goodness-of-fit measure described in the last section, I tried an experiment using U.S. aggregate consumption data. The data was taken from the Citibank economic database and consisted of aggregate consumption of durables, nondurables, and services from 1947 to 1987. (See Table 3.)

I estimated the parameters of a Cobb-Douglas utility system using three different techniques. The first technique was simply to take the average expenditure share of each good. The second technique was to estimate the regression $x_i = a_i e/p_i$, where e is the total expenditure on the three goods. I used Zellner's seemingly unrelated regression technique and imposed the normalization that $a_1 + a_2 + a_3 = 1$. (Estimating the three equations separately gave almost the same estimates.) The third technique was to determine the values of the parameters that maximized the goodness-of-fit, as measured by difference between the money metric utility and the actual expenditure. The first two methods are straightforward, but a description of the third method may be in order.

Let us derive the money metric utility function associated with the Cobb-Douglas utility function $u(x_1, x_2, x_3) = x_1^{a_1} x_2^{a_2} x_3^{a_3}$. For algebraic convenience we impose the normalization that the exponents sum to 1. The money metric utility function is defined to be the amount of money that it takes as some prices (p_1, p_2, p_3) to choose an optimal bundle that has the same utility as the bundle (x_1, x_2, x_3).

If we let m be the necessary amount of money, we have the equation

$$x_1^{a_1} x_2^{a_2} x_3^{a_3} = \left(\frac{a_1 m}{p_1}\right)^{a_1} \left(\frac{a_2 m}{p_2}\right)^{a_2} \left(\frac{a_3 m}{p_3}\right)^{a_3}$$

Solving for m we have

$$m(p, x) = a_1^{-a_1} a_2^{-a_2} a_3^{-a_3} (p_1 x_1)^{a_1} (p_2 x_2)^{a_2} (p_3 x_2)^{a_3} \tag{3}$$

(For a different derivation, see Varian (1984), page 129.) Taking logs, we can write this equation as

$$\ln m(p, x) = -a_1 \ln a_1 - a_2 \ln a_2 - a_3 \ln a_3 + a_1 \ln p_1 x_1 + a_2 \ln p_2 x_2 + a_3 \ln p_3 x_3. \tag{4}$$

We suppose that the log of the actual expenditure in period t, $\ln e^t$, is equal to the log of the expenditure minimizing amount, $\ln m(p^t, x^t)$, plus an error term representing the optimization error. Using equation (4), we have

$$\ln e^t = -a_1 \ln a_1 - a_2 \ln a_2 - a_3 \ln a_3 + a_1 \ln p_1^t x_1^t + a_2 \ln p_2^t x_2^t + a_3 \ln p_3^t x_3^t + \epsilon^t.$$

I estimated this equation using the nonlinear least squares routine in MicroTSP, imposing the restriction that $a_1 + a_2 + a_3 = 1$. The results from the three estimation methods are in Table 1.

Table 1

Estimated Parameter Values

Method	a_1	a_2	a_3
Expenditure shares	0.152	0.461	0.387
Regression	0.129	0.358	0.413
Nonlinear Least Squares	0.150	0.472	0.378

The first thing to observe is that the three methods give somewhat different answers. This is simply a consequence of the fact that the estimates which "fit the data best" depend on what measure of goodness-of-fit you use. The regression estimates that minimizes the sum of squared deviations from the observed demands will not in general be the same as the estimates that minimize the squared difference between money metric utility and actual expenditure.

It is surprising that the expenditure share method and the money metric method give very similar estimates, especially since the expenditure share estimate involves a system of equations while the money metric estimation involves only a single equation. Of course, ultimately it is a single sum-of-squares that is minimized in the regression technique, so perhaps this is not so surprising after all.

The computed values of the money metric utility function for each of the different parameters are given in Table 2, along with the percentage difference between money metric utility and the actual expenditure for each of the three different estimation methods.

Note that these percent differences are very small, at least for the expenditure share estimates and the NLS estimates. Using the expenditure share methods the largest difference is 7.4 %, and the majority of the differences are less than one percent. The average difference is 2%. This suggests that the observed aggregate demand behavior is not very different from optimizing behavior, at least when measured in units of "wasted expenditure."

Similar results hold for the nonlinear least squares estimates. Here the average value of the error is only 1.9%. The regression estimates do much poorer, resulting in an average error of about 5%.

It is worth noting that the residuals in all of the estimates are positive in each observation; this is as it should be if the optimizing model is to make any sense since the minimum expenditure to achieve a given level of utility must always be less than an arbitrary expenditure.

7. Summary

In the first part of this paper I discussed some ways to measure goodness-of-fit in a non-parametric context. It appears that the Afriat index is a reasonable measure, and that it is not difficult to calculate it's sampling distribution by Monte-Carlo methods. This sampling distribution can be used to test hypotheses in the standard way.

The second part of the paper showed how the money metric utility function can be used to construct a goodness-of-fit measure. For the data set examined here, it appears that aggregate consumption behavior is not terribly far from maximizing behavior, at least in the money-metric norm.

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Table 2 Comparison of Estimation Techniques							
Year	Actual Expenditure	(Shares) m_1	(Regress) m_2	(NLS) m_3	(Shares) $1 - m_1/e$	(Regress) $1 - m_2/e$	(NLS) $1 - m_3/e$
1947	3,855	3,568	3,117	3,600	0.074	0.191	0.066
1948	4,462	4,132	3,609	4,169	0.074	0.191	0.066
1949	4,470	4,235	3,755	4,265	0.053	0.160	0.046
1950	4,876	4,637	4,129	4,666	0.049	0.153	0.043
1951	5,534	5,249	4,658	5,286	0.052	0.158	0.045
1952	5,872	5,634	5,057	5,670	0.041	0.139	0.034
1953	6,257	6,088	5,543	6,120	0.027	0.114	0.022
1954	6,576	6,444	5,924	6,474	0.020	0.099	0.015
1955	7,285	7,149	6,585	7,177	0.019	0.096	0.015
1956	7,816	7,713	7,163	7,740	0.013	0.084	0.010
1957	8,537	8,441	7,867	8,469	0.011	0.079	0.008
1958	9,048	8,979	8,436	9,006	0.008	0.068	0.005
1959	9,939	9,881	9,318	9,904	0.006	0.063	0.004
1960	10,560	10,525	10,002	10,545	0.003	0.053	0.001
1961	11,019	10,996	10,517	11,014	0.002	0.046	0.000
1962	11,933	11,915	11,423	11,930	0.002	0.043	0.000
1963	12,785	12,767	12,290	12,777	0.001	0.039	0.001
1964	13,914	13,894	13,414	13,901	0.001	0.036	0.001
1965	15,350	15,319	14,793	15,325	0.002	0.036	0.002
1966	17,163	17,135	16,550	17,143	0.002	0.036	0.001
1967	18,581	18,554	18,038	18,555	0.001	0.029	0.001
1968	21,431	21,380	20,804	21,376	0.002	0.029	0.003
1969	24,340	24,282	23,729	24,272	0.002	0.025	0.003
1970	27,471	27,389	26,961	27,372	0.003	0.019	0.004
1971	31,294	31,124	30,829	31,082	0.005	0.015	0.007
1972	35,846	35,591	35,382	35,531	0.007	0.013	0.009
1973	42,530	42,374	41,812	42,341	0.004	0.017	0.004
1974	50,710	50,507	49,743	50,503	0.004	0.019	0.004
1975	61,383	61,131	60,316	61,113	0.004	0.017	0.004
1976	72,478	72,090	71,506	72,012	0.005	0.013	0.006
1977	85,701	84,969	84,889	84,822	0.009	0.009	0.010
1978	102,261	101,237	101,465	101,043	0.010	0.008	0.012
1979	124,104	122,988	123,047	122,804	0.009	0.009	0.010
1980	148,952	146,795	148,177	146,525	0.014	0.005	0.016
1981	180,406	177,153	179,803	176,757	0.018	0.003	0.020
1982	205,088	199,545	204,941	198,908	0.027	0.001	0.030
1983	232,873	225,046	232,866	224,161	0.034	0.000	0.037
1984	263,642	253,883	263,598	252,772	0.037	0.000	0.041
1985	294,808	281,220	294,461	279,813	0.046	0.001	0.051
1986	322,488	304,152	321,354	302,408	0.057	0.004	0.062
1987	353,848	331,047	351,875	329,083	0.064	0.006	0.070
				Mean	0.020	0.052	0.019

Table 3 U.S. Aggregate Consumption (Citibank Economic Database)						
Year	Durables p_1	Nondurables p_2	Services p_3	Durables x_1	Nondurables x_2	Services x_3
1947	27.40	26.88	16.85	20.43	90.88	50.60
1948	28.77	29.18	17.77	22.85	96.60	55.48
1949	29.95	27.85	18.45	25.05	94.85	58.42
1950	31.23	27.73	18.90	30.73	98.22	63.15
1951	31.85	29.73	19.40	29.85	109.15	69.03
1952	31.57	29.85	20.30	29.23	114.72	75.13
1953	31.77	29.38	21.40	32.67	117.83	82.13
1954	32.10	30.02	22.18	32.10	119.67	88.05
1955	33.38	30.60	23.02	38.88	124.70	94.30
1956	34.73	30.95	24.02	38.20	130.78	101.63
1957	36.50	31.90	25.02	39.65	137.10	108.55
1958	36.92	32.85	26.10	37.17	141.75	115.67
1959	38.50	33.17	26.93	42.80	148.47	125.00
1960	38.77	33.63	27.80	43.42	153.20	134.00
1961	38.88	34.08	28.40	41.90	157.40	141.80
1962	39.75	34.58	29.13	47.02	163.82	151.05
1963	40.15	34.90	29.85	51.80	169.35	160.63
1964	40.35	35.25	30.60	56.85	179.68	172.78
1965	40,92	36.10	31.43	63.48	191.85	185.40
1966	41.55	37.35	32.60	68.53	208.45	200.30
1967	42.33	38.23	33.80	70.63	216.90	216.00
1968	44.55	39.98	35.65	81.00	235.00	236.43
1969	46.10	41.98	37.70	86.22	252.18	259.43
1970	47.80	44.05	40.38	85.67	270.32	284.02
1971	50.25	45.92	43.08	97.58	283.27	310.65
1972	51.05	47.90	45.58	111.22	305.10	341.27
1973	50.83	53.75	48.10	124.72	339.55	372.98
1974	54.08	59.58	51.77	123.75	380.90	411.90
1975	61.15	65.13	56.38	135.35	416.20	461.23
1976	65.88	67.60	60.65	161.45	451.95	515.92
1977	69.15	71.03	65.45	184.50	490.45	582.25
1978	72.78	76.00	70.30	205.57	541.80	656.10
1979	78.25	83.55	75.88	218.95	613.25	734.55
1980	85.55	88.85	83.72	219.28	681.35	831.95
1981	93.65	96.78	92.30	239.88	740.58	934.70
1982	100.00	100.00	100.03	252.65	771.00	1026.97
1983	101.60	102.50	106.13	289.10	816.70	1128.75
1984	102.97	106.17	111.60	335.55	867.30	1227.63
1985	102.35	108.50	117.25	368.70	913.13	1347.52
1986	101.38	110.13	122.25	402.43	939.35	1458.05
1987	100.40	114.05	127.42	413.73	982.88	1571.22

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