ROAD SPACING & COST FORMULAE FOR STANDS OF TWO OR MORE PRODUCTS.

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CONTENTS

FOREWORD

The basis of this thesis originated with the problem of determining a means for applying the road spacing and applicable formulae suggested by Prof. D.M. Mathews in his book COST CONTROL IN THE LOGGING INDUSTRY to Northern Michigan mixed stands of hemlock logs, spruce and fir pulp, and white cedar posts. Realization that formulae based upon a single product per acre were not applicable to situations of this type. lead to this study of the subject.

The methods applied throughout this paper are those taught by Prof. Mathews and the formulae derived are merely elaborations of his originals.

ROAD SPACING FORMULA-INVOLVING TWO OR MORE PRODUCTS PER ACRE.

The prime purpose of this formula derivation is to provide a means to determine the most economical spacing of roads when two or more products, having volume units measured by various scales, are present in the stand. Because costs are associated with the units of measurement (board feet, cords, posts, shingle bolts, etc.) instead of being constant with a means of measurement common to all, such as cubic foot volume, the spacing of roads by the conventional one product per acre formula is out of order.

The total cost of roads should be born by each product depending upon its position in the stand (based uponthose factors that control road spacing). For determining the percentage of the total cost absorbed by each product, two factors should be considered: the volume per acre and the variable skidding cost per unit per station for each product. These factors govern the cost of roads per acre and when balanced with road costs, determine the economical spacing of the roads. The percentage of total road costs as determined by the "volume variable cost relation" for each product to the total "volume variable cost relationship" for all the products per acre, is the amount absorbed by any one product. It seem obvious that any other cost factor (such as: felling, bucking, hauling, etc.) which have no affect at all upon the spacing of roads should indeed not be included in a basis for determining the absorption of road costs by each product.

A total cost formula for skidding and road costs per unit is stated as follows:

$$X = CS/4 + C'S/4 + \frac{O.P.R/12.1}{VS} + \frac{O.P'R/12.1}{V'S}$$

where C is the variable cost per unit per station for the #1 product, C' the variable cost per unit per station for the number 2 product, O.P the percent road cost absorbed by the #1 product, O.P' the percent road cost absorbed by #2, V and V' the volumes of #1 and #2 respectively, ~~and~~ 12.1 acres the area served by a mile of road (R) with a ~~width~~ spacing of 100 feet (1 station), and S the spacing of roads in stations. O.P and O.P' are calculated in the following manner.

$$O.P = \frac{VC}{VC + V'C'} \quad : \quad O.P' = \frac{V'C'}{VC + V'C'}$$

By observing the skidding cost portion $S/4(C + C')$ of the total cost of skidding and roads formula, it is seen that this component varies directly while the road cost component $\frac{O.P/12.1}{VS} + \frac{O.P'/12.1}{V'S}$ varies inverse-ly with changing values of (S). For minimum costs achievements where a variable cost is increasing at an arithmetical rate and a fixed cost is decreasing at a reciprocal of a changing value, the variable component need only be equated against the fixed component to solve for the changing value of (S). *

$$\frac{S\ (C + C')}{4} = \frac{O.P\ R}{12.1\ VS} + \frac{O.P'R}{12.1\ V'S}$$

Substituting: $\frac{VC}{VC + V'C'}$ and $\frac{V'C'}{VC + V'C'}$ for O.P and O.P' respectively.

$$\frac{S\ (C + C')}{4} = \frac{\frac{VC}{VC + V'C'} \times \frac{R}{I}}{12.1\ VS} + \frac{\frac{V'C'}{VC + V'C'} \times \frac{R}{I}}{12.1V'S}$$

$$\frac{S\ (C + C')}{4} = \frac{RC}{12.1\ S(VC + V'C')} + \frac{RC'}{12.1\ S(VC + V'C')}$$

$$\frac{S^2(C + C')}{4} = \frac{R(C + C')}{12.1(VC + V'C')}$$

$$S = \sqrt{\frac{.33R}{VC + V'C'}}$$

DEMONSTRATION OF FORMULA USAGE:

A sawtimber-pulpwood tract is logged with tractors ---the pulpwood skidded by means of drays. The tract runs 5 cords and 5 M ft. B.M. per acre. Roads cost $200 per mile; variable skidding cost per sta. is 6¢/ cord and 10¢/ M ft. B.M.

* D.M. Mathews COST CONTROL IN THE LOGGING IND. McGraw-Hill Book Co.,--page 121

$S = \sqrt{\frac{.33R}{VC + V'C'}}$ substituting values into the formula:

$$S = \sqrt{\frac{.33 \times 20{,}000}{5 \times 10 + 5 \times 6}} = \sqrt{\frac{6600}{80}} = \sqrt{82.5} \text{ or } \underline{9.08 \text{ stations}}$$

Therefore, roads should be spaced about 910 ft. apart with a maximum skidding distance of 455 ft. and an average of 228 ft.

By substituting values into CS/4, C'S/4, $\frac{O.PR}{12.1VS}$, and $\frac{O.P'R}{12.1V'S}$ the cost of roads and skidding for each product can be determined.

COMPUTING O.P AND O.P':

O.P equals $\frac{VC}{VC + V'C'}$ equals $\frac{5 \times 10}{5 \times 10 + 5 \times 6}$ or 62.5%

O.P' equals $\frac{V'C'}{VC + V'C'}$ equals $\frac{5 \times 6}{5 \times 10 + 5 \times 6}$ or 37.5%

COST OF SAWTIMBER

ROAD COSTS: (R/12.1/VS) O.P

$\frac{20{,}000}{12.1 \times 5 \times 9.1} \times .625$ ----------22.75¢/M

SKIDDING COSTS: CS/4

10¢ $\times \frac{9.1}{4}$----------------------22.70¢/M 45.45¢/M

COST OF PULPWOOD

ROAD COSTS: (R/12.1 V'S) O.P'

$\frac{20{,}000}{12.1 \times 5 \times 9.1} \times .375$ ----------13.65¢/cd.

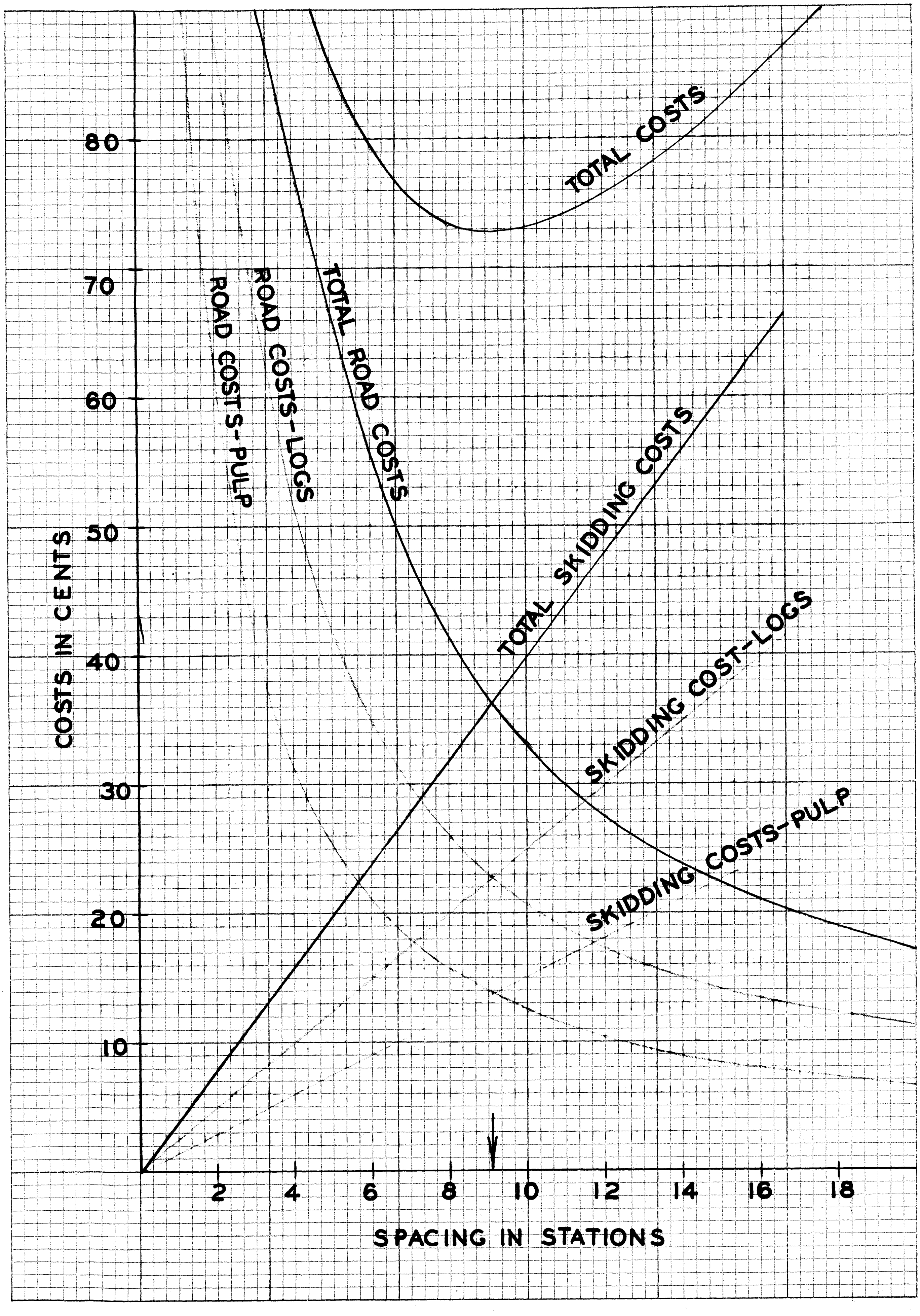
SKIDDING COSTS: C'S/4

6¢ $\times \frac{9.1}{4}$ ----------------------13.62¢/cd. 27.27¢/cd.

The total cost of skidding and the total cost of roads for both products are equal. Cutting the stand for sawtimber only would mean a road spacing of $\sqrt{\frac{6600}{5 \times 10}}$ or $\sqrt{132}$ = <u>11.5 stations</u>; if cut only for pulp, a road spacing of $\sqrt{\frac{6600}{5 \times 6}}$ or $\sqrt{220}$ = <u>14.8 stations</u>. In the event that the entire stand was cut with either of the two above road spacings, total skidding and total road costs would not be in balance; thus minimum total costs would not have been reached. A closer spacing of roads as determined by the TWO PRODUCT FORMULA merely reflects the greater unit volume per acre (compared to the two above cases) and the weighting of variable costs to give an average for the entire stand.

A graphic presentation of the skidding and road costs for each products gives a clear picture of the relative cost changes as the value for spacing varies.

Spac. stas.	LOGS			PULP			total road	total Skid	total cost.
	(R/12.1VS)O.P	CS/4	Total	(R/12.1V'S)O.P'	C'S/4	Total			
2	103.2¢	5¢	108.2¢	62.0¢	3¢	65¢	165.2	8	173.2
4	51.3	10	61.3	31.0	6	33	82.3	16	98.7
6	34.5	15	49.5	20.7	9	29.7	55.2	24	79.2
8	25.7	20	45.7	15.5	12	27.5	41.2	32	73.2
9	22.75	22.7	45.5	13.7	13.6	27.3	36.4	36.3	72.7
10	20.7	25	45.7	12.4	15	27.4	33.1	40	73.1
12	17.2	30	47.2	10.3	18	28.3	27.5	48	75.5
14	14.8	35	49.8	8.9	21	29.9	23.7	56	79.7


TOTAL COSTS
TOTAL ROAD COSTS
ROAD COSTS-LOGS
ROAD COSTS-PULP
TOTAL SKIDDING COSTS
SKIDDING COST-LOGS
SKIDDING COSTS-PULP
COSTS IN CENTS
80
70
60
50
40
30
20
10
2
4
6
8
10
12
14
16
18
SPACING IN STATIONS

The graphic presentation clearly shows the rapid fall of the cost of roads (total and for each product) until the spacing of the roads approches 900 ft. Thereafter the magnitude of the curve decreases until it closely resembles a straight line. The cost of skidding (total and for each product) rises at a steady arithmeticalrate. The point at which it crosses the road cost curve marks the spacing at which costs will be at a minimum. By examining the total cost curve, it is seen that even though nine stations may be theoretically the most economical road spacing under this set of governing conditions, spacing roads at any greater distances (within the range of variance) will have only a slight effect upon the rise of the total costs.

When each product absorbs its share of the cost of roads based upon the "volume variable cost" rela tion to the total "volume variable cost" of the stand, the spacing producing minimum costs for each product corresponds with that spacing calculated to give minimum costs for the entire stand. This indicates that the share of cost of roads absorbed by each product is in proper proportion to those factors that control road spacing. As a result, there is no overburdening or vice versus of any one product with road costs.

Formulae that are derived by using symbols for each product separately show an end result that is the same as the spacing formula derived when symbols for each product are used jointly.

SPACING ROADS BASED UPON SAWTIMBER VALUES ONLY:

COST OF SKIDDING: CS/4

COST OF ROADS: $\dfrac{\dfrac{VC}{VC + V'C'}}{12.1\ VS} \times \dfrac{R}{1}$ or $\dfrac{RC}{12.1\ S(VC + V'C')}$

Equating one against the other.

$$CS/4 = \frac{RC}{12.1\ S(VC + V'C')}$$

$$S = \sqrt{\frac{.33\ R}{VC + V'C'}}$$

SPACING ROADS BASED UPON PULP VALUES ONLY:

$$C'S/4 = \frac{RC'}{12.1\ S(VC + V'C')} \quad \text{or} \quad S = \sqrt{\frac{.33R}{VC + V'C'}}$$

The mathematical explanation of this result is: The variable cost component in the numerator of the values $\dfrac{VC}{VC + V'C'}$ and $\dfrac{V'C'}{VC + V'C'}$ (used to determine the absorption of cost of roads by each product) is always cancelled by the variable cost component of the skidding cost portion on the left hand side of the equation.

Because measurements of various types of products have very little volume relationship to each other, situations arise when the unit number of one product in the stand is quite excessive. This is quite often the case in a northern pulp-white cedar type. Often volumes run close to three cords of pulp and two hundred posts per acre. The first impression, because of the large number of units per acre characteristic of this type, might lead one to think that the spacing formula would indicate a spacing far in excess of what should actually be the case. Each product is drayed out: further study of costs reveal a skidding cost for pulp of 6¢/cord and for white cedar of .11¢/ post per station. Upon substituting these values into the spacing formula, it is seen that the spacing is not out of proportion but corresponds closely to a road spacing calculated for a stand running seven cords of pulp per acre. Products characteristically having large unit number per acre (compared with the more common units of measurement-- M ft. B.M., cords, etc.) are usually of such a nature that the variable skidding costs are very low in relation to the latter.

MODIFICATION OF ROAD SPACING FORMULA TO TAKE INTO ACCOUNT MORE THAN TWO PRODUCTS PER ACRE.

The development of road spacing formulae for three or more products per acre is done in the same manner as the original formula. C" and V" represent the variable skidding cost and the volume per acre of the third product in the stand. Equating the cost of skidding againstthe cost of roads.

$$\frac{S\ (C + C' + C'')}{4} \text{ equals } \frac{CR}{12.1\ S(VC + V'C' + V''C'')} + \frac{C'R}{12.1\ S(VC + V'C' + V''C'')} + \frac{C''R}{12.1\ S(VC + V'C' + V''C'')}$$

$$\frac{S^2\ (C + C' + C'')}{4} \text{ equals } \frac{R\ (C + C' + C'')}{12.1\ (VC + V'C' + V''C'')}$$

$$S = \sqrt{\frac{.33R}{VC + V'C' + V''C''}}$$

The spacing of roads in a stand with the number of products exceeding three, can be determined merely by adding additional values of variable cost and volume per acre for those products in the denominator of the above formula.

II
ECONOMICAL DIRECT SKIDDING DISTANCE FORMULAE.

When timber extends up to an established road, it is often desirable to determine the distance the timber must extend (based upon controlling factors) from this road to warrant the installation of spur roads in lieu of none at all. The distance at which it will be just as economical to skid directly as to install and to skid to spur roads can be determined by equating the costs of direct skidding against the alternative method.

The cost of direct skidding may be expressed as $\frac{D\,(C + C')}{2}$. Installing and skidding to spur roads will involve a "deadline" cost. This condition exists even though the timber extends up to the established road because a portion of this timber will be skidded directly instead of to the spur roads. The portion of road serving the "deadline" area is equal to one quarter of the road spacing. The cost of this "deadline" may be expressed as $\frac{r}{(V \times A)(D-d)}$ where (r) represents the cost of the "deadline", (A) is the area in acres served by 100 ft. of the "deadline", and (d) the length of the "deadline" expressed in stations.

Equating one against the other:

$$\frac{D\ (C + C')}{2} = \frac{S\ (C + C')}{4} + \frac{R(C + C')}{VC + V'C'} + \frac{\frac{VC}{VC + V'C'} \times \frac{r}{1}}{(V \times A)\ (D-d)}$$

$$+ \frac{\frac{V'C'}{VC + V'C'} \times \frac{r}{1}}{(V \times A)\ (D-d)}$$ Therefore:

$$\frac{D\ (C + C')}{2} = \frac{S\ (C + C')}{4} + \frac{R(C + C')}{VC + V'C'} + \frac{r\ (C + C')}{A\ (D-d)(VC + V'C')}$$

D equals $\frac{S}{2} + \frac{2R}{VC + V'C'} + \frac{2r}{A\ (D-d)\ (VC + V'C')}$

A CALCULATION MAKING USE OF THE ABOVE FORMULA:

Assuming the data to be the same as the previous sample problem with the exception that (R) in this case represents the cost of roads per acre, the computations are as follows:

R = R/12.1S or $\frac{20{,}000¢}{12.1 \times 9}$ or <u>183.6¢/acre</u>

d = $\frac{9 \text{ stas.}}{4}$ or <u>2.25 stations</u>

r = $\frac{2.25 \text{ sta.}}{52.8 \text{ sta.}} \times 20{,}000¢$ or <u>852¢</u>

A = $\frac{900 \text{ ft.} \times 100 \text{ ft.}}{43{,}560 \text{ sq. ft.}}$ or <u>2.07 acres</u>

Substituting values in the formula:

$$D = \frac{9}{2} + \frac{2 \times 183.6¢}{80} + \frac{2 \times 852¢}{2.07\ (D-2.25)\ (80)}$$

$$D = 4.5 + 4.59 + \frac{1704}{165.6(D-2.25)}$$

$$D = \frac{9.09D - 20.45 + 10.29}{D-2.25}$$

$$D^2 - 11.34D + 10.16 = 0$$

Solving the quadratic equation:

D equals <u>10.36 stations</u>

Therefore, if the timber extends a distance greater than 10.4 stations from the established road, the installation of spur roads will be justified by keeping costs at a minimum. This is confirmed by costs under each plan.

COST OF SKIDDING DIRECTLY:

$\frac{D\ (C + C')}{2}$ or $\frac{10.36}{2} \times 16$ ------------------ <u>82.9¢/M</u> bd. ft. - cd. unit

COST OF SKIDDING AND ROADS:

COST OF SKIDDING: $\frac{S\ (C + C')}{4}$

$\frac{9}{4} \times 16¢$ ---------------------- 36.0¢

COST OF ROADS: $\frac{R(C + C')}{VC + V'C'}$

$\frac{183.6¢ \times 16¢}{80}$ ----------------- 36.6¢

COST OF "DEADLINE":

$$\frac{r\ (C + C')}{A(D-d)(VC + V'C')}$$

$\frac{852 \times 16}{165.6(10.36-2.25)} = \frac{13,625}{1,343}$ ---- <u>9.9¢</u> <u>82.5¢/M</u> bd. ft. - cd. unit

Any decrease in the volume per acre or any decrease of the variable skidding cost due to the usage of more efficient skidding devices results in an increase of the depth of the timber from the road necessary to warrant the installation of spur roads in lieu of direct skidding to the established road. Also, any rise in the cost of road construction will have the same net results as decreasing the volume cut per acre or decreasing the variable cost of skidding.

Due to the cumbersome nature of a quadratic solution resulting from the inclusion of "deadline" in the right hand equation, a more practical formula may be desirable when making "on the spot" decisions which call only for approximate answers. The effect of "deadline" is taken into account by increasing the value derived in the formula by approximately 11%.

Equating the cost of direct skidding against the cost of skidding and road construction:

$$\frac{D\ (C + C')}{2} = \frac{S\ (C + C')}{4} + \frac{R\ (C + C')}{VC + V'C'}$$

Dividing by (C + C') and multiplying by 2:

$$D = \frac{S}{2} + \frac{2R}{VC + V'C'}$$ (Multiply the final value by 1.11)

In event that the edge of the timber is more than a quarter of the road spacing away from the established road, more consideration must be given to the "deadline" area in determining the width of the timber belt necessary to warrant the installation of roads as against direct skidding. The minimum skidding distance will be the length of the "deadline" and the maximum will be the width of the timber belt plus the "deadline". The average skidding distance is expressed as $\frac{(D + d) + d}{2}$ or $\frac{D + 2d}{2}$. The cost of the "deadline" must be absorbed by the area upon which timber still exists and is, therefore, spread against the total volume served by the continuation of this spur road into the timber.

Equating direct skidding cost against cost of skidding and spur roads:

$$\frac{(D + 2d)(C + C')}{2} = \frac{S(C + C')}{4} + \frac{R(C + C')}{VC + V'C'} + \frac{r(C + C')}{AD(VC + V'C')}$$

$$D + 2d = \frac{S}{2} + \frac{2R}{VC + V'C'} + \frac{2r}{AD(VC + V'C')}$$

$$D = \frac{S}{2} + \frac{2R}{VC + V'C'} + \frac{2r}{AD\,(VC + V'C')} - 2d$$

Utilizing the same values as before with the following exceptions: (d) equals 1000 ft.; (r) equals $\frac{1000 \text{ ft.}}{5280 \text{ ft.}} \times 20{,}000¢$ or 3790¢

Substituting values into the formula:

$$D = \frac{9}{2} + \frac{2 \times 183.6}{(10 \times 5) + (6 \times 5)} + \frac{2 \times 3790}{2.07D(L)\times 5 + 6\times 5)} - 2 \times 10$$

$$D = 4.5 + 4.59 + \frac{7580}{165.6D} - 20$$

$$D = \frac{9.09D + 45.75 - 20D}{D}$$

$$D^2 + 10.91D - 45.75 = 0$$

Solving the quadratic:

D equals 3.23 stations. The width of the timber need only be 323 ft. wide to warrant the construction of spur roads.

CHECK:
COST OF DIRECT SKIDDING: (from a belt of timber 10 stas. from the road)

$$\frac{(D + 2d)(C + C')}{2} \text{ or } \frac{(3.23 + 20)(16)}{2} \text{------} 185.8¢/M \text{ bd. ft.-cd. unit}$$

COST OF SKIDDING AND SPUR ROADS:

COST OF SKIDDING: $\frac{S(C + C')}{4}$

$\frac{9}{4} \times 16$ ---------------------36.0¢

COST OF ROADS: $\frac{R(C + C')}{VC + V'C'}$

$\frac{183¢(16)}{80}$ --------------------36.6¢

COST OF "DEADLINE": $\frac{r(C + C')}{AD(VC + V'C')}$

$\frac{3790\ (16)}{2.07 \times 3.2 \times 80}$ ----------------113.5¢ 186.1¢/M bd. ft.-cd. unit

In the case of the southern pinery, the topography is occasionally of such a nature that trucks may be driven off the roads to loading points in the timber.

This is quite often the practice in pulp operations and it might be conceivable that in a joint operation of sawtimber and pulp the latter would be picked up directly in the woods by trucks and the former skidded to the road and there loaded onto trucks. In the event that there is to be no off the road haul, the first formula would suffice. Even though hauling costs should be considered, the effect upon the final computed cost is so small that it may be disregarded. Hauling costs, however, on the spur roads are taken into account in this situation.

If roads are not constructed, the cost of hauling for pulp and the skidding of sawtimber can be expressed as; $\frac{D\,(C + C')}{2}$

In event roads are to be used the cost may be expressed as;

$$\frac{S(C + C')}{4} + \frac{\frac{VC}{VC + V'C'} \times \frac{R}{12.1}}{VS} + \frac{\frac{V'C'}{VC + V'C'} \times \frac{R}{12.1}}{V'S} +$$

$$\frac{D\,(H + H')}{2} + \frac{\frac{VC}{VC + V'C'} \times \frac{r}{I}}{VA(D-d)} + \frac{\frac{V'C'}{VC + V'C'} \times \frac{r}{I}}{V'A(D-d)}$$

or $\frac{S}{4}(C + C') + \frac{R\,(C + C')}{S12.1\,(VC + V'C')} + \frac{D\,(H + H')}{2} +$

$\frac{r\,(C + C')}{A(D-d)\,(VC + V'C')}$ where (H) is the cost of hauling #1 product per station on the spur roads and (H') the cost of hauling #2 product per station. Equating the cost of direct skidding and hauling against the cost of roads, skidding, and hauling.

$$\frac{D(C + C')}{2} = \frac{S(C + C')}{4} + \frac{R(C + C')}{S\ 12.1(VC + V'C')} + \frac{D(H + H')}{2} + \frac{r(C + C')}{A(D-d)(VC + V'C')}$$

$$\frac{D}{2} = \frac{S}{4} + \frac{R}{S\ 12.1(VC + V'C')} + \frac{D(H + H')}{2(C + C')} + \frac{r}{A(D-d)(VC + V'C')}$$

SIMPLIFICATION:

$$\frac{D\left[(C + C')-(H + H')\right]}{2(C + C')} = \frac{D}{2} - \frac{D(H + H')}{2(C + C')}$$

Multiplying by $4(C + C')$

$$2D\left[(C + C')-(H + H')\right] = S(C + C') + \frac{.33R(C + C')}{VC + V'C'} + \frac{4r(C + C')}{A(D-d)(VC + V'C')}$$

$$D = \frac{\frac{S(C + C')}{1} + \frac{.33R(C + C')}{(VC + V'C')}}{2\left[(C + C')-(H + H')\right]} + \frac{2r\ (C + C')}{\left[(C+C')-(H+H')\right]\left[A(D-d)(VC + V'C')\right]}$$

The above formula will apply when any spacing is used--if, however, the roads are placed on an economical spacing, the cost of skidding is equal to the cost of road construction and may be expressed as $2S(C + C')$.

$$D = \frac{S(C + C')}{(C + C') - (H + H')} + \frac{2r\ (C + C')}{[A(D-d)(VC + V'C')]\ [(C + C') - (H + H')]}$$

EXAMPLE: Tractors skid timber to roads where it is loaded on trucks. Trucks drive off the road and load directly from the pens.

COST DATA: (In addition to that previously stated)

C' (off road truck haul)-------4¢/cd.

C (tractor skidding)---------10¢/M

H' (hauling on roads--pulp)--- 2¢/cd. per station

H (hauling on roads--timber)- 5¢/M per station

ROAD SPACING:

$$S = \sqrt{\frac{.33R}{VC + V'C'}} \text{ or } \sqrt{\frac{.33 \times 20{,}000}{5 \times 10 + 5 \times 4}} \text{ or } \sqrt{94.7} \text{ or } \underline{9.7 \text{ Stas.}}$$

$$d = \frac{S}{4} \text{ or } \frac{9.7}{4} \text{ or } \underline{2.43 \text{ stations}}$$

$$A = \frac{970 \text{ ft.} \times 100 \text{ ft.}}{43{,}560 \text{ ft.}} \text{ or } \underline{2.23 \text{ acres}}$$

$$r = \frac{2.43}{52.8} \times 20{,}000 \text{ or } \underline{920¢}$$

SOLUTION:

$$D = \frac{9.7(4 + 10)}{(10 + 4) - (2 + 5)} + \frac{2 \times 920\ (4 + 10)}{[2.23(D-2.43)(70)]\ [(10 + 4) - (2 + 5)]}$$

$$D = \frac{135.8}{7} + \frac{920(28)}{156.1 \times 7(D-2.43)} \text{ or } 19.4 + \frac{23.6}{D-2.43}$$

$$D = \frac{19.4D - 47.1 + 23.6}{D - 2.43} \quad \text{or} \quad D^2 - 2.43D - 19.4D = -23.6$$

SOLVING THE QUADRATIC: D becomes <u>20.69 stations</u>

Thus if the timber extends a distance in excess of ~~20,700~~ 2,070 ft. from the established road, it will be more economical to build spur roads than to skid directly .

CHECK CALCULATION OF COSTS:

COST OF SKIDDING AND HAULING DIRECT.

$$\frac{D(C + C')}{2} \quad \text{or} \quad \frac{20.7}{2} \times 14¢ \text{------------------} \underline{145¢/M} \text{ bd. ft.}$$
-cd. unit

COST OF SKIDDING AND HAULING WHEN SPUR ROADS HAVE BEEN CONSTRUCTED:

COST OF ROADS AND SKIDDING: $2 \times \frac{S(C + C')}{4}$

$$\frac{9.7}{2} \times 14¢ \text{--------------------} 68¢$$

COST OF HAULING: $\frac{D(C + C')}{2}$

$$10.35 \times 7 \text{ ------------------- } 72.35¢$$

COST OF DEADLINE:

$$\frac{r\,(C + C')}{A(D-d)(VC + V'C')}$$

$$\frac{920 \times 14}{156.1(20.7-2.43)} \text{ ------------ } \underline{4.52¢} \qquad \underline{144.9¢/M} \text{ bd. ft.}$$
-cd. unit

A simplified formula may be constructed which leaves out the expression for "deadline" and thus avoids the cumbersome solution of the quadratic equation. When rough or approximate answers are required, this formula will prove quite satisfactory.

COST OF DIRECT SKIDDING AND HAULING: $\frac{D(C + C')}{2}$

COST OF HAULING AND SKIDDING WHEN ROADS ARE CONSTRUCTED: $\frac{S(C + C')}{4} + \frac{R(C + C')}{12.1S(VC + V'C')} + \frac{D(H + H')}{2}$

Equating one against the other:

$$\frac{D(C + C')}{2} = \frac{S(C + C')}{4} + \frac{R(C + C')}{12.1S(VC + V'C')} + \frac{D(H + H')}{2}$$

$$\frac{D}{2} = \frac{S}{4} + \frac{R}{12.1S(VC + V'C')} + \frac{D(H + H')}{2(C + C')}$$

Transposing and determining a common denominator:

$$\frac{D[(C + C')-(H + H')]}{2(C + C')} = \frac{S}{4} + \frac{R}{12.1S(VC + V'C')}$$

Multiplying by $4(C + C')$

$$2D[(C + C')-(H + H')] = S(C + C') + \frac{.33R(C + C')}{VC + V'C'}$$

$$D = \frac{\frac{S(C + C')}{1} + \frac{.33R(C + C')}{VC + V'C'}}{2[(C + C')-(H + H')]}$$

This formula would hold good for any spacing used. If, however, the roads could be placed on an economical spacing, then $S(C + C')$ equals $\frac{.33R(C + C')}{VC + V'C'}$

Therefore:

$$D = \frac{S(C + C')}{(C + C')-(H + H')}$$

To allow for "deadline", the value derived from this formula should be increased by approximately 11% to give an answer comprable to the more exacting solution.

III
DETERMINATION OF THE ECONOMICAL DISTANCE AT WHICH TO SPACE LANDINGS AND ROADS WHEN EACH LANDING IS LOCATED IN THE CENTER OF A RECTANGULAR SETTING.

When timber is skidded directly to roads, the average skidding distance is $\frac{S}{4}$ or 25% of the road spacing. However, when a landing is located in the center of a square setting, the average skidding distance becomes 74.6% of the external skidding distance or (74.6 ÷ 2) 37.3% of the road spacing. With square settings the distance between each landing is the same as the spacing of the roads(the relation of the spacing of landings to the roads would therfore be 100%). If this relation becomes greater or less than 100%, the settings become rectangles of various demensions--depending upon the spacing of the roads and the landings. For landings spaced at one half the road spacing(relation being 50%) the average skidding distance becomes 28.9% of (S); for a relation of 25% the average skidding distance becomes 26.1% of (S). By plotting these values of average skidding distances in terms of road spacing(P) over various percentage values of spacing of landings in percent of road spacing(Z) and drawing a smooth curve through these points, values of (P) can be readily determined for any relation of landing spacing to road spacing that might exist(values of P for every rise of Z of 10% are shown in column 2 of table). (Column 3--average skidding distance factor times spacing of roads symbal and variable skidding cost symbal for each product).*

Allowing (r) to represent the cost of road construction per station and the area of each setting to be expressed as $\frac{S^2}{4.356} \times Z$, a total cost formula for variable cost of skidding plus cost of road construction for each setting will be as follows.

$$X = .PSC + .PSC' + \frac{\frac{VC}{VC + V'C'} \times SrZ}{V \times \frac{S^2Z}{4.356}} + \frac{\frac{V'C'}{VC + V'C'} \times SrZ}{V' \times \frac{S^2Z}{4.356}}$$

$$X = .PS(C + C') + \frac{4.356r(C + C')}{S(VC + V'C')}$$

Equating one against the other:

$$.PS(C + C') = \frac{4.356r(C + C')}{S(VC + V'C')}$$

$$S^2 = \frac{4.356r}{P(VC + V'C')}$$

By substituting the values of (P) into the spacing formula the entries in column 6 are derived.

COST OF ROADS $\frac{4.356r(C + C')}{S(VC + V'C')}$ is entered in column four.

THE COST OF LANDINGS for any setting may be expressed as follows:

* D.M. Mathew's "COST CONTROL IN THE LOGGING INDUSTRY"--McGraw-Hill Book Co. pages 132-137

$$\frac{\frac{VC}{VC + V'C'} \times L}{\frac{Z}{4.356} \times S^2V} + \frac{\frac{V'C'}{VC + V'C'} \times L}{\frac{Z}{4.356} \times S^2V'} \quad \text{or} \quad \frac{4.356CL}{S^2\ Z(VC + V'C')}$$

$$+ \frac{4.356C'L}{S^2\ Z(VC + V'C')} \quad \text{or} \quad \frac{4.356L(C + C')}{S^2Z(VC + V'C')} \quad \text{or} \quad \frac{L(C + C')}{\frac{Z}{4.356} \times S^2(VC + V'C')}$$

By substituting the values of (Z) into the above formula, the entries in column five are derived.

At this point it is necessary to determine the "decrease in percent of cost of landings as landing spacing increases" and the "increase in percent of cost of roads and skidding as landing spacing increases". By comparing the "changes in cost of landings" with the "change in cost of roads and skidding" for each increase of (Z) of 10%, a "ratio of decrease in cost of landings to the increase in cost of skidding and roads" can be determined. Then by determining the ratio existing between cost of landings to costs of skidding and roads in a trial calculation using a "SPACING OF LANDINGS IN PERCENTAGE OF ROAD SPACING OF 12.5% AS STANDARD", the type setting most economical for the existing set of governing conditions(cost of landings, roads, and skidding) can be computed merely by selecting the (Z) at which this ratio exists. At any (Z) where the ratio is greater or less than the existing ratio, the minimum total cost for the existing conditions will be exceeded. Again, as in the proceeding formulae, there is a range of variance in which total costs will vary only slightly from the minimum.

Using a 12.1% spacing of landings in percentage of road spacing as standard, the percentage decrease in cost of landings for any (Z) can be determined. Determining the relation between 12.5% (Z) and 80% (Z) with a road spacing of $\sqrt{\frac{13.9r}{VC + V'C'}}$ instead of $\sqrt{\frac{17.25r}{VC + V'C'}}$ means the cost of landings when spaced at 80%(Z) instead of 12.5%(Z) is $\frac{.183 \times \frac{1}{13.0}}{.0287 \times \frac{1}{17.25}}$ or $\frac{.495}{2.38}$ = 20.6% of standard or a decrease of cost of 79.4% (column 7). Values for L, V, C, and C' are constant throughout.

Using 12.5% spacing of landings in percentage of road spacing as standard, the percentage increase in cost of roads and skidding for any value of (Z) can be determined. With cost of roads per station, variable skidding costs, and volumes constant in all cases, spacing for 12.1% (Z) is $\sqrt{17.25}$ or 4.15 stations.

COST OF SKIDDING = .2525 X 4.15------1.050

COST OF ROADS = $\frac{4.356}{4.15}$ ---------------1.050

2.100

20% (Z)--S = 16.95 = <u>4.12</u> Stations

COST OF SKIDDING = .257 X 4.12-------1.058

COST OF ROADS = $\frac{4.356}{4.12}$ --------------1.057

$\overline{2.115}$

PERCENTAGE INCREASES: $\frac{2.115}{2.100}$ = 108% or an increase of <u>8%</u> (column 8).

Column 9 & 10 are determined by noting the difference between the values in columns 7 & 8 respectively for each rise in the value of (Z) of 10%. By dividing column 9 by 10, a ratio of decrease of cost of landings to increase of cost of skidding and roads may be derived (column 11).

By plotting the values of column seven and eight, a true picture is given of the response of landing costs as against the costs of skidding and roads for each rise in value of (Z) of 10%. The landing cost curve drops very rapidly at first and then gradually flattens out after the values of (Z) exceed 100%. Meanwhile, the skidding and road cost curve rises gently, almost as a straight line, with rising values of (Z). Thus, justification for the reducing of costs of landings and the increasing of costs of skidding and roads are realized until the ratio between these two sets of costs correspond to that ratio determined by the governing factors.

(1) Spacing of landings in percentage of road spacing. Z	(2) Average skidding distance in percent of road spacing. P	(3) Cost of skidding in terms of road spacing. $.PS(C+C')$	(4) Cost of roads in terms of road spacing. $\frac{4.356r(C+C')}{S(VC+V'C')}$	(5) Cost of landings in terms of road spacing. $\frac{L(C+C')}{\frac{Z}{4.356} \times S^2(VC+V'C')}$
0.0	25.0	$0.250S(C+C')$	$\frac{4.356r(C+C')}{S(VC+V'C')}$	
12.5(1/8)	25.25	$0.2525S(C+C')$	"	$\frac{L(C+C')}{.0287S^2(VC+V'C')}$
20.0	25.7	$0.257S(C+C')$	"	$\frac{L(C+C')}{.0459S^2(VC+V'C')}$
25.0(1/4)	26.1	$0.261S(C+C')$	"	$\frac{L(C+C')}{.0573S^2(VC+V'C')}$
30.0	26.5	$0.265S(C+C')$	"	$\frac{L(C+C')}{.0689S^2(VC+V'C')}$
40.0	27.6	$0.276S(C+C')$	"	$\frac{L(C+C')}{.0925S^2(VC+V'C')}$
50.0(1/2)	28.9	$0.289S(C+C')$	"	$\frac{L(C+C')}{.115S^2(VC+V'C')}$
60.0	30.38	$0.304S(C+C')$	"	$\frac{L(C+C')}{.138S^2(VC+V'C')}$
70.0	31.95	$0.319S(C+C')$	"	$\frac{L(C+C')}{.161S^2(VC+V'C')}$
80.0	33.6	$0.336S(C+C')$	"	$\frac{L(C+C')}{.183S^2(VC+V'C')}$
90.0	35.4	$0.354S(C+C')$	"	$\frac{L(C+C')}{.206S^2(VC+V'C')}$
100.0	37.3	$0.373S(C+C')$	"	$\frac{L(C+C')}{.229S^2(VC+V'C')}$
125.0	42.0	$0.420S(C+C')$	"	$\frac{L(C+C')}{.287S^2(VC+V'C')}$
150.0	47.1	$0.471S(C+C')$	"	$\frac{L(C+C')}{.344S^2(VC+V'C')}$

ROAD & LANDING SPACING TABLE.

(6) Formula for economical spacing of roads. $S=\sqrt{\frac{4.356r}{.P(VC+V'C')}}$	(7) Decrease in cost of landings as landing spacing increases based on cost at 12.5% spacing(-%).	(8) Increase in cost of skid. and roads as landing spacing increases --cost at 12.5%spacing(+%).	(9) Differential change in cost of landings. (-%)	(10) Differential change in cost of skidding and roads. (+%)	(11) Ratio of decrease in cost of landings to increase in cost of skidding and road cons.
$S=\sqrt{\frac{17.4r}{VC+V'C'}}$					
$S=\sqrt{\frac{17.25r}{VC+V'C'}}$	100.0	100.0			
$S=\sqrt{\frac{16.95r}{VC+V'C'}}$	37.0	0.8	37.0	0.8	46 to 1
$S=\sqrt{\frac{16.6r}{VC+V'C'}}$	49.0	1.6	12.0	0.8	15 to 1
$S=\sqrt{\frac{16.4r}{VC+V'C'}}$	56.4	2.5	7.4	0.9	8 to 1
$S=\sqrt{\frac{15.8r}{VC+V'C'}}$	66.2	4.4	9.8	1.9	5 to 1
$S=\sqrt{\frac{15.1r}{VC+V'C'}}$	71.1	6.8	5.5	2.4	2.3 to 1
$S=\sqrt{\frac{14.3r}{VC+V'C'}}$	75.1	9.6	3.4	2.8	1.2 to 1
$S=\sqrt{\frac{13.6r}{VC+V'C'}}$	77.5	12.5	2.4	2.9	0.8 to 1
$S=\sqrt{\frac{13.0r}{VC+V'C'}}$	79.4	15.4	1.9	2.9	0.7 to 1
$S=\sqrt{\frac{12.3r}{VC+V'C'}}$	80.6	18.4	1.2	3.0	0.4 to 1
$S=\sqrt{\frac{11.7r}{VC+V'C'}}$	81.6	21.4	1.0	3.0	0.33 to 1
$S=\sqrt{\frac{10.37r}{VC+V'C'}}$	83.5	28.9	1.9	7.5	0.25 to 1
$S=\sqrt{\frac{9.25r}{VC+V'C'}}$	84.6	36.5	1.1	7.6	0.14 to 1

DEMONSTRATION OF METHOD:

V = 5M/acre   C = 10¢/M/station   r = 1000¢/sta.
V' = 5 cds./acre   C' = 6¢/cd./station   L = 1000¢

TRIAL CALCULATION AT VALUE OF Z OF 12.1%.

$$S = \sqrt{\frac{17.25r}{VC+V'C'}} = \sqrt{\frac{17.25 \times 1000¢}{5M \times 10¢ + 6 \text{ cds.} \times 5¢}} = \underline{14.7 \text{ stations}}$$

COST OF SKIDDING: .PS(C+C')

.2525 X 14.7(10 + 6)-----------------59.2¢

COST OF ROADS: $\frac{4.356r(C+C')}{S(VC+V'C')}$

$\frac{4.356 \times 1000 \times 16}{14.7 \times 80}$ -------------------59.3¢   118.5¢

COST OF LANDINGS: $\frac{L(C + C')}{\frac{Z}{4.356} \times S^2(VC + V'C')}$

$\frac{1000 \times 16}{.0287 \times 215.6 \times 80}$ ------------------------ 32.3¢

RATIO = $\frac{118.5}{32.3}$ = 3.6 to 1

With a road spacing of 14.7 stations, the landings will be spaced every 184 feet which is obviously out of reason. An increase of the spacing of the landings in relation to the spacing of the roads can be justified only as long as the cost reduction of the landings is 3.6 or more times the increase of the cost of roads and skidding. The most economical setting under this set of governing conditions is a setting with a value of (Z) of 40% (ratio of 5 to 1).

That a greater or lesser ratio (corresponding to a change of value of Z of 10%) is less economical can be seen by the following table.

Z	LANDINGS % cost of standard	Cost at 12.5% ¢	Cost ¢	ROADS & SKIDDING % cost of standard	Cost at 12.5% ¢	Cost ¢	TOTAL COST ¢
25	.510	32.3	16.50	1.016	118.5	120.2	136.7
30	.436	"	14.10	1.025	"	121.4	135.5
40	.338	"	10.94	1.044	"	123.6	134.5
50	.289	"	9.15	1.068	"	126.5	135.7
60	.249	"	8.05	1.096	"	129.8	137.9
70	.225	"	7.27	1.125	"	133.2	140.5
80	.206	"	6.66	1.154	"	135.6	142.3

USING A VALUE OF (Z) OF 40% THE SPACING OF ROADS WILL BE:

$$S = \sqrt{\frac{15.8r}{VC + V'C'}} = \sqrt{\frac{15.8 \times 1000}{5 \times 10 + 5 \times 6}} = \sqrt{197.5} \text{ or } \underline{14.05 \text{ stations}}$$

Therefore, roads will be spaced every <u>1400 feet</u> and landings (1400 X .4) every <u>560 feet</u>.

DETERMINING THE COSTS FOR EACH PRODUCT:

SAWTIMBER:

Skidding: PSC

.276 X 14 stas. X 10¢ --------- 38.6¢/M

Cost of roads: $\frac{4.356rC}{S(VC+V'C')}$

$\frac{4.356 \times 1000 \times 10}{14\ (80)}$ ------------38.6¢/M

Cost of Landings: $\frac{4.356LC}{S^2Z(VC+V'C')}$

$\frac{1000 \times 10}{.092 \times 197.5 \times 80}$ ------------ 6.9¢/M   92.3¢/M

PULP:

Skidding cost: .PSC'

.276 X 14 X 6¢ ---------------23.25¢/cd.

Cost of roads: $\frac{4.356rC'}{S(VC+V'C')}$

$\frac{4.356 \times 1000 \times 6}{14(80)}$ -------------23.30¢/cd.

Cost of landings: $\frac{4.356LC'}{S^2Z(VC+V'C')}$

$\frac{1000 \times 6}{.092 \times 197.5 \times 80}$ ------------ 4.12¢/cd. 50.7¢/cd.

In event that the spacing of roads can not be controlled, any reduction in cost of landings by increasing the spacing of the same must be balanced only by the increase in cost of skidding. The differential changes in cost of landings and cost of skidding will not be the same as in columns (9) and (10) of ROAD AND LANDING SPACING TABLE because the values for S become constant. For situations of this sort, reference is made to page 155 of COST CONTROL IN THE LOGGING INDUSTRY by D.M. Mathews for an applicable table.

CONCLUSION

The writer feels that the foregoing formulae indeed do not close the subject to further development of the basic spacing formula used in this thesis. Simplification by a tabular or graphic form would be of value to the average logging operator. That these formulae could be modified to cope with specific situations is quite conceivable and already the adaptation of the spacing formula to stands in which diameter classes with corresponding volumes and skidding costs for each class have been set up has been suggested as a means of departing from the use of the spacing formula based upon one set of average values for the entire stand.

This paper makes possible the following statements in regards to the ends achieved by the principles used in the derivation of these formulae.

1. The spacing of roads and landings may be computed for stands regardless of the number of products per acre and regardless of how many various units of measurement are used.

2. Each product absorbs its proportionate share of the total road cost. No one product is over-burdened or vice versus with road costs.

3. A system is devised whereby the total cost of logging a stand may be broken down into the various constituent costs for each product.