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The dominating higher order vertical modes of the internal seiche in a small lake[1]

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Abstract

The internal seiche structure of a small, shallow lake (Frains Lake, Michigan) is shown to be dominated by the higher order vertical modes. The resonant frequencies and the vertical velocity profiles of each mode were predicted from observed temperature profiles. The observed resonant frequencies and observed phase shifts in the velocity profiles of each mode corresponded closely to those predicted. Although the relative amplitude of each mode varied with depth, the higher modes (up to 7–10th) dominated and the fundamental mode was transitory.

Mortimer (1953), from an extensive analysis of the records of internal seiche motion in moderately sized lakes, showed that the period of motion could, for the most part, be explained by a simple two-layer box model, using appropriate corrections such as volumetric mean depths. Generally, the metalimnetic interface, where the temperature gradient is most pronounced, was a small portion of the total depth in these lakes. The fundamental seiche was found to be dominant; however higher horizontal harmonics have been observed. The two-layer model did not permit higher vertical modes, and they have only rarely been noted (Mortimer 1971).

In contrast, I show here that the metalimnetic region in a small, shallow lake (Frains Lake, Michigan) can be as thick as or thicker than either the hypolimnion or the epilimnion; under these conditions one would not expect the two-layer model to be applicable. In fact, the seiche structure of Frains Lake is much more complex; the internal seiches exhibit several vertical modes and the higher vertical modes dominate seiche structure.

The theoretical basis for my analysis comes from Fjeldstadt (1933) and Krauss (1966). The assumption is made that the lake basin is approximately box-shaped with a flat bottom. The Coriolis force, due to the rotation of the earth, is ignored.

The governing equation can be reduced to

$$d^2w/dz^2 = -\lambda^2 N^2 w$$

where N is the observed Brunt-Väisälä frequency; λ^2, the predicted eigenvalue; $w(z)$, the predicted vertical velocities; and z, the height from the bottom with $w = 0$ at $z = 0, h$. This is a common eigenvalue problem that can be numerically solved for λ^2 and $w(z)$ by finite difference techniques (LaZerte 1978). Each eigenvalue corresponds to the predicted reasonant frequency of an internal seiche which can be estimated by making an assumption about its effective wavelength. The associated eigenvector, $w(z)$, predicts the depth pattern of vertical velocities corresponding to that frequency.

Preliminary work on the temperature structure of Frains Lake in 1976 indicated that the major internal seiche was much longer than predicted by the two-layer model (Mortimer 1953); the period was 5–10 h as compared to the predicted 30–90 min. In addition, the oscillations in the hypolimnion and metalimnion rarely seemed to be in phase. I consequently decided to investigate the temperature regime more extensively by installing a string of thermistors in Frains Lake in 1977.

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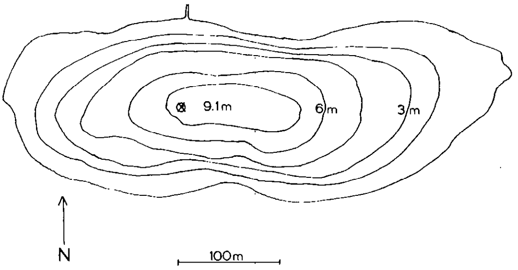

Fig. 1. Bathymetry of Frains Lake, Michigan.

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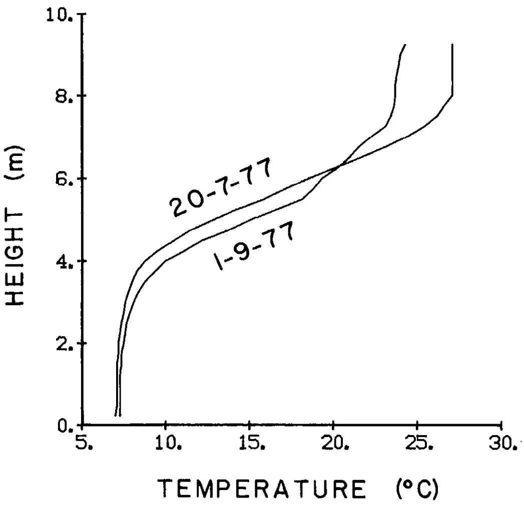

Fig. 2. Representative temperature profiles.

Methods

Frains Lake is a small eutrophic lake in the glacial drift of southeastern Michigan. The basin is relatively uniform (Fig. 1), with a mean depth of 3.5 m and surface area of about 7 ha.

The thermistor string was put at half the distance from the center of the lake to its western shore, which placed it about at the node of the second harmonic of a long-axis horizontal seiche and at the node of the fundamental horizontal short-axis seiche. The thermistors (Victory Engineering, 41A11) were mounted on PVC piping at 0.50- and 0.25-m intervals, with the greatest density in the metalimnion. A float at the surface supported the pipe, 23 thermistors, and one stable Vishay (S102) resistor (as a control for the bridge excitation voltage). The output was read through a multiplexor by a Digitec digital millivoltmeter. Long term drift in the bridge excitation voltage and DVM was always <0.1% and was removed in the data reduction. Thermistor overheat was <0.01°C. All the thermistors were initially calibrated against a Hewlett-Packard quartz crystal thermometer; no attempt was made to calibrate the units in situ.

The thermistor string was put into operation in early May and ran intermittently until the end of August 1977. Only the two longest time series are discussed here. Shorter time series were collected (LaZerte 1978), often at shorter intervals (down to 1 min), but will only be mentioned briefly.

For each channel, every datum was checked for parity and replaced by an adjacent average if necessary; outliers were treated similarly (<1% of the data was so treated). Calibration was then performed, quadratic trends removed, and spectral analyses run (Parzen 1972). Phase and autospectra were smoothed with a Parzen (2) lag window and a lag of 12% sampled size, giving confidence intervals based on 31 df (Jenkins and Watts 1968). Even more important than the confidence intervals based on a single (multivariate) time series, for both the autospectra and cumulative phase spectra to be presented here two time series separated by 11 days gave substantially the same results.

Two in situ temperature profiles at 0.25-m intervals were obtained with a stable, high resolution digital thermometer (LaZerte 1978) calibrated against a Hewlett-Packard quartz crystal thermometer and either temporal averaging over 6 h or spatial averaging profiles from both ends and the center of the lake. Because calculations based on alkalinity indicate that the temperature gradient is the major source of stability in Frains Lake, I converted temperature gradients to density gradients using Kell's (1967) data on ther-

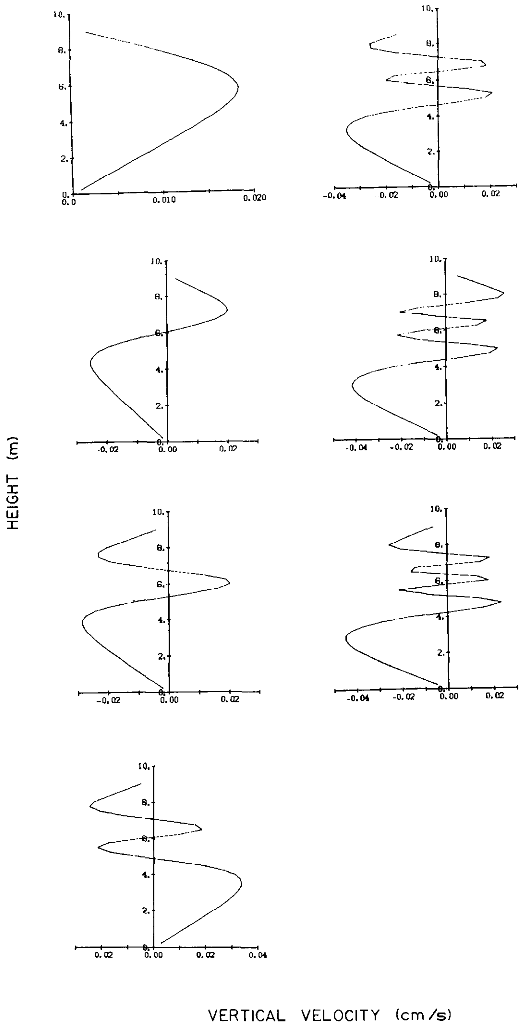

Fig. 3. First seven predicted vertical velocity profiles for 20 July 1977 temperature profile.

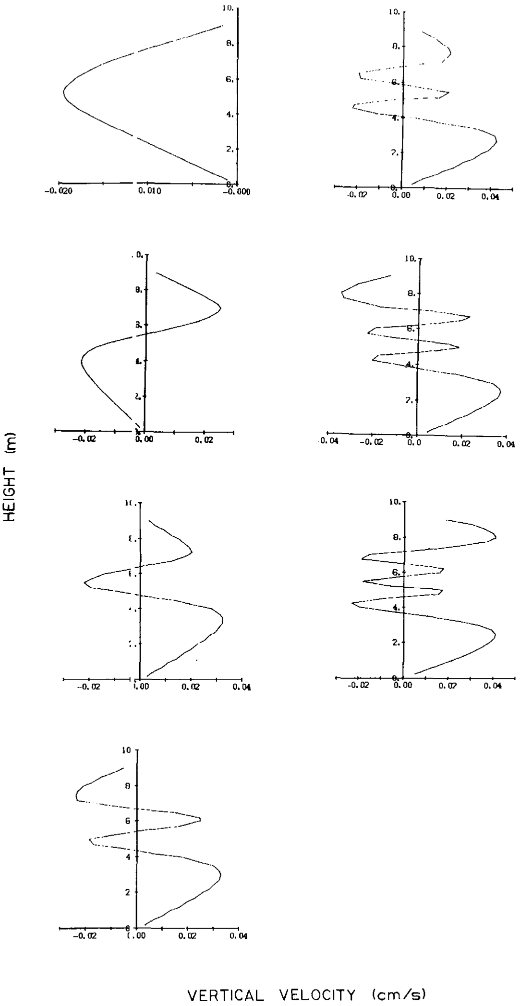

Fig. 4. As Fig. 3, but for 1 September 1977 temperature profile.

mal expansivity of pure water. These data were then used in the finite difference approximation of the reduced governing differential equation (LaZerte 1978).

Results

The two representative temperature profiles on 20 July and 1 September 1977 are presented in Fig. 2. There was a continuous transition between these profiles because of a slight cooling in August (LaZerte 1978). Of particular interest here is the relatively substantial thick-

Table 1. The first 10 predicted resonant frequencies (cph) beginning with fundamental internal mode, for two temperature profiles in Frains Lake, 1977.

20 July	1 September
1.454	1.276
0.534	0.476
0.320	0.286
0.236	0.204
0.186	0.164
0.154	0.134
0.134	0.118
0.120	0.106
0.108	0.098
0.100	0.090

ness of the metalimnion. The first seven predicted eigenvectors for these temperature profiles are given in Figs. 3 and 4. The resonant frequencies for each eigenvector were computed from the associated eigenvalues by taking two times the length of the lake at its mean depth as the effective horizontal wavelength. The first 10 values, starting with the fundamental mode, can be found in Table 1 for each date.

The first spectral analysis was performed on the 17 thermistors in the stratified water layers from 6.5 to 0.5 m off the bottom after a midsummer storm on 1 July. Each time series consisted of 425 data points at 30-min intervals and lasted for 8.8 days. The autospectra are presented in Fig. 5; the 16 phase spectra of adjacent channels were given elsewhere (LaZerte 1978).

The dominance of a low frequency band centered around 0.15 cph (cycles per hour) is apparent, but this dominance fades in the 5.25–4.5-m region and peaks of 0.18 or 0.22–0.24 cph appear. There is also prominant activity at 0.82 and 0.48 cph. The 0.32-cph peak fades at the middle heights and all frequencies except 0.15 fade in the hypolimnion.

In general, the 0.15 peak dominates in comparative magnitude, with the higher frequencies becoming progressively smaller. Temperature and velocity spectra at shorter intervals (10 min to 1 min) have not revealed any significant peaks at frequencies higher than 1 cph (LaZerte 1978); however, the rare occurrence of the fundamental mode is discernible, as will be shown later (*see Fig.* 9).

By plotting the phase shift of each major frequency mode against height above the bottom one can show which heights are 180° out of phase with others and the general pattern of phase shift through the lake at each frequency. In Fig. 6 phase shift is plotted with respect to 0.5 m vs. depth for the three dominant frequencies: 0.48, 0.32, and 0.15 cph. In general, the lower the frequency the more phase shifts there are and the earlier they occur with respect to height from the bottom.

Similar analyses were performed on the 1,019 cases of the second time series that began on 20 July and lasted 21.2 days. The autospectra for the stratified layers ranging from 5.75 to 0.5 m are presented in Fig. 7 and the corresponding phase spectra of adjacent channels were given by LaZerte (1978). The results are similar to the earlier time series except that the several low frequency peaks are highly merged with a sharp cut-off below about 0.14 cph in the hypolimnion and 0.09 cph at greater heights and that the 0.32 peak seems to have shifted to 0.28 and the 0.4 peak to 0.44 cph. As before, and especially notable in the hypolimnion, the higher frequencies are progressively lower in amplitude.

The phase shifts show a pattern similar to that reported for the earlier time series (Fig. 8). One difference is that the phase shifts begin closer to the bottom of the lake and progress further with increasing height.

Figure 9 provides the time series of the temperature fluctuation at a height of 6.5 m for the 3 days before and 1 day after the 1 July storm. At around 12 h after the beginning of the time series, there was a period of high winds (avg daily wind speed was 5.5 $\mathrm{m \cdot s^{-1}}$); internal seiche amplitudes increased, with some high frequency activity, but then rapidly died out. The much stronger 1 July storm at 75 h (avg daily wind speed was 7.8 $\mathrm{m \cdot s^{-1}}$) had a similar effect but with a stronger high frequency component, apparently the fundamental seiche. Again, this high frequency component died out rapidly.

Discussion

The observed results can be compared with those predicted in three ways: observed versus predicted frequencies; at a given frequency, corresponding to a predicted vertical mode, vertical profiles of observed versus predicted velocity; vertical profiles of phase shift. The data presented here allow comparison by the first or third way. Vertical profiles of velocity cannot be constructed without precise in situ cross-calibration of the thermistors because some thermistors drifted during the summer. One would also need

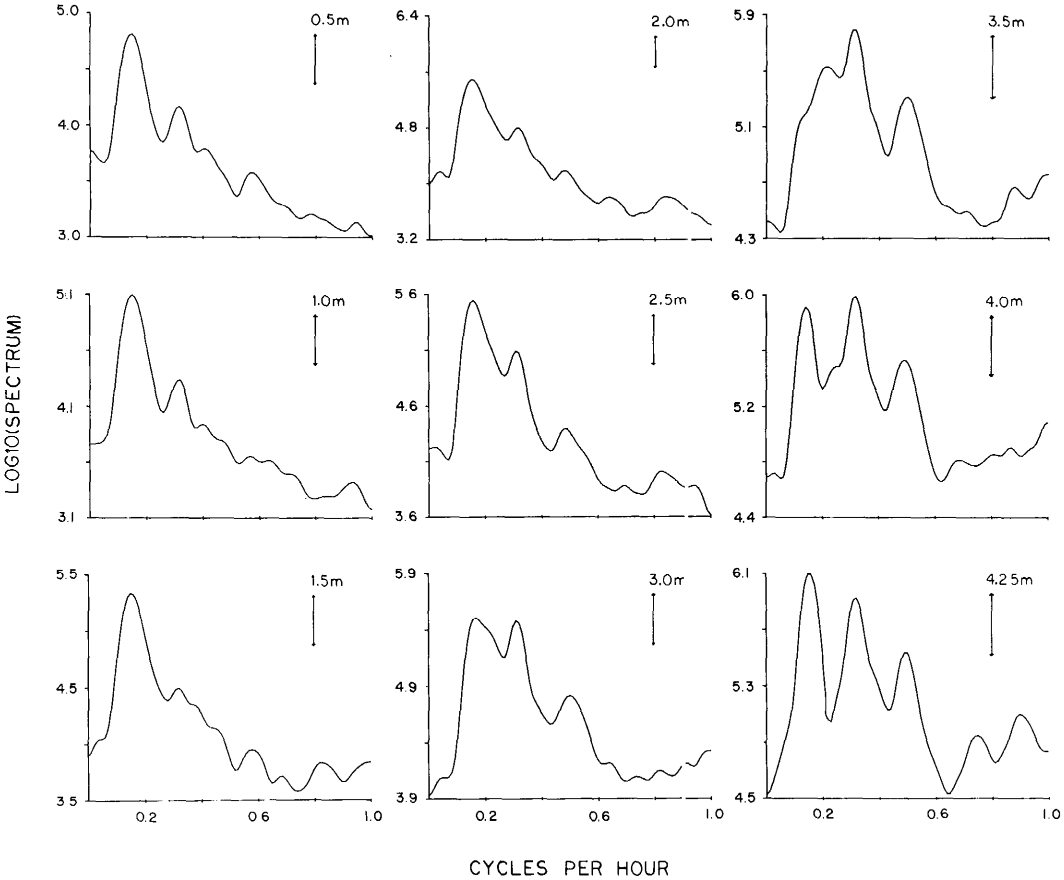

Fig. 5. Autospectra for post-1 July 1977 time series.

more frequent and detailed temperature profiles than those available to convert temperature fluctuations into vertical velocity profiles.

The three frequencies 0.48, 0.32, and 0.15 cph from the first time series, examined in comparison with the predicted frequencies based on the 20 July profile may correspond to the second, third, and sixth vertical modes. The observed phase shift at these frequencies (Fig. 5) corresponds reasonably well with that predicted for their respective modalities (Fig. 2). This type of comparison is equally successful when applied to the second time series.

The predictions of resonant frequency are reasonably close to those actually found. The phase shifts with depth predicted are quite close to those observed in the lake and have the same trends with frequency. As the thermocline erodes, it is predicted that the phase shifts will be closer to the bottom, again as found in the lake. The model also predicts the increased vertical packing of cells and the increased packing with respect to frequency as the modal frequency decreases. However, no predictions are made of the relative amplitudes at the different modal frequencies. In particular, the predicted fundamental mode does not appear to be strong except, transitionally, after storms, and there appears to be a low frequency limit of about 0.10–0.14 cph, which corresponds to vertical modes greater than the seventh to tenth. There seems to be no other possible ex-

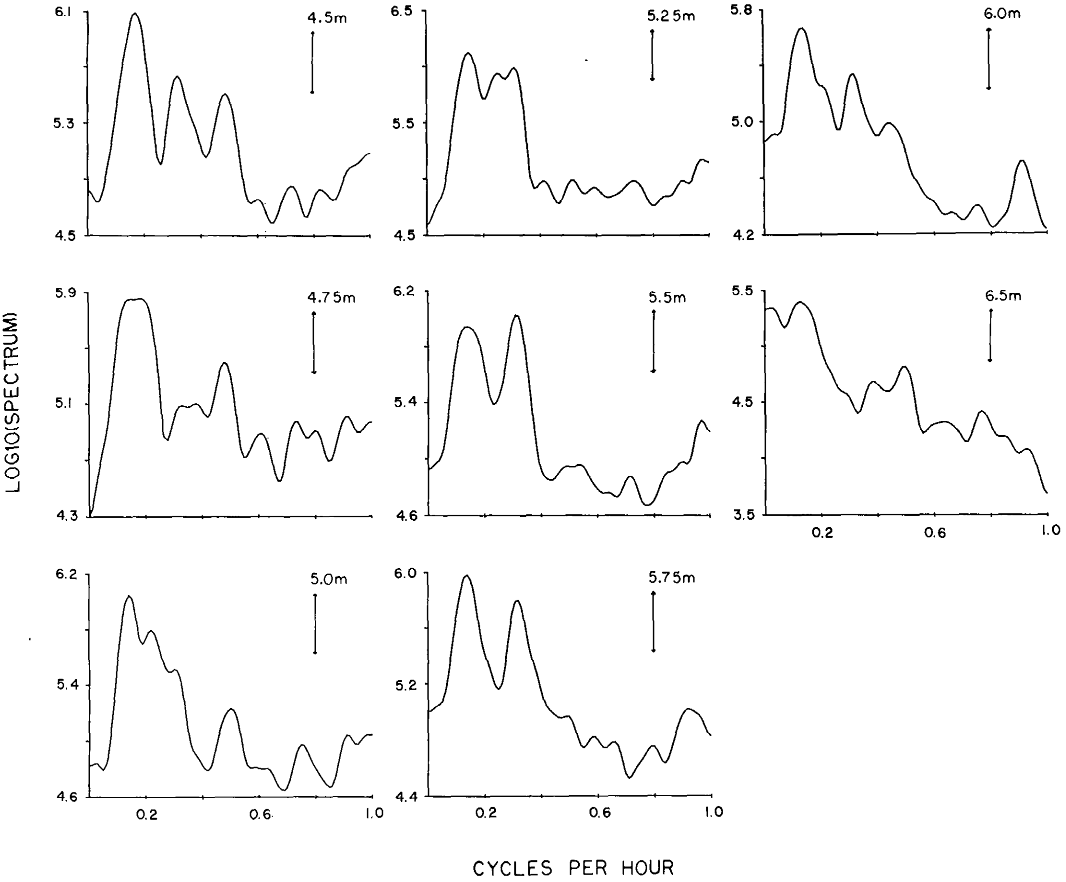

Fig. 5. Continued

planation for these observations. As higher order horizontal modes have no phase shift with depth and increase in frequency with modality, they cannot be responsible. Aliasing has also been shown to be unimportant (LaZerte 1978).

Mortimer (1953) provided an example of a second vertical mode in St. Wolfgang-See, as recorded by Exner in 1907–1908, apparent in the depth-time isotherms in the thermocline which seem to be 180° out of phase with those below. He considered the possibility of using a three-layer model for Windermere (Mortimer 1952) to predict more accurately the hypolimnetic fluctuations. Such a model would provide 180° phase shifts between the top and bottom layers. However he concluded that the model is un-

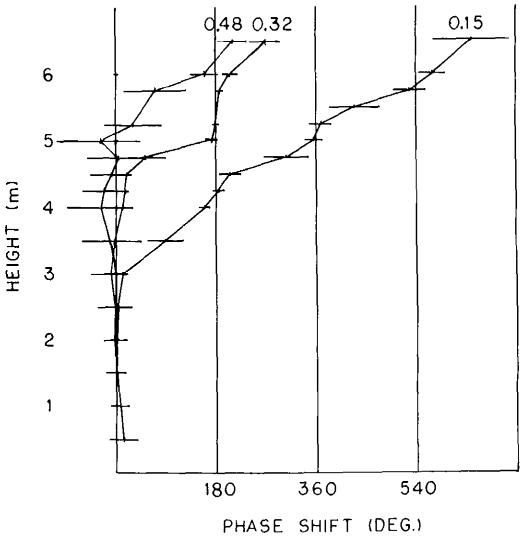

Fig. 6. Phase shifts with respect to 0.5 m for post-1 July 1977 time series.

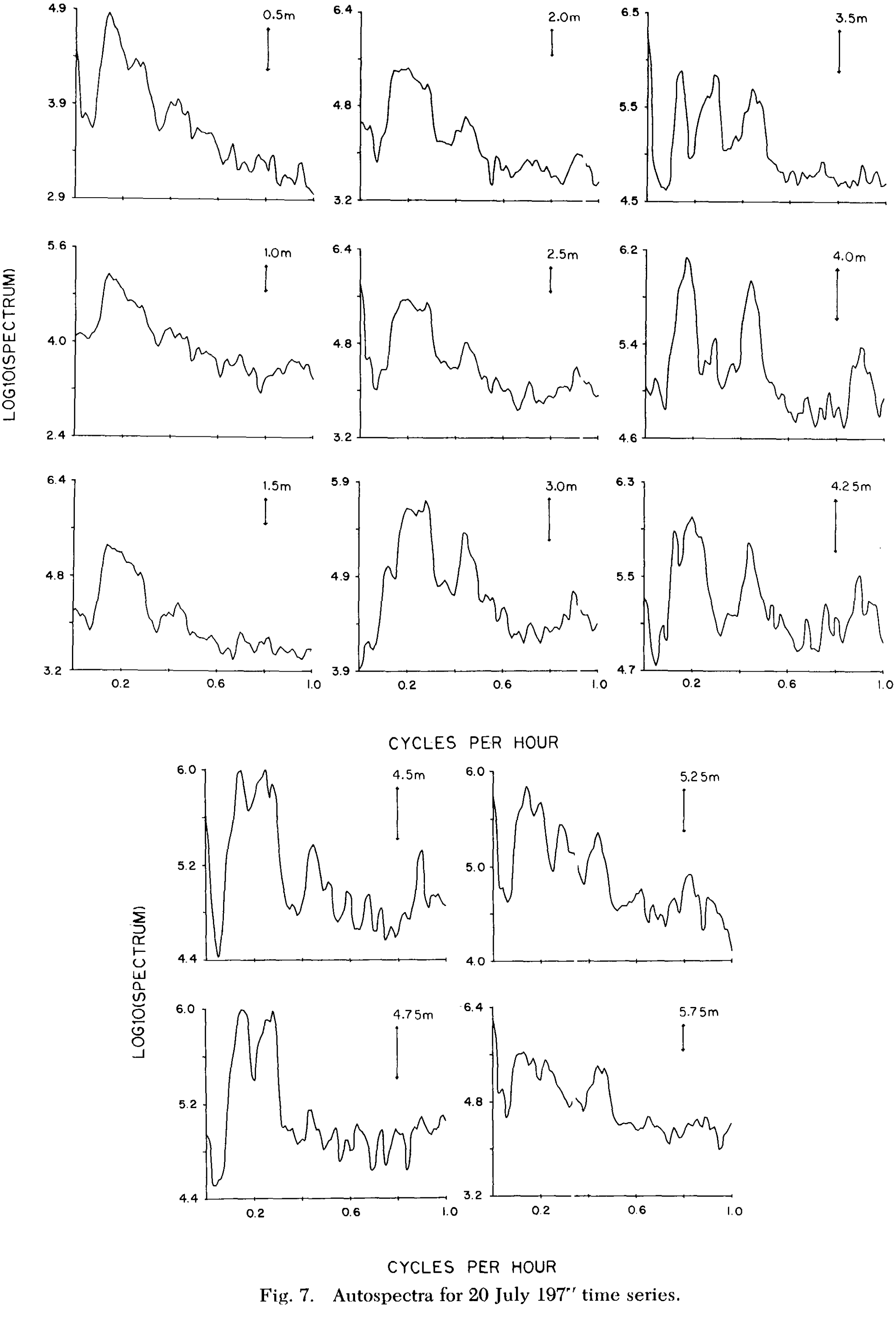

Fig. 7. Autospectra for 20 July 1977 time series.

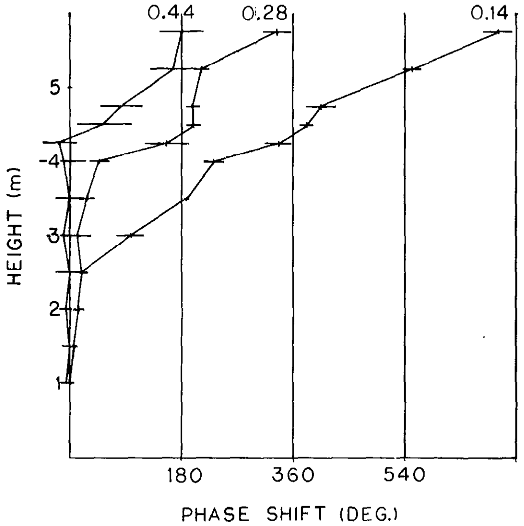

Fig. 8. Phase shifts with respect to 0.5 m for 20 July 1977 time series.

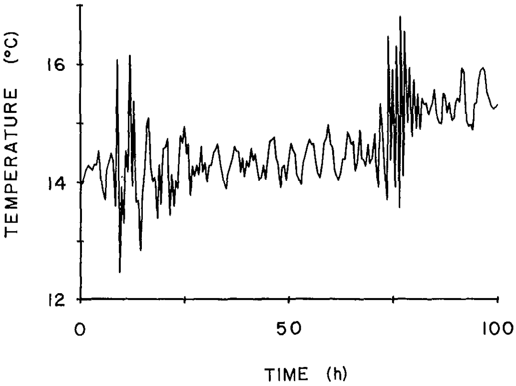

Fig. 9. Time series at 6.5 m above bottom.

necessary for understanding the metalimnetic oscillations. Bryson and Ragotskie (1960) reported that they observed an "indication" of a vertical node in the middle of the water column in Lake Mendota but gave no other information. Hale (1969) noted a periodicity in the depth between isotherms in the nearshore region of Lake Huron and suggested that this is due to a second vertical mode; however spectral analyses were not informative.

Mortimer (1971) concluded from his original study (1953) and these later accounts that higher vertical modes were not conspicuous. This is undoubtedly correct for those larger lakes where the thermocline takes up a relatively small proportion of total lake depth; in smaller and shallower lakes, as the results reported here indicate, it does not seem to hold.

The observed lower frequency limit may be associated with the inertial frequency of 0.06 cph at this latitude, or the higher modes might have increased rates of viscous dissipitation through internal shear induced by phase shift. The decay with increasing frequency is perhaps a result of the increased influence of bottom friction at higher velocity. C. S. Yih (pers. comm.) has suggested that basin morphometry may play a dominant role in resonant frequency selection, which may explain why some of the intermediate resonant modes are not pronounced. Storms do introduce large amounts of energy into the hypolimnion; however, it appears primarily in the higher modes. Whether this occurs through resonant interaction of the ephemeral high frequency internal waves or more direct mechanisms is unclear. I have no data to suggest that wind forcing is responsible for any resonant mode selection.

Garrett and Munk (1972, 1975, 1979) have developed an empirical model based on the random phase superpositions of elementary wave trains that predicts, in a very general way, a spectral peak close to the inertial frequency followed by a decline with increasing frequency. Although the mechanisms responsible for this general spectral shape are not known (Garrett and Munk 1975), one might expect that similar answers are available to both lake and ocean spectra despite the increased importance of discrete modal structure in the former.

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