Labor search, inequality, and public policy.

by

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ABSTRACT

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Chair: Dmitriy L. Stolyarov

Labor markets defy the Walrasian auctioneer: the supply of labor is persistently in excess of demand and otherwise observationally equivalent workers receive different compensation. In response, search-theoretic models of the labor market have been developed and are widely used to evaluate public policy. A key component of such models is the theoretical mechanism used to divide labor rents between an employer and employee and set the wage. Different wage setting mechanisms yield, sometimes radically, different theoretical implications for public policy, market efficiency, returns to experience, and inequality between workers. This dissertation is composed of three chapters which grapple with issues of how the wage setting mechanism is selected, the implications of the wage setting mechanism when interpreting aggregate and cross-sectional data, and the interaction of public policy with the wage setting mechanism.

In Chapter 1, *Posted and negotiated wages in markets with on-the-job search*, I model a labor market in which identical workers search on- and off-the-job and heterogenous firms choose between two wage formation mechanisms: a non-negotiable wage or a contingent wage contract that updates to match the value of workers' best-to-date outside offer when

profitable. I show that when firms must pay a per-firm cost to offer the contingent contract, a separating equilibrium arises in which only low productivity firms offer non-negotiable contracts. Modeling contract choice improves the on-the-job search model's fit with respect to observed wage dispersion, (un)employment durations and transitions, labor's share of output, and plausible values of social insurance. With only inflexible wage contracts the model struggles to achieve reasonable labor share and value of social insurance at the same time as realistically large wage dispersion. With only flexible wage contracts the model fails to provide incentive to search while unemployed and suggests implausibly low entry wages. In simulations, the mixed-contract model is able to achieve wage dispersion, labor share, employment transitions, and flow value of unemployment which are *all simultaneously* consistent with empirical observations. Surprisingly, a mixture of contracts substantially improves upon the performance of the pure ridged contract model even when most firms operate a rigid wage contract. Improved fit makes the new mixed-contract model a better candidate for welfare analysis of social insurance programs, particularly when the researcher seeks to value the impact on workers *and* on firms.

In Chapter 2, *Dead-end and career jobs: skill-specific earnings profiles in an on-the-job search equilibrium with heterogeneous wage contracts*, I include differential skill levels on the part of workers in an on-the-job search equilibrium which features heterogeneous wage contracts. Firms choose between posting a non-negotiable contract or hiring under contingent pay which matches the value of a worker's best-to-date outside offer. I provide conditions under which the market decomposes into parallel skill-specific sub-markets in each of which a separating equilibrium arises where only low productivity firms offer non-negotiable contracts. Even when skill-specific sub-markets are ex-ante identical, differences arise ex post. Higher skilled workers are more likely to receive negotiable wage offers. Consequently, high skilled workers experience lower rates of unemployment, more wage dispersion, and higher returns to experience than low skilled workers. I explore the implications for evaluating job training which increases worker skill level while unemployed. The full effect of training is expressed in wages only over time; however, effects are expressed more rapidly in higher percentiles of the wage distribution. This suggests estimating the returns to training programs, at least in the short-run, based on impacts in the right tail of the wage distribution.

Survey evidence documents heterogeneity in the wage setting mechanisms utilized by and experienced by different firms and workers. In theoretical models, different in wage negotiation and renegotiation protocols can have dramatic impact on implied lifestyle earnings and cross-sectional features such as labor share and income inequality. Empirical differences and trends in these statistics may be related to differences in the contracts used to set wages. In Chapter 3, *Empirical identification of the composition of contract types using administrative data*, I provide details on how to estimate the share of renegotiable labor contracts in an on-the-job search equilibrium which features firm's choice between a wage posting and a sequential auction contract as in Chapter 1. Identification is achieved using administrative data (only) on workers' histories of job-to-job mobility and on-the-job pay gain. I estimate that 22.06 percent of German workers and 62.88 percent of German firms have a wage posting contract. These estimates are comparable to the only existing survey evidence for Germany (Brenzel et al., 2013). Methods are applicable to a broad set of data sources, making them ideal for cross-group, cross-country, and historical analysis.

CHAPTER 1:

Posted and negotiated wages in markets with on-the-job search.

1.1 Introduction

Labor market policy influences the labor supply of workers and hiring decisions of firms. In addition to establishing a causal link between policy and outcomes, it is desirable to describe the structural underpinnings of the decisions of firms and workers that lead to these outcomes. Labor markets, however, defy the Walrasian auctioneer: the supply of labor persistently exceeds demand and otherwise observationally equivalent workers receive different compensation. Equilibrium search theories yield fruitful alternate theoretical paradigm for assessing the efficiency and design of labor market policy. However, a key component of search models is the mechanism used to divide the output of employment between an employer and employee and set the wage. Different wage setting mechanisms yield different, sometimes radically, theoretical implications for market efficiency and the distribution of output between and across workers and firms.

This paper provides a mechanism by which two canonical wage setting mechanisms - wage posting (WP) and sequential auction (SA) - exist side-by-side in a random, on-the-job search equilibrium. WP is characterized by a single and inflexible wage. In contrast, wages formed under the SA contract are contingent on a worker's best-to-date outside option.

A per-firm cost for employing under the SA contract induces firms to endogenously sort between WP and SA wage contracts such that the most productive firms select SA while all less productive firms choose WP. The cost may be thought of as reflecting the legal or human resources costs of employing workers under contingent wage contracts. This simple cost structure enables me to isolate and exploit a single mechanism which drives contract choice: differentials in rent allocation between contract types. As a result, I am able to consider a continuous distribution of firm types and optimal wage setting strategies under *both* contract types.[1]

The intuition underlying the equilibrium is clear cut. All else held equal, firms hiring under SA operate lighter wage bills, exclusive of the cost of SA, than they do when they hire under WP. SA firms offer starting wages which are contingent on history while WP firms offer the same wage to all workers. In addition, SA firms offer workers greater option value of employment than WP firms by promising to bid up wages when outside options arise and workers are willing to trade off present term wages to secure higher option values. The more productive the firm, the bigger the wedge between wage bills since they offer more lucrative future wages within the firm *and* at future employers. For a given cost, only sufficiently productive firms find that the difference in wage bills under the two contract types exceeds their willingness to pay for the right to employ under SA. This results in a low productivity sector with non-negotiable WP wage contracts and a high productivity sector with flexible SA wage contracts. This mixed-contract equilibrium nests pure-contract equilibria: under null costs all choose SA and the model becomes identical to Postel-Vinay and Robin (2002a)

[1] The environment is more general than that considered by Postel-Vinay and Robin (2004) and Holzner (2011). These papers trade off a general labor market environment for micro-foundations of the cost of SA. Each micro-founds the cost in workers' differential search effort under contract types. The general labor market considered here is at odds with such micro-foundation since a continuous distribution of productivity types and optimal wage setting strategies in the WP sector imply that workers should have search incentive in *both* sectors. Further, search incentives are not monotone in either sector. Modeling costs in this way also facilitates estimation: the model still yields a one-to-one mapping between a firm-level characteristic (turnover) and unobservable output. This is necessary when estimating the probabilities of receiving offers on-the-job, since a measure of the fraction of wage offers a worker would accept, conditional on history, is required.

and under sufficiently large costs all choose to WP and the model becomes identical to Bontemps et al. (2000).

This paper characterizes the interaction and resulting wage when a WP and a SA firm compete over the same worker and the impact of contract heterogeneity on the implied aggregate wage distribution and search incentives. An advantage to explicitly modeling wage contract heterogeneity is the ability to capture analytical and policy-relevant insight regarding firms that face information or negotiating frictions *at the same time* as an improved overall fit to aggregate data. In particular, the mixed-contract model is able to *simultaneously* rationalize data on wage dispersion, unemployment and employment duration, and the ratio of output to compensation of employees (labor share) and a good fit is achieved even when a large portion of firms hire under WP. The result is a model that is uniquely well suited to welfare analysis of labor market policy that takes into account both the impact of policy on firms' decision making when firms are not fully flexible or not fully informed and outcomes for workers.

This result is significant since the pure-WP model struggles to achieve reasonable labor share while the pure-SA mechanism fails to provide adequate incentive to search while unemployed. The former is an implication of the passive nature of competition in the WP mechanism, which is especially problematic in the right tail of the distribution. The latter is the opposite: fully contingent and back-loaded contracts extract the full value of search from the unemployed, resulting in a lack of search incentive and often implausibly low or even negative starting wages.

In the mixed-contract model developed in this paper, the WP sector, in which firms yield rents up front, puts upward pressure on entry wages in the SA sector pushing these wages toward larger and more plausible values. At the same time the WP sector restores search incentive for the unemployed. Meanwhile, the SA sector provides a long right tail of wages even when the tail of the productivity distribution is relatively light, as well as additional, moderated wage dispersion in the left tail. Simulations of the mixed contract model feature substantial wage dispersion without requiring auxiliary distributions which contradict observed short unemployment durations or observed division of output between factors. The mixed-contract model thus fits dominant features of the wage distribution at the same time as (1) implied labor share is proportional to national accounting data, (2) implied employment transitions are consistent with observed worker histories, and (3) implied flow value of unemployment is consistent with value guaranteed by social insurance programs.

The remainder of the paper proceeds as follows. Section 2.2 lays out the labor market setting under consideration, including details of contracts that are available; describes worker and firm behavior; and demonstrates that, for every fixed per-firm cost for the right to SA, there is a separating equilibrium in which low productivity firms employ under WP while higher productivity employ under SA. Section 1.3 reviews existing on-the-job search models featuring pure-WP or pure-SA with respect to empirical regularities. Existing models find it difficult to simultaneously match wage dispersion, labor flows, national accounts, and the generosity of social insurance. Section 1.4 provides comparative statics for a fixed distribution of firm types. As more firms join the WP sector, labor share increases but the distribution of wages becomes more compressed. This sets up a trade-off whereby one can achieve substantial labor share even when many firms WP, since required wage dispersion can be supplied by the remaining firms in the SA sector. Section 1.5 demonstrates that the mixed-contract model can achieve wage dispersion consistent with that observed for Germans in the later part of the 2000s at the *same time* as it matches an overall consistency with national accounts and with social insurance, when it is constrained to be consistent with observed durations of employment and unemployment for the same sample. Section 2.5 concludes. For the most part proofs and derivations are left to the appendix. In Chapter 2 I extend the dual-contract model to admit labor with heterogenous skill levels. In Chapter 3 I offer methods for identifying the composition of contract types using register data.

1.2 Model

I consider two wage setting mechanisms: wage posting (WP) and sequential auction (SA). WP is typified by a fixed wage for all workers for the duration of contracts. Specifically, each firm is constrained to select a single "posted" wage and employ all workers under this wage regardless of tenure or alternate employment offers. Under SA, each firm sets wages to match the value of the best-to-date outside option of each of its employees and retains their services by updating wages whenever profitable. Firms are fully informed about and fully flexible and responsive to workers' outside options and offer wages equal to each worker's reservation wage given her best-to-date outside option. In addition, firms update wages over the course of a contract in order to retain workers when profitable. This results in a bidding war when workers have offers from two SA employers. The SA wage setting mechanism, in this situation, is equivalent to a second price auction for the employee's services or a Bertrand competition regarding the value of employment: the worker is paid a wage which yields value equal to the highest profitable wage offer of the less productive competitor.

An innovation of this paper is to characterize the interaction and resulting wage when a WP and a SA employer compete over the same worker. I also will demonstrate that a separating equilibrium exists in which only low productivity firms employ under the WP mechanism. Intuition of the separating equilibrium rests on noting that, all else equal, employing workers under SA results in a lighter wage bill than employing under WP. A cost for SA equivalent to the difference in wage bills for the threshold firm, thus, induces less productive firms to select WP and more productive to select SA. The remainder of this section is devoted to formalizing and proving this intuition. Sections 1.4 and 1.5 detail the impact on the implied equilibrium wage distribution and value of search.

1.2.1 Setting

I consider the steady state equilibrium of a search market in which firms and workers are brought together by a sequential process of random matching. Measure N of firms operate technologies which produce flow output p per-worker. Technology is distributed continuously according to $\Gamma(p)$ on support $[\underline{p}, \bar{p}]$ with $\bar{p}$ potentially infinite. Measure M of workers search both off- and on-the-job using uniform sampling, meaning the probability of sampling a firm of productivity p or less is $\Gamma(p)$. Job offers arrive at exogenous Poisson arrival rates λ_0 when unemployed and λ_1 when employed. Workers are exogenously separated from employment contracts at Poisson arrival rate δ and discount the future at rate μ. Workers receive flow b when unemployed. Each worker has linear utility and seeks to maximize the present discounted value of wages and unemployment benefits.

Firms hire workers under one of two wage contracting mechanisms: WP or SA. If WP, the firm offers a non-negotiable wage for as long as the worker wishes or until exogenous separation. The wage offer is uniform for all workers. If SA, the firm offers a wage chosen to match the value of each workers best-to-date outside offer. The wages are updated as outside offers evolve and the SA firm bids up the worker's wage even in the event of a job-to-job transition. SA wages are thus described by the wage setting mechanism and the productivity of both the incumbent firm and best-to-date outside option.

If the firm chooses to set wages under SA, it must pay a flow cost of c. The cost is independent of firm size. The micro-foundations of this cost are left for another paper; however, one possible story is that there are legal and/or administrative fees associated with posting a vacancy in which wages will be set by SA. These fees must be paid whether or not the vacancy is filled. Note that, although firms differ in size in equilibrium, each offers an identical number of vacancies. Firm size is determined ex-post by the rate of vacancy filling and the duration for which contracts persist.

The remainder of this section proves the following proposition.

Proposition 1. *For each cost, c, there is a threshold, $\check{p}$, such that a Nash equilibrium exits in which firms with productivity less than the threshold ($p < \check{p}$) all strictly prefer WP while more productive firms ($p > \check{p}$) all strictly prefer SA. Threshold productivity firms ($p = \check{p}$) are indifferent.*

I begin by characterizing labor supply and wage schedules. I then prove that for every threshold a cost exists such that separation is consistent with a Nash equilibrium. I then observe that the mapping is continuous, that null costs are consistent with all firms selecting SA and all firms select WP for sufficiently large costs. The claim then follows from the intermediate value theorem. Conditions for uniqueness are provided in the appendix.

1.2.2 Labor supply

Workers seek to maximize the value of their current employment contract. In order to pin down labor supply to a firm of type p, I must show how workers' value maximization results in labor flow between firms.

Lemma 1. *In the mixed contract model with proposed separating equilibrium labor flows are constrained efficient.*

In other words, no worker ever rejects a job offer from a more productive employer than their current incumbent. To show this I must show 1) that the flow between WP and SA sectors is efficient and 2) that flows within sectors remain efficient in the mixed equilibrium.

The largest possible WP wage is at most equal to the productivity of the threshold firm. In the proposed separating equilibrium, the least productive SA firm has larger productivity than the most productive WP firm. So, each SA firm can profitably pay a wage at least as large as the largest possible WP wage and the SA contracting mechanism permits the firm to offer such a wage. Therefore, every SA firm must hire employees of every WP firm, if it meets them, in equilibrium, and the flow between sectors is efficient.[2]

Within the WP sector, firms continue to behave as if the whole economy posted wages, and so flows are efficient. This follows from noting that the presence of SA firms does not alter the solution to the WP firms' maximization problem since 1) profitable wage choices are only competitive against other WP firms and 2) since $\Gamma(p)$ is continuous there is no mass of WP firms at the threshold productivity. The proof due to Burdett and Mortensen (1998, pg.268) applies: more productive firms can employ workers of less productive firms at trivially greater wages and at greater profits. Finally, within the SA sector more productive firms are still able to outbid less productive firms and so the flows are efficient. In total, flows of employed workers in the separating equilibrium are efficient.

Since workers accept any wage offer originating from a more productive firm, labor supply to a p-type firm can be pinned down by the method of mass balance. In steady state, the mass of workers flowing into firms of p-type or less must be equal to the mass flowing out:

$$\underbrace{U\lambda_0\Gamma(p)}_{\text{in}} = \underbrace{[\delta + \lambda_1\bar{\Gamma}(p)](M - U)L(p)}_{\text{out}}, \tag{1.1}$$

where $\bar{\Gamma}(p) = 1 - \Gamma(p)$ is the fraction of firms with productivity greater than p, U denotes the mass of unemployed workers, and $L(p)$ is the mass of workers employed in a firm of productivity no greater than p.

Evaluating equation (2.40) at the supremum of productivity types yields steady state unemployment rate: $u = U/M = 1/(1 + \kappa_0)$, where $\kappa_0 = \lambda_0/\delta$. Also, the fraction of workers working for a firm with technology p or less is $L(p) = \Gamma(p)/(1 + \kappa_1\bar{\Gamma}(p))$, where $\kappa_1 = \lambda_1/\delta$ is the expected number of job offers per employment spell.

[2] We will see that, in equilibrium, the SA firm is actually able to hire these workers for less than their best-to-date posted wage. The reason is that the SA firm compensates its workers partially through the option value of contingent pay.

The supply of labor to a firm of type p can then be expressed as:

$$\ell(p) = \overbrace{\frac{\lambda_0 U + \lambda_1 (M-U) L(p)}{N}}^{\text{expected hiring}} \overbrace{\frac{1}{\delta + \lambda_1 \bar{\Gamma}(p)}}^{\text{expected duration}} = \frac{1+\kappa_1}{[1+\kappa_1 \bar{\Gamma}(p)]^2} \frac{M-U}{N}. \tag{1.2}$$

1.2.3 Wage choice

Wage setting in pure-WP and pure-SA equilibria with continuous productivity dispersion are well studied: the direct antecedents of the present work are Bontemps et al. (1999) and Postel-Vinay and Robin (2002a). Characterizing an equilibrium that features both kinds of wage contracts requires characterizing wages that arise when firms operating different wage-setting mechanisms interact: I will call this the "transitional wage".

To reiterate firms' strategies: WP firms are inflexible and select a single, nonnegotiable wage under which all employees are employed for the duration of employment. In contrast, SA firms are fully flexible and offer wages just large enough to offer value of employment equal to the value of employment in the best-to-date outside option of each employee, whenever profitable. In other words, SA firms employ each worker at that worker's reservation wage, noting that reservation wages are conditional on worker history and evolve over time as workers' best-to-date outside options improve.

The SA firm's optimal wage choice equates the value of employment in the SA firm at the optimal wage with the value of employment at the best-to-date outside option at that competitors optimal wage choice. Postel-Vinay and Robin (2002a) consider the case when the best-to-date outside option originates from a SA firm. When two firms which employ under SA meet, the result is a Bertrand competition or second price auction for the services of the employee. The employee ends up employed in the more productive firm in a contract which offers value equal to the maximum value the less productive firm can offer.

I characterize the nature of competition between a WP firm and a SA firm and the optimal wage choice when a SA firm employs a worker whose best-to-date outside option is a WP firm. As before, the SA firm optimally sets wages such that it offers value of employment just larger than the value of employment in the best-to-date outside option; however, the WP outside option *does not* Bertrand compete for the workers services. Thus, the SA firm need only best the value of employment in the WP firm *at the posted wage*. The optimal wage choice can be characterized for an arbitrary SA firm of type p employing a worker with best-to-date outside option from an arbitrary WP firm of type q by equating the value of employment in each contract. Note that, since I am considering a separating equilibrium, if q is WP and p is SA then $q < \check{p} \leq p$.

The value of employment at a SA firm at some wage $w_{PA}(q,p)$ consistent with best-to-date outside offer originating from a q-productivity WP competitor is:

$$\begin{aligned}
\mu V^A(w_{PA}(q,p),p) = w_{PA}(q,p)& \\
&+ \underbrace{\lambda_1[\Gamma(\check{p}) - \Gamma(q))][\mathbb{E}[V^P(w_{PP}(x),x)|q < x < \check{p}] - V^A(w_{PA}(q,p),p)]}_{\text{on-the-job wage gain due to a credible threat from a WP competitor}} \\
&+ \underbrace{\lambda_1[\Gamma(p) - \Gamma(\check{p})][\mathbb{E}[V^A(x,x)|\check{p} < x < p] - V^A(w_{PA}(q,p),p)]}_{\text{on-the-job wage gain due to a credible threat from a SA competitor}} \\
&+ \underbrace{\lambda_1[\bar{\Gamma}(p)][V^A(p,p) - V^A(w_{PA}(q,p),p)]}_{\text{job-to-job transition to a SA competitor}} \\
&+ \underbrace{\delta[V^U - V^A(w_{PA}(q,p),p)]}_{\text{unemployment shock}}, \qquad (1.3)
\end{aligned}$$

where $w_{PP}(x)$ and x are the optimal competing wage offers of x-type WP and SA firms with productivity less than p, respectively. The difference in the wage setting strategies of a WP competitor and a SA competitor is reflected in potential wages changes. If the incumbent SA firm meets a new WP competitor wages will rise only enough to just best the value of the competitor WP firm's posted wage offer. On the other hand, if the incumbent SA firm meets a new SA competitor and is able to retain the worker, it must be the case that wages rise to just best the *maximum* value which the competitor SA firm is able to offer: the value of a wage equal to the productivity of the competitor SA firm at the competitor firm. This is reflected also in the option value of moving to a more productive firm. The transition would yield value just larger than the maximum the current incumbent is able to offer.

Meanwhile, the value of employment in the employees' best-to-date outside option, a q-productivity WP firm, is:

$$
\begin{aligned}
\mu V^P(w_{PP}(q), q) = {} & w_{PP}(q) \\
& + \underbrace{\lambda_1[\Gamma(\check{p}) - \Gamma(q)][\mathbb{E}[V^P(w_{PP}(x), x) | q < x < \check{p}] - V^P(w_{PP}(q), q)]}_{\text{job-to-job transition to a WP competitor}} \\
& + \underbrace{\lambda_1[\bar{\Gamma}(\check{p})][\mathbb{E}[V^A(w_{PA}(q, x), x) | \check{p} < x] - V^P(w_{PP}(q), q)]}_{\text{job-to-job transition to a SA competitor}} \\
& + \underbrace{\delta[V^U - V^P(w_{PP}(q), q)]}_{\text{unemployment shock}}.
\end{aligned} \tag{1.4}
$$

The optimal wage choice of the p-productivity SA firm when employing a worker with best-to-date outside option from the q-type WP firm is workers reservation wage conditional on history. Thus, the optimal wage choice equates the value of these two employment contracts:

$$
V^A(w_{PA}(q, p), p) = V^A(w_{PP}(q), q) \text{ for all } q < \check{p} \leq p. \tag{1.5}
$$

The optimal wage choice can thus be expressed as the posted wage at the best-to-date outside offer and the difference in the option values of the two employment contracts:

$$
\begin{aligned}
w_{PA}(q, p) = {} & w_{PP}(q) + \\
& \underbrace{\lambda_1\left\{\bar{\Gamma}(\check{p})V^P(w_{PP}(q), q) - [\Gamma(p) - \Gamma(\check{p})]\mathbb{E}[V^A(x, x) | \check{p} < x < p] - \bar{\Gamma}(p)V^A(p, p)\right\}}_{\text{difference in option values in the SA contract and best-to-date WP outside option}}.
\end{aligned} \tag{1.6}
$$

This expression is surprisingly simple. Notably, both the WP outside option and SA incumbent promise exactly the same schedule of option values when competing against passive WP competitors and in the event of an unemployment shock. Thus, the difference in option values depends *exclusively* on the difference between how the SA employer and WP best-to-date outside option would compete with SA competitors.

Inspection of the difference in option values reveals that the WP firm offers less option value than the SA firm due to its refusal or inability to renegotiate and bid up wages in Bertrand competition. Indeed, the option value of an encounter with a SA firm when employed at a WP firm is null, since the SA firm offers exactly the reservation wage for the transition. The result is that the SA firm is always able to employ workers at wages *lower* than the wages offered by the best-to-date outside option WP firms:

$$w_{PA}(q,p) < w_{PP}(q) \text{ for all } q < \check{p} \leq p. \tag{1.7}$$

Further, a more productive SA firm offers even greater option value than a less productive one: it is able to promise a greater schedule of on-the-job raises and greater wages following job-to-job transition since it is willing to bid up wages to a larger value commensurate with its productivity. The result is that the more productive the SA firm is the larger is the wage cut for any best-to-date outside option:

$$\frac{dw_{PA}(q,p)}{dp} < 0 \text{ for all } q < \check{p} \leq p. \tag{1.8}$$

Conditions 1.7 and 1.8, which govern the transitional wage schedule, w_{PA}, are all that is required to prove that a separating equilibrium is a Nash equilibrium under an appropriate cost of SA. Appropriate costs are identified in the following section. The functional form of the full wage schedules under all combinations of contract types (WP wages, $w_{PP}(p)$; transitional wages, $w_{PA}(q,p)$; SA wages, $w_{AA}(q,p)$; and reservation entry wages from unemployment into each contract type $w_{UP}(p)$ and $w_{UA}(\cdot,p)$) is derived in the appendix. These are helpful for analysis of equilibrium properties, but are not required to establish existence of equilibrium.

1.2.4 Contract choice

If the prescribed contract choice and wage schedule are consistent with Nash equilibrium they must maximize rents for all firms. Current operating surplus for WP firms is pinned down simply as rent per worker times labor supply: $[p - w_{PP}(p)]\ell(p)$. Deriving current operating surplus for SA firms requires deriving the mass of their employees earning each wage. Following the usual solution strategy, the mass flowing in and out of such wages must balance. Note that workers willing to accept wage $w_{PA}(q,p)$ (or $w_{AA}(q,p)$ if q is a SA firm) or less must have best-to-date outside option q or less. The mass flowing into such contracts will be $U\lambda_0\Gamma(q)$ and the mass flowing out must be $[\delta + \lambda_1\bar{\Gamma}(q)](M-U)L(q)$. This yields $\ell(w(q,p)|p) = \ell(q)$.[3] The current operating surplus (exclusive of the cost of SA) for a firm of type p offering the SA contract is

$$\pi^A(p) = (p - w_{PA}(\underline{p},p))\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} (p - w_{PA}(q,p))d\ell(q) + \int_{\check{p}}^{p} (p - w_{AA}(q,p))d\ell(q). \tag{1.9}$$

If $\check{p}$ is the threshold it must be the case that the $\check{p}$-type firms are indifferent between contract types and that all less productive firms prefer WP while all more productive firms prefer SA. For threshold productivity in the interior of the support of the productivity distribution, the threshold productivity firm's willingness to pay for the right to SA is:

$$c = \pi^A(\check{p}) - \pi^P(\check{p}) = \{\mathbb{E}[w|\check{p},P] - \mathbb{E}[w|\check{p},A]\}\,\ell(\check{p}) \tag{1.10}$$

The strategies described constitute a Nash equilibrium if, when costs are equal to the

[3]Note, these results can also be found in Postel-Vinay and Robin (2002a, pg. 999-1001).

threshold firms willingness to pay, firms maximize profit by offering the prescribed the contract choice and wage schedule. The proof follows from noting that 1) a more productive WP firm always posts a larger posted wage than a less productive WP firm, 2) SA firms can hire from WP firms at lower wages, and 3) for any best-to-date outside option SA wages are decreasing in the SA firms productivity: $dw_{PA}(q, p)/dp < 0$. Details are in the appendix. An interesting implication, which is not explored here, is that any alternate pair of contract types that yield the same criteria lend themselves to similar analysis.

That a separating equilibrium exists for every choice of the cost of SA follows from the intermediate value theorem. Formal proof is in the appendix. For a fixed set of active firms, the mapping between cost, c, and threshold, $\check{p}$, is unique when marginal wage schedule under WP, $w_{PP}(\check{p})$, increases more rapidly with respect to the change in threshold than the marginal wage schedule under SA, $w_{PA}(q, \check{p})$. This requires placing restrictions on how rapidly the value of search rises with threshold productivity. Formal conditions for this criteria are presented in the appendix. Under these conditions the productivity type of the threshold firm is an increasing function of the cost of SA.

In sections 1.4 and 1.5, I analyze the properties of equilibria which feature a mixture of WP and SA employment contracts and compare them to pure-contract equilibria. When considering comparative statics, I restrict my analysis to the case in which conditions are met such that equilibria are unique. Before giving comparative statics, however, I review the properties of the nested pure-contract equilibria with respect to empirical regularities.

1.3 Equilibrium on-the-job search and relation to empirical regularities

The on-the-job search model laid out in Burdett and Mortensen (1998) propels random search as a contender for equilibrium analysis of labor markets. The model provides a solution to the

Diamond (1971) Paradox that does not require ex-ante heterogeneity to generate disperse wages and search incentive. This feature makes on-the-job search a particularly fruitful backdrop for welfare analysis. WP models describe a labor market in which informational or negotiation failures prevent firms from extracting full labor rents. The policy-relevant insight from such models is that, since firms' hands are bound in negotiations, they overreact to policy: shifting wages and/or hiring more radically than if firms were fully informed or fully flexible. For example, when uniform unemployment insurance becomes more generous the *entire* distribution of wages shifts toward higher values. As a result, changes in worker incentives affect not only what job offers are *acceptable* but also the schedule of offers that are *available*.

Substantial progress has been made toward consistency between the model and features of data. But, substantial progress also has yet to be made. Inclusion of employer heterogeneity with respect to productive capacity in the pure-WP on-the-job search model makes it possible to generate a wage distribution that matches the distribution observed in data (Bowlus et al., 1995; Bontemps et al., 2000). When productivity is distributed continuously, the model becomes quite elegant: firms play pure strategies that are the limiting cases of the mixed strategies originally studied by Burdett and Mortensen (1998) and these imply an invertible map between wages and productivity (Bontemps et al., 2000). Unfortunately, this comes at the expense of an implied distribution of productivity and an implied value of unemployment that, as I will discuss below, are implausible.

Changing the protocol of the wage setting mechanism potentially alleviates these difficulties. Instead of setting wages and remaining passive as in WP, firms may compete over wages competitively as in SA. The pure-SA model generates a thin right tail of wages, similar to that observed in the data, under every productivity distribution. It is possible to select a productivity distribution such that wage dispersion and labor share are not as dramatically at odds. Unfortunately, this solution travels too far in the competitive direction and, as I will discuss below, implies entry wages that are implausibly low and that eliminate search incentive entirely for the unemployed.

The result is that welfare analysis carried out in the pure-WP model likely overstate the impact of policy on workers, since the option value of search is extreme. Results, however, are difficult to interpret due to the disproportion of shares of output implied by the model relative to national income and product accounts. In the other extreme, pure-SA, the impact of policy is more moderate but again difficult to interpret. In particular, since search has no value of search in unemployment it is difficult to argue, theoretically, that unemployment exists at all in the model or that it should change in response to policy. Additionally, one loses the initial interesting intuition that both the set of acceptable choices and the set of offered choices are affected by policy.

1.3.1 Value of unemployment and search behavior

Inconsistencies between observed search behavior, plausible leisure values, and search incentives in the pure-WP model, stem from the lucrative search incentives provided to workers by high wages posted by firms in the tail of the distribution. If pure-WP describes the world, Hornstein et al. (2011) point out that either: (1) these wages must be extremely unlikely. This is inconsistent with observed wage data. (2) workers must be willing to forgo lesser opportunities and search a long time to secure high wages from the tail of the distribution. This would lead to a compressed wage distribution due to high reservation wages of the unemployed and long unemployment durations, both inconsistent with data. Or (3) workers must hate leisure enough to be willing to accept low or moderate wages instead of waiting to search for lucrative wages in the tail of the distribution. This is likely inconsistent with existing social programs which provide a floor for consumption even in unemployment.

The seminal empirical implementation of a pure-WP model, Eckstein and Wolpin (1990), requires that some unemployed workers have reservation wages equal to the largest observed wage, since in that model workers do not search on-the-job. These workers induce some firms to offer high wages, but the consequence is that these workers must reject all other offers and thus have counter-factually long unemployment duration (Eckstein and Wolpin, 1990). On-the-job search mitigates the problem in two ways: the maximum wage is decoupled from a reservation wage of an unemployed worker and the difference between the option values of employment and unemployment become smaller. However, search would have to be more efficient on- than off-the-job to match wage dispersion (Hornstein et al., 2011). This contradicts mobility data which shows dramatically less job switching among the employed than job finding among the unemployed.

In the other extreme, under pure-SA unemployed workers are willing to take low starting wages in the hopes of bringing the employer into Bertrand competition with another employer in the future. This offers a potential solution to the problem of Hornstein et al. (2011). In the standard SA model presented in Postel-Vinay and Robin (2002a,b), fully informed firms extract the full rents from the unemployed, making the option value of search zero for the unemployed. Without further restrictions, the option value of employment *always exceeds* that of unemployment as long as there is *any* chance of on-the-job search, in the pure-SA equilibrium. As a result, unemployed workers always accept starting wages *smaller* than the flow value of unemployment. This is in contrast to the pure-WP model where, as long as offers arrive more rapidly in unemployment, unemployed workers demand wages strictly *larger* than the flow value of unemployment. These alternate search incentives potentially induce great wage dispersion while at the same time providing no incentive to prolong search.

Unfortunately, full rent extraction leaves no incentive for labor force participation for the unemployed. From the point of view of pure-SA, unemployment and unemployment duration are simply exogenous. This contradicts data revealing that a sizeable fraction of non-employed workers actively seek work and that unemployment durations respond to policy changes. More problematic, is the relation between on-the-job offer arrival, average wages, and flow value of unemployment. The model implies that, for a given labor market history, wages are a *decreasing* function of the on-the-job offer arrival rate. As a result, without restrictions on admissible productivity distributions or the rate of on-the-job offer arrival, the model may produce average wages near or even *below* the flow value of leisure. Simulations conducted by this author suggest that the flow value of leisure is approximately equal to the average wage in the SA model. This is, again, at odds with active search behavior of the unemployed.

1.3.2 Labor share and distributions of wages and productivity

The pure-WP model also struggles to simultaneously fit wage dispersion and output data. This stems from the increasing size of the current operating surplus earned by increasingly more productive firms. All except the least productive firm pay workers strictly less than the product of labor. The difficulty arises from the implication that the most productive firms pay the smallest share. As a result, labor share in the most productive firms is exceedingly small. Coupled with the presumption of a single posted wage, the result is that fitting the observed wage dispersion requires a productivity distribution which *severely dominates* the wage distribution (Bontemps et al., 1999; Mortensen, 2003). In other words the WP model implies a counter-factually small share of output is paid to labor when the model is required to match observed wage dispersion, which features a long-thin tail of wages.

Severe dominance of the productivity distribution stems from the passive nature of competition between firms. The most productive firm can hire all workers by offering wages only minutely larger than the next most productive competitor, which in turn needs to offer wages only minutely larger than the third most productive competitor. Workers' inability to capture a greater share of rent is a result of firms' inability or refusal to counter outside offers as workers accrue them. Inability or unwillingness to negotiate on wages is, however, not a good descriptor of all employment relationships.[4] In some careers, for instance in academia, pay is often dependent on credibly demonstrating a competitor employer's willingness to hire at a high wage.

SA again provides a possible remedy. Occasional Bertrand competition between highly productive firms results in a thin tail of wages with support that matches that of the productivity distribution. Postel-Vinay and Robin (2002a) demonstrate that the required thin tail arises under all potential productivity distributions. However, the SA model fits observed data poorly in the left tail of the wage distribution without further restrictions. Full rent extraction and the consequent sizeable difference between the option value of employment and unemployment can imply large negative entry wages. This contradicts observed wage distributions and minimum wage laws which suggest that wages distribution are strictly positive.

In the literature, several remedies to the problem have been implemented. Workers can be modeled with preferences that minimize the option value of search: higher rates of time preference or increased risk aversion. Postel-Vinay and Robin (2002b) implement both. Alternately, larger wages can be arbitrarily assigned to workers by a reduced-form bargaining rule, as in Cahuc et al. (2006), or by imposing minimum wages. However, unless workers with poor labor market histories are the most effective bargainers, bargaining in the reduced form formulation applied by Cahuc et al. (2006) does little to solve the problem. Meanwhile, imposition of minimum wages resolves the issue by brute force but, at the same time, sacrifices analytical tractability since, with minimum wages, value functions under SA can only be solved by simulation.

The mixed-contract model with dual wage formation mechanisms developed here offers an additional solution. The new solution is both tractable and can exist in tandem previous solutions.

[4] It is also not sub-game perfect unless more structure is imposed.

1.4 Comparative statics of the mixed-contract model under a fixed productivity distribution

I compare models with mixed contracts of the type just developed when the cost of SA varies. Throughout, I consider a fixed set of transition hazards, $\{\lambda_0, \lambda_1, \delta\}$, discount rate, μ, and distribution of productivity, $\Gamma(p)$. The main takeaways are 1) flow of labor between employers is constrained efficient in all models; 2) search is more valuable to workers with poor employment histories when more firms WP; 3) the distribution of output among factors is more favorable to labor when more firms employ under WP; and 4) wage distributions are more disperse when more firms SA.

The third takeaway initially seems at odds with the stated need to generate a model that features WP and a large labor share. I reiterate, the problem is to develop a model that generates large labor share *and* substantial wage dispersion *at the same time*. With 3) and 4) established, I pursue the goal of matching observed wage dispersion in the following section. The intuition that will be established in this section and utilized in that section is that the larger the share of firms employing under SA in equilibrium, then the lighter the tail of the productivity distribution required to match any given level of wage dispersion.

1.4.1 Efficiency

In section 2.2, I established that, in equilibrium, flow of workers between firms is constrained efficient, meaning that an offer from a firm that is more productive than a worker's current employer is never rejected. This is somewhat surprising since SA is a typical frame for the criticism that WP is not sub-game perfect. In such a criticism deviating to SA enables a firm, in an otherwise WP equilibrium, to retain a worker who would otherwise exit to an alternate employer. Such a deviation would disturb the flow of labor to more productive firms. This intuition stands as a critique of the lack of sub-game-perfection of the pure-WP equilibrium. However, in the pure-SA equilibrium constrained efficiency returns. No lower productivity firm can profitably outbid a higher productivity firm and workers again always flow toward more productive firms.

In the present model featuring both contract types, constrained efficiency arises in the WP and SA sectors *and* flow between the sectors is efficient. The last occurs because all firms that opt to employ under SA are more productive than the most productive firm which employs under WP. The implication, somewhat unintuitively, is that the supply of labor to a firm of a given type is identical in all three models (pure-WP, pure-SA, and the mixed contract model considered here) since the flow of labor in and out of firms is, in equilibrium, identical in all three models despite differences in wage setting strategies.[5]

An alternate and fruitful mode of comparing the models is to compare the value of search and the allocation of rent between workers and employers. We have seen that the ability to tailor contracts to workers' present outside options while retaining their services down the line via contingent pay skews rent towards the employer. We have also seen how the wedge between labor rents earned by WP and SA firms can be exploited in order to derive the separating equilibrium.[6]

1.4.2 Value of search

Before proceeding to discussion of the allocation of output, I discuss the values of unemployment and search in the mixed contract equilibrium. These results build the foundation for results regarding the allocation of output.

Refer to the value of employment in a WP firm, equation 2.27. The option value of receiving an employment offer from a more productive WP firm is positive since posted

[5]Constrained efficiency, however, is not generally the case for on-the-job search. In particular, Stevens (2004) and Burdett and Coles (2010) show that when all firms post tenure-specific contracts the set of firms a worker is willing to transition to shrinks with tenure, inefficiently depressing ladder-climbing turnover.

[6]In practice, all that is required for separation is constrained efficient flows between sectors. This frames a clear heuristic argument for how more complicated contracts, such as tenure-specific contracts, might interact with WP and SA in equilibrium; however, I leave rigorous comparison for future work.

wages are not contingent on workers' labor market histories. The option value of receiving an employment offer from a SA firm behaves quite differently. Pitting the fully flexible and informed wage setting policy of SA firms against the passive wage setting strategy of WP generates *no increase in the value of employment* upon transition! The reason is that SA firms yield only enough rent to new employees to just best the highest wage offer of the workers' best-to-date outside option, and WP firms do not update their wage offers in the face of competition. The value of unemployment or employment in a WP depends only on the distribution of WP firms.

Assuming that $w_{PP}(x)$ is a differentiable function (this is verified in the appendix), the value of employment in a p-productivity WP firm is:

$$V^P(w_{PP}(p),p) = \frac{w_{PP}(p)}{\mu+\delta} + \frac{\lambda_1}{\mu+\delta}\int_p^{\check{p}} \frac{\Gamma(\check{p})-\Gamma(x)}{\mu+\delta+\lambda_1[\Gamma(\check{p})-\Gamma(x)]}\frac{dw_{PP}(x)}{dx} + + \frac{\delta V^U}{\mu+\delta}. \tag{1.11}$$

Similarly, the value of unemployment is:

$$V^U = \frac{b}{\mu} + \frac{\lambda_0}{\mu}\int_{\underline{p}}^{\check{p}} \frac{\Gamma(\check{p})-\Gamma(x)}{\mu+\delta+\lambda_1[\Gamma(\check{p})-\Gamma(x)]}\frac{dw_{PP}(x)}{dx} \tag{1.12}$$

It is easy to verify that the value in both states is larger when more firms operate the WP contract: $V^P(w_{PP}(p)/d\check{p} > 0$ and $V^U/d\check{p} > 0$. This follows since as the threshold rises a greater share of firms yield rents and the new rent yielding firms yield large rents.

Thus, we have:

Claim 1. *For a given distribution of firms, as the cost of SA increases, the option value of unemployment rises.*

Pure-SA has least option value of unemployment with $\mu V^U = b$, mixed contract value of unemployment increases in the threshold, and the value of unemployment is maximized under pure-WP. Conversely, one can rank equilibria in order of the flow value of unemployment consistent with a given set of active firms. The ranking falls in the reverse order:

Claim 2. *For a given distribution of active firms, as the cost of SA increases, the flow value of unemployment consistent with equilibrium falls.*

In other words, Claim 2 states that if two economies are each described by the same distribution of firms $\Gamma(p)$, the same set of transition hazards and discount rate, and *different* composition of wage contracts, it must be the case that workers in the economy with fewer SA contracts have a lower flow value of unemployment. I discuss how this may be at odds with modeling the economy as containing too large a share of WP contracts: with many WP contracts and realistic wage dispersion, the flow value of unemployment must be *extremely* low.

Similar logic and calculations yield:

Claim 3. *For a given distribution of active firms, as the cost of SA increases, the value of employment with best-to-date outside option a WP firm rises.*

As we will see, increasing share of firms operating WP contracts *significantly* impacts the support and shape of the wage distribution: shifting the support upward and increasing the skew of the distribution.

1.4.3 Distribution of output among factors

The model divides output into three shares: current operating surplus, payment for the right to SA, and wages. These can be interpreted in comparison to statistics from national income and product accounts: capital input, intermediate service input, and compensation of employees.

Interpretation of wages as compensation of employees is direct. Interpretation of aggregate payments for the right to SA as intermediate service input is consistent with conceiving of these as payment to a human resources department or a head hunter which is responsible for negotiating employment contracts at least cost. Under the structure of cost for the right to SA modeled in this paper, the human resources department or head hunter is equally effective regardless of the number of employees it is required to manage.

Current operating surplus may be interpreted as capital input using the following logic. It is possible to micro-found the distribution of productivity on market clearing conditions for capital input. Let there be some function $f(K) = p$ which transforms capital input K into labor productivity p with the usual conditions $f'(\cdot) > 0$ and $f"(\cdot) < 0$. The capital market is in equilibrium when all firms are indifferent between all active productive technologies: $\Pi = \pi^i(p) - rf^{-1}(p)$ for all choices of productivity $p \in [\underline{p}, \bar{p}]$ and contract $i \in \{P, A\}$, where r is the rental rate of capital. Since $\pi^i(p)$ is continuous and increasing on the intervals $[\underline{p}, \check{p})$ and $[\check{p}, \bar{p}]$ and $\pi^P(\check{p}) = \pi^A(\check{p})$ a function $f(\cdot)$ exists which is consistent with distribution of productivity $\Gamma(p)$ (or visa versa). With free entry, $\Pi = 0$ and $\pi^i(p) = rf^{-1}(p)$ for all productivities and both contract types. Here I do not explicitly micro-found the capital market.[7] However, noting that such a market can be written, I interpret current operating surplus, $\pi^i(p)$, as rents payed to capital, $rf^{-1}(p)$, in the un-modeled capital market.

In what follows, I consider the distribution of output between capital, intermediate service, and labor for a fixed distribution of active productivity types, $\Gamma(p)$, under different costs of SA. Throughout, I assume that the distribution of productivity, $\Gamma(p)$, meets the condition for uniqueness of equilibria, which is laid out in the appendix. Under this condition the threshold productivity, $\check{p}$, is increasing in the cost of SA, c, and, thus, equilibria are unique.

[7]For an example of such an exercise see the final section of Postel-Vinay and Robin (2002a).

Current operating surplus (capital input)

Since the threshold productivity is an increasing function of the cost of SA, it is straightforward to show:

Claim 4. *As the cost of SA increases, the share of output paid to capital falls.*

Consider a SA firm under an initial, low cost of SA. When costs rise the firm has two options: 1) continue to employ under SA but pay a larger share of output to intermediate service or 2) switch to WP, saving the cost of SA but paying a larger fraction of output to labor. Regardless of the firm's decision, each firm that employed under SA under the initial, low, cost earns strictly less current operating surplus under high costs. Meanwhile, the current operating surplus of firms which employed under WP even under low costs are unaffected. In aggregate, firms earn less current operating surplus under high costs of SA. Interpreting current operating surplus as payments to a capital input, as described above, gives the result. Equilibria can be ranked by decreasing share paid to capital: pure-SA with the greatest allocation to capital, mixed contract allocation to capital decreases in costs of SA, until pure-WP with least allocation to capital.

Aggregate spending on cost of SA (intermediate service input)

As the cost for the right to SA increases, some firms change wage contracting strategies, so fewer firms pay the cost. The following result arises immediately:

Claim 5. *Aggregate spending on cost of SA (intermediate service input) is*

1. *zero when all firms WP ($\check{p} = \bar{p}$),*
2. *zero when all SA ($\check{p} = \underline{p}$), and*
3. *maximized in some mixed-contract equilibrium.*

The first follows since when all firms WP ($\check{p} = \bar{p}$) none pay any cost and aggregate payments are thus zero. The second follows since $\check{p} = \underline{p}$ when $c = 0$, so all firms pay a zero cost and the aggregate payment is zero.

Compensation of employees (labor input)

As the cost for the right to SA increases, more firms offer WP contracts which immediately yield rent to workers. These firms, obviously, operate larger wage bills when the cost of SA increases. On the other hand, those that employ under SA come into Bertrand competition with other SA firms less often. This leads to a drop in their wage bill, since wages are lower for workers whose best-to-date outside option no longer competes via SA. However, this drop is counteracted by a simultaneous increase in the wage schedule for hiring from WP firms. This arises since a larger share of WP firms implies a larger value of search in unemployment and in WP firms. The second effect dominates the the first and we have:

Claim 6. *As the cost of SA, c, increases, the share of output paid to labor rises.*

The proof entails showing that the wage bill rises within every firm, thus, in total, the share of output paid to labor rises. Details of the proof are somewhat tedious and reserved for the Appendix.

1.4.4 Distribution of wages

Increasing the cost associated with SA, and therefore increasing the fraction of firms which select WP, decreases the spread and increases the skew of the distribution of wages. The impact is most starkly evident in the support of the distribution.

1.4.4.1 Support of the distribution of wages

Every SA firm offers some wages smaller than the wage offered by the least productive WP firm: $w_{PA}(\underline{p}, p) < w_{PP}(\underline{p})$ for all $p \geq \check{p}$. Every SA firm also offers some wages larger than the most productive WP firm: $w_{AA}(p,p) < w_{PP}(\check{p})$ for all p. Additionally, in all equilibria where some firms SA, both the largest *and* the smallest wages are paid by the most productive firm under the SA contract operated by this firm.

Upper bounds, $\bar{w}$, of implied wage distributions under equilibria can be ranked as follows:

- For $\check{p} < \bar{p}$ the upper bound of the implied wage distribution is equal to the upper bound of the productivity distribution: $\bar{w} = \bar{p}$
- If $\check{p} = \bar{p}$ the upper bound of the implied wage distribution is strictly less than the upper bound of the productivity distribution: $w_{PP}(\bar{p}) = \bar{w} < \bar{p}$

The first follows from noting that such a wage arises when two $\bar{p}$ productivity SA firms meet the resulting wage is $\bar{p}$ and that if such firms have positive probability the interaction must also occur with some probability and so the resulting wage must also have positive mass in the wage distribution. The second follows from observing that if all firms WP then the largest posted wage is strictly less than the largest productivity. This follows from the concavity of the posted wage schedule.[8]

Lower bounds are more interesting

- As cost of SA, c, increases the lower bound of the wage distribution, $\underline{w} = w_{UA}(\underline{p}, \bar{p})$, increases.

To see this remember that the value of unemployment is increasing in the fraction of firms which hire under WP. Since entry wages in SA firms are chosen to make workers indifferent between unemployment and accepting the wage offer entry wages in each SA firm rise: $\frac{dw_{UA}(p)}{d\check{p}} > 0$. This is true, in particular, for the most productive SA firm and this firm's entry wage is the smallest wage in the economy: $\underline{w} = w_{UA}(\bar{p})$.

[8]See the appendix for the functional form of the posted wage schedule, w_{PP}, and Bontemps et al. (1999) and Mortensen (2003) for further details.

Measuring wage dispersion as the spread between $\underline{w}$ and $\bar{w}$ models can be ranked by increasing threshold: pure-SA has most dispersion, mixed contract dispersion decreases in threshold, until pure-WP with least dispersion.

Expressions for the density of wages give a set of hazards $\{\lambda_0, \lambda_1, \delta\}$ discount rate μ and productivity distribution $\Gamma(p)$ are provided in the appendix. Figure 2.4, plots wage distributions implied by a pure-SA (top), pure-WP (bottom), and mixed-contract models (middle three). The mixed contract models features 25, 50, and 75 percent WP firms. These are decomposed into wages set under WP, wages set under SA with best-to-date outside option WP, and wages set after Bertrand competition between two SA firms.

As the fraction of WP firms climbs from zero to one, the distribution becomes more compressed. Compression is localized in the left tail, where increasing value of unemployment and employment in WP firms, due to increasing size of the WP sector, puts upward pressure on wages in SA firms. The result is increased skew of the distribution.

1.5 Comparative statics of the mixed-contract model under a fixed wage distribution

In the previous section, I compared equilibria for a fixed distribution of productivity. In practice, however, an econometrician or policy maker seeks a model which fits an observed distribution of wages and unemployment durations. The econometrician may also has auxiliary data on the relation between aggregate output and compensation of employees from national income and product accounts, and a notion of the generosity of social insurance with which the flow value of unemployment ought to be bounded.

Therefore, from the econometrician's standpoint, models ought to be compared on the plausibility of the implied distribution of productivity and implied flow value of unemployment when the observed empirical distributions of wages and unemployment durations are matched.

1.5.1 Data and calibration method

Data comes from the *Sample of Integrated Labour Market Biographies (SIAB)* from the Research Data Center (FDZ) of the German Federal Employment agency at the Institute for Employment Research (IAB). I calibrate the model to match labor market histories of West and East Germans and Berliners separately. The SIAB is a two percent sample drawn from the population of individuals in Germany who are employed and subject to social security, marginal part-time employed, a benefit recipient to the German Social Code II, official registered as a job-seeker at the German Federal Employment Agency, or participate in programs of active labor market policies. Employee data is matched with establishment data, in particular, to firm size and the rate of employee exit. The years 2006-2008 are selected as a relatively stable period which postdates the Hartz labor market reforms.

In the first stage, I estimate parameters $\{\lambda_0, \lambda_1, \delta\}$ to maximize the likelihood of observing the labor market transitions of the sampled individuals. The likelihood function and further data description are in the appendix. Table 1.1 reports parameter estimates for Germany as a whole and the three geographic regions. Hazards of job offer arrival and separation are calibrated to be consistent with these estimates. These estimates are broadly in line with Robin et al. (2004). Further details about the data and the likelihood function can be found in Appendix 1.10.

In the second stage, I make the assumption that productivity distributes according to a Pareto distribution. I then calibrate the parameters (shape and scale) of the Pareto distribution to minimize the sum of squared deviations between simulated mean and variance of log wages and observed moments for West and East Germans and Berliners. Empirical and simulated moments are both computed from data censored at the social security limit, 169 euros in West Germany and 143 in East Germany and Berlin. Mean and standard deviations of censored wages are presented in Table 1.2. Further details about the data and the simulation can be found in Appendix 1.10.

The parameters of the Pareto distribution have convenient interpretation with respect evaluating performance of the mixed-contract model. The scale parameter identifies the least productive active firm. Meanwhile, the shape parameter describes the rate at which the productivity density decays in the tail: a small scale parameter indicating a distribution with a heavy right tail. Results are presented in Table 1.3.

In evaluating model performance, I exploit auxiliary estimates of labor share, share of firms which WP, and income replaced by social insurance. The first, labor share of 63.7% in 2006, comes from the EU KLEMS Growth and Productivity Accounts. The second, 2/3 WP firms, is estimated by Brenzel et al. (2013) from a 2011 survey of German employers. To this author's knowledge, this is the only such estimate for Germany. The final, 30%, represents a lower bound calculated from the cross section of SIAB data in January 2006. See calibration appendix for details.

1.5.2 Results

In the mixed-contract model developed in this paper, the WP sector puts upward pressure on entry wages in the SA sector. Meanwhile, the SA sector provides a long right tail of wages even when the tail of the productivity distribution is light, as well as additional wage dispersion in the left tail. From the perspective of search incentives, the WP sector provides incentive to search when unemployed. Meanwhile, entry wages in the SA sector extend the distribution in the left tail, facilitating substantial dispersion without contradicting observed short unemployment durations. These results facilitate fitting wage dispersion at the same time as plausible flow value of leisure *and* labor share simultaneously.

Focusing first on entry wages, flow value of unemployment, and search incentives, recall that increasing the cost of SA increase the size of the WP sector and applies upward pressure on entry wages under a SA contract. This stems from the associated increasing value of search offered by the WP sector. The magnitude of this competition effect is substantial in reasonably calibrated models.

Figure 1.2 calibrates a pure-WP, pure-SA and mixed contract model in which 2/3 of firms select each contract type. Each model is then simulated using the same, fixed distribution of productivity. I take productivity to be distributed Pareto with shape 3.15 and scale 89.5. This choice of productivity dispersion produces a model consistent with observed mean and variance of log wages for the sample of West German males in 2006 when 2/3 of firms select the WP mechanism. I assume free entry on the firms' side and as a result, the scale parameter should be interpreted both as the minimal productivity, $\underline{p}$, and as the reservation wage of the unemployed to enter the least productive firm.

Comparing the distribution which arises under pure-SA (top panel) to that of the mixed contract model (middle panel) reveals a stark difference in the left tail. While entry wages are very low and indeed some are negative under pure-SA, competition from the WP sector in the mixed contract model mitigates this problem. The result is a distribution which is more positively skewed. Meanwhile, comparing to the distribution which arises under pure-WP (bottom panel) the mixed contract model supplies a degree of wage dispersion in the left tail which the pure-WP model can not. A substantial fraction of wages fall a substantial value below the reservation wage for employment in the least productive firm. The flow value of unemployment and mean wage are labeled for each figure. The difference between these rises when more firms hire under WP and yield rents up-front to unemployed workers, generating value of search.

Now turning toward the relationship between wage dispersion and labor's share of output. The three wage distributions depicted in figure 1.2 are simulated under the *same* productivity distribution. Consistent with the previous section, the labor share is increasing as the fraction of firms employing under WP is *increasing*. The mean of wages and of output are labeled for each distribution. This at first seems at odds with the discussion above, which claimed that large labor share was inconsistent with the pure-WP model. Note, however, that wages are *significantly* more dispersed in both left and right tail when the share firms which employ under SA increases. The key is to note that, for a given amount of *wage dispersion* the required productivity dispersion is *not as large* when more firms employ under SA. This makes it possible for simulated distributions to jointly imply reasonable labor share.

Comparison across a fixed productivity distribution, however, is not most relevant to the econometrician or the policy maker. Rather, one would like to compare models calibrated to match observed data on labor market histories and wage dispersion for a variety of mixtures of the two contract types. This entails finding a *different* distribution of productivity for every contract composition such that the model fits the data. Figure 1.3 plots the distribution of wages and productivity from models that match the mean and variance of wages observed for West Germans under pure-SA, pure-WP, and a mixed-contract model in which 2/3 of firms select the WP contract. Under pure-SA, the flow value of leisure and the average wage are nearly identical. Additionally, the wage distribution has an implausibly long left tail. By comparison, under pure-WP the flow value of unemployment is extremely low and the required distribution of output dominates the distribution of wages too severely to produce credible labor share. The mixed-contract model mitigates both problems.

Table 1.3 records the scale and shape parameters required to achieve observed wage moments under different compositions of wage contracts. As the portion employing under wage posting approaches 100%, the required distribution of productivity has an increasingly heavy tail. Implausibility of such a heavy tail is captured by implausibly low labor share, documented in Table 1.4.

Table 1.4 also records the ratio of the value of leisure to the average wage. When nearly all firms select WP, the best-fit simulation is unable to produce substantial value of leisure. The difficulty arises from the supposition that starting wages may be drawn from the full empirical distribution resulting in an implausibly large option value of search for the unemployed. Inclusion of a SA sector mitigates this problem: large wages arise only after on-the-job search and competition for a worker by two firms employing under SA. The result is that the simulated distribution of posted wages is more compressed, resulting in a more reasonably sized option value of search while unemployed.

1.6 Conclusion

This paper offers a model of on-the-job search in which firms select between hiring under a wage posting (WP) and a sequential auction (SA) contract. The mechanism described, a fixed per-firm cost for the right to hire under SA, is quite elegant. This cost structure enables me to find an equilibrium under any continuous distribution of productivity and for optimal WP strategies. The equilibrium described is separating: with some intermediate firm being indifferent between contract types, all less productive firms selecting WP, and all more productive firms selecting SA.

Compared with nearest neighbors, Postel-Vinay and Robin (2004) and Holzner (2011), the model presented here accepts general productivity distributions and allows for a WP sector in which wages are optimally set by firms and, more importantly, in which wages are disperse. This relaxes the restrictions on wage setting strategy imposed by Postel-Vinay and Robin (2004) and on the domain of firm types imposed by Holzner (2011). Further, my model nests the continuous productivity case of the pure-WP with on-the-job search (e.i. Bontemps et al. (2000)) and the pure-SA model (i.e. Postel-Vinay and Robin (2002a)) as limiting cases. This is accomplished by abstracting from the micro-foundations of the cost structure and instead directly exploiting a single, separation-driving, mechanism: the differential in wage bills for like-firms employing under different wage contracts.

The cost structure and the mechanism that it isolates could be extended to include more complex contract types. Particularly interesting candidates are tenure-contingent contracts. In principle, as long as flows between sectors of the labor market are constrained efficient it should be possible to find a similar separating equilibrium even with tenure-contingent contracts. The equilibrium plausibly features three sectors, with the least productive firms populating the WP sector, moderately productive offering tenure-contingent contracts, and the most productive offering fully contingent SA contracts. I leave full exposition of such an equilibrium to future work.

In intermediate cases of the model, when both contract types coexists, one can match employment transitions and wage dispersion, which can be observed by a policy maker or econometrician, while at the same time the implied productivity dispersion does not contradict observed labor share or plausible values of the flow value of unemployment. This is novel for a model which features WP. Further, since the value of search when unemployed depends only on the distribution of firms in the WP sector and wages in this sector need not be as disperse as the aggregate wage distribution, a large flow value of leisure is no longer at odds with moderate unemployment duration.

Also in intermediate cases featuring both contract types, distributions of wages are more positively skewed when more firms WP, mitigating a shortcoming of the pure-SA model. The main alternatives, increasing workers' impatience or risk aversion or introducing explicit bargaining power for workers, are substitutable (Postel-Vinay and Robin, 2002b; Cahuc et al., 2006)). When a mixture of contracts is considered, the model can be consistent with data for patient workers with linear utility and no bargaining power. I demonstrate that the model can match labor market histories and cross-sectional wages of Germans without being at odds with data on labor share or the share of income replaced by social insurance.

These features make the new model an excellent candidate for welfare analysis of labor market policy and social insurance. The model captures features of wage posting: in particular that policy impacts both the set of acceptable jobs *and* the characteristics of offered jobs. Particular policy changes of interest include manipulation of need-based social insurance such as food stamps, labor history contingent social insurance such as unemployment insurance, and minimum wages. Full analysis of such policy changes are reserved for future work.

1.7 Figures

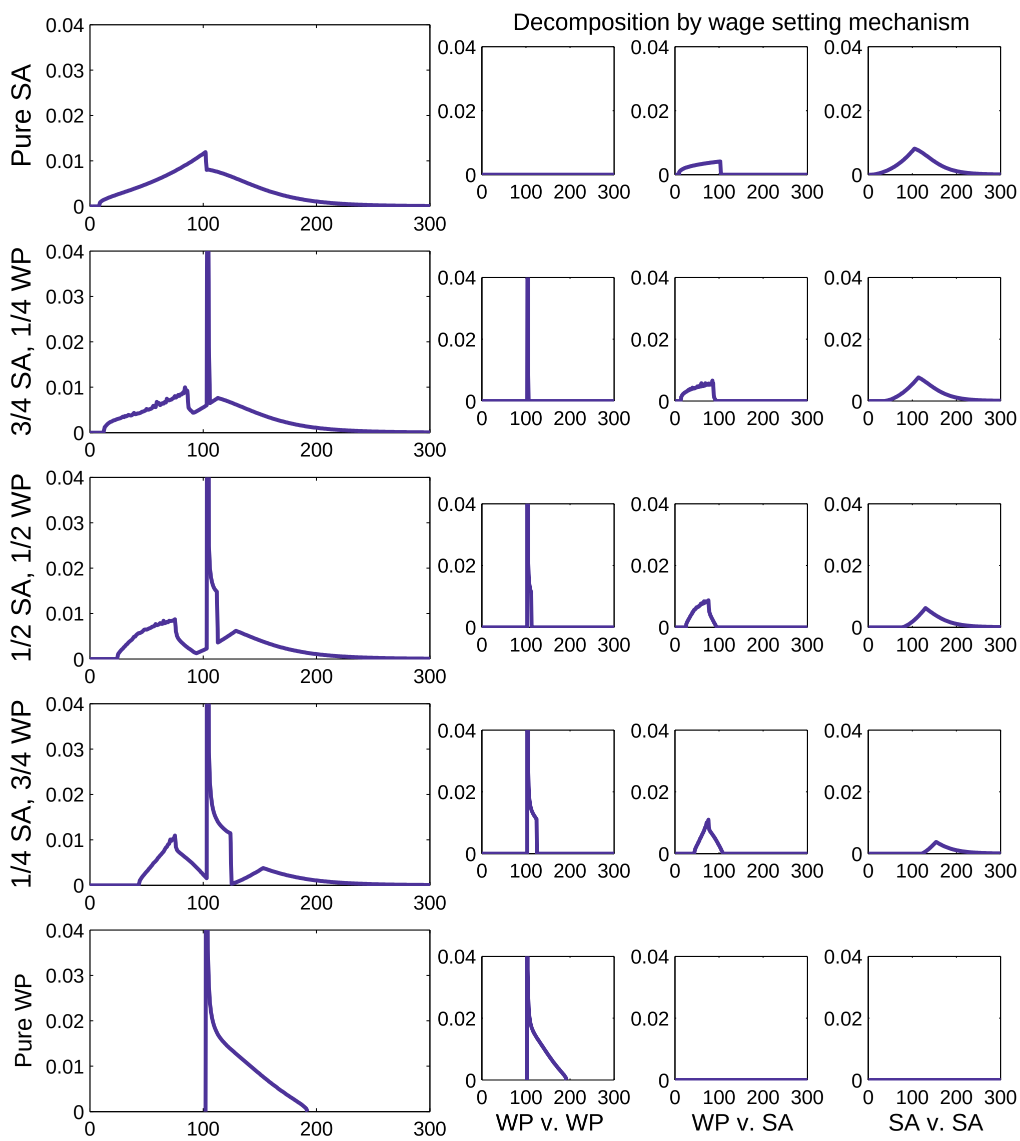

Figure 1.1: Distributions of wages for a fixed distribution of output for different proportions of posting and countering firms. The distribution is decomposed based upon whether the current employer or best-to-date outside option employees under SA. The wage distribution is more positively skewed when the WP sector is larger. The wage distribution is more disperse when the SA sector is larger.

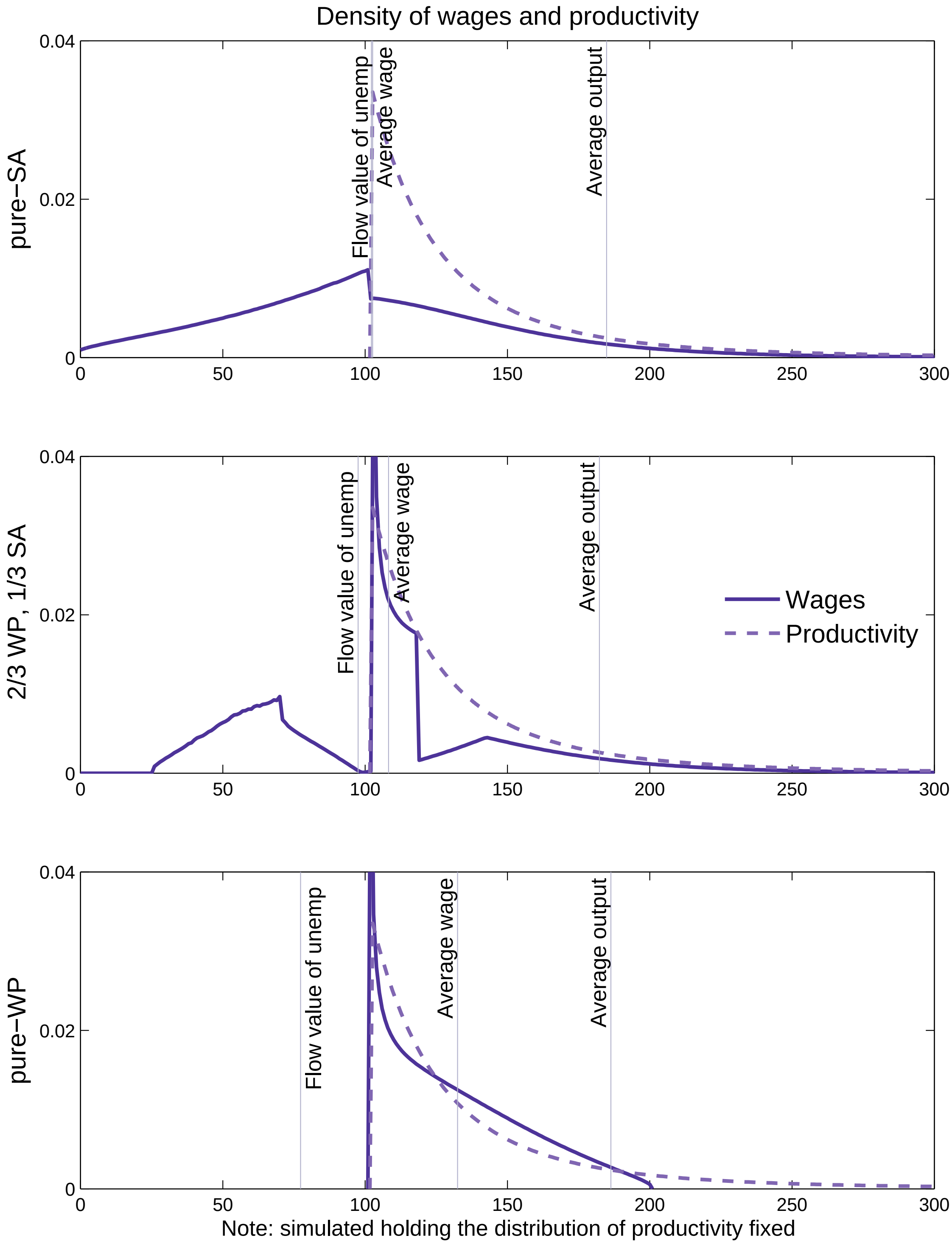

Figure 1.2: Distributions of wages for a fixed distribution of output for an economies with pure-WP, pure-SA, and an economy with 2/3 WP and 1/3 SA. Flow value of leisure and mean of wages and output are indicated by vertical bars.

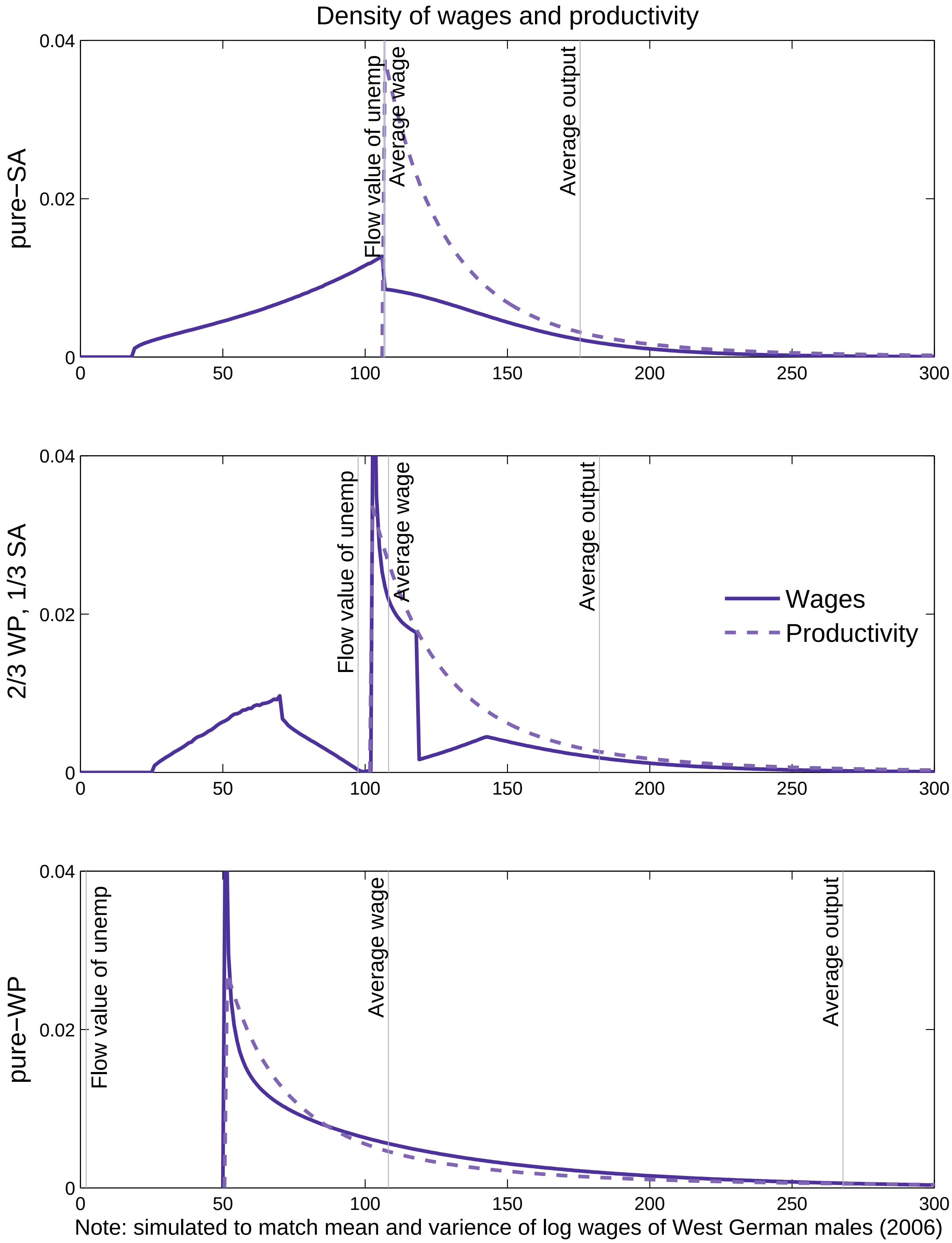

Figure 1.3: Distributions of wages for models which minimize the squared difference between mean and variance log simulated wages and mean and variance of log wages for West German males in 2006 and the productivity distribution implied by this wage distribution and transition hazards $\{\lambda_0, \lambda_1, \delta\}$ consistent with labor market histories 2006-2008. Flow value of leisure and mean of wages and output are indicated by vertical bars.

1.8 Tables

Table 1.1: Estimated parameters: job offer and separation hazards.[a]

	λ_0	λ_1	δ	k_0	k_1	N
Germany	0.830	0.180	0.074	11.157	2.412	293,499
	(0.0045)	(0.0008)	(0.0003)			
West	0.803	0.179	0.072	11.163	2.485	239,436
	(0.0049)	(0.0009)	(0.0004)			
Berlin	0.998	0.175	0.085	11.749	2.054	43,197
	(0.0125)	(0.0021)	(0.0009)			
East	0.747	0.215	0.089	8.421	2.418	10,866
	(0.0197)	(0.0046)	(0.0019)			

[a] Estimates are per annum. Standard errors in parentheses.

Table 1.2: Target moments: mean and variance of wages and log wages.

	Wages			Log Wages				percent
	mean	std. dev.	censor	mean	std. dev.	censor	N	censored
Germany	93.19	39.31					293,499	
West	97.55	39.45	169.9	4.484	0.469	5.135	239,436	8.7
Berlin	87.46	35.60	142.4	4.368	0.490	4.959	10,866	13.7
East	70.46	30.42	142.4	4.159	0.450	4.959	43,197	4.0

Table 1.3: Calibrated parameters: scale and shape of the productivity distribution.

	Percent wage posting						
	0	10	33.3	50	66.7	90	100
Scale							
West	99.804	99.690	99.074	97.985	95.507	80.741	44.621
Berlin	91.932	91.800	91.245	90.339	88.198	69.505	37.902
East	70.900	70.751	69.884	68.433	65.523	51.359	34.735
Shape							
West	3.946	3.949	3.899	3.719	3.320	2.216	1.140
Berlin	3.327	3.330	3.291	3.145	2.824	1.949	0.834
East	4.037	4.037	3.905	3.600	3.077	1.972	1.367
Cost	0	4	11	20	38	143	∞
% of aggregate output	*0*	*2.24*	*4.57*	*6.13*	*7.38*	*6.85*	*0*
% of output in $\check{p}$ firm							
West	*0*	*3.85*	*9.92*	*16.86*	*28.60*	*64.20*	*100*
Berlin	*0*	*4.15*	*10.57*	*17.72*	*29.32*	*59.53*	*100*
East	*0*	*5.34*	*13.51*	*22.58*	*37.01*	*68.09*	*100*

Table 1.4: Auxiliary moments: social insurance and national accounts.

	Percent wage posting							Germany
	0	10	33.3	50	66.7	90	100	(2006)
Flow value of unemployment as a percent of mean wages								
West	99.8	99.6	98.3	95.5	88.8	56.5	-27.3	
Berlin	99.7	99.6	97.9	94.2	85.2	39.4	-104.1	
East	99.6	99.3	97.2	93.2	85.0	50.6	7.8	
Aggregate	99.7	99.6	98.2	95.1	88.1	54.5	-24.9	>>30[a]
Labor share (%)								
West	61.0	61.6	63.1	63.3	61.7	51.9	38.5	
Berlin	56.4	57.0	58.4	58.4	56.5	47.9	33.2	
East	62.3	63.2	64.6	63.9	60.7	48.4	34.0	
Aggregate	61.0	61.7	63.1	63.2	61.4	51.2	39.0	63.7[a]

[a] Source: Authors calculations based on SIAB data in 2006. This is like a dramatic underestimate. See calibration appendix for details.

[a] Source: EU-KLEMS Growth and Productivity Accounts. http://www.euklems.net/

1.9 Proofs and Derivations

Proof of Claim 1

Separating Nash equilibrium for appropriate c and $\check{p}$

The proposed separating equilibrium is a Nash equilibrium of the labor market if each firm prefers the prescribed wage contract and wage schedule conditional on all other firms playing the assigned contract and wage schedule and labor flowing toward more productive firms. In order to prove this I must show that current operating surplus from the proposed strategies exceed current operating surplus from each firms best deviation.

Suppose WP is prescribed: A firm for which WP is prescribed must have $p < \check{p}$.

For the p-productivity firm, current operating surplus from playing optimal wage under the prescribed wage contract, WP, and the best deviation to SA can be written as

$$\pi^P(p) = [p - w_{PP}(p)]\ell(p)$$

and

$$\pi^{BD}(p) = [p - w_{PA}(\underline{p}, p)]\ell(\underline{p}) + \int_{\underline{p}}^{\grave{p}} [p - w_{PA}(q, p)] d\ell(q) - c(\check{p})$$

where $\grave{p}$ is the productivity of the most productive firm which offers a posted wage less than p (e.g. the most productive firm which the p-type firm can outbid by switching to SA).

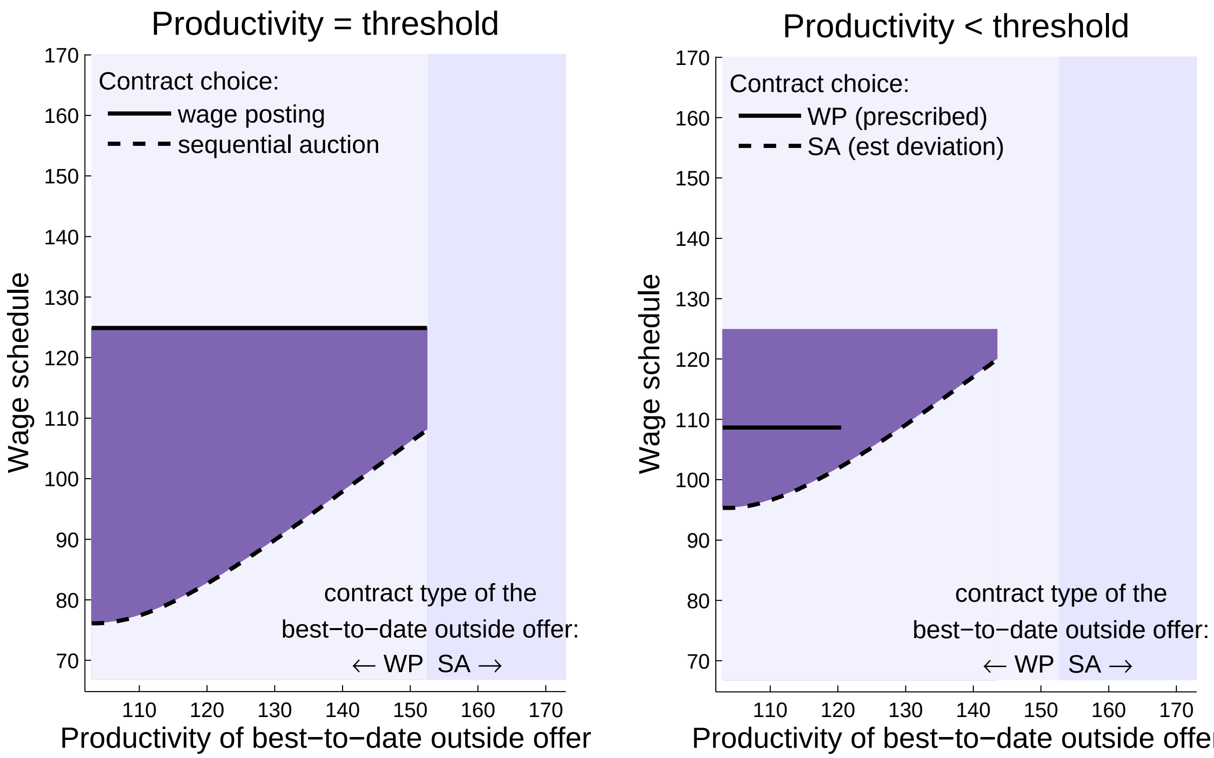

Figure 1.4: Regions over which the cost of SA and bound on the willingness to pay to SA are integrated for the threshold productivity firm and a firm of lesser productivity.

Simplifying,

$$
\begin{aligned}
\pi^{BD}(p) =& [p - w_{PP}(\check{p}) + \underbrace{w_{PA}(\underline{p}, \check{p}) - w_{PA}(\underline{p}, p)}_{<0, \text{ since } \frac{dw_{PA}(\underline{p},p)}{dp}<0}]\ell(\underline{p}) \\
&+ \int_{\underline{p}}^{\check{p}} [p - w_{PP}(\check{p}) + \underbrace{w_{PA}(q, \check{p}) - w_{PA}(q, p)}_{<0, \text{ since } \frac{dw_{PA}(q,p)}{dp}<0}] d\ell(q) \\
&- \int_{p}^{\check{p}} [\underbrace{w_{PP}(\check{p}) - w_{PA}(q, \check{p})}_{\geq 0}] d\ell(q) \\
<& [p - w_{PP}(\check{p})]\ell(\check{p}) \\
\leq& \pi^{P}(p).
\end{aligned}
$$

The last line follows from noting $w_{PP}(p)$ was the unique profit maximizing posted wage choice for the p-type firm.

In other words, the WP firm could increase its labor supply by deviating to SA. However, the firm could also increase its labor supply by the same amount by deviating to a larger posted wage. Willingness to pay for the right to SA is then strictly less than the difference between the wage bill under the deviation to SA and the deviation to a higher posted wage. This in turn is strictly less than the cost of SA. Figure 2.7 depicts wages schedules under WP and SA in the threshold firm (left) and for a less productive firm (right). The cost of SA and bound on the willingness to pay for the right to SA are represented by the shaded regions. The cost or willingness to pay are calculated as the mass in these regions weighted by the supply of labor to the firm with each possible best-to-date outside option.

Suppose SA is prescribed: A firm for which SA is prescribed must have $\check{p} \leq p$. For the p-productivity firm, current operating surplus from playing the prescribed SA wage schedule and deviating to the best posted wage.

$$\pi^A(p) = [p - w_{PA}(\underline{p}, p)]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - w_{PA}(q,p)]d\ell(q) + \int_{\check{p}}^{p} [p - w_{AA}(q,p)]d\ell(q) - c(\check{p})$$

and

$$\pi^{BD}(p) = [p - \grave{w}]\ell(\grave{w})$$

Note that $\grave{w} \geq w_{PP}(\check{p})$ since $p \geq \check{p}$. Simplifying,

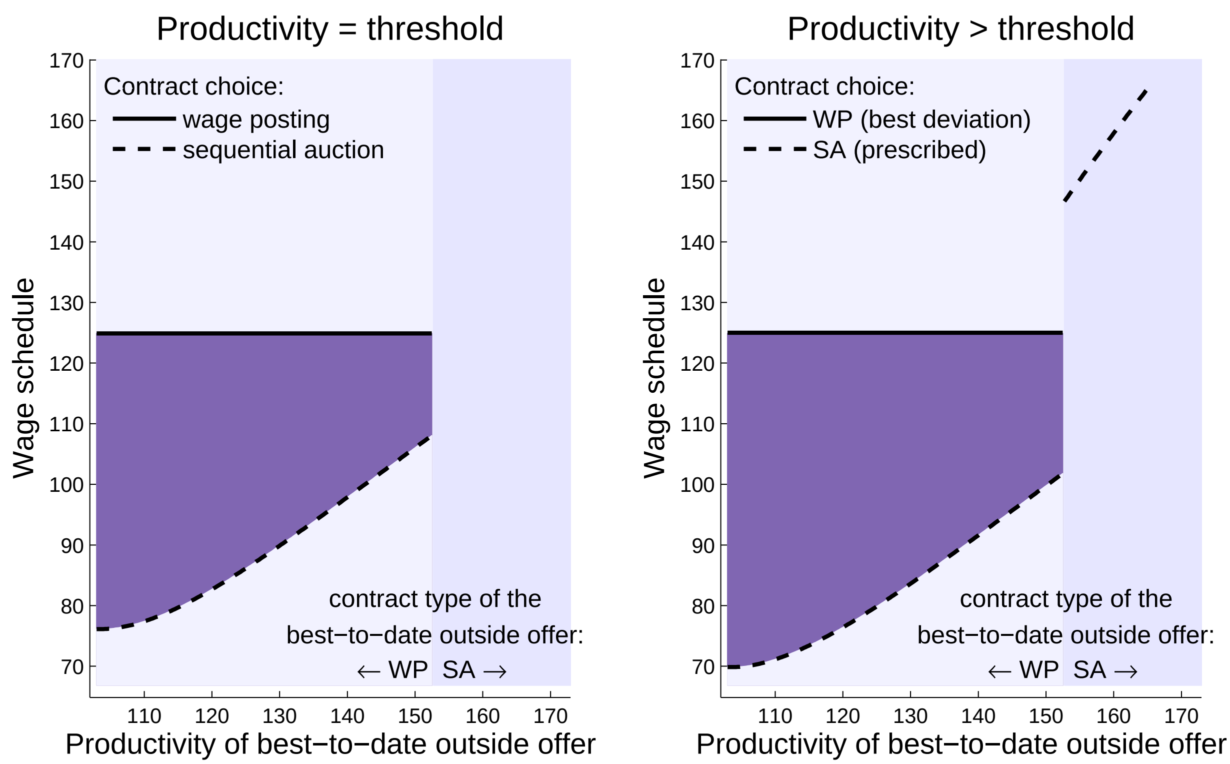

Figure 1.5: Regions over which the cost of SA and bound on the willingness to pay to SA are integrated for the threshold productivity firm and a firm of lesser productivity.

$$\begin{aligned}
\pi^{BD}(p) =& [p - \grave{w}]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - \grave{w}]d\ell(q) + \int_{\check{p}}^{\grave{w}} [p - \grave{w}]d\ell(q) \\
<& [p - \underbrace{w_{PP}(\check{p})}_{\leq \grave{w}}]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - \underbrace{w_{PP}(\check{p})}_{\leq \grave{w}}]d\ell(q) + \int_{\check{p}}^{\grave{w}} [p - \underbrace{w_{AA}(q,p)}_{<\grave{w}}]d\ell(q) \\
<& [p - w_{PA}(\underline{p},p) - \underbrace{w_{PP}(\check{p}) + w_{PA}(\underline{p},\check{p})}_{>0, \text{ since } \frac{dw_{PA}(\underline{p},p)}{dp}<0}]\ell(\underline{p}) \\
&+ \int_{\underline{p}}^{\check{p}} [p - w_{PA}(q,p) - \underbrace{w_{PP}(\check{p}) + w_{PA}(q,\check{p})}_{>0, \text{ since } \frac{dw_{PA}(q,p)}{dp}<0}]d\ell(q) \\
&+ \int_{\check{p}}^{p} [p - w_{AA}(q,p)]d\ell(q) \\
<& \pi^{A}(p).
\end{aligned}$$

The best deviation to WP involves a reduction in the SA firm's labor supply. I can find a bound on the minimum willingness to pay for the right to SA by considering only the labor supply which would arise under the *smallest possible* best deviation the SA firm might select: $w_{PP}(\check{p})$. Willingness to pay for the right to SA is then larger less than the difference between the wage bill under the deviation to WP and the wage bill for these employees under the prescribed SA contract. This in turn is strictly greater than the cost of SA. Figure **??** depicts wages schedules under WP and SA in the threshold firm (left) and for a more productive firm (right). The cost of SA and bound on the willingness to pay for the right to SA are represented by the shaded regions. The cost or willingness to pay are calculated as the mass in these regions weighted by the supply of labor to the firm with each possible best-to-date outside option.

Since no firm wishes to unilaterally deviate the pair $\{c, \check{p}\}$ form a Nash equilibrium. ■

Existence of $\check{p}$ for any c.

Since $\check{p}$ was chosen arbitrarily c is defined for any possible threshold in the support of Γ.

First consider $\Gamma(p)$ with finite support $[\underline{p}, \bar{p}]$. To show that for every cost, c, there exists a threshold, $\check{p}$, I must first extend the definition of c to include the boundaries of the support of $\Gamma(p)$.

- $c(\underline{p}) = (-\infty, 0]$ (when SA is subsidized or free all firms select SA).
- $c(\bar{p}) \supset [\ell(\bar{p})\bar{p}, \infty)$ (if the cost of SA exceeds the output of the most productive firm, $\ell(\bar{p})\bar{p}$, then no firm selects SA).

So, c is upper hemicontinuous on support $[\underline{p}, \bar{p}]$ and continuous on support $(\underline{p}, \bar{p})$. The intermediate value theorem implies that there exists at least one threshold for every cost.

The result can be generalized to Γ with infinite upper support by considering the limit as $\bar{p} \to \infty$: for every $\bar{p}$ there exists a $c = \ell(\bar{p})\bar{p}$ such that all firms SA. ■

Uniqueness of equilibria

Equilibrium is unique if $[(w_{PP}(\check{p})-\underline{p})k_1]^{-1} \geq d\Gamma(\check{p})$ for all $\check{p}$. This condition requires that the distribution of productivity be "thin enough" everywhere in the tail that the shift $\frac{dw_{PA}(q,\check{p})}{d\check{p}}$ due to indirect upward pressure on schedules $w_{PA}(q,p)$ from the now larger WP sector is dominated by the direct downward pressure on the schedule in the marginal firm due to the now larger productivity of the marginal firm. Proof, which stems from differentiating the marginal wage schedule, is available upon request.

This guarantees that $\frac{dc}{d\check{p}}$ is increasing for all $\check{p}$ in the interior of the support of $\Gamma(p)$ and the mapping from $\check{p}$ to c is one-to-one.

Wage schedules

SA vs. SA

If the best-to-date outside offer originates from a SA firm the problem is analogous to the problem considered by Postel-Vinay and Robin (2002a), since all firms with productivity greater or equal to the best-to-date outside option also SA. The least productive firm which is able make a credible threat to hire the worker is a SA firm and it is the firm which is able to offer a value equal to $V^A(w_{AA}(q,p),p)$ with a wage offer of q.

Postel-Vinay and Robin (2002a) observe that $V^A(q,q) = (q + \delta V^U)/(\delta + \mu)$. Equating $V^A(q,q)$ and $V^A(w_{AA}(q,p),p)$ identifies the reservation wage for accepting a job at the p-productivity SA employer when the best-to-date outside option is a q-productivity SA employer:

$$w_{AA}(q,p) = q - k_1 \int_q^p \bar{\Gamma}(x)dx \qquad \text{for } \check{p} < q \leq p, \tag{1.13}$$

where $k_1 = \lambda_1/(\mu+\delta)$ is the expected *discounted* number of job offers per employment spell.[9] This is pins down the portion of the optimal wage schedule for a p-productivity SA firm and best-to-date outside option a q-productivity SA firm: when $\check{p} \leq q \leq p$.

WP vs. WP

In the equilibrium, if the p-productivity firm selects WP then it must be the case that all less productive firms also select WP. The problem facing the firm is thus very similar to the problem considered by Bontemps et al. (2000). The optimal WP wage offer, $w_{PP}(p)$, maximizes the expected profit from the WP contract:

$$\pi^P(p) = \underbrace{[p - w_{PP}(p)]}_{\text{rent per worker}} \underbrace{\ell(p)}_{\text{labor supply}}, \tag{1.14}$$

Bontemps et al. (2000) show that when all firms WP and make optimal wage choices constrained efficiency of labor flows arrives and labor supply is pinned down by firms' productivity type. The intuition is that more productive firms prefer to post higher wages and continuity of $\Gamma(p)$ yields the required one-to-one mapping between $w_{PP}(p)$ and p.[10] To see that the result extends to the separating equilibrium I must verify that the mapping remains one-to-one in the presence of (more productive) SA firms. This follows from noting that all SA firms can outbid the highest profitable posted wage of the most productive WP firm and that the measure of $\check{p}$ WP firms is zero since $\Gamma(p)$ is continuous.

The solution to WP firms' maximization problem then follows from applying the envelope theorem and solving the implied differential equation:

$$w_{PP}(p) = p - [1 + \kappa_1\bar{\Gamma}(p)]^2 \int_{w_{UP}}^{p} [1 + \kappa_1\bar{\Gamma}(x)]^{-2} dx \qquad \text{for } p < \check{p}, \tag{1.15}$$

[9] Note that $k \to \kappa$ when $\mu \to 0$.

[10] See Burdett and Mortensen (1998) and Bontemps et al. (2000) for details and proofs of these results.

where w_{UP} is the reservation wage of a worker to accept a job at a WP firm from unemployment.

SA vs. WP

Returning to optimal wages for workers transitioning between contract types. Nearly all the elements of equation 2.24 are now pinned down. What remains is to solve for the value of employment in a WP firm. Integration of equation 2.27 by parts gives

$$V^P(w_{PP}(q)) = \frac{w_{PP}(q)}{\mu+\delta} + \frac{1}{\mu+\delta}\int_q^{\check{p}} \frac{k_1[\Gamma(\check{p})-\Gamma(x)]}{1+k_1[\Gamma(\check{p})-\Gamma(x)]}\frac{dw_{PP}(x)}{dx} + \frac{\delta V^U}{\mu+\delta}$$

Plugging in V^P and V^C appropriately and manipulating gives the result:

$$w_{PA}(q,p) = w_{PP}(q) - k_1\bar{\Gamma}(\check{p})\big[\check{p} - w_{PP}(q) - \int_q^{\check{p}} \tfrac{k_1[\Gamma(\check{p})-\Gamma(x)]}{1+k_1[\Gamma(\check{p})-\Gamma(x)]}\tfrac{dw_{PP}(x)}{dx}\big] - k_1\int_{\check{p}}^{p} \bar{\Gamma}(x)dx \quad \text{for } q \leq \check{p} \leq p \tag{1.16}$$

Hiring out of unemployment

Finally, the reservation wages for entering employment at a WP firm or a SA of type-p are pinned down as the wages equate the value of unemployment to the value of employment in the relevant firm. The value of unemployment (when all firms follow the proposed strategies) can be written as:

$$\mu V^U = b + \lambda_0\Big\{\underbrace{\Gamma(\check{p})[\mathbb{E}[V^P(w_{PP}(x),x)|q<x<\check{p}] - V^U]}_{\text{hired by a WP firm}} + \underbrace{[\bar{\Gamma}(\check{p})][\mathbb{E}[V^A(w_{UA}(x),x)|\check{p}<x] - V^U]}_{\text{hired by a SA firm}}.\Big\} \tag{1.17}$$

Since the SA firm selects the wage offer to yield no rents the third line is equal to zero. Equating this to the value of employment in the least productive WP firm gives the reservation wage for employment in a WP firm, $w_{UP}(\varepsilon)$, as:

$$w_{UP} = b + (k_0 - k_1) \int\limits_{w_{UP}}^{\check{p}} \frac{[\Gamma(\check{p}) - \Gamma(x)]}{1 + k_1[\Gamma(\check{p}) - \Gamma(x)]} \frac{dw_{PP}(x)}{dx}. \tag{1.18}$$

Meanwhile, when entering employment in a SA firm, reservation wages depend on the SA firm's productivity, p. However, since the value of unemployment equals the value of employment in the least productive WP firm, $V^U = V^P(\underline{p})$, and the SA firm selects the wage to match the value of the best-to-date outside option, the schedule of reservation wages is equal to the wage required to hire from the least productive firm: $w_{UC}(p) = w_{PC}(\underline{p}, p)$.

Thus all wages are pinned down as a function of worker type and labor market history. All equations have closed form solutions for the case when $\lambda_0 = \lambda_1$, since in this case $w_{UP} = b$.

Proof of Claim 6

The claim is proved if every firms wage bill weakly rises. Consider a small increase in the costs of SA. Firms are of three types: always WP, switch from SA to WP, always SA. Wage bills for always WP firms are clearly unaffected by the change in threshold productivity induced by the change in cost. This follows from noting that I am considering an identical set of active firms. Switching firms strictly increase their wage bill, this follows from noting that before the increase in costs they paid for the right to SA but after they prefer to pay larger WP wage bills in order to evade the higher cost of SA. The third category of firms requires heavier lifting.

First note that the wages of workers with best-to-date outside option a WP are set under schedule $w_{PA}(q, p)$ in each p-productivity SA firm and that the mass of such workers in each firm is $\ell(\check{p})$. Also note that $\frac{d^2 w_{PA}(q,p)}{dq d\check{p}} < 0$.[11] So the change in the wage bill associated with

[11] Proof, which follows from performing the derivatives, is available on request.

these workers when $\check{p}$ rises is at least

$$\frac{dw_{PA}(\check{p},p)}{d\check{p}} = [1 + k_1\bar{\Gamma}(\check{p})]\frac{dw_{PP}(\check{p})}{d\check{p}} + k_1 d\Gamma(\check{p})[\check{p} - w_{PP}(\check{p})] > 0.$$

Meanwhile, each p-productivity firm can lower the wage paid to employees whose best-to-date outside option was a SA firm before the cost increase and is now a WP firm. This reduces the p-productivity firms wage bill by $d\ell(\check{p})[\check{p} - w_{PP}(\check{p})]$.

The increase in the schedule of wages for workers with best-to-date outside option a WP firm dominates. This is made explicit by noting that $\frac{dw_{PP}(\check{p})}{d\check{p}} = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\Gamma(\check{p})}[\check{p} - w_{PP}(\check{p})]$ and $d\ell(\check{p}) = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\bar{\Gamma}(\check{p})}\ell(\check{p})$.

Since the wage bill for every productivity firm weakly rises the total wage bill rises. ■

Wage distributions

The distribution of wages can be derived by aggregating across firms within sectors and then by aggregating across sectors.

1.9.0.1 Distribution of wages within firms

Within WP firms the distribution of wages is a mass at the posted wage. For SA firms, however, wages are disperse. Following the discussion in section 1.2.4 and Postel-Vinay and Robin (2002a, pg. 999-1001), the distribution of wags within SA firms can be expressed as a function of the distribution of employees best-to-date outside options:

$$G(w|p) = \frac{\ell(q(w,p))}{\ell(p)} = \begin{cases} \mathbb{1}_{w \geq w_{PP}(p)}, & \text{if } p < \check{p} \\ \left[\frac{1+\kappa_1[1-\Gamma(p)]}{1+\kappa_1[1-\Gamma(q(w,p))]}\right]^2, & \text{if } \check{p} \leq p \end{cases}$$

1.9.0.2 Aggregate distribution of wages

Within the WP sector the wage distribution can be expressed as:

$$G(w|WP) = L(q(w_{PP}))$$

where $q(w_{PP})$ is the productivity of the firm which optimally posts wage w_{PP}. Meanwhile, within the SA sector the wage distribution can be expressed as:

$$G(w|SA) = \int_{w(\underline{p},\bar{p})}^{w} G(w|p)dL(p) + L(\underline{p})G(w|\underline{p}).$$

Summing the two gives the aggregate wage distribution.

1.10 Calibration

Data

Data for this paper come from the Research Data Center (FDZ) of the German Federal Employment agency at the Institute for Employment Research (IAB) *Sample of Integrated Labour Market Biographies (SIAB)* and the *IAB Establishment Panel.* The SIAB is a two percent sample drawn from the population of individuals in Germany who are employed and subject to social security, marginal part-time employed, a benefit recipient to the German Social Code II, official registered as a job-seeker at the German Federal Employment Agency, or participate in programs of active labor market policies. I restrict these data to 2006-2008. These dates are selected as a relatively stable period between the implementation of the Hartz reforms and the financial crisis. I also utilize national data compiled and distributed in the *EU KLEMS Growth and Productivity Accounts* in order to construct labor share in these years.

Methodology

The methodology of this paper follows Bontemps et al. (2000) and Postel-Vinay and Robin (2002b). This is done for comparability. These authors choose the parameter of relative risk aversion and discount rate a priori. Here I follow suit choosing linear utility and discounting approaching zero. I also take the ratio of the mass of workers to the mass of firms to be one. The remaining model parameters then are the rates of job offer arrival, the rate of separation, the value of leisure, the cost of SA $(\lambda_0, \lambda_1, \delta, b, c)$, and the distribution of productivity across firms $(\Gamma(p))$. These must be retrieved from observable data including the realized distribution of wages, worker's and firm's employment histories, and labor share.

The models of Bontemps et al. (2000) and Postel-Vinay and Robin (2002b) are fit using a two step process. The first estimates μ, δ, λ_0, and λ_1 using panel data on worker histories and the Method of Maximum Likelihood. The second exploits an invertible relation, derived from theory, between average wages within a firm and firm's productivity in order to retrieve the distribution of productivity consistent with a nonparasitically estimated distribution of the within-firm average wages. The distribution of productivity together with the Poisson arrival rates can be taken as structural parameters in order to conduct analysis of policy variation as in Vuuren et al. (2000).

The estimation procedure used in this paper repeats a similar first step using worker histories constructed from the SIAB data. The second step, however, is more complex: my model explicitly implies a *non-monotone* relation between productivity and wage in the neighborhood of the threshold productivity. As a result there is *not* an invertible map between within-firm average wages and firm productivity. I instead make a distributional assumption regarding the distribution of productivity across firms: productivity distributes Pareto. I then select parameters of the Pareto distribution to minimize the sum of squared deviations between model moments and data moments. Targeted moments are the mean and variance of log wages. The described estimation is conducted separately for East and

West Germany and Berlin.

First Stage

Data

I select the population German workers who were employed in the first two weeks of 2006. For each of these I record:

d_{ei} *length of the first employment spell*

d_{ui} *length of the first non-employment spell (if any)*

D_i *nature of the ending spells*

$= 1$ *if first employment spell is censored*

$= 2$ *if job-to-job transition ends first employment spell*

$= 3$ *if non-employment ends first employment spell and non-employment is censored*

$= 4$ *if job to non-employment to job transition*

Z_i *the fraction of firms which are prefered to the initial employer*

The first three objects are straightforward. Estimation of the likelihood requires an estimate of the fraction of job offers which are acceptable to an employed worker. I identify Z_i using the observed rate of employee exit from a firm. Let Z_i be the cumulative density of over firm-level rate of employee exit. Since, in the model, workers only make job-to-job transition to lower turnover firms, I can write:

$$Z_i = \bar{\Gamma}(p_i). \tag{1.19}$$

In Chapter 3, I demonstrate that the rate of employee exit at the firm level is a reasonable measure of worker's ranking over firms. I also compare the measure to alternatives, notably firm size.

Likelihood function

The likelihood of the quadruplet $\{d_{ei}, d_{ui}, D_i, Z_i\}$, where d_{ei} is the duration of the first employment spell, d_{ui} is the duration of the first unemployment spell (if any), and D is an indicator for the type of transition observed in the data, has 4 cases, one for each possible value of D:

First employment spell is censored (D_i=1). Since the probability of exit out of a job is $\delta + \mu + \lambda_1 Z_i$ the probability that a given employment spell lasts longer than T periods, where T is the total time of observation, is given by

$$\exp[-(\delta + \lambda_1 Z_i)T] \tag{1.20}$$

Job to job transition after first employment spell (D_i=2). The first employment spell has a duration of exactly d_{ei}, this has probability $(\delta + \lambda_1 Z_i)\exp[-(\delta + \lambda_1 Z_i)d_{ei}]$. The probability that the transition is made directly to another job is $\lambda_1 Z_i/(\delta + \lambda_1 Z_i)$. So the probability of observing a job-to-job transition after employment of duration d_{ei} is

$$[\lambda_1 Z_i]\exp[-(\delta + \lambda_1 Z_i)d_{ei}] \tag{1.21}$$

Job to truncated non-employment (D_i=3). The first employment spell has a duration of exactly d_{ei}, this has probability $(\delta + \lambda_1 Z_i)\exp[-(\delta + \lambda_1 Z_i)d_{ei}]$. The probability that the transition is made either unemployment or exit is $(\delta)/Z_i$. The probability that it is truncated is $\exp[-(\lambda_0)(T - d_{ei})]$.

$$(\delta\exp[-(\lambda_0)(T - d_{ei})])\exp[-(\delta + \lambda_1 Z_i)d_{ei}] \tag{1.22}$$

Job to non-employment to job transition (D_i=4). The first employment spell has a duration of exactly d_{ei}, this has probability $(\delta + \lambda_1 Z_i)\exp[-(\delta + \lambda_1 Z_i)d_{ei}]$. The probability that the transition is made to unemployment is $\delta/(\delta+\lambda_1 Z_i)$. The first non-employment spell has a duration of exactly d_{ui} which has a probability $\lambda_0\exp[-\lambda_0 d_{ui}]$.

$$\delta\lambda_0\exp[-(\delta+\lambda_1 Z_i)d_{ei}-\lambda_0 d_{ui}] \tag{1.23}$$

Results

The results are summarized in Table 1.1. Notably, estimated on-the-job offer rates are lower than in Postel-Vinay and Robin (2002b). This arises because I do not use on-the-job pay change to identify on-the-job offer arrival rates. I prefer to be agnostic about the source of (at least some) on-the-job pay changes, in particular those which occur on January 1 and are likely to reflect pay changes designed to keep pace with price levels rather than designed to respond to outside offers. Additionally, the likelihood is identified without on-the-job pay change data.

Second Stage

Data

For the population German workers who were employed in the first two weeks of 2006 I estimate the mean and variance of log wages paid at the beginning of this first employment spell. These are presented in Table 1.2. A notable difficulty with the SIAB data is censoring at the limit of social security contribution. Between 4 and 14 percent of employment histories include censored wage observations, as noted in the final column of Table 1.2. In simulations, I match censored empirical moments to moments from an identically censored simulation as described below.

Simulation

I posit that the productivity distribution distributes Pareto. For three regions of Germany (West, East, and Berlin) I simulate the wage distribution. Wage distributions are censored at the social security maximum (169 and 143 in west and east Germany respectively). Parameters of the Pareto distribution are selected to minimize the sum of squared errors between target moments and simulated moments:

$$SSE = (mean_{data} - mean_{sim.})^2 + (std.dev._{data} - std.dev._{sim.})^2$$

Both target and simulated moments are estimates from censored data and simulation. Simulations are conducted in MATLAB and squared deviations are minimized using Knitro. The procedure is repeated for compositions of contracts ranging from 100% WP to 100%SA.

Results

Results are summarized in Table 1.3. Decreasing weight in the tail of the required productivity distribution can be observed from the increase in the shape parameter as the composition of contracts skews toward SA. Notably, the shape is most responsive to changes in the composition of contracts when most firms select WP. As a result, inclusion of a small proportion of SA contracts dramatically improves the fit relative to the pure-WP model. It is also notable that the cost of SA, while large for the threshold firm, is never a large fraction of output. In all simulations the share of output that services the cost of SA relatively small.

Auxiliary moments

The simulated model is compared to national income and product accounts from EU KLEMS and the fraction of firms which operate under wage posting in 2011, as estimated by Brenzel et al. (2013). The fraction of wages replaced by social insurances is calculated from the

2006 cross-section of the SIAB data as the ratio of the average unemployment benefit or assistance received by the population who are unemployed in January 2006 to the average wage of employed workers in January 2006. Estimates range between 30% in West Germany and 36% in East Germany. Estimates are likely too low as I have not accounted for selection into employment and unemployment and have not adjusted for the length of unemployment spells.

CHAPTER 2:

Dead-end and career jobs: skill-specific earnings profiles in an on-the-job search equilibrium with heterogeneous wage contracts.

2.1 Introduction

Some jobs feature negotiable wages, others do not. Some jobs feature on-the-job wage growth, others do not. How do these differences shape our understanding of firm, worker, and job heterogeneity? This paper includes differential skill levels on the part of workers in an equilibrium model of on-the-job search in which some employers offer jobs with uniform, non-negotiable pay while others offer targeted, negotiable pay with potential for on-the-job wage growth. The result is the ability to explicitly assess how contract type varies with worker skill; and how, as a result, employment opportunities, wage dispersion, and return to experience differ across workers with different levels of skill. This complements a developing literature which focuses on new hires' wages and wage negotiations.[12]

I allow firms to select between two wage setting mechanisms: wage posting (WP) and sequential auction (SA). Under WP, a firm sets a non-negotiable wage and commits to it for the duration of the employment relationship, as in Burdett and Mortensen (1998) and

[12]For example Ellingsen and Rosén (2003) and Michelacci and Suarez (2006).

Bontemps et al. (2000). Under SA, pay is commensurate with the worker's best-to-date outside option and evolves as workers accrue new outside offers by a mechanism equivalent to a sequential second price auction, as in Postel-Vinay and Robin (2002a,b). In the present paper, I allow wages set under both contract types to be conditioned on worker skill level.

Chapter 1 shows that, when workers are homogeneous, the arrival rate of job offers is exogenous, and firms must pay a per-firm fee in order to set wages by means of the SA mechanism, the market is characterized by a separating equilibrium in which only high productivity firms are interested in providing the flexible contract. Such separation is consistent with the intuition that dead-end jobs - with inflexible wages and poor job-to-job transition prospects - are likely to be offered by the least productive firms while more productive firms offer career jobs - which offer the prospect of on-the-job wage gain and, often, significant job-to-job wage gain as well. This is also consistent with empirical evidence that negotiable and non-negotiable wage contracts exist side-by-side in equilibrium (Hall and Krueger, 2008, 2010; Barron et al., 2006; Brenzel et al., 2013).

Here, I extend the model to admit workers who are heterogeneous with respect to their skill, modeling higher levels of skill as yielding greater effective labor. As is the usual construct, firms are modeled as producing via a constant returns to scale technology. Firms hire under skill-specific wage contracts under both wage contract types. Costs for the right to SA are skill specific: if firms hire some skill levels under WP and others under SA they pay costs *only* for the skill levels which are employed under flexible contracts. Under these conditions skill-specific sub-markets operate and can be analyzed in parallel. Within each sub-market, contract choice follows the pattern described for a market with uniform labor: a separating equilibrium exists in which low productivity employers hire under WP and high productivity employees hire under SA.

I consider ex-ante identical sub-markets: identical distribution of productivity among potentially active firms; identical hazards of job offers and unemployment; and identical costs for the right to SA. Even when markets are ex-ante identical, differences arise ex-post. Since costs for the right to SA are larger relative to output for low-skilled workers, the share of SA firms is increasing with skill level and high-skilled workers are more likely to be employed under a SA contract. The implications are striking: low-skilled workers experience higher rates of unemployment, less log wage dispersion, and lower returns to experience than high-skilled workers.

From a policy perspective, these results are informative about how job training programs may impact workers and how they may be best assessed. Typically evaluations find little or no effect on wages immediately following training but larger effects some years out: Jacobi and Kluve (2007) give a summary of evaluation of training programs in Germany; Card et al. (2010) provide a meta-analysis of active labor market policy assessments in the United States and Europe. A lag in the response of wages could be due to several sources: training may take time to sink in, firms may learn the value of workers' training only slowly, and/or firms may wait to reward training until required to do so by market pressures. The model described in this paper provides a structural mechanism of the third type.

I consider a hypothetical experiment in which unemployed workers are provided training which is effective and increases skill level by X%. When costs of SA are identical for all skill levels, the effect of training is muted in re-employment wages due to differentials in the composition of contracts available to trained (high skill) and untrained (low skill) workers. Percentage differences in average wages are compressed at low (continuous) experience, due to the predominance of WP for untrained (low skill) workers. As workers gain more (continuous) experience, however, they move to the competitive SA tail of employers and are paid wages proportionate to their training. This biases estimates of the impact of training based on differences in average wages toward zero on a short horizon. On the other hand, the full effect of training is realized in the right tail of the wage distribution more quickly since these workers, by chance, have climbed the job ladder to employment in a contract in which wages are proportionate to skill. This suggest quantile regression methods for evaluations of job training programs on short horizons.

The remainder of this paper proceeds as follows. Section 2.2 lays out the labor market setting under consideration, including the set of contracts available; describes worker and firm behavior in parallel sub-markets; and demonstrates that, under a set of costs for the right to SA in each sub-market, each sub-market exhibits a separating equilibrium in which low productivity firms WP while higher productivity SA. Section 2.3 defines and provides comparative statics with respect to skill level for ex-ante identical markets. Section 2.4 applies the results of Section 2.3 to evaluation of job training for the unemployed. Section 2.5 concludes. For the large part, proofs and derivations are left to the appendix. Chapter 1 considers implications of the dual-contract model with respect to aggregate empirical regularities. Chapter 3 provides methods for identifying the composition of contracts using administrative data.

2.2 Model

I consider two wage setting strategies: wage posting (WP) and sequential auction (SA). The former is typified by a fixed wage for all workers of the same skill level for the duration of contracts. Specifically, each firm is constrained to select a single "posted" wage for each skill level and employ all workers of the same skill level at this wage regardless of tenure or alternate employment offers. Under the latter, firms set wages to match the value of the best-to-date outside option of each worker and retain workers' services by updating wages whenever profitable. Firms are fully informed about each worker's outside option and offer wages equal to the worker's reservation wage given skill level and labor market history. When two firms employing under SA compete for an employee this results in a bidding war. The SA wage setting mechanism in this case is equivalent to a second price auction for the employee's services or a Bertrand competition over the offered value of employment: the worker is paid a wage which yields value equal to the highest profitable wage offer of the less productive competitor. As a result, wages under SA are dependent on skill insofar as the best-to-date alternate wage offer for a worker of given skill depends on skill.

Intuition of the separating equilibrium rests on noting that, all else equal, employing workers under SA results in a lighter wage bill than employing under WP. A cost for SA equivalent to the difference in wage bills for the threshold firm induces less productive firms to select WP and more productive to select SA.

2.2.1 Setting

I consider the steady state equilibrium of a search market in which firms possessing technology, p, and workers possessing skill, ε, are brought together by a sequential process of random matching: technology and skill are revealed to the other party only after workers and firms meet. ε distributes discretely, so that there is a positive mass, M^{ε}, of workers of each skill type. Meanwhile, p distributes continuously. Skill yields effective labor, and production is constant returns to scale, meaning that technology of type p paired with a worker of skill ε produces output εp per period. Workers of type-ε receive flow εb when unemployed. This last assumption can be justified, as Postel-Vinay and Robin (2002b) point out, by presuming that more productive laborers are also more productive home producers. Each worker has linear utility and seeks to maximize the present discounted value of wages and unemployment.

Firms employ workers under one of two contracts: wage posting (WP) or sequential auction (SA). If WP, the firm offers a non-negotiable wage for as long as the worker wishes or until exogenous separation. This wage may be conditioned on skill but is uniform within each skill type. If SA, the firm offers a wage chosen to match the value of the worker's best-to-date outside offer. The SA wage updates as the outside offer evolves and, in addition, the

SA firm commits to bid up the worker's wage in the event of a job-to-job transition.[13] SA wages are thus described by the wage setting strategy and productivity of the incumbent firm as well as the wage setting strategy and productivity of the best-to-date outside option and (through these) the worker's skill level. Meanwhile, WP wages depend only on the current incumbent's productivity and the skill level of the employee.

If the firm chooses the SA contract for workers of skill type ε it must pay a flow fee of c^{ε}. The fee is independent of firm size. However, if a firm employs some, but not all, skill levels under SA, it pays costs only for the skill levels for which SA is implemented. The micro-foundations of this cost are left for another paper; however, one possible story is that there are legal and/or administrative fees associated with posting a SA vacancy for ε-skill level workers. These fees must be paid whether or not the vacancy is filled.[14] Since output and costs associates with employing one skill level are independent of employing any other, the market for labor can be decomposed into parallel, but independent, sub-markets for each skill.

In each sub-market, measure N^{ε} of firms operate technologies which produce output that distributes according to a continuous distribution $\Gamma(p|\varepsilon)$ which may be different in each skill-specific sub-market. Each productivity distribution has support $[\underline{p}^{\varepsilon}, \bar{p}^{\varepsilon}]$ where the maximal productivity $\bar{p}^{\varepsilon}$ may be infinite. Workers search for contracts both off- and on-the-job using uniform sampling, meaning the probability of a ε-type worker sampling a firm of productivity p or less is $\Gamma(p|\varepsilon)$. Job offers arrive at exogenous Poisson arrival rates λ_0^{ε} when unemployed and λ_1^{ε} when employed. Workers are exogenously separated from employment contracts at Poisson arrival rate δ^{ε} and discount the future at rate μ^{ε}.

[13]Note that this "commitment" is sub-game perfect.

[14]Note that, although firms differ in size in equilibrium, each offers an identical number of vacancies. Size is determined ex-post by the rate of vacancy filling and the duration for which contracts persist.

2.2.2 Equilibrium in each sub-market

Chapter 1 demonstrates that, when workers are homogeneous, a separating equilibrium arises when firms face a per-firm cost for implementing the flexible SA contract. The result applies here to each of the ε-skill sub-markets.

Claim 1. *For every set of skill-specific costs, $\{c^{\varepsilon}\}$, there exists a set of skill-specific thresholds, $\{\check{p}^{\varepsilon}\}$, such that within every skill-specific sub-market an equilibrium exits in which firms with productivity less than the skill-specific threshold all prefer WP while more productive firms all strictly prefer SA. The threshold productivity firm is indifferent.*

Proof is in the appendix. The main steps are: first, to show that the flow of labor between firms is constrained efficient; second, that a SA firm can hire from a WP firm at a reduced wage; and third, that the magnitude of this wage cut is increasing in the SA firm's productivity.

The first requires that if an employee receives a job offer from a firm more productive than her incumbent employer there is always a wage offer which is profitable to the approaching firm and acceptable to the worker. Further, such a wage offer is always available in the sense that equilibrium wage setting strategies do not constrain the firm and prevent it from making said wage offer. This holds in the proposed equilibrium and is shown in the appendix. From this we can infer that $V^P(w^1, p^1, \varepsilon) < V^A(w^2, p^2, \varepsilon) < V^A(w^3, p^3, \varepsilon)$ if $w^1 < w^2 < w^3$ and $p^1 < p^2 < p^3$, where $V^i(w, p, \varepsilon)$ is the value of employment in a WP ($i = P$) or SA ($i = A$) contract in a p-productivity firm at wage w for a ε-skilled worker.

Second, the optimal wage strategy of a p-productivity SA firm when hiring from a q-productivity WP firm is to select a wage, $w_{PA}(q, p, \varepsilon)$, such that the value of employment in each contract is identical:

$$\underbrace{V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)}_{\text{value in the SA firm}} = \underbrace{V^P(w_{PP}(q,\varepsilon),q,\varepsilon)}_{\text{value in the WP firm}}$$

where $w_{PP}(q,\varepsilon)$ is the optimal posted wage choice of the WP firm. The appendix provides value functions and shows that the optimal wage offer is

$$w_{PA}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) + \tag{2.24}$$

$$\underbrace{\lambda_1^{\varepsilon}\big\{\bar{\Gamma}(\check{p}^{\varepsilon}|\varepsilon)V^P(w_{PP}(q,\varepsilon),q,\varepsilon) - [\Gamma(p|\varepsilon) - \Gamma(\check{p}^{\varepsilon}|\varepsilon)]\mathbb{E}[V^A(\varepsilon x,x,\varepsilon)|\check{p} < x < p] - [1-\Gamma(p|\varepsilon)]V^A(\varepsilon p,p,\varepsilon)\big\}}_{\text{difference in option values in the SA contract and best-to-date WP outside option}}.$$

where $V^P(w_{PP}(q,\varepsilon),q,\varepsilon)$ is the value of employment in a q-productivity WP firm at wage $w_{PP}(q,\varepsilon)$, and $V^A(\varepsilon x,x,\varepsilon)$ is the value of employment in a x-productivity SA firm at wage εx. WP firms do not alter their wage offers when facing competition. Meanwhile, competition between two SA firms results in the less productive firm offering a wage equal to the worker's output. The more productive firm then hires the worker under a wage contract which provides exactly the same value.

Note that the difference in option values in equation 2.24 is negative, since $q < x < p$ and therefore $w_{PP}(q,\varepsilon) < \varepsilon x < \varepsilon p$, when $w_{PP}(q,\varepsilon)$ is affordable to the q-productivity WP firm, so $V^P(w_{PP}(q,\varepsilon),q,\varepsilon) < V^A(\varepsilon x,x,\varepsilon) < V^A(\varepsilon p,p,\varepsilon)$. This shows that, within every sub-market, otherwise identical firms are able to operate lighter wage bills under SA than under WP. This arises for two reasons: 1) the SA firm hires each worker at a wage no greater than is required to outbid the best-to-date outside option, and 2) the SA firm partially compensates each worker with a commitment to aggressively bid up wages upon future job-to-job transition. The second means that an SA firm can hire a worker from every WP firm at a *wage cut*.

This wage cut, and therefore the wedge between wage bills, is larger for more productive SA firms since the option value associated with the firms' commitment to bid up wages is increasing in productivity: $V^A(w,x,\varepsilon) \leq V^A(w,y,\varepsilon)$ whenever $x \leq y$. This means that for any worker history the optimal wage offered by a more productive SA firm is lower. Observe that the difference in option values in equation 2.24 is increasing in absolute value as p increases. This is sufficient to show that if the cost of countering is equal to the willingness to pay for the right to SA for the threshold productivity firm:

$$c^{\varepsilon} = \{\mathbb{E}[w|\check{p}, P, \varepsilon] - \mathbb{E}[w|\check{p}, A, \varepsilon]\}\, \ell(\check{p}|\varepsilon) \tag{2.25}$$

where $\ell(\check{p}|\varepsilon)$ is the steady state level of employment in the threshold firm, then separation is an equilibrium.

When costs of SA are null or subsidized all firms select SA and under sufficiently high costs all firms select WP. Existence of a separating equilibrium for any choice of costs the follows from the intermediate value theorem. Equilibria are unique if the distribution of firms is sufficiently sparse in the right tail. Conditions for uniqueness are not restrictive and are given in the appendix.

2.2.3 Skill-specific value and wage functions

With the equilibrium characterized, I derive skill-contingent wage functions for each skill type. It is instructive to begin by considering the value function of a worker employed in a WP firm. The value of employment at a p-productivity WP firm can be written as:

$$\begin{aligned}
\mu^{\varepsilon} V^{P}(w_{PP}(p,\varepsilon),\varepsilon) = {} & w_{PP}(p,\varepsilon) \\
& + \underbrace{\lambda_1^{\varepsilon}[\Gamma(\check{p}^{\varepsilon}|\varepsilon) - \Gamma(p|\varepsilon)]\{\mathbb{E}[V^{P}(w_{PP}(x,\varepsilon),x,\varepsilon)|q < x < \check{p}^{\varepsilon}] - V^{P}(w_{PP}(p,\varepsilon),\varepsilon)\}}_{\text{job-to-job transition to a more productive WP firm}} \\
& + \underbrace{\lambda_1^{\varepsilon}[\bar{\Gamma}(\check{p}^{\varepsilon}|\varepsilon)]\{\mathbb{E}[V^{A}(w_{UA}(x,\varepsilon),\varepsilon)|\check{p} < x] - V^{P}(w_{PP}(p,\varepsilon),\varepsilon)\}]}_{\text{job-to-job transition to a more productive SA firm}} \\
& + \underbrace{\delta^{\varepsilon}\{\mathbb{E}[V^{U}(\varepsilon) - V^{P}(w_{PP}(p,\varepsilon),\varepsilon)\}]}_{\text{unemployment shock}},
\end{aligned} \tag{2.26}$$

where $w_{PP}(p,\varepsilon)$ is the optimal wage choice of the p-productivity WP firm.

The option value of receiving an employment offer from a more productive WP firm is positive since posted wages are not contingent on labor market history of workers and are, therefore, typically in excess of what would be required to induce transition: e.g. they are typically larger than the reservation wage conditional on labor market history. The option value of receiving an employment offer from a SA firm behaves quite differently. The fully flexible, informed, and rent extracting wage setting policy of SA firms coupled with the passive wage setting strategy of WP firms has interesting implications. In particular, since SA firms yield only enough rent to new employees to best the highest wage offer of the workers' best-to-date outside option and since WP firms do not update their wage offers in the face of competition, the gain in value when transitioning from employment in a WP firm to a SA firm is exactly zero!

The value of employment in a WP firm is, surprisingly, *independent* of the distribution of firms in the tail of the productivity distribution which comprises the SA sector. Assuming that $w_{PP}(x, \varepsilon)$ is a differentiable function (I will verify that it is momentarily), the value of employment in a p-productivity WP firm is:

$$V^P(w_{PP}(p,\varepsilon),\varepsilon) = \frac{w_{PP}(p,\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon} + \frac{\lambda_1^\varepsilon}{\mu^\varepsilon + \delta^\varepsilon} \int_p^{\check{p}^\varepsilon} \frac{\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon + \lambda_1^\varepsilon[\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)]} \frac{dw_{PP}(x,\varepsilon)}{dx} + \frac{\delta^\varepsilon V^U(\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon}. \tag{2.27}$$

The first term is the discounted flow value of receiving wage $w_{PP}(p, \varepsilon)$. The second and third terms are the discounted option value of search and of falling to unemployment. Similarly, the value of unemployment for each skill level is:

$$V^U(\varepsilon) = \frac{\varepsilon b}{\mu^\varepsilon} + \frac{\lambda_0^\varepsilon}{\mu^\varepsilon} \int_{\underline{p}^\varepsilon}^{\check{p}^\varepsilon} \frac{\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon + \lambda_1^\varepsilon[\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)]} \frac{dw_{PP}(x,\varepsilon)}{dx} \tag{2.28}$$

The first term is the discounted flow value of non-employment and the second the option value of search.

It is easy to verify that the value in both states is larger when more firms operate the

WP contract: $\frac{dV^P(w_{PP}(p,\varepsilon)}{d\check{p}^\varepsilon} > 0$ and $\frac{dV^U(\varepsilon)}{d\check{p}^\varepsilon} > 0$. Section 2.3 shows that the relation between threshold productivity and the value of unemployment and employment in a WP contract impacts wage distributions and wage-experience profiles in an interesting and intuitive way.

The reservation wage for employment in a WP firm in the ε-skill sub-market, $w_{UP}(\varepsilon)$, equates the value of unemployment with the value of employment in the least productive active firm in the ε-skill sub-market:

$$V^U(\varepsilon) = V^P(w_{PP}(\underline{p}^\varepsilon), \underline{p}^\varepsilon, \varepsilon).$$

So, the reservation wage for employment in a WP firm is

$$w_{UP}(\varepsilon) = \varepsilon b + (k_0^\varepsilon - k_1^\varepsilon) \int_{w_{UP}(\varepsilon)/\varepsilon}^{\check{p}^\varepsilon} \frac{[\Gamma(\check{p}^\varepsilon|\varepsilon) - \Gamma(x|\varepsilon)]}{1 + k_1^\varepsilon[\Gamma(\check{p}|\varepsilon) - \Gamma(x|\varepsilon)]} \frac{dw_{PP}(x,\varepsilon)}{dx}, \tag{2.29}$$

where $k_0 = \lambda_0^\varepsilon/(\delta^\varepsilon + \mu^\varepsilon)$ and $k_1 = \lambda_1^\varepsilon/(\delta^\varepsilon + \mu^\varepsilon)$. Assuming the fixed cost of entry for firms into each sub-market is zero, the lowest WP wage and the output of a ε-skill worker matched with the least productive active firm in each sub-market should be identical. This allows me to substitute $w_{UP}(\varepsilon)/\varepsilon$ for $\underline{p}^\varepsilon$ as the lower bound of integration. Note that whenever job offers arrive more quickly off-the-job than on-the-job the reservation wage exceeds the flow value of non-employment, since taking a job implies forgoing some potential job offers.

Now, I derive the optimal wage schedule for WP firms and check that it is, indeed, differentiable. In the equilibrium, if the p-productivity firm selects WP then it must be the case that all less productive firms also select WP. The problem facing the firm is thus very similar to the problem considered by Bontemps et al. (2000). The optimal posted wage offer, $w_{PP}(p)$, maximizes the expected profit from the posted contract:

$$\pi^P(p|\varepsilon) = \underbrace{[\varepsilon p - w_{PP}(p,\varepsilon)]}_{\text{rent per worker}} \underbrace{\ell(p|\varepsilon)}_{\text{labor supply}}, \tag{2.30}$$

Bontemps et al. (2000) show that when all firms WP, make optimal wage choices and productivity distributes according to a continuous distribution, labor supply is pinned down by firms' productivity type via mass-balance. More productive firms prefer to post higher wages and continuity of $\Gamma(p)$ yields the required one-to-one mapping between $w_{PP}(p,\varepsilon)$ and p.[15] The result extends to the separating equilibrium, since the mapping remains one-to-one in the presence of (more productive) SA firms. Every SA firm can outbid the highest profitable posted wage of every WP firm and the measure of $\check{p}$ posting firms is zero since $\Gamma(p|\varepsilon)$ is continuous in every sub-market. Labor supply is thus:

$$\ell(p|\varepsilon) = \frac{1+k_1^\varepsilon}{[1+k_1^\varepsilon\bar{\Gamma}(p|\varepsilon)]^2}\frac{M^\varepsilon - U^\varepsilon}{N^\varepsilon}. \tag{2.31}$$

where M^ε, U^ε, and N^ε are the mass of workers, non-employed, and firms in each sub-market. Details are provided in the appendix.

The solution to posting firms' maximization problem follows from applying the envelope theorem and solving the implied differential equation:

$$w_{PP}(p,\varepsilon) = \varepsilon p - [1+k_1^\varepsilon\bar{\Gamma}(p|\varepsilon)]^2 \int_{w_{UP}/\varepsilon}^{p} [1+k_1^\varepsilon\bar{\Gamma}(x|\varepsilon)]^{-2}dx \qquad \text{for } p < \check{p}, \tag{2.32}$$

See Bontemps et al. (2000, pg. 315-316) for explicit details. Now, one can easily check that $w_{PP}(p,\varepsilon)$ is indeed differentiable in p.

Turning to the value of employment in a SA firm. Full ex-post rent extraction implies that an employee in an SA firm receives the value of the best-to-date outside option. The value of employment in a SA firm is either $V^P(w_{PP}(q),\varepsilon)$ if the best-to-date outside option

[15]See Burdett and Mortensen (1998) and Bontemps et al. (2000) for details and proofs of these results.

is WP ($q < \check{p}$),

$$V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon) = V^P(w_{PP}(q),\varepsilon) \qquad \text{if } q < \check{p} \tag{2.33}$$

as was previously noted in the derivation of equation 2.24, or $V^A(\varepsilon q, q, \varepsilon)$ if the best-to-date outside option is SA, $\check{p} \leq q$,

$$V^A(w_{AA}(q,p,\varepsilon),p,\varepsilon) = V^A(\varepsilon q,q,\varepsilon) \qquad \text{if } \check{p} \leq q. \tag{2.34}$$

The appendix provides the value function for employment in a SA firm and shows that the value of employment in a q-productivity SA firm at the largest profitable wage, εq, has a surprisingly simple expression:

$$V^A(\varepsilon q, q, \varepsilon) = \frac{\varepsilon q + \delta V^U(\varepsilon)}{\mu^\varepsilon + \delta^\varepsilon},$$

as shown by Postel-Vinay and Robin (2002b).

Note also that the value of employment with sufficiently robust labor market history (when best-to-date outside option is a SA firm) is independent of the distribution of WP firms! This is perhaps less surprising than independence of the value of employment in a WP firm from the distribution of SA firms, since typically the value of employment does not depend on the distribution of firms which are unable to make attractive wage offers. However, employment in a SA firm alone is not sufficient for independence, since when the best-to-date outside offer is a WP firm the wage is set to match value of employment in the WP firm.

Wages set under SA contracts as a function of labor market history - best-to-date outside offer, q, and incumbent employer, p - and worker skill level are:

$$w_{PA}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) \tag{2.35}$$
$$-k_1^\varepsilon\bar{\Gamma}(\check{p}^\varepsilon|\varepsilon)\big[\check{p}^\varepsilon - w_{PP}(q,\varepsilon) - \int_q^{\check{p}^\varepsilon} \tfrac{k_1^\varepsilon[\Gamma(\check{p}|\varepsilon)-\Gamma(x|\varepsilon)]}{1+k_1^\varepsilon[\Gamma(\check{p}|\varepsilon)-\Gamma(x|\varepsilon)]}\tfrac{dw_{PP}(x,\varepsilon)}{dx}\big] - k_1^\varepsilon\int_{\check{p}^\varepsilon}^{p}\bar{\Gamma}(x|\varepsilon)dx \quad \text{for } q \leq \check{p} \leq p$$

and

$$w_{AA}(q,p,\varepsilon) = \varepsilon\{q - k_1^\varepsilon\int_q^p \bar{\Gamma}(x|\varepsilon)dx\} \qquad \text{for } \check{p}^\varepsilon < q \leq p. \tag{2.36}$$

The first simplifies wage equation 2.24. The second solves equation 2.34 as shown by Postel-Vinay and Robin (2002b, pg. 2339-2340).

Finally, when entering employment in a SA firm, reservation wages depend on both ε and the SA firm's productivity p. However, since the value of unemployment equals the value of employment in the least productive wage posting firm, $V^U(\varepsilon) = V^P(\underline{p},\varepsilon)$, and the countering firm selects the wage to match the value of the best-to-date outside option, the schedule of reservation wages for each skill type is equal to the wage required to hire that skill-type from the least productive firm: $\underline{w}_{UC}(p,\varepsilon) = w_{PC}(\underline{p}^\varepsilon, p,\varepsilon)$.

Thus all wages are pinned down as a function of worker type and labor market history. All equations have closed form solutions for the case when $\lambda_0 = \lambda_1$, since in this case $\underline{w}_{UP} = \varepsilon b$.

2.2.4 Steady state wage distribution

Within each sub-market, the distribution of wages can be expressed as a function of the wage schedules just derived and the distribution of workers across firms and across wage offers within firms.

Since workers never reject an employment offer from a more productive firm the method of mass balance pins down the distribution of labor across productivity as:

$$L(p|\varepsilon) = \frac{\Gamma(p|\varepsilon)}{1 + \kappa_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon)}. \tag{2.37}$$

The mass of workers employed in a firm at wages less than or equal to w can also be derived by the method of mass balance. For WP firms the mass is null everywhere except the posted wage where it is equal to the employment of ε-skilled workers in the firm. This follows since WP firms offer only a single wage for each skill level. Within SA firms, however, workers willing to accept wage $w_{iA}(q, p, \varepsilon)$ or less must have best-to-date outside option q or less (where $i = P$ if $q < \check{p}$ and $i = A$ if $q \geq \check{p}$). The mass flowing into such contracts must be $U\lambda_0\Gamma(q)$ and the mass flowing out must be $[\delta + \lambda_1\bar{\Gamma}(q)](M - U)L(q)$. This yields $\ell(w(q,p)|p) = \ell(q)$.[16] Within each firm wages are distributed according to:

$$G(w|p) = \begin{cases} \mathbb{1}_{w \geq w_{PP}(p)}, & \text{if } p < \check{p} \\ \frac{\ell(q(w,p))}{\ell(p)} = \left[\frac{1+\kappa_1[1-\Gamma(p)]}{1+\kappa_1[1-\Gamma(q(w,p))]}\right]^2, & \text{if } \check{p} \leq p \end{cases}$$

Aggregating requires summing the mass employed at wages less than or equal to w in each firm weighted by that firm's fraction of aggregate employment. So we have

$$G(w) = \int_{w(\underline{p},\bar{p})}^{w} G(w|p)dL(p) = \int_{w(\underline{p},\bar{p})}^{w} \ell(q(w,p))d\Gamma(p).$$

where $w(\underline{p}, \bar{p})$ is the smallest wage offered by the most productive firm. This wage is offered to hire from the least productive competitor. Noting that if p is a WP firm then $q(w,p) = p$ at the posted wage. The distribution of wages for a single skill-level is plotted in figure 2.1. The bottom panel decomposes this distribution by labor market history.

The wage distribution exhibits three modes. The left-most are entry wages in SA firms:

[16] Note, these results can also be found in Postel-Vinay and Robin (2002a).

wages for workers with best-to-date outside option a WP firm or unemployment. SA firms are able to employ these workers at low wages by promising future on-the-job wage growth and lucrative job-to-job transition prospects. This is a direct consequence of the difference in option value supplied by SA and WP firms. Recall that the separating equilibrium is induced by difference in expected wage bills for the threshold WP and SA firm in equilibrium.

Wages in the middle mode are offered by WP firms. Typically these firms yield rent when hiring, since they are constrained to offer a single wage to all employees. Additionally, wages are high since WP firms which are committed to not to attempt to retain workers when they transition to alternate employment and, as a result, offer poor job-to-job transition prospects. As a result wages in WP firms are higher than entry wages in SA firms.

Finally, wages in the right tail are set by competition between two SA firms. Workers in this tail earn wages that are (sometimes very) near the output produced in their employment match.

2.3 Comparative statics with respect to skill type

Turning to the relation between worker skill, contract type, and wages, I show that differences in workers' opportunities arise even in ex-ante identical skill-specific sub-markets. Ex-post differences in the composition of contracts play out in the distribution of wages, returns to experience, and employment opportunities of workers with different skill levels. Before proceeding, I define ex-ante identical sub-markets:

The cost of SA is identical in each sub-market. The distribution of potential firms in every sub-market is identical: $\Gamma(p)$. Vacancy posting is uniform, meaning that the probability of posting a vacancy in the ε-sub-market is equal to the portion of workers who are ε-type. This results in equal market tightness in all sub-markets: $M^{\varepsilon}/N^{\varepsilon} = M/N$ in all sub-markets. The hazards and discount rate (λ_0, λ_1, and δ, and μ) are identical across sub-markets, consistent with uniform market tightness.

It is possible that some firms are not active in all skill markets. Denote $\underline{p}^{\varepsilon}$ as the least productive firm which is able to profitably hire a ε-skilled worker. Firms of p-productivity less than $\underline{p}^{\varepsilon}$ are inactive in the ε-skill sub-market. The mass of active firms in the ε-skill sub-market is then $N^{\varepsilon} = \bar{\Gamma}(\underline{p}^{\varepsilon})N$. Similarly, the arrival rates of acceptable wage offers are $\bar{\Gamma}(\underline{p}^{\varepsilon})\lambda_0$ and $\bar{\Gamma}(\underline{p}^{\varepsilon})\lambda_1$. Finally, the distribution of active firms is described by $\Gamma(p|\varepsilon) = [\Gamma(p) - \Gamma(\underline{p}^{\varepsilon})]/\bar{\Gamma}(\underline{p}^{\varepsilon})$.[17] Without loss of generality define $\varepsilon = 1$ as the maximal skill type and $\Gamma(p|1) = \Gamma(p)$.

Before proceeding I establish the following useful benchmark:

Claim 2. *If sub-markets are **nearly** ex-ante identical but the cost of SA is **proportional to skill level**, $c^{\varepsilon} = \varepsilon c^1$, then:*

- *threshold productivity and the set of active firms is identical in all sub-markets:*

$$\check{p}^{\varepsilon} = \check{p} \text{ and } \Gamma(p|\varepsilon) = \Gamma(p) \; \forall \; \varepsilon,$$

- *wages decompose into the product of skill and labor market history:*

$$w(q, p, \varepsilon) = \varepsilon w(q, p, 1) \; \forall \; \varepsilon.$$

Proof of Claim 2 is mildly tedious and presented in the appendix. Direct implications are that the proportion of firms that hire under SA, hire under WP, or are inactive is identical in all sub-markets.[18]

[17] Note ex-ante identical sub-markets imply somewhat restrictive assumptions on the un-modeled matching function.

[18] An interesting additional consequence is that when thresholds are identical log wages decompose into a return to skill and a return to labor market history.

$$ln(w(q, p, \varepsilon)) = ln(\varepsilon) + \ln(w(q, p, 1)). \tag{2.38}$$

The implication is that a regression of log wages on skill, labor market history, and the interaction should yield an insignificant coefficient on the interaction term(s). This is consistent with the classic results of Mincer (1974) but not more recent results, for example Braga (2014).

Empirically, as Braga (2014) points out, the discrepancy in results stems from (mis)measurement of labor market history. Braga (2014) shows that, in U.S. data, results are substantively different if history is measured as actual versus potential experience. The specification suggested by this and other job-ladder

Observe that if the costs of SA are identical in all sub-markets then they are larger relative to output in lower skilled sub-markets. The direct implication is the following foundational result:

Claim 3. *If sub-markets are ex-ante identical then threshold productivity is decreasing in skill type:* $\varepsilon < 1 \;\Rightarrow\; \check{p}^1 \leq \check{p}^\varepsilon$.

This follows from the monotonicity of $\check{p}$ in c in each sub-market.

Juxtaposition of the case of constant costs (Claim 3) with that of proportional costs (Claim 2) is central to the interpretation of the remaining results of this section. I seek comparative statics for differentials in worker skill. Claim 3 informs that lower skill markets feature higher threshold productivity. Claim 2 allows me to compare these to counterfactual sub-markets for low-skill workers in which threshold productivity is constrained to be identical to the threshold for the high skilled. As a result, a comparative static with respect to threshold productivity can be reinterpreted as a comparative static with respect to skill level.

When comparative statics regard wages a notion of proportionality is required. I call a statistic, $X(\varepsilon)$, proportional to skill if $X(\varepsilon) = \varepsilon X(1)$. If $X(\varepsilon) > \varepsilon X(1)$, I call this larger than proportionate or disproportionately large and vice versa. Claim 2 describes the case when wages are proportional to skill. Note that the special cases of pure-SA and pure-WP equilibria simultaneously satisfy Claim 3 and Claim 2: null costs of SA are consistent with pure-SA while infinite costs are consistent with pure-WP. All intermediate cases, however, reveal interesting ex-post disparities in the composition of contract types, composition of employer types, employment, wage levels and dispersion, and entry wages and return to experience.

For each outcome, I begin by considering the simplified case in which the chance of

models would be to measure labor market history, $\ln(w(q, p, 1))$, as *continuous employment experience* (eg. the time since the *most recent* spell of unemployment) or as the *number and quality of wage offers in the current employment spell*. The measure of Braga (2014) comes much nearer to these definitions.

receiving a wage offer is identical on- and off-the job: $\lambda_1 = \lambda_0$. This case is simplest because identical offer arrival rates pin down the reservation wage for employment in a posting firm as the flow value of leisure, shutting down equilibrium feedback from the threshold productivity, $\check{p}^\varepsilon$, to the minimum productive productivity, $\underline{p}^\varepsilon = w_{UP}(\varepsilon)/\varepsilon$. The results in the simplified case isolate the effect of the *composition* of contract types when the set of active firms is identical in all sub-markets.

I then consider the total effect when offer arrival is more likely for the unemployed, $\lambda_1 < \lambda_0$, as is likely the case in reality. The situation becomes both more interesting, more realistic, and a great deal more complex when the arrival rate of offers off-the-job exceeds the arrival rate on-the-job. Complexity arises since differentials in threshold productivity induced by skill differentials now feed back into the distribution of active firms. This produces interesting comparative statics with respect to unemployment and utilization. To maintain tractability, I consider the case where offer arrival rates are sufficiently similar off- and on-the-job. In this case feedback through the distribution of active firms is minor.

2.3.1 Entry wages and return to experience

I begin by considering entry wages.

Claim 4. *Entry wages for low-skill workers are disproportionately large,* $\varepsilon w_{PA}(q, p, 1) < w_{PA}(q, p, \varepsilon)$ *and* $\varepsilon w_{PP}(p, 1) = w_{PP}(p, \varepsilon)$.

To see this refer first to the wage schedule for employees of SA firms with best-to-date outside option a WP firm, equation 2.29, which under ex-ante identical sub-markets, simplifies to:

$$w_{PA}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) \tag{2.39}$$

$$-k_1\bar{\Gamma}(\check{p}^{\varepsilon})\Big[\check{p}^{\varepsilon} - w_{PP}(q,\varepsilon) - \int_{q}^{\check{p}^{\varepsilon}} \tfrac{k_1[\Gamma(\check{p})-\Gamma(x)]}{1+k_1[\Gamma(\check{p})-\Gamma(x)]}\tfrac{dw_{PP}(x,\varepsilon)}{dx}\Big] - k_1\int_{\check{p}^{\varepsilon}}^{p}\bar{\Gamma}(x)dx \quad \text{for } q \le \check{p} \le p$$

It is straightforward to show that the wage schedule each SA firm offers when hiring workers with best-to-date outside options at WP firms is increasing in the threshold productivity: $\frac{dw_{PA}(q,p,\varepsilon)}{d\check{p}^{\varepsilon}} > 0$. From Claim 3 we know that if the threshold is identical in the sub-markets for two skill levels 1 and ε then $w_{PA}(q,p,\varepsilon) = \varepsilon w_{PA}(q,p,1)$. From this we have that, if the threshold is higher in the ε sub-market, then wages in SA firms when the best-to-date outside offer is a WP firm are disproportionately large for low-skilled workers: $w_{PA}(q,p,\varepsilon) > \varepsilon w_{PA}(q,p,1)$.

Now refer to the schedule of wages offered to employees of WP firms, equation 2.32. When the arrival of wage offers is constant across employment states the minimal active productivity is identical in all sub-markets. Inspection of the WP wage schedule then immediately reveals that with ex-ante identical sub-markets and identical sets of active firms wages set under WP are proportionate to skill: $w_{PP}(q,p,\varepsilon) = \varepsilon w_{PP}(q,p,1)$. However, when offer arrival is more likely off-the-job than on-the-job it is easy to show that $w_{UP}(\varepsilon) > \varepsilon w_{UP}(1)$ and as a result $w_{PP}(p,\varepsilon) > \varepsilon w_{PP}(p,1)$ for $p < \check{p}^1$.

So, entry wages are disproportionately large regardless of initial employer (strictly when $\lambda_0 > \lambda_1$). Finally, when offer arrival rates are sufficiently similar, wages set under WP are more prevalent in low skilled sub-markets since the threshold productivity is higher and feedback to the set of active firms is minimal, as is shown later in this section. This means that newly employed low-skilled workers are more likely to be employed in WP contracts. Since entry wages in WP firms are always larger than entry wages in SA firms this reinforces the result.

I now consider wages of workers with sufficient experience to a have best-to-date outside option at a SA firm.

Claim 5. *For workers with sufficient labor market experience wages are proportionate to skill, and* $\varepsilon w_{AA}(q, p, 1) = w_{AA}(q, p, \varepsilon)$.

Wages set by Bertrand competition between two SA firms are proportionate to skill. To see this refer to the wage schedule set by SA firms when hiring workers with best-to-date outside options SA firms, equation 2.36. Quick inspection reveals that this is independent of threshold productivity: $w_{AA}(q, p, \varepsilon) = \varepsilon w_{AA}(q, p, 1)$. With sufficient labor market experience a worker will be employed in a SA firm with best-to-date outside option a SA firm and be employed at wages set under wage schedule 2.36. So, sufficiently experienced workers are compensated proportionately to their skill level.

The implication for returns to continuous employment are striking.

Claim 6. *Low-skilled workers experience lower returns to continuous employment experience.*

This is a direct consequence of inflated entry wages coupled with proportionate long term prospects. The result is reinforced since the share of job offers originating from SA firms is smaller for low-skilled workers, thus, on average, it takes longer for low-skilled workers to climb to the lucrative right tail of the wage distribution characterized by Bertrand competition between two SA employers.

Figure 2.2 plots the wage-experience profile (top) and return to experience (bottom) for workers of type 0.5, 0.75, and 1, when the job offer arrival rate is constant on- and off-the-job. Solid lines plot simulated histories while hatched lines plot counterfactual histories for a labor market in which the threshold productivity is identical in all sub-markets. Consistent with Claim 4, entry wages are disproportionately high for low skilled workers. The larger the difference in skill, the greater the disproportion of wages. As continuous experience increases, however, workers of all skill levels climb the job ladder toward the SA sector. As a result, the premium low skill workers earn early in their careers dies out with experience. In the long run, low skill workers wages fall behind and receive wages that are disproportionately low. This occurs because it takes low skill workers longer to climb the job ladder to the smaller SA sector in the low skilled sub-market. As a result of disproportionately high entry wages and greater average experience required to achieve employment with best-to-date outside option a SA firm, low skill workers experience flatter wage-experience profiles compared to high skilled workers.

Figure 2.3 plots the wage-experience profile (top) and return to experience (bottom) for workers of type 0.5, 0.75, and 1, when the job offer arrival rates are $\lambda_0 = 2$ and $\lambda_1 = 0.2$ per year. These are roughly consistent with labor market histories of Germans 2006-2008 documented in Chapter 3. Solid lines plot simulated histories while hatched lines now plot counterfactual histories for a labor market in which the set of active firms is identical for all skill levels (the previous case). Consistent entry wages are, as before, disproportionately high for low skilled workers and wage-experience profiles follow a similar pattern. Overall, differences are extremely moderate. The overall features of differential return to experience are moderately accentuated.

Table 2.1 records differentials in the proportion of contract types with respect to skill level. As discussed, the likelihood of receiving an offer from a WP firm is decreasing in skill level. Meanwhile, the value of unemployment, average entry wages and average wages are decreasing disproportionately slowly with respect to skill level.

Differentials in the distributions of employment for different skill levels complicate proof of the remaining. Clearly these are analogous to the simpler case and their proof is maintained for sufficiently small differences in offer arrival rates. This is treated in the appendix.

Meanwhile, table 2.2 records statistics analogous to table 2.1 as well as the unemployment rate and fraction of firms which are inactive in each sub-market. A substantial fraction of firms do not operate in the low skill labor market. The result is unemployment rates which are increasing as skill declines.

2.3.2 Average wages and wage dispersion

Next, I turn to average wages.

Claim 7. *Average wages for low-skill workers are disproportionately large:* $\varepsilon\mathbb{E}[w|1] < \mathbb{E}[w|\varepsilon]$.

The result is not straightforward to obtain. On the one hand, for the vast majority of labor market histories, wages weakly rise when threshold productivity rises. The logic above implies that wages fall disproportionately slowly with respect to a decline in skill for these histories. On the other hand, for labor market histories for which the best-to-date outside option changes strategy from SA to WP when the threshold rises the wage *falls dramatically*. Again, this implies that, for a small difference in skill, workers with identical labor market histories but slightly larger skill will have *dramatically larger wages*. The second occurs since the less skilled individual's wages are *not* set by Bertrand competition between two SA firms while the more skilled individuals wages are. The claim is proved if in a market with homogeneous workers every firms' wage bill weakly rises when threshold productivity rises. This will imply that for each firm the percentage change in average wages falls (weakly) more slowly than skill level. Proof that this is indeed the case when offer arrival is constant on- and off- the job is tedious and presented in the appendix. The result is maintained if offer arrivals are sufficiently similar; since feedback from threshold productivity to minimum productivity is minor.

Claim 4 and claim 6 have additional implications for wage dispersion. Figure 2.4 plots the distribution of log wages implied by the model for a worker of skill level 1, 0.75 and 0.5 in ex-ante identical sub-markets. 10^{th}, 50^{th}, and 90^{th} percentiles are indicated by the vertical bars. Distributions of wages for less skilled individuals are more compressed. Compression is particularly pronounced for wages in SA employers with best-to-date outside option unemployment or a WP employer (the left mass). This portion of the distribution is shifted disproportionately toward larger wages for low skilled individuals, since the value of unemployment and employment in a WP contract are disproportionately high for these individuals. Starting wages are more generous due to this shift in the left tail of wages and the increased odds of becoming employed in a WP firm out of unemployment (the middle mass). Notably, however, distribution in the right tail, where wages are set by SA firms facing best-to-date outside option a SA firm, is proportional to skill.

2.3.3 Employment

Turning to employment, the following result follows from a slower than proportionately declining reservation wage for employment in the WP sector as skill falls.

Claim 8. *Low-skilled workers experience higher rates of unemployment.*

Since reservation wages are not proportionately low, offers from the least productive firms are rejected (equivalently, these firms exit the low skilled market) resulting in inflated unemployment for low skilled workers.

2.3.4 Composition of contract types and employers

Claim 9. *Low-skilled workers are less likely to be employed in SA contracts.*

This is a direct implication of higher threshold productivity. When on- and off-the-job arrival rates are identical the corollary is clearly that low skilled workers are more likely to be employed in WP contracts. Allowing for differences in on- and off-the-job arrival rates requires considering a third category, unemployment. In this case low-skilled workers are more likely to be unemployed, as just noted, or employed in a WP contract than their high-skilled counterparts. Conditioning on employment, low skilled workers less likely to be employed in a SA contract so long as the on- and off-the-job arrival rates are sufficiently similar.

Finally turning to the composition of employers.

Claim 10. *1. Some low productivity firms employ no low-skilled workers.*

2. The fraction of workers employed at firms of productivity less than p is increasing in skill level.

3. Firms that employ both high-skilled and low-skilled workers employ a disproportionately large mass of low-skilled workers.

Firms' inability to direct vacancies toward high skill workers motivates these. Now consider that in the low skilled sub-market the value of unemployment is disproportionately high relative to the high skilled sub-market. Since the arrival rate of offers is less frequent on-the-job this induces a disproportionately large reservation wage for employment in a WP firm in the low skill sub-market. As a result, some of the least productive firms are effectively "priced out" of the low skilled sub-markets and these firms employ no low-skilled workers. This is the corollary of claim 8.

The second and third result are proved in the appendix. Intuition rests on noting that low skill workers reject the least productive employers and, thus, the distribution of their employment is skewed toward more productive firms.

2.4 Application: evaluating job training

Many countries offer, and sometimes require, job training as an activation policy for the unemployed. Evaluation of these policies has been extensive but is complicated by considerable econometric difficulty. In particular, training programs are not typically designed with pseudo-experimental evaluation in mind. This results in difficulty obtaining data on suitable "control" individuals to serve as counterfactuals in empirical assessment. In addition, even when policy is designed or implemented in a way where a control group can be identified, estimates of treatment effects may be attenuated due to spill-overs from the behavior of trained individuals to the behavior of untrained individuals (Heckman et al., 1999).

Despite econometric difficulties, job training programs are extensively studied in part because many training programs are accompanied with a mandate for cost-benefit analysis. Studies typically find little or no impact of training on re-employment wages in the short run but often find sizeable positive impact in the medium and long run (Card et al., 2010).

Differences in results between the short and the longer run are not well explained by a lack of suitable or suitably matched control group. Why should a more suitable control group appear over time? Differences in results with respect to time horizon are also not easily explained by spill-overs. Typically one suspects spill-overs attenuate impacts and spill-over is likely to be more prevalent in the longer term.

A lag in the impact of training, however, can be explained by structural models of learning or labor market behavior in which training takes time to sink in for workers, be recognized by firms, or be rewarded by firms even if it is immediately recognized. This paper provide a mechanism of the third type: firms recognize training immediately, however, differences in labor markets for low skilled (untrained) versus high skilled (trained) workers result in attenuated expression of training in wages for short durations of continuous employment experience.

This suggests that the policy evaluator must defer evaluation in order to observe the full effect of training in average wages from which the effect of training on productivity can be inferred. This helps explain the relative plethora of null results obtained from evaluations with short time horizons.

However, if the mechanism described contributes substantively to patterns in the data, there is another way. As discussed, as workers move into the SA sector, and in particular when their best-to-date outside option is a SA firm, wages are proportional to skill level (training). This may take a long time and even in steady state average wages are disproportionately high for low skilled (untrained) workers. However, the policy evaluator can selectively observe workers who have climbed the job ladder by targeting workers whose wages are in the right tail of the distribution.

Under the hypothesis that the model presented here is the true model, statistics obtained from the right tail of the wage distribution are less attenuated at all time horizons and reveal the *true* impact of training on productivity of training at reasonably short horizons.

2.4.1 Example

Consider training unemployed individuals of skill type 0.75. Suppose that training is effective and after training 100% of trained individuals are of skill type 1: an increase in skill of 33.3 percent. Suppose also that the policy maker has an experimental design. Finally, suppose that there are no general equilibrium effects of training. The last assumption imposes that both before and after training the effective labor produced by a worker of a given skill level is identical and that the arrival rate of job offers and distribution of productivity are also invariant to the presence of the training program.

I simulate such an experiment for 5000 trained and 5000 untrained workers. I set hazards to be $k_0 = 2$ and $k_1 = 0.2$ per year, consistent with estimates from German workers in 2006-2008 obtained in Chapter 3. I model productivity as having a Pareto distribution and let the shape and scale be 3.73 and 113 respectively. These imply an unemployment rate of 9.38 and mean and standard deviation of the wage distribution 114.88 and 51.98 for trained workers 87.39 and 37.64 for untrained workers. These are broadly consistent with labor market histories of West German males 2005-2008. I then simulate continuous spells of employment experience for each individual for 15 years.

Estimates of the percent increase in skill from a regression at 1, 2, and 3 years of continuous experience, the typical horizons tested in the literature, are 27.6, 28.1, and 29.7. These are underestimates of 3-5 percentage points of the true effect on productivity. The percent difference in average wages between the trained and untrained group are plotted in the top panel of figure 2.5. Indeed it takes more than 10 years for the percentage differences in wages to reflect the increase in skill from training.

The policy evaluator may have more success at short horizons by considering statistics which focus on the right tail of the wage distribution, where wages are set competitively through competition between SA employers. This suggests analyzing percentiles of the distribution. The difficulty is to be sure of when the X percentile individual is sure to have best-to-date outside option a SA firm. For the simulation conducted here, this occurs within 2 years for the 95th percentile and 4 years for the 90th percentile. After this point the percent difference in wages at the Xth percentile reveal the true impact of training on productivity.

An unfortunate difficulty is that the time until the X percentile individual has best-to-date outside option a SA firm is longer for the untrained control group since more firms hire the untrained under WP. As a result the X percentile diverges radically from the true estimate just before becoming informative. Fortunately, the time horizon is larger for lower percentiles and the researcher can compare estimates based on percentiles and on average wages to rule out gross overstatements.

2.5 Conclusion

This paper considers an on-the-job search model in which heterogenous firms and workers form employment matches under one of two contract types: wages posting (WP) or sequential auction (SA). Workers are characterized by their skill and firms by their productivity. In equilibrium, more productive firms select the SA contract for all worker types while less productive firms select the WP contract when employing under the SA contract is costly. The model nests pure-contract equilibria which feature only WP, e.g. Bontemps et al. (2000), and only SA, e.g. Postel-Vinay and Robin (2002b). Intermediate cases, which feature both contract types simultaneously, deliver a new look at differences in employment, wage dispersion, and returns to experience for workers of different skill levels.

Even in ex-ante identical sub-markets differences in outcomes arise. Costs of SA are a larger portion of the output produced by low skilled workers. Consequently, fewer firms select the SA contract in low-skilled sub-markets. As a result, low skilled workers experience higher rates of unemployment, less log wage dispersion, and lower returns to experience than high skilled workers.

A particularly interesting finding is that entry wages for low-skilled workers are disproportionately large, relative to skill. This should impact the methods a policy evaluator selects when evaluating job training programs. Over short horizons, differences in average wages of trained and untrained groups understate the differences in skill produced by training. Differences in higher quantiles, however, fully reflect skill differences. These results suggest quantile regression methods for evaluations of job training programs over short horizons.

2.6 Figures

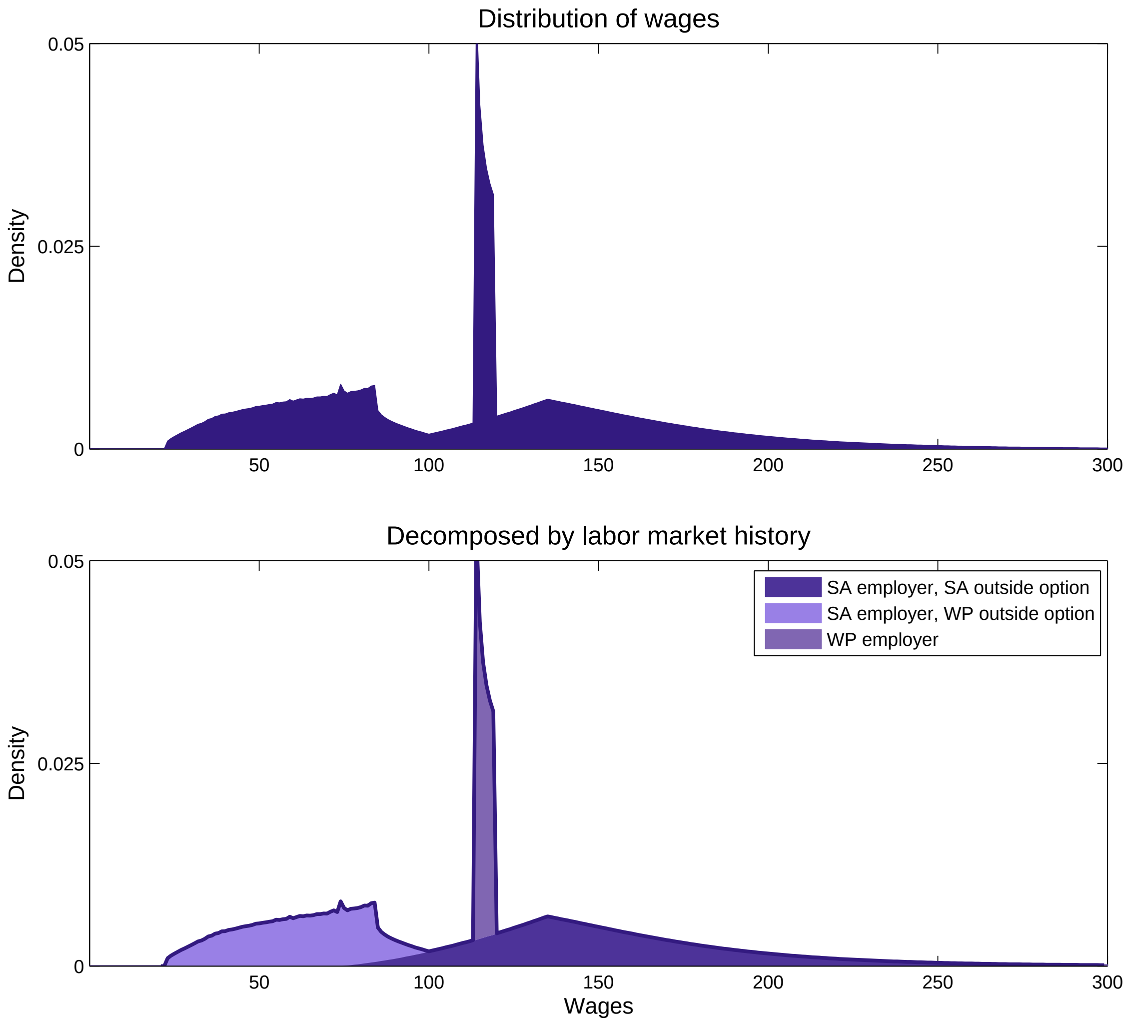

Figure 2.1: Distribution of wages (top) for a single skill type. Distribution decomposed by labor market history (bottom). The wage distribution exhibits three modes. The left most are wages in SA firms for workers with best-to-date outside option a WP firm or unemployment. Wages in the middle mode are offered by WP firms. Wages in the right tail are set by competition between two SA firms.

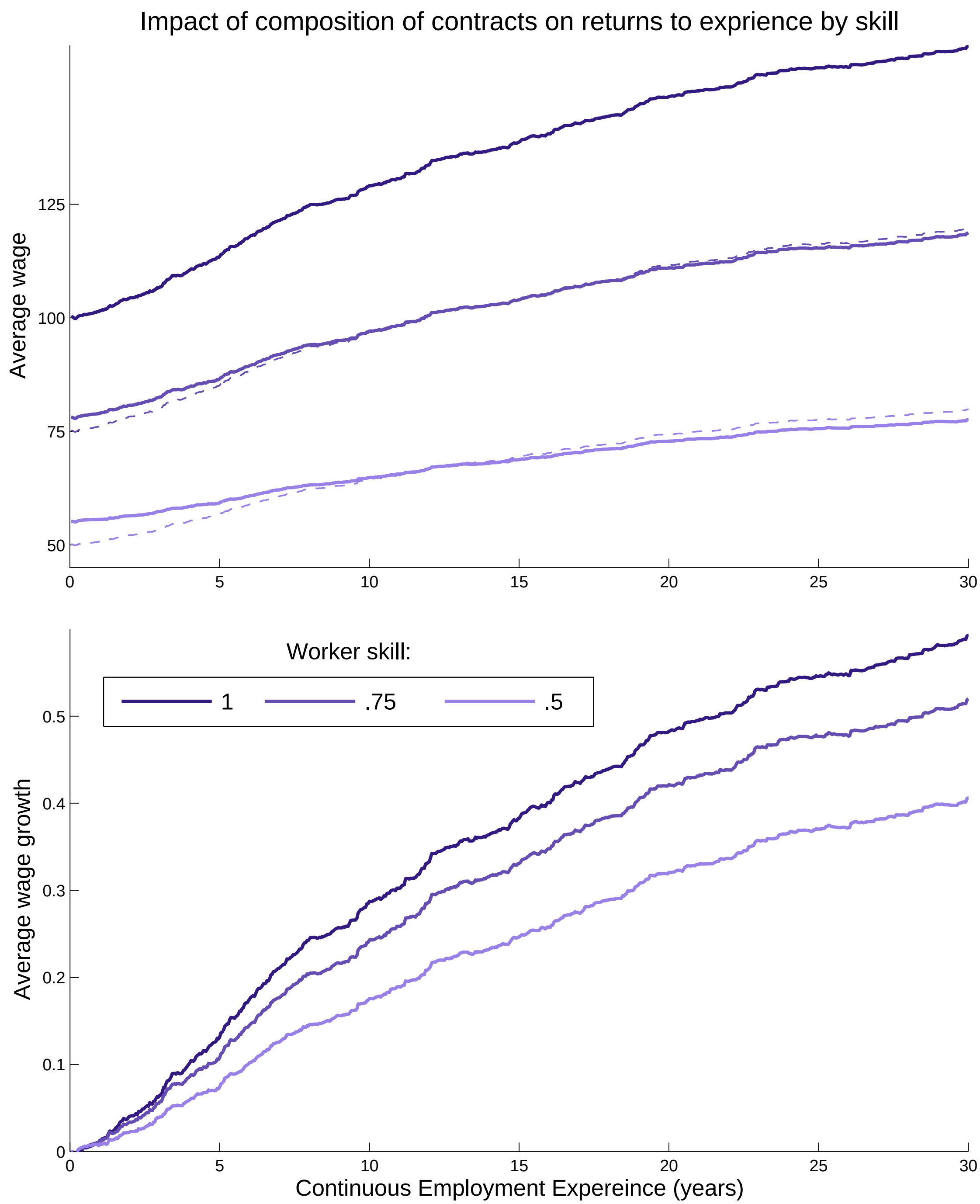

Figure 2.2: Average wages and average wage growth by skill level and continuous employment experience. Compression in the left tail of the wage distribution coupled with greater odds of initial employment in a WP firm result in higher than proportional starting wages for low skilled workers. As workers climb the job lader the odds of employment in a SA firms facing best-to-date outside option a SA firm increase. Wages for such a history *are* proportionate to skill. The over all effect is that more skilled workers experience greater wage growth.

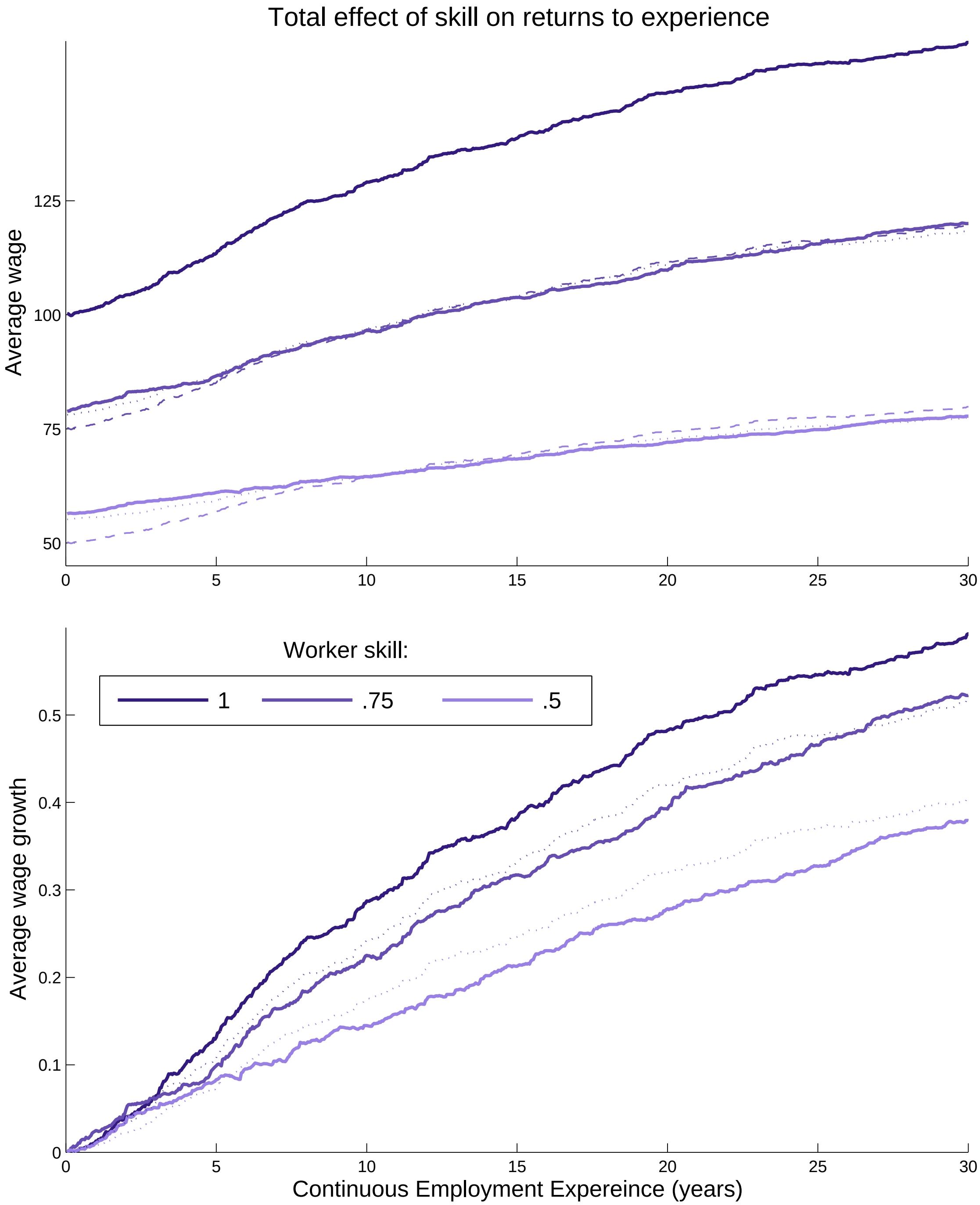

Figure 2.3: Average wages and average wage growth by skill level and continuous employment experience. Compression in the left tail of the wage distribution coupled with greater odds of initial employment in a WP firm result in higher than proportional starting wages for low skilled workers. As workers climb the job lader the odds of employment in a SA firms facing best-to-date outside option a SA firm increase. Wages for such a history *are* proportionate to skill. The over all effect is that more skilled workers experience greater wage growth.

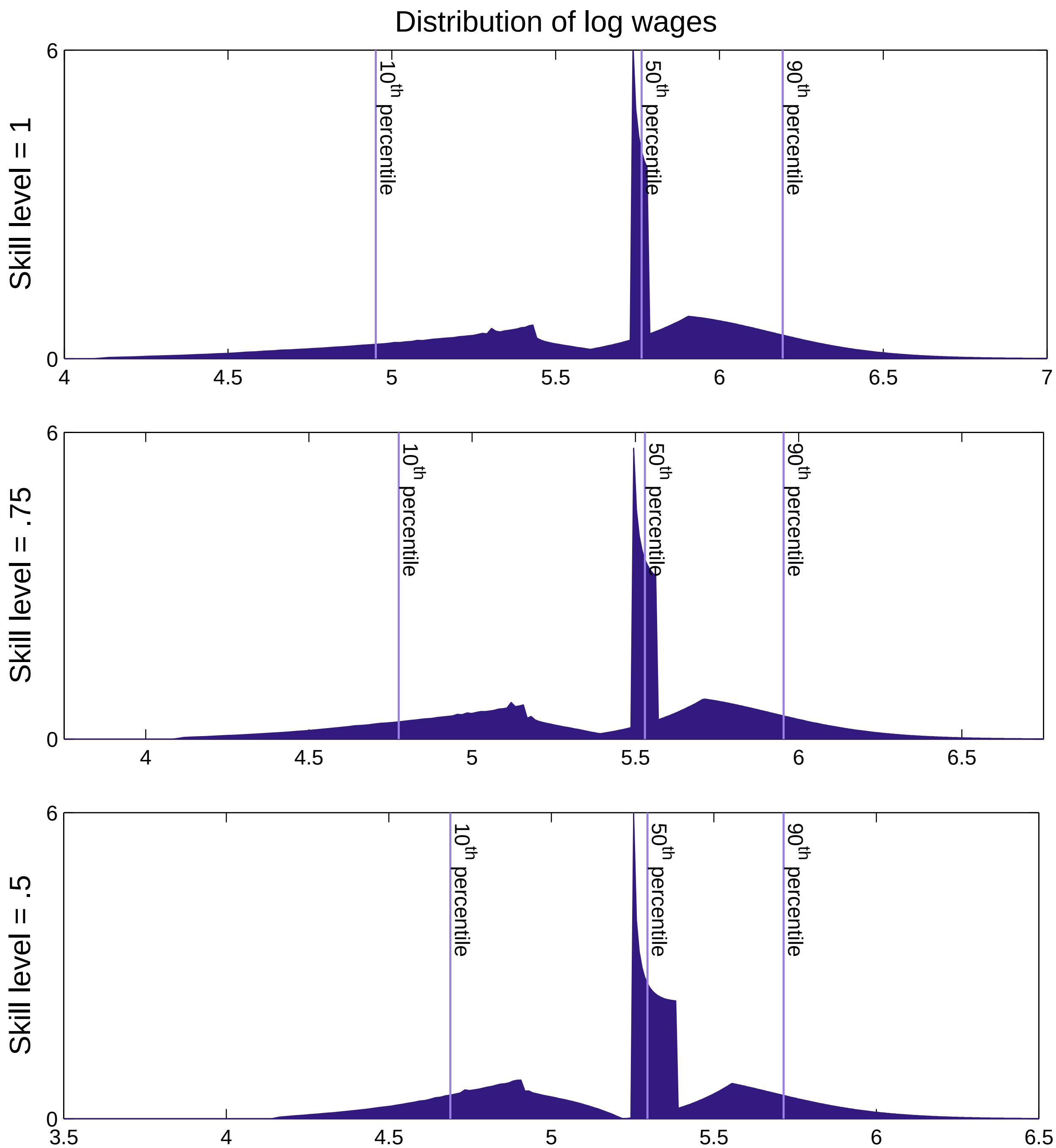

Figure 2.4: Distributions of wages for skill levels 1, .75, and .5. 10^{th}, 50^{th}, and 90^{th} percentiles are indicated by the vertical bars. Distributions of wages for less skilled individuals are more compressed. Compression is particularly pronounced in starting wages in SA firms which is shifted disproportionately toward larger wages for low skilled individuals. Starting wages are again more generous due to the increased odds of becoming employed in a WP firm out of unemployment. Notably, however, mass in the right tail, where wages are set by SA firms facing best-to-date outside option a SA firm, is not affected by skill.

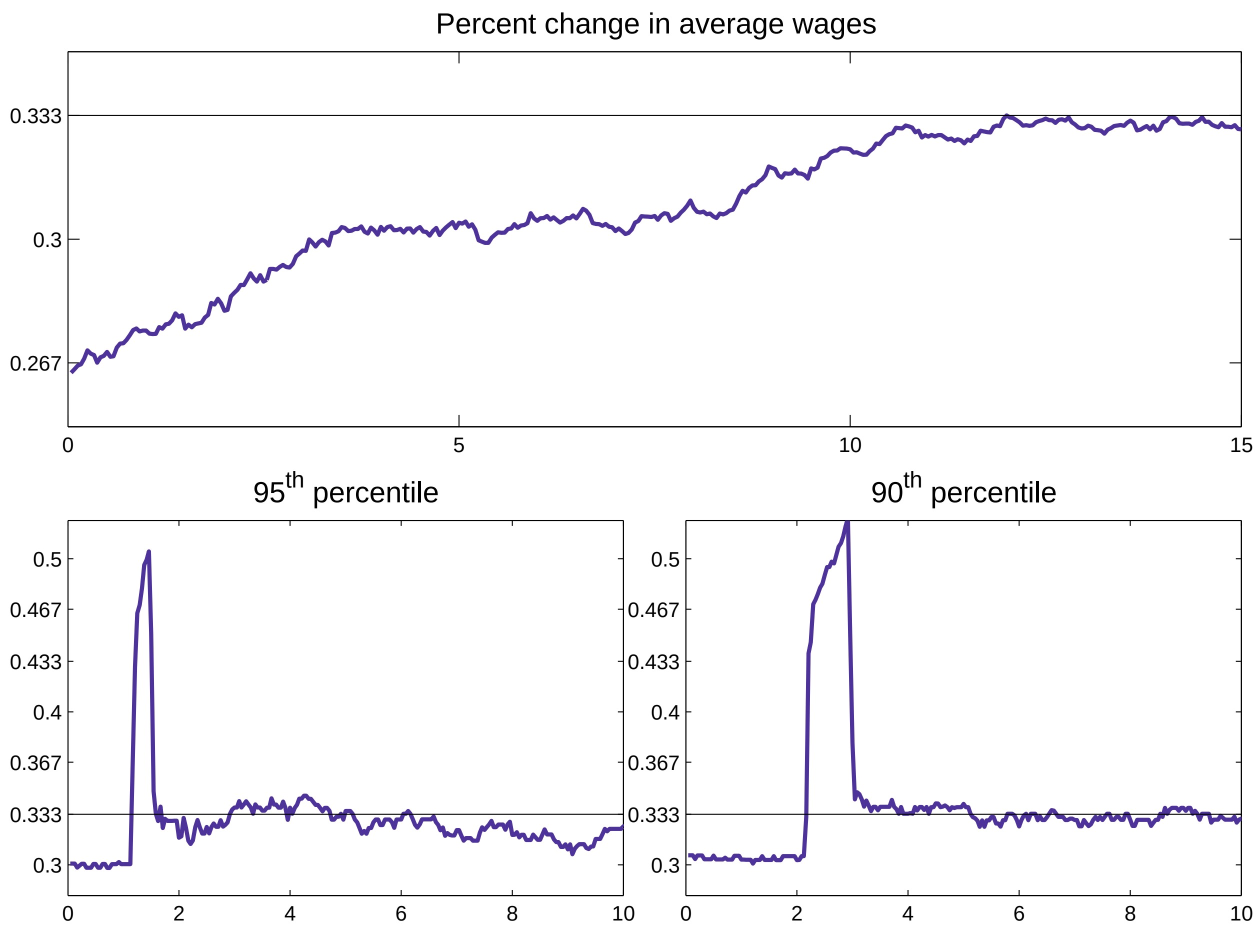

Figure 2.5: The difference in average wages understates the impact of training even at high experience levels. Differences in high percentiles reveal the value of training on much shorter horizons.

2.7 Tables

Table 2.1: Market composition average wages and value of unemployment: equally efficient search off- and on-the-job

	Worker skill: .5	.75	1
Average wage	59.57	87.17	114.88
(fraction of full skill)	*51.86%*	*75.88%*	*100%*
Average entry wage	55.17	78.10	100.29
(fraction of full skill)	*55.01%*	*77.88%*	*100%*
Average entry — 10 years continuous experience	64.55	96.58	129.11
(fraction of full skill)	*50.00%*	*74.8%*	*100%*
μ(Val. Nonemp.)	59.57	85.51	113.62
(fraction of full skill)	*50.53%*	*75.26%*	*100%*
Firms			
Inactive	0.00%	0.00%	0.00%
Posting	68.08%	56.18%	47.76%
Countering	31.92%	43.82%	52.24%
Workers			
Unemployed	9.38%	9.38%	9.38%
Posting	37.90%	26.84%	20.73%
Countering	62.10%	73.16%	79.27%

Table 2.2: Market composition average wages and value of unemployment: more efficient search off-the-job

	Worker skill:		
	.5	.75	1
Average wage	57.77	87.39	114.88
(fraction of full skill)	*50.29%*	*76.07%*	*100%*
Average entry wage	56.39	78.82	100.29
(fraction of full skill)	*56.22%*	*78.60%*	*100%*
Average entry — 10 years continuous experience	64.87	97.10	129.11
(fraction of full skill)	*50.24%*	*75.19%*	*100%*
μ(Val. Nonemp.)	58.53	86.18	113.62
(fraction of full skill)	*51.51%*	*75.85%*	*100%*
Firms			
Inactive	6.33%	1.63%	0.00%
Posting	62.13%	54.55%	47.76%
Countering	31.54%	43.82%	52.24%
Workers			
Unemployed	9.95%	9.52%	9.38%
Posting	38.19%	26.72%	20.73%
Countering	61.81%	73.28%	79.27%

2.8 Proofs and Derivations

Proof of Claim 1

I begin by proving that for every threshold a cost exists such that separation is consistent with a Nash equilibrium. I then observe that the mapping is continuous, that null costs are consistent with all firms selecting SA and all firms select WP for sufficiently large cots. The claim then follows from the intermediate value theorem.

Labor supply

Workers seek to maximize the value of their current employment contract. In order to pin down labor supply to a firm of type p I must show how workers' value maximization results in labor flow between firms.

As a starting point, in the pure-WP and pure-SA equilibria Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002a), respectively, show that higher productivity firms are able and prefer to offer higher value contracts in equilibrium.

Now note that the largest possible WP wage is at most equal to the marginal product of the threshold firm. Next, the least productive SA firm has larger marginal product since it is, in the proposed separating equilibrium, more productive. So, each SA firm can profitably pay a wage at least as large as the largest possible WP wage and, therefore, must hire employees of WP firms (when it meets them) in equilibrium.[19] Therefore, the flow between sectors is efficient.

Within the WP sector firms behave as if the whole economy posted wages, and so flows are efficient. This follows from noting that the presence of SA firms does not alter the solution to the posting firms' maximization problem since 1) profitable wage choices are

[19]We will see that, in equilibrium, the countering firm is actually able to hire these workers for less than their best-to-date posted wage. The reason is that the countering firm compensates its workers partially through the option value of contingent pay.

only competitive against other WP firms and 2) since $\Gamma(p)$ is continuous there is no mass of WP firms at the threshold productivity. The proof due to Burdett and Mortensen (1998) applies: more productive firms can employ workers of less productive firms a trivially greater wages and at greater profits. Finally, within the SA sector higher productivity firms are still able to outbid lesser productivity firms and so the flows are efficient. This shows that, in total, flows of employed workers in the separating equilibrium are efficient.

Since workers accept any wage offer originating from a more productive firm, labor supply to a p-type firm can be pinned down by the method of mass balance. In steady state, the mass of workers flowing into firms of p-type or less must be equal to the mass flowing out:

$$\underbrace{U^{\varepsilon}\lambda_0^{\varepsilon}\Gamma(p|\varepsilon)}_{\text{in}} = \underbrace{[\delta^{\varepsilon} + \lambda_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon))](M^{\varepsilon} - U^{\varepsilon})L(p|\varepsilon)}_{\text{out}}, \tag{2.40}$$

where $\bar{\Gamma}(p|\varepsilon) = 1 - \Gamma(p|\varepsilon)$ is the fraction of firms with productivity greater than p, U denotes the mass of unemployed workers, and $L(p|\varepsilon)$ is the mass of workers employed in a firm of productivity no greater than p.

Evaluating equation (2.40) at the supremum of productivity types yields steady state unemployment rate: $u = U^{\varepsilon}/M^{\varepsilon} = 1/(1 + \kappa_0^{\varepsilon})$, where $\kappa_0^{\varepsilon} = \lambda_0^{\varepsilon}/\delta^{\varepsilon}$. Also, the fraction of workers working for a firm with technology p or less is

$$L(p|\varepsilon) = \frac{\Gamma(p|\varepsilon)}{1 + \kappa_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon)}, \tag{2.41}$$

where $\kappa_1^{\varepsilon} = \lambda_1^{\varepsilon}/\delta^{\varepsilon}$ is the expected number of job offers per employment spell.

The supply of labor from ε-type workers to a firm of type p can then be expressed as:

$$\ell(p|\varepsilon) = \overbrace{\frac{\lambda_0^{\varepsilon}U^{\varepsilon} + \lambda_1^{\varepsilon}(M^{\varepsilon} - U^{\varepsilon})L(p|\varepsilon)}{N^{\varepsilon}}}^{\text{hiring}} \overbrace{\frac{1}{\delta^{\varepsilon} + \lambda_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon)}}^{\text{ex-post expected duration}} = \frac{1 + \kappa_1^{\varepsilon}}{[1 + \kappa_1^{\varepsilon}\bar{\Gamma}(p|\varepsilon)]^2}\frac{M^{\varepsilon} - U^{\varepsilon}}{N^{\varepsilon}}. \tag{2.42}$$

Note that the flow of workers between employers is constrained efficient.

Wage choice

The SA firms's optimal wage choice equates the value of employment in the SA firm at the optimal wage with the value of employment at the best-to-date outside option at that competitors optimal wage choice. The optimal wage offer equates the value of employment in the SA firm with the value of employment in the WP firm at the optimal posted wage.

$$V^P(w_{PA}(q,p,\varepsilon),p,\varepsilon) = V^P(w_{PP}(q,\varepsilon),q,\varepsilon) \tag{2.43}$$

The value of employment at a SA firm at some wage $w_{PA}(q,p)$ is consistent with best-to-date outside offer originating from a q-productivity WP competitor is:

$$\begin{aligned}
\mu^\varepsilon V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon) = w_{PA}(q,p,\varepsilon) \\
&+ \underbrace{\lambda_1^\varepsilon[\Gamma(\check{p}|\varepsilon) - \Gamma(q|\varepsilon))][\mathbb{E}[V^P(w_{PP}(x,\varepsilon),x,\varepsilon)|q < x < \check{p}] - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{on-the-job wage gain due to a credible threat from a WP competitor}} \\
&+ \underbrace{\lambda_1^\varepsilon[\Gamma(p|\varepsilon) - \Gamma(\check{p}|\varepsilon)][\mathbb{E}[V^A(\varepsilon x,x,\varepsilon)|\check{p} < x < p] - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{on-the-job wage gain due to a credible threat from a SA competitor}} \\
&+ \underbrace{\lambda_1^\varepsilon[\bar{\Gamma}(p|\varepsilon)][V^A(\varepsilon p,p,\varepsilon) - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{job-to-job transition to a SA competitor}} \\
&+ \underbrace{\delta^\varepsilon[V^U(\varepsilon) - V^A(w_{PA}(q,p,\varepsilon),p,\varepsilon)]}_{\text{unemployment shock}},
\end{aligned} \tag{2.44}$$

where $w_{PP}(x)$ and x are the optimal competing wage offers of x-type WP and SA firms with productivity less than p, respectively.

Meanwhile, the value of employment in the employees best-to-date outside option, a

q-productivity WP firm, is:

$$\begin{aligned}\mu^{\varepsilon}V^{P}(w_{PP}(q,\varepsilon),q,\varepsilon) &= w_{PP}(q,\varepsilon) \\ &+ \underbrace{\lambda_1^{\varepsilon}[\Gamma(\check{p}|\varepsilon)-\Gamma(q|\varepsilon)][\mathbb{E}[V^{P}(w_{PP}(x,\varepsilon),x,\varepsilon)|q<x<\check{p}]-V^{P}(w_{PP}(q,\varepsilon),q,\varepsilon)]}_{\text{job-to-job transition to a WP competitor}} \\ &+ \underbrace{\lambda_1^{\varepsilon}[\bar{\Gamma}(\check{p}|\varepsilon)][\mathbb{E}[V^{A}(w_{PA}(q,x,\varepsilon),x,\varepsilon)|\check{p}<x]-V^{P}(w_{PP}(q,\varepsilon),q,\varepsilon)]}_{\text{job-to-job transition to a SA competitor}} \\ &+ \underbrace{\delta^{\varepsilon}[V^{U}(\varepsilon)-V^{P}(w_{PP}(q,\varepsilon),q,\varepsilon)]}_{\text{unemployment shock}}. \end{aligned} \tag{2.45}$$

The optimal wage choice can thus be expressed as the posted wage at the best-to-date outside offer and the difference in the option values of the two employment contracts:

$$\begin{aligned} &w_{PA}(q,p,\varepsilon) = w_{PP}(q,\varepsilon)+ \\ &\underbrace{\lambda_1\left\{\bar{\Gamma}(\check{p}^{\varepsilon}|\varepsilon)V^{P}(w_{PP}(q,\varepsilon),q,\varepsilon)-[\Gamma(p|\varepsilon)-\Gamma(\check{p}^{\varepsilon}|\varepsilon)]\mathbb{E}[V^{A}(\varepsilon x,x,\varepsilon)|\check{p}<x<p]-\bar{\Gamma}(p|\varepsilon)V^{A}(\varepsilon p,p,\varepsilon)\right\}}_{\text{difference in option values in the SA contract and best-to-date WP outside option}}. \end{aligned} \tag{2.46}$$

Contract choice

Current operating surplus for WP firms is pinned down simply as rent per worker times labor supply: $[p - w_{PP}(p)]\ell(p)$. Deriving current operating surplus for SA firms requires deriving the fraction of their employees earning each wage. Following the usual solution strategy, the mass flowing in and out of such wages must balance. Note that workers willing to accept wage $w_{PA}(q,p)$ (or $w_{AA}(q,p)$ if q is a SA firm) or less must have best-to-date outside option q or less. The mass flowing into such contracts will be $U\lambda_0\Gamma(q)$ and the mass flowing out must be $[\delta + \lambda_1\bar{\Gamma}(q)](M-U)L(q)$. This yields $\ell(w(q,p)|p) = \ell(q)$.[20] The current operating

[20]Note, these results can also be found in Postel-Vinay and Robin (2002a).

surplus (exclusive of the cost of countering) for a firm of type p offering the SA contract is

$$\pi^A(p|\varepsilon) = \int_{\underline{p}^\varepsilon}^{\check{p}^\varepsilon} (p - w_{PA}(q,p,\varepsilon))d\ell(q|\varepsilon) + \int_{\check{p}^{\varepsilon+}}^{p} (p - w_{AA}(q,p,\varepsilon))d\ell(q|\varepsilon). \tag{2.47}$$

For threshold productivity in the interior of the support of the productivity distribution, the threshold productivity firms willingness to pay for the right to SA is:

$$\begin{aligned} c^\varepsilon &= \pi^A(\check{p}^\varepsilon|\varepsilon) - \pi^P(\check{p}^\varepsilon|\varepsilon) \\ &= \int_{\underline{p}^\varepsilon}^{\check{p}^\varepsilon} [w_{PP}(\check{p}^\varepsilon,\varepsilon) - w_{PA}(q,\check{p}^\varepsilon,\varepsilon)]d\ell(q|\varepsilon) \\ &= \{\mathbb{E}[w|\check{p},P,\varepsilon] - \mathbb{E}[w|\check{p},A,\varepsilon]\}\,\ell(\check{p}|\varepsilon) \end{aligned}$$

Separating Nash equilibrium for appropriate $\{c^\varepsilon\}$ and $\{\check{p}^\varepsilon\}$

The proposed separating equilibrium is a Nash equilibrium of the labor market if each firm prefers the prescribed wage contract and wage schedule conditional on all other firms playing the assigned contract and wage schedule and labor flowing toward more productive firms. In order to prove this I must show that current operating surplus from the proposed strategies exceed current operating surplus from each firms best deviation.

Suppose WP is prescribed: A firm for which WP is prescribed must have $p < \check{p}^\varepsilon$. For the p-productivity firm, current operating surplus from playing optimal wage under the prescribed wage contract, WP, and the best deviation to SA can be written as

$$\pi^P(p|\varepsilon) = [p - w_{PP}(p,\varepsilon)]\ell(p|\varepsilon)$$

and

$$\pi^{BD}(p|\varepsilon) = \int_{\underline{p}}^{\grave{p}} [p - w_{PA}(q,p,\varepsilon)]d\ell(q|\varepsilon) - c^{\varepsilon}$$

where $\grave{p}$ is the productivity of the most productive firm which offers a posted wage less than p (e.g. the most productive firm which the p-type firm can outbid by switching to SA). Simplifying and suppressing worker skill type for notational convenience

$$\begin{aligned}
\pi^{BD}(p) =& \int_{\underline{p}}^{\grave{p}} [p - w_{PP}(\check{p}) + \underbrace{w_{PA}(q,\check{p}) - w_{PA}(q,p)}_{<0,\text{ since } \frac{dw_{PA}(q,p)}{dp}<0}]d\ell(q) - \int_{p}^{\check{p}} [\underbrace{w_{PP}(\check{p}) - w_{PA}(q,\check{p})}_{\geq 0}]d\ell(q) \\
<& [p - w_{PP}(\check{p})]\ell(\grave{p}) \\
\leq& \pi^P(p).
\end{aligned}$$

The last line follows from noting $w_{PP}(p,\varepsilon)$ was the unique profit maximizing posted wage choice for the p-type firm when hiring skill level ε workers.

In other words, the WP firm could increase its labor supply by deviating to SA. However, the firm could also increase its labor supply by the same amount by deviating to a larger posted wage. Willingness to pay for the right to SA is then strictly less than the difference between the wage bill under the deviation to SA and the deviation to a higher posted wage. This in turn is strictly less than the cost of SA. Figure 2.7 depicts wages schedules under WP and SA in the threshold firm (left) and for a less productive firm (right). The cost of SA and bound on the willingness to pay for the right to SA are represented by the shaded regions. The cost or willingness to pay are calculated as the mass in these regions weighted

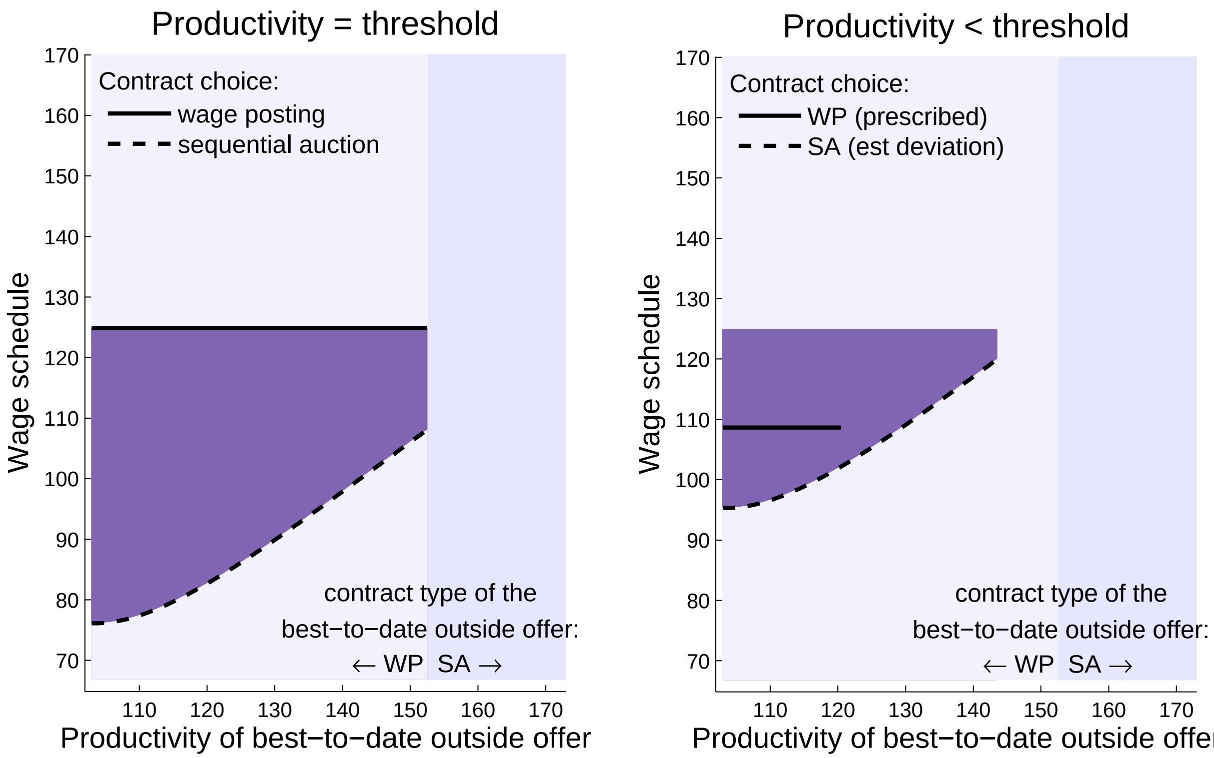

Figure 2.6: Regions over which the cost of SA and bound on the willingness to pay to SA are integrated for the threshold productivity firm and a firm of lesser productivity.

by the supply of labor to the firm with each possible best-to-date outside option.

Suppose SA is prescribed: A firm for which SA is prescribed must have $\check{p}^{\varepsilon} \leq p$. For the p-productivity firm, current operating surplus from playing the prescribed SA wage schedule and deviating to the best posted wage.

$$\pi^{A}(p|\varepsilon) = \int_{\underline{p}}^{\check{p}^{\varepsilon}} [p - w_{PA}(q, p, \varepsilon)] d\ell(q|\varepsilon) + \int_{\check{p}}^{p} [p - w_{AA}(q, p, \varepsilon)] d\ell(q|\varepsilon) - c^{\varepsilon}$$

and

$$\pi^{BD}(p) = [p - \grave{w}]\ell(\grave{w}|\varepsilon)$$

Note that $\grave{w} \geq w_{PP}(\check{p}^{\varepsilon})$ since $p \geq \check{p}$. Simplifying and suppressing skill level,

$$\begin{aligned}
\pi^{BD}(p) =& \int_{\underline{p}}^{\check{p}} [p - \grave{w}] d\ell(q) + \int_{\check{p}}^{\grave{w}} [p - \grave{w}] d\ell(q) \\
<& \int_{\underline{p}}^{\check{p}} \underbrace{[p - w_{PP}(\check{p})]}_{<(p - w_{PA}(q,p) - c),\text{ since } \frac{dw_{PA}(q,p)}{dp} < 0} d\ell(q) + \int_{\check{p}}^{\grave{w}} \underbrace{[p - \grave{w}]}_{<p - w_{AA}(q,p)} d\ell(q) \\
<& \pi^{A}(p).
\end{aligned}$$

The best deviation to WP involves a reduction in the SA firm's labor supply. I can find a bound on the minimum willingness to pay for the right to SA by considering only the labor supply which would arise under the *smallest possible* best deviation the SA firm might select: $w_{PP}(\check{p}, \varepsilon)$. Willingness to pay for the right to SA is then larger less than the difference between the wage bill under the deviation to WP and the wage bill for these employees under the prescribed SA contract. This in turn is strictly greater than the cost of SA. Figure **??** depicts wages schedules under WP and SA in the threshold firm (left) and for a more productive firm (right). The cost of SA and bound on the willingness to pay for the right to SA are represented by the shaded regions. The cost or willingness to pay are calculated as the mass in these regions weighted by the supply of labor to the firm with each possible best-to-date outside option.

Since no firm wishes to unilaterally deviate the pair $\{c^{\varepsilon}, \check{p}^{\varepsilon}\}$ form a Nash equilibrium. ■

Existence of $\check{p}$ for any c.

Again, suppressing skill level:

Since $\check{p}$ was chosen arbitrarily c is defined for any possible threshold in the support of Γ.

First consider $\Gamma(p)$ with finite support $[\underline{p}, \bar{p}]$. To show that for every cost, c, there exists

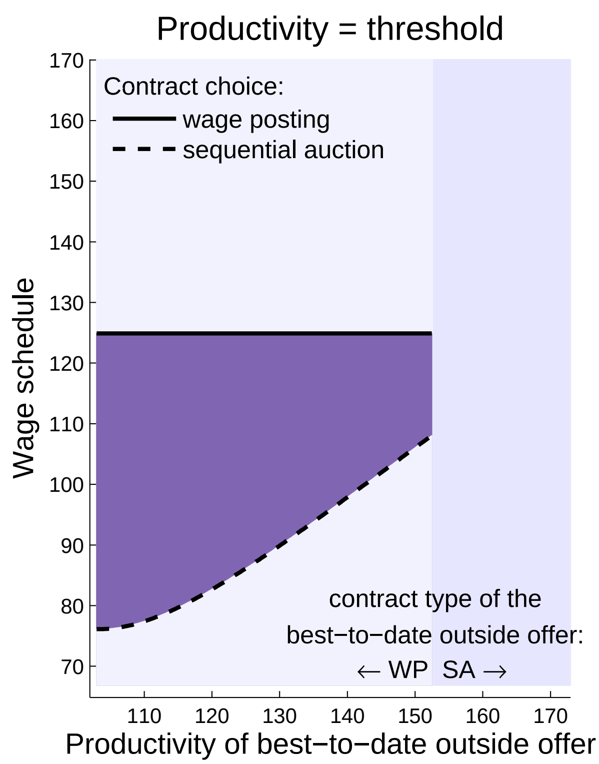

Figure 2.7: Regions over which the cost of SA and bound on the willingness to pay to SA are integrated for the threshold productivity firm and a firm of lesser productivity.

a threshold, $\check{p}$, I must first extend the definition of c to include the boundaries of the support of $\Gamma(p)$.

- $c(\underline{p}) = (-\infty, 0]$ (when SA is subsidized or free all firms select SA).

- $c(\bar{p}) \supset [\ell(\bar{p})\bar{p}, \infty)$ (if the cost of SA exceeds the output of the most productive firm, $\ell(\bar{p})\bar{p}$, then no firm selects SA).

So, c is upper hemicontinuous on support $[\underline{p}, \bar{p}]$ and continuous on support $(\underline{p}, \bar{p})$. The intermediate value theorem implies that there exists at least one threshold for every cost.

The result can be generalized to Γ with infinite upper support by considering the limit as $\bar{p} \to \infty$: for every $\bar{p}$ there exists a $c = \ell(\bar{p})\bar{p}$ such that all firms SA. ■

Uniqueness of equilibria

Equilibrium is unique if $[(w_{PP}(\check{p})-\underline{p})k_1]^{-1} \geq d\Gamma(\check{p})$ for all $\check{p}$. This condition requires that the distribution of productivity be "thin enough" everywhere in the tail that the shift $\frac{dw_{PA}(q,\check{p})}{d\check{p}}$ due to indirect upward pressure on schedules $w_{PA}(q,p)$ from the now larger WP sector is dominated by the direct downward pressure on the schedule in the marginal firm due to the now larger productivity of the marginal firm. Proof, which stems from differentiating the marginal wage schedule, is available upon request.

This guarantees that $\frac{dc}{d\check{p}}$ is increasing for all $\check{p}$ in the interior of the support of $\Gamma(p)$ and the mapping from $\check{p}$ to c is one-to-one.

Proof of Claim 7

The claim is proved if every firms wage bill weakly rises. Consider a small increase in the costs of SA. Firms are of three types: always WP, switch from SA to WP, always SA. Wage bills for always WP firms are clearly unaffected by the change in threshold productivity induced by the change in cost. This follows from noting that I am considering an identical set of active firms. Switching firms strictly increase their wage bill, this follows from noting that before the increase in costs they paid for the right to SA but after they prefer to pay larger WP wage bills in order to evade the higher cost of SA. The third category of firms requires heavier lifting.

First note that the wages of workers with best-to-date outside option a WP are set under schedule $w_{PA}(q,p)$ in each p-productivity SA firm and that the mass of such workers in each firm is $\ell(\check{p})$. Also note that $\frac{d^2 w_{PA}(q,p)}{dq d\check{p}} < 0$.[21] So the change in the wage bill associated with these workers when $\check{p}$ rises is at least

$$\frac{dw_{PA}(\check{p},p)}{d\check{p}} = [1 + k_1\bar{\Gamma}(\check{p})]\frac{dw_{PP}(\check{p})}{d\check{p}} + k_1 d\Gamma(\check{p})[\check{p} - w_{PP}(\check{p})] > 0.$$

[21]Proof, which follows from performing the derivatives, is available on request.

Meanwhile, each p-productivity firm can lower the wage paid to employees whose best-to-date outside option was a SA firm before the cost increase and is now a WP firm. This reduces the p-productivity firms wage bill by $d\ell(\check{p})[\check{p} - w_{PP}(\check{p})]$.

The increase in the schedule of wages for workers with best-to-date outside option a WP firm dominates. This is made explicit by noting that $\frac{dw_{PP}(\check{p})}{d\check{p}} = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\Gamma(\check{p})}[\check{p} - w_{PP}(\check{p})]$ and $d\ell(\check{p}) = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\bar{\Gamma}(\check{p})}\ell(\check{p})$.

Since the wage bill for every productivity firm weakly rises the total wage bill rises. ■

Proof of Claims 4-10 when $\lambda_0 > \lambda_1$

The mass of firms posting acceptable job offers is $N^\varepsilon = \bar{\Gamma}(\underline{p}^\varepsilon)N^1$. The result is that the arrival rate of *acceptable* job offers are diminished proportionally to the fraction of firms which are priced out of the market: $\lambda_0^\varepsilon = \bar{\Gamma}(\underline{p}^\varepsilon)\lambda_0^1$ and $\lambda_1^\varepsilon = \bar{\Gamma}(\underline{p}^\varepsilon)\lambda_1^1$ off- and on-the-job respectively.[22]

Decrease in the arrival of acceptable offers yields the result that unemployment rates are higher in low skilled sub-markets: mass balance implies that

$$u_\varepsilon = \frac{1}{1 + k_0\bar{\Gamma}(\underline{p}^\varepsilon)} = u_1 \frac{1 + k_0}{1 + k_0\bar{\Gamma}(\underline{p}^\varepsilon)} > u_1. \tag{2.48}$$

This gives the third result. Further, the distribution of employment across productivity in markets with $\varepsilon < 1$ is:

$$L(p|\varepsilon) = \frac{\Gamma(p) - \Gamma(\underline{p}^\varepsilon)}{\bar{\Gamma}(\underline{p}^\varepsilon)[1 + k_1\bar{\Gamma}(p)]} < L(p|1). \tag{2.49}$$

Equation 2.49 gives a somewhat non-intuitive result which must be discussed. Low skilled workers spend additional time in unemployment because value of search is relatively higher. This results in low skilled workers obtaining starting jobs which are on average higher up the

[22]NOTE: these are strong assumptions on the matching function. Deviations from these are left to future work.

job ladder. Producing a first order stochastically dominant distribution of employment as described by equation 2.49. However, due to additional unemployment the mass of low skilled workers employed in firms with productivity less than or equal to p exceeds the analogous mass for high-skilled workers: $(1-u^{\varepsilon})L(p|\varepsilon) > (1-u^{1})L(p|1)$.

And finally the mass of workers employed in a firm of type p:

$$\begin{aligned}
\ell(p|\varepsilon) &= \frac{1+k_1\bar{\Gamma}(\underline{p}^{\varepsilon})}{[1+k_1\bar{\Gamma}(p)]^2}\frac{M-U^{\varepsilon}}{N^{\varepsilon}} \\
&= \frac{1+k_1}{[1+k_1\bar{\Gamma}(p)]^2}\frac{M-U}{N}\left[\left(\frac{1+k_0}{1+k_1}\right)\left(\frac{1+k_1\bar{\Gamma}(\underline{p}^{\varepsilon})}{1+k_0\bar{\Gamma}(\underline{p}^{\varepsilon})}\right)\right] > \ell(p|1).
\end{aligned} \tag{2.50}$$

These give the forth result. Intuitively this arises from the larger pool of unemployed workers which each firm can hire from in the low-skilled sub-market.

Somewhat surprisingly, these differences in steady state distributions have a relatively benign impact on firm's wage choices. This can be seen clearly by applying the notion of acceptable job offer to equations 2.29, 2.32, 2.24, and 2.36. First note that, whenever the distribution of productivity occurs so does the hazard of a job offer. The result is that the impact of some firms being priced out of the market is offset. Furniture, shifts in labor supply do not depend on firm productivity (for active firms) resulting in a similar wage choice for posting firms:

$$\underline{w}_{UP}(\varepsilon) = \varepsilon b + (k_0 - k_1)\int_{\underline{w}_{UP}(\varepsilon)}^{\check{p}^{\varepsilon}} \frac{\Gamma(\check{p}^{\varepsilon}) - \Gamma(x)}{1+k_1[\Gamma(\check{p}^{\varepsilon}) - \Gamma(x)]} dw_{PP}(x,\varepsilon)$$

$$w_{PP}(p,\varepsilon) = \varepsilon\left\{p - [1+k_1\bar{\Gamma}(p)]^2 \int_{w_{UP}(\varepsilon)}^{p} [1+k_1\bar{\Gamma}(x)]^{-2}dx\right\}$$

$$
\begin{aligned}
w_{PC}(q,p,\varepsilon) = w_{PP}(q,\varepsilon) + \lambda_1^{\varepsilon}\big\{&\bar{\Gamma}(\check{p}^{\varepsilon})V^P(w_{PP}(q,\varepsilon),\varepsilon) \\
&- [\Gamma(p) - \Gamma(\check{p}^{\varepsilon})]\mathbb{E}[V^C(\varepsilon x, x, \varepsilon)|\check{p}^{\varepsilon} < x < p] \\
&- \bar{\Gamma}(p)V^C(\varepsilon p, p, \varepsilon)\big\}.
\end{aligned}
$$

and

$$
w_{CC} = \varepsilon\left\{q - k_1\int_q^p \bar{\Gamma}(x)dx\right\}
$$

The new result is that reservation wages for employment in a wage posting firm rise since the value of unemployment rises more than the value of employment in the minimum posting firm due to the difference in offer arrival rates across states. In addition, the effect "trickles up" the posted wage distribution (to see this inspect equation 2.32). The result overall is that differences in wage growth for different skill types are amplified compared to the case of equal job-offer hazards. Now low skilled workers earn disproportionately higher wages in all entry-level positions in WP firms and in entry positions at SA firms.

CHAPTER 3:

Empirical identification of the composition of contract types using administrative data.

3.1 Introduction

Some workers work in jobs with fixed, non-negotiable pay while others work in jobs where pay is negotiable and significant on the job wage growth is possible.[23] Theory developed in Chapters 1 and 2 of this dissertation suggests a wide range of implications of contract heterogeneity: from patterns in life-cycle wage changes, to both explained and residual inequality, to factor utilization and the share of output captured by labor. Unfortunately, existing survey evidence on wage contract heterogeneity is not well suited to evaluate of many of these predictions. Analyses using survey data are restricted to the time and population surveyed.

This paper provides a method of identifying the share of workers employed in a renegotiable contract in an on-the-job search model in which some, but not all, employers renegotiate. Identification is achieved using administrative data (only) on workers' labor market histories. Specifically, I consider a model in which heterogeneous employers may offer wage

[23] Hall and Krueger (2008, 2010) report results of a survey of workers asked wheatear they negotiated at the beginning of the present employment relationship. Barron et al. (2006); Brenzel et al. (2013) report results of a survey of firms which were asked if they would renegotiate the contract of the most recent employee if said employee received an alternate employment offer.

contracts of two types: wage posting (WP) and sequential auction (SA). Under WP, a firm sets a non-negotiable wage and commits to it for the duration of the employment relationship, as in Burdett and Mortensen (1998) and Bontemps et al. (2000). Under SA, pay is commensurate with the worker's best-to-date outside option and evolves as workers accrue new outside offers by a mechanism equivalent to a sequential second price auction, as in Postel-Vinay and Robin (2002a,b).

In Chapter 1, I show that when the arrival rate of job offers is exogenous and firms must pay a per-firm fee in order to set wages by means of the SA mechanism, the market is characterized by a separating equilibrium in which only high productivity firms are interested in providing the flexible contract. The intuition underlying the equilibrium is clear cut. All else held equal, employers hiring under SA operate lighter wage bills, exclusive of the cost of SA, than they do when they hire under WP. SA employers offer starting wages which are contingent on history while WP employers offer the same wage to all workers. In addition, SA employers offer workers greater option value of employment than WP employers by promising to bid up wages when outside options arise and workers are willing to trade off present term wages to secure higher option values. The more productive the job, the bigger the wedge between wage bills since the employer can offer more lucrative future wages within the employment relationship *and* at future employers. For a given cost, only sufficiently productive firms find that the difference in wage bills under the two contract types exceeds their willingness to pay for the right to employ under SA.

The model produces several results which are of interest in empirical applications. In section 3.2, I quickly recap important features of the model of equilibrium on-the-job search with contract heterogeneity laid out in Chapter 1. I provide more detail on these motivational results regarding the relation between job change and wage growth over the life-cycle and the distribution of wages and rents. In addition, I highlight features of the theoretical model that facilitate empirical identification.

In empirical analysis, I use register data from Germany. Section 3.3 describes the data and the mapping between objects in the model, namely the "firm," and appropriate empirical analogues. I discuss the pros and cons of linked employer-employee data in this context. Section 3.4 describes worker mobility and pay change and then mobility and pay change data when linked to establishment level measures of employer quality.

Section 3.5 discusses the empirical strategy, which is based exclusively on worker data. Identification is achieved from the duration dependence of job switching behavior, following Ridder and van den Berg (2003), and from the prevalence of job switching relative to on-the-job pay gain, using the Simulated Generalized Method of Moments. Section 3.6 presents the estimated share of workers employed in renegotiable contracts. In the preferred estimation, I find that 22.06 percent of German workers and 62.88 percent of German firms had a WP contract in the period 2006-2008. This is roughly comparable to the only existing survey evidence, to this author's knowledge: Brenzel et al. (2013) document that just over two thirds of establishments surveyed in 2011 report that they would renegotiate the contract of the most recently hired employee if said employee received an alternate wage offer.

Section 3.7 revisits patterns over the life cycle and in linked data, comparing patterns found in the data in section 3.4 to simulations of the fitted model. In general, simulations are more extreme than patterns found in the data. In simulations, job switching is much more prevalent early in the life-cycle or while employed in a low-ranked firm than in the data. Simulated job switching also falls off more quickly. Simulated on-the-job pay gain also exhibits more dramatic increases early in the life-cycle and when moving up firm rank than does the data. Patterns in the data may be muted for many reasons. Empirical life-cycles can not reasonably be interpreted as data drawn from a stationary, steady state. As will be discussed, the model's notion of firm may not be well approximated by the data's notion of establishment. Still, the picture revealed in the data generally matches that in the theory in sign, if not in magnitude. Section 3.8 concludes and suggests avenues of future research.

3.2 Model

I consider an equilibrium of a search market in which identical workers search for employment in heterogeneous firms both on- and off-the-job. Workers may be employed under either a wage posting (WP) or sequential auction (SA) contract. WP is typified by a fixed level of pay that is invariant over the duration of the employment relationship and across individuals employed at the same firm. In contrast, the SA contract pays each worker exactly her reservation wage given her labor market history. Reservation wages differ across workers within the same firm because workers have different best-to-date alternative offers for employment. Further, as a worker accumulates better and better outside options, pay increases in a renegotiable contract. In Chapter 1, I show that when the arrival rate of job offers is exogenous and employers must pay a firm-level fee to operate the SA contract the most productive firms select the SA contract while less productive firms all select the WP contract.

Specifically, firms are characterized by their productive technology: a p-technology firm produces p units of output per worker. Technology or firm type is distributed according to an exogenous and differentiable distribution $\Gamma(p)$. Workers search randomly with balanced matching, meaning that the probability of drawing a firm of type p is equal to the density $\Gamma(p)$. Job offers arrive according to exogenous Poisson processes governed by hazard parameters λ_0 when unemployed and λ_1 when employed. Existing jobs are destroyed according to a Poisson process with hazard δ. Workers discount the future at rate μ. This stylized environment is the same as that considered in the continuous productivity version of the Burdett and Mortensen (1998) WP model due to Bontemps et al. (2000) and in the SA model of Postel-Vinay and Robin (2002a). For any cost for the right to SA some threshold firm, $\check{p}$, is indifferent between contract types. More productive firms all SA and less productive all WP. For extreme values of the cost of the SA contract these models are nested cases.

A full derivation of the model and proof of the separating equilibrium can be found in

Chapter 1. Here I highlight two sets of results. First, results which motivate the effort to identify the composition of contract types in equilibrium. Second, results which facilitate identification.

3.2.1 Motivating results

The first result pertains to distribution of output:

Motivating Result 1. *For a fixed distribution of firm qualities,* $\Gamma(p)$*, an increase in the share of SA contracts decreases labor's share of output.*

Chapter 1 provides proof of this result. The intuition lies in noting that WP firms yield rent to their employees, while SA firms pay only enough to outbid the best-to-date outside option. This alone is not quite enough. SA firms also partially compensate employees with the promise to bid up wages in future firms. Thus a firm will, on average, pay less for its employees if it employs under a SA contract. When a greater share of firms WP in equilibrium each yields more rent to its employees. In addition, higher rents available in the WP sector drive up average wages in the SA sector.[24]

The second regards wage inequality between identical workers:

Motivating Result 2. *For a fixed distribution of firm qualities,* $\Gamma(p)$*, an increase in the share of SA contracts increases inequality.*

Consider the spread between a pair of quantiles, for example the 25th and 75th percentile of the wage distribution. With sufficiently few SA firms these are relatively close together. As the market shifts toward SA contracts, however, the quantiles shift toward more extreme values. The shift is most pronounced in the lower quantile. The result stems from similar market pressures as the previous: with fewer WP firms to compete with, SA firms can extract more rent from workers with poor labor market histories.

[24] One can prove that average wages rise in every SA firm. The proof is presented in Chapter 1.

With these two results together, one might wonder if trends in labor share and income inequality might be related via the wage contracting mechanism. Perhaps firms have shifted toward offering the SA contract in recent decades? Methods presented in this paper offer the potential for future research in this direction.

A third result relates to patterns observable from worker histories alone:

Motivating Result 3. *Workers who have been more recently unemployed are more likely to change their job than receive a pay increase. As continuous experience accumulates, the hazard of on-the-job pay gain overtakes the hazard of job-to-job transition.*

The result is a straightforward implication of the job-ladder model. Greater continuous employment experience is associated with a higher position on the job ladder. Since only the highest ranking firms offer the SA contract experience is associated with such a contract. Thus, inexperienced workers are more likely to switch employers than receive an on-the-job pay gain. As continuous experience accumulates, the odds that a given worker is employed in an SA contract rise, and thus the odds of on-the-job pay gain also rise. With sufficient experience, job-to-job mobility and on-the-job pay gain evolve as the first and second order statistic of the accumulated draws.

The pattern generated is consistent with well documented patterns in the literature on the evolution of wages over the life-cycle, for example Topel and Ward (1992). Job-to-job mobility is more prevalent at low levels of experience. At higher levels, on-the-job pay gain and job-to-job transition feature roughly equally. In the following section, I document a similar pattern for German males. The documented patterns lend credulity to the model. Further, given differentials in wages across sub-groups of the population, an interesting question of future research is what, if any, of these differences can be attributed to differentials in the types of wage contracts offered to different types of workers. Again, methods presented in this paper offer the potential to investigate these relationships.

3.2.2 Useful results for identification

Before continuing further a detailed description of the model features that facilitate identification is required. I start with job mobility and pay change. I then discuss the distribution of firms, workers, and workers' best-to-date outside across productive technologies.

The hazard of job separation is simply δ. The hazards of job-to-job mobility and on-the-job pay gain are slightly more involved. Job-to-job mobility occurs if an offer arrives from a firm more productive than the current employer, p. The hazard of this event is $\lambda_1[1-\Gamma(p)]$. Thus, after a given interval of time, t, the probability of transition to nonemployment and transition between employers:

$$\mathbb{P}(\text{sep. to nonemp.}|p,t) = \delta \exp[-(\delta + \lambda_1[1-\Gamma(p)])t] \tag{3.51}$$

and,

$$\mathbb{P}(\text{job-to-job move}|p,t) = \lambda_1[1-\Gamma(p)] \exp[-(\delta + \lambda_1[1-\Gamma(p)])t] \tag{3.52}$$

On-the-job pay gain occurs if an employee resides in a SA firm and receives an outside offer from a firm more productive than the best-to-date outside offer, q, but less productive than the incumbent, p. Thus the hazard of on-the-job pay gain is $\lambda_1[\Gamma(p)-\Gamma(q)]$ if $p \geq \check{p}$. On the other hand, if the incumbent is WP, no on-the-job pay gain is possible and the hazard is exactly zero. Given that an employment spell lasts exactly t periods the probability that at least one on-the-job pay change occurred in this period is:

$$\mathbb{P}(\text{on-the-job gain}|q,p,t) = \mathbf{1}_{p \geq \check{p}} \{1 - exp[-\lambda_1[\Gamma(p)-\Gamma(q)]t]\} \tag{3.53}$$

Equations 3.51 and 3.52, which describe the odds of separation and job-to-job mobility are fairly standard. Equation 3.53 requires discussion. Ignoring the indicator we have the probability of on-the-job wage gain as in Postel-Vinay and Robin (2002b). It is straight forward to observe that as the share of firms that employ under WP the fraction of workers who experience on-the-job pay gain in a given interval of time falls.

Figure 3.1 illustrates the share of workers who experience job-to-job mobility and on-the-job pay gain for a given pair of hazards λ_1 and δ as the share of WP firms increases from zero to one. Obviously, the fraction experiencing job-to-job transition are unaffected while the fraction with at least one on-the-job pay gain fall. Figure 3.1 also illustrates that for a given set of hazards the share of workers experiencing on-the-job pay gain (that is explicable by the model) is bounded above by the share that can be explained by a market in which all firms SA. In general, it is difficult to generate a share of on-the-job pay gains which greatly exceeds the share of job-to-job transitions.

Turning to distributions in the cross-section. The model is an equilibrium job-ladder. Following Burdett and Mortensen (1998), I can solve for the steady state distribution of labor across firm quality types via the method of mass balance. In the steady state, flows in and out of firms of type p must balance. The flow out of firms with productivity less than or equal to p is $(\delta + \lambda_1[1-\Gamma(p)])(M-U)L(p)$ where M is the mass of employed workers, U the mass of unemployed workers, $L(p)$ the share of workers employed at firms less than or equal to p, δ and λ_1 the rates of job destruction and arrival on-the-job offers. The flow into such firms is $U\lambda_0\Gamma(p)$. Noting that the steady state unemployment rate is $U/M = \delta/(\delta+\lambda_0)$ and manipulating yields the steady state distribution of workers across job quality:

$$L(p) = \frac{\Gamma(p)}{1 + k_1[1-\Gamma(p)]}, \tag{3.54}$$

where $k_1 = \lambda_1/\delta$. So, in steady state I have a fixed relation between the distribution of firms across quality, $\Gamma(p)$, and the distribution of workers across quality, $L(p)$. Thus, identifying the share of firms offering a SA contract since $s_F = \Gamma(\check{p})$ is equivalent to identifying the share of workers employed in a SA contract $s_w = L(\check{p}) = \Gamma(\check{p})/(1 + k_1[1-\Gamma(\check{p})])$ whenever

k_1 is correctly identified. Note also that the relation between $\Gamma(p)$ and $L(p)$ is independent of the rate of job offer arrival in unemployment, λ_0; the size of the labor force, M; and the unemployment rate, U/M.

Finding the distribution of best-to-date outside offers is more involved. First I need the mass of workers employed in a firm of type p:

$$\ell(p) = \overbrace{\frac{\lambda_0 U + \lambda_1 (M-U) L(p)}{N}}^{\text{expected hiring}} \overbrace{\frac{1}{\delta + \lambda_1 \bar{\Gamma}(p)}}^{\text{expected duration}} = \frac{1+\kappa_1}{[1+\kappa_1 \bar{\Gamma}(p)]^2} \frac{M-U}{N}. \tag{3.55}$$

Similar logic reveals that the mass of workers with best-to-date outside option q is $\ell(q)$. Finally, the share of workers in a p-type firm with best-to-date outside option q is the ratio of these:

$$G(q|p) = \frac{\ell(q)}{\ell(p)} = \left(\frac{1 + k_1[1-\Gamma(q)]}{1 + k_1[1-\Gamma(p)]} \right)^2. \tag{3.56}$$

3.3 Data

3.3.1 SIAB

The Sample of Integrated Labor Market Biographies (SIAB) is a two percent random sample drawn from the Integrated Employment Biographies (IEB) of the Institute for Employment Research (IAB) of the German Federal Employment Agency. IEB contains records of all individuals in Germany that have at least one of the following employment statuses: employed and subject to social security, marginal part-time employment, benefit receipt under German Social Code III or II, official registration as job-seeking at the German Federal Employment Agency, and planned participation in programs of active labor market policies. Employment status is recorded on a daily basis.

The data extends since 1975 for West-Germans and 1993 for East-Germans. The 2% random sample is a random drawn from listing of all individuals recorded in the IEB data.

This sampling facilitates the current project in two important ways. First, for each sampled individual I observe the complete labor market history since 1975 (or since 1993). Second, restricting the data to a particular subgroup, for example individuals employed in the first two weeks of 2006, yields a 2% random sample of individuals in said subgroup.

This data contains the beginning and ending date of each employment relationship, average daily wages during that relationship, basic job characteristics (occupation, industry), and basic demographics (age, sex, German/non-German nationality). Information on employment qualifications and schooling also exist, but with lower fidelity.[25] Additionally, data contain the beginning and ending date of interaction with the Employment Agency: registration as unemployed, receipt of unemployment assistance, or planned participation in an active labor market policy.

From the SIAB, I select males between the ages of 25 and 55 individuals who were fulltime employed at a job paying at least 13 Euros per day in the first two weeks of 2006.[26] This criteria yields a sample of 192,644 records. For each I can construct a complete labor market history for the time period 1975-2008 (1993-2008 if East German). In primary analysis, I focus on the period 2006 to 2008. I select a short interval in order to reasonably approximate a steady state. The window 2006-2008 post-dates the implementation of the Hartz reforms and predates the Financial Crisis.

3.3.2 BHP and IAB Establishment Panel

The Establishment History Panel (BHP) is composed of a yearly cross-section of all establishments in Germany which have at least one employee subject to social security on June 30th. The cross-sections extend back to 1975 for West-German and 1993 for East-German

[25] Qualifications can be, but are not always, reported by employers. Schooling information is primarily collected by the Employment Agency when providing services, and is thus available for a selected sample of individuals.

[26] 13 Euros per day is the minimum daily wage for which the employment relation is required to be reported to the employment agency.

establishments. The BHP contains information on establishment size, quartiles of the within-establishment wage distribution, and quartiles of the within-establishment age distribution on June 30th of each year. Additionally, outflows from the establishment since the preceding June 30th are also available.

Every employment spell recorded in the SIAB can be linked to employer characteristics if the employment spell contains a June 30th. In analysis of linked employer-employee data, I link employment spells to the characteristics of the employing establishment with a one year lag. This results in 187,607 linked employer-employee records at initial observation, or a link for 97.39% of employees. For workers who move establishments within the observation period I am able link 75.08% to the second employing establishment as well. A missing link implies that the establishment in question employed no employees subject to social security in the preceding year. Thus, missing establishments are likely to be small and young. The link is nearly complete for initial establishments. In all analysis considering both initial and second establishment I will note the likely bias due to missing links.

In addition to administrative data, the IAB collects survey data from establishments. The IAB Establishment Panel is an annually conducted representative survey of establishments targeting topics related to labor demand. The survey has been conducted since 1993 in West Germany and since 1996 in East Germany. The survey contains data sufficient for constructing value-added per employee. Establishments in the IAB Establishment Panel can also be linked to their administrative records in the BHP; however, the SIAB and IAB Establishment Panel are not designed to overlap and are not linkable. Still, it is possible to impute characteristics observed in the IAB Establishment Panel in the SIAB via administrative characteristics that are linkable to both data.

3.3.3 What is a "firm"?

In keeping with the literature on equilibrium on-the-job search, I have thus far equated productive technology, p, and "firm"; however, the verbal equivalence is misleading. Before approaching the data a detailed discussion of the notion of and relation between "firm", "establishment", and productive technology is merited.

In the model a "firm" is a unit which possesses a particular productive technology that produces p units of output per worker (per period). The production function is constant return to scale. This stylized description facilitates derivation of this and other models of on-the-job search in equilibrium. In particular, constant returns to scale leave "firms" equally eager to hire regardless of current size. Size is pinned down by imposing steady state on the equilibrium as in equation 3.55. An implication is that "firms" possessing less productive technology are smaller, have greater outflow of employees given their size, and have workers with lower tenure. One might look for these features in data.

This notion of firm; however, is dissimilar to what is observed as an establishment or a firm in administrative data. A typical firm or establishment in administrative data will contain workers producing with many different productive technologies (for instance workers in different occupations). These are grouped together in the data as a single taxpaying unit. Further, there may be complementarities between different types of labor and there may be returns to scale, which make this unit optimal for reasons not modeled here. The distinction between firm and establishment in the data is a distinction based on the geography in which production occurs and not on the type of production which is occurring. Thus, aggregating establishments into firms or disaggregating firms into establishments does not get one any closer to the notion of "firm" described in the model.

One might wonder if the predictions about "firms" in the model pertain to actual establishments or firms in real life. For instance, are more productive firms bigger? The answers to these questions are, at best, mixed. The following section documents which of the expected correlations are present in the BHP and IAB Establishment Panel.

Given that worker data can be linked to establishment characteristics, one might consider an estimation strategy conditional firm covariates. With such a strategy one might construct the empirical *c.d.f.* of the distribution of firms across technology or quality and then use this as an estimator of $\Gamma(p)$ in equations 3.51, 3.52, and 3.53. Estimation could then be conducted via Maximum Likelihood or Generalized Method of Moments.

There are, however, several problems with this approach. First, what measures technology or firm quality? Size, rate of employee exit, age of employees, some moment of the establishment level pay distribution? Some index of these? Second, an estimator constructed from 3.51, 3.52, and 3.53 will be non-linear in firm quality. Estimates of the hazards of separation and on-the-job offer arrival will, as a result, be biased in the presence of measurement error (even classical measurement error).[27] Further, an accurate estimate of the share of WP contracts depends on an accurate estimate of these hazards. One might consider explicitly modeling errors in the measure of technology or firm quality; however, resulting estimates would depend heavily on assumptions regulating the distribution of errors. Further, given the precarious mapping between theoretical firm and empirical establishment, classical measurement errors are an unlikely and best case scenario. Third, even if a satisfactory proxy for incumbent firm quality is located, a proxy for the quality of the best-to-date outside option is still illusive.

These problems point toward methods which are not conditional on firm quality. Identification can be achieved without firm covariances by exploiting the information revealed in mobility behavior (Ridder and van den Berg, 2003). One selects the parameters to match the average rate of transition in a given interval. Essentially, this amounts to integrating out firm type from equations 3.51, 3.52, and 3.53. In this paper, I perform the integration by simulation methods and estimate the model using the Simulated Generalized Method of

[27] See Chen et al. (2011) for a survey of related results.

Moments. Details are presented in section 3.5. An additional advantage to non-conditional methods is that they impose fewer requirements on the data and thus admit a wider array of data sets.

3.4 Descriptive analysis

The primary empirical analysis of this paper is conducted using SIAB data. As I will discuss in section 3.5, these employee level data contain the baseline required features: high frequency data on employer identity and wage level. Although some establishment level characteristics are linkable, I forgo explicit use of these when estimating parameters of the model due to concern first, that the mapping between an establishment in the data and the notion of a "firm" in the model are faulty and second, due to concern about measurement error in these variables. Before moving on to structural estimation; however, I begin in this section by describing patterns observable in worker data alone and patterns that can be seen in linked employer-employee data.

3.4.1 Employee histories

As stated, for each individual I observe employment status and employer on a daily basis. I also observe the average daily wage paid during each reported period. Appropriate treatment of these objects requires discussion. In particular, what constitutes a job loss versus a job-to-job transition? What constitutes a wage gain?

For each individual, SIAB contains the employer's report of the beginning and ending date of an employment relationship. Reports are required to be (re)submitted yearly such that a yearly report covers one calendar year (January 1 to December 31). The typical report is a yearly report. Changes in employer and changes in the specifics of an existing employment relationship require submitting a new report of the employment relationship.

An employer who looses an employee mid-year submits a report of the termination of the employment relationship. An employer of a newly hired employee submits a report of the new hire. An employer who changes an existing employees contract re-submits a report of that employment relationship.

I repeat analyses on the sample divided by regions: East and West Germany and Berlin. I investigate these subgroups in order to see if there is meaningful variation in the target moments and thus in estimated parameters. As a prior, one might expect that East Germany, in which unionization is more prevalent, would feature a greater share of WP contracts. By contrast, as an urban labor market, Berlin may be expected to have more churn: higher rates of both job-to-job mobility and job loss.

Decomposition of the market requires strict assumptions: firms and workers may not match with workers or firms in any other sub-market. Just over 0.52 percent of the sample which originates in West Germany transitions to employment in one of the other regions within the three years of observation. For East Germany and Berlin the percents are 2.33 and 5.51. These regions are smaller. By comparison, just under one-forth of workers switch broad occupation categories in the same time period. Decomposition by occupation is common in the literature.

3.4.1.1 Job-to-job transition

Testing theoretical implications requires a measurement of the incidence of voluntary job-to-job mobility. Figure 3.2 records the duration of overlapping employment spells or elapsed non-employment between employment spells for the first job-to-job transition (if any) observed. The vast majority of job-to-job transitions take place within one day. However, the literature typically considers a job-to-job transition with less than or equal to 15 days of non-employment as a voluntary job-to-job mobility. 15 days allows for exactly two weeks of non-employment while 17 days allows for two weeks and three weekends. Allowing for the additional weekend is in some sense more practical: allowing the job mover to complete old job on Friday and begin the new job on Monday. However, as one can see from Figure 3.2 the difference is not substantial. The vast majority of job movers do so without any intervening days of nonemployment.

Applying the 17 day criteria, table 3.1 records percent of the population who are job stayers for the entire three years of observation, job-to-job movers, and job losers. Just under two-thirds of workers experience no transition in the three observed years. Under the 17 day criteria, job changing and job losing are almost equally likely. Table 3.1 also records the mean duration of the initial employment spell and subsequent unemployment spell (if any) in years.

Dividing the sample into West and East Germany and Berlin reveals that East Germans are more likely to lose a job in these years and that Berliners are both more likely to lose a job and to change jobs than the national average. These observations are consistent with the West Germany economy being stronger than the East German and with a greater level of churn in an urban labor market. Differences are, however, not large.

The 17 day window, although in keeping with the literature, is not a completely satisfactory definition of voluntary movement. Some job losers may quickly secure new employment. Some job changers may voluntarily opt for prolonged non-employment between jobs, taking an opportunity for vacation. As noted, SIAB data contains data on direct interaction between workers and the Employment Agency as well as employment reports submitted on workers behalf by the employer. Table 3.2 records the fraction of job stayers, job changers, and job losers who receive unemployment benefits, register as unemployed, or register for services of the Employment Agency any time in the 17 days before or after the final day of employment with the initially observed employer. Dividing job leavers into job-to-job changers and job losers using the 17 day criterion, 9.86 percent of job changers and 50.18 of job losers register as unemployed.

Importantly, in Germany employers are required to give advance notification of job separation and workers are required to register upcoming job separations with the Employment Agency. Failure to register an upcoming separation results in penalties should the worker subsequently seek unemployment assistance. Thus, registration is likely to be fairly complete. Antidotally, even workers for whom contracts are renewed on a yearly basis are expected to and often do register upcoming cessation of the current years contract. Thus the 9.86 percent of workers who move between employers rapidly but also register as unemployed may overstate the share who should be considered job losers.

In an effort to distinguish, table 3.3 records the distribution of log wage changes following job-to-job mobility in 17 days or less and in 18 days or greater. Wages are adjusted to 2005 Euros via a cubic spline run through the yearly deflator provided by the IAB to create a daily deflator. Failure to adjust to real wages will make job-to-job transitions appear more lucrative if they follow longer employment or non-employment spells.

Job movers with 17 days or fewer non-employment are further divided into those who register as unemployed and those who do not. Median real log wage changes following rapid job-to-job transition are approximately zero while protracted non-employment results in a median log wage loss of 1.97 percent. Notably, the distributions primarily differ in the weight of the left tail: workers with protracted unemployment are more likely to experience wage loss but are almost as likely to experience wage gain in excess of 5 or 10 percent. Dividing rapid job movers into those who register as unemployed and those who do not reveals that the distribution of real log wage changes for those who register unemployment is quite similar to that of workers who experience protracted unemployment. This suggests a stricter definition of voluntary mobility than is typically applied. In this paper, I estimate the model under both criteria.

3.4.1.2 On-the-job pay change

Testing theoretical implications also requires a measurement of the incidence of on-the-job pay gain. Recall that employers are required to submit a report for all employees on a yearly basis. As a result, nearly every employment relationship which continues for more than a year includes an on-the-job pay change. A great number of these "pay changes", however, may be spurious features of the data. Consider that even if the real wage is unchanged, differences in the number of days worked from one year to the next will produce small differences in the average daily wage recorded in the data. Consider also, that even if an employer intends to maintain the same real wage by implementing a yearly wage adjustment, the employer is not able to precisely measure inflation when implementing this wage adjustment. Importantly for the present research, mid-year changes in an employment contract require re-reporting the employment relationship.

Figure 3.3 illustrates the fraction of contracts which are re-registered with a pay change each month. The solid portion of each bar represents changes made on the first of the month while the hollow portion represents changes made on all other days of the month. New Years pay changes are by far the most prevalent. The next most prevalent date of pay change is on the first day of a quarter. Are all these pay changes equivalent?

Table 3.6 presents the median and distribution of real log wage changes that occur on New Years, the first of the quarter, first of the month, a Monday, and on other days. New Years pay changes are notably more clustered about zero than the rest with a median real pay loss of 0.75 percent.[28] Mid year pay changes are significantly more positively skewed. Just under a quarter of mid-year pay changes result in a pay gain of at least 10 percent. By comparison under 8 percent of year-end pay changes are as lucrative. Pay changes occurring at the quarter, month, or week are associated with modest median real pay gains of 1.19,

[28]Note: workers may appear in the table twice. If a worker has both a new years and a non-new years pay change then both are recorded. The remaining categories are mutably exclusive.

2.07, and 0.53 percent respectively. These correspond to likely breaks between pay periods. Pay changes occurring mid week and not on a monthly or quarterly schedule are much more negatively skewed resulting in a median pay loss of 1.94 percent.

Pay changes in table 3.6 are computed as the difference between initial pay level and the second observed pay level. Both pay levels are deflated to 2005 Euros on the day the pay level is first observed. This start-to-start definition of real pay change is non-trivial: note that a nominal pay gain may be a real pay loss if the time elapsed before the pay change is substantial. Implicitly, utilizing this start-to-start definition imposes that the value to the employee of the initial pay level includes the value of an anticipated New Years pay change intended to keep pace with inflation. Clustering of New Years pay changes around zero supports this assumption.

Alternately, if workers do not anticipate yearly inflation-indexed pay gain then the relevant pay change to consider is rather end-to-start which is equivalent to nominal pay change. Table 3.7 repeats table 3.6 for nominal wage changes. Distributions are more difficult to interpret due to differing average durations in the categories of date of pay change.

Tables 3.6 and 3.7 record the share of the sample which receive a real and nominal (respectively) pay gain at some point in the three years of observation and the duration elapsed before this pay gain. By construction, nominal pay gain is more likely and duration until nominal pay gain is shorter since every real pay gain will also be a nominal pay gain. As with mobility behavior, differences across regions are not substantial.

3.4.1.3 Life Cycle

A first test of the hypothesis that only highly ranked firms offer on-the-job pay gain is to match transition and pay gains observed in the 2006-2008 window to each employee's history of non-employment prior to 2006. For each worker I record the year in which he or she was most recently not employed. Figure 3.4 plots the relation between job-to-job mobility (defined as transition within the 17 day window) and on-the-job pay gain (defined as real pay gain) and the time since the last nonemployment spell. As predicted by a basic job-ladder model, job-to-job mobility declines with time since the last non-employment spell. Also, as predicted by the model considered here with heterogeneous contracts, the rate of job switching exceeds that of on-the-job pay gain at low levels of continuous experience (short duration since most recent non-employment). As time since the last non-employment spell increases the rate of on-the-job pay gain catches up to that of job-to-job mobility.

3.4.2 Establishment characteristics

Although the mapping between the model's "firm" and establishments observed in the BHP and IAB Establishment is likely imperfect, characteristics of real life establishments may capture a noisy measure of the quality of the modeled firm, p.

Taken most literally, p is measured as value added per employee at the establishment level. The BHP Establishment Panel asks establishments - excluding banks and financial institutions, insurance companies, and the public sector - to report total sales in the previous year, value of intermediate inputs used in the previous year, total wages paid in the previous month, and current and one year lagged number of employees. From these I can construct:

$$\begin{aligned} \text{Value added} &= \text{Sales} - \text{Intermediate inputs} \\ \text{Value added per employee} &= \text{Value added/Average employees} \\ \text{Labor share} &= \frac{12 * \text{Wages in past month} * \frac{\text{Average employees}}{\text{Current employees}}}{\text{Value added}}. \end{aligned}$$

Table 3.8 presents value added per employee and labor share for Germany surveyed firms in as a whole and by region. Labor share figures are large relative to aggregate figures reported in the EU KLEMS database. This cannot be accounted for by the omission of banks and financial institutions, insurance companies, and the public sector since these sectors have larger than average labor share in national accounts (OMahony and Timmer, 2009). Western establishments produce more per employee but retain a similar share of output. Establishments within Berlin retain a larger share of output. Excluding the smallest establishments reveals that these yield the least rents and are generally less productive per employee. That these firms are less productive is consistent with a job-ladder theory; however, the same theory would suggest that the smallest firms yield the largest rent. This suggests against utilizing firm size as a proxy for firm firm quality. Table 3.9 reports the correlation between constructed survey measures of value added and labor share. It is interesting to note that more productive firms retain a larger share of output. This is consistent the typical job-ladder models which feature firm heterogeneity.

If the theory is taken literally, value added per person is exactly the measure of firm quality intended to be captured in p. Unfortunately, survey data used to construct value added per employee is not linkable to employee histories. Still, I am able to impute value added per employee using administrative measures that can be linked to both survey and worker history data: size, employee age distribution, percentiles of the within-establishment wage distribution, and rate of employee exit. Theory suggests that larger establishments, establishments with older workers, and lower rates of employee exit are better employers. Regardless of the wage formation mechanism, theory predicts that establishments with high wages in the high within-establishment percentiles are better employees. Table 3.10 presents the correlation between surveyed and administrative characteristics in liked data. Notably the sign of correlation is as predicted by a job-ladder model which features a substantial portion wage posting (even the low percentiles of the within-establishment wage distribution are positively related to value added per employee).

Using linked survey and administrative data for establishments, I can construct a linear imputation of value added. Table 3.11 reports regression coefficients of the following model in 2005, 2006, and 2007:

$$
\begin{aligned}
log(\text{value added per employee}) = \beta_0 &+ \beta_1(\text{establishment size}) \\
&+ \beta_2(\text{establishment size})^2 \\
&+ \beta_3(\text{fraction of employees who exit}) \\
&+ \beta_4 log(75^{th} \text{ percentile of wages}) \\
&+ \epsilon. \qquad (3.57)
\end{aligned}
$$

Again, consistent with a job-ladder model, coefficients are positive on age and the 75^{th} percentile of wages and negative on the fraction of employees who exit the firm. Value added and the log 75^{th} percentile of wages have the strongest, most statistically significant, and most time invariant relationship with a one percentage point increase in the 75^{th} percentile of wages predicting a one percentage point increase in value added in the sample of firms larger than five employees. The relation between employee exit and value added is also strong, although somewhat less stable across years. Note, employee age does not substitutively alter point estimates of the other coefficients and has statistically insignificant coefficient itself when included.

Although value added per employee is the most literal interpretation of p imputation via firm-level administrative variables may not be the best measure of firm quality. This method relies heavily on correct measurement of value added at the establishment level and, as noted, this may not be the relevant unit of analysis. One would prefer to know value added on the job or match level. An alternative strategy would be to capture the greatest share of covariation between administrative characteristics which are predicted to have monotone relationships with the rank of a firm from an employee's perspective. Again, candidate characteristics are size, employee age distribution, percentiles of the within-establishment wage distribution, and rate of employee exit.

Principal component analysis identifies the set of orthogonal vectors that describe the correlation in a set of observations of correlated variables. The number of components is less than or equal to the number of variables with each component describing a successively smaller fraction of the remaining correlation. The first principal component, thus, contains the largest possible correlation of the variables explicable by a single vector. Theory suggests that establishment size, highest wages, and employee age are all unambiguously positively related to quality. Meanwhile the rate of employee exit is unambiguously negatively related. The principal component of these variables, therefore, suggests itself as a measure of firm quality.

Using principal component analysis, I construct a measure of firm quality as the greatest shared correlation between the 75^{th} percentile of wages, establishment size, age of co-workers, and turnover. Table 3.12 presents the correlation of these measures in the population of German establishments in 2005, 2006 and 2007. Consistent with a job ladder model high rates of employee exit is negatively correlated with the other measures of establishment quality; however, correlation is not strong between any of the measures.

Table 3.13 presents the factor loadings and percent of correlation explained by the first principal component in each year from 2005 to 2007. The first principal component explains just over 30 percent of the correlation among all establishments and just over a third of correlation among establishments with at least five employees. The 75^{th} percentile of wages and the portion of employees exiting the firm in a given year have the highest loadings with the loading on wages dominating in the sample of establishments larger than five employees. Again, consistent with a job-ladder model, the loading on turnover is negative while the loading on the remaining establishment characteristics is positive. Loadings on 75^{th} percentile of wages and the portion of employees exiting the firm compare with large coefficients on these characteristics in the value added per employee regression. Thus, the first factor might be interpreted as an additional measure of firm quality, which is still related to value added per employee.[29] Factor loadings and explained variation are stable over time.

3.4.3 Linked employer-employee data

I now link these measures of firm quality to employee transition behavior. Theory predicts that employees of establishments with a low ranking in the distribution of imputed value added or firm quality as measured by the first principle component should not experience on-the-job pay gain and should be likely to transition between jobs. Theory also predicts that job-to-job movers should move to higher ranked establishments.

Figures 3.5 and 3.6 plot the share of workers who experienced job-to-job mobility (defined as transition with the 17 day window) and on-the-job pay gain (defined as real pay gain) by decile of imputed value added and of the first principal component. The picture is broadly consistent with theoretical predictions. Employees of lower ranked firms are more likely to move job-to-job than employees of higher ranked firms. The reverse is true of on-the-job pay change.

Table 3.16 presents the fraction of job-to-job movers (defined as transition within the 17 day window) and of job to protracted nonemployment to job movers who move to more highly ranked firms. 54 and 58 percent of job-to-job movers climb a job ladder defined by these measures as compared to less than 50 percent of movers who pass through nonemployment. The difference is statistically significant, but is not stark. Figure 3.7 and table 3.17 present the corresponding pictures for each of the raw establishment level characteristics: size, employee age, rate of employee exit, and 75^{th} percentile of wages.

[29] Table 3.14 presents the second principle component. The second component loads heavily on establishment size. This likely picks up returns to scale in some technologies which are abstracted from in the theoretical model. Table 3.15 presents the third principle component. This component does not have a readily available interpretation.

3.5 Estimation strategy

3.5.1 Identifying Assumption

For the reasons discussed in section 3.3, I deliberately avoid use of proxies for establishment quality in estimation of parameters. The estimation strategy employed relies only on transition and duration data. This is recorded with high fidelity in the German register. Identification requires one assumption:

Assumption 1. *There exists some ranking of establishments such that if $p' > p$ then a worker given the opportunity to work for a firm with ranking p or p' always chooses to work for the firm with ranking p'.*

In addition to being robust to mismeasurement of firm quality due to measurement error or miss-mapping between "establishment" in the data and "firm" in the theory, the methods used here are robust to the functional form of the distribution of firm quality so long as it is without mass points. This is because all estimates are constructed simply from order statistics and the steady-state distribution of workers across employer rank and rank of workers' best-to-date outside-option conditional on employer rank.

This method provides a more hands-off approach to the issue of measuring firm quality; however, it is still reasonably restrictive. Identification requires that some ranking of firm quality exists, is agreed upon by all observed workers, and that workers climb the job-ladder described by this ranking at every available opportunity. These assumptions preclude explicit wage-tenure contracts and heterogenous preferences.[30]

[30] Wage-tenure contracts cause workers to decrease the set of acceptable alternate employers as tenure increases so that some high-tenure workers will reject offers from more highly ranked employers than their encumbers employer. If wage-tenure contracts or mechanisms which induced similar behavior are prevalent in the data, estimates of the on-the-job offer arrival rate obtained using the methods of this paper are attenuated. For examples of models with wage-tenure contracts see Stevens (2004), Burdett and Coles (2010), and others.

3.5.2 Moments

Estimation targets means of the following variables:

d_{ei}	*duration of the initial employment spell*
d_{pi}	*duration of the initial pay level*
$jcen_i$	*indicator for first employment spell censored*
jtj_i	*indicator for job-to-job transition ends first employment spell*
jn_i	*indicator for non-employment ends first employment spell*
r_i	*indicator for at least one positive wage change in the initial employment spell*

Constructing these requires a definition of what constitutes a job-to-job transition and what constitutes a pay-gain. For the primary analysis, I define a job-to-job transition as any change of employer with 17 days or fewer of intervening nonemployment. An on-the-job pay gain is defined as a pay change which results in a real valued increase in daily pay relative to the start of observation. Empirical moments are recorded in table 3.18. Alternate definitions of job-to-job transitions and on-the-job pay gain are considered as robustness checks.

3.5.3 Estimation procedure

I estimate the model by minimizing the sum of squared errors between simulated and empirical moments. Simulations contain a cross-section of 10,000 simulated employment histories. Workers' employer type is drawn from the theoretical steady state distribution of workers across ranks of employer quality given some parameter choice for λ_1 and δ, equation 3.54. Workers' best-to-date outside option is drawn from the theoretical steady state distribution of outside offers given incumbent employer type, equation 3.56.

Three years (1095 days) of employment history is then simulated for each worker. Each day, each worker may receive an new job opportunity, or may become separated from her employer. These events occur with Poisson probabilities λ_1 and δ. These are to be estimated.[31]

If the worker receives a new job opportunity it may come from a more productive firm than the incumbent - with $[1-\Gamma(p)]$ probability - or a more productive firm than the best-to-date outside option - with $[1-\Gamma(q)]$ probability. Note again, that the functional form of Γ is irrelevant so long as it is a differentiable *c.d.f.*. In the first case, I record a job-to-job transition. If the second case but not the first holds - if the new draw quality is between q and p - the worker may experience an on-the-job raise; however, the raise occurs only if incumbent employer quality is above some threshold, $\check{p}$. Note that cardinal value of the threshold, $\check{p}$, is not substantive. $s_F \in [0,1]$ is the share of employers with quality greater than this threshold. $s_w = \delta s_F/(\delta + \lambda_1[1-s_F])$ is the parameter of interest, which is to be estimated.

The average time to job and pay change and shares of employees experiencing job change, job loss and pay change are means of rates 3.52 and 3.53 appropriately weighted by densities 3.54 and 3.56. These integrals are tedious. In estimation, I simulate these means and match simulated moments to empirical moments. In particular, simulation allows me to integrate out the impact of firm quality on transition and pay change, matching aggregate incidence of these as opposed to employee level incidence. From the three years of simulated data, I compute the fraction which experience no employment change, job-to-job transition, job-to-nonemployment transition, and on-the-job pay gain in the initial employer. Note that the first three of these sum to one by construction. I also record simulated initial employment and initial pay level durations. These are the analogues to the empirical moments recorded in table 3.18.[32]

[31] At daily frequency and plausible parameter values it is extremely unlikely to simulate more than one shock per day. Daily is, however, only an approximation to the continuous time process in which the model is formulated.

[32] Simulations are conducted in Matlab. All code is available upon request. Contact by email: donigerc@umich.edu

I estimate an optimally weighted, over-identified model, minimizing the sum of squared errors between simulated moments and empirical moments:

$$\hat{\theta} = argmin_{\theta} \left\{ m(\theta)' W m(\theta)) \right\}, \tag{3.58}$$

where $m(\theta) = \frac{1}{n} \sum_i \left[S(\theta) - d(x_i) \right]$ is the vector of moment conditions. $S(\theta)$ is the vector of simulated moments conditional on θ and $d(x_i)$ is the vector of data for each observation. W is the optimal weight matrix.

Estimation is conducted in two steps. First, I minimize the sum of squared errors with W as the identity matrix and estimate the optimal weight matrix as:

$$\hat{W} = \hat{\Omega}^{-1} = \left[\frac{1}{n} \sum_i (S(\theta) - d(x_i))'(S(\theta) - d(x_i) \right]^{-1}. \tag{3.59}$$

I then repeat the minimization using the estimated weighting matrix.

Standard errors are computed by means of numerical derivatives. The variance covariance matrix is:

$$N\mathbb{V}(\hat{\theta}) = (D'\Omega D)^{-1}(D'W\Omega W D)(D'\Omega D)^{-1} \tag{3.60}$$

where $D = \frac{dm(\cdot)}{d\theta}|_{\theta=\theta_0}$. Numerical derivatives are computed from a 1 percent deviation around the point estimate of the parameter value. Since the optimal weight matrix is such that $W = \Omega^{-1}$ equation 3.61 simplifies to:

$$N\mathbb{V}(\hat{\theta}) = (D'\Omega D)^{-1}. \tag{3.61}$$

Empirical moments and covariances are computed in STATA. Simulation and estimation are conducted in MATLAB. Optimal values are found using `fminsearch`.[33]

[33] All `.do` an `.m` files are available upon request. Contact by email: donigerc@umich.edu

3.6 Results

Table 3.19 reports the estimated share of workers, s_W, employed in and share of establishments, s_F, employing under wage posting contracts. I estimate that 22.06% of employees work in a WP contract. Table 3.19 also reports the estimated hazards of on-the-job offer arrival, λ_1, and job separation, δ. Given these an estimated 62.88 percent of firms utilize the WP contract. This estimate is consistent with the only survey evidence on contract renegotiation available from Germany: Brenzel et al. (2013) find that two-thirds of German establishments surveyed in 2011 report that they would not renegotiate the contract of the most recently hired employee in the face of an outside offer.

Table 3.19 also reports estimates in each of the three regions: West and East Germany and Berlin. Not surprisingly, on-the-job offer arrival is higher in West Germany and Berlin. Meanwhile job destruction is more likely in the East and Berlin. Berliners are most likely to work in a WP contract followed by East Germans and then West Germans. It is somewhat surprising to see the rigid wage be more prevalent in the urban market. An explanation is that high rates of job destruction negate some of the option value of the renegotiable contract, making it less attractive to both workers and employers.[34]

3.6.1 Robustness checks

Are results robust to definitions of job-to-job transition and on-the-job pay gain? Table 3.3 suggests that rapid job changers who register as unemployed with the Employment Agency are more like job movers who experience protracted unemployment than they are like other rapid job movers. These workers experience pay loss as large and as disperse in the left tail as workers for whom job finding takes substantially longer. Redefining a job-to-job movement as job finding within 17 days and without registering as unemployed results just under 10 percent fewer recorded job-to-job transitions. Table 3.20 records the new set of

[34]Note: shares are not constrained to have weighted average equal to the full population estimate.

target moments.

Since there are now proportionately more raises as compared to job-to-job transitions, one should expect that the estimated share of wage posting contracts will be smaller. Additionally, one would expect the hazard of on-the-job offer arrival to be smaller and the hazard of job separation to be larger. Table 3.21 reports the value of λ_1, δ, and s_W implied by the tighter definition of job-to-job mobility. As expected, the hazard of on-the-job offer arrival is lower and of separation is higher than under the original definition of job-to-job mobility. A smaller, but still sizeable, fraction of firms are estimated to offer a nonnegotiable contract. Again, this is not surprising as an equal large empirical share of raise receivers must be accounted for despite fewer on-the-job offers. The overall patterns between regions of Germany remain similar however.

One might also question defining an on-the-job pay gain as a nominal gain relative to the start of observation. An alternate definition might be nominal pay change or instantaneous real pay change. Table 3.22 records a set of target moments when on-the-job pay gain is defined in nominal rather than real terms. The new definition implies a larger number of on-the-job pay changes are raises. The increase in the share of workers earning a pay gain is substantial. Under the new definition of pay gain the point estimates suggest one hundred percent renegotiable pay contracts for the full population and East and West Germany. Only for Berlin do I obtain an interior solution. Due to proximity to the corner solution and to the potential corner solution, I do not report standard errors.

3.7 Simulations

As a test of external validity of these estimates I simulate year employment of up to 12 years and compare these to their empirical counterpart plotted in Figure 3.8. Figure 3.8 plots simulated and empirical odds of pay-gain and job-to-job transition conditional on year of most recent nonemployment. Relative to data the simulated series are more extreme. Simulated job-to-job mobility is initially higher and falls off more quickly than the empirical series while simulated on-the-job pay gain starts of lower and overtakes the empirical series. Although magnitudes are too extreme, the overall trends are similar. The model and simulation abstracts from all other sources mobility and pay gain. These include, but are not limited to, business cycle fluctuations which occurred in the period 1994-2006; duration dependence in job mobility and pay gain steaming from un-modeled features such as human capital accumulation;

Similarly, figure 3.9 plots simulated and empirical odds of on-the-job pay-gain and job-to-job transition conditional on decile of firm rank. The left two panels rank establishments of workers in the SIAB on imputed value added and on the first principal component as in figures 3.5 and 3.6. The right panel produces the analogous figure from simulated data. As with life-cycle predictions, simulation is more extreme than data with respect to firm rank. Job-to-job transitions out of firms with the lowest ranking empirically are not as common as predicted by the simulation. Meanwhile, unlike simulation some workers make job-to-job transitions out of even the most highly ranked firms. Also, contrary to simulation some employees firms with a low ranking empirically still experience on-the-job pay gain. Still, empirical patterns and simulation agree in sign, if not magnitude. Transition is less likely if an employee is in a highly ranked firm and on-the-job pay gain is more likely. Discrepancies between simulation and data could stem from mismeasurement of firm quality and/or from unmodeled features such as complementarities between workers or returns to scale.

3.8 Conclusion

The model developed in this dissertation produces several results which are of interest in empirical applications. On a micro level, as workers climb the job lader they become increasingly likely to reside in a labor contract that is renegotiable and features on-the-job pay gain. This produces patterns similar to those described in Topel and Ward (1992): early in careers job switching is the most important source of wage gain, but later on-the-job wage gain features more prominently. Chapter 2 shows how differences in these patterns across subgroups in the labor market may be partially attributable to differences in contract types. More productive subgroups experience lower unemployment and greater wage dispersion and return to experience even if the pool of employers is the same. The result arises from renegotiating being proportionately less costly for high-skilled labor and thus a greater proportion of contracts are renegotiable for more productive labor types. Changing composition of contracts may also partially explain trends on an aggregate level. Chapter 1 shows that in markets with a greater proportion of renegotiable contracts labor captures a smaller share of output and wages are more disperse. Given the well documented differentials in wages, employment and returns to experience across subgroups in the labor market and trends in labor share and income inequality in recent decades a natural question is how much, if any, of these phenomena can be attributed to differentials and changes in the share of renegotiable of wage contracts. This paper provides a method of identifying the share of wage contracts in a market which are renegotiable, facilitating future evaluation of these relationships.

This paper provides a method of identifying the share of workers employed in a renegotiable contract in an on-the-job search model in which some, but not all, employers renegotiate. Identification is achieved using administrative data (only) on workers' labor market histories. The strategy enables historical analysis of the relation between contract composition and aggregate variables such as labor's share, inequality, and employment, which have exhibited trends in recent decades. The German register data, used in the present paper, are available since 1975 for West Germany and 1993 for East Germany. Methods presented here may also be used to estimate the share of renegotiable contracts in sub-populations and, thus, to ases if differentials may be attributable to a prevalence of one or the other contract type. Additionally, many other European countries also maintain similar, suitable register data and panel data, such as the Panel Study of Income Dynamics, may also be acceptable. Thus, methods presented here are also suitable for cross-country comparisons.

3.9 Figures

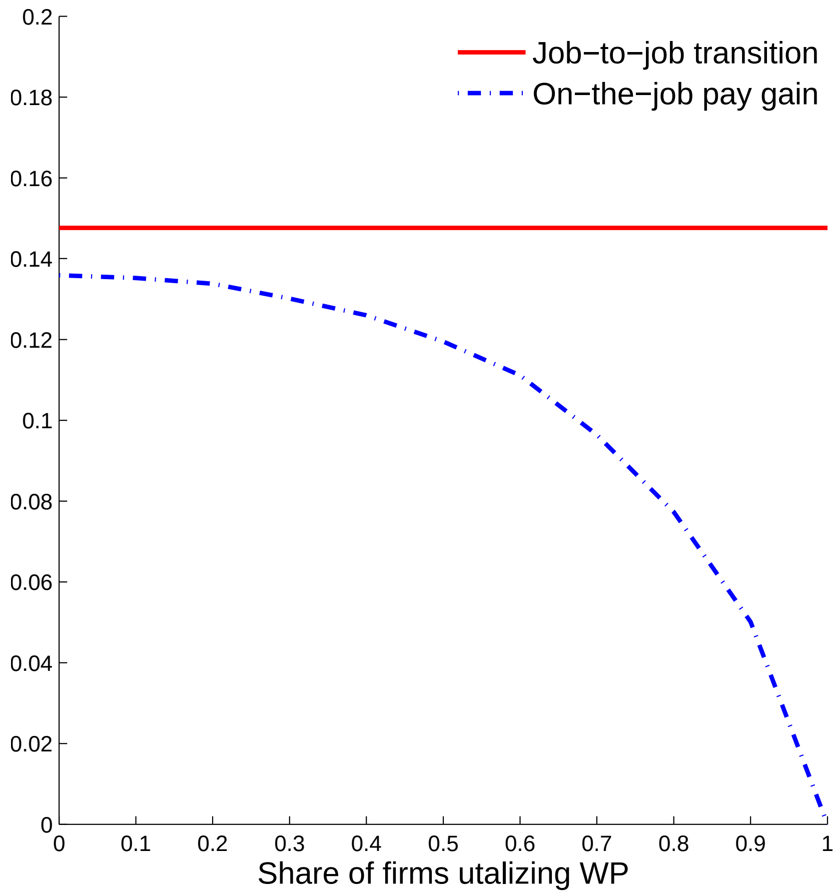

Figure 3.1: **Job-to-job transition versus on-the-job pay gain** As the share of contracts which are SA declines to zero so to does the fraction of workers who experience on-the-job pay gain.

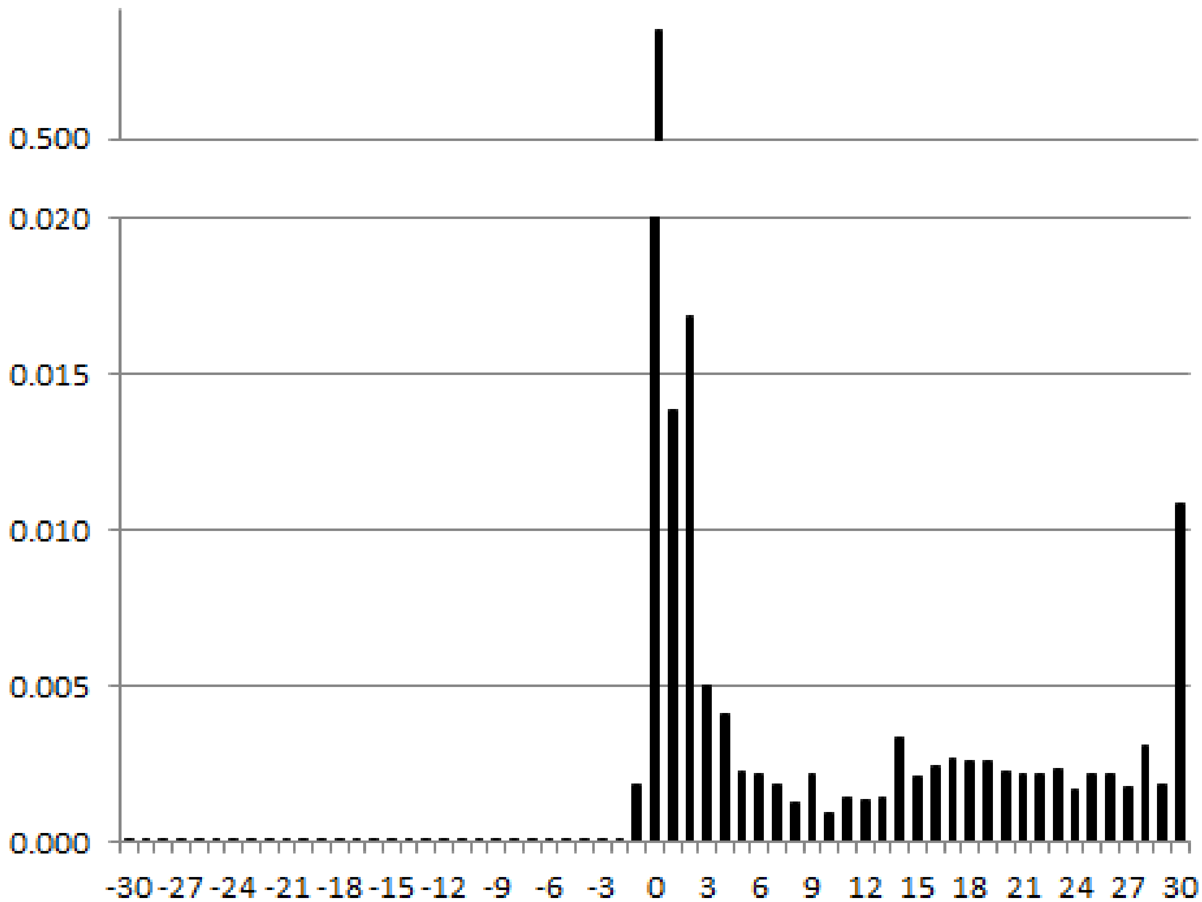

Figure 3.2: **Elapsed days of non-employment between jobs** The vast majority of employees who change jobs do so with exactly zero days of overlap/interveining non-employment.

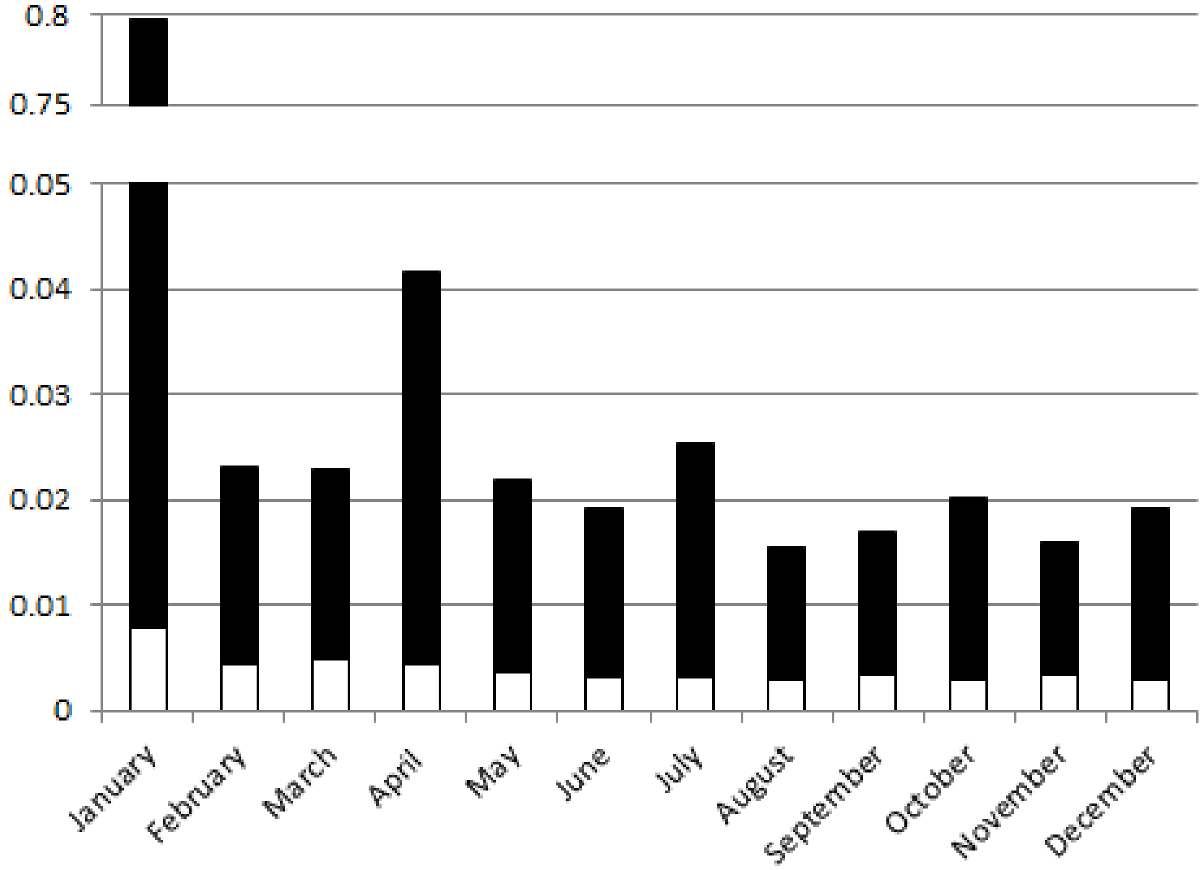

Figure 3.3: **Timing of on-the-job pay change** Most pay changes occur on new-years. Mid-year pay changes are clustered on the first of the quarter and the first of the month.

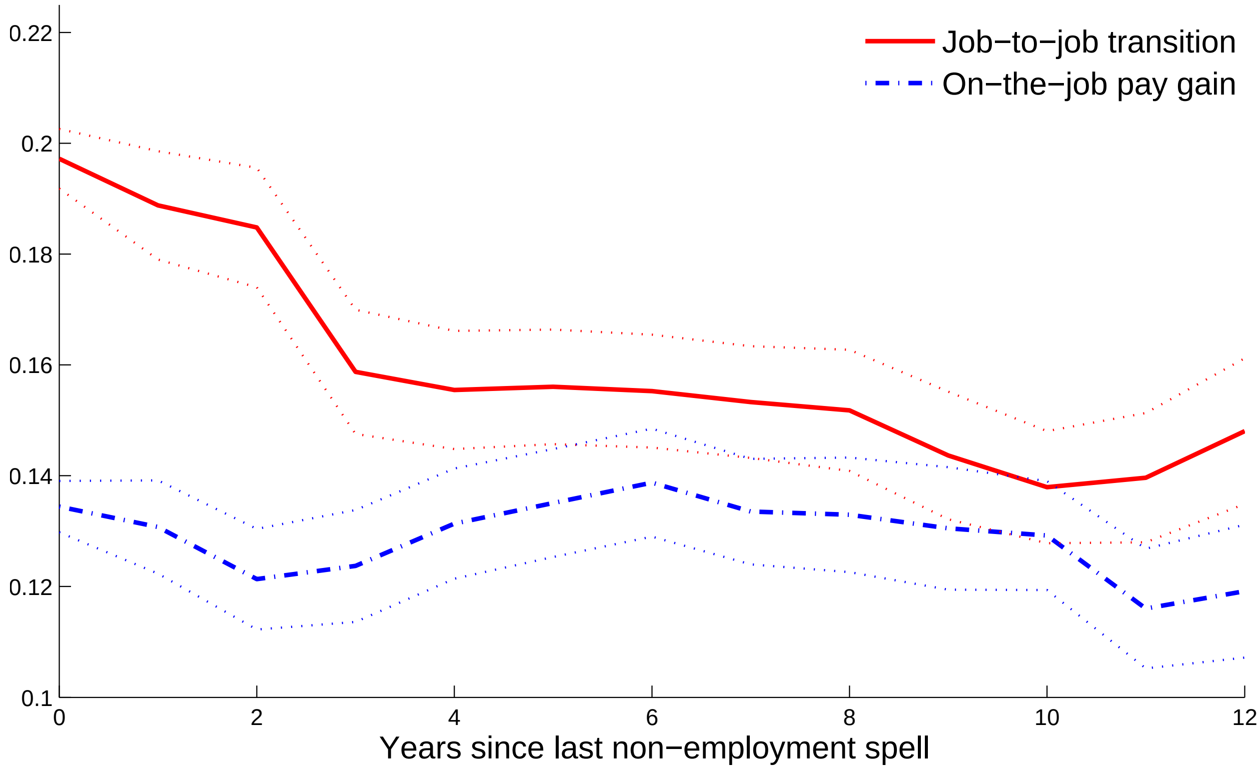

Figure 3.4: **Mobility and pay gain with continuous experience** Job-to-job mobility declines with time since the last non-employment spell. The rate of job switching exceeds that of on-the-job pay gain at low levels of continuous experience. As time since the last non-employment spell increases the rate of on-the-job pay gain catches up to that of job-to-job mobility.

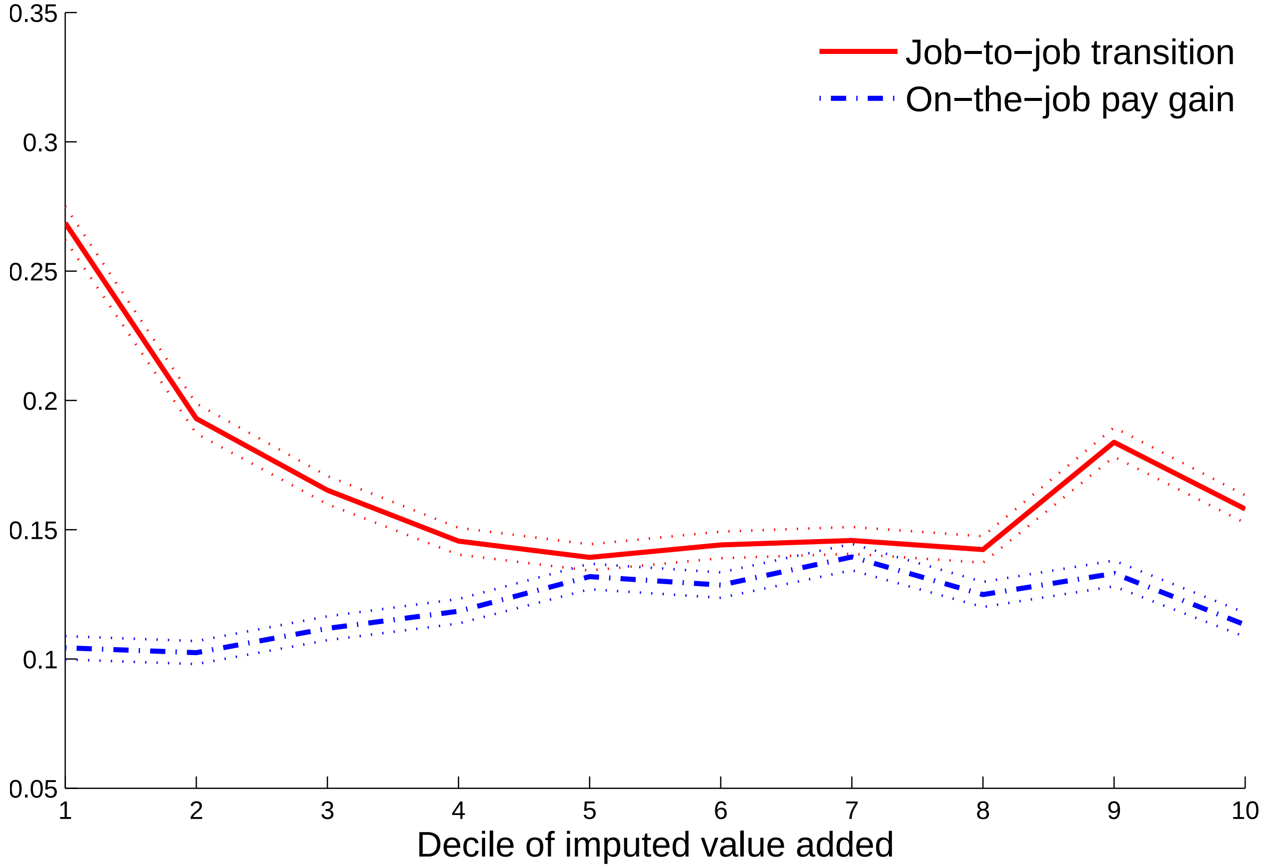

Figure 3.5: **Mobility and pay gain with establishment quality (imputed value added)** The more highly ranked an establishment then the less likely is job-to-job transition and the more likely on-the-job pay gain.

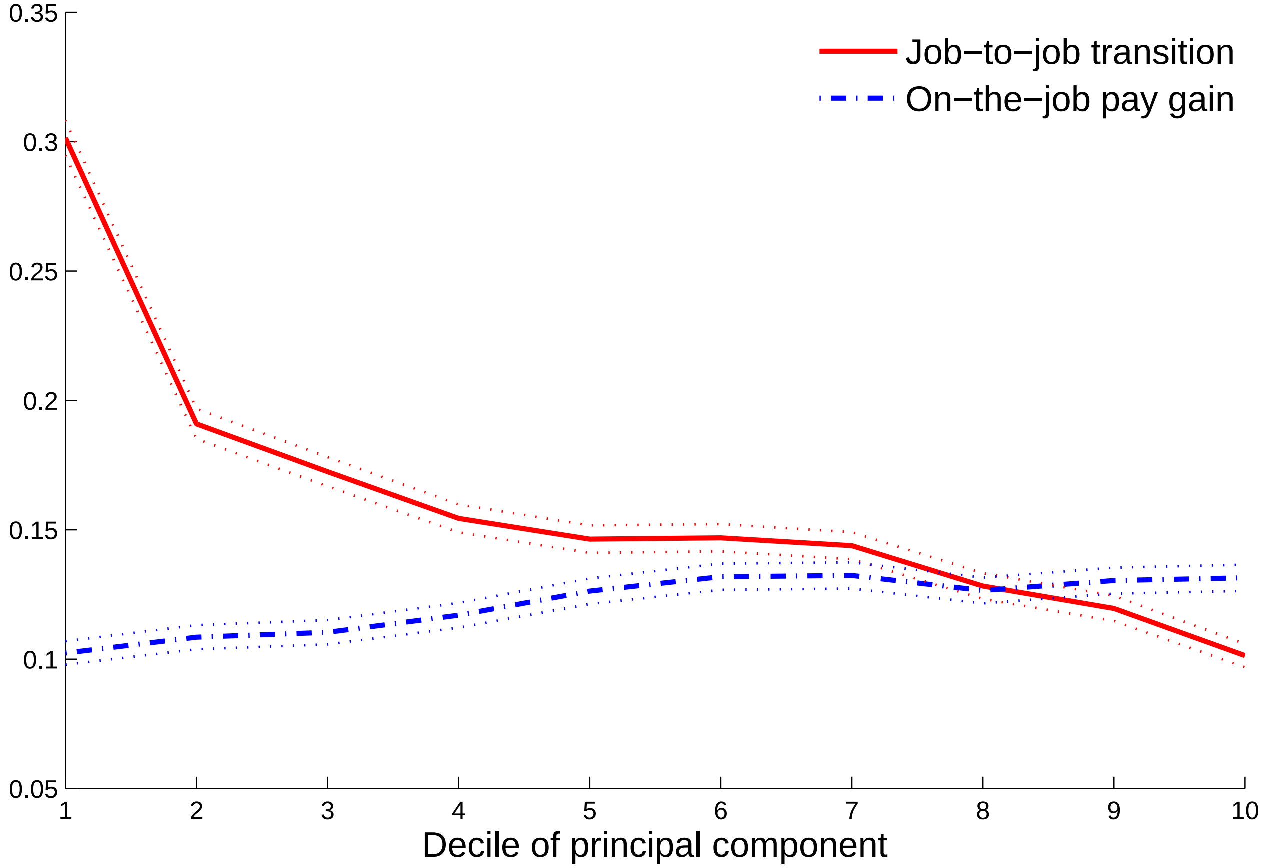

Figure 3.6: **Mobility and pay gain with establishment quality (first principal component)** The more highly ranked an establishment then the less likely is job-to-job transition and the more likely on-the-job pay gain.

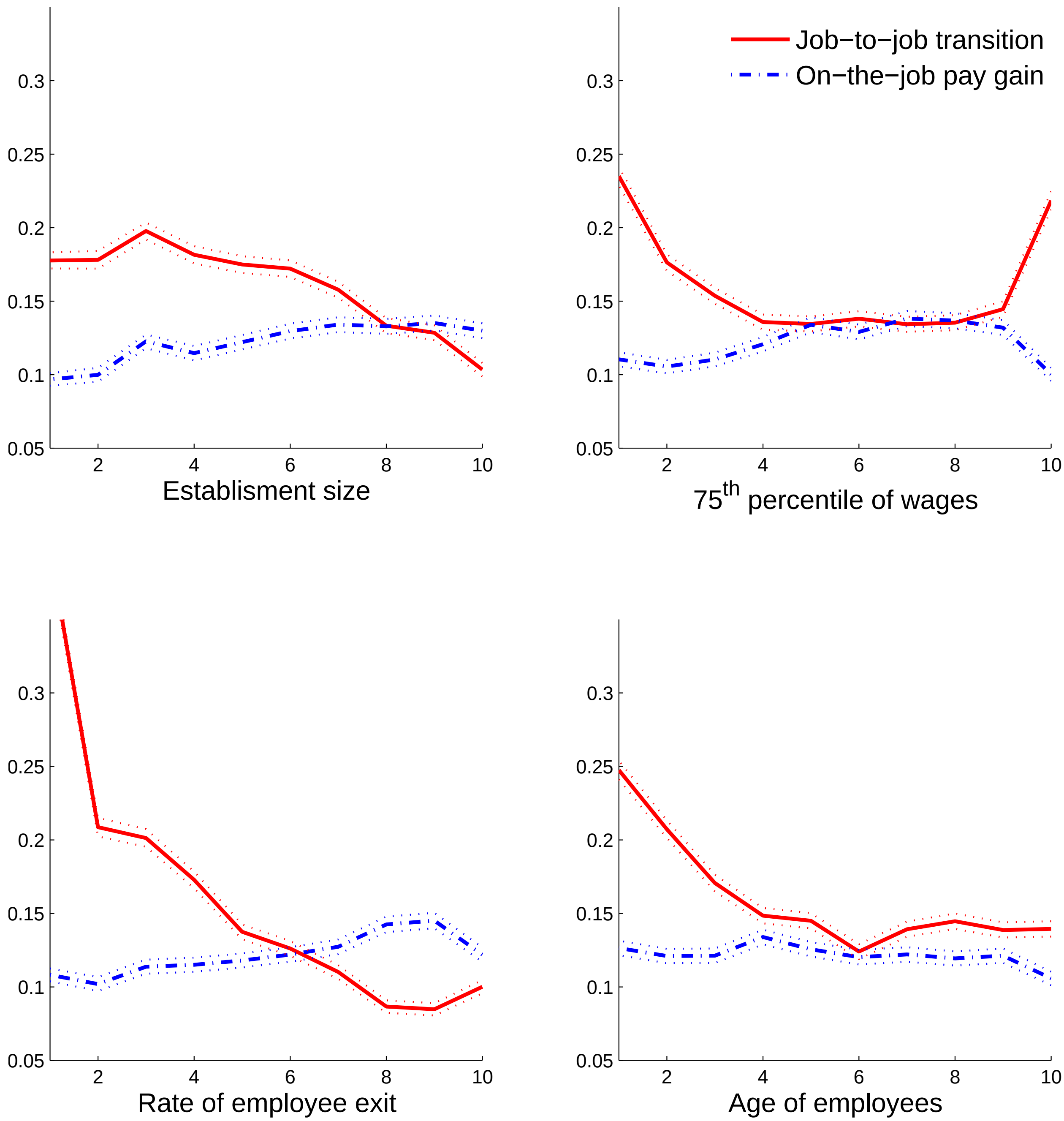

Figure 3.7: **Mobility and pay gain with establishment quality (raw measures of quality)** The more highly ranked an establishment then the less likely is job-to-job transition and the more likely on-the-job pay gain.

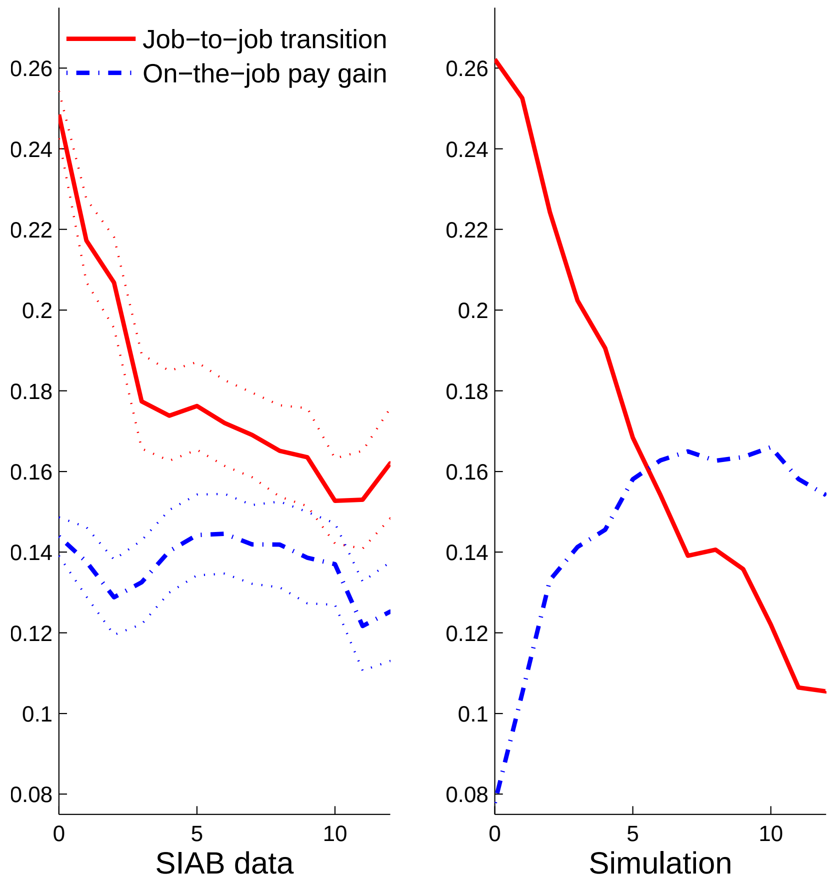

Figure 3.8: Mobility and pay gain with experience: Simulation v. Data. Relative to data the simulated series are more extreme. Simulated job-to-job mobility is initially higher and falls off more quickly than the empirical series while simulated on-the-job pay gain starts of lower and overtakes the empirical series. Although magnitudes are too extreme the overall trends are similar.

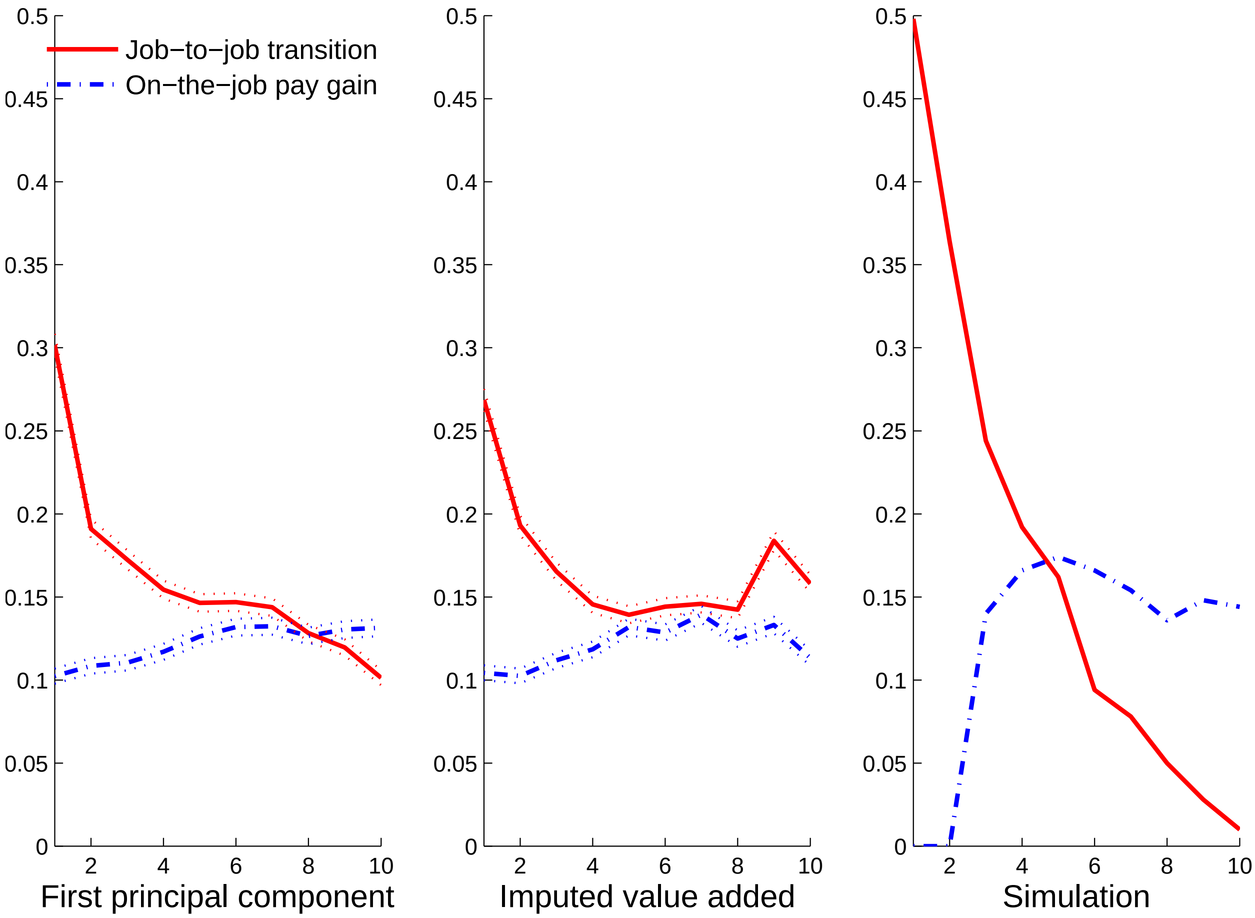

Figure 3.9: Mobility and pay gain with firm rank: Simulation v. Data. Relative to data the simulated series are more extreme. Simulated job-to-job mobility is initially higher and falls off more quickly than the empirical series while simulated on-the-job pay gain starts of lower and overtakes the empirical series. Although magnitudes are too extreme the overall trends are similar.

3.10 Tables

Table 3.1: Employment transitions and duration.

	N	% job stayers	% job movers		mean duration (years)	
			to job < 18 days	to non-employment	employment	non-employment
Total	192,589	66.57	17.09	16.34	2.409	0.405
West	157,622	67.67	16.91	15.43	2.434	0.398
East	29,579	62.28	17.48	20.24	2.304	0.410
Berlin	5,388	57.98	20.12	21.90	2.243	0.534

Table 3.2: Percent of workers who register as unemployed.

	N	Job stayer	Job mover: to job < 18 days	Job mover: to non-employment
Total	192,589	0.40	9.86	50.18
West	157,622	0.33	9.38	47.11
East	29,579	0.87	12.27	61.65
Berlin	5,388	$(\cdot)$	10.15	55.34

$(\cdot)$ indicates too few observations for disclosure.

Table 3.3: Real log wage changes after transition between employers.

	N	median Δ log wage (%)	% obs. such that Δ log wage $\leq$ -0.1	-0.05	0	0.05	0.1
$\leq$ 17 days non-emp.	32,907	-0.06	0.1685	0.2618	0.5019	0.6315	0.7266
w/o unemp. reg.	29,664	0.19	0.1487	0.2422	0.4943	0.6276	0.7252
w/ unemp. reg.	3,243	-2.74	0.3497	0.4413	0.5720	0.6673	0.7401
$>$ 17 days non-emp.	20,144	-1.97	0.3299	0.4102	0.5510	0.6584	0.7405

Note: to be consistent with privacy restrictions, median pay changes are computed as the average pay change in a small window around the true median pay change.

Table 3.4: Real log wage changes on-the-job.

	N	median Δ log wage (%)	% obs. such that Δ log wage $\leq$ -0.1	-0.05	0	0.05	0.1
New years	160,366	-0.75	0.0573	0.1575	0.5594	0.8249	0.9245
Mid year	42,978	0.75	0.0885	0.1941	0.4664	0.6581	0.7774
Quarter	15,708	1.19	0.0646	0.1681	0.4517	0.6512	0.7887
1st of the month	19,883	2.07	0.0918	0.1805	0.4217	0.6109	0.7298
Monday	5,630	0.53	0.0973	0.2341	0.4803	0.6368	0.7542
Other days	5,757	-1.94	0.1317	0.2837	0.6192	0.8023	0.8822

Note: to be consistent with privacy restrictions, median pay changes are computed as the average pay change in a small window around the true median pay change.

Table 3.5: Nominal log wage changes on-the-job.

	N	median Δ log wage (%)	% obs. such that Δ log wage $\leq$ -0.1	-0.05	0	0.05	0.1
New years	160,366	1.54	0.0387	0.0945	0.2800	0.7377	0.8902
Mid year	42,978	4.13	0.0573	0.1057	0.2777	0.5377	0.7079
Quarter	15,708	4.59	0.0392	0.0802	0.2670	0.5265	0.7095
1st of the month	19,883	5.27	0.0642	0.1088	0.2621	0.4966	0.6633
Monday	5,630	4.36	0.0627	0.1124	0.2870	0.5215	0.6750
Other days	5,757	1.74	0.0754	0.1529	0.3396	0.6743	0.8270

Note: to be consistent with privacy restrictions, median pay changes are computed as the average pay change in a small window around the true median pay change.

Table 3.6: Incidence of and duration until real on-the-job pay gain (excluding new years).

	N	% workers with on-the-job real pay gain	duration of initial pay level (years)
Total	192,589	12.53	2.241
West	157,622	12.54	2.265
East	29,590	12.52	2.141
Berlin	5,392	12.29	2.093

Table 3.7: Incidence of and duration until nominal on-the-job pay gain (excluding new years).

	N	% workers with on-the-job nominal pay gain	duration of initial pay level (years)
Total	192,589	16.70	2.200
West	157,622	16.70	2.224
East	29,590	16.82	2.099
Berlin	5,392	16.05	2.058

Table 3.8: Labor share, value added, and value added per employee.

	All Establishments			Estab. with $\geq$ five empl.		
	mean	Std. Err.	N	mean	Std. Err.	N
Labor share						
Total	0.6910	0.0218	27,077	0.8032	0.0350	21,760
West	0.6968	0.0274	16,156	0.8101	0.0428	13,393
East	0.6815	0.0172	9,907	0.7917	0.0237	7,609
Berlin	0.5698	0.0290	1,014	0.6154	0.0239	758
Value added per employee						
Total	46,479	843	27,105	50,617	1,181	21,782
West	49,090	1,055	16,174	52,930	1,441	13,408
East	36,326	716	9,917	40,129	834	7,616
Berlin	41,293	2,321	1,014	43,961	2,187	758

Table 3.9: Covariance of labor share and value added.

	All Establishments		Estab. with $\geq$ five empl.	
	labor share	ln(value added)	labor share	ln(value added)
	2005 , N= 9,758		2005 , N= 7,731	
ln(value added)	-0.1549*	1	-0.1947*	1
ln(val. add. per emp.)	-0.4261*	0.6585*	-0.4430*	0.6615*
	2006 , N= 9,113		2006 , N= 7,209	
ln(value added)	-0.1628*	1	-0.2100*	1
ln(val. add. per emp.)	-0.4547*	0.6638*	-0.4816*	0.6666*
	2007 , N= 9,093		2007 , N= 7,130	
ln(value added)	-0.1055*	1	-0.1417*	1
ln(val. add. per emp.)	-0.3533*	0.6640*	-0.3956*	0.6554*

* indicates significance at the 1% level

Table 3.10: Covariance of survey and administrative measures.

All Establishments	labor share	value added	value added per emp.	size (survey)
size (admin)	-0.0025	0.8623*	0.0380*	0.9619*
p. of employees who exit	0.0295*	-0.0433*	-0.0465*	
log wages, p25	-0.0220*	0.1451*	0.2283*	
log wages, p50	-0.0158	0.1485*	0.2391*	
log wages, p75	-0.0101	0.1459*	0.2361*	
employee ages, p25	-0.0186*	-0.0012	0.0314*	
employee ages, p50	-0.0029	0.0165	0.0440*	
employee ages, p75	0.0138	0.0297*	0.0550*	
first factor of employee ages	-0.0017	0.0166	0.0478*	
Estab. with $\geq$ five empl.	**labor share**	**value added**	**value added per emp.**	**size (survey)**
size (admin)	-0.0076	0.8620*	0.0330*	0.9624*
p. of employees who exit	0.0500*	-0.0514*	-0.0505*	
log wages, p25	-0.0501*	0.1615*	0.2337*	
log wages, p50	-0.0495*	0.1641*	0.2449*	
log wages, p75	-0.0479*	0.1612*	0.2443*	
employee ages, p25	-0.014	0.0075	0.0426*	
employee ages, p50	-0.0079	0.0132	0.0362*	
employee ages, p75	0.001	0.016	0.0334*	
first factor of employee ages	-0.0066	0.0133	0.0412*	

* indicates significance at the 1% level

Table 3.11: Linear prediction of value added.

	All Establishments		
	2005	2006	2007
ln(value added per person)			
estab. size	0.0002 **	0.0002 **	0.0002 ***
(estab. size)^2	0.0000 **	0.0000 **	0.0000 *
portion of employees exiting	-0.2893 ***	-0.2330 ***	-0.2870 ***
ln(75^{th} percentile of wages)	0.6601 ***	0.6532 ***	0.6687 ***
constant	7.6774 ***	7.6754 ***	7.6775 ***
N	8,380	7,665	7,610
R-squared	0.1584	0.1603	0.1841

	Estab. with $\geq$ five empl.		
	2005	2006	2007
ln(value added per person)			
estab. size	0.0000	0.0001	0.0001
(estab. size)^2	0.0000	0.0000	0.0000
portion of employees exiting	-0.3912 ***	-0.1872 **	-0.2849 ***
ln(75^{th} percentile of wages)	1.0602 ***	0.9845 ***	0.9837 ***
constant	5.9515 ***	6.2016 ***	6.3003 ***
N	6,823	6,250	6,156
R-squared	0.2234	0.0000	0.2512

***, **, and * indicate significance at the 1, 5 and 10% level.

[a] Age is measured as the first principle component of quartiles of the age distribution at the establishment level.

Table 3.12: Correlation of administrative establishment characteristics.

	turnover	size	wage (p75)	employee age
turnover	1			
size	-0.0285*	1		
wage (p75)	-0.1928*	0.0956*	1	
employee age	-0.1109*	0.0335*	0.1525*	1

[a] Age is measured as the first principle component of quartiles of the age distribution at the establishment level.

Table 3.13: First principal component.

	All Establishments			Estab. with $\geq$ five empl.		
	2005	2006	2007	2005	2006	2007
Loading						
Portion of employees exiting	-0.5901	-0.6001	-0.5991	-0.5458	-0.5620	-0.5676
ln(75th percentile of wages)	0.6093	0.5822	0.5766	0.6199	0.6068	0.6041
Age[a]	0.3775	0.4284	0.4316	0.4929	0.5033	0.4989
Size	0.3714	0.3426	0.3497	0.2738	0.2501	0.2529
% explained correlation	30.21	30.49	30.46	33.33	33.96	33.98
N	1,742,927	1,737,634	1,755,278	933,362	941,015	953,850

[a] Age is measured as the first principle component of quartiles of the age distribution at the establishment level.

Table 3.14: Second principal component.

	All Establishments			Estab. with $\geq$ five empl.		
	2005	2006	2007	2005	2006	2007
Loading						
Portion of employees exiting	0.3003	0.3055	0.3116	0.2777	0.2634	0.2557
75^{th} percentile of wages	0.2570	0.3132	0.3313	0.0200	0.0533	0.0648
Age[a]	-0.6159	-0.5601	-0.5667	-0.2349	-0.2343	-0.2587
Size	0.6815	0.7035	0.6870	0.9313	0.9343	0.9292
% explained correlation	25.31	25.56	25.67	24.63	24.75	24.76
N	1,742,927	1,737,634	1,755,278	933,362	941,015	953,850

[a] Age is measured as the first principle component of quartiles of the age distribution at the establishment level.

Table 3.15: Third principal component.

	All Establishments			Estab. with $\geq$ five empl.		
	2005	2006	2007	2005	2006	2007
Loading						
Portion of employees exiting	0.4028	0.3370	0.3370	0.5707	0.4968	0.4666
75^{th} percentile of wages	-0.3444	-0.4374	-0.4466	-0.1543	-0.2575	-0.2941
Age[a]	0.6523	0.6280	0.6076	0.8057	0.8248	0.8252
Size	0.5419	0.5484	0.5637	0.0364	0.0815	0.1219
% explained correlation	23.47	23.16	23.06	22.34	21.78	21.77
N	1,742,927	1,737,634	1,755,278	933,362	941,015	953,850

[a] Age is measured as the first principle component of quartiles of the age distribution at the establishment level.

Table 3.16: Movers who change to ...

	after jtj transition		after juj transition		
	obs.	%	obs.	%	tstat
Larger first principal component	20,779	54.69	6,702	47.25	7.4934
Larger imputed value per employee	20,779	58.43	6,702	49.16	9.3485

Table 3.17: Movers who change to ...

	after jtj transition		after juj transition		
	obs.	%	obs.	%	tstat
Lower percent exits	20,979	52.89	6,826	47.77	5.2185
Larger size	20,979	52.28	6,826	51.55	0.7438
Older co-workers	20,779	53.82	6,702	47.02	6.8563
Larger wages (p75)	20,979	62.30	6,826	49.71	12.8488

Table 3.18: Targeted empirical moments.

	duration of		Fraction of workers with ...				N
	employment	initial pay	pay raise	job staying	job changing	job losing	
Total	2.4090	2.2409	0.1253	0.6657	0.1709	0.1634	192,589
West	2.4344	2.2647	0.1254	0.6767	0.1691	0.1542	157,622
East	2.3039	2.1407	0.1252	0.6228	0.1748	0.2024	29,590
Berlin	2.2427	2.0932	0.1229	0.5798	0.2012	0.2190	5,392

Table 3.19: Share of wage posting contracts and transition hazards.

	Yearly hazards			Share wage posting		
	λ_1	δ	k_1	firms (s_F)	workers (s_W)	N
Total	0.3434	0.0689	4.9840	0.6288	0.2206	192,644
	(0.0010)	(0.0003)	(0.0267)	(0.0171)	(0.0111)	
West	0.3996	0.0647	6.1762	0.6764	0.2256	157,662
	(0.0006)	(0.0003)	(0.0264)	(0.0071)	(0.0055)	
East	0.3487	0.0868	4.0173	0.6244	0.2489	29,590
	(0.0011)	(0.0008)	(0.0410)	(0.0224)	(0.0213)	
Berlin	0.4929	0.0956	5.1559	0.7932	0.3839	5,392
	(0.0020)	(0.0023)	(0.1229)	(0.0253)	(0.0349)	

Standard errors in parenthesis

Table 3.20: Alternate definition of job-to-job transition.

	duration of		Fraction of workers with ...				N
	employment	initial pay	pay raise	job staying	job changing	job losing	
Total	2.4090	2.2409	0.1253	0.6657	0.1540	0.1802	192,589
West	2.4344	2.2647	0.1253	0.6657	0.1540	0.1700	157,622
East	2.3039	2.1407	0.1252	0.6228	0.1533	0.2238	29,590
Berlin	2.2427	2.0932	0.1229	0.5798	0.1808	0.2394	5,392

Table 3.21: Robustness to definition of job-to-job mobility.

	Yearly hazards			Share wage posting		
	λ_1	δ	k_1	firms (s_F)	workers (s_W)	N
Total	0.2602	0.0757	3.4373	0.4206	0.1406	192,644
	(0.0003)	(0.0004)	(0.0201)	(0.0599)	(0.0279)	
West	0.2645	0.0722	3.6634	0.4948	0.1736	157,662
	(0.0014)	(0.0006)	(0.0424)	(0.1068)	(0.0549)	
East	0.2418	0.0980	2.4673	0.4388	0.1840	29,590
	(0.0007)	(0.0020)	(0.0525)	(0.1506)	(0.0932)	
Berlin	0.3147	0.1066	2.9522	0.6300	0.3011	5,392
	(0.0040)	(0.0025)	(0.0681)	(0.1200)	(0.1063)	

Standard errors in parenthesis

Table 3.22: Alternate definition of on-the-job pay raise.

	duration of		Fraction of workers with ...				N
	employment	initial pay	pay raise	job staying	job changing	job losing	
Total	2.4090	2.2001	0.1670	0.6657	0.1709	0.1634	192,589
West	2.4344	2.2239	0.1670	0.6767	0.1691	0.1542	157,622
East	2.3039	2.0990	0.1682	0.6228	0.1748	0.2024	29,590
Berlin	2.2427	2.0584	0.1605	0.5798	0.2012	0.2190	5,392

Table 3.23: Robustness to definition of on-the-job pay raise.

	Yearly hazards			Share wage posting		
	λ_1	δ	k_1	firms (s_F)	workers (s_W)	N
Total	0.3699	0.0688	5.3765	0	0	192,644
West	0.3972	0.0648	6.1296	0	0	157,662
East	0.3506	0.0866	4.0485	0	0	29,590
Berlin	0.4549	0.0958	4.7484	0.6168	0.4549	5,392

Standard errors omitted due to the corner solution.

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