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Jonathan B. Cohn
McCombs School of Business
University of Texas

Uday Rajan
Stephen M. Ross School of Business
University of Michigan

Günter Strobl
Frankfurt School of Finance and Management

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Jonathan Cohn[†] Uday Rajan[‡] Günter Strobl[§]

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Abstract

We study a model in which an issuer can manipulate the information obtained by a credit rating agency (CRA) seeking to screen and rate the quality of its asset. In some equilibria, more intense CRA screening leads to more manipulation by the issuer, since it improves the payoff to surviving the screening. As a result, the CRA may optimally abandon screening, even though the direct marginal cost of screening is zero. This result suggests that strategic disclosure and issuer moral hazard may have played an important role in recent ratings failures.

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[†]McCombs School of Business, University of Texas, Austin; `jonathan.cohn@mccombs.utexas.edu`

[‡]Stephen M. Ross School of Business, University of Michigan, Ann Arbor; `urajan@umich.edu`

[§]Frankfurt School of Finance and Management; `g.strobl@fs.de`

1 Introduction

The failure of credit ratings to predict defaults of mortgage-backed securities in the lead-up to the financial crisis has raised questions about the role of credit rating agencies (CRAs) in the economy. In principle, these "information intermediaries" produce information that allows investors to more accurately price assets. Importantly, they rely on the issuers of the assets for much of the information on which their ratings are based. Issuers seeking favorable ratings may have an incentive to manipulate the information that a CRA observes. A full assessment of the effects of CRA screening on the quality of ratings should therefore account for the effects of a CRA's actions on issuer incentives.

We construct a model in which an issuer has a financial asset (or claim) to sell in each of two periods. A CRA observes a signal about the quality of the asset, and produces a rating based on this signal. To account for the reliance of the CRA on the issuer for information, we allow the issuer to manipulate the CRA's signal. In turn, the CRA can invest resources in scrutinizing the asset in order to reduce the likelihood that such manipulation succeeds. We exhibit equilibria in which more intense CRA scrutiny in the first period strengthens the issuer's incentive to manipulate. The resulting increase in manipulation effectively undoes the positive direct effect of greater scrutiny on rating accuracy. Therefore, the optimal level of CRA scrutiny can be low (potentially zero), even if the CRA cares about ratings accuracy and the direct cost of more intense CRA scrutiny is small.

Many commentators have argued that the apparent lack of CRA diligence leading up to the financial crisis was due to CRA negligence, or possibly even complicity with issuers attempting to defraud investors. Our results suggest that a CRA might choose a relatively lax investigative policy even if it is concerned about ratings accuracy. Because of issuers' strategic disclosure incentives, greater diligence may not yield much improvement in the accuracy of ratings. Moreover, attempting to infer the degree of CRA diligence from observed rating accuracy may be difficult. Finally, our results raise doubts that regulatory changes forcing CRAs to exert greater investigative effort would actually lead to more accurate ratings.

Our model has two periods. In each period, an issuer has an asset with either high or low value. A CRA obtains a high or low signal of the asset value, but the issuer may be able to manipulate the signal that the CRA observes. The CRA always observes a high signal when the value of the asset is high. If the value of the asset is low, the CRA observes a low signal if the issuer does not manipulate, but may observe a high signal if the issuer does manipulate. The probability that the CRA observes the high signal in this case (i.e., manipulation is successful) decreases with the level of screening that the CRA exercises (discussed in more detail below). After observing its signal, the CRA issues either a high or low rating. The issuer then sells its asset to rational expectations-forming investors operating in a perfectly competitive market. Finally, the asset's payoff is realized, and the period ends.

There are two types of issuer in our model. An "opportunistic" type issuer can manipulate the CRA's signal, while an "honest" type issuer cannot. This structure allows us to capture reputational incentives for honest behavior in reduced form, and can be justified by assuming that some issuers face an unobserved high cost of engaging in manipulation.[1] It also appears broadly consistent with the way at least one of the major rating agencies perceives the issuers whose securities it rates. In a 2002 statement to the SEC, Moody's concluded that "Most issuers operate in good faith and provide reliable information to the securities markets, and to us. Yet there are instances where we may not believe that the numbers provided or the representations made by issuers provide a full and accurate story."[2] The opportunistic issuer's objective at any point in time is to maximize its discounted cash flow from selling assets. An issuer knows its type, which does not change over time. Other agents only know the distribution of issuer types.

The CRA chooses a level of costly screening in each period.[3] We assume that the CRA chooses its first period screening intensity before the issuer's manipulation decision, and that all agents observe this level of screening. We think of this level of screening as a policy choice. Agents become aware of the CRA's screening policy through a variety of sources, including its myriad interactions with issuers. We allow the CRA to choose its optimal screening intensity in the second period after observing actions and outcomes in the first period.

The CRA's objective at each point in time is to minimize the discounted value of its current and future losses. Its loss in each period consists of two components. The first component is the direct cost of screening, which is increasing and convex in screening intensity. The second one is the likelihood that it makes a rating mistake (i.e., that it rates a low-valued asset high or vice versa). This preference for accurate ratings can be justified by the threat of regulatory sanctions, lawsuits, and reputational costs. The weight that the CRA places on the second

[1] The adverse selection approach to modeling reputations was introduced by Kreps and Wilson (1982) and Milgrom and Roberts (1982).

[2] From the Written Statement of Raymond W. McDaniel, President, Moody's Investors Service Before the United States Securities and Exchange Commission, November 21, 2002.

[3] In our model, the CRA bears the marginal cost of screening, rather than passing it on to the issuer; implicitly, this assumes that the CRA's effort is difficult to verify or contract upon. Along these lines, in 2008 New York Attorney General Andrew Cuomo and the major rating agencies agreed to a plan that requires fees to be set up front, before a rating agency does any work.

component (relative to the first) captures the importance of these costs.

Assuming that the CRA's screening is imperfect, an opportunistic issuer benefits from manipulating the CRA's signal when it has a low-valued asset because it can sell its asset at a higher price when the CRA's rating is high. In the second period, there are no countervailing incentives, and the opportunistic issuer manipulates with probability one when it has a low-valued asset. Taking into account its belief about the probability that the issuer is opportunistic given the events of the first period, the CRA then chooses its screening intensity in the second period (knowing that the opportunistic issuer will manipulate with probability one in the second period when it has a low-valued asset). The greater the weight that the CRA places on the issuer being opportunistic, the more intensely it screens in the second period.

As the honest type issuer never manipulates the CRA's signal, it never has an asset rated high that subsequently turns out to have low value. Therefore, if the issuer's asset is rated high in the first period and its value proves to be low, it is revealed to be the opportunistic type. Being revealed as such eliminates the opportunistic issuer's ability to pool with the honest type in the second period, and therefore reduces the expected price at which it can sell its asset in the second period. The risk of this "reputational loss" counters the opportunistic issuer's incentive to manipulate in the first period. As a result, the opportunistic issuer may manipulate with probability less than one in the first period.

A high rating allows the issuer to charge a higher price for its asset because it provides certification of the asset's value.[4] An increase in the CRA's screening intensity increases the likelihood that the asset's value is high when the CRA's rating is high, and therefore increases the benefit of a high rating. Put differently, the certification benefit of a high rating to the issuer is more valuable when investors know that the CRA is scrutinizing the asset more carefully. This provides the insight for our first main result: Greater screening intensity in the first period results in more manipulation by the opportunistic issuer (assuming that the opportunistic type manipulates with probability less than one). This equilibrium effect undoes the benefit of more intense screening. That is, the probability of a ratings mistake, which depends on both the probability that the issuer manipulates and the CRA's screening intensity, does not change with the level of screening intensity. An immediate implication of this result is that it may not be possible to infer a CRA's screening intensity from the accuracy of its ratings. This result also suggests that regulatory policies forcing CRAs to screen more intensely may not

[4] In our model, the CRA's rating contains new information not previously available to the market. Empirically, Kliger and Sarig (2000) show that when Moody's refined its ratings categories in 1982, both bond and stock prices of affected firms reacted (in opposite directions). See their paper for additional references on whether ratings contain new information.

yield improvements in ratings accuracy.

Under some circumstances (for example, when ratings mistakes are very costly), the CRA optimally chooses a high level of screening in the first period, accepting that the opportunistic issuer will manipulate with probability one. However, under other circumstances (in particular, when inaccurate ratings are less costly), the CRA optimally chooses not to screen at all in the first period, even though the marginal cost of increasing screening intensity is zero. This is our second main result, and offers a new explanation based on issuer rather than CRA moral hazard for why CRAs might put little effort into screening.

Our paper contributes to the growing literature on the role of credit rating agencies and the phenomenon of ratings inflation. In papers by Bolton, Freixas, and Shapiro (2012), Skreta and Veldkamp (2009), and Sangiorgi and Spatt (2011) inflated ratings emerge from ratings shopping—an issuers' ability to conceal unfavorable ratings. Mathis, McAndrews, and Rochet (2009) demonstrate in a dynamic model that reputational concerns are insufficient to prevent ratings inflation if the flow income from new ratings is high enough in a given period. In a related vein, Bar-Isaac and Shapiro (2012) analyze ratings quality over the business cycle and show that agencies issue less accurate ratings when their income from fees is high, competition in the labor market for analysts is tough, and default probabilities for the rated securities are low. Opp, Opp, and Harris (2013) argue that ratings inflation may result from regulatory distortions when credit ratings are used for regulatory purposes such as bank capital requirements, and Fulghieri, Strobl, and Xia (2013) demonstrate how the possibility that CRAs can issue unsolicited credit ratings may lead to ratings inflation by allowing CRAs to extract more from issuers in exchange for issuing a favorable rating. Bouvard and Levy (2012) and Frenkel (2013) consider a CRA that has an incentive to maintain two reputations—with investors for stringency, and with issuers for leniency. The latter in turn leads to ratings inflation.

While these papers share some important features with ours, they do not address the question of how the CRA's policies influence issuers' incentives to manipulate the information on which the CRA bases its ratings and hence the accuracy of those ratings. We find that poor screening by the CRA can be optimal once these effects are taken into account. A word of caution is in order though: In our model, "poor screening" is not equivalent to "ratings inflation." Indeed, we show that under some conditions better screening is completely unwound by strategic disclosure on the part of the issuer, and so does *not* lead to a better quality of credit ratings.

Our paper is also related to the literature on performance manipulation. Important contributions to this literature include work by Stein (1989), Goldman and Slezak (2006), and Crocker and Slemrod (2007). These papers show that concerns about short-term stock prices can lead to “signal-jamming” equilibria in which managers take costly actions to manipulate their performance reports, even though market participants correctly anticipate the extent of manipulation and take it into account when pricing the firm’s stock. However, in contrast to our analysis, these papers take the manager’s manipulation cost as exogenous and do not consider the effect of the investors’ screening intensity on the manager’s manipulation strategy. In this respect, our approach is closer in spirit to Strobl (2013), who demonstrates that stricter disclosure requirements can lead to more earnings manipulation.

A number of papers in the agency literature have shown that more information can hurt the principal by reducing the agent’s incentive to work hard in order to prove his worth (e.g., Holmström, 1999; Dewatripont, Jewitt, and Tirole, 1999). Similarly, Prendergast (1993), Brandenburger and Polak (1996), and Prat (2005) show that greater transparency about a manager’s actions can have detrimental effects because it induces the manager to act according to how a talented manager is expected to act and to disregard useful private information. A similar phenomenon arises in Cohn and Rajan (2013), who show that more stringent corporate governance can exacerbate the agency problem caused by the manager’s reputational concerns. Our analysis complements these papers by identifying a new channel through which more information can have detrimental effects. In our model, a better screening technology influences the issuer’s incentive to misbehave because of its effect on the value of a high rating.

Finally, our paper is related to the broader literature on reputation as an incentive mechanism. This literature is enormous, and we will not do it justice here. Firms have been shown to face reputational concerns in many aspects of their business, including repaying debt (Diamond, 1989), fighting new entrants (Kreps and Wilson, 1982; Milgrom and Roberts, 1982), not holding up suppliers (Banerjee and Duflo, 2000), meeting earnings targets (Fisher and Heinkel, 2008) and producing quality products (Cabral, 2000; Hörner, 2002). Reputation is also known to matter for underwriters (Chemmanur and Fulghieri, 1994a), banks (Chemmanur and Fulghieri, 1994b), and workers (Tadelis, 1999). For reputation to be interesting from an economist’s viewpoint, the benefit of “cheating” (not repaying debt, for example) must be weighed against the cost of a lost reputation. These papers show that costs of reputation loss can be large enough to ensure “good behavior.”

2 Model

An issuer and a credit rating agency (CRA) each live for two periods, indexed by $t = 1, 2$. In each period, the issuer wishes to sell a one-period financial claim with stochastic cash flows. For example, the claim may be a corporate bond or an asset-backed security. Since the claim is a financial asset for investors, we refer to it as an asset throughout. In each period, the cash flow from that period's asset will be revealed to the market at the end of the period. A period is therefore broadly interpreted to be the length of time in which the market finds out about the realized cash flows from an asset.

At the beginning of the period, the value of the asset is unknown to all parties. The realized cash flow v_t may be high (v^h) or low (v^ℓ). There is no discounting within each period, so we alternately refer to v as the value of the asset in that period. Without loss of generality, we normalize v^h to 1 and v^ℓ to 0. In the first period, $v_1 = v^h$ with probability $\eta_1 \in (0, 1)$ and $v_1 = v^\ell$ with probability $1 - \eta_1$. Therefore, η_1 is also the expected value of the asset. We allow for persistence of asset values across the two periods, to account for intrinsic characteristics of the issuer that may lead to a correlation in values across time. Specifically, we allow the probability that $v_2 = 1$ in the second period to depend on the realized value of v_1 in the first period. Let $\eta_2^i = \text{Prob}(v_2 = v^h \mid v_1 = v^i)$ be the probability of a high cash flow in the second period, given that the cash flow in the first period was v^i, for $i \in \{h, \ell\}$. Then, we assume that $\eta_2^h \geq \eta_1$ and $\eta_2^\ell = \frac{\eta_1(1-\eta_2^h)}{1-\eta_1} \leq \eta_1$. This choice of η_2^ℓ ensures that the asset continues to have an unconditional expected value of η_1 at time 2. If $\eta_2^h = \eta_1$, there is no persistence across periods, while if $\eta_2^h > \eta_1$, there is positive persistence.

In each period $t = 1, 2$, the following sequence of events occurs. First, the CRA establishes a screening intensity, $\alpha_t \in [0, 1]$, that is observed by all parties. Second, the issuer applies for a rating. In the process, the issuer generates a signal g_t that is observed by the CRA but not by investors. The issuer can choose to manipulate the signal. If the cash flow is v^h, manipulation has no effect and signal $g_t = g^h$ is generated. If the cash flow is v^ℓ and the issuer manipulates, the CRA obtains signal $g_t = g^\ell$ with probability α_t and signal $g_t = g^h$ with probability $1 - \alpha_t$. In the absence of manipulation, $g_t = g^\ell$ with probability one. Third, the CRA assigns a rating r_t to the asset. We assume throughout that the CRA has no discretion in manipulating its rating once the signal g_t has been obtained. That is, the CRA must assign a rating $r_t = r^i$ on receiving signal $g_t = g^i$, for $i \in \{h, \ell\}$.[5] This restriction allows us to focus on the CRA's

[5]The CRA may be subject to lawsuits from investors if it assigns a high rating r^h on receiving signal g^ℓ, the low signal. Witness, for example, the recent Justice Department suit against S&P (see "U.S. Sues S&P Over

screening decisions without having to consider the possibility that the CRA is strategic in its rating decisions. Fourth, investors observe the rating r_t and purchase the asset at a price p_t. Investors are perfectly competitive so the asset is fairly-valued (i.e., p_t is the expected payoff of the asset conditional on investors' information, including r_t). Fifth, the cash flow from the asset v_t is observed.[6]

The structure of the game between the issuer and the CRA captures the idea that the issuer may want to hide information from the CRA. Rather than modeling the signal as a message sent from the issuer to the CRA, we model it in reduced form, with the likelihood of successful garbling depending on α_t, the screening intensity of the CRA.[7] Since the issuer has the incentive to hide bad news but reveal good news, we model the signal as asymmetric, with cash flow v^h always generating signal g^h. In what follows, an issuer that does not manipulate information is said to disclose truthfully, whereas an issuer that manipulates is said to lie.

The issuer has two "integrity" types, honest (H) and opportunistic (O). The prior probability that the issuer has type H is μ_1. The integrity type is privately known to the issuer, and reflects intrinsic features about the issuer and its cash flow sources that make manipulation feasible or infeasible. We assume that the honest issuer always reports truthfully, or equivalently has an infinite cost of manipulating the signal. This could be because it suffers a direct disutility from lying, faces a high reputational cost to lying and being found out, or finds lying to be technologically difficult (e.g., some firms are more transparent than others). As a result, in evaluating the asset of an honest issuer, the CRA's signal is consistent with the true value of the asset.

In contrast, an opportunistic issuer has zero manipulation cost, and no direct disutility from lying. Therefore, it will manipulate the CRA's information whenever it can thereby increase its own payoff. We allow the O-type issuer to mix between manipulating and not manipulating its signal, with σ_t denoting the probability that the O-type issuer manipulates in period t. Figure 1 displays the full sequence of events in each period.

Next, we describe the payoffs that the various parties earn in the two periods. In aggregating payoffs across periods, all parties place a weight of one on first period payoffs and of $\delta \geq 0$ on

Ratings," *Wall Street Journal*, February 5, 2013). Conversely, an issuer may sue the CRA if it assigns a low rating r^ℓ despite having received signal g^h, the high signal.

[6]As manipulation has no effect if the asset value is high, we could equivalently assume the issuer privately knows the true value at the start of the period. An issuer with a high-valued asset will never want to manipulate, so such an assumption would leave the model and analysis unchanged.

[7]We have analyzed an alternate version of the model in which the issuer does send a message to the CRA. This version is analytically less tractable because the CRA's and investors' information sets can differ, increasing the number of states. Numerical examples suggest that the main insights of the paper continue to hold in the alternate model.

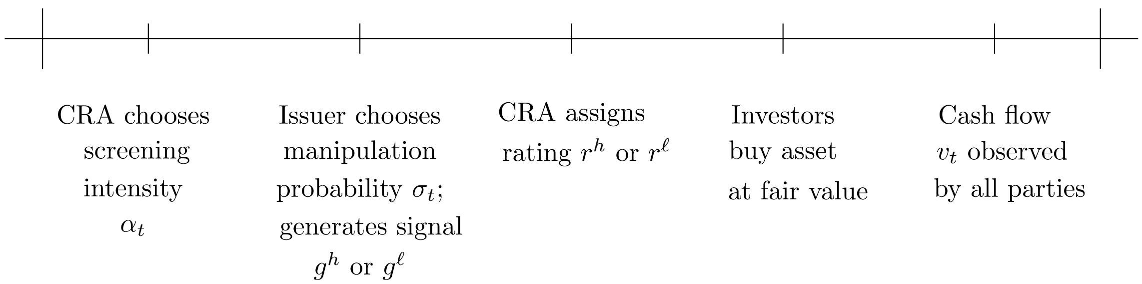


Figure 1: Sequence of events in each period $t = 1, 2$

second period payoffs. Here, δ may be interpreted as a discount factor, in which case $\delta \leq 1$ is natural. More broadly, δ just reflects the importance of period 2 as compared to period 1. If one thinks of period 2 as representing in reduced form multiple future periods after period 1, it may be reasonable to consider the case in which $\delta > 1$.

Investors obtain a payoff of $v_t - p_t$ in period t. The issuer obtains an unmodeled benefit of $B > v^h$ from selling the asset and investing the proceeds in a project, and its payoff at time t is $B - v_t + p_t$. This ensures that the issuer always sells its asset. As B and v_t are exogenous, we can consider the issuer's payoff in period t to be p_t without loss of generality. This results in an overall payoff in the game to the issuer of $p_1 + \delta\, p_2$.

The CRA minimizes its total cost. In each period, its cost has two components. The direct cost of screening the issuer with intensity α_t is $c(\alpha_t)$, where c is strictly increasing and strictly convex, with $c(0) = c'(0) = 0$. The CRA also incurs an indirect reputational cost whenever it makes a rating error, that is, whenever an asset has value v^h but is rated r^ℓ or vice versa. Since an asset with value v^h always generates signal g^h, an error by the CRA can only occur when the asset has low value and the CRA issues a rating r^h. The cost of the CRA at time t may then be written as $\psi_t = \lambda \,\mathrm{Prob}(v_t = v^\ell, r_t = r^h) + c(\alpha_t)$, and its cost in the overall game is $\Psi = \psi_1 + \delta\, \psi_2$. Here, $\lambda \geq 0$ is a parameter that determines the relative importance of the cost of making an error versus the direct cost of screening.

We consider perfect Bayesian equilibria of this game. Since the strategy of the honest type is fixed (it never manipulates) as is the rating assignment process for the CRA, an equilibrium can be described by the screening strategy of the CRA and the strategy of the O-type issuer in each period t, and by the corresponding beliefs. Note that the period 2 strategy is state-contingent in the sense that actions can depend on outcomes in period 1. Further, the presence of the H-type ensures that there are no unreached information sets in the game.

2.1 Single-Period Case

Suppose first that there is only one period. Let η_1 denote the probability that $v = v^h$ and $p^i = p_1(r^i)$ the market price of an asset with rating r^i, $i \in \{h, \ell\}$. As the honest issuer always generates a truthful signal, in any equilibrium it must be that $p^h > p^\ell$. Fix the CRA's screening intensity at α and consider the O-type issuer. Suppose it manipulates its information with probability σ. If the asset value is v^h, manipulation has no effect and it obtains a rating r^h. If the asset value is v^ℓ, it obtains a rating r^ℓ with probability $1 - \sigma(1-\alpha)$ and a rating r^h with probability $\sigma(1-\alpha)$. That is, with probability $\sigma(1-\alpha)$, it is successful in its manipulation attempt. Its expected payoff is therefore $\eta_1 p^h + (1-\eta_1)\left((1-\sigma(1-\alpha))\, p^\ell + \sigma(1-\alpha)\, p^h\right) = (\eta_1 + (1-\eta_1)\sigma(1-\alpha))\, p^h + (1-\eta_1)(1-\sigma(1-\alpha))\, p^\ell$. As $p^h > p^\ell$, it is immediate that for any $\alpha < 1$, the unique best response is to manipulate with probability one.

Recall that the H-type firm manipulates with probability zero (i.e., it always generates a truthful signal). Therefore, if the CRA obtains signal g^h, with probability $\frac{\mu_1\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)(1-\alpha)}$ the firm is of type H and the true cash flow is v^h. With probability $\frac{(1-\mu_1)\eta_1}{\eta_1+(1-\mu_1)(1-\eta_1)(1-\alpha)}$, the firm has type O and the true cash flow is v^h. Finally, with probability $\frac{(1-\mu_1)(1-\eta_1)(1-\alpha)}{\eta_1+(1-\mu_1)(1-\eta_1)(1-\alpha)}$, the firm has type O and the true cash flow is v^ℓ. Following a signal g^h, the posterior probability that the asset's payoff is v^h is therefore $\frac{\eta_1}{\eta_1+(1-\mu_1)(1-\eta_1)(1-\alpha)}$.

Given the strategies of the H-type and O-type issuer, the CRA only makes a rating mistake if it issues a high rating r^h for an asset whose value turns out to be v^ℓ. The probability of an error is therefore $(1-\mu_1)(1-\eta_1)(1-\alpha)$. The CRA's problem is:

$$\min_{\alpha} \; \lambda(1-\mu_1)(1-\eta_1)(1-\alpha) + c(\alpha). \tag{1}$$

Observe that if the CRA chooses $\alpha = 1$, its total cost is $c(1)$, and if it chooses $\alpha = 0$, its total cost is $\lambda(1-\mu_1)(1-\eta_1)$. Therefore, when $c(1) > \lambda(1-\mu_1)(1-\eta_1)$, the CRA strictly prefers $\alpha = 0$ to $\alpha = 1$. This condition then implies that, in the single-period game, the CRA chooses $\alpha < 1$. We assume that a slightly stronger version of this condition holds. Specifically, noting that $\eta_2^\ell \leq \eta_1$, we assume that $c(1) > \lambda(1-\mu_1)(1-\eta_2^\ell)$. This ensures that the optimal choice of α is less than one, even in the two-period case when the probability of the high cash flow v^h is η_2^ℓ. This allows us to rule out corner solutions in which the CRA reaches a maximum level of scrutiny in both the one- and two-period cases.

Assumption 1. $c(1) > \lambda(1-\mu_1)(1-\eta_2^\ell)$.

The first-order condition for the CRA's problem is $c'(\alpha) = \lambda(1-\eta_1)(1-\mu_1)$. Let $\phi(\mu, \eta) = (c')^{-1}(\lambda(1-\eta)(1-\mu))$ for any CRA belief μ (where μ is the probability the firm has type H) and probability of high cash flow η. Note that $\phi > 0$ as long as $\mu < 1$. Although ϕ depends on λ as well, for convenience we suppress this variable in the notation.

Proposition 1. *In the single-period game, there is a unique perfect Bayesian equilibrium. It has the following properties:*

(i) The CRA chooses a screening intensity of $\hat{\alpha} = \phi(\mu_1, \eta_1)$.

(ii) The H*-type issuer always discloses truthfully, and the* O*-type issuer manipulates with probability one.*

(iii) The price of the asset is 0 *if the rating is* r^ℓ *and* $\frac{\eta_1}{\eta_1+(1-\mu_1)(1-\eta_1)(1-\hat{\alpha})}$ *if the rating is* r^h.

Since c is strictly convex, it follows that the optimal screening intensity $\hat{\alpha}$ decreases in both μ_1 and η_1. This is intuitive: given the strategies of each type of issuer, the CRA's rating is incorrect only when the issuer has type O and its true cash flow is v^ℓ. An increase in μ_1 reduces the likelihood of the O-type issuer, and an increase in η_1 reduces the likelihood of a low cash flow. Therefore, both of these effects lead to reduced screening.

Observe that the price of the asset given a high rating increases in $\hat{\alpha}$. This price is just the posterior probability that the asset has a high value, given a rating r^h. In the single-period model, therefore, an increase in $\hat{\alpha}$ leads directly to an increase in the informativeness of a high rating.

3 Equilibrium

In this section, we solve for the perfect Bayesian equilibrium in the two-period case. In the spirit of backward induction, we start with period 2. We then consider the optimal strategy of the O-type issuer in period 1, keeping fixed the CRA's screening intensity α_1. Finally, we turn to the optimal choice of α_1.

3.1 Equilibrium Strategies in Period 2

Period 2 is the last period of the game, so the equilibrium strategies are similar to those in the single-period case, characterized in Proposition 1. One important difference is that the prior belief of the CRA at the start of period 2 depends on the outcome observed in period 1. Suppose the rating assigned in period 1 was r^i and the cash flow at the end of the period was v^j, for $i, j \in \{h, \ell\}$. We let s^{ij} denote the state at the end of period 1 and μ_2^{ij} the posterior probability that the issuer has type H in state s^{ij}. Recall that η_2^i denotes the probability of a high cash flow in period 2, given that the cash flow in period 1 was v^i.

As in the one-period game, the H-type issuer always generates a truthful signal in the second period, and the O-type issuer generates a truthful signal when the cash flow is v^h and manipulates with probability one when the cash flow is v^ℓ.

Lemma 1. *Suppose that in period 2, the CRA's screening intensity in state s^{ij} is $\alpha_2^{ij} < 1$. Then, in equilibrium, in state s^{ij}:*

(i) The O-type issuer manipulates with probability one.

(ii) The price of the asset is $p_2^{ij} = \frac{\eta_2^i}{\eta_2^i + (1-\mu_2^{ij})(1-\eta_2^i)(1-\alpha_2^{ij})}$ if $g_2 = g^h$, and $p_2^{ij} = 0$ if $g_2 = g^\ell$.

(iii) The expected payoff of the O-type issuer in period 2 is $\pi_2^{ij} = \eta_2^i \left(1 + \frac{\mu_2^{ij}(1-\eta_2^i)(1-\alpha_2^{ij})}{\eta_2^i + (1-\mu_2^{ij})(1-\eta_2^i)(1-\alpha_2^{ij})}\right)$.

Given the best response of the issuer, the CRA in turn chooses a screening intensity based on its prior belief at the start of period 2.

Lemma 2. *In period 2, the CRA chooses the screening intensity $\hat{\alpha}_2^{ij} = \phi(\mu^{ij}, \eta_2^i)$ in state s^{ij}.*

From Proposition 1, it follows that, in each state s^{ij}, there is a unique equilibrium in period 2, characterized by the screening intensity $\hat{\alpha}_2^{ij}$ and by the fact that the O-type issuer manipulates with probability one.

Now consider period 1. The H-type issuer truthfully discloses the value of its asset. Suppose the O-type issuer manipulates with some probability $\sigma_1 \in [0, 1]$, and for now suppose the CRA and investors believe that the O-type issuer manipulates with probability $\tilde{\sigma}_1$. Of course, in equilibrium, it must be that $\tilde{\sigma}_1 = \sigma_1$. Recall that manipulation only has an effect on the CRA's signal if the asset has value v^ℓ. If the cash flow is v^h, therefore, the CRA always obtains signal g^h and assigns rating r^h. However, if the cash flow is v^ℓ and the O-type issuer manipulates, the CRA sometimes errs and assigns a rating r^h. The outcomes in period 1 and the associated issuer and investor beliefs are specified in Table 1.

State s^{ij}	Events in period 1	Probability	Posterior belief μ_2^{ij}
s^{hh}	r^h, v^h	η_1	μ_1
$s^{h\ell}$	r^h, v^ℓ	$(1-\eta_1)(1-\mu_1)\sigma_1(1-\alpha_1)$	0
$s^{\ell\ell}$	r^ℓ, v^ℓ	$(1-\eta_1)(1-(1-\mu_1)\sigma_1(1-\alpha_1))$	$\frac{\mu_1}{\mu_1+(1-\mu_1)(1-\tilde{\sigma}_1(1-\alpha_1))}$

Table 1: Events and beliefs in period 1

The cash flow at the end of period 1 is observed by both the CRA and investors. To see its effect on posterior beliefs, consider the case in which the cash flow is v^h (state s^{hh}). Since manipulation has no effect on the signal generated in this state, the posterior probability that the issuer has type H is just $\frac{\mu_1\eta_1}{\mu_1\eta_1+(1-\mu_1)\eta_1} = \mu_1$. Conversely, in state $s^{h\ell}$, the low cash flow realization in conjunction with a high rating immediately reveals that the issuer is of type O. Indeed, as we will show below, the fact that the issuer will be unmasked and revealed to be of type O in state $s^{h\ell}$ is a key factor in the issuer's choice of its optimal manipulation strategy in period 1. Finally, in state $s^{\ell\ell}$, the rating r^ℓ is consistent with the observed cash flow v^ℓ. Observe that $\mu_2^{\ell\ell} > \mu_2^{hh} = \mu_1 > \mu_2^{h\ell}$. Thus, in state $s^{\ell\ell}$, the market revises its prior beliefs upward.

3.2 Optimal Manipulation Strategy of the O-Type Issuer in Period 1

In this section, we fix the CRA's screening intensity at time 1, α_1, and describe the best response of the issuer. The H-type issuer continues to generate a truthful signal; that is, it generates signal g^i when the cash flow is v^i, for $i \in \{h, \ell\}$. Suppose that the CRA and investors believe that the O-type issuer manipulates the signal with probability $\tilde{\sigma}_1$. The price of the asset in period 1 following a high rating r^h depends on both α_1 and $\tilde{\sigma}_1$, and we write it as $p_1(\alpha_1, \tilde{\sigma}_1) = \frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)\tilde{\sigma}_1(1-\alpha_1)}$. When $r_1 = r^\ell$, the price in period 1 is zero.

Consider the best response of the O-type issuer. Suppose first that it is truthful with probability one. There are two possible outcomes at the end of period 1. First, with probability η_1, the first-period cash flow is v^h. In this case, the issuer obtains $p_1(\alpha_1, \tilde{\sigma}_1)$ with probability one. Entering period 2, the state is s^{hh}. Since the issuer will manipulate with probability one at this time, its second-period payoff is $\pi_2^{hh} = (\eta_2^h + (1-\eta_2^h)(1-\alpha_2^{hh}))\, p_2^{hh}$, so that the overall payoff in the game is $\Pi^h = p_1(\alpha_1, \tilde{\sigma}_1) + \delta\, \pi_2^{hh}$. Second, with probability $1-\eta_1$, the first-period cash flow is v^ℓ. The issuer obtains zero in period 1 and a payoff of $\pi_2^{\ell\ell} = (\eta_2^\ell + (1-\eta_2^\ell)(1-\alpha_2^{\ell\ell}))\, p_2^{\ell\ell}$ in period 2. Note that $p_2^{\ell\ell}$ depends on $\alpha_1, \tilde{\sigma}_1$ and $\alpha_2^{\ell\ell}$. The overall payoff in the game is $\delta\, \pi_2^{\ell\ell}$.

Therefore, if the O-type issuer does not manipulate in period 1, its expected payoff in the game is

$$\Pi^n = \eta_1 \Pi^h + \delta(1-\eta_1)\, \pi_2^{\ell\ell}. \tag{2}$$

Next, suppose the O-type issuer manipulates with probability one. Again, with probability η_1 the first-period cash flow is v^h and the firm obtains the two-period payoff Π^h. If the first-period cash flow is v^ℓ, there are two further possibilities. With probability α_1, the CRA obtains signal g^ℓ and assigns rating r^ℓ. This leads to a payoff of zero in period 1 and of $\pi_2^{\ell\ell}$ in period 2. With probability $1-\alpha_1$, the CRA assigns rating r^h. Then, the issuer obtains $p_1(\alpha_1, \tilde{\sigma}_1)$ in period 1. However, the state entering period 2 is $s^{h\ell}$, and the issuer obtains an expected payoff of $\pi_2^{h\ell} = \eta_2^\ell$ in period 2 (Lemma 1 (iii)).[8] Overall, if the O-type issuer manipulates in period 1, its expected payoff is

$$\Pi^m = \eta_1 \Pi^h + (1-\eta_1)\left(\alpha_1 \delta \pi_2^{\ell\ell} + (1-\alpha_1)(p_1 + \delta \pi_2^{h\ell})\right). \tag{3}$$

It is optimal for the O-type issuer to manipulate if $\Pi^m \geq \Pi^n$, that is, if $p_1(\alpha_1, \tilde{\sigma}_1) \geq \delta(\pi_2^{\ell\ell} - \pi_2^{h\ell})$. If the issuer chooses to manipulate, there are three possibilities: (i) the realized cash flow is v^h (and therefore manipulation has no effect), (ii) the cash flow is v^ℓ, the manipulation is unsuccessful, and the CRA assigns a rating r^ℓ, and (iii) the cash flow is v^ℓ, the manipulation is successful, and the CRA assigns a rating r^h. In the first two cases, the issuer obtains the same payoff as in the case without manipulation. In the last case, the issuer gains $p_1(\alpha_1, \tilde{\sigma}_1)$ in period 1 by manipulating. However, its payoff in period 2 is reduced by $\pi_2^{\ell\ell} - \pi_2^{h\ell}$, because after manipulating the rating, the state entering period 2 is $s^{h\ell}$ rather than $s^{\ell\ell}$. Manipulation is optimal if the gain $p_1(\alpha_1, \tilde{\sigma}_1)$ outweighs the cost $\delta(\pi_2^{\ell\ell} - \pi_2^{h\ell})$.

We show in Lemma 3 below that there is a threshold $\bar{\delta}$ such that it is optimal to manipulate if $\delta \leq \bar{\delta}$. The threshold discount factor $\bar{\delta}$ depends on the likelihood of high cash flow, η_1, the prior probability that the firm is of type H, μ_1, the market's belief about the O-type issuer's strategy, $\tilde{\sigma}_1$, and the screening intensity of the CRA in periods 1 and 2. The complete expression for $\bar{\delta}$ is shown in equation (11) in the proof of Lemma 3.

Lemma 3. *Fix the screening intensity of the CRA at time 1, α_1, and consider the perfect Bayesian equilibrium of the continuation game. Then, there exists a threshold $\bar{\delta}$ such that:*

[8] Recall that in state $s^{h\ell}$, the issuer is known to be of type O. That is, the prior belief at the start of period 2 that the issuer has type H is $\mu_2^{h\ell} = 0$. Therefore, the expected payoff of the O-type issuer in state $s_2^{h\ell}$ is just η_2^ℓ, and is independent of the screening intensity in that state, $\alpha_2^{h\ell}$.

(i) The O-type issuer's best response is to manipulate if $\delta < \bar{\delta}$ and to send a truthful signal if $\delta > \bar{\delta}$. If $\delta = \bar{\delta}$, the O-type issuer is indifferent between manipulating and not manipulating.

(ii) $\bar{\delta}$ is strictly increasing in η_1, α_1 and $\alpha_2^{\ell\ell}$. Further, as $\eta_2^{\ell} \to 0$ or $\eta_2^{\ell} \to 1$, $\bar{\delta} \to \infty$. That is, the issuer manipulates with probability one in period 1.

Suppose the O-type issuer manipulates with probability one. Then, it sometimes succeeds in obtaining a high rating when its cash flow is low. This leads to a higher payoff in period 1 than if it had been truthful and been assigned a low rating. However, at the end of period 1, investors observe a low cash flow, so the firm is now revealed as the O type. This results in a reduced period-2 payoff. If, on the other hand, the issuer were truthful in period 1, investors would assign a positive probability to the issuer being of type H, thereby increasing the issuer's period-2 payoff. Manipulation is worthwhile for the O-type issuer, if the discount factor is sufficiently low (that is, if the firm is sufficiently impatient).

Now, fix α_1 and consider the continuation game that follows after α_1 has been chosen. Let $\hat{\sigma}_1$ denote the level of manipulation by the O-type issuer in the equilibrium of this continuation game. In this equilibrium, the beliefs of the CRA and investors must coincide with the actual strategy of the O-type issuer, so that $\tilde{\sigma}_1 = \hat{\sigma}_1$. We assume that δ is neither too low (otherwise the issuer will not care about its second period payoff and will always manipulate in period 1) nor too high (otherwise the issuer will not care about its first period payoff and will never manipulate).

Assumption 2. $\frac{\eta_1}{1-\eta_2^{\ell}} < \delta < \frac{1}{1-\eta_2^{\ell}}$.

Note that $\frac{1}{1-\eta_2^{\ell}} > 1$, so we allow for the possibility that agents place greater weight on the second period than on the first period.

We characterize the equilibrium of the continuation game in terms of $\hat{\sigma}_1$. We begin by showing that the equilibrium is unique and that, under Assumption 2, the O-type issuer manipulates the signal g_1 with strictly positive probability.

Proposition 2. *Fix α_1 and consider the continuation game that follows.*

(i) The continuation game has a unique perfect Bayesian equilibrium.

(ii) *In the equilibrium of the continuation game, the O-type issuer manipulates with strictly positive probability in period 1, that is,* $\hat{\sigma}_1 > 0$.

The intuition for part (ii) of the proposition is as follows. Suppose that investors conjecture that the O-type issuer never manipulates. Then they would be willing to pay a high price (specifically, a price of one) for the asset upon observing rating r^h in the first period. This creates a strong incentive for the O-type issuer to manipulate the signal g_1 in an effort to get a high rating. As long as the issuer cares sufficiently about the first-period cash flow (specifically, as long as $\delta < 1/(1-\eta_2^\ell)$), it will manipulate, in contradiction to the investors' conjecture. Thus, in equilibrium it cannot be that $\hat{\sigma}_1 = 0$.

Proposition 2 implies that there are two possible types of equilibria in the continuation game. The first type is characterized by $\hat{\sigma}_1 = 1$, that is, the O-type issuer manipulates with probability one. In such an equilibrium, a small increase in α_1 implies that the O-type issuer continues to manipulate with probability 1. Conditional on having a low cash flow of v^ℓ in period 1, the firm is successful at manipulation (that is, obtains a first-period rating r^h) with probability $1-\alpha_1$. This probability strictly decreases in α_1. Therefore, in this type of equilibrium, on average, the market's posterior beliefs over firm types becomes more informative as the screening intensity α_1 increases.

In the second type of equilibrium, $\hat{\sigma}_1 \in (0,1)$. Here, we show that the O-type firm manipulates with a greater probability when α_1 increases (that is, $\hat{\sigma}_1$ increases in α_1). Specifically, the increased manipulation by the O-type firm exactly unwinds the effect of better screening by the CRA in period 1: the probability of obtaining a rating r^h in period 1 when the cash flow is v^ℓ is invariant to small changes in α_1.

Proposition 3. *Fix* α_1 *and consider the equilibrium of the continuation game in which* $\hat{\sigma}_1 \in (0,1)$. *Then:*

(i) *The O-type issuer manipulates with greater probability when* α_1 *increases, that is,* $\frac{\partial \hat{\sigma}_1}{\partial \alpha_1} > 0$.

(ii) *Over some range of* α_1*, the probability of obtaining a rating* r^h *when the cash flow is* v^ℓ *does not depend on* α_1*, that is,* $(1-\alpha_1)\hat{\sigma}_1$ *is invariant to small changes in* α_1.

Proposition 3 provides one of the key insights of this paper. The intuition behind part (i) is as follows. For the O-type issuer, an increase in the screening intensity α_1 has two effects. First, it implies a greater payoff in period 1 if manipulation is successful (that is, if the issuer obtains a rating r^h even when its cash flow is low). Intuitively, a high rating by the CRA provides certification to investors about the quality of the issuer's asset, and the value of this certification increases as the CRA's screening intensity increases.

Second, an increase in α_1 decreases the likelihood that manipulation is successful. Now, if $\hat{\sigma}_1 \in (0,1)$, the O-type issuer is indifferent between manipulating and not manipulating. Notice that if it manipulates and is unsuccessful, it obtains a payoff of zero in period 1, and the realized state is $s^{\ell\ell}$. The same payoff and state are attained if it does not manipulate. Therefore, for the issuer to be indifferent between manipulating and not manipulating, it must also be indifferent between being successful and being unsuccessful at manipulation. In equilibrium, this indifference is maintained for small changes in α_1 (since $\hat{\sigma}_1$ then remains between zero and one). Thus this second effect is neutral for small changes in α_1. What remains then is the first effect, an increase in the certification effect due to increased CRA scrutiny. This effect leads to an increase in the probability of manipulation.

Part (ii) of the proposition directly implies that a small increase in the screening intensity in period 1 has no effect on the quality of ratings. In particular, the probability that an issuer with low cash flow survives the scrutiny of the CRA and obtains a high rating does not change. An increase in α_1 causes the opportunistic issuer to manipulate more often so that it exactly offsets the effect of more screening by the CRA. As a result, the posterior distribution over types is not affected by the increase in α_1.

3.3 Optimal Screening Intensity in Period 1

We now turn to the CRA's optimal choice of screening intensity in period 1, $\hat{\alpha}_1$. Given the results in Proposition 3, it seems useful to start by examining the conditions under which the O-type issuer is indifferent between manipulating and not manipulating the signal g_1. The following lemma provides necessary and sufficient conditions for the existence of an interior solution $\hat{\sigma}_1 \in (0,1)$.

Lemma 4. *(i) Suppose $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)} < \delta(1-\eta_2^\ell)$. Then, there exists an $\bar{\alpha} \in (0,1)$ such that, in the equilibrium of the continuation game, $\hat{\sigma}_1 \in (0,1)$ if $\alpha_1 < \bar{\alpha}$ and $\hat{\sigma}_1 = 1$ if $\alpha \geq \bar{\alpha}$.*

(ii) Suppose $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)} \geq \delta(1-\eta_2^\ell)$*. Then, for all* $\alpha_1 \geq 0$*,* $\hat{\sigma}_1 = 1$ *in the equilibrium of the continuation game.*

The condition in Lemma 4 (i) is satisfied when η_1 and μ_1 are sufficiently low. For example, if $\mu_1 = 0$, the condition becomes $\delta > \frac{\eta_1}{1-\eta_2^\ell}$. As μ_1 increases, the range of η_1 for which the condition holds shrinks. If $\mu_1 = 1$, the condition cannot be satisfied—it directly violates Assumption 2. Similarly, if $\eta_1 = 0$, the condition is always satisfied. As η_1 increases, the range of μ_1 satisfying the condition shrinks, and at $\eta_1 = 1$ it is violated for all μ_1.

Now let $\bar{\alpha} = 0$ if the condition in part (ii) of Lemma 4 is satisfied. Then, it follows that $\hat{\sigma}_1 \in (0,1)$ whenever $\alpha_1 \in [0,\bar{\alpha})$, and $\hat{\sigma}_1 = 1$ whenever $\alpha_1 \in [\bar{\alpha},1]$. In determining the optimal choice of α_1, we can therefore proceed in two steps. First, find the optimal α_1 separately over the regions $[0,\bar{\alpha})$ and $[\bar{\alpha},1]$. Then, find the overall optimum across the two regions.

First, consider the case in which $\alpha_1 \in [0,\bar{\alpha})$. Notice that the cost to the CRA in period 1 is

$$\psi_1 = \lambda(1-\eta_1)(1-\mu_1)\sigma_1(1-\alpha_1) + c(\alpha_1). \tag{4}$$

From Proposition 3 (ii), we know that a small change in α_1 has no effect on the error probability, but affects the screening cost $c(\alpha_1)$ in period 1. Further, note that α_1 only influences second-period payoffs by altering either the probability of the different states or the posterior beliefs about the type of the firm in each state. As Table 1 shows, α_1 affects these quantities only through the term $\sigma_1(1-\alpha_1)$. Moreover, from Proposition 3, we know that $\sigma_1(1-\alpha_1)$ is invariant to small changes in α_1. Therefore, the choice of α_1 over this region has no effect on any outcome variables in period 2. So the period-2 cost ψ_2 is invariant to the choice of α_1. Therefore, it is immediate that, over this region, the optimal value of α_1 is zero. Thus, the CRA does not screen at all in period 1. The intuition for this result is that changes in α_1 are exactly offset by the issuer's increased manipulation activity, so the CRA optimally saves on screening costs by choosing $\alpha_1 = 0$.

Second, consider the case in which $\alpha_1 \in [\bar{\alpha},1]$. In this region, $\hat{\sigma}_1 = 1$. Therefore, increasing α_1 cannot exacerbate the issuer's manipulation intensity. The optimal value of α_1 is found by a suitable first-order condition. The period-1 cost of the CRA is given by equation (4). In period 2, there are three possible states, as shown in Table 1. The CRA's period-2 cost in state s^{ij} is given by

$$\psi_2^{ij} = \lambda(1-\eta_2^j)(1-\mu_1)(1-\hat{\alpha}_2^{ij}) + c(\hat{\alpha}_2^{ij}). \tag{5}$$

Taking into account the probabilities over the states s^{ij}, the CRA's expected cost over the two periods is

$$\Psi(\alpha_1) = \psi_1 + \delta \left(\eta_1 \psi_2^{hh} + (1-\mu_1)(1-\eta_1)(1-\alpha_1)\psi_2^{h\ell} + (1-(1-\mu_1)(1-\eta_1)(1-\alpha_1))\psi_2^{\ell\ell} \right). \tag{6}$$

As we show in the proof of Lemma 5, the first-order condition yields the implicit equation

$$c'(\alpha_1) = \lambda(1-\eta_1)(1-\mu_1)\left[1 + \delta\left(\frac{c(\hat{\alpha}_2^{h\ell}) - c(\hat{\alpha}_2^{\ell\ell})}{\lambda} - (1-\eta_2^{\ell})(\hat{\alpha}_2^{h\ell} - \hat{\alpha}_2^{\ell\ell})\right)\right]. \tag{7}$$

This is an implicit equation, because $\hat{\alpha}_2^{\ell\ell}$ on the right-hand side is a function of α_1. Denote the solution to this equation by α_h. We call α_h the "heavy" screening intensity.

Lemma 5. *The CRA's optimal choice of α_1 over the region $[\bar{\alpha}, 1]$ is given by the heavy screening intensity α_h.*

Before characterizing the optimal screening intensity in period 1, we show that the heavy screening level α_h is always lower than $\hat{\alpha}$, the optimal screening intensity in the single-period equilibrium. While not the focus of the paper, this comparison illustrates a trade-off that the CRA faces when it chooses its screening level in period 1, even holding the issuer's manipulation decision fixed.

Proposition 4. *The heavy screening intensity in the first period of the two-period game is strictly lower than the optimal screening intensity in the single-period game, that is, $\alpha_h < \hat{\alpha}$.*

Proposition 4 shows that, even at the heavy screening intensity, the CRA chooses to screen less than in the single-period game. In the two-period game, screening in period 1 has a dual role. First, it directly affects the error rate in the first period. Second, it affects the extent to which the CRA learns about the integrity type of the firm. Knowledge about the integrity type is valuable because it leads to more refined screening in period 2. Recall that the CRA screens more intensely in period 2 when it believes that the firm is likely to be of type O (that is, $\alpha_2^{h\ell} > \alpha_2^{hh} > \alpha_2^{\ell\ell}$). Now suppose (as we did in the calculation of α_h) that the O-type issuer manipulates with probability one. By choosing a lower screening level in period 1, the CRA increases the likelihood that the issuer's manipulation attempt in the first period is successful. However, successful manipulation reveals that the firm is of type O. This allows the CRA to provide better screening in period 2. As a result, compared to the single-period case, the intensity of screening is lower in the two-period case.

The overall optimal screening intensity in period 1, $\hat{\alpha}_1$, may now be found as follows. If $\bar{\alpha} = 0$, the optimal intensity is α_h. If $\bar{\alpha} > 0$, the optimal intensity is equal to zero if $\Psi(0) < \Psi(\alpha_h)$, and equal to α_h otherwise. The following proposition provides sufficient conditions for the CRA to not screen in the first period.

Proposition 5. *Suppose* $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)} < \delta(1-\eta_2^\ell)$. *Then, there exists a* $\bar{\lambda} > 0$ *such that if* $\lambda < \bar{\lambda}$, *it is optimal for the CRA to choose a screening intensity of* $\hat{\alpha}_1 = 0$, *whereas if* $\lambda > \bar{\lambda}$, *it is optimal to choose a screening intensity of* $\hat{\alpha}_1 = \alpha_h$. *If* $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)} \geq \delta(1-\eta_2^\ell)$, *it is always optimal for the CRA to choose* $\hat{\alpha}_1 = \alpha_h$.

Proposition 5 is another of the main results of the paper. Intuitively, it is optimal for the CRA to abandon screening in period 1 if the reputational cost from errors is sufficiently low and an additional condition on η_1 and μ_1 is satisfied. The condition on η_1 and μ_1 in Proposition 5 is the same as the condition in Lemma 4. As discussed earlier, it generally holds when η_1 and μ_1 are both sufficiently low. It is important to note that the direct marginal cost of screening is zero when there is no screening (since $c'(0) = 0$). Nevertheless, the endogenous strategic response of the issuer to an increase in the screening level implies that not screening issuers can be optimal for the CRA.

The parameter λ can be interpreted as a measure of market discipline imposed on the CRA. If the market penalizes the CRA for mistakes, λ is high; otherwise, λ is low. Further, one can expect λ to be low in situations in which the market does not learn the value of the asset at the end of period 1, or learns this value with noise. In such situations, the CRA will reduce the screening intensity in period 1.

Proposition 5 implies that when the probability of a high cash flow, η_1, is low, it is optimal to not screen in period 1. This is in stark contrast to the single-period case in which the screening intensity strictly increases as η_1 decreases. The intuition is that when η_1 is low, the benefit to the O-type issuer of manipulating the signal g_1 is also low. Therefore, for a wide range of parameter values of α_1, the O-type issuer manipulates only partially (that is, with a probability of less than one). As a result, over this range, increases in α_1 only encourage further manipulation. From Proposition 3, we know that a small increase in the screening intensity when $\hat{\sigma}_1 \in (0, 1)$ does not affect the probability that the CRA makes an error in the first period or the probability distribution of states in the second period. Thus, an increase in α_1 does not benefit the CRA. As screening is costly, it is therefore optimal to not screen.

Observe that even when the screening intensity in period 1 is zero, the credit rating is still informative about the value of the asset. This is because the honest issuer always discloses the value of its asset truthfully. Specifically, when $\alpha_1 = 0$, the posterior probability that the asset has high value when the CRA receives a high signal is $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)(1-\sigma_1)} > \eta_1$. Conversely, when the CRA receives a low signal, the asset has a low value for sure.

3.4 Comparative Statics

We explore the comparative statics of the optimal screening intensity in period 1, $\hat{\alpha}_1$, in a series of numerical examples.

Example 1: *Changes in η_1 and μ_1.*

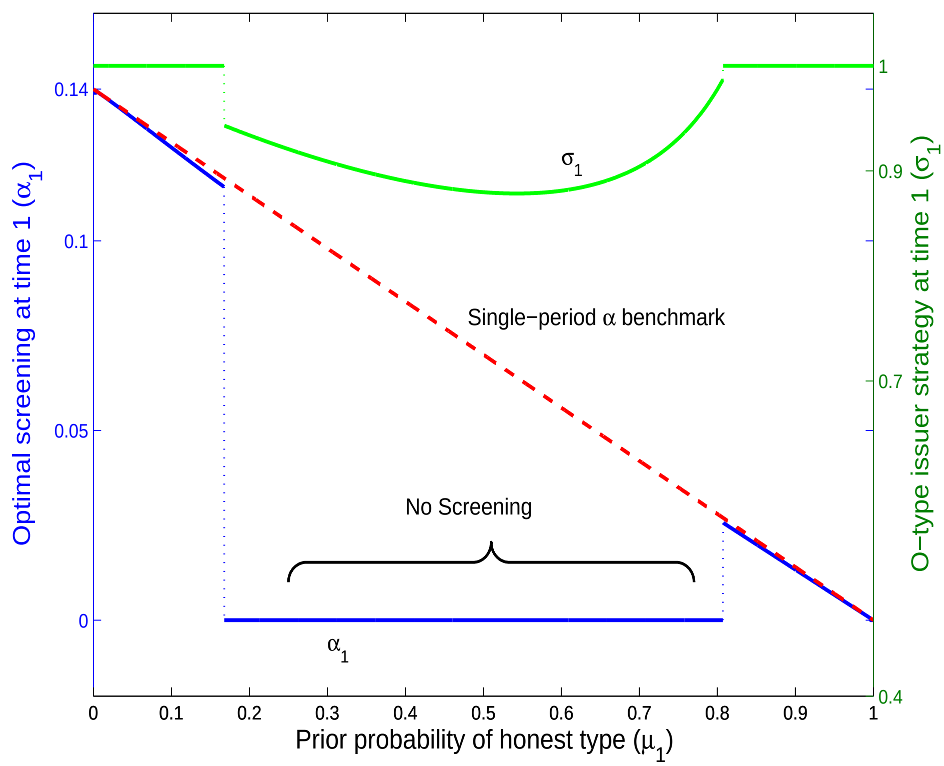
Let $\delta = 1$, $\lambda = 0.4$ and $c(\alpha) = \alpha^2$. We vary η_1 and μ_1 and determine the optimal level of α_1. For simplicity, we assume that $\eta_2^h = \eta_2^\ell = \eta_1$ (i.e., there is no persistence in asset values).

Comparative statics with respect to μ_1.

First, we set $\eta_1 = \eta_2^h = \eta_2^\ell = 0.3$ and determine the optimal screening intensity in period 1 and the equilibrium strategy of the O-type issuer as a function of μ_1. The results are displayed in Figure 2. Given these parameter values, the condition $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)} < \delta(1-\eta_1)$ is equivalent to $\mu_1 > 0.8183$.

We find that for $\mu_1 \in [0.17, 0.82]$, the CRA optimally chooses to not screen in period 1 (i.e., $\hat{\alpha}_1 = 0$). When $\mu_1 < 0.17$ or $\mu_1 > 0.82$, the CRA chooses the heavy screening level α_h. The intuition is as follows. When μ_1 is low, there is little opportunity for O-type issuers to pool with H-type issuers in the second period. Thus, the O-type issuer's incentive to refrain from manipulation in the first period is weak, and it manipulates with high probability (possibly one) even if the screening intensity is low. The CRA's choice of α_1 has little effect on the issuer's behavior in this case, so it chooses a relatively high α_1 in order to better screen out O-type issuers.

When μ_1 is high, the benefit from manipulation is high, as, conditional on observing a high rating r^h, investors place high weight on the issuer being of type H and therefore on the asset having a high value. Again, the O-type issuer manipulates with high probability (possibly one) even if α_1 is low, and so the CRA chooses a positive α_1 to screen out some O-type issuers.



For this figure, we set $\delta = 1$, $\lambda = 0.4$, $c(\alpha) = \alpha^2$, and $\eta_1 = 0.3$. We then determine the optimal α_1 and corresponding σ_1 as μ_1 changes.

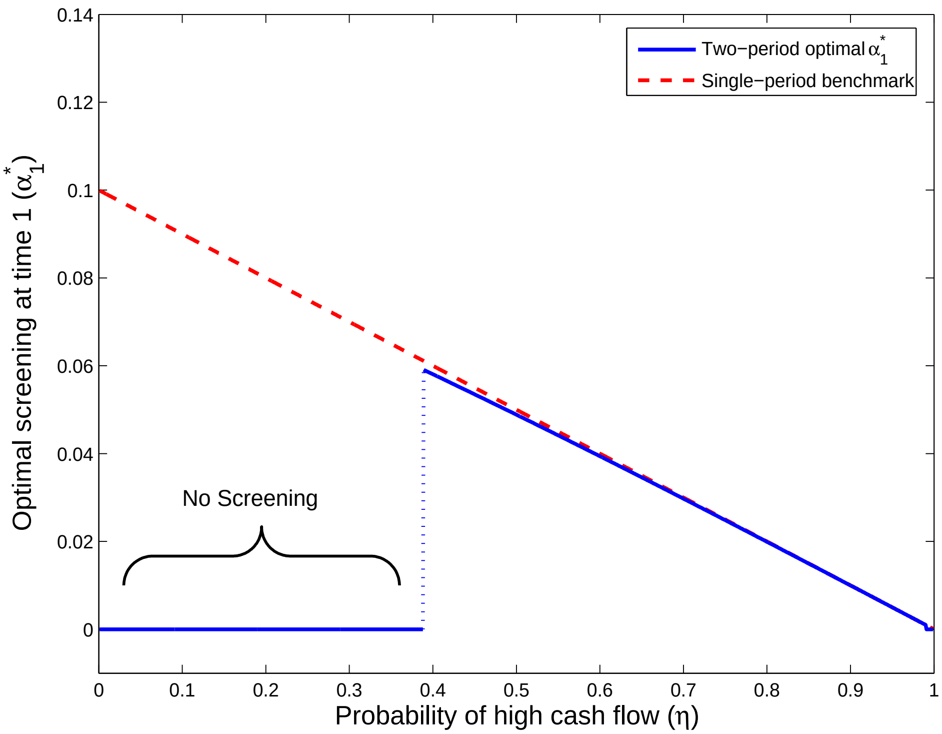
Figure 2: The optimal screening intensity, $\hat{\alpha}_1$, and the equilibrium strategy of the O-type issuer, $\hat{\sigma}_1$, as a function of μ_1

Notice that for both low and high values of μ_1, the CRA chooses a screening intensity high enough to ensure that the O-type issuer manipulates with probability 1 in the first period.

For intermediate values of μ_1, it is costly for O-type issuers to be exposed in period 1, and the gains from successfully manipulating the signal are limited. Therefore, an O-type issuer manipulates only partially for a wide range of parameter values of α_1. In this case, increases in α_1 are offset by more manipulation, leading the CRA to optimally choose $\hat{\alpha}_1 = 0$. Given the lack of screening, in equilibrium the O-type issuer manipulates with probability close to (but strictly less than) 1 in the first period. It is a general feature of our base model that in the first period, in equilibrium the O-type issuer manipulates with probability 1 if and only if the CRA has chosen a non-zero screening intensity.

Comparative statics with respect to η_1.

Next, we set $\mu_1 = 0.5$ and consider variations in η_1. Given these parameter values, the condition $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)} < \delta(1-\eta_1)$ reduces to $\eta_1 < \sqrt{2}-1 \approx 0.4142$. We numerically compute the optimal value of α_1 for $\eta_1 \in [0, 1]$. From Proposition 5, we know that if $\eta_1 > 0.4142$, the optimal value of α_1 is $\alpha_h > 0$. If $\eta_1 < 0.4142$, the optimal level depends on whether or not $\lambda < \bar{\lambda}$.



For this figure, we set $\delta = 1$, $\lambda = 0.4$, $c(\alpha) = \alpha^2$, and $\mu_1 = 0.5$. We then determine the optimal α_1 as η_1 changes.

Figure 3: The optimal screening intensity $\hat{\alpha}_1$ as a function of η_1

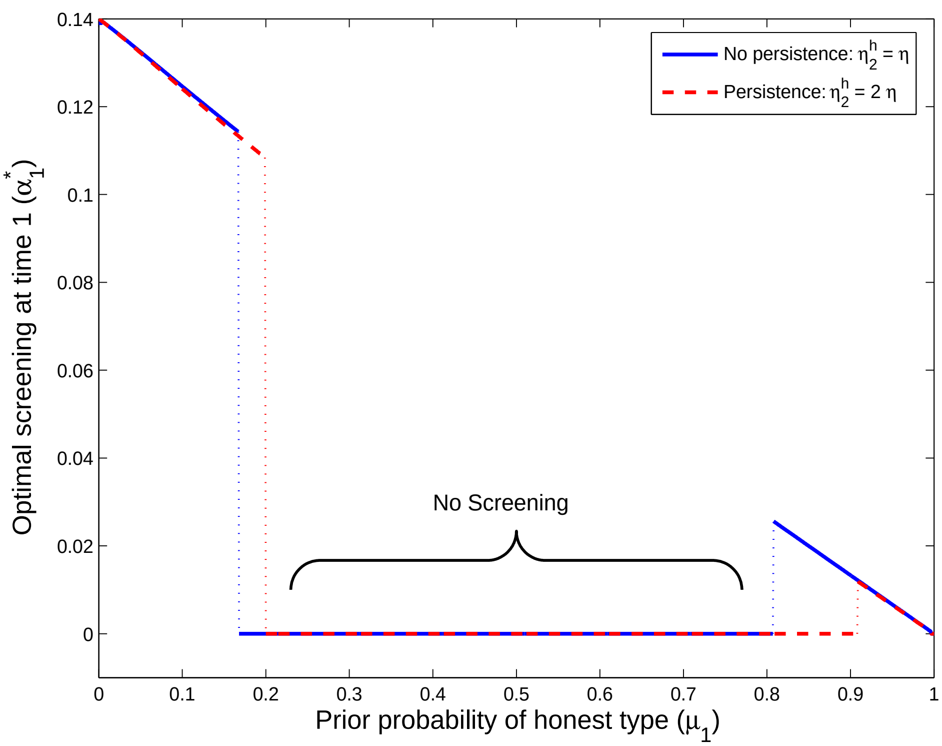
We find that if $\eta_1 < 0.38$, the CRA optimally chooses to not screen (i.e., $\hat{\alpha}_1 = 0$). If $\eta_1 > 0.38$, the CRA optimally chooses heavy screening (i.e., $\hat{\alpha}_1 = \alpha_h$). The intuition is that when η_1 is low, the benefit from manipulation to the O-type issuer is low, so all else equal, the issuer manipulates less. However, as η_1 increases, the benefit from manipulation increases and, given the parameter values, at $\eta_1 \approx 0.38$, the issuer manipulates with probability one. At this point, increasing α_1 does not lead to a further increase in $\hat{\sigma}_1$, so the CRA chooses a strictly positive screening intensity. As a result, at $\eta_1 \approx 0.38$, $\hat{\alpha}_1$ jumps discretely from 0 to about

0.06. Thereafter, it falls in η_1, as expected. Note also that $\alpha_h < \hat{\alpha}$, where $\hat{\alpha}$ is the optimal screening level in the one-period game.

These results are displayed in Figure 3. We do not show the $\hat{\sigma}_1$, the optimal strategy of the O-type issuer in period 1. It remains the case that when $\hat{\alpha}_1 > 0$, the O-type issuer manipulates with probability 1 in the first period, and when $\hat{\alpha}_1 = 0$, it manipulates with probability strictly less than 1.

Example 2: *Effects of persistence in asset quality.*

Let $\eta_1 = 0.3$, $\delta = 1$, $\lambda = 0.4$ and $c(\alpha) = \alpha^2$. We examine the CRA's optimal first period screening intensity, $\hat{\alpha}_1$, for different values of μ_1 when $\eta_2^h = 0.3$ (no persistence) and when $\eta_2^h = 0.6$ (strong persistence).



For this figure, we set $\delta = 1$, $\lambda = 0.4$, $c(\alpha) = \alpha^2$, and $\eta_1 = 0.3$. We then determine the optimal α_1 as μ_1 changes.

Figure 4: The optimal screening intensity $\hat{\alpha}_1$ as a function of μ_1 in the case with and without persistence in asset quality

As shown in Figure 4, the optimal screening intensity remains zero for a large set of intermediate values of μ_1. When it is nonzero, it mostly matches the optimal screening level in the no-persistence case.

Persistence in asset quality has different effects on the issuer's and the CRA's strategy. Holding fixed the CRA's screening intensity in period 1, persistence implies that the O-type issuer is more likely to manipulate in period 1. Recall that the O-type issuer manipulates if $p_1(\alpha_1, \tilde{\sigma}_1) \geq \delta(\pi_2^{\ell\ell} - \pi_2^{h\ell})$. Persistence implies that, after a low outcome in period 1, the probability of a high outcome in period 2 is low. This reduces the difference in payoffs between the states $s^{\ell\ell}$ and $s^{h\ell}$ in period 2, making manipulation more profitable.

Holding fixed the strategy of the O-type issuer, persistence in asset quality makes it more valuable for the CRA to smoke out the O-type issuer so that it can fine tune its screening level in period 2. This requires the screening intensity in period 1 to be low, because the O-type issuer is only caught if it successfully manipulates the signal g_1 (i.e., if it obtains a high rating when its cash flow is low).

These countervailing forces result in screening strategy shown in Figure 4. The broad pattern is similar to the pattern in the case without persistence, with no screening over a large intermediate range of values for μ_1. For low values of μ_1 (in the figure, between 0.17 and 0.23), the CRA is willing to engage in heavy screening to encourage the O-type issuer to manipulate with probability one. For some high values of μ_1 (in the figure, between 0.81 and 0.91), it is optimal to abandon screening in period 1.

4 Extensions

In this section, we consider two extensions to the basic model. First, we consider the case in which the CRA commit to (state-dependent) screening intensities at the beginning of period 1. Second, we add a small direct cost of manipulation for the O-type issuer and examine its effects on the equilibrium.

4.1 Commitment to Screening Intensity in Period 2

Suppose that the CRA can commit to a screening intensity in each state at the beginning of period 1. What effect does this commitment have on the manipulation probability of the O-type issuer? Consider an equilibrium in which the O-type issuer manipulates with a probability of less than one in period 1. We show that an increase in the screening intensity in state $s^{\ell\ell}$ leads to an increase in $\hat{\sigma}_1$. However, changes in the screening intensity in the other two states leave $\hat{\sigma}_1$ unchanged.

Proposition 6. *Suppose that the CRA's period-2 state-contingent screening intensities are known at the beginning of period 1 and consider an equilibrium in which* $\hat{\sigma}_1 \in (0,1)$*. Then,* $\hat{\sigma}_1$ *increases in the screening intensity in state* $s^{\ell\ell}$*, and is independent of the screening intensities in states* $s^{h\ell}$ *and* s^{hh}*. That is,* $\frac{\partial \hat{\sigma}_1}{\partial \alpha_2^{\ell\ell}} > 0$ *and* $\frac{\partial \hat{\sigma}_1}{\partial \alpha_2^{h\ell}} = \frac{\partial \hat{\sigma}_1}{\partial \alpha_2^{hh}} = 0$*.*

An increase in $\alpha_2^{\ell\ell}$ decreases the payoff of the O-type issuer in state $s^{\ell\ell}$, directly increasing his incentive to manipulate the signal g_1 in period 1. This result complements the result in Proposition 3 that when the O-type issuer is indifferent between manipulating and not manipulating, an increase in α_1 increases the probability of manipulation. A change in α^{hh} has no effect on the extent of manipulation in period 1. This is because manipulation itself has no effect if the first-period cash flow is v^h. Finally, in state $s^{h\ell}$, the issuer is known to be of type O. Therefore, the issuer's expected payoff in period 2 is η_2^{ℓ}, regardless of the screening intensity in that state. As a result, changes in the screening intensity do not affect $\hat{\sigma}_1$.

One implication of this result is that, if the CRA can commit to its period-2 screening intensities in advance, it will choose to screen with a lower intensity in state $s^{\ell\ell}$, as compared to the no-commitment case. It chooses the same screening intensities in states $s^{h\ell}$ and s^{hh} as it does without commitment.

4.2 Manipulation Cost for O-type Issuer

The cost of manipulation arises endogenously in the model as a result of the issuer's reputational concerns. We now consider the possibility of a small direct cost that the opportunistic issuer incurs if it manipulates the CRA's signal. This cost could represent the time and effort involved in manipulation, as well as any distortions in issuer policy required to carry out the manipulation.

First, suppose that the game only lasts one period and that all issuers are opportunistic, so that the direct cost is the only cost of manipulating (i.e., there is no reputational cost). Like a reputational cost, the direct cost would constrain the issuer's incentive to manipulate. However, with only the direct cost, the equilibrium probability that the issuer manipulates decreases rather than increases with the CRA's screening intensity.[9] This result is reversed in our base model because the opportunistic type incurs a cost whenever it is successful in manipulating, in the form of a lower payoff in period 2. Thus the endogenous reputational cost of manipulation is key to the paper's results.

[9] Details of this case are omitted for brevity.

Next, we return to our base model with two periods with endogenous reputational costs. Suppose that, in addition, the O-type issuer incurs a small direct cost of manipulation, m. The manipulation cost has effects on the outcomes in both periods. In period 2, if η_2^i is low, the O-type issuer plays a mixed strategy and manipulates with a probability of less than one, because the potential benefit from manipulation is low. The manipulation cost reduces the period-2 payoff by $\sigma_2^{ij}\, m$, where σ_2^{ij} is the equilibrium manipulation probability in state s^{ij}. For high values of η_2^i, the O-type issuer continues to manipulate with probability one in period 2, regardless of the realized state. In this case, the difference $\pi_2^{\ell\ell} - \pi_2^{h\ell}$ remains unchanged. However, the benefit of manipulation in period 1 falls to $p_1(\alpha_1, \tilde{\sigma}_1) - m$, so that the O-type issuer is less likely to manipulate. This results in an increase in the parameter range for which the low screening level is optimal.

Consider equilibria in which the O-type issuer mixes between manipulation and truth-telling in period 1. If the manipulation cost is sufficiently small, an increase in the screening intensity α_1 again leads to an increase in the probability of manipulation, $\hat{\sigma}_1$. However, the increase in $\hat{\sigma}_1$ is smaller than it would be in the absence of a manipulation cost, so that the optimal screening intensity is no longer zero. Instead, the CRA chooses a small but positive screening intensity in this parameter range.

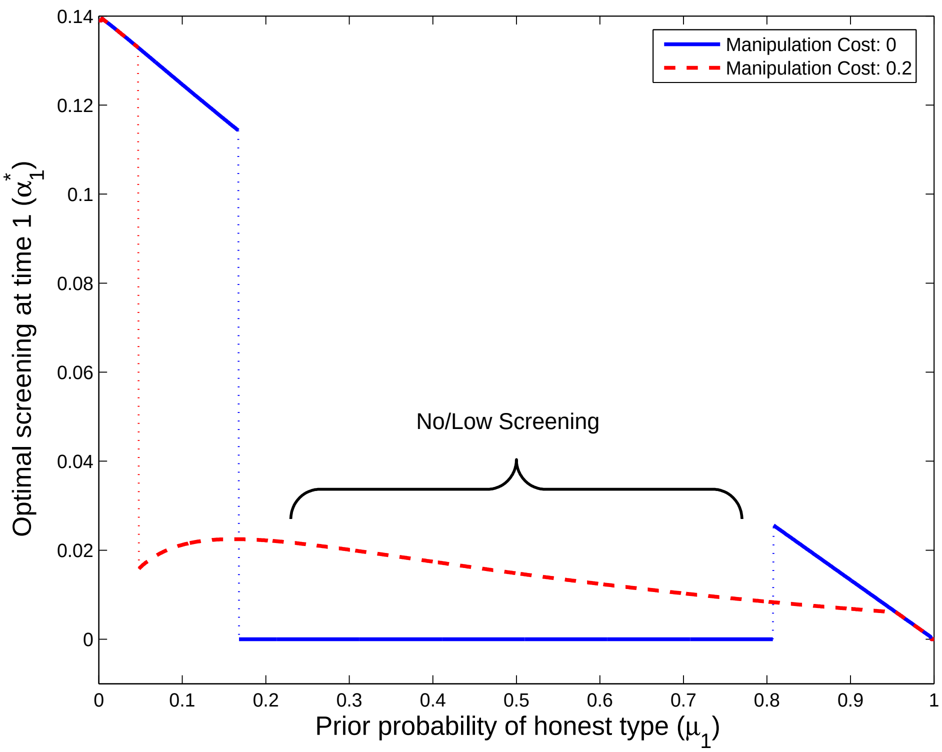
Example 3: *Manipulation cost for the O-type issuer*

As before, we set $\eta_1 = 0.3$, $\delta = 1$, $\lambda = 0.4$, and $c(\alpha) = \alpha^2$. For simplicity, we assume that $\eta_2^h = \eta_2^\ell = \eta_1$ (i.e., there is no persistence in asset quality). We examine the CRA's optimal first-period screening intensity, $\hat{\alpha}_1$, for different values of μ_1 when $m = 0$ (no direct cost of manipulation) and when $m = 0.2$ (a substantial direct cost of manipulation).

As shown in Figure 5, the range of values of μ_1 for which a low screening intensity is optimal increases from $[0.17, 0.81]$ (when $m = 0$) to $[0.05, 0.95]$ (when $m = 0.2$). Further, the low screening level is strictly positive, being about 0.023 when $\mu_1 = 0.15$.

5 Empirical Implications

We now discuss some empirical implications of our model. First, suppose that there is an exogenous increase in the intensity of screening by the CRA at a given time. Such an increase could be triggered by regulatory changes or short-term political pressure that might correspond to an increase in the λ parameter in the model. Our model predicts that there will be weakly more manipulation by issuers, that is, more observed attempts by issuers to game the ratings



For this figure, we set $\delta = 1$, $\lambda = 0.4$, $c(\alpha) = \alpha^2$, and $\eta_1 = 0.3$. We then determine the optimal α_1 as μ_1 changes.

Figure 5: Effect of changing μ_1 on α_1 with and without the O-type incurring a cost of manipulation

system. Such gaming may be observable after the fact through sources such as regulatory audits and court records.

Second, suppose that there is an expectation that the screening intensity of the CRA will change in the future. For example, new online technologies may improve computing and communications, making it easier to analyze and transfer information. This reduces the CRA's cost of obtaining and scrutinizing information from the issuer. Our model predicts that an increase in future screening intensity will also lead to weakly more manipulation by the issuer today.

Our model also has implications for rating agencies' screening decisions and the accuracy of credit ratings. Our analysis implies that a CRA should exert less effort scrutinizing new issues when it rates new assets or when new issuers are numerous. As Figure 2 shows, the CRA abandons screening in period 1 when it is sufficiently unsure about the type of the issuer (that is, when μ_1 takes intermediate values rather than values close to 0 or 1). In practice, a CRA will be unsure about the issuer type either when it is rating a new class of securities (so the ability of the issuer to manipulate information is unknown) or when there is a flood of new issuers on the market. This could explain why CRA ratings of mortgage-backed securities in the 2000s appear to have been more lax than those of corporate issuers. Note also that our model predicts discontinuous jumps as μ_1 changes. So, starting from intermediate beliefs about the issuer's type, small amounts of learning can lead to large positive jumps in observed CRA scrutiny.

One way to interpret the CRA's screening intensity is in terms of the transparency of the ratings process. The key feature of screening in our model is to make manipulation less likely to succeed. A more transparent process is easier for an issuer to manipulate, either by distorting information or by slightly changing the financial claim being issued to meet minimum requirements for a higher rating. Rating transparency does vary over time. For example, in the 2000s, S&P was relatively transparent about the ratings model it used for mortgage-backed securities (see, for example, the description in Griffin and Tang (2011)). Thus, rating transparency may be a useful proxy for the extent of CRA screening.

Our next prediction addresses changes in the CRA's screening intensity over time. Our analysis shows that screening intensity of a new issuer or new class of securities should increase over time. In the model, the CRA screens intensely in the second period because it is the final period of the game: there are no future periods over which the issuer has reputational concerns. In reality, there is generally not a pre-determined terminal period beyond which an issuer ceases to sell assets. However, the CRA learns about the issuer over time as it rates more of the issuer's securities and observes their realized values. As a result of this learning process, the weight that the CRA places on the issuer being honest will tend towards either zero or one over time. Thus, even if one treats the first period as always being the current period (as opposed to comparing periods 1 and 2), the CRA's screening intensity should increase over time as it learns more about the issuer's type. This is a sharp prediction, as one would generally expect more intense screening with newer classes of assets about which less is known.

Finally, our model predicts that CRA screening will be more intense when expected asset quality is high (see Figure 3). This implication must be interpreted carefully, as ratings in our model are binary (high or low). In reality, ratings scales tend to have many values. However, certain thresholds tend to particulary salient for certain types of securities. For example, mortgage-backed securities issuers focus on obtaining AAA ratings. Thus one can imagine that splitting rating categories into AAA and non-AAA captures much of the distinction in MBS quality. The inability of many institutions to hold non-investment grade securities creates a natural split in the corporate bond market. According to our model, CRA scrutiny should be more intense when the "correct" rating for a security is likely to be just above the threshold than when it is likely to be just below the threshold.

6 Conclusion

We argue that strategic disclosure by issuers is an important friction to consider in the ratings process. Our broad message is that the quality of credit ratings depends both on the quality of screening and on the type and discloser strategy of the issuer. With an honest issuer, the CRA can glean the quality of an asset with little effort, so accurate ratings will obtain with little contribution from the CRA. Once an issuer is known to be opportunistic, a CRA will provide intensive screening, but the quality of ratings will remain somewhat noisy because the issuer attempts to manipulate the information revealed. In-between, while the CRA is trying to learn more about issuer type, it provides little screening and ratings are even noisier.

In our model, information manipulation by an opportunistic issuer can sometimes completely unwind the effect of better screening by a credit rating agency. The result is that better screening may simply lead to a greater cost for the CRA without a corresponding benefit in terms of more informative credit ratings. In dealing with a new issuer, it is optimal to reduce screening to allow the CRA to learn about the type of the issuer, and allow the market to generate information about the quality of assets through time. We should therefore expect that screening intensity for new issuers and assets is low for some period of time.

Appendix: Proofs

Proof of Proposition 1

First, we show that the strategies exhibited for the CRA and the O-type firm are mutual best responses, given that the beliefs of investors satisfy Bayes' rule. We then show that the equilibrium is unique.

(i) Given the strategies of the H and O type firms, the CRA chooses α to minimize $\lambda(1-\mu_1)(1-\eta_1)(1-\alpha)+c(\alpha)$. The first-order condition is $c'(\alpha)=\lambda(1-\mu_1)(1-\eta_1)$, which yields $\hat{\alpha}=\phi(\mu_1,\eta_1)$. Assumption 1 implies that $\hat{\alpha}<1$. As $c(\cdot)$ is strictly convex, the second-order condition is satisfied.

(ii) The H-type issuer always reports truthfully, by assumption. Consider the strategy of the O-type issuer. Suppose it manipulates with probability σ. As shown in the text, its payoff can be written as

$$\pi_O = [\eta_1+(1-\eta_1)\sigma(1-\alpha)]p^h+(1-\eta_1)(1-\sigma(1-\alpha))p^\ell \tag{8}$$

$$= \eta_1 p^h+(1-\eta_1)p^\ell+\sigma(1-\alpha)(p^h-p^\ell). \tag{9}$$

Given the strategy of the CRA, $p^h=\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)(1-\hat{\alpha})}>p^\ell=0$. Therefore, it is immediate that $\sigma=1$ is a best response.

(iii) The pricing rule for the asset follows from the strategies of the CRA and issuer, and the fact that investors are competitive so the asset is priced at fair value.

Finally, suppose there is some other perfect Bayesian equilibrium. Since the issuer plays a strict best response given $\hat{\alpha}$, it must be that in the other equilibrium the CRA chooses some other level of α, say $\tilde{\alpha}$. Suppose the O-type issuer manipulates with probability $\tilde{\sigma}$. Then, the price of the asset given a high signal is $\tilde{p}^h=\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)\tilde{\sigma}(1-\tilde{\alpha})}>0$, and the price given a low signal is $\tilde{p}^\ell=0$. Therefore, it is again a strict best response for the O-type issuer to manipulate with probability 1. But if the O-type issuer manipulates with probability 1, $\hat{\alpha}$ is a unique best response. Therefore, there is no other perfect Bayesian equilibrium. ■

Proof of Lemma 1

(i) Consider any state s^{ij}, and suppose the O-type issuer manipulates with probability σ. Then, the price of the asset at time 2 in that state following a high rating is $p_2^{ij}=\frac{\eta_2^i}{\eta_2^i+(1-\eta_2^i)(1-\mu_2^{ij})\sigma(1-\alpha_2^{ij})}$.

Since a low rating always corresponds to a low cash flow, the price of the asset following a low rating is 0.

Now, if the cash flow is high, manipulation has no effect and a high rating is generated, so the O-type issuer earns p_2^{ij}. Suppose the cash flow is low. In each state s^{ij}, we have $\mu_2^{ij} < 1$ (from Table 1) and $\alpha_2^{ij} < 1$ (by assumption in the statement of the lemma). Therefore, $p_2^{ij} > 0$. The O-type issuer earns $(1-\alpha_2^{ij})p_2^{ij} > 0$ by manipulating, and 0 from being truthful. Therefore, it is optimal to manipulate with probability 1.

(ii) The price of the asset in each state follows from setting the manipulation probability of the O-type issuer to 1.

(iii) The O-type issuer earns the price with probability $\eta_2^i+(1-\eta_2^i)(1-\alpha_2^{ij})$. Therefore, its payoff is $(\eta_2^i+(1-\eta_2^i)(1-\alpha_2^{ij}))\frac{\eta_2^i}{\eta_2^i+(1-\mu_2^{ij})(1-\eta_2^i)(1-\alpha_2^{ij})}+(1-\eta_2^i)\alpha_2^{ij}\times 0 = \eta_2^i\left(\frac{\eta_2^i+(1-\eta_2^i)(1-\alpha_2^{ij})}{\eta_2^i+(1-\mu_2^{ij})(1-\eta_2^i)(1-\alpha_2^{ij})}\right) = \eta_2^i\left(1+\frac{\mu_2^{ij}(1-\eta_2^i)(1-\alpha_2^{ij})}{\eta_2^i+(1-\mu_2^{ij})(1-\eta_2^i)(1-\alpha_2^{ij})}\right)$. ■

Proof of Lemma 2

The proof mirrors the proof of Proposition 1 part (i). In state s_2^{ij}, the CRA believes that the probability of the issuer having the H-type is μ_2^{ij}. Suppose the CRA chooses a screening intensity α. The H-type issuer always reports truthfully, and from Lemma 1, the O-type issuer manipulates with probability 1. Given these strategies, an error occurs only if the CRA assigns a rating r^h and the cash flow is v^ℓ. The probability of this event is $(1-\mu_2^{ij})(1-\eta_2^i)(1-\alpha)$. Therefore, the CRA optimally chooses

$$\hat{\alpha}_2^{ij} \quad = \quad \arg\min_{\alpha} \; \lambda(1-\mu_2^{ij})(1-\eta_1)(1-\alpha)+c(\alpha).$$

The first-order condition yields $c'(\alpha) = \lambda(1-\mu_2^{ij})(1-\eta_1)$, so that $\hat{\alpha}_2^{ij} = \phi\left(\mu_2^{ij},\eta_2^i\right)$. As $c(\cdot)$ is convex, it is immediate that the second-order condition is satisfied. ■

Proof of Lemma 3

(i) It is optimal for the type O issuer to manipulate when $\Pi^m \geq \Pi^n$; that is, when $p_1(\alpha_1,\tilde{\sigma}_1) \geq \delta(\pi_2^{\ell\ell}-\pi_2^{h\ell})$. Further, we have $p_1 = \frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)\tilde{\sigma}_1(1-\alpha_1)}$, $\pi_2^{\ell\ell} = \eta_2^\ell\left(1+\frac{\mu_2^{\ell\ell}(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})}{\eta_2^\ell+(1-\mu_2^{\ell\ell})(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})}\right)$, and $\pi_2^{h\ell} = \eta_2^\ell$. Therefore, $\pi_2^{\ell\ell}-\pi_2^{h\ell} = \frac{\mu_2^{\ell\ell}\eta_2^\ell(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})}{\eta_2^\ell+(1-\mu_2^{\ell\ell})(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})}$.

Making the relevant substitutions, manipulation is strictly preferable if

$$\delta\times\frac{\mu_2^{\ell\ell}\eta_2^\ell(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})}{\eta_2^\ell+(1-\mu_2^{\ell\ell})(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})} \quad < \quad \frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)\tilde{\sigma}_1(1-\alpha_1)}. \tag{10}$$

Define $\bar{\delta}$ as follows. Notice that $\bar{\delta}$ depends on $\eta_1, \mu_1, \tilde{\sigma}_1, \alpha_1$ and $\alpha_2^{\ell\ell}$.

$$\bar{\delta} = \frac{\eta_1[\eta_2^\ell + (1-\mu_2^{\ell\ell})(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})]}{[\eta_1 + (1-\eta_1)(1-\mu_1)\tilde{\sigma}_1(1-\alpha_1)]\mu_2^{\ell\ell}\eta_2^\ell(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell})} \tag{11}$$

It is now immediate that manipulation is strictly preferable when $\delta < \bar{\delta}$, and truth-telling is strictly preferable when $\delta > \bar{\delta}$.

(ii) Consider the expression for $\bar{\delta}$ in equation (11). Recall that $\mu_2^{\ell\ell} = \frac{\mu_1}{\mu_1+(1-\mu_1)(1-\tilde{\sigma}_1(1-\alpha_1))}$ is independent of η_1 and $\alpha_2^{\ell\ell}$. Then, by inspection, $\bar{\delta}$ is increasing in η_1 and $\alpha_2^{\ell\ell}$. Further, as $\eta_2^\ell \to 0$ the second term in the large parentheses goes to infinity, with all other terms remaining finite in each case. Therefore, $\bar{\delta} \to \infty$. When $\eta_2^\ell \to 1$, it is important to recognize that $\eta_2^\ell \leq \eta_1$ implies that $\eta_1 \to 1$ as well. Under these conditions, the first term in the large parentheses goes to ∞ while all other terms remain finite, so again $\bar{\delta} \to \infty$.

Finally, consider changes in α_1. Notice that $\mu_2^{\ell\ell}$ is decreasing in α_1. We can write $\bar{\delta} = A(\mu_2^{\ell\ell}, \alpha_2^{\ell\ell}, \eta_2^\ell)\ \frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)\tilde{\sigma}_1(1-\alpha_1)}$, where $A(\mu_2^{\ell\ell}, \alpha_2^{\ell\ell}, \eta_2^\ell)$ is decreasing in $\mu_2^{\ell\ell}$, and therefore increasing in α_1, and the term $\frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)\tilde{\sigma}_1(1-\alpha_1)}$ is also increasing in α_1. Therefore, $\bar{\delta}$ is increasing in α_1.

■

Proof of Proposition 2

(i) Fix α_1, and let $\hat{\sigma}_1$ denote a strategy for the type-O issuer played in a perfect Bayesian equilibrium of the continuation game. Define $G(\sigma_1) = p_1(\sigma_1, \alpha_1) - \delta(\pi_2^{\ell\ell} - \eta_2^\ell)$, where $\pi_2^{\ell\ell}$ in turn depends on σ_1. Substituting in for p_1 and $\pi_2^{\ell\ell}$, we can write

$$G(\hat{\sigma}_1) \quad = \quad \frac{\eta_1}{\eta_1 + (1-\eta_1)(1-\mu_1)\hat{\sigma}_1(1-\alpha_1)} - \frac{\delta\eta_2^\ell(1-\eta_2^\ell)(1-\hat{\alpha}_2^{\ell\ell})\mu_2^{\ell\ell}}{\eta_2^\ell + (1-\eta_2^\ell)(1-\mu_2^{\ell\ell})(1-\hat{\alpha}_2^{\ell\ell})}, \tag{12}$$

where $\hat{\alpha}_2^{\ell\ell}$ represents the equilibrium screening intensity of the CRA at time 2 in state $s^{\ell\ell}$. We show that $G(\cdot)$ is strictly decreasing in $\hat{\sigma}_1$. It is immediate that the first term on the RHS of equation (12) is decreasing in $\hat{\sigma}_1$. Consider the second term on the RHS, and ignore the minus sign for now. The derivative of this term with respect to $\mu_2^{\ell\ell}$ is

$$\frac{\delta\eta_2^\ell(1-\eta_2^\ell)\left(\eta_2^\ell(1-\hat{\alpha}_2^{\ell\ell}) + (1-\eta_2^\ell)(1-\mu_2^{\ell\ell})(1-\hat{\alpha}_2^{\ell\ell})^2 - \eta_2^\ell\mu_2^{\ell\ell}\,\frac{d\hat{\alpha}_2^{\ell\ell}}{d\mu_2^{\ell\ell}}\{1-(1-\eta_2^\ell)(1-\mu_2^{\ell\ell})(1-\hat{\alpha}_2^{\ell\ell})\}\right)}{(\eta_2^\ell(1-\hat{\alpha}_2^{\ell\ell}) + (1-\eta_2^\ell)(1-\mu_2^{\ell\ell})(1-\hat{\alpha}_2^{\ell\ell}))^2}$$

Note that $\hat{\alpha}_2^{\ell\ell} = \phi(\mu_2^{\ell\ell}, \eta_2^{\ell\ell})$ is a decreasing function of $\mu_2^{\ell\ell}$. Since $\hat{\alpha}_2^{\ell\ell}$ is strictly decreasing in $\mu_2^{\ell\ell}$ and $\hat{\alpha}_2^{\ell\ell} < 1$, it follows that the second term on the RHS of equation (12) is strictly increasing in $\mu_2^{\ell\ell}$. Now, recall that $\mu_2^{\ell\ell} = \frac{\mu_1}{\mu_1+(1-\mu_1)(1-\hat{\sigma}_1(1-\alpha_1))}$ is strictly increasing in $\hat{\sigma}_1$. It then follows that the second term on the RHS of equation (12), ignoring the minus sign, is strictly increasing in $\hat{\sigma}_1$. That is, taking account of the minus sign, this term too is strictly decreasing in $\hat{\sigma}_1$. Therefore, $G(\cdot)$ is strictly decreasing in $\hat{\sigma}_1$.

As $G(\cdot)$ is strictly decreasing in $\hat{\sigma}_1$, it follows that in equilibrium $\hat{\sigma}_1 = 0$ only if $G(0) \leq 0$, $\hat{\sigma}_1 = 1$ only if $G(1) \geq 0$, and $\hat{\sigma}_1 \in (0,1)$ only if $G(\hat{\sigma}_1) = 0$. Because $G(\cdot)$ is strictly decreasing in $\hat{\sigma}_1$, the equilibrium is unique.

(ii) Suppose that, in an equilibrium of the continuation game after α_1 has been chosen, $\hat{\sigma}_1 = 0$. Then, it must be that $\tilde{\sigma}_1 = 0$ as well. Then, $\mu_2^{\ell\ell} = \mu_1$. Consider the expression for $\bar{\delta}$ in equation (11). Substituting in $\tilde{\sigma}_1 = 0$ and $\mu_2^{\ell\ell} = \mu_1$, we have

$$\bar{\delta} \quad = \quad \frac{1}{(1-\eta_2^{\ell})(1-\alpha_2^{\ell\ell})}.$$

Now, from Assumption 2, $\delta < \frac{1}{1-\eta_2^{\ell}}$ and from Assumption 1, $\alpha_2^{\ell\ell} < 1$. Therefore, it follows that $\delta < \bar{\delta}$ for all values of μ_1, so that from Lemma 3, it is a best response for the firm to manipulate with probability 1. That is, there cannot be an equilibrium of the continuation game in which $\hat{\sigma}_1 = 0$, so that in any continuation equilibrium $\hat{\sigma}_1 > 0$. ∎

Proof of Proposition 3

(i) Suppose that for a given α_1, $\hat{\sigma}_1(\alpha_1) \in (0,1)$. Then, recalling the definition of $G(\hat{\sigma}_1)$ in equation (12) in the proof of Proposition 2, it must be that $G(\hat{\sigma}_1) = 0$. Denote $z = \hat{\sigma}_1(1-\alpha_1)$. Notice that $\mu_2^{\ell\ell} = \frac{\mu_1}{\mu_1+(1-\mu_1)(1-z)}$, which we write as $\mu_2^{\ell\ell}(z)$. This further implies that $\hat{\alpha}_2^{\ell\ell}$ is a function of z. Substituting $z = \hat{\sigma}_1(1-\alpha_1)$ into the expression for $G(\hat{\sigma}_1)$ in equation (12) and setting $G(\hat{\sigma}_1) = 0$, we have

$$\frac{\eta_1}{\eta_1 + (1-\eta_1)(1-\mu_1)z} \quad = \quad \frac{\delta\eta_2^{\ell}(1-\eta_2^{\ell})(1-\hat{\alpha}_2^{\ell\ell}(z))\mu_2^{\ell\ell}(z)}{\eta_2^{\ell} + (1-\eta_2^{\ell})(1-\mu_2^{\ell\ell}(z))(1-\hat{\alpha}_2^{\ell\ell}(z))}. \tag{13}$$

In the proof of Proposition 2 (i), we have shown that the term on the RHS of equation (13) is strictly increasing in $\mu_2^{\ell\ell}$. Further, $\mu_2^{\ell\ell}(z)$ is strictly increasing in z, so that the RHS of equation (13) is strictly increasing in z. The LHS is clearly decreasing in z.

Observe that α_1 and $\hat{\sigma}_1$ do not directly affect equation (13), except through z. Suppose $\hat{\sigma}_1 \in (0,1)$ so that equation (13) holds for some $z = \hat{z}$. As the left-hand side is strictly decreasing in z and the right-hand side is strictly increasing, it follows that $\hat{z}$ is a unique solution to equation (13). Now, consider a small change in α_1. As $\hat{z}$ is unique, it follows that the change in α_1 cannot change $\hat{z}$. By construction, it must be that $\hat{\sigma}_1$ is such that $(1-\alpha_1)\hat{\sigma}_1 = \hat{z}$ and $\hat{\sigma}_1 \in [0,1]$ is an equilibrium strategy for type O. Consider any α_1 such that the equilibrium mixing probability $\hat{\sigma}_1$ lies in $(0,1)$. It follows that $\hat{\sigma}_1(\alpha_1) = \frac{\hat{z}}{1-\alpha_1}$ is strictly increasing in α_1.

(ii) For the O-type issuer, the probability of obtaining a rating r^h when the cash flow is v^ℓ is $(1-\alpha_1)\hat{\sigma}_1$. Consider any equilibrium in which $\hat{\sigma}_1(\alpha_1) \in (0,1)$. Then, equation (13) holds for some $\hat{z}$. As mentioned in the proof of part (i), $\hat{z}$ is invariant to changes in α_1; that is, $(1-\alpha_1)\hat{\sigma}_1$ is invariant to small changes α_1 (if there is a large change in α_1, then the type O-firm may no longer be indifferent between manipulating and truthful disclosure). ■

Proof of Lemma 4

As in the proof of Proposition 3, define $z = \hat{\sigma}_1(1-\alpha_1)$. Let $f(z) = \frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)z}$ and let $g(z) = \frac{\delta\eta_2^\ell(1-\eta_2^\ell)(1-\hat{\alpha}_2^{\ell\ell}(z))\mu_2^{\ell\ell}(z)}{\eta_2^\ell+(1-\eta_2^\ell)(1-\mu_2^{\ell\ell}(z))(1-\hat{\alpha}_2^{\ell\ell}(z))}$. It is immediate that $f(z)$ is strictly decreasing in z and in the proof of Proposition 3, we have shown that $g(z)$ is strictly increasing in z.

Now, there are two possibilities:

(i) $f(1-\alpha_1) > g(1-\alpha_1)$. In this case, $G(\sigma_1) > 0$ for any $\sigma_1 \in [0,1]$, where $G(\sigma_1)$ is defined in the proof of Proposition 3, so in equilibrium $\hat{\sigma}_1(\alpha_1) = 1$.

(ii) There exists a $\hat{z} \in (0, 1-\alpha_1]$ such that $f(\hat{z}) = g(\hat{z})$. Then, in equilibrium $\hat{\sigma}_1(\alpha_1) = \frac{\hat{z}}{(1-\alpha_1)}$.

Now, Proposition 2 implies that $f(0) > g(0)$. Observe that $f(1) = \frac{\eta_1}{\eta_1+(1-\eta_1)(1-\mu_1)}$. Further, $\mu_2^{\ell\ell}(1) = 1$ and $\hat{\alpha}_2^{\ell\ell}(1) = 0$. Therefore, $g(1) = \delta(1-\eta_2^\ell)$. Therefore, if

$$\frac{\eta_1}{\eta_1 + (1-\eta_1)(1-\mu_1)} \quad \geq \quad \delta(1-\eta_2^\ell), \tag{14}$$

it follows that for any $\alpha_1 \geq 0$, in equilibrium $\hat{\sigma}_1(\alpha_1) = 1$, proving part (ii) of the Lemma.

For part (i), suppose instead, that

$$\frac{\eta_1}{\eta_1 + (1-\eta_1)(1-\mu_1)} \quad < \quad \delta(1-\eta_2^\ell). \tag{15}$$

Then, there exists some $\hat{z} \in [0,1]$ such that $f(\hat{z}) = g(\hat{z})$. Define $\alpha_a = 1-\hat{z}$. Then, by definition $\hat{\sigma}_1(\alpha_a) = 1$. Further, as the function $\frac{\hat{z}}{1-\alpha_1}$ is increasing in α_1, it follows $\hat{\sigma}_1(\alpha_1) \in (0,1)$ for all $\alpha_1 < \alpha_a$ and $\hat{\sigma}_1(\alpha_1) = 1$ for all $\alpha_1 > \alpha_a$. ■

Proof of Lemma 5

The objective function of the CRA is $\Psi = \psi_1 + \delta\psi_2$. Taking an expectation over states at time 2, we have

$$\Psi = \psi_1 + \delta\{\eta_1\psi_2^{hh} + (1-\eta_1)[(1-\mu_1)\sigma_1(1-\alpha_1)\psi_2^{h\ell} + (1-(1-\mu_1)\sigma_1(1-\alpha_1))\psi_2^{\ell\ell}]\} \quad (16)$$

Here,

$$\begin{aligned} \psi_2^{hh} &= \lambda(1-\eta_2^h)(1-\mu_1)(1-\phi(\mu_1)) + c(\phi(\mu_1)) \\ \psi_2^{h\ell} &= \lambda(1-\eta_2^\ell)(1-\phi(1)) + c(\phi(1)). \end{aligned}$$

That is, neither ψ_2^{hh} nor $\psi_2^{h\ell}$ depend on σ_1 or on α_1.

Further,

$$\psi_2^{\ell\ell} = \lambda(1-\eta_2^\ell)(1-\mu_2^{\ell\ell})(1-\hat{\alpha}_2^{\ell\ell}) + c(\hat{\alpha}_2^{\ell\ell}).$$

For the rest of this proof, for convenience we write $\alpha_2^{\ell\ell}$ for $\hat{\alpha}_2^{\ell\ell}$. Also, recall that $\mu_2^{\ell\ell} = 1 - \frac{\mu_1}{\mu_1+(1-\mu_1)(1-\sigma_1(1-\alpha_1))}$.

Using $z = \hat{\sigma}_1(1-\alpha_1)$, we can write

$$\begin{aligned} \Psi &= c(\alpha_1) + \lambda(1-\eta_1)(1-\mu_1)z \\ &\quad + \delta\eta_1\psi_2^{hh} + \delta(1-\eta_1)\{(1-\mu_1)z\psi_2^{h\ell} + (1-(1-\mu_1)z)\psi_2^{\ell\ell}(z)]\}. \end{aligned}$$

We now have $\frac{\partial\Psi}{\partial\alpha_1} = c'(\alpha_1) + \frac{\partial\Psi}{\partial z}\frac{\partial z}{\partial\alpha_1}$. Consider the term $\frac{\partial\Psi}{\partial z}$. We have

$$\frac{\partial\Psi}{\partial z} = \lambda(1-\eta_1)(1-\mu_1) + \delta(1-\eta_1)(1-\mu_1)\{\psi_2^{h\ell} - \psi_2^{\ell\ell}\} + \delta(1-\eta_1)\{1-(1-\mu_1)z\}\frac{\partial\psi_2^{\ell\ell}}{\partial z} \quad (17)$$

Now,

$$\begin{aligned} \frac{\partial\psi_2^{\ell\ell}}{\partial z} &= c'(\alpha_2^{\ell\ell})\frac{\partial\alpha_2^{\ell\ell}}{\partial z} - \lambda\left((1-\eta_2^\ell)\left(1-\frac{\mu_1}{\mu_1+(1-\mu_1)(1-z)}\right)\right)\frac{\partial\alpha_2^{\ell\ell}}{\partial z} \\ &\quad -\lambda(1-\eta_2^\ell)\frac{\mu_1(1-\mu_1)}{[\mu_1+(1-\mu_1)(1-z)]^2}(1-\alpha_2^{\ell\ell}). \end{aligned} \quad (18)$$

Recall that $\alpha_2^{\ell\ell}$ satisfies $c'(\alpha_2^{\ell\ell}) = \lambda(1-\eta_2^\ell)\left(1-\frac{\mu_1}{\mu_1+(1-\mu_1)(1-z)}\right)$, so we can simplify $\frac{\partial\psi_2^{\ell\ell}}{\partial z} = -\lambda(1-\eta_2^\ell)\frac{\mu_1(1-\mu_1)}{[\mu_1+(1-\mu_1)(1-z)]^2}(1-\alpha_2^{\ell\ell})$.

Substituting into equation (17), we get

$$\begin{aligned}\frac{\partial \Psi}{\partial z} &= \lambda(1-\eta_1)(1-\mu_1)+\delta(1-\eta_1)(1-\mu_1)\{\psi_2^{h\ell}-\psi_2^{\ell\ell}\}\\ &\quad -\lambda\delta(1-\eta_1)\{1-(1-\mu_1)z\}(1-\eta_2^{\ell})\frac{\mu_1(1-\mu_1)}{[\mu_1+(1-\mu_1)(1-z)]^2}(1-\alpha_2^{\ell\ell}).\end{aligned}$$

Note that $\mu_1+(1-\mu_1)(1-z)=1-(1-\mu_1)z$. Therefore,

$$\begin{aligned}\frac{\partial \Psi}{\partial z} &= \lambda(1-\eta_1)(1-\mu_1)+\delta(1-\eta_1)(1-\mu_1)\{\psi_2^{h\ell}-\psi_2^{\ell\ell}\}\\ &\quad -\lambda\delta(1-\eta_1)\frac{\mu_1(1-\mu_1)(1-\eta_2^{\ell})}{[\mu_1+(1-\mu_1)(1-z)]}(1-\alpha_2^{\ell\ell})\\ \text{Or, } \frac{1}{\lambda(1-\eta_1)(1-\mu_1)}\frac{\partial \Psi}{\partial z} &= 1+\frac{\delta}{\lambda}\{\psi_2^{h\ell}-\psi_2^{\ell\ell}\}-\delta\frac{\mu_1(1-\eta_2^{\ell})}{[\mu_1+(1-\mu_1)(1-z)]}(1-\alpha_2^{\ell\ell}).\end{aligned}$$

Observe that the last term can be written as $\delta(1-\eta_2^{\ell})\mu_2^{\ell\ell}(1-\alpha_2^{\ell\ell})$. Using this and substituting in for $\psi_2^{h\ell}$ and $\psi_2^{\ell\ell}$, we have

$$\frac{1}{\lambda(1-\eta_1)(1-\mu_1)}\frac{\partial \Psi}{\partial z} = 1+\delta\left\{\frac{c(\alpha_2^{h\ell})-c(\alpha_2^{\ell\ell})}{\lambda}\right\}-\delta(1-\eta_2^{\ell})(\alpha_2^{h\ell}-\alpha_2^{\ell\ell}).$$

That is,

$$\frac{\partial \Psi}{\partial z} = \lambda(1-\eta_1)(1-\mu_1)[1+\delta\left\{\frac{c(\alpha_2^{h\ell})-c(\alpha_2^{\ell\ell})}{\lambda}\right\}-\delta(1-\eta_2^{\ell})(\alpha_2^{h\ell}-\alpha_2^{\ell\ell})]. \tag{19}$$

Now, when $\sigma_1=1$, we have $\frac{\partial z}{\partial \alpha_1}=-1$. Therefore, when $\sigma_1=1$,

$$\frac{\partial \Psi}{\partial \alpha_1} = c'(\alpha_1)-\lambda(1-\eta_1)(1-\mu_1)\left[1+\delta\left\{\frac{c(\alpha_2^{h\ell})-c(\alpha_2^{\ell\ell})}{\lambda}\right\}-\delta(1-\eta_2^{\ell})(\alpha_2^{h\ell}-\alpha_2^{\ell\ell})\right] \tag{20}$$

Recall that $\alpha_2^{h\ell}>\alpha_2^{\ell\ell}$, so $c(\alpha_2^{h\ell})>c(\alpha_2^{\ell\ell})$. Further, $\alpha_2^{h\ell}-\alpha_2^{\ell\ell}<1$. Further, under Assumption 2, $\delta(1-\eta_2^{\ell})\leq 1$. Therefore, $\delta(1-\eta_2^{\ell})(\alpha_2^{h\ell}-\alpha_2^{\ell\ell})<1$, so that

$$\lambda(1-\eta_1)(1-\mu_1)\left[1+\delta\left\{\frac{c(\alpha_2^{h\ell})-c(\alpha_2^{\ell\ell})}{\lambda}\right\}-\delta(1-\eta_2^{\ell})(\alpha_2^{h\ell}-\alpha_2^{\ell\ell})\right] > 0.$$

That is, $\frac{\partial \Psi}{\partial z}>0$.

Denote $h(\alpha_1)=\frac{\partial \Psi}{\partial z}=\lambda(1-\eta_1)(1-\mu_1)[1+\delta\{c(\alpha_2^{h\ell})-c(\alpha_2^{\ell\ell})\}/\lambda-\delta(1-\eta_2^{\ell})(\alpha_2^{h\ell}-\alpha_2^{\ell\ell})]$.

Then, we can write equation (20) as

$$\frac{\partial \Psi}{\partial \alpha_1} = c'(\alpha_1) - h(\alpha_1).$$

Further, $c'(0) = 0$ whereas $h(0) > 0$. Also, $c'(1) > c(1)$ (because $c(\cdot)$ is convex) and $c(1) > h(1)$. Therefore, the optimal value of α_1 is given by the first-order condition $c'(\alpha_1) = h(\alpha_1)$, or

$$c'(\alpha_1) = \lambda(1-\eta_1)(1-\mu_1)\left[1 + \delta\left\{\frac{c(\alpha_2^{h\ell}) - c(\alpha_2^{\ell\ell})}{\lambda}\right\} - \delta(1-\eta_2^{\ell})(\alpha_2^{h\ell} - \alpha_2^{\ell\ell})\right]. \quad (21)$$

■

Proof of Proposition 4

Recall that α_h satisfies equation (7),

$$c'(\alpha_1) = \lambda(1-\eta_1)(1-\mu_1)[1 + (\delta/\lambda)\{c(\alpha_2^{h\ell}) - c(\alpha_2^{\ell\ell})\} - \delta(1-\eta_2^{\ell})(\alpha_2^{h\ell} - \alpha_2^{\ell\ell})\}].$$

Here, $\alpha_2^{\ell\ell}$ is in turn a function of α_1.

Similarly, from Proposition 1, $\hat{\alpha}$ satisfies the equation $c'(\hat{\alpha}) = \lambda(1-\mu_1)(1-\eta_1)$. Therefore, if

$$c(\alpha_2^{h\ell}) - c(\alpha_2^{\ell\ell}) < \lambda(1-\eta_2^{\ell})(\alpha_2^{h\ell} - \alpha_2^{\ell\ell}), \quad (22)$$

it follows that $c'(\alpha_1) < c'(\hat{\alpha})$ so that $\alpha_1 < \hat{\alpha}$. We show in the remainder of the proof that equation (22) holds.

Consider the function $w(x) = c(x) - \lambda(1-\eta_2^{\ell})x$. The derivative is $w'(x) = c'(x) - \lambda(1-\eta_2^{\ell})$. Recall that $\phi(\mu, \eta_1) = c'^{-1}(\lambda(1-\eta_1)(1-\mu))$. Therefore, $w'(x) = 0$ for $w = \phi(0, \eta_2^{\ell})$ and $w'(x) < 0$ for all $x < \phi(0, \eta_2^{\ell})$. That is, $w(\cdot)$ is strictly decreasing in the range $[0, \phi(0, \eta_2^{\ell}))$.

Observe that $\phi(0, \eta_2^{\ell})$ is the highest value of α that will be chosen by CRA in the one-shot game when the probability of a high cash flow is η_2^{ℓ}, and corresponds to the screening level when $\mu = 0$; that is, the CRA believes the firm has the O-type with probability 1. Further, $\alpha_2^{h\ell} = \phi(0, \eta_2^{\ell})$, and for any α_1, we have $\alpha_2^{\ell\ell} = \phi\left(\frac{\mu_1}{\mu_1 + (1-\mu_1)(1-\sigma_1(1-\alpha_1))}\right) < \phi(0, \eta_2^{\ell})$. Since $w(\cdot)$ is strictly decreasing in the range $[0, \phi(0, \eta_2^{\ell}))$, it follows that $w(\alpha_2^{\ell\ell}) > w(\alpha_2^{h\ell})$. This directly implies that equation (22) holds. ■

Proof of Proposition 5

(i) Suppose $\frac{\eta_1}{\eta_1 + (1-\eta_1)(1-\mu_1)} < \delta(1-\eta_2^{\ell})$. The proof proceeds in two steps.

Step 1: If $\hat{\sigma}_1(\alpha_h) < 1$, then the optimal α_1 is 0.

Proof of Step 1: Suppose that $\hat{\sigma}_1(\alpha_h) < 1$. From Proposition 3 part (i), it follows that $\hat{\sigma}_1(0) < 1$.

Recall that the overall two-period cost to the CRA is

$$\Psi(\alpha_1) \;\; = \;\; \psi_1 + \delta[\eta_1 \psi_2^{hh} + \psi_2^{\ell\ell} + (1-\mu_1)(1-\eta_1)(1-\alpha_1)\{\psi_2^{h\ell} - \psi_2^{\ell\ell}\}].$$

Let $z = \hat{\sigma}_1(1-\alpha_1)$. From Proposition 3 part (ii), for any α_1 such that $\hat{\sigma}_1(\alpha_1) < 1$, we know that z is a constant. That is, $\frac{\partial z}{\partial \alpha_1} = 0$.

Now, from the proof of Lemma 5, we have $\frac{d\Psi}{d\alpha_1} = c'(\alpha_1) + \frac{\partial \Psi}{\partial z}\frac{\partial z}{\partial \alpha_1} = c'(\alpha_1) > 0$ whenever z is constant. With a slight abuse of notation, we write the CRA's objective function as $\Psi(\alpha, \sigma)$. Then, $\Psi(0, \hat{\sigma}_1(0)) < \Psi(\alpha_1, \hat{\sigma}_1(\alpha_1))$ for any α_1 such that $\hat{\sigma}_1(\alpha_1) < 1$.

Next, we need to show that $\Psi(0, \hat{\sigma}_1(0)) < \Psi(\alpha_1, 1)$ for any α_1 such that $\hat{\sigma}_1(\alpha_1) = 1$. Observe that $\frac{\partial \Psi}{\partial \sigma_1} = (1-\alpha)\frac{\partial \Psi}{\partial z}$. As observed in the proof of Lemma 5, under Assumption 2, it follows that $\frac{\partial \Psi}{\partial z} > 0$. Therefore, $\frac{\partial \Psi}{\partial \sigma_1} > 0$.

Then, $\Psi(\alpha_h, \hat{\sigma}_1(\alpha_h)) \leq \Psi(\alpha_h, 1)$, because the error rate is strictly lower when σ_1 is lower. Further, by definition, $\Psi(\alpha_h, 1) < \Psi(\alpha_1, 1)$ for any $\alpha_1 \neq \alpha_h$. Therefore, $\Psi(\alpha_h, \hat{\sigma}_1(\alpha_h)) < \Psi(\alpha_1, 1)$ for any α_1 such that $\sigma_1(\alpha_1) = 1$.

But we know that $\Psi(0, \hat{\sigma}_1(0)) < \Psi(\alpha_1, 1)$ for any α_1 such that $\hat{\sigma}_1(\alpha_1) = 1$. Therefore, the CRA minimizes its overall cost by choosing $\alpha_1 = 0$ whenever $\hat{\sigma}_1(\alpha_h) < 1$.

Step 2: There exists $\bar{\lambda} > 0$ such that $\alpha_h < \bar{\alpha}$ for all $\lambda < \bar{\lambda}$, where $\bar{\alpha}$ is as defined in Lemma 4.

Proof of Step 2: Consider equation (7) that defines α_h. We can write this equation as

$$c'(\alpha_1) \;\; = \;\; (1-\eta_1)(1-\mu_1)[\lambda + (\delta\{c(\hat{\alpha}_2^{h\ell}) - c(\hat{\alpha}_2^{\ell\ell})\} - \delta\lambda(1-\eta_2^{\ell})(\hat{\alpha}_2^{h\ell} - \hat{\alpha}_2^{\ell\ell})\}].$$

The only terms on the right-hand side of the equation that contain α_1 are those that include $\hat{\alpha}_2^{\ell\ell}$.

From the first-order condition on the CRA's problem in period 2, in each state ij, $\hat{\alpha}_2^{ij}$ decreases with λ and goes to 0 as λ goes to 0. Thus the right-hand side of (23) goes to zero as $\lambda \to 0$. Therefore, the left-hand side must also go to zero as $\lambda \to 0$; that is, $\alpha_h \to 0$ as $\lambda \to 0$. By continuity, there must exist then $\lambda_0 > 0$ such that, if $\lambda < \lambda_0$, then $\alpha_h < \bar{\alpha}$.

At $\lambda = \lambda_0$, we have $\hat{\sigma}_1(\alpha_h) = 1$. Observe that the expression $u(\lambda) = \Psi(\alpha_h) - \Psi(0)$ is decreasing in λ. There are two possibilities:

1. $\Psi(\alpha_h) < \Psi(0)$ when $\lambda = \lambda_0$. Then, it is optimal to choose $\alpha_1 = 0$ for $\lambda < \lambda_0$ and $\alpha = \alpha_h$ for $\lambda \geq \lambda_0$. In this case, set $\bar{\lambda} = \lambda_0$.

2. $\Psi(\alpha_h) > \Psi(0)$. As λ increases, there exists some $\bar{\lambda}$ such that $\Psi(\alpha_h) = \Psi(0)$. Then, for $\lambda < \bar{\lambda}$, it is optimal to choose $\alpha_1 = 0$, and for $\lambda > \bar{\lambda}$, it is optimal to choose $\alpha_1 = \alpha_h$.

(ii) Suppose $\frac{\eta_1}{\eta_1 + (1-\eta_1)(1-\mu_1)} > \delta(1-\eta_2^\ell)$. Then, from Lemma 4 (ii), for all $\alpha_1 \geq 0$, in the equilibrium of the continuation game $\hat{\sigma}_1(\alpha) = 1$. It follows that it is optimal to set $\alpha_1 = \alpha_h$.

■

Proof of Proposition 6

Treating $\hat{\alpha}_2^{\ell\ell}$ as fixed, we can solve the O-type issuer's indifference condition (equation (13) in the proof of Proposition 3) for σ_1 in closed form:

$$\sigma_1 = \frac{\eta_1\{1-(1-\eta_2^\ell)[(1-\alpha_2^{\ell\ell})(1+\delta\eta_2^\ell)\mu_1 + \alpha_2^{\ell\ell}]\}}{(1-\alpha_1)(1-\mu_1)\{\delta\mu_1\eta_2^\ell(1-\eta_1)(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell}) + \eta_1[1-\alpha_2^{\ell\ell}(1-\eta_2^\ell)]\}}$$

This is invariant to $\alpha_2^{h\ell}$ and α_2^{hh}. Taking a derivative with respect to $\alpha_2^{\ell\ell}$, we have

$$\frac{\partial\sigma_1}{\partial\alpha_2^{\ell\ell}} = \frac{\eta_1\eta_2^\ell(1-\eta_2^\ell)(\eta_1+\delta\eta_2^\ell)\mu_1}{(1-\alpha_1)(1-\mu_1)\{\delta\mu_1\eta_2^\ell(1-\eta_1)(1-\eta_2^\ell)(1-\alpha_2^{\ell\ell}) + \eta_1[1-\alpha_2^{\ell\ell}(1-\eta_2^\ell)]\}^2}.$$

This is unambiguously positive. ■

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